Nonlinear Dynamics: Mathematical and Computational Approaches

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✓ 4.6 Flows II: Unit test » Take unit 4 test Download Exam PDF

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Important things to remember when taking tests:

- **Tests auto-save your answers!** Don't worry if you close the page or walk away from the computer, all of your selections will be remembered when you come back.
- When you are finished, click the "Submit" button at the bottom of the test. **Be careful: submitting tests cannot be undone.**

Instructions

You may use any course materials, websites, books, computer programs, calculators, etc. for this test. Just don't ask another person for the answers or share your answers with other people. Be aware that simply typing the question text into google is unlikely to get you directly to the right answer; you're going to have to read what you find there in order to extract that answer, and the course videos are probably a faster way to do that.

"Experts" notes clarify situations that haven't been covered in this course, but that may introduce subtleties into the exam answers. Do not worry about them unless you understand the terms and issues in those notes.

If you have questions about this test, please email us at nonlinear@complexityexplorer.org rather than posting on the forum.

Question 1

How many fixed points does the damped pendulum have?

- A. One and it's stable.
- B. Two: one stable and one unstable.
- ✔ C. An infinite number of both stable and unstable ones.
- D. One stable one and an infinite number of unstable ones.
- E. None

Question 2

What is the definition of an unstable fixed point of a dissipative dynamical system?

- A. A point in state space to which a trajectory converges.
- B. A point in state space where the dynamics are stationary (i.e., if you put it there, it'll stay there).
- C. A point in state space where the dynamics are stationary and around which any perturbation will shrink.
- ✔ D. A point in state space where the dynamics are stationary and around which any perturbation will grow.
- E. None of the above.

Question 3

What are possible sign configurations of the eigenvalues of an unstable fixed point in a 2D system (disregarding complex ones, which we'll get to later)?

- A. Both positive
- B. Both negative
- C. One positive, one negative
- ✔ D. Either A or C above
- E. Either B or C above
- F. Either A or B above

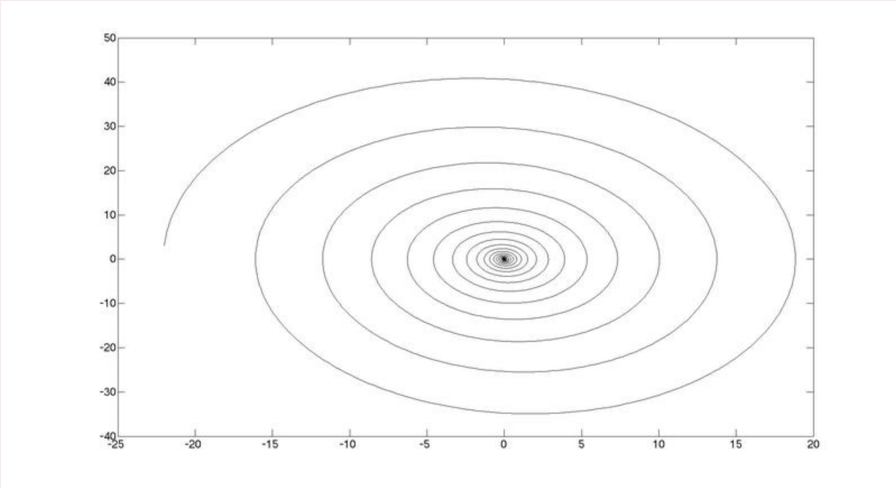
Question 4

Only dissipative dynamical systems have attractors.

- ✔ True
- False

Question 5

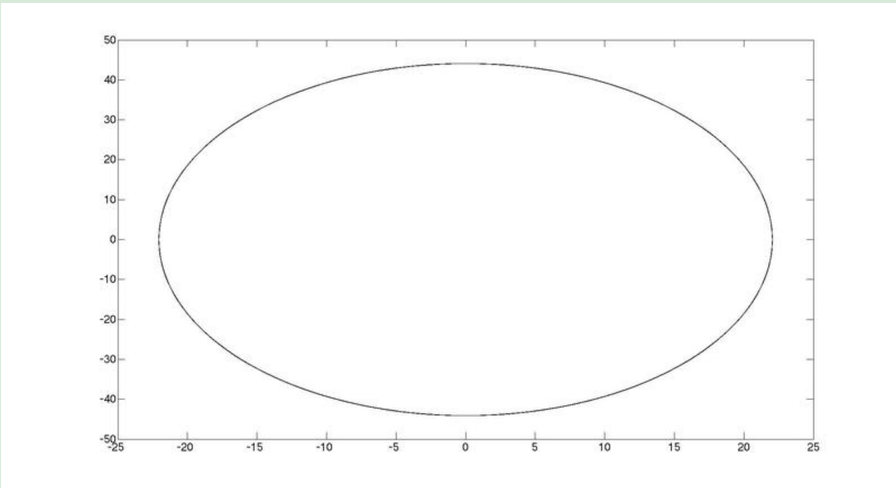
What dynamical system is this state-space trajectory from? (the initial condition here is at (-22,2))



- A. The simple harmonic oscillator (SHO)
- B. The pendulum
- C. The damped SHO
- D. The damped pendulum

