Assingment 04

EE-527 Machine Learning Laboratory

Group Members

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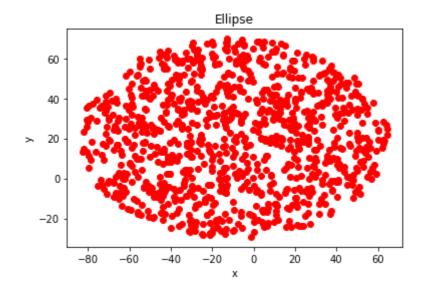
In [1]:

```
# Importing all necessary libraries
import numpy as np
import matplotlib.pyplot as plt
import random
import math
```

Problem 1: Generation of Points within an Ellipse

```
In [2]:
```

```
import random
import matplotlib.pyplot as plt
import numpy as np
# Ellipse Class
class ellipse:
# Initializes the ellipse class
  def init (self, origin x, origin y, major axis, minor axis):
      self.origin_x = origin_x
      self.origin y = origin y
      self.major axis = major axis
      self.minor axis = minor axis
# Checks whether the point lies inside the ellipse or not
  def isInsideEllipse(self, x_value, y_value):
      if ((x_value - self.origin_x)**2)/((self.major_axis/2)**2) + ((y value - self
          return True
      else:
          return False
# Execution begins here
# define an ellipse with given constraints
e = ellipse(origin x = -10, origin y = 20, major axis = 150, minor axis = 100)
points= []
# Loop continues till we get 1000 valid points inside the ellipse
while len(points) < 1000:
    lower limit x = e.origin x - (e.major axis/2)
    upper_limit_x = e.origin_x + (e.major_axis/2)
    lower_limit_y = e.origin_y - (e.minor_axis/2)
    upper limit y = e.origin y + (e.minor axis/2)
   new point x = random.uniform(lower limit x, upper limit x)
   new_point_y = random.uniform(lower_limit_y, upper_limit_y)
    # if generated point lies inside ellipse then append point
    if e.isInsideEllipse(new_point_x, new_point_y) is True:
        points.append([new_point_x, new_point_y])
x = [points[i][0] for i in range(len(points))]
y = [points[i][1] for i in range(len(points))]
plt.scatter(x,y, color ='red')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Ellipse')
plt.show()
```



Problem 2: Generation of Points within 10 dimensional hypersphere

Execution takes more than 30 seconds since it is in 10 dimensions (have patience)

```
In [3]:
```

```
# Returns true if the generated point lies inside the hypersphere
def isInsideHyperSphere(radius, center, new_point):
    # calculate the respective dimenstion difference and square and store into an a
    # let's say we are storing into differ (for 3 dimension) = [(x1-x2)^2, (y1-y2)^2]
    # hence for 10 dimesnsion it can be written in shorthand
    differ = (center - new point)**2
    # if the following equation is less than zero means point is inside hypersphere
    # if greater outside the hypersphere
    if sum(differ) - (radius**2) < 0:</pre>
        return True
    else:
        return False
# Generates the points inside hypersphere
def generateInsideHypersphere(radius, center, total points):
    # To store the generated valid points
    S = []
    valid points = 0
    # Iterate until we get required number of points
    while valid points < total points:</pre>
        new point = np.array([center[i] + np.random.uniform(-1*radius, radius) for
        if isInsideHyperSphere(radius, center, new point) is True:
            S.append(new point)
            valid points += 1
    # points in 3 dimension look like [x1,y1,z1], [x2,y2,z2]
    # can be generalized in 10 dimension
    S = np.array(S)
    return S
# Execution begins here
# Given fixed values
radius = 100
# the number of elements in the center is deciding factor to know how many dimension
center = np.array([-1, -2, -1, 0, 0, 0, 3, 4, 9, 0])
total points = 1000
# Points will be finally store into points variable
points = generateInsideHypersphere(radius, center, total_points)
# Points can be fetched as follows, along each dimension
# x = np.array([points[i][0] for i in range(len(points))])
# y = np.array([points[i][1] for i in range(len(points))])
```

