Lab Session 5

MA581: Numerical Computations Lab R. Alam August 30, 2021

1. Since $\int_0^1 \frac{4}{1+x^2}dx = \pi$ one can compute an approximate value for π using numerical integration of the given function. Use the midpoint, trapezoid, and Simpson composite quadrature rules to compute the approximate value for π for various stepsizes h. Compare the accuracy of the rules with each other (based on the known value of π). Is there any point beyond which decreasing h yields no further improvement?

2. Use midpoint, trapezoid, and Simpson composite quadrature rules to verify or refute the following conjectures.

   (a) $\int_0^1 \frac{e^{-9x^2} + e^{-1024(x-1/4)^2}}{\sqrt{\pi}} dx = 0.2$

   (b) $\int_0^1 \sqrt{x}\log(x)dx = -\frac{4}{9}$.

3. The intensity of diffracted light near a straight edge is determined by the values of the Fresnel integrals

$$C(x) := \int_0^x \cos\left(\frac{\pi t^2}{2}\right) dt \quad \text{and} \quad S(x) := \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt.$$

   Use quadrature routine to evaluate these integrals for enough values of x to draw a smooth plot of $C(x)$ and $S(x)$ over the range $0 \leq x \leq 5$. You may use MATLAB function `quad` which uses adaptive quadrature to compute these integrals.

4. Planck's theory of blackbody radiation leads to the integral $\int_0^\infty \frac{x^3}{e^x - 1}dx$.

   Truncate the infinite interval of integration and use a composite quadrature rule, such as trapezoid or Simpson. You will need to do some experimentation or analysis to determine where to truncate the interval, based on the usual trade-off between efficiency and accuracy.

   Truncate the interval and use MATLAB function `quad` to evaluate the integral. Compare the efficiency and accuracy of the three methods.

5. The period of a simple pendulum is determined by the complete elliptic integral of the first kind

$$K(x) := \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - x^2\sin^2\theta}}.$$

   Use MATLAB function `quad` to evaluate this integral for enough values of x to draw a smooth plot of $K(x)$ over the range $0 \leq x \leq 1$.

*** End ***