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Self-Insurance and Self-Protection: A Nonexpected Utility Analysis

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Abstract

This paper studies the comparative statics of self-insurance and self-protection for individuals with rank-dependent expected utility preferences. In particular, proportional wealth risks, background risk, and limited liability are considered. Limited liability has a major impact on self-insurance and self-protection. It can reverse seemingly robust comparative static results for the case of self-insurance and can eliminate some puzzling ambiguities for the case of self-protection.

Key words: Self-Insurance, Self-Protection, Rank-Dependent Utility, Limited Liability

1. Introduction

Decision makers can often reduce exposure to risk not just by the purchase of market insurance but also through expenditures on self-insurance (reductions in the size of a loss) or through expenditures on self-protection (reductions in the probability of a loss). Both of these are intuitively thought of as risk-reducing activities and thus as substitutes for market insurance, an intuition that is at the basis of the moral-hazard problem (i.e., the purchasers of market insurance tend to engage in fewer of those activities). The first systematic analysis of self-insurance and self-protection and their interaction with market insurance was made by Ehrlich and Becker [1972]. While their results for the case of self-insurance are unsurprising, they found that market insurance and self-protection can be complements. Thus, contrary to the moral-hazard intuition, the presence of market insurance may, in fact, increase self-protection activities relative to the situation where market insurance is unavailable.

Subsequently, Dionne and Eeckhoudt [1985], Hiebert [1989], Briys and Schlesinger [1990], Briys, Schlesinger, and Schulenburg [1991], and Sweeney and Beard [1992] have added to this literature. All the results have been derived within the expected utility hypothesis. This paper uses rank-dependent expected utility (anticipated utility), which is an important class of nonexpected utility, and considers proportional wealth risks, background risks, and limited liability. It shows that many of the comparative static results that hold for expected utility carry over to rank-dependent expected utility, and it extends the comparative statics analysis to proportional wealth risks. It also shows that limited liability may reverse the relation between the activity level of self-insurance and increases in wealth. Further, the paper shows that, despite the well-known ambiguities of the relation between risk aversion and the self-protection effort, for limited liability the relation is straightforward for expected utility and for rank-dependent expected utility.

2. Rank-dependent expected utility

Most studies of economic decision problems involving risk or uncertainty use expected utility theory. Experimental economics, however, reports a vast body of empirical evidence for violations of the axioms of this theory (see, e.g., Allais [1953], Camerer [1989], Conlisk [1989], and MacCrimmon and Larsson [1979]). To deal with this problem, decision theory came up with more general preference functionals that allow for a weakening of those axioms that are at odds with experimental evidence (see Fishburn [1988] for an overview).

One of the well-established generalizations of expected-utility theory is rank-dependent (anticipated) expected utility. It has been described and axiomatized by Quiggin [1982], Yaari [1987], Segal [1989], and Chew [1989] and has been proved useful in theories of lottery design (Quiggin [1991b]), theory of insurance markets (Doherty and Eeckhoudt [1992]), cost-benefit analysis and portfolio theory (Chew and Herk [1990]), and in explaining bandwagon effects in two-party majority voting (Chew and Konrad [1992]). The rank-dependent expected-utility $E_g u$ of random wealth V is given by

$$E_g u = \int_V u(v)\, d(g \circ F)(v),$$

where the probability transformation function g:[0, 1] $\rightarrow$ [0, 1] is continuous, increasing, and onto. For the case with only two possible outcomes v_1 and v_2, with $v_1 < v_2$ and probabilities p and $(1 - p)$, respectively, rank-dependent expected utility reduces to

$$E_g u = g(p)\, u(v_1) + (1 - g(p))\, u(v_2). \tag{1}$$

The function u plays a role much like a von Neumann Morgenstern utility function which is continuous, and increasing in outcomes v. The function g transforms probabilities of the true probability function of possible outcomes into subjective probabilities. By its nature, a concave g function enlarges the probabilities of outcomes with low wealth and reduces the probabilities of outcomes with high wealth. For instance, in the case with two outcomes in (1), if g is concave, the weight $g(p)$ of utility of the low outcome is increased compared to expected utility $[g(p) \geq p]$ and, of course, the weight of utility of the high outcome is reduced

$[1 - g(p) \leq 1 - p]$. A concave g function could be interpreted as describing pessimism. Similarly, a convex g function could be interpreted as describing optimism; it increases the probability weights of good outcomes and reduces the probabilities of bad outcomes.