Problem 3: Generation of points within oriented ellips

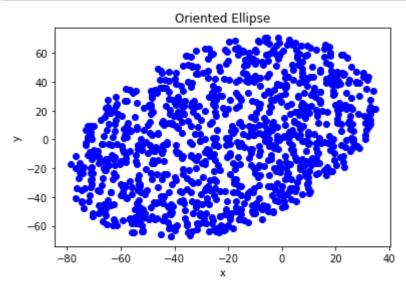
Randomly generate 2D ellipse of axes major axis makes an angle of 2D points, inside an oriented eo and centered at c with the horizontal axis

```
In [4]:
```

```
# Ellipse Class
class ellipse:
# Initializes the ellipse class
  def init (self, origin x, origin y, major axis, minor axis):
      self.origin x = origin x
      self.origin y = origin y
      self.major axis = major axis
      self.minor axis = minor axis
# Checks whether the point lies inside the ellipse or not
  def isInsideEllipse(self, x value, y value):
      if ((x value - self.origin x)**2)/((self.major axis/2)**2) + ((y value - self
          return True
      else:
          return False
# Execution begins here
# define an ellipse with given constraints
origin_x, origin_y = -10, 20
major axis, minor axis = 150, 100
e = ellipse(origin x, origin y, major axis, minor axis)
points= []
lower limit x = e.origin x - (e.major axis/2)
upper_limit_x = e.origin_x + (e.major_axis/2)
lower limit y = e.origin y - (e.minor axis/2)
upper limit y = e.origin y + (e.minor axis/2)
# Loop continues till we get 1000 valid points inside the ellipse
while len(points) < 1000:
    lower limit x = e.origin x - (e.major axis/2)
    upper_limit_x = e.origin_x + (e.major_axis/2)
    lower limit y = e.origin y - (e.minor axis/2)
    upper limit y = e.origin y + (e.minor axis/2)
    new_point_x = round(random.uniform(lower_limit_x, upper_limit_x),5)
    new_point_y = round(random.uniform(lower_limit_y, upper_limit_y),5)
    # if generated point lies inside ellipse then append point
    if e.isInsideEllipse(new_point_x, new_point_y) is True:
        points.append([new_point_x, new_point_y])
x = np.array([points[i][0] for i in range(len(points))])
y = np.array([points[i][1] for i in range(len(points))])
point matrix = []
point matrix.append(x)
point_matrix.append(y)
theta = math.pi/3
rotation matrix = np.array([[np.cos(theta), -1*np.sin(theta)], [np.sin(theta), np.c
point_matrix = np.array(point_matrix)
result = rotation_matrix.dot(point_matrix)
x_new = []
y \text{ new} = []
```

```
x_new = np.array([result[0][i] for i in range(len(result[0]))])
y_new = np.array([result[1][i] for i in range(len(result[0]))])

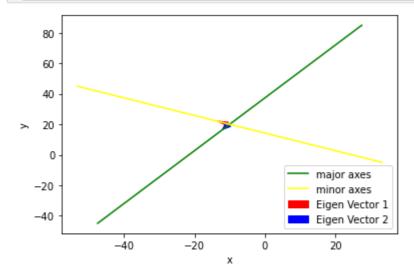
plt.scatter(x_new, y_new, color ='blue')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Oriented Ellipse')
plt.show()
```



Problem 4: Covariance Matrix Computation

In [5]:

```
cov mat = np.cov(x new, y new)
# previous major axes
major axes x = np.array([origin x - major axis/2, origin x + major axis/2])
major axes y = np.array([origin y, origin y])
# previous minor axes
minor axes x = np.array([origin x, origin x])
minor_axes_y = np.array([origin_y - minor_axis/2, origin_y + minor_axis/2])
major axes x new, major axes y new = [], []
minor axes x new, minor axes y new = [], []
# update major and minor axes due to rotation of an angle = pi/3
for i in range(2):
   major axes x new.append((major axes x[i]-origin x)*np.cos(math.pi/3) - (major a
   major axes y new.append((major axes x[i]-origin x)*np.sin(math.pi/3) + (major a
    minor axes x new.append((minor axes x[i]-origin x)*np.cos(math.pi/3) - (minor a
    minor axes y new.append((minor axes x[i]-origin x)*np.sin(math.pi/3) + (minor a
eigen_values, eigen_vectors = np.linalg.eig(cov mat)
vec 1 = eigen vectors[:, 0]
vec 2 = eigen vectors[:, 1]
plt.arrow(origin_x, origin_y, vec_1[0], vec_1[1], color='red', label = 'Eigen Vecto'
plt.arrow(origin_x, origin_y, vec_2[0], vec_2[1], color='blue', label = 'Eigen Vect
plt.plot(major axes x new, major axes y new, color = 'green', label = 'major axes')
plt.plot(minor axes x new, minor axes y new, color = 'yellow', label = 'minor axes'
plt.xlabel('x')
plt.ylabel('y')
plt.legend(loc = 4)
plt.show()
k = [3, 4, 5]
```

