

EE-527: Machine Learning Laboratory

Assignment 1: Python Basics

Group Members:

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1. Print 'Hello World!'.

In [1]:

```
print('Hello World')
```

Hello World

2. User input two numbers a and b. Perform the following algebraic operations $c = a+b$, $d = a-b$, $e = a*b$, $f = a/b$ and $g = a \% b$ and print their results.

In [2]:

```
a = int(input('Enter a :'))
b = int(input('Enter b :'))
print(f'{a} + {b} : {a+b}')
print(f'{a} - {b} : {a-b}')
print(f'{a} * {b} : {a*b}')
print(f'{a} / {b} : {a/b}')
print(f'{a} % {b} : {a%b}')
```

```
Enter a :4
Enter b :6
4 + 6 : 10
4 - 6 : -2
4 * 6 : 24
4 / 6 : 0.6666666666666666
4 % 6 : 4
```

3. Print the factorial of a positive number 'a' given as a user input.

In [3]:

```
# Returns the factorial of given number
def factorial(number):

    result = 1
    if number == 0:
        return result
    else:
        factor = 2
        while factor <= number:
            result *= factor
            factor += 1
        return result

a = int(input('Enter a positive number :'))
print(factorial(a))
```

```
Enter a positive number :2
2
```

4. Write a function to print all prime numbers in an interval [a,b]. Interval is to be obtained as a user input.

In [4]:

```
# checks whether a number is prime or not
def isPrime(number):
# 1 is neither prime nor composite
    if number == 1:
        return False
# 2 and 3 both are prime numbers
    elif number == 2 or number == 3:
        return True
# if number is completely divisible by 2 or 3 then number is not prime
    elif number % 2 == 0 or number % 3 == 0:
        return False
    else:
        divisor = 5
        while divisor**2 <= number:
            if number % divisor == 0 or number % (divisor + 2) == 0:
                return False
            divisor += 5
        return True

# prints prime number in the given range
def primeInRange(start, end):
    for number in range(start, end+1):
        if isPrime(number):
            print(number)

# Execution begins here
a = int(input('Enter a : '))
b = int(input('Enter b : '))
primeInRange(a, b)
```

Enter a : 2

Enter b :15

2
3
5
7
11
13

5. User input two numbers a and b. Print their lowest common multiple (LCM).

In [5]:

```
# extended euclidean algorithm to find GCD of two numbers
def gcd(num_1, num_2):
    if num_2 == 0:
        return num_1
    else:
        return gcd(num_2, num_1 % num_2)

# returns the LCM of two given numbers
def LCM(num_1, num_2):
    # since product of two numbers is same as product of LCM and GCD of those numbers
    return (num_1 * num_2) / gcd(num_1, num_2)

# Execution begins here
a = int(input('Enter a : '))
b = int(input('Enter b : '))
print(f'LCM({a},{b}) : {LCM(a,b)}')
```

```
Enter a : 4
Enter b : 15
LCM(4,15) : 60.0
```

6. Create a list of length n = 15. Sort in descending order and print the sorted list as well as the sorted indices. Use bubble sort algorithm.

In [6]:

```
import random

list_of_15 = [random.randint(1,100) for i in range(15)]

print(f'Unsorted List : {list_of_15}')

number_of_elements = 15

index = [i for i in range(15)]
for i in range(number_of_elements-1):
    for j in range(number_of_elements-i-1):
        # Inversion of bubble sort (since the output needed in descending order)
        if list_of_15[j] < list_of_15[j+1] and i != j:
            list_of_15[j], list_of_15[j+1] = list_of_15[j+1], list_of_15[j]
            index[j], index[j+1] = index[j+1], index[j]

print(f'Sorted List (Desceding) : {list_of_15}')
print(f'Sorted index of numbers : {index}')
```

```
Unsorted List : [61, 71, 81, 21, 89, 56, 28, 44, 9, 27, 49, 38, 95, 7
1, 14]
Sorted List (Desceding) : [95, 89, 81, 71, 71, 61, 56, 49, 44, 38, 28,
27, 21, 14, 9]
Sorted index of numbers : [12, 4, 2, 1, 13, 0, 5, 10, 7, 11, 6, 9, 3,
14, 8]
```

