Quiz-1

MA-581 : Numerical Computations Lab **Time : 90 minutes** **20 marks**

August 30, 2021

1. Suppose we wish to compute e^x for a given $x \in \mathbb{R}$. Then the Taylor series

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

can be used to write an algorithm. Let s_n denote the first n terms of the series. Then $s_n \to e^x$ as $n \to \infty$ for all $x \in \mathbb{R}$. Hence a naive algorithm is to compute s_n for large enough n. This is implemented by the following MATLAB function.

```
function s = myexp(x,tol)
%  myexp computes an approximation s of exp(x)
%  up to a given tolerance tol.
%
s=1; term=1; k=1;
while abs(term)>tol*abs(s)
sum=s; term=term*x/k;
s=sum+term; k=k+1;
end
```

For positive x, and also small negative x, this program works quite well. For $x = 5 : 5 : 20$, compute e^x using `myexp` with `tol = 1e-8` and the MATLAB function `exp`. Prepare a table with four columns showing

$$\mathtt{x},\ \mathtt{myexp(x, 1e-8)},\ \mathtt{exp(x)},\ \mathtt{abs(exp(x) - myexp(x, 1e-8))}$$

and explain your results. Repeat the experiment for $x = -.5, -2, -10, -20$, and explain the possible cause for inaccurate results.

Since $e^x = 1/e^{-x}$, a modification of `myexp` together with a machine independent stopping criterion yield the following algorithm:

```
function s = goodexp(x)
%  stable computation of exp(x)
%  up to machine precision.
%
if x < 0, v = -1; x = abs(x); else v = 1; end
sum = 0; s = 1; term = 1;  k = 1;
while s ~= sum
sum = s; term = term*x/k;
s = sum + term; k = k+1;
end
if v < 0, s = 1/s; end
```

Explain the essential difference between `myexp` and `goodexp`. Explain why the stopping criterion in `goodexp` works in finite precision arithmetic. Will the stopping criterion work in infinite precision arithmetic? Now repeat the above experiment for `goodexp`, prepare a table and explain your results. **7 marks**

2. Collect 101 consecutive daily close prices of Tata Consultancy Services (TCS) stock from the BSE website

   https://www.bseindia.com/markets/equity/EQReports/StockPrcHistori.aspx?flag=0&type=ETF

   Choose any 101 consecutive daily close prices in 2021. You should mention the period you have chosen in your answer.

   (a) Construct the interpolating polynomial $p(x)$ through every fifth point, that is, let $x = 0 : 5 : 100$ and y denote the stock prices on days $0, 5, 10, ..., 100$. Plot $p(x)$ and the daily stock price data (t, s), where s is the stock prices on days $t = 0 : 100$, in a single plot. What does the plot say about using $p(x)$ to estimate/predict the stock prices s?. Can you identify days when $p(x)$ approximates s better than the other days? What is the maximum interpolation error? Is the Runge phenomenon evident in your plot?

   7 marks

   (b) Plot the natural cubic spline (use MATLAB command `spline`) with interpolating nodes $0 : 5 : 100$ instead of the interpolating polynomial $p(x)$, along with the daily data. Answer the same two questions as in (a). Compare the two approaches of representing the data and write your comments. **6 marks**

Submission instruction: Your answers should be in a single file in pdf or doc format. You may copy and paste your MATLAB output (final results including plots) in a doc file and write your answer/comments for each question in the doc file. You may convert the doc file to a pdf file and upload the same on Teams. You may also upload the doc file. The submission option will remain active till 10:30 AM, 30th August (Monday) 2021, after which you will NOT be able to submit the assignment. **Upload your MATLAB programs (script files including functions called by the main program, if any). Also upload the data file, if any, used in your program.**

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