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Financial Modeling, Actuarial Valuation and Solvency in Insurance

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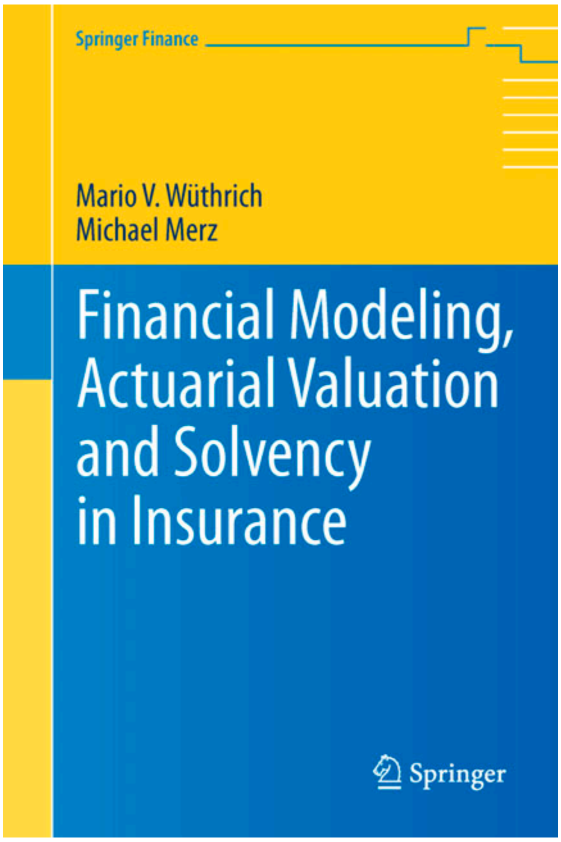
Book review



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Financial Modeling, Actuarial Valuation and Solvency in Insurance, by Mario V. Wüthrich and Michael Merz, Springer (2013). ISBN 978-3-642-31392-9.

The proliferation of books and articles describing quantitative financial models from a theoretical perspective creates a niche for authors like Mario Wüthrich and Michael Merz (WM) to explain a subset of models useful in practice. While this volume targets insurance modelling with a particular focus on addressing the challenges of both sides of the balance sheet, quantitative analysts supporting other types of portfolios (e.g. pension funds, commercial bank portfolios, bond funds—to name a few) will find useful content.

The authors start the book with an ambitious agenda:

> We define a comprehensive mathematical framework that adequately describes price formation and captures the corresponding risk factors that influence the stability of the financial industry. (p. 1)

> … we derive a full risk measurement model for an insurance company that has a non-life insurance portfolio and a life-time annuity portfolio … (p. 1)

> … we need to perform a so-called *full-balance-sheet approach*, which means that we display *all* asset and liability positions *simultaneously*. (p. 3)

Many books (and papers) provide guidance on portfolio asset risk and valuation models for financial institutions in general and insurance companies in specific; however, few publications address a total balance sheet (i.e. assets and liabilities) perspective. This book reflects a new and promising trend wherein academics and practitioners marry financial risk theory and actuarial risk theory. WM go one step further and identify the few theoretical approaches that work in practice. They present a useful menu of recommendations replete with discussions of strengths and weaknesses.

Another strength of this book is the mostly self-contained nature of the content. While a basic understanding of finance, risk, stochastic modelling and the relevant math and statistics is essential to absorbing this book's content, both newly hired quantitative analysts and quantitative veterans will find in this book the full range of models, concepts and guidance to develop a full-balance sheet portfolio risk and valuation model for an insurance company. Even though the authors do not provide a comprehensive treatment of models and approaches applicable to pension fund balance sheets and commercial bank balance sheets, quantitative analysts working at these financial institutions will find useful information, also.

In particular, I found the sections focused on foundational concepts (e.g. stochastic discounting, interest-rate modelling and actuarial modelling) some of the best-targeted, self-contained explanations for new students I have read in this area. This book mostly eliminates the need to refer regularly to other books and papers to develop a comprehensive portfolio risk and valuation model. WM's inclusion of liability modelling introduces the reader to a new and productive direction for portfolio modelling.

