Lab Session 6

MA581: Numerical Computations Lab | R. Alam | September 06, 2021

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1. Let $a > 0$. Then the square root $\alpha = \sqrt{a}$ is the zero of $f(x) = x^2 - a$. The Newton's method yields
$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right), \; n = 0, 1, 2, \ldots.$$
This scheme converges globally, that is, $x_n \longrightarrow \sqrt{a}$ for any $x_0 > 0$. To verify global convergence, compute $\sqrt{173373}$ for various values of x_0. Compare your computed result with the result obtained by using MATLAB command `sqrt(173373)`. Determine the number of iterations required to achieve $|x_n - \sqrt{173373}| \leq \texttt{tol}$, for $\texttt{tol} = 10^{-8}, 10^{-12}, 10^{-16}$. Do the results show quadratic order of converegnce?

2. This problem discusses inverse interpolation which gives another method to find the zero of a function. Let $f : [a, b] \to \mathbb{R}$ be continuous and has only one zero at α in the interval, that is, $f(\alpha) = 0$ and $f(x) \neq 0$ for $x \neq \alpha$. Also assume that f has an inverse. Let $x_0, x_1, \ldots, x_n$ be $n+1$ distinct nodes in $[a, b]$ with $f(x_j) = y_j$, $j = 0 : n$. Construct an interpolating polynomial $p_n(x)$ for $f^{-1}(x)$ by taking your data points as (y_j, x_j), $j = 0 : n$. Write a MATLAB program for constructing $p_n(x)$. Observe that $f^{-1}(0) = \alpha$, the zero we are trying to find. Then, approximate the zero α, by evaluating the interpolating polynomial for f^{-1} at 0, that is, $p_n(0) \approx \alpha$. Use this method to find an approximation to the solution of $\log x = 0$ using the following data:

x	0.4	0.8	1.2	1.6
$\log x$	-0.92	-0.22	0.18	0.47

Next solve $\log(x) = 0$ using Newton's method with tolerance $\texttt{tol} = 10^{-6}$ and compare the result with that obtained by inverse interpolation. Estimate the errors for both the methods.

3. In neutron transport theory, the critical length of a fuel rod is determined by the solutions of the equation $\cot(x) = (x^2 - 1)/(2x)$. Use a zero finder (your own program) to determine the smallest positive solution of this equation. Compare your result with that obtained by using MATLAB function `fzero`.

4. Consider the problem of finding the smallest positive solution of the nonlinear equation $\cos(x) + 1/(1 + e^{-2x}) = 0$. Investigate, both theoretically and empirically, the following iterative schemes for solving this problem using the starting point $x_0 := 3$. For each scheme, you should show that it is indeed an equivalent fixed-point problem, determine analytically whether it is locally convergent and its expected convergence rate, and then implement the method to confirm your results.
   (a) $x_{k+1} = \arccos(-1/(1 + e^{-2x_k}))$.
   (b) Newton's method.

5. The natural frequencies of vibration of a uniform beam of unit length, clamped on one end and free on the other, satisfy the equation $\tan(x)\tanh(x) + 1 = 0$. Use a zero finder (your own program) to determine the smallest positive solution of this equation. Compare your result with that obtained by using MATLAB function `fzero`.

6. The vertical distance y that a parachutist falls before opening the parachute is given by the equation $y = \log(\cosh(t\sqrt{gk}))/k$, where t is the elapsed time in seconds, $g = 9 \cdot 8065\ m/s^2$ is the acceleration due to gravity, and $k = 0.00341\ m^{-1}$ is a constant related to air resistance. Use a zero finder (your own program) to determine the elapsed time required to fall a distance of 1 km. Compare your result with that obtained by using MATLAB function `fzero`.

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