7. Repeat the previous program for sorting in ascending order. Use numpy array instead of list.

In [7]:

```
import numpy as np

numbers = np.array([random.randint(1,100) for i in range(15)])
print(f'Unsorted Array : {numbers}')

index = [i for i in range(15)]

for i in range(number_of_elements-1):
    for j in range(number_of_elements-i-1):
        if numbers[j] > numbers[j+1] and i != j:
            # Swapping the numbers if numbers at j index is greater than j+1 th index
            numbers[j] , numbers[j+1] = numbers[j+1] , numbers[j]
            # swapping the indices
            index[j] , index[j+1] = index[j+1] , index[j]

print(f'Sorted Array (Ascending) : {numbers}')
print(f'Sorted index of numbers : {index}')
```

```
Unsorted Array : [25 26 96 47 75 71 86 56 93 72 89 43 10 63 57]
Sorted Array (Ascending) : [10 25 26 43 47 56 71 72 57 63 75 86 89 93
96]
Sorted index of numbers : [12, 0, 1, 11, 3, 7, 5, 9, 14, 13, 4, 6, 10,
8, 2]
```

8. Print a matrix $M \in \mathbb{R}^{m \times n}$ having random values in the given range $[-2, 5]$. m and n are to be given as a user input.

In [8]:

```
import random

# Execution begins here
m = int(input('Enter m : '))
n = int(input('Enter n : '))

# Stores the entrie matrix
matrix = np.array([[random.randrange(-2,6) for x in range(n)] for y in range(m)])

print(matrix)
```

```
Enter m : 4
Enter n : 6
[[ 5  2  3  2  0  2]
 [ 3  4  2  0  4  1]
 [ 2  3  4 -1 -2  5]
 [-2  0  3  2  3  4]]
```

9. Program to multiply two random matrices $M1 \in \mathbb{R}^{m \times n}$, $M2 \in \mathbb{R}^{n \times p}$ (Don't use built-in functions). Compare the result obtained with the built-in function.

In [9]:

```
import random
import numpy as np

# Function to multiply two matrices
def multiply(mat_1, mat_2, row_1, column_1, row_2, column_2):
    result = np.array([[0 for x in range(column_2)] for y in range(row_1)])
    for i in range(row_1):
        for j in range(column_2):
            for k in range(row_2):
                result[i][j] += mat_1[i][k] * mat_2[k][j]
    return result

# Function to check the equality between two matrices
def checkEquality(matrix_1, matrix_2):
    # Checks for the dimensions first
    if len(matrix_1) == len(matrix_2):
        for i in range(len(matrix_1)):
            for j in range(len(matrix_1[i])):
                if matrix_1[i][j] != matrix_2[i][j]:
                    return False
        else:
            return True
    else:
        return False

# execution begins here
# first matrix of 2 x 3
row_1 = 2
column_1 = 3
mat_1 = np.array([[random.randrange(-2,6) for x in range(column_1)] for y in range(
print(f'Matrix 1 : \n{mat_1}'))

#second matrix of 3 x 4
row_2 = 3
column_2 = 4
mat_2 = np.array([[random.randrange(-2,6) for x in range(column_2)] for y in range(
print(f'Matrix 2 : \n{mat_2}'))

# using User-defined function
func_product = multiply(mat_1, mat_2, row_1, column_1, row_2, column_2)
print(f'From User Defined Function : \n{func_product}')

# Using in-built function
np_product = np.dot(mat_1, mat_2)
print(f'From Built in Function : \n{np_product}'))

# Check for the equality of results
if checkEquality(func_product, np_product):
    print('Both are same.')
else:
    print('Both are different')
```

```
Matrix 1 :
[[ 5 -2  3]
 [-1  0  2]]
Matrix 2 :
[[ 4  2  5 -2]
```

```
[ 5  0 -2  2]
[-1  0 -1  2]]
From User Defined Function :
[[ 7 10 26 -8]
 [-6 -2 -7  6]]
From Built in Function :
[[ 7 10 26 -8]
 [-6 -2 -7  6]]
Both are same.
```

