Measuring Value

We now consider the two primary sources of value for the insured:

- The ability to deploy capital that it would have otherwise had to set aside for an insurable event.
- Reduced volatility of earnings and the resulting reduction in required return.

We review the first source of value creation in Section X.1 and the second source in Section X.2.

In our review, we assume the following:

- There is a fixed amount of capital, C, available to the firm.
- The firm requires minimum capital, C_m, to continue as a going concern.
- We denote deployed capital, i.e., the working capital, as C_w.
- The firm is subject to the loss events that result in aggregate claim amounts, X. We present distribution of claim amounts in Figure X[1].
  - For simplicity, we assume information symmetry as respects X. That is, the insurer and the insured use the same distribution for X.
  - We denote the maximum probable claim amount as X_m.
  - The scale of X is a function of C_w.

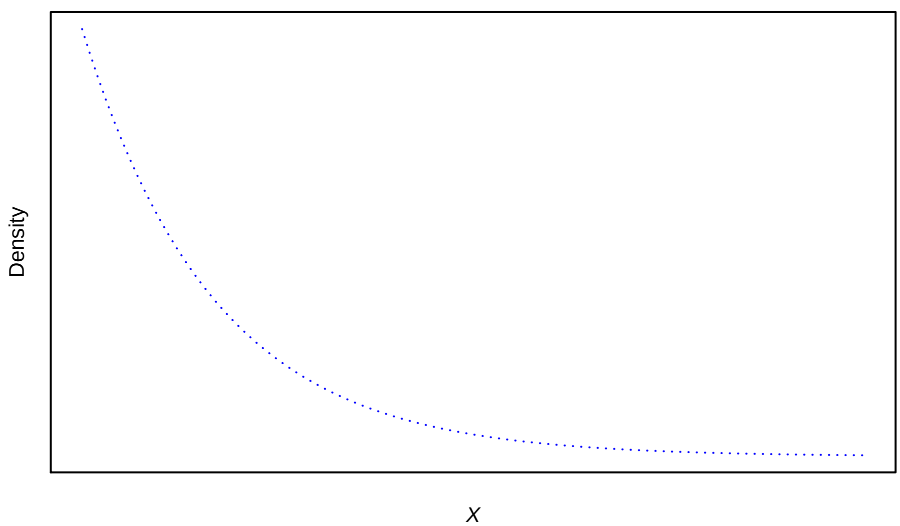


Figure 1: Claim (X) Distribution

- The firm has an enterprise risk management (ERM) strategy underlying its capital allocation. Its ERM strategy dictates that it can absorb a claim at the p^{th} percentile of the distribution of X which we denote as X_p.
- The firm is able to generates a return on capital of r_c.
- We denote the risk free rate r_f.

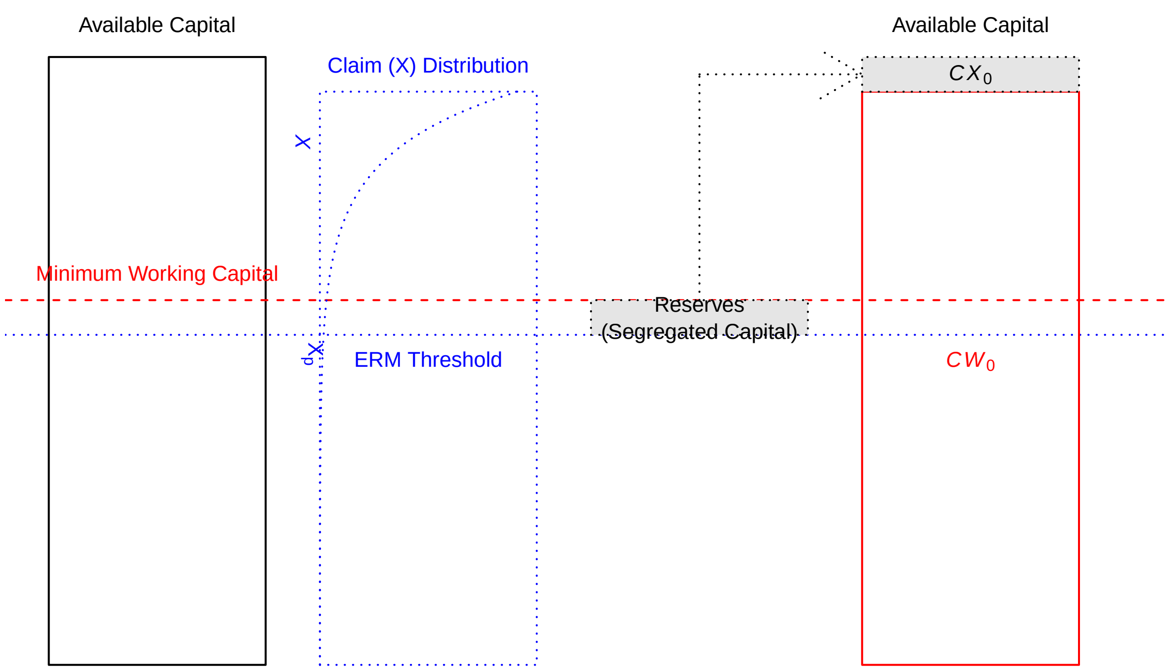
[1]This is an illustrative claim distribution that we present to support further development of value measurement.

