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Self-Insurance, Self-Protection and Market Insurance within the Dual Theory of Choice

CHRISTOPHE COURBAGE
Lombard Odier & Cie, Geneva, Switzerland

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Abstract

As demonstrated by Ehrlich and Becker [1972], Expected Utility Theory predicts that market insurance and self-insurance are substitutes, whilst surprisingly, market insurance and self-protection could be complements. This article examines the robustness of this conclusion, as well as its extensions under the Dual Theory of Choice [Yaari, 1987]. In particular, the non-reliability of self-insurance activities, background risk and asymmetric information are considered.

Key words: market insurance, self-insurance, self-protection, Dual Theory

1. Introduction

Many empirical contradictions of the independence axiom [see, e.g., Allais, 1953; Ellsberg, 1961] have led economists to call into question the global validity of Expected Utility (EU) models and to develop new theories of choice under risk. The question is then whether existing results are robust to new models of behaviour under risk. An important class is the Rank Dependant Expected Utility's (RDEU) developed by Quiggin [1982], Chew [1983], Yaari [1987], Allais [1988] and Segal [1989]. The study of optimal insurance demand has known, under these alternatives models, a large success [see Karni, 1992; Machina, 1995; Doherty and Eeckhoudt, 1995; Dupuis and Langlais, 1997; Jaleva, 1997]. Yet analysis of two other risk protections tools, self-insurance (reducing the severity of loss) and self-protection (reducing the probability of loss), have not shown the same developments [except in Konrad and Skaperdas's article, 1993]. The present article tries to fill this gap.

In their reference paper, Ehrlich and Becker [1972] examined, within the EU hypothesis, the interaction between market insurance, self-insurance and self-protection. In line with intuition based on the moral hazard problem, they showed that market insurance and self-insurance are substitutes. Yet, surprisingly, the analysis of self-protection led to different results since they derived that market insurance and self-protection could be complements depending on the level of the probability of loss. Thus, the presence of market insurance may, in fact, increase self-protection activities relative to situation where market insurance is unavailable. This work has led, under EU, to many discussions and extensions [Dionne and Eeckhoudt, 1985; Briys and Schlesinger, 1990; Briys, Schlesinger and Schulenburg, 1991, among others]. The aim of this article is to study the robustness of Ehrlich and Becker's [1972] results, as well as some of their extensions under an alternative model of decision.

We focus on just one rule, Yaari's Dual Theory (DT). While EU assigns a value to a prospect by taking a transformed expectation that is linear in probabilities but non-linear in wealth, DT provides the counterpoint since it reverses the transformation (DT is linear in wealth but non-linear in probabilities). This model allows individuals to subjectively distort the probabilities. So, by reversing the structure of the decision rule, DT provides a convenient test of the robustness of existing results under EU. This theory has been particularly used to test results on production decisions under uncertainty [Demers and Demers, 1990], insurance decisions [Doherty and Eeckhoudt, 1995] and portfolio decisions [Hadar and Seo, 1995]. Konrad and Skaperdas [1993] studied the properties of self-insurance and self-protection under RDEU. Our work differs from theirs in that we are specifically concerned with the interactions between market insurance, self-insurance and self-protection.

The paper is organised as follows. The next section introduces the Dual Theory of Yaari. Sections 3 and 4 discuss respectively self-insurance and self-protection activities in relation to market insurance. Section 5 extends the previous results to a situation of non-reliability of self-insurance activity, background risk and asymmetric information. The final section offers a conclusion.

2. Yaari's Dual Theory

Consider that wealth is represented by a vector $W = (w_1, \ldots, w_n)$ with the probability distribution $p = (p_1, \ldots, p_n)$, and such that $w_1 < \cdots < w_n$. Using $u(\cdot)$ as a transformation of wealth, EU is expressed by:

$$EU(W) = \sum_{i=1}^{n} p_i u(w_i).$$

Risk aversion is denoted by $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

Yaari's Dual Theory provides the counterpoint to EU, since it is linear in wealth, but non-linear in probabilities. Probabilities are weighted by a transformation function $h(\cdot)$, which is defined on the cumulative distribution function over wealth. Thus DT can be represented as:

$$DT(W) = \sum_{i=1}^{n} h_i(p) w_i,$$

with

$$h_i(p) = f\left(\sum_{j=1}^{i} p_j\right) - f\left(\sum_{j=1}^{i-1} p_j\right), \quad \text{where } f(0) = 0,\ f(1) = 1 \quad \text{and} \quad f'(\cdot) > 0.$$

If $n = 2$, $\quad DT(W) = f(p_1)w_1 + (1 - f(p_1))w_2.$

Under DT, the attitude to risk is conveyed entirely in the properties of the transformation function $f(\cdot)$. An individual will be considered as risk averse when f is concave,[1] i.e. $f'' < 0$. This property describes a pessimistic behaviour as the individual reduces subjectively the objective probability of good events and increases that of bad events. Conversely, an optimistic individual underweights low outcome and overweights high outcome.