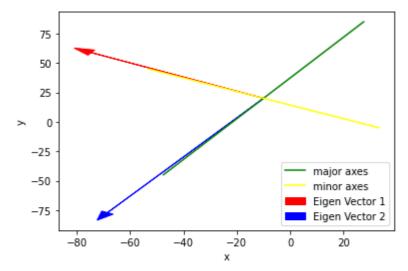


In [6]:

```
length_1 = k[0]*(eigen_values[0]**0.5)
length_2 = k[0]*(eigen_values[1]**0.5)

v_1 = vec_1*length_1
v_2 = vec_2*length_2

plt.arrow(origin_x, origin_y, v_1[0], v_1[1], color='red', label = 'Eigen Vector 1'
plt.arrow(origin_x, origin_y, v_2[0], v_2[1], color='blue', label = 'Eigen Vector 2
plt.plot(major_axes_x_new, major_axes_y_new, color = 'green', label = 'major axes')
plt.plot(minor_axes_x_new, minor_axes_y_new, color = 'yellow', label = 'minor axes'
plt.xlabel('x')
plt.ylabel('y')
plt.legend(loc = 4)
plt.show()
```



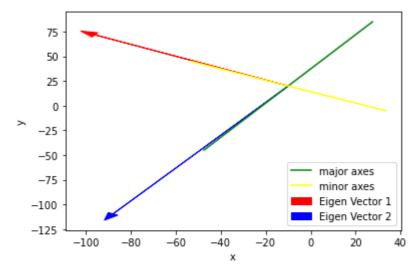
For k = 4

In [7]:

```
length_1 = k[1]*(eigen_values[0]**0.5)
length_2 = k[1]*(eigen_values[1]**0.5)

v_1 = vec_1*length_1
v_2 = vec_2*length_2

plt.arrow(origin_x, origin_y, v_1[0], v_1[1], color='red', label = 'Eigen Vector 1'
plt.arrow(origin_x, origin_y, v_2[0], v_2[1], color='blue', label = 'Eigen Vector 2
plt.plot(major_axes_x_new, major_axes_y_new, color = 'green', label = 'major axes')
plt.plot(minor_axes_x_new, minor_axes_y_new, color = 'yellow', label = 'minor axes'
plt.xlabel('x')
plt.ylabel('y')
plt.legend(loc = 4)
plt.show()
```



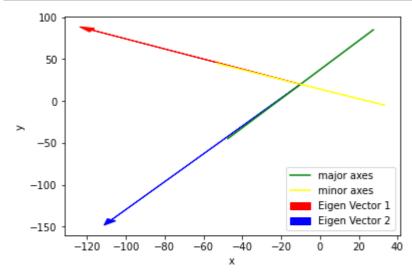
For k = 5

In [8]:

```
length_1 = k[2]*(eigen_values[0]**0.5)
length_2 = k[2]*(eigen_values[1]**0.5)

v_1 = vec_1*length_1
v_2 = vec_2*length_2

plt.arrow(origin_x, origin_y, v_1[0], v_1[1], color='red', label = 'Eigen Vector 1'
plt.arrow(origin_x, origin_y, v_2[0], v_2[1], color='blue', label = 'Eigen Vector 2
plt.plot(major_axes_x_new, major_axes_y_new, color = 'green', label = 'major axes')
plt.plot(minor_axes_x_new, minor_axes_y_new, color = 'yellow', label = 'minor axes')
plt.xlabel('x')
plt.ylabel('y')
plt.legend(loc = 4)
plt.show()
```



Problem 4: Distribution estimation

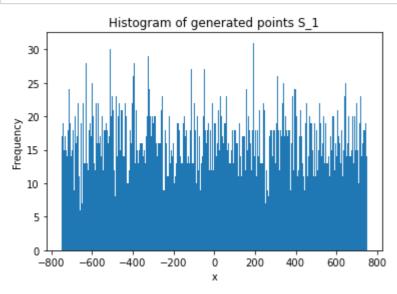
Necessary user defined function that will be required for calculations