For rank-dependent expected utility, a preference functional characterized by $(\hat{g}, \hat{u})$ is (weakly) more risk averse in the Arrow-Pratt sense (cf. Pratt [1964]) than the one characterized by (g, u), if $\hat{g} = \Psi(g)$ and $\hat{u} = \Phi(u)$, where Φ and Ψ are both concave (see Chew, Karni, and Safra [1987]). Note that rank-dependent expected utility reduces to well-known types of preference functionals once g or u are linear; when u is linear, (1) reduces to Yaari's [1987] dual theory, or, rank-dependent expected wealth

$$E_g v \equiv \int_V v \, d(g \circ F)(v) = g(p)\, v_1 + (1 - g(p))\, v_2, \tag{2}$$

where the second equation describes the case where V has only the two outcomes. The preference functional (2) expresses risk aversion if g is concave. This could be called risk aversion in probabilities or pessimism.

For a linear g function, rank-dependent expected utility becomes expected utility—i.e.,

$$Eu \equiv \int_V u(v) \, dF(v). \tag{3}$$

Finally, if u and g are both linear, (1) becomes expected wealth

$$Ev \equiv \int_V v \, dF(v), \tag{4}$$

which is obviously also a special case of (2) and (3).

3. Self-insurance[1]

We consider an individual who maximizes according to a rank-dependent expected utility preference functional. Consider a risk-averse individual endowed with exogenous wealth w. This wealth is subject to a random loss. The loss is zero with probability $1 - p$ and equal to $s > 0$ with probability p.

The individual may undertake self-insurance activities that reduce the size of a loss. Let y denote expenditure for self-insurance. Its effect is described by the differentiable function $s(y)$, which relates the loss size to the level of self-insurance activity, with $s'(y) < 0$—i.e., the higher the expenditure on self-insurance, the lower the loss.

Define $v_1(y) \equiv w - s(y) - y$ and $v_2(y) \equiv w - y$, the two possible outcomes of final wealth.[2] The individual's rank-dependent expected utility for random wealth with a bivariate distribution is

$$E_g u = g(p)\, u_1 + (1 - g(p))\, u_2, \tag{5}$$

where $u_1 \equiv u(v_1(y))$, and $u_2 = u(v_2(y))$ for brevity. The first-order condition for maximizing (5) with respect to y is

$$d(E_g u)/dy = -g[1 + s'(y)]\, u_1' - (1 - g)\, u_2' = 0. \tag{6}$$

When $E_g u$ in (5) is strictly concave in y, there will be positive self-insurance activity y^* if $s'(0) < -\dfrac{[1 - g(p)]u'(w) + g(p)u'(w - s(0))}{g(p)\, u'(w - s(0))}$ (<0).[3] Assuming $u'' < 0$, the right side of the inequality is higher (and thus positive self-insurance is more likely) the greater the potential loss ($s(0)$), the higher the probability of loss (p), and the more the probability of loss is psychologically weighted by the individual, as measured by $g(.)$.

Note that the self-insurance activity does not alter the probability of the loss occurring. The g-function is also invariant with respect to the choice of y. The self-insurance problem of an individual with rank-dependent expected utility, with the probability transformation function g, and with loss probability p is equivalent to a self-insurance problem of an expected-utility maximizer with a von Neumann Morgenstern function u and a loss probability $g(p)$ (see Quiggin [1991a] for a formal treatment). Therefore, many of the comparative static results for self-insurance that have been derived for expected utility carry over to rank-dependent expected utility. We consider three examples:

1. *The effect of a global efficiency loss.* Ehrlich and Becker [1972] defined a productivity loss to be a change of the loss-reduction function $s(y)$ to $\hat{s}(y)$ with $\hat{s}'(y) > s'(y)$ for all $y > 0$ (recall that s' is negative). They show that a global productivity loss of self-insurance leads to a reduction in self-insurance expenditure. Given the equivalence pointed out, this result also holds under rank-dependent utility.
2. *An income increase.* Ehrlich and Becker [1972] also show that an income increase decreases (keeps constant, increases) the demand for self-insurance if absolute risk aversion is decreasing (constant, increasing) with increases in wealth. Given the insight provided by Chew, Karni, and Safra [1987] that the Arrow-Pratt definition of absolute risk aversion carries over to rank-dependent expected utility, this result also carries over to rank-dependent expected utility.
3. *A risk aversion increase.* Consider now the question of how an increase in risk aversion affects the optimal level of self-insurance. It turns out that the expected utility results of Dionne and Eeckhoudt [1985] and Briys and Schlesinger ([1990], p. 459n.) also carry over to rank-dependent expected utility.

