EE-527: Machine Learning Laboratory

Sun 23 Jan 2022

Assignment 2

1. Generate a set of points around a line $y = ax + b$
   (a) Choose $a = 2$ and $b = 3$
   (b) Select the range for x as $[-10, 10]$ and generate $n = 100$ values for x in that interval.
   (c) Compute the values of y for each x as $y_i = 2x_i + 3$.
   (d) Plot the line $y = 2x + 3$ in black color.
   (e) Generate a set of n points around the line using the equation

$$y_i = 2x_i + 3 + \sigma\mathcal{N}(0, 1) \tag{1}$$

   where σ is the standard deviation and $\mathcal{N}(0, 1)$ is the zero-mean unity-variance normal distribution
   (f) Show the scatter plot of these noisy points (in red color) on the same graph generated in step (d).

2. Plot the average error surface E for different values of a and b in the interval of $[-10 : 0.1 : 10]$.
   (a) Vary both a and b in steps of 0.1 in the interval $[-10, 10]$
   (b) Compute the element-wise error as $e_i = y_i - \hat{y}_i$ where $\hat{y}_i = ax_i + b$ and y_i is computed using equation 1
   (c) Compute the average error as

$$E = \frac{1}{n}\sum_{i=i}^{n} {e_i}^2 \tag{2}$$

   (d) Compute the average error values for all combinations of a and b.
   (e) Plot the error surface with the values of a along x-axis, that of b along y-axis and E along z-axis.

3. Solve for a and b using Pseudo-inverse based approach on the points generated in question 1.

4. Solve for a and b using the Gradient Descent approach where the values of $\mathbf{p} = (a, b)^T$ in the $(k+1)^{th}$ iteration is updated as

$$\mathbf{p}_{k+1} = \mathbf{p}_k - \eta \nabla_{\mathbf{p}} E|_{\mathbf{p}=\mathbf{p}_k} \tag{3}$$

Vary the update rate η and the initial values (a_0, b_0) and note the final solution after 100 iterations. Plot the trajectory of the solutions (a_k, b_k) for varying (a_0, b_0, η) on the contour plot of E on (a, b) plane.

5. Consider the multi-modal function given by

$$z = 1.7 * exp\left[-\left\{\frac{(x-3)^2}{10} + \frac{(y-3)^2}{10}\right\}\right] + exp\left[-\left\{\frac{(x+5)^2}{8} + \frac{(y+5)^2}{8}\right\}\right] + 2 * exp\left[-\left\{\frac{x^2}{4} + \frac{y^2}{5}\right\}\right] + 1.5 * exp\left[-\left\{\frac{(x-4)^2}{18} + \frac{(y+4)^2}{16}\right\}\right] + 1.2 * exp\left[-\left\{\frac{(x+4)^2}{18} + \frac{(y-4)^2}{16}\right\}\right] \tag{4}$$

Display the surface plot and contour plot of the above function in the search space given by $\mathbf{S}_{min} = [x_{min}, y_{min}]^T = [-10, -10]^T$ and $\mathbf{S}_{max} = [x_{max}, y_{max}]^T = [10, 10]^T$.