In [9]:

```
# Generate points for S 1
def generateNumbers(x_min, x_max,number_of_points):
   points = np.random.randint(x_min, x_max, number_of_points)
    return points
# Calculating Probability distribution function
# Will store x values in 0th index and cumulative value till that point in 1th inde
def generateCDF(pmf):
   cdf = []
    cum sum = 0
    for index in range(len(pmf)):
        # make a fresh list to store cdf at a particular point
        cdf at x = []
        # first append the x value
        cdf at x.append(pmf[index][0])
        cum sum += pmf[index][1]
        # then append the corresponding cdf value
        cdf_at_x.append(cum_sum)
        # finally append it to the cdf function
        cdf.append(cdf_at_x)
   cdf = np.array(cdf)
    return cdf
```

```
at x_min, x_max = -750, 750
number_of_points = 5000
bin size = 5
```

```
# Generates points within given range and returns
def generatePoints(x_min, x_max,number_of_points):
    points = np.random.randint(x min, x max, number of points)
    return points
# Execution begins here
x \min, x \max = -750, 750
number_of_points_P = 5000
bin size = 5
# Store the generated points into S 1
S_1 = generatePoints(x_min, x_max, number_of_points_P)
# plot the histogram for the generated points
plt.hist(S 1, bins = int((x max-x min)/bin size))
plt.xlabel('x')
plt.ylabel('Frequency')
plt.title('Histogram of generated points S 1')
plt.show()
# Calculate and store x values and bin frequncies
hist, bin_edges = np.histogram(S_1, bins = int((x max-x min)/bin size))
# probability mass function stores as (xi, Pi) where xi is event and Pi is probabil
pmf = [[(int(bin_edges[i]+bin_edges[i+1]))/2, hist[i]/number_of_points_P] for i in
# probability mass function of P
pmf P = np.array(pmf)
# cumulative distribution of P
cdf P = generateCDF(pmf P)
# separate out the values and store into individual arrays x values and probability
# x values = np.array([pmf[i][0] for i in range(len(pmf))])
# prob values = np.array([pmf[i][1] for i in range(len(pmf))])
```



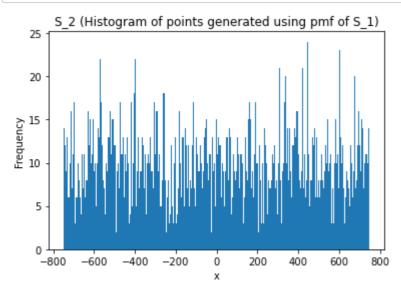
Problem 5: Data Generation and Distribution estimation

```
# Generates random number uniform distribution
def generateFromP(x_min, x_max, pmf, cdf, number_of_points):
             points = []
             # loop untill we generate required number of points
             while len(points) < number of points:</pre>
                           # First generate a random value uniformly i.e. p~U(0,1)
                           random value = np.random.uniform(0,1)
                          index = 0
                           for index in range(len(pmf)):
                                        # if we hit cdf valeu greater than random value then generate x from th
                                        if cdf[index][1] > random value:
                                                     # if we get the random value is even less than the first cdf value
                                                     if index == 0:
                                                                   x = ((random_value - 0)*(cdf[index][0] - x_min)/pmf[index][1])
                                                     else:
                                                                   # since we are at (r+1) i.e. index so our calculation will be i
                                                                   x = ((random\ value\ -\ cdf[index-1][1])*(cdf[index][0]\ -\ cdf[index][0])*(cdf[index][0]\ -\ cdf[index][0])*(cdf[index][0]\ -\ cdf[index][0])*(cdf[index][0]\ -\ cdf[index][0]\ -\ cdf[index]
                                                     # append the obtained x into points and break from loop to generate
                                                     points.append(x)
                                                     break
             points = np.array(points)
             return points
```

```
at x_min, x_max = -750, 750
number_of_points = 3000
bin size = 10
```

In [12]:

```
x \min, x \max = -750, 750
number_of_points_Q = 3000
bin size = 5
# Store the generated points into S 2
S_2 = generateFromP(x_min, x_max, pmf_P, cdf_P, number_of_points_Q)
# plot the histogram for the generated points
plt.hist(S 2, bins = int((x max-x min)/bin size))
plt.xlabel('x')
plt.ylabel('Frequency')
plt.title('S 2 (Histogram of points generated using pmf of S 1)')
plt.show()
# Calculate and store x values and bin frequncies
hist, bin_edges = np.histogram(S_2, bins = int((x_max-x_min)/bin_size))
# probability mass function stores as (xi, Pi) where xi is event and Pi is probabil
pmf = [[(int(bin edges[i]+bin edges[i+1]))/2, hist[i]/number of points Q] for i in
# probability mass function of Q
pmf Q = np.array(pmf)
# cumulative distribution of PQ
cdf Q = generateCDF(pmf Q)
```



