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Energy Loss Distribution of a High Energetic Charged Particle in a Thin Gaseous Medium

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INTRODUCTION

When a relativistic charged particle passes through a gaseous medium it can interact in many ways. In general electromagnetic interaction is used for detection of charged particles due to its higher probability of interaction. When a charged particle goes through a gaseous medium it suffers multiple inelastic collisions with the atoms or molecules in the medium. The inelastic collision with the atoms or the valence electrons of the atoms causes loss of energy of the incoming particles. And the majority of the collisions that account for the energy deposition leads to excitation and ionization of atoms in the medium.

ENERGY LOSS DUE TO IONIZATION AND EXCITATION

This energy loss due to ionization and excitation is statistical in nature. The expression for average differential energy loss per unit length (also called stopping power) was derived by Hans Bethe (1932) and Felix Bloch (1933) using quantum formulation is as follow:

$$-\left\langle \frac{dE}{dX} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[ln \left(\frac{2m_e c^2 \beta^2 E_m}{I^2(1-\beta^2)} \right) - 2\beta^2 \right] \tag{1}$$

where

- N_a is Avogadro's number,
- m_e and e are the electron mass and charge,
- Z, A and ρ are the atomic number and mass, and the density of the medium, respectively,
- I is its effective ionization potential,
- c is the speed of light in vaccum
- z is the charge in terms of electronic charge and β is the velocity with respect to c of the incoming particle

- r_e is the classical electron radius ($r_e = e^2/4\pi\epsilon_0 m_e c^2$) and ϵ_0 is the permittivity of the free space
- and E_m is the maximum energy transfer allowed to the electron in each interaction.

E_m can be calculated with simple two-body relativistic kinematics i.e., from energy and momentum conservation.

$$E_m = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + \left(\frac{m_e}{M}\right)^2 + \gamma\left(\frac{m_e}{M}\right)} \tag{2}$$

here $\gamma = 1/\sqrt{1-\beta^2}$, M is the mass of incoming particle.

For $M >> m_e$, E_m can be approximated as

$$E_m = 2m_e c^2 \beta^2 \gamma^2 \tag{3}$$

Effective Ionisation Potential

The effective ionization potential is the main parameter of the energy loss formula, and is essentially the average orbital frequency $\bar{\nu}$ times the Planck's constant, $h\bar{\nu}$. It is theoretically a logarithmic average of ν weighted by the so called oscillator strengths of the atomic levels. In practice, this is a very difficult quantity to calculate since the oscillator strengths are unknown for most materials. Instead, values of I for several materials have been deduced from actual measurements of dE/dx and semi-empirical formula for I vs Z fitted to the points. But a good approximation can be taken as $I = I_0 Z$, where $I_0 = 12eV$[1].

The Bethe-Bloch formula clearly expresses that energy loss does depends upon the mass of the projectile rather it depends only on β. In the plot of $< dE/dX >$ as function of momentum of the projectile as shown in the Fig.1 in a Ar chamber with density ($\rho = 1.66 \times 10^{-3} g/cm^3$), we can observe

- a fast decrease at the beginning, this is due to the term β^{-2}.
- then we get a region of minima called region of minimum ionization. At this region the energy loss is minimum due to very less interaction of projectile with the matter. Energy loss at minimum ionizing region are nearly same for both the charged particles.

The logarithm term in the Bethe-Bloch formula is dominated at very high momentum of particle. In the Fig.2, relativistic rise of the energy loss in Argon at NTP is shown with respect to the momentum of proton. The vertical axis shows that relative energy loss with respect to the minimum ionizing region. The experimentally fitted data is taken from the article [2] and the other

