

## 1 Machine Configuration

Attribute	Value
Hostname	barolo.cs.wisc.edu
OS	Ubuntu 20.04.5 LTS
Compiler	g++ (gcc version 9.4.0), OpenMP version 4.5
Compile command	<i>(Makefile included in root directory of zip file)</i> <code>icc *.cpp -Wall -O3 -o conjugate_gradients_mkl -qopenmp -mkl</code>
CPU	AMD EPYC 7451 24-Core Processor
Cache configuration	L1-3MiB, L2-24MiB, L3-128MiB
Memory bandwidth	~150 GiB/s ( <a href="#">Source,2</a> ). The machine has 256GB of memory spread across (8 out of 16) slots each housing 32GB stick. This information was retrieved using <code>sudo lshw -class memory</code>
Number of threads	24 cores * 2 threads/core = 48 threads
Dependencies	GCC, OpenMP, ICC (source /s/intelcompilers-2019/bin/iccvars.sh intel64)

Changes to the base code (LaplaceSolver\_I\_5)

- Added MKL variants for InnerProduct, Norm and Copy kernels
- Instrumented timers for each invocation for every kernel

```
// Algorithm : Line 2
timerLaplacian1.Restart(); ComputeLaplacian(matrix, x, z); timerLaplacian1.Pause();
timerSaxpy1.Restart(); Saxpy(z, f, r, -1); timerSaxpy1.Pause();
timerNorm1.Restart(); float nu = Norm(r); timerNorm1.Pause();

// Algorithm : Line 3
if (nu < nuMax) return;

// Algorithm : Line 4
timerCopy1.Restart(); Copy(r, p); timerCopy1.Pause();
timerIP1.Restart(); float rho=InnerProduct(p, r); timerIP1.Pause();
```

Fig 1: Excerpt to illustrate the instrumented timers for per-kernel measurements

## 2 MKL variant replacement for Conjugate Gradient

I added the MKL variants for InnerProduct, Norm and Copy kernels in the Conjugate Gradient algorithm as shown in the code snippets below:

```
35 float InnerProduct(const float (&x)[XDIM][YDIM][ZDIM], const float (&y)[XDIM][YDIM][ZDIM])
36 {
37     #ifdef DO_NOT_USE_MKL
38         double result = 0.;
39         #pragma omp parallel for reduction(+:result)
40         for (int i = 1; i < XDIM-1; i++)
41             for (int j = 1; j < YDIM-1; j++)
42                 for (int k = 1; k < ZDIM-1; k++)
43                     result += (double) x[i][j][k] * (double) y[i][j][k];
44         return (float) result;
45     #else
46         return
47         cblas_sdot(
48             XDIM * YDIM * ZDIM, // Specifies the number of elements in vectors x and y
49             &x[0][0][0], // Array, size at least (1 + (n-1)*abs(incx))
50             1, // Specifies the increment for the elements of x
51             &y[0][0][0], // Array, size at least (1 + (n-1)*abs(incy))
52             1 // Specifies the increment for the elements of y
53         );
54     #endif
55 }
```

Fig 2: Inner product MKL variant using *cblas\_sdot*

```
10 float Norm(const float (&x)[XDIM][YDIM][ZDIM])
11 {
12     #ifdef DO_NOT_USE_MKL
13         float result = 0.;
14         #pragma omp parallel for reduction(max:result)
15         for (int i = 1; i < XDIM-1; i++)
16             for (int j = 1; j < YDIM-1; j++)
17                 for (int k = 1; k < ZDIM-1; k++)
18                     result = std::max(result, std::abs(x[i][j][k]));
19         return result;
20     #else
21         int idx =
22         cblas_isamax( // Rajesh: Replaced for MKL variant
23             XDIM * YDIM * ZDIM, // Specifies the number of elements
24             &x[0][0][0], // Array, size at least (1 + (n-1)*abs(incx))
25             1 // Specifies the increment for the elements of x
26         );
27         // Retrieve element from index
28         int i = idx / (XDIM * YDIM);
29         int j = (idx - (i * XDIM * YDIM)) / XDIM;
30         int k = idx - (i * XDIM * YDIM) - (j * XDIM);
31         return std::abs(x[i][j][k]);
32     #endif
33 }
```