Capital Deployment

In order to measure of the value of insurance related to the firm's ability to deploy capital, we compare rates of return both with and without insurance.

Rate of Return without Insurance

The firm's ERM strategy will require that it set aside a reserve in case it was to experience a capital depletion event that reduces. We denote this reserve as C_x, to indicate that the reserve is a seggregation of capital to absorb realizations of X that reduce C below C_m. We present that capital allocation approach in Figure X.



Using the naught subscript to represent this base case, the rate of return is then as follows:

$$R_0 = \frac{r_c \times CW_0 + r_f \times CX_0 - X}{C}$$

The numerator is the sum of the return on working capital, the return on the reserve less the value of the claim amount. The inclusion of the random variable X indicates that the return is a function of the realized value of the loss event.

Rate of Return with Insurance

If instead, the firm purchases insurance to cover a portion of loss event for a certain premium, using the subscript, I, to represent the "insurance case," then the rate of return is as follows:

$$R_I = \frac{r_c \times (CW_I - P) + r_f \times CX_I - X_{ret} - P}{C}$$

where X_{ret} represents portion of the distribution of claims values that the firm retains. We express X_{ret} using the convention used to specify insurance policies. That convention specifies the layer as the limit of the insurance payment, L, in excess of the attachment point, AP of the coverage. We allow X_{ret} (i.e. to L) vary between 0 (equal to the no insurance case) and X_m (in which case, the firm fully transfers the risk). We observe that for $0 < X_{ret} < (X_p - C_m)$, $C_x = 0$.

P is the sum of the expected loss $E[X - X_{ret}]$ and an insurance charge. The insurance charge contemplates the insurer for underwriting the exposure and provides for a return on its capital. Therefore we expect the insurance charge to be a function of $Var(X - X_{ret})$. However, the firm only observes premiums quoted under various options. As such, it is not concerned with the function the insurer uses to develop its risk charge.

Value Creation

We observe that the purchase insurance creates value when:

$$\begin{aligned}
R_I &> R_0 \\
r_c \times (C - P - CX_I) + r_f \times CX_I - X_{ret} - P &> r_c \times CW_0 + r_f \times CX_0 - X \\
P + X_{ret} &< r_c \times (C - P - CX_I - CW_0) + r_f \times (CX_I - CX_0) + X \\
P &< r_c \times (C - P - CX_I - CW_0) + r_f \times (CX_I - CX_0) + X - X_{ret}
\end{aligned}$$

To simplify this equation:

- For the rational firm (where $r_c > r_f$[2]) purchases insurance such that it need not set aside any capital. Therefore, $CX_I = 0$ and $CW_I = C - P$.
- We note that the expected value of X in high excess layers is nearly 0. That is, $E[X] \approx 0$ and $E[X_{ret}] \approx 0$. Moreover, $E[X - X_{ret}] \approx 0$. As a result, taking expectations, our value equation reduces to:

$$\begin{aligned}
P &< r_c \times (C - P - CW_0) + r_f \times (-CX_0) \\
P &< r_c \times (-P) + r_c \times CX_0 + r_f \times (-CX_0) \\
P \times (1 + r_c) &< CX_0 \times (r_c - r_f) \\
P &< CX_0 \times \frac{(r_c - r_f)}{(1 + r_c)}
\end{aligned}$$

The equation has a straightforward intuitive interpretation that the insurance transaction creates value when the premium is less than the excess return on capital that would be earned on the reserve reduced for the return on capital if the premium amount were also deployed as capital.

[2]Without this condition there is no economic value to the firm's existence.