3. Self-insurance

Let us consider an individual with an initial wealth w_0 subject to a risk of partial loss L with probability q $(0 < q < 1)$. This individual maximises his wealth according to Yaari's Dual Theory preference functional and he is risk averse. The individual may undertake self-insurance activities that reduce the size of the loss, should it occur. Let x denote the level of self-insurance. Its effect is described by the differentiable function $L(x)$ such as $0 \leq L(\cdot) \leq w_0$, which relates the size of the loss to the level of self-insurance activity, with $L'(\cdot) < 0$ and $L''(\cdot) > 0$.[2] The cost of this activity is represented by a monotonic increasing and convex function ($c'(\cdot) > 0$ and $c''(\cdot) \geq 0$).[3] In addition to self-insurance activities, the individual is also able to purchase a co-insurance[4] policy. The individual pays a premium P to have a proportion α of loss insured. He is free to choose α between 0 and 1. Using DT, the value of this wealth prospect is:

$$H = f(q)[w_0 - c(x) - P - (1-\alpha)L(x)] + (1 - f(q))[w_0 - c(x) - P],$$

with

$$P = \alpha(1+\lambda)qL(x).$$

Let λ represent the loading factor to allow for transaction costs and profit. We assume perfect information, i.e. the insurance company is able to directly observe self-insurance activity and prices the premium accordingly.

To know whether self-insurance and market insurance are substitutes or complements,[5] we examine the consumer behaviour with respect to self-insurance, following changes in the "price" of insurance, λ.

By deriving the valuation function with respect to α, we obtain the optimal insurance purchase. The value of the wealth prospect H is linear in α. Then only a corner solution arises [as shown by Doherty and Eeckhoudt, 1995]. Insurance demand will depend on the sign of the following equation:

$$\frac{\partial H}{\partial \alpha} = L(x)[f(q) - (1+\lambda)q].$$

As $L(x) \geq 0$, the sign of $\frac{\partial H}{\partial \alpha}$ depends on $f(q) - (1+\lambda)q$.

$$\text{If } 1+\lambda < f(q)/q, \quad \text{full coverage is optimal } (\alpha = 1). \tag{1}$$

$$\text{If } 1+\lambda > f(q)/q, \quad \text{zero coverage is optimal } (\alpha = 0). \tag{2}$$

For small values of λ, the individual purchases full insurance. But over a critical value of the loading factor he switches to zero coverage. As Doherty and Eeckhoudt [1995] noted, this behaviour of "all" or "nothing" seems validated empirically.

Turning to self-insurance, the optimal demand is given by:

$$\frac{\partial H}{\partial x} = -L'(x) f(q) - c'(x) + \alpha L'(x)[f(q) - (1+\lambda)q] = 0. \tag{3}$$

It can easily be shown that the second-order condition is verified:

$$\frac{\partial^2 H}{\partial x^2} = -L''(x)[f(q)(1-\alpha) + (1+\lambda)q] - c''(x) < 0.$$

Given the discontinuity of the market insurance demand, we get from (3):

$$(1+\lambda)q = -\frac{c'(x)}{L'(x)} \quad \text{if } (1+\lambda) < \frac{f(q)}{q}. \tag{4}$$

$$f(q) = -\frac{c'(x)}{L'(x)} \quad \text{if } (1+\lambda) > \frac{f(q)}{q}. \tag{5}$$

Let $\hat{x}_0$ and $\hat{x}_1$ be respectively the solutions of Eqs. (4) and (5). Equation (4) says that for low values of λ, and given risk aversion behaviour, $\hat{x}_0$ is dependent of λ. Hence, differentiating (3) with respect to λ and $\hat{x}_0$ gives:

$$\operatorname{sgn}\left(\frac{d\hat{x}_0}{d\lambda}\right) = \operatorname{sgn}\left(\frac{\partial^2 H}{\partial x\, \partial \lambda}\right) = \operatorname{sgn}(-L'(x)q) > 0.$$

As the price of insurance increases, the optimal self-insurance demand gets higher. Yet once λ reaches high levels, self-insurance demand becomes independent of λ (see (5)) as the individual does not purchase insurance anymore (see (2)). In order to definitely conclude on substitutability, the comparison of $\hat{x}_0$ and $\hat{x}_1$ is required.

