Nonlinear Dynamics: Mathematical and Computational Approaches

Lead instructor: Liz Bradley

Important things to remember when taking tests:

- **Tests auto-save your answers!** Don't worry if you close the page or walk away from the computer, all of your selections will be remembered when you come back.
- When you are finished, click the "Submit" button at the bottom of the test. **Be careful: submitting tests cannot be undone.**

Instructions

You may use any course materials, websites, books, computer programs, calculators, etc. for this test. Just don't ask another person for the answers or share your answers with other people. Be aware that simply typing the question text into google is unlikely to get you directly to the right answer; you're going to have to read what you find there in order to extract that answer, and the course videos are probably a faster way to do that.

"Experts" notes clarify situations that haven't been covered in this course, but that may introduce subtleties into the exam answers. Do not worry about them unless you understand the terms and issues in those notes.

If you have questions about this test, please email us at nonlinear@complexityexplorer.org rather than posting on the forum.

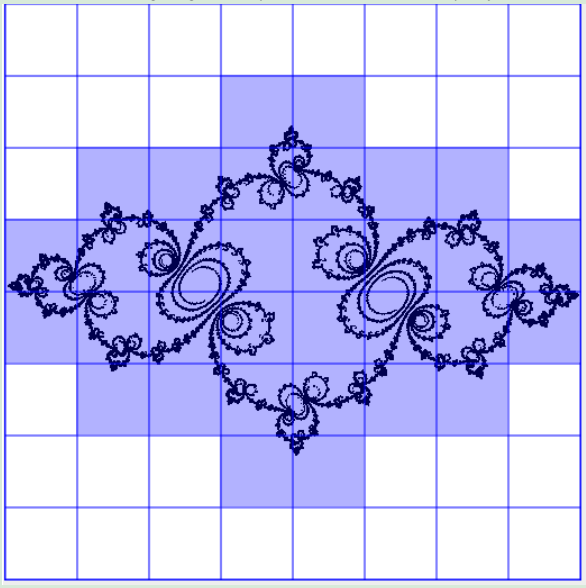
Question 1

What's the difference between capacity dimension and correlation dimension?

- Capacity dimension measures the number of boxes needed to cover the points in a set, whereas correlation dimension measures the clustering of those points.
- Calculating correlation dimension is much slower than calculating capacity dimension.
- ✖ Both of them measure exactly the same thing; the only difference is the algorithm used to calculate them.
- Sauer et al. proposed the correlation dimension, whereas Grassberger and Procaccia proposed the box-counting dimension.

Question 2

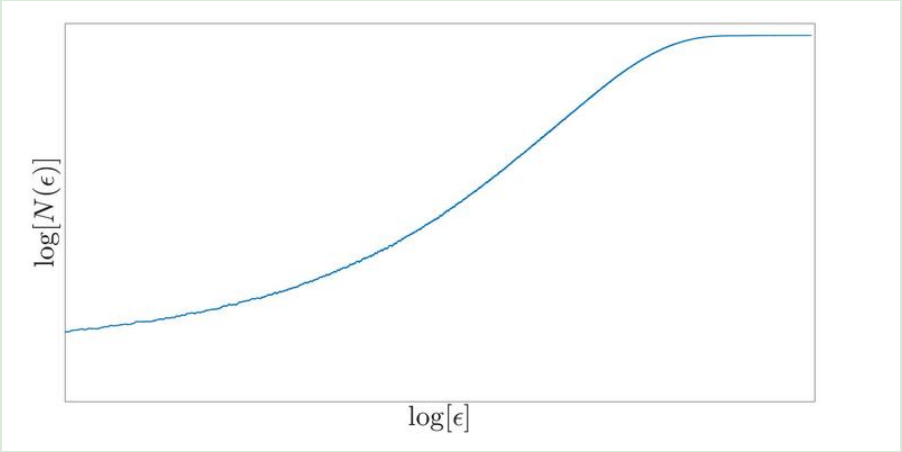
Consider the following image of a step in the calculation of the capacity dimension of the Julia set that was discussed in this unit:



What is $N(\epsilon)$ in this image?

- 12
- 0.125^2
- 0.125
- ✔ 32

Question 3



Does this curve have a scaling region?

