

Assignment-1

ee24btech11043-Muura Rajesh Kumar Reddy

- 1) Let f be a one-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(x) = 1, f(y) \neq 1, f(z) \neq 2$ determine $f^{-1}(1)$.
(1981 – 2Marks)
- 2) Let R be the set of real numbers and $f : R \rightarrow R$ be such that for all x and y in R $|f(x) - f(y)| \leq |x - y|^3$. Prove that $f(x)$ is a constant.
(1988 – 2Marks)
- 3) Find the natural number ' a ' for which $\sum_{k=1}^n f(a+k) = 16(2^n - 1)$, where the function ' f ' satisfies the relation $f(x+y) = f(x)f(y)$ for all natural numbers x, y and further $f(1) = 2$.
(1992 – 6Marks)
- 4) Let $\{x\}$ and $[x]$ denotes the fractional and integral part of a real number x respectively. Solve $4\{x\} = x + [x]$.
(1994 – 4Marks)
- 5) A function $f : IR \rightarrow IR$, where IR is the set of real numbers, is defined by $f(x) = \frac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$. Find the interval of values α for which f is onto. Is the function one-to-one for $\alpha = 3$? Justify your answer.
(1996 – 5Marks)
- 6) Let $f(x) = Ax^2 + Bx + C$ where A, B, C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers $2A, A+B$ and C are all integers. Conversely, prove that if the numbers $2A, A+B$ and C are all integers then $f(x)$ is an integer whenever x is an integer.
(1998 – 8Marks)

F MACTH THE FOLLOWING

1. Let the fuction defined in column I have domain $(-\frac{\pi}{2}, \frac{\pi}{2})$ and range $(-\infty, \infty)$ (1992 – 2Marks)

COLUMN I

COLUMN II

- | | |
|---|---|
| <p>(A) $1 + 2x$</p> <p>(B) $\tan x$</p> | <p>(p) onto but not one-one</p> <p>(q) one-one but not onto</p> <p>(r) one-one and onto</p> <p>(s) neither one-one nor onto</p> |
|---|---|

2. Let $f(x) = \frac{x^2-6x+5}{x^2-5x+6}$ (2007 – 6Marks)
- Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

COLUMN I

I. COLUMN II

- | | |
|---|--|
| <p>(A) If $-1 < x < 1$, then $f(x)$ satisfies</p> <p>(B) If $1 < x < 2$, then $f(x)$ satisfies</p> <p>(C) If $3 < x < 5$, then $f(x)$ satisfies</p> <p>(D) If $x > 5$, then $f(x)$ satisfies</p> | <p>(p) $0 < f(x) < 1$</p> <p>(q) $f(x) < 0$</p> <p>(r) $f(x) > 0$</p> <p>(s) $f(x) < 1$</p> |
|---|--|

This section contains 4 questions. Each questions has 2 matching lists: LIST-I and LIST-II. Four options are given representing matching of elements from LIST-I and LIST-II. Only one of these four option corresponding to a correct matching.

3. Let $E_1 = \{x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0\}$ and $E_2 = \{x \in E_1 : \sin^{-1}(\log_e(\frac{x}{x-1})) \text{ is a real number } \}$.

(Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $[-\frac{\pi}{2}, \frac{\pi}{2}]$).

Let $f : E_1 \rightarrow R$ be the function defined by $f(x) = \log_e(\frac{x}{x-1})$ and $g : E_2 \rightarrow R$ be the function defined by $g(x) = \sin^{-1}(\log_e(\frac{x}{x-1}))$ (JEEAdv.2018)

II. LIST-I

III. LIST-II

- | | |
|---|--|
| <p>P The range of f is</p> <p>Q The range of g contains</p> <p>R The domain of f contains</p> <p>S The domain of g is</p> | <p>1) $(-\infty, \frac{1}{1-e}] \cup [\frac{e}{e-1}, \infty)$</p> <p>2) $(0, 1)$</p> <p>3) $[-\frac{1}{2}, \frac{1}{2}]$</p> <p>4) $(-\infty, 0) \cup (0, \infty)$</p> <p>5) $(-\infty, \frac{e}{e-1}]$</p> <p>6) $(-\infty, 0) \cup (\frac{1}{2}, \frac{e}{e-1})$</p> |
|---|--|

The correct option is:

- | | |
|--|--|
| (a) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 1$ | (b) $P \rightarrow 3; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$ |
| (c) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 6$ | (d) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$ |

I Integer Value Correct type

- | | |
|---|---|
| <p>1) Let $f : [0, 4\pi] \rightarrow [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is (JEEAdv.2014)</p> | <p>2) The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is . (JEEAdv.2018)</p> <p>3) Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is (JEEAdv.2018)</p> |
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Section-B JEE Main/ AIEEE

- 1) The domain of $\sin^{-1}[\log_3(\frac{x}{3})]$ is [2002]
- a) $[1, 9]$
 - b) $[-1, 9]$
 - c) $[-9, 1]$
 - d) $[-9, 1]$
- 2) The function $f(x) = \log(x + \sqrt{x^2 + 1})$, is [2003]
- (a) neither an even nor an odd function
 - (b) an even function
 - (c) an odd function
 - (d) a periodic function.
- 3) Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is [2003]
- (a) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
 - (b) $(a, 2)$
 - (c) $(-1, 0) \cup (a, 2)$
 - (d) $(1, 2) \cup (2, \infty)$.