Problem 6: Comparing the Distributions

```
In [13]:
```

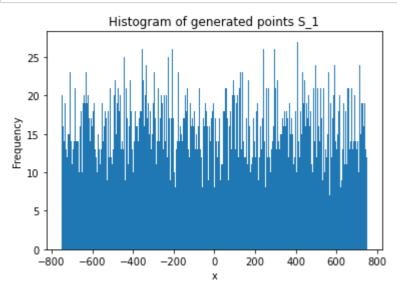
```
# Calculate number of bins
number_of_bins = int((x_max-x_min)/bin_size)
# store the multiple and sqaure root into BC sequence for final summation
BC_sequence = np.array([(pmf_P[i][1]*pmf_Q[i][1])**0.50  for i in range(number_of_bi
# sum all the elements obtained in BC sequence
BC_of_PQ = sum(BC_sequence)
# Print result
print('Result')
print(f'\nx_min : \{x_min\} \tx_max : \{x_max\} \tbin size : \{bin size\}')
print(f'\nFor P\nn : {number_of_points_P} \n')
print(f"\nFor Q\nn' : {number_of_points_Q} \n")
print(f'Bhattacharya Coefficient : {BC_of_PQ}')
Result
x min : -750 x max : 750
                                bin size : 5
For P
n: 5000
For Q
n': 3000
Bhattacharya Coefficient: 0.9830786882176388
```

Comparing Distribution #2:

```
at x_min, x_max = -750, 750
number_of_points = 6000
bin_size = 4
```

In [14]:

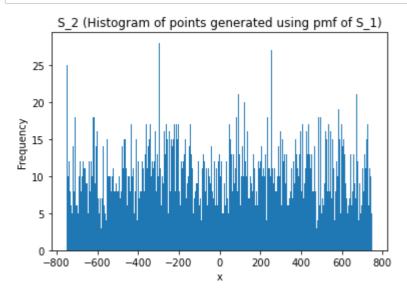
```
# Execution begins here
x_{min}, x_{max} = -750, 750
number of points P = 6000
bin size = 4
# Store the generated points into S_1
S_1 = generatePoints(x_min, x_max, number_of_points_P)
# plot the histogram for the generated points
plt.hist(S 1, bins = int((x max-x min)/bin size))
plt.xlabel('x')
plt.ylabel('Frequency')
plt.title('Histogram of generated points S_1')
plt.show()
# Calculate and store x values and bin frequncies
hist, bin edges = np.histogram(S 1, bins = int((x max-x min)/bin size))
# probability mass function stores as (xi, Pi) where xi is event and Pi is probabil
pmf = [[(int(bin_edges[i]+bin_edges[i+1]))/2, round(hist[i]/number_of_points_P, 5)]
# probability mass function of P
pmf P = np.array(pmf)
# cumulative distribution of P
cdf P = generateCDF(pmf P)
```



```
at x_min, x_max = -750, 750
  number_of_points = 4000
  bin_size = 4
```

In [15]:

```
x \min, x \max = -750, 750
number_of_points_Q = 4000
bin size = 4
# Store the generated points into S 2
S_2 = generateFromP(x_min, x_max, pmf_P, cdf_P, number_of_points_Q)
# plot the histogram for the generated points
plt.hist(S 2, bins = int((x max-x min)/bin size))
plt.xlabel('x')
plt.ylabel('Frequency')
plt.title('S 2 (Histogram of points generated using pmf of S 1)')
plt.show()
# Calculate and store x values and bin frequncies
hist, bin_edges = np.histogram(S_2, bins = int((x_max-x_min)/bin_size))
# probability mass function stores as (xi, Pi) where xi is event and Pi is probabil
pmf = [[(int(bin edges[i]+bin edges[i+1]))/2, round(hist[i]/number of points Q, 5)]
# probability mass function of Q
pmf Q = np.array(pmf)
# cumulative distribution of Q
cdf Q = generateCDF(pmf Q)
```