Fig 3: Norm MKL variant using *cblas\_isamax*

```

8 void Copy(const float (&x)[XDIM][YDIM][ZDIM], float (&y)[XDIM][YDIM][ZDIM])
9 {
10 #ifdef DO_NOT_USE_MKL
11 #pragma omp parallel for
12     for (int i = 1; i < XDIM-1; i++)
13         for (int j = 1; j < YDIM-1; j++)
14             for (int k = 1; k < ZDIM-1; k++)
15                 y[i][j][k] = x[i][j][k];
16 #else
17     cblas_scopy( // Rajesh: Replaced for MKL variant
18                 XDIM * YDIM * ZDIM, // Specifies the number of elements in vectors x and y
19                 &x[0][0][0], // Array, size at least (1 + (n-1)*abs(incx))
20                 1, // Specifies the increment for the elements of x
21                 &y[0][0][0], // Array, size at least (1 + (n-1)*abs(incy))
22                 1 // Specifies the increment for the elements of y
23             );
24 #endif
25 }
26

```

Fig 3: Copy MKL variant using *cblas\_scopy*

The above changes were verified for functional correctness (in Fig 4) by ensuring consistency in the residual norm values. The performance results of these optimizations are presented in Fig 5.

```

Conjugate Gradients terminated after 256 iterations; residual norm (nu) = 0.0009755
[Total Laplacian 1 Time : 1699.12ms]
[Total Laplacian 2 Time : 5174.41ms]
[Total Copy 1 Time : 6.78585ms]
[Total Copy 2 Time : 133.596ms]
[Total InnerProduct 1 Time : 1.09858ms]
[Total InnerProduct 2 Time : 278.671ms]
[Total InnerProduct 3 Time : 195.368ms]
[Total Norm 1 Time : 1.93664ms]
[Total Norm 2 Time : 359.697ms]
[Total Saxpy 1 Time : 29.794ms]
[Total Saxpy 2 Time : 251.9ms]
[Total Saxpy 3 Time : 1.43302ms]
[Total Saxpy 4 Time : 363.156ms]
[Total Saxpy 5 Time : 836.455ms]

```

Fig 4: Functional verification upon MKL changes using OMP\_THREADS=48

	A	B	C	D	E	F	G	H
1	Invocation count	Kernel	Line# in algorithm	Non-MKL		MKL		MKL Speedup Formula: [(E-G)/G]
2				Time (ms) per- iter with OMP threads = 1	Time (ms) per-iter with OMP threads = 48	Time (ms) per- iter with OMP threads = 1	Time (ms) per-iter with OMP threads = 48	
3	1	ComputeLaplacian	2	0.70090625	0.919644531	4.730163934	6.6371875	
4	2	ComputeLaplacian	6	97.52265625	28.19394531	101.8438525	20.21253906	
5	1	<b>Copy (MKL: scopy)</b>	<b>4</b>	0.087192578	0.026504453	0.088671311	0.026507227	0%
6	2	<b>Copy (MKL: scopy)</b>	<b>13</b>	8.748085938	1.726910156	8.638729508	0.521859375	231%
7	1	<b>InnerProduct (MKL: sdot)</b>	<b>4</b>	0.075975391	0.006111484	0.025920902	0.004291328	42%
8	2	<b>InnerProduct (MKL: sdot)</b>	<b>6</b>	19.03613281	2.297605469	6.310204918	1.088558594	111%
9	3	<b>InnerProduct (MKL: sdot)</b>	<b>13</b>	18.62167969	1.928109375	5.932868852	0.76315625	153%
10	1	<b>Norm (MKL: isamax)</b>	<b>2</b>	0.032246445	0.00236809	0.035829713	0.007565	-69%
11	2	<b>Norm (MKL: isamax)</b>	<b>8</b>	7.56546875	1.060300781	8.888483607	1.405066406	-25%
12	1	Saxpy	2	0.100804297	0.069908594	0.118718033	0.116382813	
13	2	<b>Saxpy (MKL: saxpy)</b>	<b>8</b>	9.368359375	2.19846875	8.895245902	0.983984375	123%
14	3	Saxpy	9-12	0.053708984	0.008149297	0.043254098	0.005597734	
15	4	<b>Saxpy (MKL: saxpy)</b>	<b>16</b>	9.231171875	2.184070313	8.807213115	1.418578125	54%
16	5	Saxpy	16	14.65300781	3.393824219	12.08270492	3.267402344	
17	Total runtime as sum of kernels:			185.7973964	44.01592082	166.4418613	36.45867613	21%

Fig 5: Performance results from using MKL optimizations for Conjugate Gradient

The MKL variants of the kernels that were replaced are indicated in bold on “column B” in Fig 5. The bottomline of the above results is that we see an overall speed-up in execution time of about 21%. Individually, the MKL counterparts of the kernels consistently outperforms baseline except in the case of Norm. This can likely be attributed to the fact that *cblas\_i?amax* is more generic in function than we need for our purposes in the algorithm. The variations in MKL performance between independent invocations of the same kernel (say InnerProduct) can be attributed to three factors (i) statistical variation due to measurements (ii) background processes running on the system (iii) variations in the input data for each of the invocations.