Risk Premium Rate

Approach 1: Penalty function on Rate of Return (R_0 and R_1),

I write 2 possible ways to apply the penalty function, we can pick a better one

Without insurance

I only assume X and r_c are random variables

r_c follows a lognormal distribution or any other reasonable distributions

About X, I first define a variable $X_{original}$ with $E(X_{original}) \approx 0$, and $var(X_{original}) \neq 0$

$$X = \begin{cases} X_{original} & X_{original} <= CX_0 \\ X_{original} + (1 + F_{penalty}) \times (X_{original} - CX_0) & X_{original} > CX_0 \end{cases}$$

Then, we can calculate $E(r_c)$, $var(r_c)$, then the $E(r_0)$, $var(r_0)$

$$\begin{aligned} E(R_0) &= E(\frac{r_c \times CW_0 + r_f \times CX_0 - X}{C}) \\ &= \frac{CW_0 \times E(r_c)}{C} - \frac{E(X)}{C} + \frac{r_f \times CX_0}{C} \\ &= \frac{CW_0 \times E(r_c)}{C} + \frac{r_f \times CX_0}{C} \\ &= \frac{(C - CX_0) \times E(r_c)}{C} + \frac{r_f \times CX_0}{C} \end{aligned}$$

$$\begin{aligned} var(R_0) &= var(\frac{r_c \times CW_0 + r_f \times CX_0 - X}{C}) \\ &= \frac{CW_0^2 \times var(r_c)}{C^2} + \frac{var(X)}{C^2} - \frac{2 \times CW_0 \times cov(X, r_c)}{C^2} \\ &= \frac{(C - CX_0)^2 \times var(r_c)}{C^2} + \frac{var(X)}{C^2} \end{aligned}$$

Then, we can use Sharpe Ratio ($\frac{E(\ R) - r_f}{\sigma_R}$), which shows every unit of volatility would bring how much of the risk premium rate of return. we can calculate the optimal reserve amount(CX_0) using Sharpe Ratio as criterion at this step.
The higher the Sharpe Ratio, the better the investment is.

$$\begin{aligned} Sharpe\ Ratio &= \frac{E(\ R_0) - r_f}{\sigma_{R_0}} \\ &= \frac{(C - CX_0) \times E(r_c) + r_f \times CX_0 - r_f * C}{C} / (\frac{(C - CX_0)^2 \times var(r_c) + var(x)}{C^2})^{0.5} \\ &= \frac{(C - CX_0) \times (E(r_c) - r_f)}{((C - CX_0)^2 \times var(r_c) + var(x))^{0.5}} \end{aligned}$$

With Insurance

Then we calculate $E(r_I)$, $var(r_I)$

$$\begin{aligned}
E(R_I) &= E(\frac{r_c \times (CW_I - P) + r_f \times CX_I - X_{ret} - P}{C}) \\
&= \frac{(CW_I - P) \times E(r_c)}{C} - \frac{E(X_{ret})}{C} + \frac{r_f \times CX_I - P}{C} \\
&= \frac{(CW_I - P) \times E(r_c))}{C} + \frac{r_f \times CX_I - P}{C} \\
&= \frac{(C - P) \times E(r_c)) - P}{C}
\end{aligned}$$

$$\begin{aligned}
var(R_I) &= var(\frac{r_c \times (CW_I - P) + r_f \times CX_I - X_{ret} - P}{C}) \\
&= \frac{(CW_I - P)^2 \times var(r_c)}{C^2} + \frac{var(X_{ret})}{C^2} - \frac{2 \times (CW_I - P) \times cov(X_{ret}, r_c)}{C^2} \\
&= \frac{(C - P)^2 \times var(r_c)}{C^2} + \frac{var(X_{ret})}{C^2}
\end{aligned}$$

Then We can calculate the Sharpe Ratio under the with Insurance case

$$\begin{aligned}
Sharpe\ Ratio &= \frac{E(\ R_I) - r_f}{\sigma_{R_0}} \\
&= \frac{(C - P) \times E(r_c)) - P - C \times rf}{C} / (\frac{(C - P)^2 \times var(r_c) + var(X_{ret})}{C^2})^{0.5} \\
&= \frac{C \times (E(r_c) - r_f) - P \times (1 + E(r_c))}{((C - P)^2 \times var(r_c) + var(X_{ret}))^{0.5}}
\end{aligned}$$

Finally, we can simplify and compare these 2 Sharpe Ratio. I haven't gone through strict calculation yet, but via simple comparison, we can find that with the increase of var(x), the 'with insurance' Sharpe Ratio will decrease more slowly than 'without insurance', which shows the value of insurance.

P.S. When we compare the expected returns of 2 cases, the result would be exactly the same as the result in Value Creation Section. So I didn't attach it one more time.