Calculating Bhattacharya Coefficient and Results

```
In [16]:
```

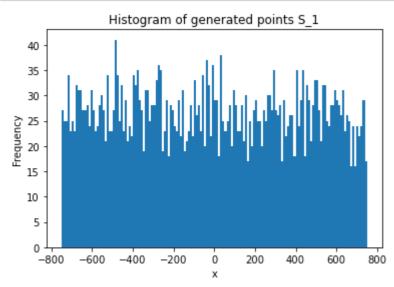
```
# Calculate number of bins
number_of_bins = int((x_max-x_min)/bin_size)
# store the multiple and sqaure root into BC sequence for final summation
BC_sequence = np.array([(pmf_P[i][1]*pmf_Q[i][1])**0.50  for i in range(number_of_bi
# sum all the elements obtained in BC sequence
BC_of_PQ = sum(BC_sequence)
# Print result
print('Result')
print(f'\nx_min : \{x_min\} \tx_max : \{x_max\} \tbin size : \{bin size\}')
print(f'\nFor P\nn : {number_of_points_P} \n')
print(f"\nFor Q\nn' : {number_of_points_Q} \n")
print(f'Bhattacharya Coefficient : {BC_of_PQ}')
Result
x min : -750 x max : 750
                                bin size : 4
For P
n: 6000
For Q
n': 4000
Bhattacharya Coefficient: 0.9851296908710356
```

Comparing Distribution #3:

```
at x_min, x_max = -750, 750
number_of_points = 4000
bin_size = 10
```

In [17]:

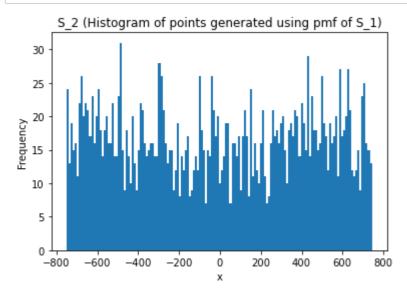
```
# Execution begins here
x_{min}, x_{max} = -750, 750
number_of_points_P = 4000
bin size = 10
# Store the generated points into S 1
S_1 = generatePoints(x_min, x_max, number_of_points_P)
# plot the histogram for the generated points
plt.hist(S 1, bins = int((x max-x min)/bin size))
plt.xlabel('x')
plt.ylabel('Frequency')
plt.title('Histogram of generated points S_1')
plt.show()
# Calculate and store x values and bin frequncies
hist, bin edges = np.histogram(S 1, bins = int((x max-x min)/bin size))
# probability mass function stores as (xi, Pi) where xi is event and Pi is probabil
pmf = [[(int(bin_edges[i]+bin_edges[i+1]))/2, round(hist[i]/number_of_points_P, 5)]
# probability mass function of P
pmf P = np.array(pmf)
# cumulative distribution of P
cdf P = generateCDF(pmf P)
```



```
at x_min, x_max = -750, 750
number_of_points = 2500
bin size = 10
```

In [18]:

```
x \min, x \max = -750, 750
number_of_points_Q = 2500
bin size = 10
# Store the generated points into S 2
S_2 = generateFromP(x_min, x_max, pmf_P, cdf_P, number_of_points_Q)
# plot the histogram for the generated points
plt.hist(S 2, bins = int((x max-x min)/bin size))
plt.xlabel('x')
plt.ylabel('Frequency')
plt.title('S 2 (Histogram of points generated using pmf of S 1)')
plt.show()
# Calculate and store x values and bin frequncies
hist, bin_edges = np.histogram(S_2, bins = int((x_max-x_min)/bin_size))
# probability mass function stores as (xi, Pi) where xi is event and Pi is probabil
pmf = [[(int(bin edges[i]+bin edges[i+1]))/2, round(hist[i]/number of points Q, 5)]
# probability mass function of Q
pmf Q = np.array(pmf)
# cumulative distribution of Q
cdf Q = generateCDF(pmf Q)
```



Calculating Bhattacharya Coefficient and Results

```
In [19]:
```

```
# Calculate number of bins
number_of_bins = int((x_max-x_min)/bin_size)
# store the multiple and sqaure root into BC sequence for final summation
BC_sequence = np.array([(pmf_P[i][1]*pmf_Q[i][1])**0.50  for i in range(number_of_bi
# sum all the elements obtained in BC sequence
BC_of_PQ = sum(BC_sequence)
# Print result
print('Result')
print(f'\nx_min : \{x_min\} \tx_max : \{x_max\} \tbin size : \{bin size\}')
print(f'\nFor P\nn : {number_of_points_P} \n')
print(f"\nFor Q\nn' : {number_of_points_Q} \n")
print(f'Bhattacharya Coefficient : {BC_of_PQ}')
Result
x min : -750 x max : 750
                                bin size : 10
For P
n: 4000
For Q
n': 2500
```