Question 6

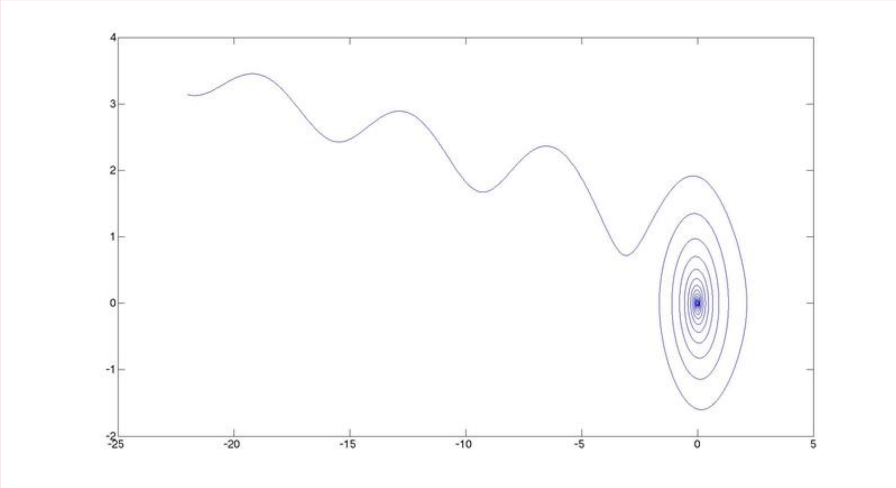
What dynamical system is this state-space trajectory from?



- A. The SHO
- B. The damped SHO
- C. The damped pendulum

Question 7

What dynamical system is this state-space trajectory from? (the initial condition is at (-22, 3.2))



- A. The SHO
- B. The pendulum
- ✖ C. The damped SHO
- **D. The damped pendulum**

Question 8

What are possible sign configurations of the eigenvalues of a saddle point in a 2D system? *(Experts*: disregard complex ones.)

- A. Both positive
- B. Both negative
- ✔ C. One positive and one negative
- D. A and B above
- E. B and C above
- F. A and C above

Question 9

What is the shape of this matrix? [1 2 3 4 5]

- A. 5x1
- ✔ B. 1x5
- C. undefined
- D. It isn't a matrix

Question 10

How many eigenvalues does a 3x3 matrix have? (*Experts*: count all complex and repeated ones individually.)

- A. 1
- B. 2
- ✔ C. 3
- D. 4
- E. 6
- Undefined

Question 11

What are the eigenvalues of:

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Compute these by hand — i.e, do not use an eigenvalue calculator.

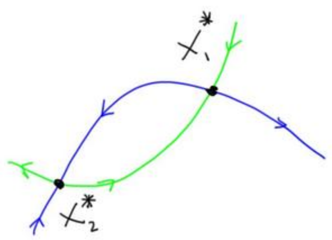
- ✔ $s_1 = -1, s_2 = 5$
- $s_1 = 1, s_2 = -5$
- $s_1 = 2, s_2 = 5$

Question 12

Why is it a bad idea to use linear mathematics on a nonlinear system?

- You may get the wrong answer.
- Linearization is a good approximation of a nonlinear system, but only locally.
- Linearization makes the math easy, but you shouldn't use it where it's not valid.
- ✔ All of the above.

Question 13



In the picture above, x_1^* and x_2^* are fixed points. Are there any heteroclinic orbits in that picture?

- ✔ Yes
- No

Question 14

In the picture in question 13, the stable manifold of x_1^* includes...

- A. The blue curve (to the right of x_2^*)
- ✔ B. The green curve (to the right of x_2^*)
- C. Neither A nor B.
- D. Both A and B.

Question 15

In the picture in question 13, the unstable manifold of x_2^* includes...

- ✖ A. The blue curve (to the right of x_2^*)
- B. The green curve (to the right of x_2^*)
- C. Neither A nor B.
- D. Both A and B.

Question 16

Stable and unstable manifolds are locally tangent to the stable and unstable eigenvectors near the fixed point.

- ✔ True
- False

Question 17

If a dynamical system has a periodic orbit, there is no stable manifold transverse to that periodic orbit.

- True
- ✔ False

Question 18

Why was the title of Lorenz's 1963 paper ("Deterministic Nonperiodic Flow") provocative?

- At that time, people didn't think that flows could be deterministic.
- At that time, people didn't think that flows could have periodic dynamics.
- At that time, people thought that nonperiodic dynamics didn't exist.
- ✔ At that time, people thought that deterministic flows could only have periodic behavior.

Question 19

What system do the Lorenz equations model?

- ✔ A chunk of fluid heated from below.
- A mass on a nonlinear spring.
- The orbit of Pluto's moon, Charon.
- The turbulence patterns in an eddy in a stream.

Question 20

What is the dimension of the Lorenz equations?

- 1
- 2
- ✔ 3
- 4
- Undefined

Question 21

Changing the r parameter in the Lorenz equations does not cause bifurcations in the dynamics.

- True
- ✔ False

You got **18** out of **21** questions correct