Proposition 1: *Assume that the wealth prospect and choice set of an individual does not change. A (weak) increase in risk aversion leads to a (weak) increase in self-insurance.*

Proof: Suppose that $\hat{g} = \Psi(g)$ and $\hat{u} = \Phi(u)$, where Φ and Ψ are both concave. For $\hat{g}$ to be a probability transformation function, Ψ is, of course, a bijective self-mapping on [0, 1]. We get

$$\begin{aligned}\frac{d}{dy}(E_{\hat{g}}\,\hat{u})\mid_{y^*} &= -\Psi(g)\,[1 + s'(y^*)]\,\Phi'(u_1(y^*))u_1'(y^*)\\ &\quad - (1 - \Psi(g))\,\Phi'(u_2(y^*))u_2'(y^*)\\ &= \Psi(g)\Phi'(u_1(y^*))u_2'(y^*)(1 - g)/g\\ &\quad - (1 - \Psi(g))\Phi'(u_2(y^*))u_2'(y^*)\\ &= u_2'(y^*)\left[\frac{\Psi(g)(1 - g)}{g}\,\Phi'(u_1(y^*)) - (1 - \Psi(g))\Phi'(u_2(y^*))\right],\end{aligned}$$

where the second equality follows by the use of (6). Since $u_1(y^*) < u_2(y^*)$, by concavity of Φ, $\Phi'(u_1(y^*)) \geq \Phi'(u_2(y^*))$, and similarly, by the concavity and bijectivity of Ψ on [0, 1], $\Psi(g) \geq g$ and $(1 - \Psi(g)) \leq 1 - g$. Therefore, for $(\hat{g}, \hat{u})$, $d(E_{\hat{g}}\,\hat{u})/dy \mid_{y^*} > 0$.■

3.1. Background risk

Instead of a sure exogenous wealth w, suppose exogenous wealth W is random with a low and a high outcome, $\underline{w}$ and $\bar{w}$, whose probabilities are q and $(1 - q)$, respectively. Again, the self-insurable risk is a bivariate random variable $S(y)$; $S(y) = s(y)$ with probability p, and $S(y) = 0$ with probability $(1 - p)$. Suppose that S and W are stochastically independent. The four possible outcomes for final wealth with their respective probabilities are

$$\begin{array}{lll} v_1 \equiv \underline{w} - y - s(y) & \text{with} & p_1 = pq \\ v_2 \equiv \bar{w} - y - s(y) & \text{with} & p_2 = p(1 - q) \\ v_3 \equiv \underline{w} - y & \text{with} & p_3 = (1 - p)q \\ v_4 \equiv \bar{w} - y & \text{with} & p_4 = (1 - p)(1 - q). \end{array}$$

Briys and Schlesinger [1990] show that, under the Ross [1981] definition of increases in risk aversion, more risk-averse individuals demand more self-insurance, even if there is a stochastically independent background risk. Their result implies that risk-neutral individuals invest less in self-insurance than risk-averse expected utility maximizers. This result generalizes to rank-dependent expected utility.

Proposition 2: *Suppose $s'' > 0$. For stochastically independent S and W, a risk-averse rank-dependent expected utility maximizer undertakes more self-insurance than an individual who maximizes expected wealth.*

Proof: Consider the following:

$$
\begin{array}{lcccl}
(Ev): & \Sigma_{i=1}^{4} p_i v_i & \rightarrow & \Sigma_{i=1}^{4} p_i u_i & :(Eu) \\
 & \downarrow & \searrow & \downarrow & \\
(E_g v): & \Sigma_{i=1}^{4} g_i v_i & \rightarrow & \Sigma_{i=1}^{4} g_i u_i & :(E_g u)
\end{array}
$$

The preference functionals $E_g u$, $E_g v$, Eu and Ev are defined in (1) – (4), and g and u are assumed to be concave.