10. File operations :write

1. Generate a set of $n=100$ random points $X = \{x_i\}$, $i = 1, \dots, n$, $x_i \in \mathbb{R}^D$

In [10]:

```
import numpy as np
import random
```

```
n = 100
```

```
x_i = [[random.randint(0,100) for i in range(10)] for j in range(100)]
print(x_i)
```

```
[[15, 81, 77, 46, 25, 43, 97, 34, 46, 65], [57, 79, 91, 62, 90, 33, 5
9, 46, 34, 48], [25, 92, 95, 81, 60, 45, 90, 51, 15, 71], [15, 79, 67,
35, 19, 31, 79, 59, 19, 11], [3, 16, 53, 86, 60, 34, 96, 12, 97, 99],
[95, 90, 41, 56, 67, 14, 51, 34, 62, 73], [11, 98, 0, 57, 81, 68, 100,
2, 16, 53], [28, 35, 42, 69, 78, 42, 59, 5, 5, 22], [74, 24, 56, 56, 5
8, 16, 16, 31, 43, 92], [23, 100, 47, 69, 84, 90, 2, 84, 50, 4], [88,
100, 61, 41, 38, 86, 14, 83, 24, 58], [56, 46, 54, 97, 40, 3, 49, 87,
77, 12], [61, 71, 49, 44, 45, 87, 96, 81, 86, 40], [77, 52, 72, 11, 5,
60, 97, 87, 41, 0], [26, 26, 12, 98, 94, 36, 25, 49, 93, 29], [1, 95,
10, 65, 79, 0, 25, 94, 75, 0], [0, 76, 56, 61, 91, 42, 33, 9, 99, 70],
[89, 8, 76, 10, 52, 3, 76, 82, 8, 1], [47, 99, 62, 61, 85, 76, 4, 83,
90, 7], [80, 4, 83, 42, 92, 64, 4, 26, 38, 77], [90, 4, 58, 73, 92, 4
5, 3, 2, 47, 81], [54, 30, 5, 92, 98, 96, 13, 44, 99, 27], [22, 29, 1,
83, 71, 57, 95, 76, 84, 26], [27, 10, 2, 66, 63, 26, 60, 81, 33, 43],
[97, 32, 14, 66, 82, 81, 51, 1, 74, 68], [78, 97, 1, 49, 15, 63, 1, 6
5, 71, 42], [37, 67, 39, 62, 78, 32, 27, 52, 93, 15], [53, 43, 43, 62,
50, 78, 80, 79, 2, 76], [44, 94, 92, 84, 71, 69, 21, 95, 34, 12], [4,
92, 34, 42, 79, 100, 83, 87, 85, 18], [54, 72, 96, 51, 0, 79, 4, 4, 2,
86], [96, 22, 48, 49, 15, 9, 83, 60, 73, 13], [48, 80, 12, 76, 81, 16,
39, 80, 23, 84], [40, 68, 52, 12, 13, 90, 34, 3, 42, 91], [73, 39, 4,
19, 86, 28, 26, 35, 50, 7], [29, 71, 15, 73, 6, 65, 88, 70, 52, 19],
[69, 24, 68, 40, 85, 2, 45, 94, 67, 67], [81, 34, 27, 45, 93, 99, 68,
26, 97, 43], [34, 35, 27, 62, 95, 51, 84, 51, 86, 75], [100, 86, 84, 2
0, 17, 82, 39, 80, 34, 69], [41, 20, 49, 17, 64, 17, 29, 64, 37, 54],
[4, 34, 28, 57, 37, 14, 28, 85, 100, 71], [36, 16, 30, 24, 97, 44, 92,
75, 8, 21], [21, 3, 99, 71, 17, 25, 22, 18, 78, 83], [71, 13, 43, 28,
60, 27, 42, 56, 100, 9], [79, 35, 71, 4, 88, 83, 19, 74, 84, 49], [76,