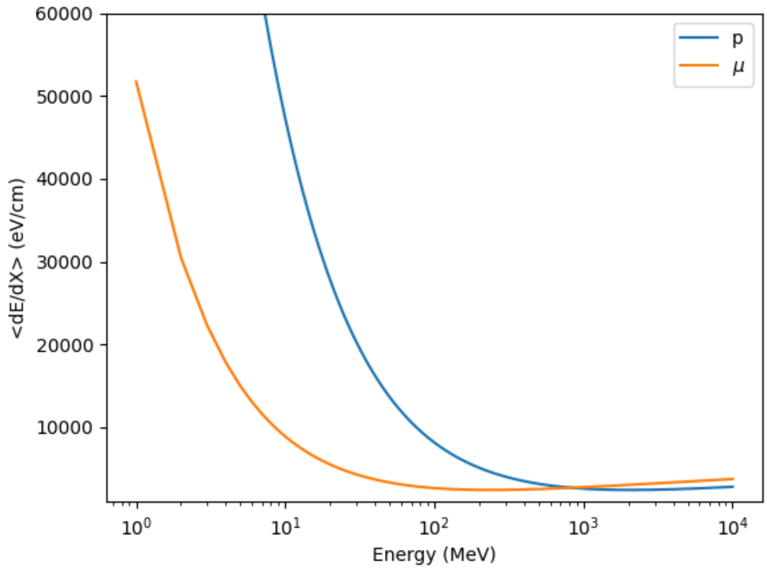

FIG. 1: Energy loss in unit length in Argon computed for energy of incident proton and muon

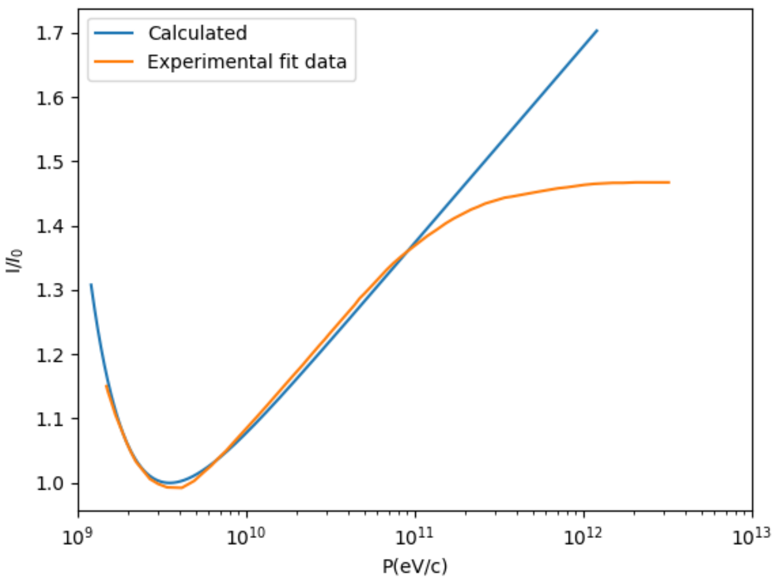

FIG. 2: Relativistic rise of the energy loss in Argon at NTP at the function of proton momentum

curve is obtained from the theoretical calculation from the Bethe-Bloch formula. The curve seems converging upto some 100 GeV/c of proton. Then in the experimental data we got a saturation in the energy loss. This is due to the fact that the medium get polarized by the electromagnetic field produced by the high energetic particle and this polarization effect increases with energy of the particle. Polarization act as a shield to the interaction of electromagnetic field between the projectile and the electrons at distant collisions. This effect was not considered in the eq.1. Gradually, eq.1 is modified to get accurate result for different energetic particles which is not discussed in this report.

δ ELECTRON

In a single collision E_m is the maximum allowed energy transfer to the electron as discussed above. So, now in a single collision the electron on the way of the incident charged particle may receive any energy value in the range 0 to E_m. This amount of energy transfer depends on the distance of the closest approach[3]. If the distance of closest approach is large compared to the size of the atom (a distant collision), the result can be excitation or ionization of the atom. If the distance of closest approach is of the order of the atomic dimensions, (a close collision) the interaction involves ejection of an electron from the atom with considerable energy or can be delta-electrons. This phenomenon is also statistical in nature. So, an approximate value of the probability of an electron receiving an energy E in the range X is given by

$$P(E) = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \frac{X}{E^2} \tag{4}$$

Number of electron having energy in the range E to E+dE is

$$P(E)dE = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \frac{X}{E^2} dE = \frac{K}{\beta^2} \frac{Z\rho}{A} \frac{X}{E^2} dE \tag{5}$$

where K is constant ($K = 0.1535 MeV cm^2 mol^{-1}$). Electrons ejected above a few keV are called delta electrons. These are electrons with sufficiently large energy which can further ionize other atoms on their way as in general noble gas has ionization energy in the order of ∼0eV. Number of electrons having energy greater than E_0 upto E_m can be calculated as

$$N(E \geq E_0) = \int_{E_0}^{E_m} P(E)dE \tag{6}$$

The plot for $N(E \geq E_0)$ vs E_0 is plotted in the Fig.3 in 1cm of Argon at NTP and the maximum allowed energy for 1GeV/c proton. This is basically the number of δ-electron ejected at an energy larger than or equal to E_0. From the graph we can see around 72 electrons has energy greater than 3eV when a 1GeV/c proton passes through 1cm of Ar chamber. The value of N at $E_0 = E_m$ is very small and asymptotically going to zero.