Before I review each major section, I will comment on what makes a good quantitative reference book in order to place WM's work in context. In the past 25 years, we have seen explosive growth in the number of finance-related publications. These finance-related books generally fall into five categories:

(1) Highly technical and specialized
(2) Technical and more general
(3) Practitioner-level technical
(4) Non-technical
(5) Polemical

Unfortunately, there are few books in the 'actionable' categories 2 and 3. Most books are in categories 1, 4 and 5, and they don't help us to build, extend or improve financial models. A disturbing trend in finance (and business discussions generally) highlighted in these many (and there are too many) books and conference presentations is the unintelligent use of mathematics. Either a book is too technical, too specialized and too focused on abstruse proofs or it dispenses with equations altogether, resulting in a fairly shallow treatment of the subject material. Financial modelling and risk modelling are quantitative disciplines. Even senior managers at financial institutions should have a basic quantitative understanding of their portfolios' and balance sheets' value and risk. WM have written an exemplary book in terms of providing enough maths to adequately explain a concept or model while refraining from bashing the reader with too much technical prowess. They often add insightful remarks, which pre-emptively address common questions, concerns and confusion.

An example is in Chapter 3, where the authors introduce the concepts underlying affine term structure models. Part of this discussion focuses on state-price deflators, φ_t, applied to the dynamics of spot rate, r_t (with ε_t as the random part of the spot rate process) as follows:

$$\varphi_t = \exp\left\{ -\sum_{s=1}^{t}\left[r_{s-1} + \frac{1}{2}\lambda_s^2 r_{s-1}^2 \right] + \sum_{s=1}^{t}\lambda_s r_{s-1}\varepsilon_s \right\}$$

3.11 (from p. 39)

Note that this discrete-time characterization, clarifies the relationship between spot rates, the market price of risk and the state-price deflator. This equation highlights how the parameterization of λ_t impacts the tractability of the final functional form used to generate a model incorporating state-price deflators into valuation of a portfolio's security. The mathematics—particularly in discrete time—effectively conveys the trade-offs and the functional relationships. Later in Chapter 5, the authors provide examples for asset price calculations. The combination of mathematical foundations with examples makes the material easy to absorb.

My only (minor) complaint about this book concerns its organization, and I highlight the issues with a few examples:

(1) *Properties of 'good' risk measures*: Many papers (particularly Delbaen (2000)) have highlighted desirable properties of risk measures (monotonicity, translation invariance, positive homogeneity, subadditivity, convexity). The authors provide their own take on this topic and summarize the key points reasonably well. The problem is this discussion happens on pages 262–264 of a 432-page book. These properties should be at the beginning or at the end in an appendix (or maybe in its current position *plus* either the front or the back.)
(2) *Limitations of the Vasicek model*: In Chapter 5, WM skilfully describe how to use a discrete-time, one-factor Vasicek term structure model. Inexplicably, their practical advice on the limitations of this model does not appear until p. 325 (183 pages following the introduction of the model after several non-related discussions).
(3) *Overall structure is a little hard to follow*: The primary challenge with the authors' structure is it mixes discussions that would be better kept separate and in some cases, the sequencing of topics is hard to track. Here is the actual structure:
    (a) Valuation principles
    (b) Actuarial
    (c) Solvency
    (d) Special topics

My recommendation would be to change the structure as follows:

(a) Basic concepts
(b) Interest-rate modelling
(c) Actuarial modelling
(d) Solvency modelling
(e) Optimization

The remainder of this review will follow my recommended structure in order to make it easier to concisely emphasize the authors' contributions.

Basic concepts

As I have discussed, most risk modelling focuses on the asset side of the balance sheet. Recent market-wide illiquidity events (e.g. Lehman shock in 2008) have put a spotlight on the other side of a financial institution's balance sheet—liabilities. In the case of insurance companies, meeting the company's actuarial obligations is an essential part of portfolio risk modelling. The authors highlight the importance of this shift to a full-balance sheet treatment and their book skilfully wrestles with the primary challenge for full-balance sheet risk modelling—valuing asset and liabilities with the same measure.

The authors adopt market-consistent values when market values are not available. They quote a useful definition from Article 75 in the Solvency II guidelines (p. 3):

> 'assets shall be valued at the amount for which they could be exchanged between knowledgeable willing parties in an arm's length transaction' and 'liabilities shall be valued at the amount for which they could be transferred, or settled, between knowledgeable willing parties in an arm's length transaction'.

They introduce an important, basic concept in this context: Solvency per a so-called *accounting condition* can be assessed from the market (or market-consistent) asset values exceeding corresponding market (or market-consistent) liabilities values.

Much of this book focuses on developing tractable, robust and credible models for determining the range of possible asset and liability values. The authors make the important point that even if a risk model is consistent across both asset and liability models, it need not be a robust and credible full-balance sheet risk model. For example, flat interest-rate term structures may be used to value both assets and liabilities; however, richer term structure models are needed to capture richer (and impactful) interest-rate environments. The challenge in full-balance sheet risk modelling lies in not only choosing the same (consistent) measures across asset classes and liability classes, but also choosing models, which provide useful and diagnosable output. Given the enormous literature on valuation and risk modelling, practitioners will appreciate the authors' focus on a subset of models, which work in practice.