36, 18, 73, 57, 41, 94, 38, 54, 32], [88, 18, 70, 11, 31, 39, 33, 90,
60, 82], [67, 37, 26, 10, 0, 64, 5, 79, 2, 27], [48, 92, 21, 94, 51, 7
2, 89, 38, 37, 81], [30, 70, 17, 84, 19, 93, 17, 30, 40, 10], [20, 61,
94, 59, 18, 33, 97, 24, 4, 80], [6, 63, 99, 24, 99, 23, 80, 48, 14, 9
7], [100, 63, 77, 48, 91, 30, 93, 29, 89, 53], [89, 99, 61, 45, 53, 2
9, 72, 31, 90, 39], [54, 40, 96, 68, 9, 82, 22, 40, 10, 71], [36, 57,
37, 79, 62, 80, 19, 93, 23, 91], [67, 9, 17, 80, 88, 75, 70, 41, 100,
50], [98, 71, 16, 76, 86, 20, 6, 64, 50, 92], [75, 8, 27, 98, 50, 74,
43, 59, 88, 91], [7, 18, 77, 52, 29, 81, 74, 18, 70, 96], [89, 95, 95,
28, 54, 39, 1, 8, 66, 87], [94, 20, 63, 84, 18, 30, 9, 55, 64, 67], [3
2, 27, 29, 51, 75, 38, 8, 93, 98, 8], [32, 27, 28, 71, 84, 85, 12, 6,
24, 53], [88, 45, 16, 69, 52, 22, 81, 41, 45, 22], [52, 39, 22, 45, 5
5, 31, 75, 88, 40, 93], [58, 18, 74, 36, 94, 48, 13, 34, 67, 97], [81,
76, 89, 33, 96, 41, 25, 74, 20, 64], [55, 14, 50, 72, 45, 14, 95, 2, 9
5, 65], [85, 67, 96, 89, 64, 15, 27, 99, 57, 0], [21, 78, 48, 74, 7, 1
4, 54, 47, 18, 48], [92, 19, 41, 20, 14, 18, 10, 10, 23, 38], [34, 37,
4, 97, 79, 43, 5, 30, 66, 88], [44, 7, 26, 36, 13, 43, 82, 2, 10, 54],
[83, 55, 71, 1, 77, 70, 23, 59, 49, 32], [54, 77, 55, 8, 23, 94, 50, 9
8, 45, 76], [39, 53, 78, 63, 45, 21, 75, 100, 24, 8], [65, 25, 22, 79,
44, 4, 7, 22, 90, 2], [13, 6, 29, 59, 17, 23, 52, 88, 56, 3], [96, 77,
64, 65, 59, 0, 43, 30, 42, 12], [98, 15, 15, 46, 24, 54, 8, 76, 77, 2
2], [59, 28, 0, 37, 95, 56, 55, 99, 13, 60], [68, 1, 62, 68, 13, 74, 9
1, 46, 68, 72], [18, 48, 85, 61, 58, 49, 6, 43, 74, 16], [68, 59, 1, 8
1, 13, 3, 74, 41, 69, 34], [44, 98, 53, 91, 45, 17, 22, 56, 92, 3], [2
8, 79, 88, 45, 20, 85, 24, 3, 42, 45], [60, 73, 78, 21, 9, 41, 24, 88,
```



```
55, 56], [76, 19, 20, 37, 18, 96, 28, 9, 67, 95], [2, 52, 90, 73, 7, 6
7, 20, 52, 44, 80], [78, 40, 38, 93, 81, 100, 4, 45, 27, 32], [77, 39,
90, 45, 28, 40, 57, 78, 1, 56], [91, 15, 12, 77, 53, 47, 2, 35, 61, 9
3], [58, 66, 35, 48, 35, 90, 38, 81, 85, 100], [74, 48, 11, 23, 38, 6
8, 65, 59, 70, 62], [56, 33, 62, 59, 85, 81, 37, 41, 5, 57], [55, 49,
22, 17, 87, 69, 45, 31, 70, 51], [95, 78, 34, 16, 84, 80, 64, 87, 30,
21], [99, 88, 11, 78, 14, 29, 51, 49, 75, 29]]
```