By evaluating $\partial H/\partial x$ when $\alpha = 1$ at $\hat{x}_1$, we obtain:

$$\left.\frac{\partial H}{\partial x}\right|_{\substack{x=1 \\ x=\hat{x}_1}} = L'(x)[f(q) - (1+\lambda)q] < 0 \quad \text{since } 1+\lambda \le f(q)/q,$$

leading to $\hat{x}_0 < \hat{x}_1$.

Clearly, market insurance and self-insurance are substitutes. An increase in the price of insurance pushes up self-insurance activity. The result obtained by Ehrlich and Becker [1972] under EU carries over to DT.

4. Self-protection

We now assume that the individual can invest in self-protection activities y that reduce the probability of loss, but do not affect the size of the loss L, should it occur. The probability of the loss is a decreasing function of the level of self-protection whose marginal productivity is increasing, i.e. $q'(y) < 0$ and $q''(y) > 0$. The cost of self-protection is given by $c(y)$, where $c'(y) > 0$ and $c''(y) \geq 0$. We assume the same hypothesis on the insurance contract, as in the previous section. The valuation function is given by:

$$U = f(q(y))[w_0 - c(y) - P - (1-\alpha)L] + (1 - f(q(y))[w_0 - c(y) - P],$$

with

$$P = \alpha(1+\lambda)q(y)L.$$

Deriving the valuation function with respect to α gives the optimal insurance purchase:

$$\frac{\partial U}{\partial \alpha} = L[f(q(y)) - (1+\lambda)q(y)]. \tag{6}$$

$$\text{If } 1+\lambda > f(q(y))/q(y), \quad \text{zero coverage is optimal } (\alpha = 0).$$

$$\text{If } 1+\lambda < f(q(y))/q(y), \quad \text{full coverage is optimal } (\alpha = 1).$$

The optimal level of self-protection must verify the following first-order condition:

$$\frac{\partial U}{\partial y} = -f'(q(y))q'(y)L - c'(y) + \alpha L q'(y)[f'(q(y)) - (1+\lambda)] = 0. \tag{7}$$

The second-order optimality condition requires:

$$\begin{aligned}\frac{\partial^2 U}{\partial y^2} = {} & -(1-\alpha)(f''(q(y))q'(y)^2 + f'(q(y))q''(y))L - c''(y) \\ & - \alpha(1+\lambda)Lq''(y) < 0.\end{aligned}$$

This condition is not always satisfied for a pessimistic individual ($f(\cdot)$ concave). Nevertheless, we admit $f''(q(y))q'(y)^2 + f'(q(y))q''(y) \geq 0$ in order to satisfy it.

We proceed as in the previous section to stress the interaction between the two tools. Given (6), (7) writes as:

$$q'(y)f'(q(y)) = -\frac{c'(y)}{L} \quad \text{if } 1+\lambda < f(q(y))/q(y). \tag{8}$$

$$(1+\lambda)q'(y) = -\frac{c'(y)}{L} \quad \text{if } 1+\lambda > f(q(y))/q(y). \tag{9}$$

Let $\hat{y}_0$ and $\hat{y}_1$ be respectively the solutions of (8) and (9). Differentiating (7) with respect to λ and $\hat{y}_0$ gives:

$$\operatorname{sgn}\left(\frac{d\hat{y}_0}{d\lambda}\right) = \operatorname{sgn}\left(\frac{\partial^2 U}{\partial y\, \partial \lambda}\right) = \operatorname{sgn}(-Lq'(y)) > 0.$$

When the loading factor is relatively small, market insurance and self-protection are substitutes. Once an upper limit is passed, self-protection becomes independent of λ (see (9)). As for self-insurance, the comparison of $\hat{y}_0$ and $\hat{y}_1$ is required to definitely conclude.