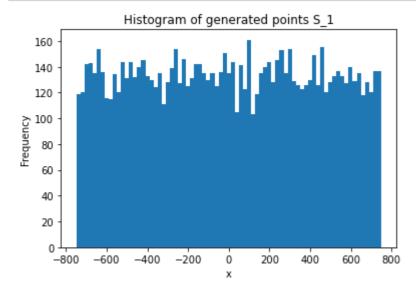
Comparing Distribution #4:

Bhattacharya Coefficient : 0.991368718937178

```
at x_min, x_max = -750, 750
number_of_points = 10000
bin_size = 20
```

In [20]:

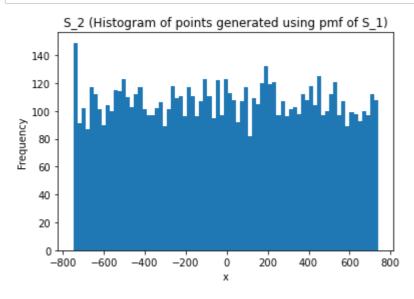
```
# Execution begins here
x_{min}, x_{max} = -750, 750
number_of_points_P = 10000
bin size = 20
# Store the generated points into S 1
S_1 = generatePoints(x_min, x_max, number_of_points_P)
# plot the histogram for the generated points
plt.hist(S 1, bins = int((x max-x min)/bin size))
plt.xlabel('x')
plt.ylabel('Frequency')
plt.title('Histogram of generated points S_1')
plt.show()
# Calculate and store x values and bin frequncies
hist, bin edges = np.histogram(S 1, bins = int((x max-x min)/bin size))
# probability mass function stores as (xi, Pi) where xi is event and Pi is probabil
pmf = [[(int(bin_edges[i]+bin_edges[i+1]))/2, round(hist[i]/number_of_points_P, 5)]
# probability mass function of P
pmf P = np.array(pmf)
# cumulative distribution of P
cdf P = generateCDF(pmf P)
```



```
at x_min, x_max = -750, 750
  number_of_points = 8000
  bin_size = 20
```

In [21]:

```
x \min, x \max = -750, 750
number_of_points_Q = 8000
bin size = 20
# Store the generated points into S 2
S_2 = generateFromP(x_min, x_max, pmf_P, cdf_P, number_of_points_Q)
# plot the histogram for the generated points
plt.hist(S 2, bins = int((x max-x min)/bin size))
plt.xlabel('x')
plt.ylabel('Frequency')
plt.title('S 2 (Histogram of points generated using pmf of S 1)')
plt.show()
# Calculate and store x values and bin frequncies
hist, bin_edges = np.histogram(S_2, bins = int((x_max-x_min)/bin_size))
# probability mass function stores as (xi, Pi) where xi is event and Pi is probabil
pmf = [[(int(bin edges[i]+bin edges[i+1]))/2, round(hist[i]/number of points Q, 5)]
# probability mass function of Q
pmf Q = np.array(pmf)
# cumulative distribution of Q
cdf Q = generateCDF(pmf Q)
```



Calculating Bhattacharya Coefficient and Results

In [22]:

```
# Calculate number of bins
number_of_bins = int((x_max-x_min)/bin_size)
# store the multiple and squure root into BC sequence for final summation
BC_sequence = np.array([(pmf_P[i][1]*pmf_Q[i][1])**0.50  for i in range(number_of_bi
# sum all the elements obtained in BC sequence
BC_of_PQ = sum(BC_sequence)
# Print result
print('Result')
print(f'\nx_min : {x_min} \tx_max : {x_max} \tbin size : {bin_size}')
print(f'\nFor P\nn : {number_of_points_P} \n')
print(f"\nFor Q\nn' : {number_of_points_Q} \n")
print(f'Bhattacharya Coefficient : {BC_of_PQ}')
Result
bin size : 20
For P
n: 10000
```

For Q n' : 8000

Bhattacharya Coefficient : 0.9982280778965426