Risk Premium Rate

The second source of value created by insurance results from the reduction in the volatility of returns for insurance buyers. Investors will demand a lower return from firms with reduced earning volatility.

As we did in measuring the value through capital deployment, we first calculate the Sharpe Ratio without insurance (i.e., the base case) and then compare to the Sharpe Ratio for the insurance buyer. In this analysis, we focus on the change in Sharpe Ratio.

$$\text{Sharpe Ratio} = \frac{E(R) - r_f}{\sigma_R}$$

The Sharpe Ratio relates the risk premium required (numerator) to return volatility (denominator). More specifically, the Sharpe Ratio indicates the risk premium required for every unit of volatility.

Sharpe Ratio without insurance

As with the discussion in Section capital-deployment, we use the naught subscript to represent the base case. We can calculate $E(R_0)$, $var(R_0)$ as presented below under the assumption that the risk free rate, r_f, is fixed and that that X and r_c are independent.

$$\begin{aligned}
E(R_0) &= E(\frac{r_c \times CW_0 + r_f \times CX_0 - X}{C}) \\
&= \frac{CW_0 \times E(r_c)}{C} - \frac{E(X)}{C} + \frac{r_f \times CX_0}{C} \\
&= \frac{CW_0 \times E(r_c)}{C} + \frac{r_f \times CX_0}{C} \\
&= \frac{(C - CX_0) \times E(r_c)}{C} + \frac{r_f \times CX_0}{C}
\end{aligned}$$

$$\begin{aligned}
var(R_0) &= var(\frac{r_c \times CW_0 + r_f \times CX_0 - X}{C}) \\
&= \frac{CW_0^2 \times var(r_c)}{C^2} + \frac{var(X)}{C^2} - \frac{2 \times CW_0 \times cov(X, r_c)}{C^2} \\
&= \frac{(C - CX_0)^2 \times var(r_c)}{C^2} + \frac{var(X)}{C^2}
\end{aligned}$$

$$\begin{aligned}
\text{Sharpe Ratio}_0 &= \frac{E(R_0) - r_f}{\sigma_{R_0}} \\
&= \frac{(C - CX_0) \times E(r_c) + r_f \times CX_0 - r_f \times C}{C} / (\frac{(C - CX_0)^2 \times var(r_c) + var(x)}{C^2})^{0.5} \\
&= \frac{(C - CX_0) \times (E(r_c) - r_f)}{((C - CX_0)^2 \times var(r_c) + var(x))^{0.5}}
\end{aligned}$$

Sharpe Ratio With Insurance

We can also calculate the Sharpe Ratio for the insurance buyer, using the subsript I to represent this case. We calculate $E(r_I)$ and $var(r_I)$

$$
\begin{aligned}
E(R_I) &= E(\frac{r_c \times (CW_I - P) + r_f \times CX_I - X_{ret} - P}{C}) \\
&= \frac{(CW_I - P) \times E(r_c)}{C} - \frac{E(X_{ret})}{C} + \frac{r_f \times CX_I - P}{C} \\
&= \frac{(CW_I - P) \times E(r_c))}{C} + \frac{r_f \times CX_I - P}{C} \\
&= \frac{(C - P) \times E(r_c)) - P}{C}
\end{aligned}
$$

$$
\begin{aligned}
var(R_I) &= var(\frac{r_c \times (CW_I - P) + r_f \times CX_I - X_{ret} - P}{C}) \\
&= \frac{(CW_I - P)^2 \times var(r_c)}{C^2} + \frac{var(X_{ret})}{C^2} - \frac{2 \times (CW_I - P) \times cov(X_{ret}, r_c)}{C^2} \\
&= \frac{(C - P)^2 \times var(r_c)}{C^2} + \frac{var(X_{ret})}{C^2}
\end{aligned}
$$

Now we calculate the Sharpe Ratio under the insurance case:

$$
\begin{aligned}
\text{Sharpe Ratio}_I &= \frac{E(R_I) - r_f}{\sigma_{R_0}} \\
&= \frac{(C - P) \times E(r_c)) - P - C \times rf}{C} / (\frac{(C - P)^2 \times var(r_c) + var(X_{ret})}{C^2})^{0.5} \\
&= \frac{C \times (E(r_c) - r_f) - P \times (1 + E(r_c))}{((C - P)^2 \times var(r_c) + var(X_{ret}))^{0.5}}
\end{aligned}
$$

Approach 1

The next step should be the comparison of Sharpe Ratios under 2 cases. However, due to the complexity of the Sharpe Ratios formula, it would be hard to compare the Sharpe Ratios of 2 cases directly to get a clear result. Instead, I compare the variance.