By evaluating $\partial U/\partial y$ when $\alpha = 1$ at $\hat{y}_1$, we obtain:

$$\left.\frac{\partial U}{\partial y}\right|_{\substack{\alpha=1\\ y=\hat{y}_1}} = Lq'(y)[f'(q(y)) - (1+\lambda)]. \tag{10}$$

The function f' corresponds to the slope of f, which is increasing and concave in q. Therefore, as stressed by Konrad and Skaperdas [1993], for a loss that occurs with a high probability, $f' < 1$. If the loss occurs with a low probability, then $f' > 1$.

Hence, from (10), for a high level of the probability of loss $\hat{y}_1 < \hat{y}_0$, leading to complementarity between market insurance and self-protection.

This result has an intuitive interpretation. When the individual is fully insured, as the wealth is the same in both states, the only incentive to increase self-protection is to make the premium decrease. Whereas, when the individual stops purchasing insurance, he will undertake self-protection only in order to reduce the realisation of the loss. Yet as the individual "transforms" the probability, the perception of the variation of the occurrence of the loss depends on the level of the probability. As a matter of fact, as f is concave, the higher the probability of loss is, the smaller the impact of the variation of q on f is. For high values of q, the individual underestimates the variation of q. Hence self-protection activity is not perceived to strongly reduce the occurrence of the loss. Conversely, for low values of q, the individual overestimates the variation of q. Self-protection is perceived as strongly reducing the occurrence of the loss. As self-protection is a costly activity, the individual has more incentive to practice it for a low probability of loss than for a high one.

For a high probability of loss, when the individual stops purchasing insurance, self-protection is discouraged because its marginal gain with insurance is superior to the one without insurance.

If Ehrlich and Becker [1972] exhibited the same result under EU, they could not interpret the importance of the level of the probability of loss. The specific properties of DT allow filling this gap.

5. Extensions

In this section, extensions and developments of the previous results are considered. As the methodology used is the same as in Sections 3 and 4, the calculations are developed in the appendices.

1. *Non-reliability of self-insurance.* The self-insurance mechanism can be compared with a sprinkler system: it might be inoperative or be destroyed during a fire. Under EU, Briys, Schlesinger and Schulenburg [1991] showed that market insurance and self-insurance could be complements for non-reliable self-insurance activity. DT leads to the same puzzle and does not provide more insights than EU.

2. *Background risk*. Under DT, the introduction of background risk makes it possible to restore interior solutions for insurance [see Doherty and Eeckhoudt, 1995]. It also modifies numerous classical insurance results under EU [see Doherty and Schlesinger, 1983; Gollier and Pratt, 1996, among others]. It is then legitimate to wonder if the previous results are still valid in the presence of background risk. It turns out that market insurance and self-insurance are also substitutes in the presence of an independent background risk. If we consider a non-independent background risk either non-negatively or non-positively correlated, this result is still valid. A positive correlation tends to increase the demand for insurance whereas a negative correlation acts as a substitute for insurance and tends to decrease the demand for insurance [see Doherty and Eeckhoudt, 1995, p. 170]. Yet, the levels of self-insurance associated with these demands as well as their position remain the same.
3. *Asymmetric information*. For some contracts, the premium is completely independent of prevention activity. The French system of natural catastrophes insurance is a good illustration as the contribution rate, fixed by the government, is the same for everybody in the whole country. Moreover, insurance companies are not necessarily able to directly observe the voluntary actions of individuals and thus cannot price the premium accordingly. If the consideration of asymmetric information does not modify our result on self-insurance, it always leads to substituability between self-protection and market insurance. This point is easily explainable. As the premium is independent of self-protection activity, the individual, when he is fully insured, no longer has incentive to practice self-protection activity since the impact on the premium is nil.

6. Conclusion

This paper has reconsidered the relationships existing between market insurance and respectively self-insurance and self-protection in the context of Yaari's Dual Theory. The results for EU on self-insurance carry over to DT. Market insurance and self-insurance are substitutes, even with background risk. They can be complements when reliability of self-insurance activity is not guaranteed. The generally ambiguous link between market insurance and self-protection carry over also to DT. However, this result is easily explainable by the role of the transformation function in under- or overestimating probabilities and their variation.

This paper also considered the situation where the insurance company may not price the premium according to effective self-insurance and self-protection activities. Naturally, in that case market-insurance and self-protection are substitutes.

