Lab Session 2

MA581 : Numerical Computations Lab R. Alam August 9, 2021

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1. The purpose of this exercise is to illustrate anomaly in automatic computation. On my computer, MATLAB produces

$$\begin{aligned}(\frac{4}{3} - 1) * 3 - 1 &= -2.2204 \times 10^{-16} \\ 5 \times \frac{(1+\exp(-50)) - 1}{(1+\exp(-50)) - 1} &= \mathbf{NaN} \\ \frac{\log(\exp(750))}{100} &= \mathbf{Inf}\end{aligned}$$

   Try on your machine. Can you explain the reason behind these anomalies?

2. Consider $(\beta, t, L, U) = (10, 8, -99, 99)$ and the normalized floating-point numbers

$$\begin{aligned} x &= 0.23371258 \times 10^{-4} \\ y &= 0.33678429 \times 10^{2} \\ z &= -0.33677811 \times 10^{2} \end{aligned}$$

   Use MATLAB command `round` with $t = 8$ to calculate $\mathrm{fl}(x + \mathrm{fl}(y + z))$ and $\mathrm{fl}(\mathrm{fl}(x + y) + z)$. Is $\mathrm{fl}(x + \mathrm{fl}(y + z)) = \mathrm{fl}(\mathrm{fl}(x + y) + z)$?

   Next, calculate $x+y+z$ in exact arithmetic and determine the relative errors in calculating $\mathrm{fl}(x + \mathrm{fl}(y + z))$ and $\mathrm{fl}(\mathrm{fl}(x + y) + z)$.

3. Consider the power series for $\sin x$ given by

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

   Here is a Matlab function that uses this series to compute $\sin x$.

```
function s = powersin(x)
%
% POWERSIN(x) tries to comptue sin(x) from a power series
%
  s = 0;
  t = x;
  n = 1;
  while s + t ~= s;
   s = s + t;
   t = -x.^2/((n+1)*(n+2)).*t;
   n = n + 2;
  end
```

   When does the while loop terminate? Answer each of the following questions for $x = \pi/2, 11\pi/2, 21\pi/2$ and $31\pi/2$.

   - How accurate is the computed result?

- How many terms are required?
- What is the largest term in the series?

What is your conclusion about the use of floating point arithmetic and power series to evaluate functions?

4. Show how to rewrite the following expressions to avoid cancellation for the indicated arguments. Evaluate these expressions using 5-digits decimal system (use MATLAB command `round`).

   (a) $\sqrt{x+1} - \sqrt{x};\ x \approx 0$.

   (b) $\sin x - \sin y; x \approx y$.

   (c) $(1 - \cos x)/\sin x;\ x \approx 0$.

   (d) $c = (a^2 + b^2 - 2ab\cos\theta)^{1/2}; a \approx b; \theta \approx 0$.

5. Consider the function $f(x) = (e^x - 1)/x$, which arises in various applications. By L'Hopital's rule, we know that $\lim_{x\to 0} f(x) = 1$.

   (a) Compute the values of $f(x)$ for $x = 10^{-n}$ for $n = 1, 2, \ldots, 16$. Do your results agree with theoretical expectations? Explain why.

   (b) Now perform the computation in part (a) again, this time using the mathematically equivalent formulation $f(x) = (e^x - 1)/\log(e^x)$ (evaluate as indicated without simplification). If this works any better, can you explain why?

6. Consider the recurrence $x_{k+1} = 111 - (1130 - 3000/x_{k-1})/x_k,\ x_0 = 11/2,\ x_1 = 61/11$. In exact arithmetic the x_k form a monotonically increasing sequence that converges to 6. Implement the recurrence in MATLAB and compare the computed x_{34} with the true value 5.998 (correct to four digits). Try to explain what you see.

7. Find the smallest value of p for which the expression calculated in double precision arithmetic at $x = 10^{-p}$ has no correct significant digits (no correct digits in the mantissa). (Hint: First find the limit of the expression as $x \to 0$.)

   (a) $\dfrac{\tan x - x}{x^3}$

   (b) $\dfrac{e^x + \cos x - \sin x - 2}{x^3}$.

*** End ***