Lab Examination 1

MA-581 : Numerical Computations Lab **Full marks. 25** **Time: 90 minutes**
Date: November 08, 2021

Answer all questions.

1. Suppose that we have the following data gathered from timing the execution of an algorithm operating on matrices (n is the size of the matrix and t is the execution time)

n	5	55	105	155	205	255	305	355	405	455	505
t	0.0004	0.0012	0.0024	0.0046	0.0080	0.0120	0.0168	0.0232	0.0320	0.0682	0.0604

   Find a third-degree polynomial $p(n)$ expressing the execution time t in terms of the size n of square matrices by solving the LSP $Ax = b$. Also plot the data points (n, t) and the polynomial $p(n)$ in a single plot. Does the plot show that $p(n)$ provides a good fit?

   5 marks

2. Determine the polynomials $p(x)$ of degree 10 that best fits the Runge function

$$f(x) = 1/(1 + 25x^2)$$

   at 100 equally-spaced points x between -1 and 1. Plot the data points and $p(x)$ in a single plot. Do you observe Runge phenomenon? **5 marks**

3. **Planetary Orbit:** Consider the quadratic form $q(x, y) := ax^2 + bxy + cy^2 + dx + ey + f$. The set $\mathcal{C} := \{(x, y) \in \mathbb{R}^2 : q(x, y) = 0\}$ is a *conic section*. Cutting the surface $z = q(x, y)$ by the place $z = 0$, one obtains the conic $\mathcal{C}$. The contour $\mathcal{C}$ can be generated using `meshgrid` and `contour` commands (type `doc contour` for more information) as follows:

```
u = linspace(a0, b0, n); v = linspace(c0, d0, m);
[X, Y] = meshgrid(u, v); Z = q(X, Y); contour(X, Y, Z, [0 0])
```

   Suppose that a planet follows an elliptical orbit. Here are ten observations of its position in (x, y) plane:

$$\begin{aligned} x &= \begin{bmatrix} 1.02 & 0.95 & 0.87 & 0.77 & 0.67 & 0.56 & 0.44 & 0.30 & 0.16 & 0.01 \end{bmatrix} \\ y &= \begin{bmatrix} 0.39 & 0.32 & 0.27 & 0.22 & 0.18 & 0.15 & 0.13 & 0.12 & 0.13 & 0.15 \end{bmatrix} \end{aligned}$$

   (a) Determine the conic section that fits this data (in the sense of least squares). This can be done by setting one of the coefficients, say, $f = 1$ and solving the resulting 10-by-5 over determined LSP using backslash command. Plot the orbit (that is the conic) with x on the x-axis and y on the y-axis. Superimpose the ten data points on the plot. Write a single MATLAB script that implements the above task.

   8 marks

   (b) Can it be said that the LSP is nearly rank deficient? Your next task is to illustrate the effect of small perturbation in the data to the orbit of the planet. Perturb the

data x and y by adding to each component a random number uniformly distributed in the interval $[-0.005, 0.005]$. You may use the command

```
x = x + .005*(2*rand(n,1)-1); y = y + .005*(2*rand(n,1)-1).
```

Compute the new coefficients resulting from the perturbed data. Plot the new orbit on the same plot of the old orbit. Write a MATLAB script that implements the job. Run the script a couple of times and comment on your results. **7 marks**

Submission instruction: Your answers should be in a single file in pdf or doc format and upload the file on Teams. You may copy your numerical results (but not intermediate results) and plots from MATLAB to a doc file and upload the same. Also, you should submit your MATLAB programs. **The submission option will remain active till 10:50 AM, 8th November 2021, after which you will NOT be able to submit the answer.**