The aim of this article was not to stress the superiority of a decision model over another; but rather to cultivate their differences to test the robustness of the existing results. As a consequence, we can conclude that self-insurance and self-protection results, in conjunction with market insurance, are robust for the relaxation of the EU hypothesis.

Appendix A: Market insurance and risky self-insurance

Uncertainty about self-insurance activities is introduced by considering the loss, if it occurs, as $L(\varepsilon x)$, where ε, the random prospect, takes values 0 and 1 with probabilities p and

$(1-p)$ respectively. We assume full information about ε and that this information is taken into account in the insurance premium.

The wealth valuation function of the individual is given by:

$$T = f(pq)[w_0 - c(x) - (1-\alpha)L(0) - P] + (f(q) - f(pq))[w_0 - c(x) - (1-\alpha)L(x) - P] + (1 - f(q))[w_0 - c(x) - P],$$

with

$$P = \alpha(1+\lambda)q(pL(0) + (1-p)L(x)).$$

Deriving the valuation function with respect to α gives the optimal insurance purchase:

$$\frac{\partial T}{\partial \alpha} = f(pq)L(0) + (f(q) - f(pq))L(x) - (1+\lambda)q(pL(0) + (1-p)L(x)).$$

$$\text{If } 1+\lambda < \frac{pqL(0) + (f(q) - f(pq))L(x)}{pqL(0) + (q - pq)L(x)}, \quad \text{full coverage is optimal } (\alpha = 1).$$

$$\text{If } 1+\lambda > \frac{pqL(0) + (f(q) - f(pq))L(x)}{pqL(0) + (q - pq)L(x)}, \quad \text{zero coverage is optimal } (\alpha = 0).$$

The optimal level of self-protection must verify the following first-order condition:

$$\frac{\partial T}{\partial x} = -\alpha(1+\lambda)q(1-p)L'(x) - c'(x) - (f(q) - f(pq))(1-\alpha)L(x) = 0.$$

We easily show that the second-order conditions are satisfied.

By evaluating the first derivative of T in $\alpha = 0$ and $\alpha = 1$, we define the optimal self-insurance demand associated with the different levels of insurance:

$$\left.\frac{\partial T}{\partial x}\right|_{\alpha=1} = -c'(x) - (1+\lambda)q(1-p)L'(x) = 0. \tag{11}$$

$$\left.\frac{\partial T}{\partial x}\right|_{\alpha=0} = -c'(x) - (f(q) - f(pq))L'(x) = 0. \tag{12}$$

Let $\bar{x}_0$ and $\bar{x}_1$ be respectively the solutions of Eqs. (11) and (12). Unfortunately, we cannot usefully compare these two levels. Consider then the following example:

$$q = 0.4,\ p = 0.375,\ L(0) = 2,\ L(x) = L(0)e^{-x},\ c(x) = 0.2x \quad \text{and} \quad f(q) = \sqrt{q}.$$

We obtain $\bar{x}_0 = 1.32$ and $\bar{x}_1 = 1.03$. From this numerical example, the two tools can be considered as complements.

Appendix B: Background risk and self-insurance

We suppose that the background risk is independent of self-insurance activity. Initial wealth takes the value w_1 with a probability p and w_2 with a probability $1 - p$, such that $w_1 < w_2$. In order to define the valuation function, we rank the wealth levels in descending order. The four wealth levels with their respective probabilities are shown in Table 1:

Table 1. Wealth and the associated probability.

Wealth	Probability
$A = w_1 - c(x) - \alpha(1 + \lambda)qL(x) - (1 - \alpha)L(x)$	pq
$B = w_1 - c(x) - \alpha(1 + \lambda)qL(x)$	$p(1 - q)$
$C = w_2 - c(x) - \alpha(1 + \lambda)qL(x) - (1 - \alpha)L(x)$	$(1-p)q$
$D = w_2 - c(x) - \alpha(1 + \lambda)qL(x)$	$(1 - p)(1 - q)$

Following Doherty and Eeckhoudt (1995), three cases are possible.