The accounting solvency condition is not sufficient for assessing an insurance company's risk; we also need to determine the ability of the insurance company to meet its future obligations given a reasonable assessment of likely macroeconomic and demographic dynamics. This is the insurance contract solvency condition. The authors rightly point out the insurance contract condition is harder to model using a full-balance sheet stochastic model, which does the following:

(1) Incorporates all random future values of both assets and liabilities.
(2) Avoids arbitrage (at least in theory) using consistent valuation approaches for both assets and liabilities.

Having laid out an ambitious scope for this book, WM proceed to provide self-contained tutorials on key components:

(1) State-price deflators
(2) Stochastic discounting
(3) Spot rate models

Interest-rate modelling

In the chapters related to interest rates (2–4), WM highlight a range of approaches that can be used to model the stochastic evolution of a yield curve. They point out that the range of models generally fall into three broad categories (p. 19):

(1) Direct macroeconomic factor-based
(2) Purely statistical
(3) Consistent, no-arbitrage-based

In fact, this threefold description applies to more than just interest-rate modelling (similar discussions occur in the equity factor and credit literature). The authors emphasize 'We focus on [3] and give interpretations to the factors in terms of economic variables whenever possible. Moreover, statistical methods are used for model calibration'. This sentiment reflects best-practice mixing of academic and practitioner viewpoints. Said differently, some models directly use macroeconomic factors, others rely on a statistical approach extracting latent factors and some models derive from a no-arbitrage framework. No-arbitrage models are typically recast as either direct-factor or as latent-factor models; however, a direct-factor or latent-factor model need not reflect a no-arbitrage assumption.

Another insightful discussion deals with choosing a discrete-time measure (e.g. discrete-time bank account numeraire (pp. 26–27)) vs. a continuous-time measure. The authors emphasize the trade-off between the ease with which discrete-time models can be economically interpreted vs. the mathematical challenges arising from clear definitions of the shortest possible time interval.

Once the authors cover the foundation of state-price deflators and discount modelling, they spend a number of pages on the Vasicek model for the term structure of interest rates. While often criticized in the past for allowing negative, nominal short-term interest rates, recent negative nominal short-term interest rates in Japan indicate Vasicek's model always captured the realm of possible states. Swiss interest rates provide another example of how a supposed weakness of Vasicek's model turns out to be a strength. The authors demonstrate the power of this model, which is timely given its newfound applicability in the post-crisis, ultra-low interest-rate environment.

Again the authors start simple—one-factor—and then finish with a realistic discrete-time multi-factor Vasicek model, demonstrating how to obtain estimators using the Kalman filter technique (starts on p. 72). The authors are particularly effective in communicating a concise and understandable approach (pp. 75–76):

Step 1: Anchoring—Choose a fixed starting value.

Step 2: Forecasting the measurement system—Obtain at time t the observable prediction error for the continuously compounded spot rate $R(t)$

Step 3: Using Bayesian inference in the transition system —Use this prediction error to update the unobservable risk factor $Y(t)$

Step 4: Forecasting the transition system—Obtain the forecast for the unobservable risk factors.

Step 5: Estimating parameters—Maximize the likelihood function over the parameters defined in the structure developed in the first 4 steps using a given data sample.

Given space constraints, I will not cover other recipes the authors provide; however, this step-by-step explanation is just one of many useful nuggets of practical guidance found in this book.

Asset modelling

Once the basic concepts for valuation are in place, the authors turn to the nuts and bolts of financial asset modelling (Chapter 5). They (rightly) begin with a general cash flow valuation approach (again using the Vasicek model to make the discussion concrete) and quickly cover defaultable coupon bonds, financial market implications and generic derivatives.

They distinguish two cases for defaultable coupon bond modelling (p. 137):

Case 1: Independence of the default variable and the state-price deflator.

Case 2: Correlation between the default variable and the state-price deflator

Unfortunately, the authors only touch on the crucial differences between these cases. Interested readers should look at Bohn and Stein (2009) for more details.

On defaultable bonds, the book is uncharacteristically inadequate. The authors could have gone into more detail on the asset classes they do cover and should probably have included more asset classes—particularly given the recent push for insurance companies (and other large financial institutions) to broaden their portfolio allocations. Readers new to this area will want to supplement this section with Bohn and Stein (2009) or Lando (2004).

