ASSIGNMENT-1

EE24BTECH11043 - Murra Rajesh Kumar Reddy

1) Let f be a one-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(x) = 1, f(y) \neq 1, f(z) \neq 2$ determine $f^{-1}(1)$. $(1981 - 2Marks)$
2) Let R be the set of real numbers and $f : R \to R$ be such that for all x and y in R $|f(x) - f(y)| \leq |x - y|^3$ .Prove that $f(x)$ is a constant. $(1988 - 2Marks)$
3) Find the natural number $'a'$ for which $\sum_{k=1}^{n} f(a + k) = 16(2^n - 1)$, where the function $'f'$ satisfies the relation $f(x + y) = f(x) f(y)$ for all natural numbers x, y and further $f(1) = 2$. $(1992 - 6Marks)$
4) Let $\{x\}$ and $[x]$ denotes the fractional and integral part of a real number x respectively. Solve $4\{x\} = x + [x]$. $(1994 - 4Marks)$
5) A function $f : IR \to IR$, where IR is the set of real numbers, is defined by

$$f(x) = \frac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2} \tag{1}$$

. Find the interval of values α for which f is onto. Is the function one-to-one for $\alpha = 3$? Justify your answer. $(1996 - 5Marks)$
6) Let $f(x) = Ax^2 + Bx + C$ where A, B, C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers $2A, A + B$ and C are all integers. Conversly, prove that if the numbers $2A, A + B$ and C are all integers then $f(x)$ is an integer whenever x is an integer. $(1998 - 8Marks)$

F Match the Following

1) Let the function defined in column I have domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and range $(-\infty, \infty)$ $(1992 - 2Marks)$

Column I	Column II
	(p) onto but not one-one
	(q) one-one but not onto
(A) $1 + 2x$	(r) one-one and onto
(B) $\tan x$	(s) neither one-one nor onto

2) Let

$$f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6} \tag{2}$$

Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS. $(2007 - 6Marks)$

Column I	Column II
(A) If $-1 < x < 1$, then $f(x)$ satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then $f(x)$ satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then $f(x)$ satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then $f(x)$ satisfies	(s) $f(x) < 1$

This section contains 4 questions. Each questions has 2 matching lists: LIST-I and LIST-II. Four options are representing matching of elements from LIST-I and LIST-II. Only one of these four option corresponding to correct matching.

3) Let $E_1 = \{x \in R : x \neq 1\}$ and $\frac{x}{x-1} > 0$ and $E_2 = \{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$ is a real number $\}$.
(Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$).
Let $f : E_1 \to R$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$ and $g : E_2 \to R$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$ (*JEEAdv*.2018)

LIST-I	LIST-II
	a) $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
	b) $(0, 1)$
(P) The range of f is	c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(Q) The range of g contains	d) $(-\infty, 0) \cup (0, \infty)$
(R) The domain of f contains	e) $(-\infty, \frac{e}{e-1}]$
(S) The domain of g is	f) $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right)$

The correct option is:

a) P→ 4; Q→ 2; R→ 1; S→ 1
b) P→ 4; Q→ 2; R→ 1; S→ 6
c) P→ 3; Q→ 3; R→ 6; S→ 5
d) P→ 4; Q→ 3; R→ 6; S→ 5

I Integer Value Correct type

1) Let $f : [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation

$$f(x) = \frac{10 - x}{10} \tag{3}$$

is (*JEEAdv*.2014)

2) The value of

$$\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}} \tag{4}$$

is . (*JEEAdv*.2018)

3) Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is (*JEEAdv*.2018)

Section-B JEE Main/ AIEEE

1) The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is |2002|
   a) $[1, 9]$
   b) $[-1, 9]$
   c) $[-9, 1]$
   d) $[-9, 1]$

2) The function $f(x) = \log\left(x + \sqrt{x^2 + 1}\right)$, is |2003|
   a) neither an even nor an odd function
   b) an even function
   c) an odd function
   d) a periodic function.

3) Domain of definition of the function

$$f(x) = \frac{3}{4 - x^2} + \log_{10}\left(x^3 - x\right) \tag{5}$$

is |2003|

a) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
b) $(a, 2)$
c) $(-1, 0) \cup (a, 2)$
d) $(1, 2) \cup (2, \infty)$.