Briys and Schlesinger [1990] show that a risk-averse expected utility maximizing individual (upper right corner) chooses more self-insurance than a risk-neutral individual (upper left). Their proof is applicable for $Ev \rightarrow Eu$ and for $E_g v \rightarrow E_g u$; the transformed probability weights $g_1, \ldots g_4$ can simply be considered as subjective probability weights on the four possible outcomes. If $Ev \rightarrow E_g v$ implies an increase in self-insurance, then a change $Ev \rightarrow E_g u$ does so *a fortiori*. If $Ev \rightarrow E_g v$ is ambiguous, then $Ev \rightarrow E_g u$ is also ambiguous, as $Ev \rightarrow E_g v$ is a special case of $Ev \rightarrow E_g u$.

Under rank-dependent expected utility, the probability weights $p_1, \ldots p_4$ are transformed and the transformation depends on whether $v_2 \gtreqless v_3$. Suppose $v_2 < v_3$. Rank-dependent expected-utility weights become

$$
\begin{aligned}
g_1 &= g(pq) \\
g_2 &= g(p) - g(pq) \\
g_3 &= g(p + (1 - p)q) - g(p) \\
g_4 &= 1 - g(1 - (1 - p)(1 - q)),
\end{aligned}
$$

where, of course, $\sum_{i=1}^{4} g_i = 1$. The first-order condition for y for an individual who maximizes $\sum_{i=1}^{4} g_i v_i$ is

$$-1 - [g_1 + g_2]s'(y) = 0. \tag{7}$$

For a risk-neutral individual who maximizes expected wealth, this condition becomes

$$-1 - p\, s'(y) = 0. \tag{8}$$

Self-insurance (y^g) of the $E_g v$-maximizing individual is higher than self-insurance (y^n) of the risk-neutral individual iff $-s'(y^n) > -s'(y^g)$, or, iff

$$g_1 + g_2 > p. \tag{9}$$

Note that $g_1 + g_2 = g(pq) + g(p) - g(pq) = g(p) > p$ by concavity of g. Suppose now that $v_2 > v_3$. In this case $E_g v$ becomes $\sum_{i=1}^4 g_i v_i$, where

$$\begin{aligned} g_1 &= g(pq) \\ g_2 &= g(pq + (1 - p)q + (1 - q)p) - g(pq + (1 - p)q) \\ &= g(q + p(1 - q)) - g(q) \\ g_3 &= g(q) - g(qp) \\ g_4 &= 1 - g(q + p(1 - q)). \end{aligned}$$

The marginal condition (9) for these weights becomes

$$g_1 + g_2 = g(pq) + g(q + p(1 - q)) - g(q) > p.$$

This is fulfilled for $p = 0$ and $p = 1$ with equality. Further, $g_1 + g_2$ is concave on $[0, 1]$ in p, as

$$\frac{d^2}{(dp)^2}(g_1 + g_2) = q^2 g''(pq) + (1 - q)^2 g''(q + p(1 - q)) \leq 0.$$

Concavity of $g_1 + g_2$ on $p \in [0, 1]$ together with $g_1 + g_2 = p$ for $p = 0$ and $p = 1$ implies that $g_1 + g_2 \geq p$ for all $p \in (0, 1)$.■

3.2. Proportional wealth risk

Consider the case of a multiplicative risk. Suppose that the size of the loss, wealth, and the effect of self-insurance expenditure are linked by strict proportionality in the following sense. Let ε be the fraction of wealth used for self-insurance, i.e., $y = \varepsilon w$. Let $s = \lambda w$ be the loss that occurs with probability p. The loss is proportional to wealth and is a function of the fraction of wealth spent on loss reduction, i.e., $\lambda = \lambda(\varepsilon)$ with $\lambda' < 0$. Marginal effectiveness of self-insurance is assumed to be nonincreasing, i.e., $\lambda'' \geq 0$.

There are many examples for proportional risks. Suppose that wealth is invested in a portfolio of assets. If the returns to these assets are risky, and the structure of the portfolio is not changed for increasing wealth, then risk and portfolio size are positively correlated and may increase proportionally. Similarly, the same degree of loss reduction will require higher expenditure if the portfolio size increases. If, e.g., all wealth is invested in houses and the source of risk is an earthquake, then, to reduce the size of the loss in all houses in case of an earthquake requires the same effort in each single house. The additional costs incurred for making all houses equally safe are approximately proportional to the number of houses. This raises the question of how an increase in wealth changes the fraction of wealth used for self-insurance when the wealth increase also increases the size of the possible loss. The question seems to be even more natural than the question of how an increase in expected wealth for unaltered risk (i.e., a shift of the wealth distribution to the right) changes self-insurance expenditure.