Actuarial modelling

The authors begin Part II of their book with a description of actuarial modelling (Chapter 6). The first point is liquidity and the importance of reliable market prices to build replicating portfolios for insurance liabilities. WM distinguish a *financial filtration* for describing economic and financial market information from an *insurance technical filtration* for describing insurance-related variables (e.g. weather, natural hazards and demographics) They use a simple actuarial model of insurance liabilities, which decouples financial filtration from insurance technical filtration. While a linkage between financial and insurance technical filtrations likely exists in practice, this independence assumption is reasonable as a first approximation. A later section introduces a more realistic, but less tractable model. Here again, the authors do an excellent job of introducing key concepts with a simpler model, while later providing a more complex, but often more useful model. They emphasize that in practice, insurance liabilities are a function of <u>both</u> financial variables and insurance technical variables (p. 158).

Insurance liability cash flows are then divided into hedgeable and non-hedgeable portions and valued appropriately. The insurance technical risk is priced with the ‘best estimate’ discounted liability incorporating an appropriate conditional expectation.

This actuarial modelling approach sets the stage for creating a valuation portfolio (referenced with the odd abbreviation VaPo) (p. 169). Despite the lack of market prices of insurance liabilities, the authors assume they can calculate *market-consistent* values via basis financial instruments to produce a VaPo, which can be compared on the same measuring scale as an insurance company’s assets valued with more accepted market values and mark-to-model approaches.

Solvency modelling

Once a modeller addresses valuation of an insurance company’s portfolio of assets and its liabilities in a consistent manner, the next step is evaluating the solvency of the insurance company i.e. do the assets cover the liabilities? In this section (Chapters 7–9 and part of 10), the authors provide a thorough overview of a *full-balance sheet approach* to solvency modelling.

The authors point out that because ‘market values do not have an absolute significance’, other valuation methods can be used successfully; however, whatever valuation method is used in this context, it must be consistently applied to both sides of the balance sheet (p. 170). In practice, this glaringly obvious point is often ignored.

The first step in this process is constructing a suitable VaPo. The authors outline an easy-to-follow recipe:

Step 1: Choose an appropriate basis financial portfolio.

Step 2: Replace insurance technical liabilities with their best estimates at a particular time t.

Step 3: Map the VaPo to a monetary value at time t.

The chapter provides details and examples for both life and non-life insurance companies It goes on to expand the model to cover more realistic, complicated, path-dependent portfolios requiring the construction of an approximate VaPo, which induces asset-liability modelling risk, and which consequently must be handled separately (p. 198).

VaPo construction reflects expected insurance liabilities, which leads to best-estimate reserves for outstanding liabilities; however, in order to price insurance liabilities on a risk-adjusted basis, a modeller should calculate margin for non-hedgeable insurance technical risk and potential shortfalls. The best-estimate reserves plus this extra margin constitutes *risk-adjusted reserves.*

In practice, risk-averse decision-makers require a premium for uncertainty so that the VaPo estimate requires an adjustment to estimate a *protected* VaPo. Chapter 8 is dedicated to constructing a protected VaPo. The steps are similar to constructing a VaPo except in step 2, the insurance technical liabilities are replaced by what the authors call *probability distorted* conditional expectations. (In other risk literature, probability distorted is also called risk-neutral – regardless of the naming convention, probabilities are 'distorted' or 'adjusted' to account for the risk aversion of an investor looking to buy the insurance liabilities.) This *distortion or* adjustment of real-world probabilities results in a 'price'. This notion of a 'price' for these liabilities is a critical part of consistently modelling assets and liabilities.

Given the central part of the valuation portfolio in this modelling approach, I will provide some of the math presented in the book. The authors distort (adjust) real-world probabilities to determine a protected valuation portfolio ($\mathrm{VaPo}^{\mathrm{prot}}$).

$$\mathrm{VaPo}_t^{\mathrm{prot}}(\mathbf{X}) = \sum_{k \in J} \frac{1}{\varphi_t^T} E\left[\varphi_k^T \Lambda^{(k)} | \Im_t\right] \mathfrak{A}^{(\mathfrak{k})} = \sum_{k \in J} \Lambda^{(k)} \mathfrak{A}^{(\mathfrak{k})}$$

8.3 (from p. 206)

where $\mathbf{X} \equiv$ Insurance liability cash flow; $k \equiv$ Counter for basis financial instruments in financial market J; $J \equiv$ Financial market; $\varphi_t^T \equiv$ State-price deflator for insurance technical variables back to time t; $\varphi_k^T \equiv$ State-price deflator for insurance technical variables for the kth event; $\Lambda^{(k)} \equiv$ T-adapted k insurance technical variables; $\Im_t \equiv$ Insurance technical events; $\mathfrak{A}^{(\mathfrak{k})} \equiv$ Basis financial instrument k (note financial market J is made up of the all k basis financial instruments).