2. Write the points to a csv (https://en.wikipedia.org/wiki/Comma-separated_values
(https://en.wikipedia.org/wiki/Comma-separated_values)) file

In [11]:

```
import csv
import pandas as pd

df = pd.DataFrame(x_i)
df.to_csv("points.csv", index=False)
```

11. File operations:read

1. Read the csv (https://en.wikipedia.org/wiki/Comma-separated_values
(https://en.wikipedia.org/wiki/Comma-separated_values)) file generated in the previous program to a matrix. Each column of the matrix should represent a vector

In [12]:

```
import csv
import pandas as pd
import numpy as np
```

```
df_1=pd.read_csv("points.csv")
df_1=df_1.T
print(df_1)
```

```
      0    1    2    3    4    5    6    7    8    9    ...   90   91   92   93   94
95  \
0    15   57   25   15    3   95   11   28   74   23   ...    2   78   77   91   58
74
1    81   79   92   79   16   90   98   35   24  100   ...   52   40   39   15   66
48
2    77   91   95   67   53   41    0   42   56   47   ...   90   38   90   12   35
11
3    46   62   81   35   86   56   57   69   56   69   ...   73   93   45   77   48
23
4    25   90   60   19   60   67   81   78   58   84   ...    7   81   28   53   35
38
5    43   33   45   31   34   14   68   42   16   90   ...   67  100   40   47   90
68
6    97   59   90   79   96   51  100   59   16    2   ...   20    4   57    2   38
65
7    34   46   51   59   12   34    2    5   31   84   ...   52   45   78   35   81
59
8    46   34   15   19   97   62   16    5   43   50   ...   44   27    1   61   85
70
9    65   48   71   11   99   73   53   22   92    4   ...   80   32   56   93  100
62
```

```
      96   97   98   99
0    56   55   95   99
1    33   49   78   88
2    62   22   34   11
3    59   17   16   78
4    85   87   84   14
5    81   69   80   29
6    37   45   64   51
7    41   31   87   49
8     5   70   30   75
9    57   51   21   29
```

[10 rows x 100 columns]

2. Compute the following: $C = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)(x_i - \mu)$, where $\mu = \frac{1}{n} \sum_{i=1}^n x_i$, $i = 1, \dots, n$, $x_i = [x_{i1}, \dots, x_{i10}]^T$ is a column vector

In [13]:

```
import numpy as np

sigma_xi = 0
for i in range(len(matrix)):
    sigma_xi += sum(matrix[i])

mu = sigma_xi/len(matrix)
row_vector = np.array([])

for i in range(len(matrix)):
    row_vector = np.append(row_vector, sum(matrix[i]) - mu)

c = (np.dot(row_vector, row_vector.transpose()))/len(matrix)
print(c)
```

3.1875

12. Define a class for a complex number $a + jb$. Define memeber functions to do basic operations conjugate, absolute value, addition, subtraction, multiplication, division and angle. Define two complex numbers c_1 , c_2 and print the results of the following operations $c_1 + c_2$, $c_1 - c_2$, $c_1 * c_2$, c_1/c_2 , $|c_1|$, $|c_2|$, $\angle c_1$, $\angle c_2$.

