Algorithmic Aspects of Roman $\{2\}$-domination in Graphs

Submitted in the fulfillment for the requirements of the degree of

Bachelor of Technology

in

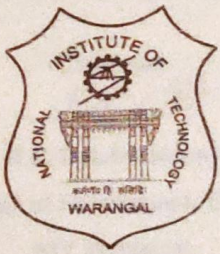
Computer Science and Engineering

by

Pratyush Kamal Chaudhary (157148)
Avinash Bunde (157108)
Rajendra Kujur (157150)

Under the Guidance of

Dr. P. Venkata Subba Reddy
Assistant Professor, Department of CSE
NIT Warangal



Department of Computer Science and Engineering

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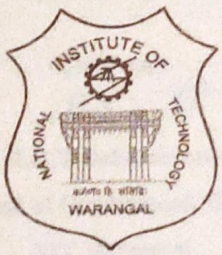

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Dissertation Approval for B.Tech.

This Project Work entitled

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Avinash Bunde

Rajendra Kujur

is approved for the Degree of

B. Tech in Computer Science and Engineering

Examiners

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.....

.....

Supervisor

.....

Dr. P Venkat Subba Reddy

Assistant Professor, Department of CSE

Chairman

.....

Dr. R.B.V. Subramaanyam

Head of Department, CSE

NIT Warangal

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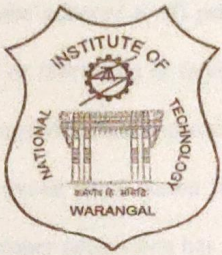


CERTIFICATE

This is to certify that the project titled **"Algorithmic Aspects of Roman {2}-domination in Graphs"** is a bonafide work carried out by **Pratyush Kamal Chaudhary (Roll No. 157148), Avinash Bunde (Roll No. 157108), Rajendra Kujur (Roll No. 157150)** in fulfilment of the requirements for the award of the degree of **B.Tech.** in **Computer Science and Engineering** and submitted to the Department of Computer Science and Engineering, National Institute of Technology, Warangal.

PVSReddy 17/5/2019

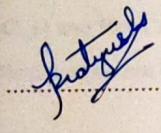
Dr. P. Venkata Subba Reddy
Project Guide
Department of CSE
NIT Warangal

RBV Subramanyam

Dr. R. B. V. Subramanyam
Head of the Department
Department of CSE
NIT Warangal

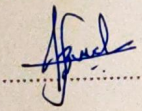
DECLARATION

We declare that this written submission represents our ideas in our own words and where others' ideas or words have been included, we have adequately cited and referenced the original sources. We also declare that we have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea / data / fact / source in our submission. We understand that any violation of the above will be the cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.



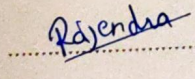
Pratyush Kamal Chaudhary

Roll no. 157148



Avinash Bunde

Roll no. 157108



Rajendra Kujur

Roll no. 157150

Date: 17/5/19

Abstract

A Roman $\{2\}$-dominating function (R2DF) on a graph $G = (V, E)$ is defined as a function $f : V(G) \rightarrow \{0, 1, 2\}$, which satisfies the condition that for any vertex $u \in V$ having $f(u) = 0$, there is at least one vertex v adjacent to u where $f(v) = 2$ or there are at least two vertices x,y adjacent to u where $f(x) = f(y) = 1$.

The weight of a R2DF f is the value $f(V) = \sum_{v \in V} f(v)$.The minimum weight of a R2DF f on a graph G is the Roman $\{2\}$-domination number $\gamma_{R2}(G)$. Finding γ_{R2} for a general graph G is proved to be an NP-complete problem.

In this paper we will begin our discussion with upper bound and exact results obtained for γ_{R2} in some special classes of Graphs. In particular, we present the exact value of γ_{R2} in Square of Cycle Graphs (C_{n^2}) and Rook's Graph. We also give a tight upper bound of γ_{R2} in King's Graph.

Next we present a linear time dynamic programming algorithm for finding $\gamma_{R2}(T)$ in a tree T.

Keywords: Roman $\{2\}$-domination, NP-complete, Square of Graphs, Cycle Graphs, Rook's Graph, King's Graph, dynamic programming, tree.

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CHAPTER 1

Introduction

Graph theory, a branch of discrete mathematics, is the study of graphs. A graph can be considered as a mathematical structure modelling the pairwise relations between various objects. In a graph, the objects are usually represented by nodes while the relation between them is represented by an edge. Formally, a graph is an ordered pair $G = (V, E)$ where V denotes the set of vertices, while E denotes the set of edges, each connecting a pair of vertices. An edge $e \in E$ is an ordered pair $e = (u, v)$ which indicates there is a relation between vertices $u \in V$ and $v \in V$.

A dominating set of a graph G is the subset $D \subseteq V$ such that for any vertex $u \notin D$, there exists a vertex $v \in D$ which is adjacent to u. The minimum size of a dominating set $|D|$ of a graph G is called domination number $\gamma(G)$ of the graph.

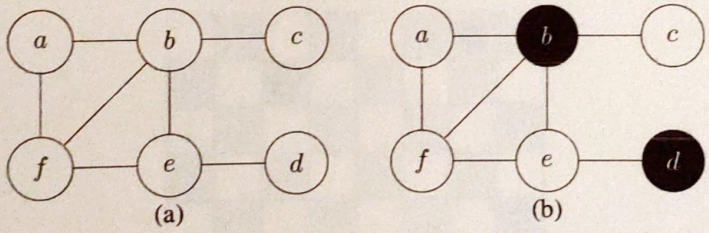

Figure 1.1: (a) Graph G (b) Graph G with γ set

In Figure 1.1, the set of vertices $D = \{b, d\}$ form a minimum dominating set. Note that there can be more than one minimum dominating set for a graph, in Figure 1.3, $\{b, e\}$ is also a minimum DS. Therefore, $\gamma(G) = 2$.