- ✔ Yes
- No

Question 4

How could one compute the capacity dimension of a set from the curve in question 3?

- The y intercept of a line fitted to the scaling region (if one exists).
- ✖ The slope of a line fitted to the scaling region (if one exists).
- The lowest value of $\log N(\epsilon)$ where the curve flattens out
- The lowest value of $\log N(\epsilon)$ where the curve becomes a line
- None of the above - something's really wrong with this curve!

Question 5

When calculating capacity dimension, why does the curve on a plot of $\log(N(\epsilon))$ flatten out for high values of $\log(1/\epsilon)$?

- Because one ϵ-ball is covering the whole set.
- ✔ Because each ϵ-ball is covering a single point.
- Because there's a power-law relationship between these two quantities.

Question 6

Download amplitude.dat. Use your delay-coordinate embedding code from the unit homework to embed this time series using m=2 and $\tau = 8$. Use your capacity dimension calculator from the unit 9 homework to calculate N(0.5) for this embeded trajectory. What do you get?

- 0.5
- 4516
- 2089
- ✖ 664
- 944

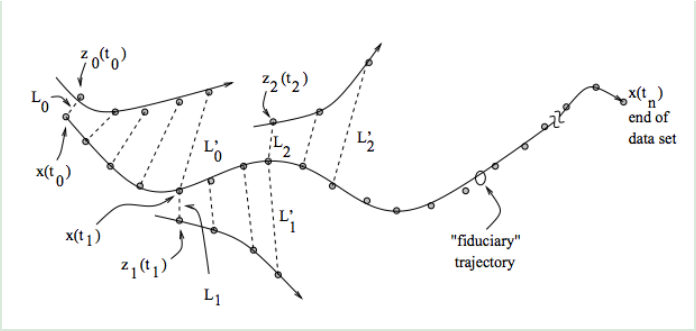
Question 7

The correlation sum $C(\epsilon)$ is proporional to $\epsilon^{d_{correlation}}$.

- ✔ True
- False

Question 8

What algorithm for calculating Lyapunov exponents from data is schematized here?



- Kantz's algorithm
- ✔ Wolf's algorithm
- Takens's algorithm

Question 9

Wolf's algorithm for calculating Lyapunov exponents requires that you know the system equations.

- True
- ✔ False

Question 10

You can calculate Lyapunov exponents directly from scalar data from a dynamical system using Kantz's algorithm, without any other preprocessing steps.

- True
- ✔ False

Question 11

This question and the two that follow concern the difference between Wolf's algorithm for calculating Lyapunov exponents and Kantz's algorithm for calculating Lyapunov exponents. Please mark each assertion as true or false.

Wolf's algorithm follows a number of trajectory points over time and uses their paths to determine how the dynamical system stretches the state space, whereas Kantz's algorithm just follows *pairs* of trajectory points to accomplish the same purpose.

- True
- ✔ False

Question 12

Kantz's algorithm works with equations whereas Wolf's algorithm works with data.

- ✖ True
- False

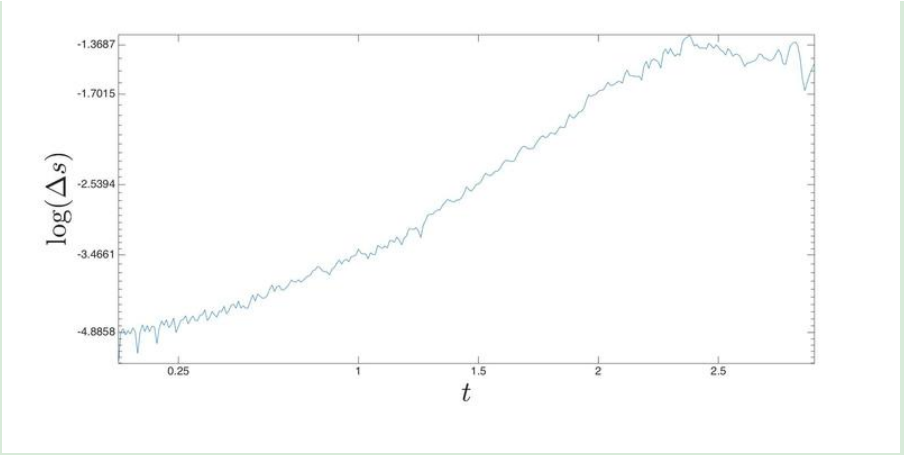
Question 13

Wolf's algorithm gives you all n of the Lyapunov exponents of an n-dimensional dynamical system, whereas Kantz's algorithm only gives you the largest one.