Approach 2: Penalty function on Rate of Capital (R_c), unfinished

This approach would be more difficult in calculation

Without insurance

I only assume X and r_c are random variables
About X, I assume a distribution with $E(X) \approx 0$, and $var(X) \neq 0$
Then, we assume r_c follows a conditional lognormal distribution based on X

$$r_c | X \backsim \begin{cases} lognormal(\mu_0, \sigma_0) & X <= CX_0 \\ lognormal(\mu_0(1 - \frac{X - CX_0}{CW_0}), \sigma_0/(1 - \frac{X - CX_0}{CW_0})) & X > CX_0 \end{cases}$$

Then, we can calculate $E(r_c)$, $var(r_c)$, then the $E(r_0)$, $var(r_0)$

$$E(r_c) = E(E(r_c|X))$$

$$var(r_c) = E(var(r_c|X)) + var(E(r_c|X))$$

$$\begin{aligned}
E(R_0) &= E(\frac{r_c \times CW_0 + r_f \times CX_0 - X}{C}) \\
&= \frac{CW_0 \times E(r_c)}{C} - \frac{E(X)}{C} + \frac{r_f \times CX_0}{C} \\
&= \frac{CW_0 \times E(E(r_c|X))}{C} - \frac{E(X)}{C} + \frac{r_f \times CX_0}{C} \\
&= \frac{CW_0 \times E(E(r_c|X))}{C} + \frac{r_f \times CX_0}{C}
\end{aligned}$$

$$\begin{aligned}
var(R_0) &= var(\frac{r_c \times CW_0 + r_f \times CX_0 - X}{C}) \\
&= \frac{CW_0^2 \times var(r_c)}{C^2} + \frac{var(X)}{C^2} - \frac{2 \times CW_0 \times cov(X, r_c)}{C^2} \\
&= \frac{CW_0^2 \times (E(var(r_c|X)) + var(E(r_c|X)))}{C^2} + \frac{var(X)}{C^2} - \frac{2 \times CW_0 \times (E(r_c \times X) - E(E(r_c|X)) \times E(X))}{C^2} \\
&= \frac{CW_0^2 \times (E(var(r_c|X)) + var(E(r_c|X)))}{C^2} + \frac{var(X)}{C^2} - \frac{2 \times CW_0 \times (E(r_c \times X))}{C^2}
\end{aligned}$$

With Insurance

$$r_c|X_{ret} \backsim \begin{cases} lognormal(\mu_0, \sigma_0) & X_{ret} <= CX_I \\ lognormal(\mu_0(1 - \frac{X - CX_0}{CW_0}), \sigma_0/(1 - \frac{X - CX_0}{CW_0})) & X_{ret} > CX_I \end{cases}$$

Then we calculate $E(r_I)$, $var(r_I)$

$$\begin{aligned}
E(R_I) &= E(\frac{r_c \times (CW_I - P) + r_f \times CX_I - X_{ret} - P}{C}) \\
&= \frac{(CW_I - P) \times E(r_c)}{C} - \frac{E(X_{ret})}{C} + \frac{r_f \times CX_I - P}{C} \\
&= \frac{(CW_I - P) \times E(E(r_c|X_{ret}))}{C} - \frac{E(X_{ret})}{C} + \frac{r_f \times CX_I - P}{C} \\
&= \frac{(CW_I - P) \times E(E(r_c|X_{ret}))}{C} + \frac{r_f \times CX_I - P}{C}
\end{aligned}$$

$$\begin{aligned}
var(R_I) &= var(\frac{r_c \times (CW_I - P) + r_f \times CX_I - X_{ret} - P}{C}) \\
&= \frac{(CW_I - P)^2 \times var(r_c)}{C^2} + \frac{var(X_{ret})}{C^2} - \frac{2 \times (CW_I - P) \times cov(X_{ret}, r_c)}{C^2} \\
&= \frac{(CW_I - P)^2 \times (E(var(r_c|X_{ret})) + var(E(r_c|X_{ret})))}{C^2} + \frac{var(X_{ret})}{C^2} - \frac{2 \times (CW_I - P) \times (E(r_c \times X_{ret}) - E(E(r_c|X}{C^2} \\
&= \frac{(CW_I - P)^2 \times (E(var(r_c|X_{ret})) + var(E(r_c|X_{ret})))}{C^2} + \frac{var(X_{ret})}{C^2} - \frac{2 \times (CW_I - P) \times (E(r_c \times X_{ret})}{C^2}
\end{aligned}$$