First, the generalization of the Arrow-Pratt measure of relative risk aversion for rank-dependent expected utility has to be stated. As with the measure of absolute risk aversion, the Arrow-Pratt definition carries over here. Relative risk aversion is said to be increasing, constant, or decreasing in wealth, if $-v\frac{u''(v)}{u'(v)}$ increases, is constant or decreases with v, respectively.

Proposition 3: *Suppose that a loss of $\lambda(\varepsilon)w$ occurs with fixed probability p. If the individual has increasing (constant, decreasing) relative risk aversion, then $d\varepsilon^*/dw > (=, <)\ 0$.*

Proof: The two possible final levels of wealth now are

$$v_1(\varepsilon) \equiv w - \lambda(\varepsilon)w - \varepsilon w = w(1 - \lambda(\varepsilon) - \varepsilon) \tag{10}$$

and

$$v_2(\varepsilon) \equiv w - \varepsilon w = w(1 - \varepsilon). \tag{11}$$

The first-order condition (6) for rank-dependent expected utility becomes

$$d(E_g u)/d\varepsilon = -gu'(v_1)(1 + \lambda') - (1 - g)u'(v_2) = 0. \tag{12}$$

Using the implicit function theorem, we get

$$[gu'(v_1)\lambda''w - gu''(v_1)(1 + \lambda')^2w^2 - (1 - g)u''(v_2)w^2]\,\frac{d\varepsilon}{dw}$$
$$= g\left[-v_1\frac{u''(v_1)}{u'(v_1)}\right]u'(v_1)(1 + \lambda') + (1 - g)\left[-v_2\frac{u''(v_2)}{u'(v_2)}\right]u'(v_2). \tag{13}$$

The bracket on the left side is positive for $\lambda'' \geq 0$. Therefore, the sign of $d\varepsilon/dw$ equals the sign of the right side. For constant relative risk aversion, this equals zero by (12). For decreasing relative risk aversion it is negative; for increasing relative risk aversion it is positive.■

3.3. Limited liability

We explore now a variation of the basic model of self-insurance. In this self-insurance reduces the size of a loss, as in the conventional model, but, depending on the amount of self-insurance effort, utility in the event of loss may be independent of expenditures on self-insurance. Medical care to the uninsured (especially in case of catastrophic illness), unemployment compensation, other social safety nets, as well as to personal and business bankruptcy, can all be thought of as forms of insurance where insurance benefits are almost independent of individual premiums for a wide range of possible premiums.

Suppose that, if the loss were to occur, the individual would get at least a minimum final wealth, i.e., if $w - s - y < c$, the individual gets the *minimum final wealth* c instead of $w - s - y$. This c is the liability limit of the individual. We assume that the individual makes his expenditure on self-insurance before a damage occurs. Therefore, these amounts are really fully paid. It is only the damage to which limited liability refers. We do not pursue the question of who eventually bears the burden of the damage that exceeds the liability limit; we assume here that the bearer of these costs has no possibility of observing the self-insurance expenditure of an individual. Therefore, the individual's actual choice of self-insurance (and, therefore, the excess loss in case of a loss) has no secondary effects apart from the portfolio effects considered here.[4]

The impact of limited liability on self-insurance is dramatic and very similar to the effect of the possibility of bankruptcy on insurance demand as derived by Sinn [1982]. The individual's problem becomes to choose y to maximize

$$gu(max(c, w - s(y) - y)) + (1 - g)u(w - y). \tag{14}$$

For sufficiently high $s(0)$ compared to w, and sufficiently moderate productivity of the self-insurance technology, this problem does not have an interior solution. The first units of self-insurance expenditure reduce the loss $s(y)$. However, these units do not reduce the economic loss this individual incurs. Only if the optimal $y^* > 0$ that solves the problem of maximizing (5) without liability limit is such that

$$gu(w - s(y^*) - y^*) + (1 - g)u(w - y^*) > gu(c) + (1 - g)u(w), \tag{15}$$

is the expenditure on self-insurance positive. This implies a change of comparative static results for self-insurance.

