

CS6140 Assignment 4

$$(2.3) \quad LL_{\text{Regularized}} = \frac{\lambda}{2} \|w\|^2 + \sum_{i=1}^N \left\{ \ln(1 + e^{-y_i w^T x_i}) \right\}$$

$$= \frac{\lambda}{2} \|w\|^2 + \sum_{i=1}^N \left\{ y_i \cdot \ln O_i + (1 - y_i) \ln(1 - O_i) \right\}$$

$$\frac{\partial LL}{\partial w} = \lambda \|w\| + \sum_{i=1}^N \left\{ y_i (1 - O_i) \times x_i + (1 - y_i) (-1) \times O_i x_i \right\}$$

$$= \lambda \|w\| + \sum_{i=1}^N \left\{ y_i x_i - y_i O_i x_i - O_i x_i + y_i O_i x_i \right\}$$

$$= \lambda \|w\| + \sum_{i=1}^N (y_i - O_i) x_i$$

$$w_0 = w_0 - \overset{\text{learning rate}}{\eta} \sum_{i=1}^N (y_i - O_i) x_i$$

$$w_v = w_v + \frac{\lambda}{c} w_v - \eta \sum_{i=1}^N (y_i - O_i) x_i$$

Gradient Descent Algorithm

2.5 Yes, it is possible to kernelize equation 2.

$$LL(w) = \sum_{i=1}^N \ln(\text{sigmoid}(y_i w^T x_i)) - \frac{\lambda}{2} \|w\|^2$$

$$\ln(\text{sigmoid}(x)) \leq x - H(x)$$

$$\text{where } H(x) = -x \log x - (1-x) \log(1-x)$$

$$\rightarrow LL(w) \leq LL(w, \alpha)$$

$$\text{where } LL(w, \alpha) = \sum_{i=1}^N \alpha_i y_i w^T x_i - H(\alpha_i) - \frac{\lambda}{2} \|w\|^2$$

$$\star w(\alpha) = \lambda^{-1} \sum_{i=1}^N \alpha_i y_i x_i$$

$$LL(\alpha) = \frac{1}{2\lambda} \sum_{i=1}^N \sum_{j=1}^N \alpha_i \alpha_j y_i y_j x_i^T x_j - \sum_{i=1}^N H(\alpha_i)$$

Dual Form

Solution learned from "A comparison of numerical optimizers for logistic regression"
by Thomas P. Minka 10/22/2003
and provided Andrew Ng video.