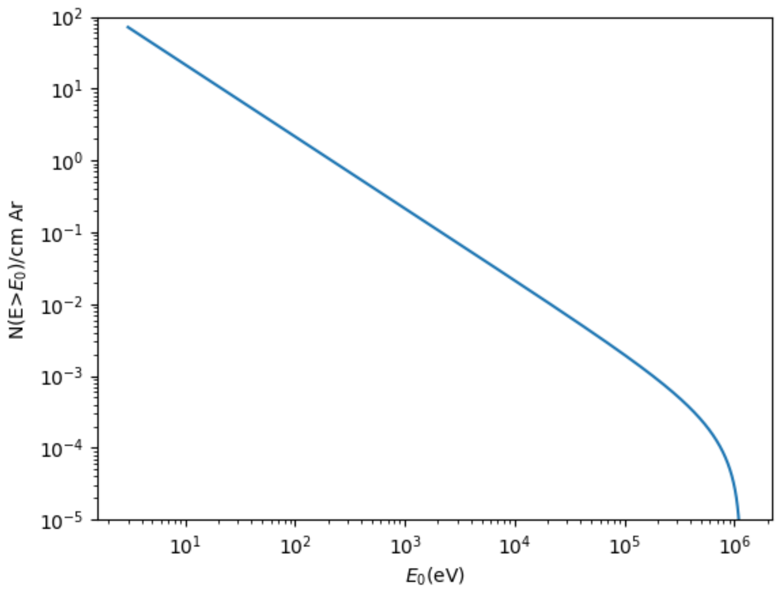

FIG. 3: Number of δ electrons ejected at an energy larger than or equal to E_0, in 1cm of argon at NTP with 1GeV/c proton

ENERGY LOSS DISTRIBUTION

The quantity $< dE/dX > \delta x$ is the average energy loss due to ionization and excitation in a layer of the medium with thickness δx. The real energy loss will fluctuate around the average value from event to event. As in a thin material the total energy loss is due to small number of interaction with the electrons with very wide range of possible range of energy transfer. As we have also seen the number electrons ejected with certain energy is also statistical in nature. The experimental data was taken by Franzen and Cochran[4] for energy loss distribution in an argon-carbon dioxide counter as shown in the Fig. 4. The shape of the curve can be fitted to Landau distribution which is as follow:

$$f(\lambda) = Ae^{-\frac{1}{2}(\lambda+e^{-\lambda})} \tag{7}$$

where λ represent the normalised deviation from the most probable energy loss (ΔE_{mp}) and can be expressed as follow:

$$\lambda = \frac{\Delta E - \Delta E_{mp}}{\xi} \tag{8}$$

where $\xi = KZ\rho X/A\beta^2$ is the average energy loss given by the first term in the Bethe-Bloch formula. A non-linear fitting was done to fit the curve with function $F(\Delta E; A, \Delta E_{mp}, \xi)$ and value of most

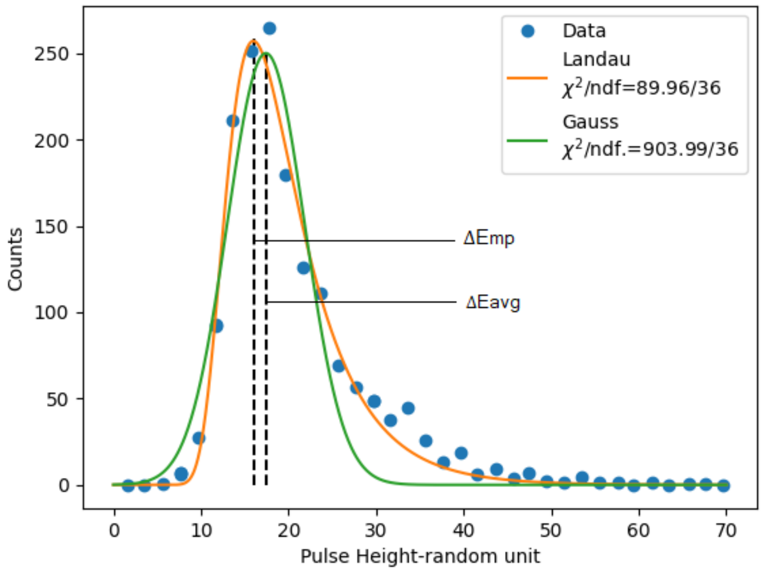

FIG. 4: Energy loss distribution in an argon-CO_2(70-30%) chamber

probable energy loss (ΔE_{mp}) was calculated. Similarly a non-linear fit was done to calculate the average energy loss with gaussian distribution:

