Energy Loss Distribution of a High Energetic Charged Particle in a Thin Gaseous Medium

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May 2, 2022

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Bethe-Bloch Formula

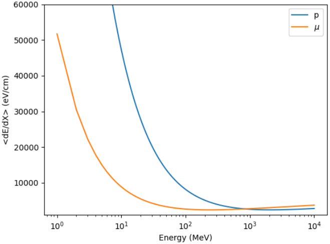
The expression for average differential energy loss per unit length due to ionisation and excitation is given by:

$$-\left\langle \frac{dE}{dX} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[ln \left(\frac{2m_e c^2 \beta^2 E_m}{I^2(1-\beta^2)} \right) - 2\beta^2 \right] \tag{1}$$

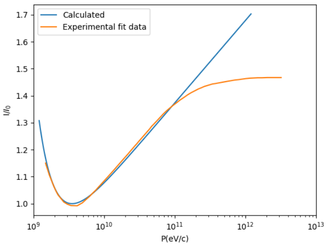
where

- $I = I_0 Z$ is the average ionisation potential[1]
- $E_m = 2m_e c^2 \beta\gamma$ is the maximum energy transfer allowed in each interaction

$< dE/dX >$ vs E and p



(a) Average energy loss per unit length in Ar chamber at NTP with respect to projectile's energy



(b) Experimental data from the report[4] for energy loss of proton in an argon chamber at NTP

Figure: Stopping power

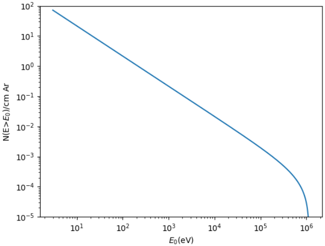
δ electron

Number of electron having energy in the range E to E+dE is

$$P(E)dE = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \frac{X}{E^2} dE = \frac{K}{\beta^2} \frac{Z\rho}{A} \frac{X}{E^2} dE \quad (2)$$

Number of electrons having energy greater than E_0 upto E_m can be calculated as

$$N(E \geq E_0) = \int_{E_0}^{E_m} P(E)dE \quad (3)$$



Energy Loss Distribution

Landau distribution:

$$f(\lambda) = Ae^{-\frac{1}{2}(\lambda + e^{-\lambda})} \qquad (4)$$

where

$$\lambda = \frac{\Delta E - \Delta E_{mp}}{\xi} \qquad (5)$$

Fit function:
$F(\Delta E; A, \Delta E_{mp}, \xi)$

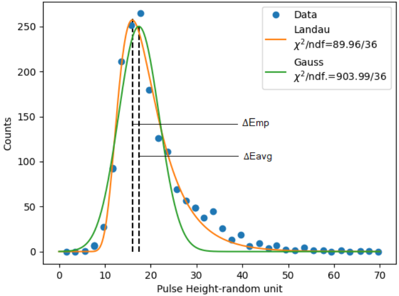


Figure: Energy loss distribution in an Ar-CO_2(70-30%) chamber[2]

Gaussian distribution:

$$G(\Delta E; A, \Delta E_{avg}, \xi) = Ae^{-\frac{1}{2}\left(\frac{\Delta E - \Delta E_{avg}}{\xi}\right)^2} \qquad (6)$$

Non-linear Least Squares

Consider a set of m data points, (x_1, y_1), (x_2, y_2),..., (x_m, y_m). Fit function: y = f(x,β) where $\beta = (\beta_1, \beta_2, ..., \beta_n)$ and m $\geq$ n.

$$S = \sum_{i=1}^{m} r_i^2 = \sum_{i=1}^{m} (y_i - f(x_i, \beta))^2 \tag{7}$$

To minimise S, the gradient should be zero,

$$\frac{\partial S}{\partial \beta_j} = 2 \sum_{i=1}^{n} r_i \frac{\partial r_i}{\partial \beta_j} = 0 \tag{8}$$

Iteration:

$$\beta_j \approx \beta_j^{k+1} = \beta_j^k + \Delta\beta_j \tag{9}$$

The first order Taylor expansion about β^k

$$f(x_i,\beta) \approx f(x_i,\beta^k) + \sum_j^n \frac{\partial f(x_i,\beta^k)}{\partial \beta_j}(\beta_j - \beta_j^k) = f(x_i,\beta^k) + \sum_j^n J_{ij}\Delta\beta_j \quad (10)$$

The residuals are $\Delta y_i = y_i - f(x_i,\beta^k)$ and

$$r_i = y_i - f(x_i,\beta) = (y_i - f(x_i,\beta^k)) + (f(x_i,\beta^k) - f(x_i,\beta)) \approx \Delta y_i - \sum_{s=1}^n J_{is}\Delta\beta_s \quad (11)$$

Substituting these equations in eq.8, we get

$$\sum_{i=1}^m \sum_{s=1}^n J_{ij}J_{is}\Delta\beta_s = \sum_{i=1}^m J_{ij}\Delta y_i \quad (12)$$

$$(J^T J)\Delta\beta = J^T \Delta y \quad (13)$$

References

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Thank You