- True
- ✔ False

Question 14

Consider the following plot of the stretching factor produced by Kantz's algorithm on an experimental time series. Real-world effects (noise, etc.) make this plot less clean than the ones that you have seen in the videos and quizzes.

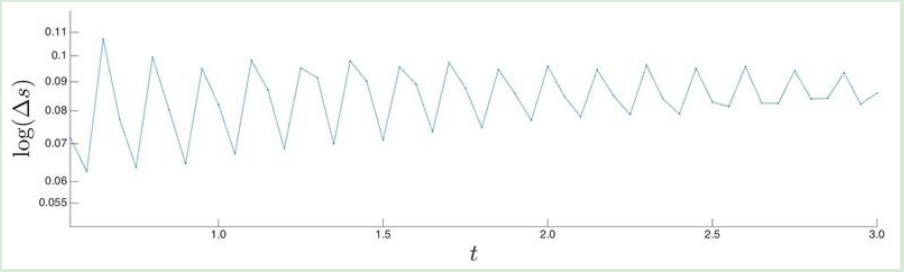


What is the Lyapunov exponent of this trajectory?

- ✔ Roughly 1.7646
- Roughly -1.7646
- Roughly -4.8858
- Roughly -1.4
- Roughly 0.25
- Roughly 2.25

Question 15

Consider the following plot of the stretching factor produced by Kantz's algorithm on a different experimental time series.



What is the Lyapunov exponent of this trajectory?

- Roughly 0.02
- Roughly -0.02
- Roughly 0.07
- Roughly 0.1
- Roughly 0.4
- Roughly 3.0
- ✔ Undefined; there's no scaling region in this plot.

Question 16

It's just as easy, from the standpoint of algorithmic complexity and data requirements, to calculate λ_2 from data as it is to calculate λ_1.

- True
- ✔ False

Question 17

Low-pass filters remove all the low-frequency components of a scalar time-series data set.

- True
- ✔ False

Question 18

The Farmer/Sidorowich filtering strategy leverages the attractor topology to remove noise.

- True
- ✔ False

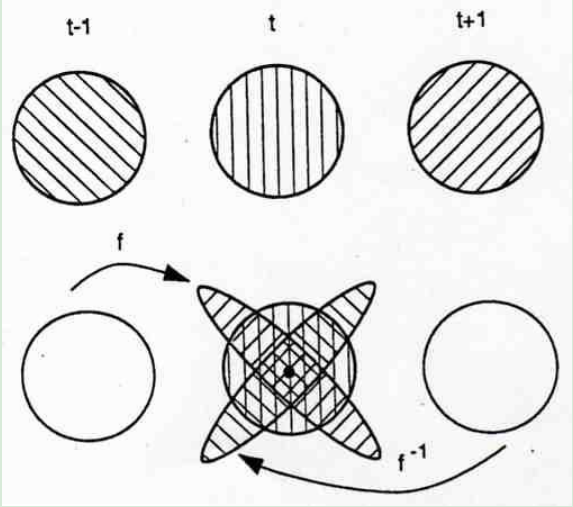
Question 19

Chaos and noise are easy to tell apart.

- ○ True
- ✔ ● False

Question 20

This is a schematic of a strategy that leverages the stable and unstable manifold structure of an attractor to remove noise from a measured trajectory.



- ✔ ● True
- ○ False

Question 21

Which of these properties can be exploited to effectively remove noise from a chaotic signal?

- ○ A. The attractor topology.
- ○ B. The stable and unstable manifolds.
- ○ C. The frequency spectrum.
- ○ All of A-C above.
- ○ None of A-C above.
- ✔ ● A & B above.
- ○ A & C above.
- ○ B & C above.

You got 17 out of 21 questions correct