$$G(\Delta E; A, \Delta E_{avg}, \xi) = Ae^{-\frac{1}{2}(\frac{\Delta E - \Delta E_{avg}}{\xi})^2} \tag{9}$$

The experimental fit curve, the Landau and Gaussian fit all are shown in the Fig.4. The x-axis represents the energy loss with arbitrary unit which is proportional to the energy loss and y-axis represents the count of particle for a certain energy loss. The tail in the right side of the peak is due to high energetic δ electrons. The tail will increase as we increase the width of the detector which will decrease the resolution of the detector.

NON-LINEAR LEAST SQUARE FIT

Consider a set of m data points, (x_1, y_1), (x_2, y_2),..., (x_m, y_m). Fit function: y = f(x,β) where $\beta = (\beta_1, \beta_2, ..., \beta_n)$ and m $\geq$ n. We have to find the best value β such that curve best fits the given data points. For that we have to minimise the sum of residuals with respect to β, now, we can define

$$S = \sum_{i=1}^{m} r_i^2 = \sum_{i=1}^{m} (y_i - f(x_i, \beta))^2 \tag{10}$$

To minimise S, the gradient should be zero,

$$\frac{\partial S}{\partial \beta_j} = 2\sum_{i=1}^{n} r_i \frac{\partial r_i}{\partial \beta_j} = 0 \tag{11}$$

Iteration:

$$\beta_j \approx \beta_j^{k+1} = \beta_j^k + \Delta\beta_j \tag{12}$$

The first order Taylor expansion about β^k

$$f(x_i, \beta) \approx f(x_i, \beta^k) + \sum_{j}^{n} \frac{\partial f(x_i, \beta^k)}{\partial \beta_j}(\beta_j - \beta_j^k) = f(x_i, \beta^k) + \sum_{j}^{n} J_{ij}\Delta\beta_j \tag{13}$$

The residuals are $\Delta y_i = y_i - f(x_i, \beta^k)$ and

$$r_i = y_i - f(x_i, \beta) = (y_i - f(x_i, \beta^k)) + (f(x_i, \beta^k) - f(x_i, \beta)) \approx \Delta y_i - \sum_{s=1}^{n} J_{is}\Delta\beta_s \tag{14}$$

Substituting these equations in eq.11, we get

$$\sum_{i=1}^{m}\sum_{s=1}^{n} J_{ij}J_{is}\Delta\beta_s = \sum_{i=1}^{m} J_{ij}\Delta y_i \tag{15}$$

In the matrix form we get as

$$(J^T J)\Delta\beta = J^T \Delta y \tag{16}$$

from here we can update β by adding $\Delta\beta$ to β^k and the iteration continuous till norm of $\Delta\beta < \epsilon$. The final β is the fitted parameter we are looking for.

With this we can calculate the value of ΔE_{mp} and ΔE_{avg} from the above graph4.

CONCLUSION

Energy loss due to ionization and excitation in an Ar chamber for different energy of proton and muon was plotted and concluded that energy loss at minimum ionizing region was nearly same for both the particles. Number of delta electrons ejected with energy above an energy (E_0) was calculated by integration with simpson method. And the statistical nature of energy deposition in medium was studied and fitted with non-linear fit by using Landau and Gaussian distribution to identify the most probable energy loss and average energy loss respectively.

[1] F. Sauli, *Principles of operation of multiwire proportional and drift chamber* (1977).

[2] M. Aderholz, P. Lazeyras, I. Lehraus, R. Matthewson, and W. Tejessy, Nuclear Instruments and Methods **118**, 419 (1974), ISSN 0029-554X, URL `https://www.sciencedirect.com/science/article/pii/0029554X74906466`.

[3] C. Lippmann, *Detector physics of resistive plate chamber* (2003).

[4] W. FRANZEN and L. COCHRAN (Wiley, New York, 1956), chap. Pulse ionization chambers and proportional counters, a.h. snell ed.