Case 1. $1 - \dfrac{w_2 - w_1}{L(x)} < \alpha < 1.$

Given that $A < B < C < D$, the valuation function writes as:

$$\begin{aligned} M = {} & f(pq)[w_1 - c(x) - (1 - \alpha)L(x) - P] + (f(p) - f(pq))[w_1 - c(x) - P] \\ & + (f(q + p - pq) - f(p))[w_2 - c(x) - (1 - \alpha)L(x) - P] \\ & + (1 - f(q + p - pq))[w_2 - c(x) - P], \end{aligned}$$

with

$$P = \alpha(1 + \lambda)qL(x).$$

Case 2. $0 \leq \alpha < 1 - \dfrac{w_2 - w_1}{L(x)}.$

Given that $A < C < B < D$, the valuation function writes as:

$$\begin{aligned} N = {} & f(pq)[w_1 - c(x) - (1 - \alpha)L(x) - P] \\ & + (f(q) - f(pq))[w_2 - c(x) - (1 - \alpha)L(x) - P] \\ & + (f(q + p - pq) - f(q))[w_1 - c(x) - P] \\ & + (1 - f(q + p - pq))[w_2 - c(x) - P], \end{aligned}$$

with

$$P = \alpha(1 + \lambda)qL(x).$$

Case 3. $\alpha = 1 - \dfrac{w_2 - w_1}{L(x)}$.

Given that $A < C = B < D$, the valuation function writes as:

$$V = (w_1 - w_2)(f(pq) + f(q + p - pq)) + w_2 - c(x) - (1 + \lambda)qL(x).$$

The introduction of a background risk authorises a corner solution [see Doherty and Eeckhoudt, 1995]. The optimal demand is:

(i) $\alpha = 0$ if $\lambda > \frac{f(q)}{q} - 1$.
(ii) $\alpha = 1 - \frac{w_2 - w_1}{L(x)}$ if $\frac{f(pq) - f(p) + f(q+p-pq)}{q} - 1 < \lambda < \frac{f(q)}{q} - 1$.
(iii) $\alpha = 1$ if $\lambda < \frac{f(pq) - f(p) + f(q+p-pq)}{q} - 1$.

These results are only valid if $w_2 - w_1 < L(x)$. Let λ_0, λ_m and λ_1 be respectively the values of the loading factor for (i), (ii) and (iii).

Let $\breve{x}(0)$, $\breve{x}(m)$ and $\breve{x}(1)$ be the levels of self-insurance activity corresponding respectively to insurance demand (i), (ii) and (iii) and given by:

$$\frac{\partial N}{\partial x} = -f(q)L'(x) - c'(x) = 0.$$
$$\frac{\partial V}{\partial x} = -c'(x) - (1 + \lambda_m)qL'(x) = 0.$$
$$\frac{\partial M}{\partial x} = -c'(x) - (1 + \lambda_1)qL'(x) = 0.$$

To compare these levels, we evaluate the first-order condition on x at the different insurance levels, giving:

$$\left.\frac{\partial M}{\partial x}\right|_{x=\breve{x}(1)} = L(x)[(1 + \lambda_1)q - f(q)].$$

We know that $(1 + \lambda_1)q < f(pq) - f(p) + f(q + p - pq)$, and from the concavity of f and for $p < q$ we have $f(pq) - f(p) + f(q + p - pq) < f(q)$. Then $\frac{\partial N}{\partial x}|_{x=\breve{x}(1)} > 0$, which implies $\breve{x}(0) > \breve{x}(1)$.

$$\left.\frac{\partial M}{\partial x}\right|_{x=\breve{x}(m)} = qL(x)[\lambda_m - \lambda_1] < 0, \text{ which implies } \breve{x}(m) > \breve{x}(1).$$
$$\left.\frac{\partial N}{\partial x}\right|_{x=\breve{x}(m)} = L(x)[(1 + \lambda_m)q - f(q)] > 0, \text{ which implies } \breve{x}(0) > \breve{x}(m).$$

So $\breve{x}(0) > \breve{x}(m) > \breve{x}(1)$.

Appendix C: Asymmetric information

Self-insurance

In the insurer is not able to observe individual self-insurance activity, the valuation function is:

$$K = f(q)[w_0 - c(x) - P - (1-\alpha)L(x)] + (1 - f(q))[w_0 - c(x) - P],$$

with

$$P = \alpha(1+\lambda)qL.$$

Deriving the valuation function with respect to α gives the optimal insurance purchase:

$$\frac{\partial K}{\partial \alpha} = -(1+\lambda)qL + f(q)L(x). \tag{13}$$

$$\text{If } 1+\lambda > f(q)L(x)/qL, \quad \text{zero coverage is optimal } (\alpha = 0).$$

$$\text{If } 1+\lambda < f(q)L(x)/qL, \quad \text{full coverage is optimal } (\alpha = 1).$$