$$
\begin{aligned}
var(R_1) &< var(R_0) \\
\frac{(C - P)^2 \times var(r_c) + var(X_{ret})}{C^2} &< \frac{(C - CX_0)^2 \times var(r_c) + var(X)}{C^2} \\
\frac{(C - P)^2 \times var(r_c) + var(X_{ret})}{C^2} &< \frac{(C - CX_0)^2 \times var(r_c) + var(X)}{C^2} \\
\frac{((C - P)^2 - (C - CX_0)^2) \times var(r_c)}{C^2} &< \frac{var(X) - var(X_{ret})}{C^2}
\end{aligned}
$$

Insurance will release free capital from reserve to operation. With the insurance, the variance caused by the loss would decrease (from $\frac{var(X)}{C^2}$ to $\frac{var(X_{ret})}{C^2}$) because the insurance can cover the loss, the variance caused by the operation would increase (from $(\frac{C-CX_0}{C})^2 \times var(r_c)$ to $(\frac{C-P}{C})^2 \times var(r_c)$) because of the increase of operation capital.

Generally, the decrease of loss-caused variance would be much larger than the increase of operation-caused variance, which results in that "$var(R_I) < var(R_0)$" always hold.

As we know from the definition of Sharpe Ratio, when $var(R_I) < var(R_0)$, if $E(R_I) > E(R_0)$, then Sharpe Ratio_I would be always larger than Sharpe Ratio_0, which is a sufficient but not the necessary condition.

Besides, we can derive that: As the increase of $var(x)$, the 'with insurance' Sharpe Ratio will decrease more slowly than 'without insurance', which proves the significance of insurance in reducing risk too.

Approach 2

Another approach is assuming the r_c to be a constant number to simplify the calculation, then $E(r_c) = r_c$, $var(r_c) = 0$.

$$
\begin{aligned}
\text{Sharpe Ratio}_I &> \text{Sharpe Ratio}_0 \\
\frac{C \times (E(r_c) - r_f) - P \times (1 + E(r_c))}{((C-P)^2 \times var(r_c) + var(X_{ret}))^{0.5}} &> \frac{(C - CX_0) \times (E(r_c) - r_f)}{((C-CX_0)^2 \times var(r_c) + var(x))^{0.5}} \\
\frac{C \times (r_c - r_f) - P \times (1 + r_c)}{var(X_{ret}))^{0.5}} &> \frac{(C - CX_0) \times (r_c - r_f)}{var(x))^{0.5}} \\
P \times (1 + r_c) &< C \times (r_c - r_f) - (C - CX_0) \times (r_c - r_f) \times (\frac{var(x_{ret})}{var(x)})^{0.5} \\
P &< C \times \frac{(r_c - r_f)}{(1 + r_c)} - (C - CX_0) \times \frac{(r_c - r_f)}{(1 + r_c)} \times (\frac{var(x_{ret})}{var(x)})^{0.5} \\
P &< \frac{(r_c - r_f)}{(1 + r_c)} \times (C \times (1 - (\frac{var(x_{ret})}{var(x)})^{0.5}) + CX_0 \times (\frac{var(x_{ret})}{var(x)})^{0.5})
\end{aligned}
$$

When $E(R_I) > E(r_0)$, $P_1 < CX_0 \times \dfrac{(r_c - r_f)}{(1 + r_c)}$

When Sharpe Ratio$_I$ > Sharpe Ratio$_0$, $P_2 < \frac{(r_c - r_f)}{(1+r_c)} \times (C \times (1 - (\frac{var(x_{ret})}{var(x)})^{0.5}) + CX_0 \times (\frac{var(x_{ret})}{var(x)})^{0.5})$

$$
\begin{aligned}
\text{Range of P}_2 - \text{Range of P}_1 &= \frac{(r_c - r_f)}{(1 + r_c)} \times (C \times (1 - (\frac{var(x_{ret})}{var(x)})^{0.5}) + CX_0 \times (\frac{var(x_{ret})}{var(x)})^{0.5}) - CX_0 \times \frac{(r_c - r_f)}{(1 + r_c)} \\
&= (C \times (1 - (\frac{var(x_{ret})}{var(x)})^{0.5}) + CX_0 \times (\frac{var(x_{ret})}{var(x)})^{0.5} - CX_0) \times \frac{(r_c - r_f)}{(1 + r_c)} \\
&= (C \times (1 - (\frac{var(x_{ret})}{var(x)})^{0.5}) - CX_0 \times (1 - (\frac{var(x_{ret})}{var(x)})^{0.5})) \times \frac{(r_c - r_f)}{(1 + r_c)} \\
&= (C - CX_0) \times (1 - (\frac{var(x_{ret})}{var(x)})^{0.5})) \times \frac{(r_c - r_f)}{(1 + r_c)} \\
&> 0
\end{aligned}
$$

The above equation compare the range of P under the 2 conditions. Since the $C > CX_0$, when $E(R_I) > E(r_0)$, Sharpe Ratio$_I$ > Sharpe Ratio$_0$ will always hold.

This is caused by that "$var(R_I) < var(R_0)$" always hold, so even when $E(R_I) < E(r_0)$, Sharpe Ratio$_I$ is still possible to be larger than Sharpe Ratio$_0$