

# ASSIGNMENT-1

EE24BTECH11043 - Murra Rajesh Kumar Reddy

- 6) Let  $f$  be a one-one function with domain  $\{x, y, z\}$  and range  $\{1, 2, 3\}$ . It is given that exactly one of the following statements is true and the remaining two are false  $f(x) = 1, f(y) \neq 1, f(z) \neq 2$  determine  $f^{-1}(1)$ . (1981 – 2Marks)
- 7) Let  $R$  be the set of real numbers and  $f : R \rightarrow R$  be such that for all  $x$  and  $y$  in  $R$   $|f(x) - f(y)| \leq |x - y|^3$ . Prove that  $f(x)$  is a constant. (1988 – 2Marks)
- 8) Find the natural number ' $a$ ' for which  $\sum_{k=1}^n f(a+k) = 16(2^n - 1)$ , where the function ' $f$ ' satisfies the relation  $f(x+y) = f(x)f(y)$  for all natural numbers  $x, y$  and further  $f(1) = 2$ . (1992 – 6Marks)
- 9) Let  $\{x\}$  and  $[x]$  denotes the fractional and integral part of a real number  $x$  respectively. Solve  $4\{x\} = x + [x]$ . (1994 – 4Marks)
- 10) A function  $f : IR \rightarrow IR$ , where  $IR$  is the set of real numbers, is defined by

$$f(x) = \frac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$$

. Find the interval of values  $\alpha$  for which  $f$  is onto. Is the function one-to-one for  $\alpha = 3$ ? Justify your answer. (1996 – 5Marks)

- 11) Let  $f(x) = Ax^2 + Bx + C$  where  $A, B, C$  are real numbers. Prove that if  $f(x)$  is an integer whenever  $x$  is an integer, then the numbers  $2A, A + B$  and  $C$  are all integers. Conversely, prove that if the numbers  $2A, A + B$  and  $C$  are all integers then  $f(x)$  is an integer whenever  $x$  is an integer. (1998 – 8Marks)

## F MATCH THE FOLLOWING

- 1) Let the function defined in column I have domain  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  and range  $(-\infty, \infty)$  (1992 – 2Marks)

## COLUMN I

## COLUMN II

- (A)  $1 + 2x$   
(B)  $\tan x$

- (p) onto but not one-one  
(q) one-one but not onto  
(r) one-one and onto  
(s) neither one-one nor onto

- 2) Let

$$f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$$

Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the  $4 \times 4$  matrix given in the ORS. (2007 – 6Marks)

## COLUMN I

## COLUMN II

- (A) If  $-1 < x < 1$ , then  $f(x)$  satisfies  
(B) If  $1 < x < 2$ , then  $f(x)$  satisfies  
(C) If  $3 < x < 5$ , then  $f(x)$  satisfies  
(D) If  $x > 5$ , then  $f(x)$  satisfies

- (p)  $0 < f(x) < 1$   
(q)  $f(x) < 0$   
(r)  $f(x) > 0$   
(s)  $f(x) < 1$

This section contains 4 questions. Each questions has 2 matching lists: LIST-I and LIST-II. Four options are representing matching of elements from LIST-I and LIST-II. Only one of these four option corresponding to correct matching.

- 3) Let  $E_1 = \{x \in R : x \neq 1\}$  and  $\frac{x}{x-1} > 0$  and  $E_2 = \{x \in E_1 : \sin^{-1}(\log_e(\frac{x}{x-1})) \text{ is a real number } \}$ .  
( Here, the inverse trigonometric function  $\sin^{-1} x$  assumes values in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$  ).

Let  $f : E_1 \rightarrow R$  be the function defined by  $f(x) = \log_e(\frac{x}{x-1})$  and  $g : E_2 \rightarrow R$  be the function defined by  $g(x) = \sin^{-1}(\log_e(\frac{x}{x-1}))$  (JEEAdv.2018)

## LIST-I

## LIST-II

- (P) The range of  $f$  is  
(Q) The range of  $g$  contains  
(R) The domain of  $f$  contains  
(S) The domain of  $g$  is

- 1)  $(-\infty, \frac{1}{1-e}] \cup [\frac{-e}{e-1}, \infty)$   
2)  $(0, 1)$   
3)  $[-\frac{1}{2}, \frac{1}{2}]$   
4)  $(-\infty, 0) \cup (0, \infty)$   
5)  $(-\infty, \frac{e}{e-1}]$   
6)  $(-\infty, 0) \cup (\frac{1}{2}, \frac{e}{e-1})$

The correct option is:

- (a)  $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 1$   
(b)  $P \rightarrow 3; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$   
(c)  $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 6$   
(d)  $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$

## I INTEGER VALUE CORRECT TYPE

- 1) Let  $f : [0, 4\pi] \rightarrow [0, \pi]$  be defined by  $f(x) = \cos^{-1}(\cos x)$ . The number of points  $x \in [0, 4\pi]$  satisfying the equation

$$f(x) = \frac{10 - x}{10}$$

is

(JEEAdv.2014)

- 2) The value of

$$\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$$

is .

(JEEAdv.2018)

- 3) Let  $X$  be a set with exactly 5 elements and  $Y$  be a set with exactly 7 elements. If  $\alpha$  is the number of one-one functions from  $X$  to  $Y$  and  $\beta$  is the number of onto functions from  $Y$  to  $X$ , then the value of  $\frac{1}{5!}(\beta - \alpha)$  is (JEEAdv.2018)

## SECTION-B JEE MAIN/ AIEEE

- 1) The domain of  $\sin^{-1}[\log_3\left(\frac{x}{3}\right)]$  is [2002]

- a)  $[1, 9]$
- b)  $[-1, 9]$
- c)  $[-9, 1]$
- d)  $[-9, 1]$

- 2) The function  $f(x) = \log(x + \sqrt{x^2 + 1})$ , is [2003]

- a) neither an even nor an odd function
- b) an even function
- c) an odd function
- d) a periodic function.

- 3) Domain of definition of the function

$$f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$$

is

[2003]

- a)  $(-1, 0) \cup (1, 2) \cup (2, \infty)$
- b)  $(a, 2)$
- c)  $(-1, 0) \cup (a, 2)$
- d)  $(1, 2) \cup (2, \infty)$ .