The optimal level of self-insurance must verify the following first-order condition:

$$\frac{\partial K}{\partial x} = -(1-\alpha)f(q)L'(x) - c'(x) = 0. \tag{14}$$

The second-order condition is given by:

$$\frac{\partial^2 K}{\partial x^2} = -(1-\alpha)f(q)L''(x) - c''(x) < 0.$$

Given (13), (14) writes as:

$$-c'(x) = 0 \qquad \text{if } 1+\lambda < f(q)L(x)/qL. \tag{15}$$

$$f(q)L'(x) = -c'(x) \quad \text{if } 1+\lambda > f(q)L(x)/qL. \tag{16}$$

Let $\bar{x}_0$ and $\bar{x}_1$ be respectively the solutions of (15) and (16). Note that both are independent of the price of insurance.

By evaluating $\partial K/\partial x$ when $\alpha = 1$ at $\bar{x}_1$, we obtain:

$$\left.\frac{\partial K}{\partial x}\right|_{\substack{\alpha=1 \\ x=\bar{x}_1}} = f(q)L'(x) < 0, \quad \text{leading to } \bar{x}_0 < \bar{x}_1.$$

Self-protection

In this case, the valuation function is:

$$B = f(q(y))[w_0 - c(y) - P - (1-\alpha)L] + (1 - f(q(y))[w_0 - c(y) - P],$$

with

$$P = \alpha(1+\lambda)qL.$$

Deriving the valuation function with respect to α gives the optimal insurance purchase:

$$\frac{\partial B}{\partial \alpha} = L[f(q(y)) - (1+\lambda)q]. \tag{17}$$

$$\text{If } 1+\lambda > f(q(y))/q, \quad \text{zero coverage is optimal } (\alpha - 0).$$

$$\text{If } 1+\lambda < f(q(y))/q, \quad \text{full coverage is optimal } (\alpha - 1).$$

The optimal level of self-protection must verify the following first-order condition:

$$\frac{\partial B}{\partial y} = -f'(q(y))q'(y)(1-\alpha)L - c'(y) = 0. \tag{18}$$

The second-order optimality condition requires:

$$\frac{\partial^2 B}{\partial y^2} = -(1-\alpha)(f''(q(y))q'(y)^2 + f'(q(y))q''(y)L - c''(y) < 0.$$

As in Section 4, we admit $f''(q(y))q'(y)^2 + f'(q(y))q''(y) \geq 0$ in order to satisfy it.

Given (17), (18) writes as:

$$-c'(y) = 0 \qquad \text{if } 1+\lambda < f(q(y))/q. \tag{19}$$

$$f'(q(y))q'(y) = -c'(y) \quad \text{if } 1+\lambda > f(q(y))/q. \tag{20}$$

Let $\bar{y}_0$ and $\bar{y}_1$ be respectively the solutions of (19) and (20). By evaluating $\partial B/\partial y$ when $\alpha = 1$ at $\bar{y}_1$, we obtain:

$$\left.\frac{\partial B}{\partial y}\right|_{\substack{\alpha=1\\ y=\bar{y}_1}} = f'(q(y))q'(y) < 0, \text{ leading to } \bar{y}_1 > \bar{y}_0.$$

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Notes

1. Note that this characterisation of risk aversion corresponds to "strong" risk aversion [see Chateauneuf and Cohen, 1994], i.e. risk aversion to any increase in risk in the sense of Rothschild and Stiglitz [1970] under EU. As shown by Cohen [1995] and Chateauneuf and Cohen [1994] "strong" risk aversion under DT leads to "weak" risk aversion [i.e. risk aversion in the sense of Arrow, 1965 and Pratt, 1964 under EU]. See also Dupuis and Langlais [1997].
2. We assume that reduction in the size of loss becomes more difficult as self-insurance activities increase, which is quite a natural assumption.
3. This assumption ensures that the second-order conditions are necessarily satisfied.
4. We consider a co-insurance contract for tractable reason. The same result applies for a deductible insurance.
5. As in Ehrlich and Becker [1972] and Briys, Schlesinger and Schulenburg [1991], we are here referring to what are called gross substitutes and gross complements, i.e. any pair of goods (i, j) for which $\partial x_i/\partial p_j > 0$, where x_i corresponds to the Marshalian demand for good i and p_j the price of good j corresponds to the former; any pair for which the opposite holds is called gross complements.

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