Proposition 4: *Consider a strictly risk-averse individual with wealth $w < s(0) + c$ who chooses $y = 0$. Suppose that $g(p)s'(y)\,|_{y=0} < -1$. Then there exists a wealth increase such that the optimal level of self-insurance is positive.*

Proof: Consider a wealth increase from w to $\hat{w} = s(0) + c$. For this case,

$$d(E_g u)/dy\,|_{y=0} = -g(1 + s'(y)\,|_{y=0})u'(c) - (1 - g)u'(\hat{w}) > 0$$
by $u'(c) > u'(\hat{w})$.■

Proposition 4 shows that an increase in wealth does not necessarily lead to a reduction of self-insurance, even for strictly decreasing absolute risk aversion, but, on the contrary, self-insurance expenditure may go up by a discrete jump. This wealth effect is important to recognize: as with market insurance with limited liability (see Sinn [1982]), high income or wealth can overcome the allocative distortion in self-insurance that is caused by the limited liability.

4. Self-protection

Suppose now that the individual cannot influence the size of the loss; instead, the probability of loss, $p(x)$, is a decreasing function of his expenditures x so that $p'(x) \leq 0$. Denote initial wealth again by w and the size of the loss by s. The problem, like that first examined by Ehrlich and Becker [1972], but this time with rank-dependent expected utility, is to choose the level of *self-protection* x so as to maximize

$$E_g u = g(p(x))\, u(w - s - x) + (1 - g(p(x)))\, u(w - x). \tag{16}$$

Dionne and Eeckhoudt [1985] provide examples where greater risk aversion reduces self-protection, and Ehrlich and Becker [1972] show that complementarity between insurance and self-protection is possible. Briys and Schlesinger [1990] show more generally in an expected utility framework why a risk-averse individual may undertake more or less self-protection than a risk-neutral individual does, and thus they provide an intuition for the ambiguity.

In general, as rank-dependent expected utility reverts to expected utility, the ambiguity carries over to rank-dependent expected utility. However, two issues may be of interest, which lead to less ambiguous results.

4.1. Dual-choice theory

For a bivariate outcome with self-protection, Yaari's rank-dependent expected wealth in (2) becomes

$$E_g v \equiv g(p(x))(w - x - s) + (1 - g(p(x))(w - x). \tag{17}$$

We show

Proposition 5: *Suppose $p(x)$ is convex. Consider two individuals with utility (17) with concave g-functions g and $\hat{g}$ respectively, where $\hat{g} = \psi(g)$ is a concave function of g. The $\hat{g}$-type individual undertakes more (less) self-protection than the g-type individual if $\psi' g'(p(x^*)) > (<)\ g'(p(x^*))$, where x^* is the optimal level of self-protection chosen by the g-type individual.*

Proof: The first-order condition for the g-type individual is

$$-g'p's = 1.$$

For the $\hat{g}$-type individual it is

$$-\psi'g'p's = 1.$$

Therefore, self-protection is higher (lower) for the $\hat{g}$-type individual if $\Psi'g'(p(x^*)) > (<) \, g'(p(x^*))$.■

The result in proposition 5 shows that the previously noticed ambiguous effects of increases in risk aversion on self-protection extend to the case of rank-dependent expected utility even if the u function is linear.[5] However, the condition $\Psi'g' > (<) \, g'$ has an intuitive meaning. For Ψ being strictly concave and onto, $\lim_{g\to 0} \Psi'(g) > 1$, and $\lim_{g\to 1} \Psi'(g) < 1$. But $g \to 0$ occurs for $p \to 0$ and $g \to 1$ occurs for $p \to 1$. Therefore, these conditions imply that, for a loss that occurs with a small probability, self-protection effort is higher for more risk-averse $E_g v$-maximizers, and, for an almost certain loss, self-protection effort is lower for more risk-averse $E_g v$-maximizers.

4.2. *Limited liability*

We consider now a variation of the basic model of self-protection and introduce the limited liability that was discussed in section 3. Suppose the probability of loss, $p(x)$, is a decreasing function of expenditures x on self-protection so that $p'(x) \leq 0$. However, if the loss were to occur, the individual would get at least a minimum level of final wealth, i.e., if $w - s - x < c$, the individual gets the *minimum wealth* c instead of $w - s - x$. Consider the case $w - s < c$. The individual's problem is to maximize

$$E_g u = g(p(x))\, u(c) + (1 - g(p(x)))\, u(w - x). \tag{18}$$

The optimal interior choice, denoted by x^*, satisfies the first-order condition

$$-g'p'(x^*)\,[u(w - x^*) - u(c)] - (1 - g(p(x^*)))\, u'(w - x^*) = 0. \tag{19}$$

