Lab Session 4

MA581: Numerical Computations Lab R. Alam August 23, 2021

Note: To be consistent with MATLAB, consider polynomial interpolation in which indexing of data (nodes and values) starts with 1 rather than 0. Thus we denote the data points as $(x_1, f_1), \ldots, (x_n, f_n)$. The interpolating polynomial $p_n(x)$ will be of degree $n-1$.

1. Consider the Runge function $g(x) := 1/(1+25x^2)$ for $x \in [-1, 1]$. Interpolate g at n equally spaced points in the interval $x \in [-1, 1]$ using `barycent` function. Plot $\|g - p_n\|_\infty$ vs. n on "`semilogy`" axes for $n = 5, 10, 15, \ldots, 45$. You should estimate $\|g - p_n\|_\infty$ by taking maximum of $|g(x) - p(x)|$ at 1000 equispaced points in $[-1, 1]$. Separately plot (in single plot) $f(x), p_{15}(x)$ and $p_{20}(x)$. Also plot the error $E_n(x) := |f(x) - p_n(x)|$ as a function of x for $n = 15, 25, 35$.

   Next, consider the Chebyshev nodes $x_j := \cos\left(\frac{(2j-1)\pi}{2n}\right)$ for $j = 1 : n$. Repeat the experiment above for the Chebyshev nodes. What is your observation?

   Finally, consider the function $h(x) := e^{\cos(6x)}$ for $x \in [0, 2\pi]$ and repeat the experiment above by considering equispaced nodes and Chebyshev nodes in $[0, 2\pi]$. The Chebyshev nodes in $[0, 2\pi]$ are given by $x_j := \pi/2\big(1 + \cos\left(\frac{(2j-1)\pi}{2n}\right)\big) = \pi \cos^2\left(\frac{(2j-1)\pi}{4n}\right)$ for $j = 1 : n$.

2. Consider the Runge function $g(x) := 1/(1+25x^2)$ for $x \in [-1, 1]$. Consider the Lebesgue function $\lambda_n(x) := \sum_{j=1}^{n} |\ell_j(x)|$ and the Lebesgue constant $\Lambda_n := \max_{x\in[-1,1]} |\lambda_n(x)|$. Now, consider equally spaced nodes and plot (in separate plots) the Lebesgue function and the Lebesgue constant for $n = 10 : 5 : 60$. Repeat the experiment for Chebyshev nodes. (To compute Λ_n, consider 1000 equally spaced points and compute the maximum of $\lambda_n(x)$ at these points.) What do you observe about the growth of Λ_n for equally spaced nodes and Chebyshev nodes? How does the growth of Λ_n relate to the following estimates? For equispaced nodes $\Lambda_n \sim \dfrac{2^n}{en \log n}$ and for Chebyshev nodes $\Lambda_n \leq \dfrac{2}{\pi} \log(n+1) + 1$. This can be checked by plotting Λ_n and $\dfrac{2^n}{en \log n}$ for equispaced nodes, and Λ_n and $\dfrac{2}{\pi} \log(n+1) + 1$ for Chebyshev nodes for various values of n.

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