Assignment-1

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6) Let f be a one-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(x) = 1, f(y) \neq 1, f(z) \neq 2$ determine $f^{-1}(1)$. $(1981 - 2Marks)$
7) Let R be the set of real numbers and $f : R \to R$ be such that for all x and y in R $|f(x) - f(y)| \leq |x - y|^3$ .Prove that $f(x)$ is a constant. $(1988 - 2Marks)$
8) Find the natural number $'a'$ for which $\sum_{k=1}^{n} f(a + k) = 16(2^n - 1)$, where the function $'f'$ satisfies the relation $f(x + y) = f(x)f(y)$ for all natural numbers x, y and further $f(1) = 2$. $(1992 - 6Marks)$
9) Let $\{x\}$ and $[x]$ denotes the fractional and integral part of a real number x respectively. Solve $4\{x\} = x + [x]$. $(1994 - 4Marks)$
10) A function $f : IR \to IR$, where IR is the set of real numbers, is defined by $f(x) = \frac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$. Find the interval of values α for which f is onto. Is the function one-to-one for $\alpha = 3$? Justify your answer. $(1996 - 5Marks)$
11) Let $f(x) = Ax^2 + Bx + C$ where A, B, C are real numbers. Prove that if $f(x)$ is an inteher whenever x is an integer, then the numbers $2A, A + B$ and C are all integers. Conversly, prove that if the numbers $2A, A + B$ and C are all integers then $f(x)$ is an integer whenever x is an integer. $(1998 - 8Marks)$

F Match the following

1. Let the function defined in column I have domain $(-\frac{\pi}{2}, \frac{\pi}{2})$ and range $(-\infty, \infty)$ $(1992 - 2Marks)$

Column I	Column II
	(p) onto but not one-one
	(q) one-one but not onto
(A) $1 + 2x$	(r) one-one and onto
(B) $tanx$	(s) neither one-one nor onto

2) Let $f(x) = \frac{x^2-6x+5}{x^2-5x+6}$ $(2007 - 6Marks)$
Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the $4 * 4$ matrix given in the ORS.

Column I	I. Column II
(A) If $-1 < x < 1$, then $f(X)$ satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then $f(x)$ satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then $f(x)$ satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then $f(x)$ satisfies	(s) $f(x) < 1$

This section contains 4 questions. Each questions has 2 matching lists: LIST-I and LIST-II. Four options are give representing matching of elements from LIST-I and LIST-II. Only one of these four option corresponding to a correct matching.

3) Let $E_1 = \{x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0\}$ and $E_2 = \{x \in E_1 : \sin^{-1}(\log_e(\frac{x}{x-1})) \text{ is a real number }\}$.
(Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $[-\frac{\pi}{2}, \frac{\pi}{2})$.
Let $f : E_1 \to R$ be the function defined by $f(x) = \log_e(\frac{x}{x-1})$ and $g : E_2 \to R$ be the function defined by $g(x) = \sin^{-1}(\log_e(\frac{x}{x-1}))$ $(JEEAdv.2018)$

II. LIST-I	III. LIST-II
	1) $(-\infty, \frac{1}{1-e}] \cup [\frac{e}{e-1}, \infty)$
	2) $(0, 1)$
P The range of f is	3) $[-\frac{1}{2}, \frac{1}{2}]$
Q The range of g contains	4) $(-\infty, 0) \cup (0, \infty)$
R The domain of f contains	5) $(-\infty, \frac{e}{e-1}]$
S The domain of g is	6) $(-\infty, 0) \cup (\frac{1}{2}, \frac{e}{e-1})$

The correct option is:

(a) P→ 4; Q→ 2; R→ 1; S→ 1
(b) P→ 3; Q→ 3; R→ 6; S→ 5
(c) P→ 4; Q→ 2; R→ 1; S→ 6
(d) P→ 4; Q→ 3; R→ 6; S→ 5

I Integer Value Correct type

1) Let $f : [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is $(JEEAdv.2014)$
2) The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{log_4 7}}$ is . $(JEEAdv.2018)$
3) Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is $(JEEAdv.2018)$

Section-B JEE Main/ AIEEE

1) The domain of $\sin^{-1}[\log_3(\frac{x}{3}]$ is |2002|
   a) $[1, 9]$
   b) $[-1, 9]$
   c) $[-9, 1]$
   d) $[-9, 1]$
2) The function $f(x) = log(x + \sqrt{x^2 + 1})$, is |2003|
   (a) neither an even nor an odd function
   (b) an even function
   (c) an odd function
   (d) a periodic function.
3) Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$,is |2003|
   (a) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
   (b) $(a, 2)$
   (c) $(-1, 0) \cup (a, 2)$
   (d) $(1, 2) \cup (2, \infty)$.