Now consider a more risk-averse agent with utility function $\hat{u} = \Phi(u)$, where Φ is increasing and concave, and a probability transformation function $\hat{g}(p) = \Psi(g(p))$, with Ψ: $[0, 1] \to [0, 1]$ increasing, concave and onto. The first derivative with respect to x, evaluated at x^*, is

$$\Delta \equiv -\psi' g' p'(x^*)[\Phi(u(w - x^*)) - \Phi(u(c))] - (1 - \psi)\Phi'(u(w - x^*))\, u'(w - x^*). \tag{20}$$

By concavity of Φ, $\Phi(u(w - x^*)) - \Phi(u(c)) \geq \Phi'(u(w - x^*))\ [u(w - x^*) - u(c)]$. Therefore,

$$\frac{\Delta}{\Phi'(u(w - x^*))} \geq -\psi' g' p'(x^*)\ [u(w - x^*) - u(c)] - (1 - \psi)u'(w - x^*). \tag{21}$$

By the properties of Ψ, $\Psi' \geq (1 - \Psi)/(1 - g)$. Therefore,

$$\frac{\Delta}{\Phi'(u(w - x^*))} \geq [-g' p'(u(w - x^*) - u(c)) - (1 - g)u'(w - x^*)]\ (1 - \psi)/(1 - g) = 0.$$

Thus, the optimal amount of self-protection chosen by a more risk-averse agent must be greater than x^*. We summarize this result as

Proposition 6: *If, for given rank-dependent expected utility, a level of self-protection is chosen such that $w - s - x^* \leq c$, then an increase in risk aversion leads to an increase in self-protection undertaken.*

To see the effects of the level of wealth w and the level of limited liability c on the optimal choice of self-protection, totally differentiate (19) to get

$$\frac{dx^*}{dw} = \frac{g' p'(x^*) u'(w - x^*) + (1 - g(p(x^*)))u''(w - x^*)}{D}, \tag{22}$$

and

$$\frac{dx^*}{dc} = \frac{-g' p'(x^*)\ u'(c)}{D}, \tag{23}$$

where $D = -(g'' p' + g' p'')[u(w - x^*) - u(c)] + 2g' p' u'(w - x^*) + (1 - g(p(x^*)))u''(w - x^*)$ is negative since it is the second-order condition for a maximum.[6] Then, by the other properties of $p(.)$, $g(.)$ and $u(.)$, we have $\frac{dx^*}{dw} > 0$ and $\frac{dx^*}{dc} < 0$. Thus, an increase in exogenous wealth w increases expenditure on self-protection, and an increase in the minimum level of final wealth reduces self-protection.

The second effect is consistent with the moral hazard intuition: when bankruptcy laws become laxer (c is increased) or the poverty limit in social security is increased, the severity of a loss is reduced. Agents become less careful in avoiding this loss. The effect of wealth is not as evident, especially since we usually associate greater wealth with more risk-taking activities. Note, however, that an increase in wealth is similar to a decrease in the minimum level of wealth since both increase the difference of utility in the two states. Seen this way, the effects of endowment wealth w and of minimum final wealth are essentially the same in this model and can be summarized as follows:

Proposition 7: *An increase in the difference between exogenous wealth and the minimum wealth level increases self-protection.*

Condition (19) can be rearranged as follows:

$$\frac{u'(w - x^*)}{u(w - x^*) - u(c)} = \frac{-g'p'(x^*)}{1 - g(p(x^*))}. \tag{24}$$

The right side of (24) represents the percentage change in the rank-dependent probability of not facing the minimum wealth level. The left side of (24) has been called *boldness* by Aumann and Kurz [1977]. It measures an agent's attitude toward risking all her fortune ($w - x$ in this instance) against a small possible gain.[7] This concept, or its inverse called *fear of ruin* by Aumann and Kurz, appears to capture elements of risk aversion in several other economic situations.[8] Thus, according to (24), the optimal level of self-protection is chosen so as to equate boldness in the good state to the percentage change in the probability of avoiding ruin.