In [14]:

```
import math

# Class named complex number
class ComplexNumber:

# Initializes the class
    def __init__(self, real, imaginary):
        self.real = real
        self.imaginary = imaginary

# Returns the conjugate of given complex number
    def conjugate(self):
        result = ComplexNumber(0,0)
        result.real = self.real
        result.imaginary = (-1)*self.imaginary
        return result

# Returns the absolute value of given complex number
    def absolute(self):
        return (self.real**2 + self.imaginary**2)**0.5

# Returns the sum of two given complex numbers
    def addition(self, number):
        result = ComplexNumber(0,0)
        result.real = self.real + number.real
        result.imaginary = self.imaginary + number.imaginary
        return result

# Returns the difference between two complex numbers
    def subtraction(self, number):
        result = ComplexNumber(0,0)
        result.real = self.real - number.real
        result.imaginary = self.imaginary - number.imaginary
        return result

# Returns the product of given two complex numbers
    def multiplication(self, number):
        result = ComplexNumber(0,0)
        result.real = self.real*number.real - self.imaginary*number.imaginary
        result.imaginary = self.imaginary*number.real + self.real*number.imaginary
        return result

# Returns the division of two given complex numbers
    def division(self, number):
        result = ComplexNumber(0, 0)
        denominator = number.real**2 + number.imaginary**2
        result.real = (self.real*number.real + self.imaginary*number.imaginary)/denominator
        result.imaginary = (self.imaginary*number.real - self.real*number.imaginary)/denominator
        return result

# Displays the complex number
    def display(self):
        if self.imaginary < 0:
            print(f'{self.real}{self.imaginary}j')
        else:
            print(f'{self.real}+{self.imaginary}j')

# Execution begins here
```

```

c1 = ComplexNumber(5,-6)
print('First Number : ',end='')
c1.display()
c2 = ComplexNumber(3,8)
print('Second Number : ',end='')
c2.display()

# Conjugate
c3 = ComplexNumber(0,0)
c3 = c1.conjugate()
print('Conjugate of first number, : ',end='')
c3.display()
c3 = ComplexNumber(0,0)
c3 = c2.conjugate()
print('Conjugate of second Number : ',end='')
c3.display()

# Absolute
print(f'Absolute of first number : {c1.absolute()}')
print(f'Absolute of second number : {c2.absolute()}')

# Addition
c3 = ComplexNumber(0,0)
c3 = c1.addition(c2)
print('Addition of the two numbers : ',end='')
c3.display()

# Subtraction
c3 = ComplexNumber(0,0)
c3 = c1.subtraction(c2)
print('Difference between the two numbers : ',end='')
c3.display()

# Multiplication
c3 = ComplexNumber(0,0)
c3 = c1.multiplication(c2)
print('Product of the two numbers : ',end='')
c3.display()

# Division
c3 = ComplexNumber(0,0)
c3 = c1.division(c2)
print('Division of the two numbers : ',end='')
c3.display()

# Angles
print(f'Angle of First number in degrees: {math.degrees(math.atan(c1.imaginary/c1.real))}')
print(f'Angle of Second number in degrees : {math.degrees(math.atan(c2.imaginary/c2.real))}')

```

```

First Number : 5-6j
Second Number : 3+8j
Conjugate of first number, : 5+6j
Conjugate of second Number : 3-8j
Absolute of first number : 7.810249675906654
Absolute of second number : 8.54400374531753
Addition of the two numbers : 8+2j

```

Difference between the two numbers : $2-14j$
Product of the two numbers : $63+22j$
Division of the two numbers : $-0.4520547945205479-0.7945205479452054j$
Angle of First number in degrees: -50.19442890773481
Angle of Second number in degrees : 69.44395478041653