This expands on the equation used to determine a non-protected valuation portfolio.

$$\mathrm{VaPo}_t(\mathbf{X}) = \sum_{k \in J} E\left[\Lambda^{(k)} | \Im_t\right] \mathfrak{A}^{(\mathfrak{k})} \quad 7.5 \text{ (from p. 172)}$$

An interesting series of discussions in the context of the VaPo concerns the relationship of financial variables and insurance technical variables. The authors' simplest model rests on the assumption that these two sets of variables are independent. The authors emphasize that assuming independence across these two variable sets does not imply that assets and insurance liabilities are independent from the same set of economic and financial variables. Rather, the independence assumption eliminates excessive model complexity while still retaining some correlated behaviour across assets and liabilities as a function of their relationships to the same set of economic and financial variables.

Once we can model both the asset and liability sides of the balance sheet, we have the tools to calculate solvency. WM consider two possibilities:

Possibility 1: *Accounting solvency condition*—Asset value at time t is equal to or greater than the risk-adjusted reserves at time *t*.

Possibility 2: *Insurance contract solvency condition*—Asset value is equal to or greater than the risk-adjusted liabilities at time $t + 1$.

The insurance contract solvency condition requires determining the availability of assets to cover liabilities in the future. Regulators typically require that both solvency conditions be met even though one could argue that meeting only the second condition is sufficient.

The authors insightfully point out that with a conditional (i.e. conditioned on a level of risk tolerance) risk measure ρ_t, an insurance company's business plan (in terms of assets relative to liabilities) is *acceptable* if $\rho_t(AD_{t+1}) \leq 0$ where AD $(t + 1)$ = Risk-adjusted liabilities $(t + 1)$ – Asset value $(t + 1)$.

An *acceptability* condition is weaker than accounting solvency. In my own experience, this condition should be tied back to a quantitative risk-appetite statement, although the authors do not address this subject. The authors contextualize the problem of a positive asset deficit in risk terms by showing how a risk measure applied to an asset deficit can provide an indication of the likelihood that future shortfalls cannot be covered (p. 272). Typically, insurance companies look to allocations, which meet both solvency and acceptability conditions.

Throughout Chapter 8, the authors discuss liquidity as a separate and difficult problem, which they do not address in these models. The solvency modelling assumes sufficient liquidity. In practice, liquidity constraints should be added to solvency constraints for comprehensive risk management.

At the end of the book in Sections 10.3–10.6, the authors discuss more realistic characterizations of insurance company modelling culminating in a description of a comprehensive stochastic asset-liability model. While I would have changed the position of this discussion so that it flowed more naturally with the development of the modelling approaches, the examples are realistic enough to be useful, but not so complicated as to be incomprehensible.

Optimization

Assuming an insurance company has liabilities given by $\mathrm{VaPo}^{\mathrm{prot}}$ and a particular asset portfolio driven by the same economic and financial variables, senior executives at an insurance company can potentially use optimization algorithms to determine portfolio allocation. In Section 9.5 of Chapter 9, the authors address this difficult topic.

WM set up the optimization problem with the constraints presented in the context of solvency: Accounting and acceptability (for a given risk measure). They point out the well-known fact that mean standard deviation approaches lead to analytically tractable optimization solutions despite most insurance executives desire to capture skew and heavy tails in their targeted risk measures and related optimization approaches. Unfortunately, the more useful risk measures (e.g. expected shortfall), which capture a distribution's higher moments, generally require numerical solutions for optimization. Sadly, the authors only address mean standard deviation in this section though they do have a good section on estimating covariance matrices (pp. 333–336). (See Rockafellar and Uryasev (2000), Bertsimas *et al.* (2004) and Goldberg *et al.* (2013) for good discussions of optimization approaches with respect to the portfolio tail-risk measure, Expected Shortfall.)

Concluding remarks

In summary, WM deliver a readable book on full-balance sheet stochastic modelling of insurance companies with judicious presentation of mathematical equations and insightful remarks throughout the text. The authors set themselves up to achieve the implicit objective of building a useful model when they say ‘Modelers always face the (difficult) trade-off between complexity and simplicity … Often, [modelers] cannot model all the real world features, however, they should capture the essential risk drivers in an appropriate way, so that these can be analyzed, understood and managed’ (p. 1). WM achieve a balance of this trade-off. The mixing of simple models, realistic-but-not-too-complex models and real-life examples make this book essential for any student or quantitative analyst looking to understand how to adapt finance and risk theories to the challenging world of insurance risk and solvency modelling.

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