

# **Algorithmic Aspects of Roman {2}-domination in Graphs**

**Submitted in the fulfillment for the requirements of the degree of**

**Bachelor of Technology**

**in**

**Computer Science and Engineering**

**by**

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**Under the Guidance of**

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**Department of Computer Science and Engineering**

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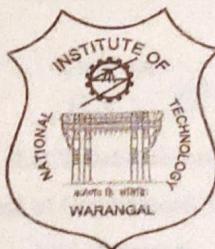
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# Dissertation Approval for B.Tech.

*This Project Work entitled*

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**B. Tech in Computer Science and Engineering**

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**CERTIFICATE**

This is to certify that the project titled “Algorithmic Aspects of Roman {2}-domination in Graphs” is a bonafide work carried out by **Pratyush Kamal Chaudhary (Roll No. 157148)**, **Avinash Bunde (Roll No. 157108)**, **Rajendra Kujur (Roll No. 157150)** in fulfilment of the requirements for the award of the degree of **B.Tech.** in **Computer Science and Engineering** and submitted to the Department of Computer Science and Engineering, National Institute of Technology, Warangal.

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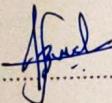
## **DECLARATION**

We declare that this written submission represents our ideas in our own words and where others' ideas or words have been included, we have adequately cited and referenced the original sources. We also declare that we have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea / data / fact / source in our submission. We understand that any violation of the above will be the cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.



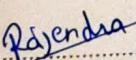
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## Abstract

A Roman  $\{2\}$ -dominating function (R2DF) on a graph  $G = (V, E)$  is defined as a function  $f : V(G) \rightarrow \{0, 1, 2\}$ , which satisfies the condition that for any vertex  $u \in V$  having  $f(u) = 0$ , there is at least one vertex  $v$  adjacent to  $u$  where  $f(v) = 2$  or there are at least two vertices  $x, y$  adjacent to  $u$  where  $f(x) = f(y) = 1$ .

The weight of a R2DF  $f$  is the value  $f(V) = \sum_{v \in V} f(v)$ . The minimum weight of a R2DF  $f$  on a graph  $G$  is the Roman  $\{2\}$ -domination number  $\gamma_{R2}(G)$ . Finding  $\gamma_{R2}$  for a general graph  $G$  is proved to be an NP-complete problem.

In this paper we will begin our discussion with upper bound and exact results obtained for  $\gamma_{R2}$  in some special classes of Graphs. In particular, we present the exact value of  $\gamma_{R2}$  in Square of Cycle Graphs ( $C_{n^2}$ ) and Rook's Graph. We also give a tight upper bound of  $\gamma_{R2}$  in King's Graph.

Next we present a linear time dynamic programming algorithm for finding  $\gamma_{R2}(T)$  in a tree  $T$ .

**Keywords:** Roman  $\{2\}$ -domination, NP-complete, Square of Graphs, Cycle Graphs, Rook's Graph, King's Graph, dynamic programming, tree.

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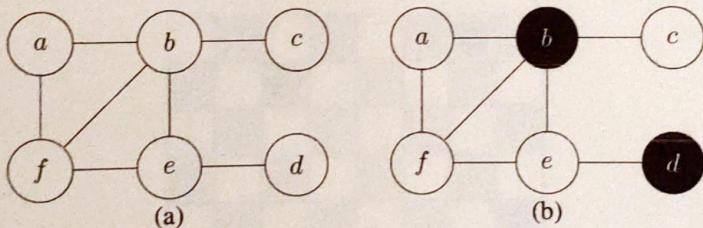
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# CHAPTER 1

## Introduction

Graph theory, a branch of discrete mathematics, is the study of graphs. A graph can be considered as a mathematical structure modelling the pairwise relations between various objects. In a graph, the objects are usually represented by nodes while the relation between them is represented by an edge. Formally, a graph is an ordered pair  $G = (V, E)$  where  $V$  denotes the set of vertices, while  $E$  denotes the set of edges, each connecting a pair of vertices. An edge  $e \in E$  is an ordered pair  $e = (u, v)$  which indicates there is a relation between vertices  $u \in V$  and  $v \in V$ .

A dominating set of a graph  $G$  is the subset  $D \subseteq V$  such that for any vertex  $u \notin D$ , there exists a vertex  $v \in D$  which is adjacent to  $u$ . The minimum size of a dominating set  $|D|$  of a graph  $G$  is called domination number  $\gamma(G)$  of the graph.



**Figure 1.1:** (a) Graph  $G$  (b) Graph  $G$  with  $\gamma$  set

In Figure 1.1, the set of vertices  $D = \{b, d\}$  form a minimum dominating set. Note that there can be more than one minimum dominating set for a graph, in Figure 1.3,  $\{b, e\}$  is also a minimum DS. Therefore,  $\gamma(G) = 2$ .