13. Plot the function $y = 3x + 2$ with $x \in [-10, 10]$.

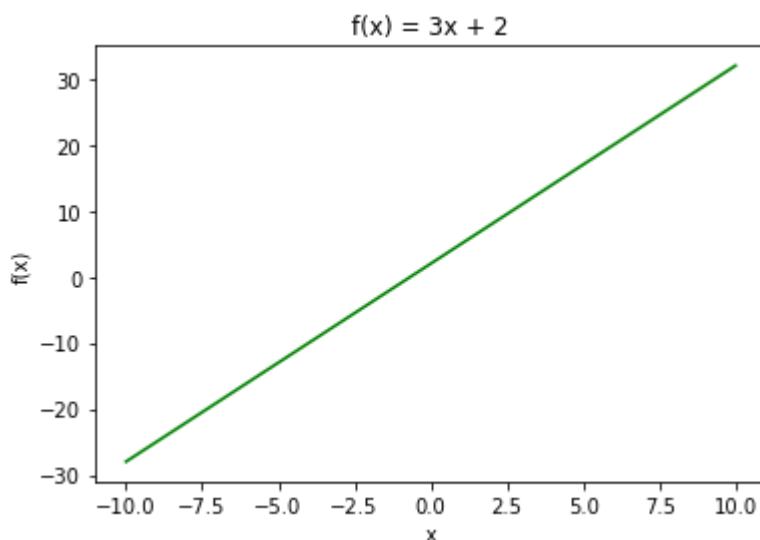
In [15]:

```
import numpy as np
import matplotlib.pyplot as plt

# calculates the function value and returns
def f(x):
    return (3*x + 2)

# Divides the points into 10000
x = np.linspace(-10, 10, 10000)
plt.plot(x, f(x), color = 'green')

plt.xlabel('x')
plt.ylabel('f(x)')
plt.title('f(x) = 3x + 2')
plt.show()
```



14. Scatter plot

1. Generate a set of $n = 100$ points, $X = \{x_i\}$, $i = 1, \dots, n$, $x_i \in \mathbb{R}^2$ within an ellipse $\frac{(x - \mu_x)^2}{a^2} + \frac{(y - \mu_y)^2}{b^2} = 1$ with $\mu_x = 5$, $\mu_y = -5$ and has a major axis $2a = 10$ and minor axis $2b = 5$

In [16]:

```
import random
import matplotlib.pyplot as plt

# Ellipse Class
class ellipse:

# Initializes the ellipse class
    def __init__(self, origin_x, origin_y, major_axis, minor_axis):
        self.origin_x = origin_x
        self.origin_y = origin_y
        self.major_axis = major_axis
        self.minor_axis = minor_axis

# Checks whether the point lies inside the ellipse or not
    def isInsideEllipse(self, x_value, y_value):
        if ((x_value - self.origin_x)**2)/((self.major_axis/2)**2) + ((y_value - self
            return True
        else:
            return False

# Execution begins here
# define an ellipse with given constraints
e = ellipse(origin_x=5, origin_y=-5, major_axis=10, minor_axis=5)

points= []

# Loop continues till we get 100 valid points inside the ellipse
while len(points) < 100:
    lower_limit_x = e.origin_x - (e.major_axis/2)
    upper_limit_x = e.origin_x + (e.major_axis/2)
    lower_limit_y = e.origin_y - (e.minor_axis/2)
    upper_limit_y = e.origin_y + (e.minor_axis/2)
    new_point_x = random.uniform(lower_limit_x, upper_limit_x)
    new_point_y = random.uniform(lower_limit_y, upper_limit_y)

# if generated point lies inside ellipse then append point
    if e.isInsideEllipse(new_point_x, new_point_y) is True:
        points.append([new_point_x, new_point_y])

print(points)
```

```
[[5.599869729990869, -6.613480023827052], [5.027940971196518, -7.04149
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```

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2. Scatter plot all the points.

In [17]:

```
import matplotlib.pyplot as plt

x = [points[i][0] for i in range(len(points))]
y = [points[i][1] for i in range(len(points))]
plt.scatter(x,y)
plt.xlabel('x')
plt.ylabel('y')
plt.title('Ellipse')
plt.show()
```