The idea of individuals influencing the probability distribution they face through their own actions (which is behind the model of self-protection) does not only concern individual decision making. It is potentially more important for interactive situations like tournaments (e.g., Rosen [1986]) and rent-seeking (Hillman and Katz [1984], Schlesinger and Konrad [1993], and Skaperdas and Li [1993]) and contests in general (Dixit [1987] and Skaperdas and Li [1993]). In such situations, for an agent the probability of winning depends not just on his own activities but also on the activities of the other participants. In other words, contest situations are the game-theoretic equivalent of self-protection. Thus, any positive results for self-protection could be applicable to contest situations as well. In particular, the model of this paper is relevant for contests where the utility of the losers is fixed or where it is independent of the level of effort expended on the contest. Skaperdas [1991] examined such a contest where the more risk averse players made more effort and as a result had a higher probability of winning.

5. Conclusions

This paper reconsidered self-insurance and self-protection in a framework with rank-dependent expected utility. The comparative static results for expected utility on self-insurance basically carry over to rank-dependent expected utility. Risk-averse individuals have a higher demand for self-insurance, even with background risk. Self-insurance demand in case of multiplicative risks increases (decreases) with wealth if the individual has increasing (decreasing) relative risk aversion. The generally ambiguous results on self-protection carry over also to rank-dependent expected utility, even for Yaari's [1987] special case. However, for risks that occur with very small or very large probabilities, the comparative statics of increases in risk aversion are qualitatively determined.

The paper emphasizes the role of limited liability for self-insurance and self-protection. It shows that limited liability changes many of the comparative static results on self-insurance and self-protection. Self-insurance effort becomes discontinuous and may increase with an increase in wealth, regardless of whether absolute risk aversion is decreasing or increasing. Moreover, limited liability turns out to be crucial for the relation of the level of self-protection and risk aversion. If the minimum wealth that determines limited liability is high enough, then, an increase in risk aversion increases the amount of self-protection that is undertaken.

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Notes

1. In this section we use the framework of Briys and Schlesinger [1990] but add the rank-dependent expected-utility preference hypothesis.
2. Implicitly we assume here that $w - s(y) - y \geq 0$. Later the case with $w - s(y) - y < 0$ that leads to the concept of limited liability will be discussed.
3. Rank-dependent expected utility would be strictly concave in y if $u(.)$ were strictly concave and $s''(.) \geq 0$, as can be seen from the following expression:

$$d^2(E_g u)/(dy)^2 = -g(p)s''(y)\, u_1' + g(p)[1 + s'(y)]^2 u_1'' + [1 - g(p)]u_2''.$$

Note that $s''(.) \geq 0$ implies that reductions in the size of the loss become more difficult as self-insurance activities increase, a very plausible condition.

4. For instance, if the individual with limited liability is a firm, a change of self-insurance expenditure changes the credit default risk of the firm and, in general, changes the credit market conditions the firm faces. However, if banks cannot observe actual expenditure on self-insurance, then the credit conditions—e.g., the rate of interest and the credit line—are exogenous to the firm's self-insurance decision. The bank will assume that the firm chooses the self-insurance expenditure that is optimal from the firm's perspective *given* the credit conditions.
5. This result is also connected to McGuire, Pratt, and Zeckhauser's [1991] paper, in which there is a critical p^* such that for loss probabilities below p^* self-protection is higher for the more risk-averse individuals; the opposite is true for loss probabilities above p^*. In our case there is a critical g^*, or equivalently, a critical $\Psi(g^*)$, such that $\Psi'(g) > 1$ for $g < g^*$ and $\Psi'(g) < 1$ for $g > g^*$.
6. Since $p'(.)$ is nonpositive and no restrictions have been imposed on $p''(.)$ in general, risk aversion is neither necessary nor sufficient for $D < 0$. However, assuming that decreases in the probability of loss become increasingly more difficult (i.e., $p''(.) > 0$) makes more sense than assuming $p''(.) < 0$; in the former case, risk aversion would be sufficient for $D < 0$.
7. The reader is referred to Aumann and Kurz [1977] or to Roth [1979, 1989] for formal derivations of this interpretation and illuminating discussions of the concept.
8. Roth ([1979, pp. 49–52, and especially [1989]) employs this concept in the analysis of Nash bargaining and its connection to alternating offers bargaining (the Nash bargaining solution equates the boldness of the agents involved relative to the disagreement point). Aumann and Kurz [1977] considered the concept because of its applicability in the problem they analyze in that paper (i.e., voting for taxes).

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