On the Value of High-Excess Commercial Insurance

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Insurance serves both as a source of loss reimbursement and contingent capital. The expected value of claims (i.e., loss reimbursement value) for the insured in high excess layers is minimal, so the principal value of insurance in those layers is as a source of contingent capital. This paper describes an approach to measure this source of value.

# Introduction

In this paper, we provide a basis to measure the value provided by high-excess layer insurance policies. Our focus in this paper is the highest excess layers purchased by large commercial firms. Insured can compare the value and the premium for the insurance policy to understand whether the insurance purchase is accretive.

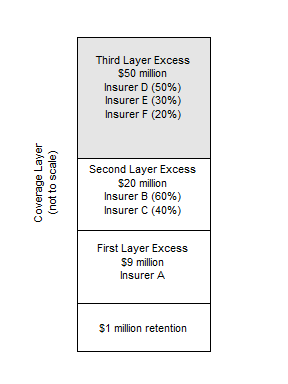
## Commercial Insurance Programs

Commercial insurance programs are often depicted as “towers” (See Figure ). Extending this analogy, risk retained by the insured typically comprised the lowest floors of the tower . The middle floors include commercially-insured working layers. High excess layer insurance resides in the penthouse.

Each of these layers has a cost to it. Insurers establish premiums which are sufficient to cover claim costs, expenses and return a profit to its shareholder.

The profit provision will be proportional to the volatility of the risk. That is, the required supporting insurer capital is directly proportional to the volatility of the business. As a result, more volatile business will require additional profit to satisfy the insurer’s shareholder. Consistent with this notion, we ignore expenses and refer to the profit provision as a capital charge in describing the cost for various insurance layers below.

* The cost of the lowest layers will have a reasonable degree of predictability. Because costs are predictable, the value of insurance is minimal, and insureds will, therefore, typically retain this risk.[[1]](#footnote-20) The cost for these layers will principally be the expected loss with a minimal capital charge for volatility. There may also be frictional costs associated with risk retention, such as increased audit and actuarial fees and claims costs from legal counsel and third-party claim administrators.
* The middle layers provide coverage for claims related to events that occur with a degree of regularity. The amount of claims may not be reasonably predictable in a single year but would be predictable over a longer time horizon. The insurance coverage provides value to the insured by smoothing claim volatility . There may be multiple insurers in the higher working layers.
* Insureds generally do not expect to access coverage from the highest layers. The expected claims will likely be close to $0. The variance of claims in this layer, however, may be significant. Insurance in this layer smooths claims volatility . As a result of the need to smooth volatility across risks, there are often multiple insurers in these layers, each with a pro-rata share. These high excess layers are the focus of this paper.



Example Insurance Tower

## The Value of High Excess Layers

Despite minimal expected claim values, premiums in these layers can be significant. Sometimes buyers refer to these layers as “sleep insurance”; that is, the insurance allows the buyer to sleep at night. In this paper, we describe an approach that the buyer can consider to understand whether there is *financial value*[[2]](#footnote-23) to the insurance transaction.

To assess the value, we should understand that the primary benefit of this layer is not in the loss reimbursement but the protection of capital that this layer of insurance provides to the insured.

* Insureds often consider the insurance in this layer as part of its capital structure rather than as an operational cost. The insurance allows the firm to deploy the capital that it would otherwise need to set aside to weather an extreme loss event.
* In addition to this protection of capital, insurance will also reduce earning volatility and potentially improve the attractiveness of the firm to investors.

In this paper, we will quantify these benefits of insurance.

## Literature Search

Most research related to capital in the insurance transaction focuses on the return on capital rather than the protection of the capital. The limited research related to the insured’s preference focuses on utility theory (as a function of wealth), and that research is generally presented in the context of personal insurance coverages (such as homeowners).

Capital protection and wealth maximization are related concepts, and utility functions (generically) provide a basis to assess alternatives. This is not dissimilar to the approach that we present in this paper. However, there are differences to the research we present:

* Utility functions are typically abstract, whereas the measurement approach we present in this paper is more specifically defined.
* We frame our approach in the context of commercial insurance and present two value considerations for a commercial insured.

We list the papers that we reviewed in Appendix .

## Presentation Outline

In this paper, we consider the two primary sources of value for the insured:

* The ability to deploy capital that it would have otherwise had to set aside for an insurable event. We present our review of this source of value in Section .
* Reduced volatility of earnings and the resulting reduction in required return. We present our review of this source of value in Section .

We use the Sharpe Ratio in our analysis in Section . As we will discover from this analysis, the measured value from the Sharpe Ratio will the value related to capital deployment.

# Capital Deployment

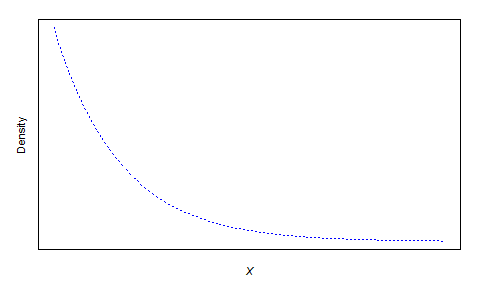
In this section, we describe how high-excess insurance allows the insured to deploy capital that it would otherwise need to set aside to weather an extreme loss event.

The analysis we present does not relate to the entire capital structure of the firm but rather only that portion allocated to activities that are subject to the risk of claims. In order to measure the value of insurance related to the firm’s ability to deploy that capital, we compare rates of return when the firm “funds” the claims risk without insurance (i.e., with capital) and with insurance.

## Funding Risk using Capital

We first present an analysis to demonstrate why a firm would find it necessary to set aside capital; we do not intend this analysis to be a prescriptive approach for determining the amount of capital to set aside. In our review, we assume the following:

* There is a fixed amount of capital, , available to the firm.
* The firm requires a minimum level of capital, , to continue as a going concern.
* We denote deployed capital, i.e., the working capital, as .
* The firm is subject to the loss events that result in aggregate claim amounts, . We present the distribution of claim amounts in Figure [[3]](#footnote-28).
  + For simplicity, we assume information symmetry as respects . That is, the insurer and the insured use the same distribution for .
  + We denote the maximum probable claim amount as .
  + The scale of is a function of .



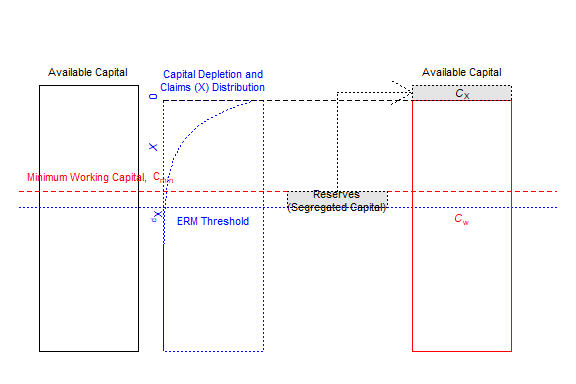
Illustrative Claim (X) Distribution

* The firm has an enterprise risk management (ERM) strategy underlying its capital allocation. Its ERM strategy dictates that it can absorb a claim at the percentile of the distribution of , which we denote as .
* The firm is able to generate a return on capital of . For purposes of determining the value of insurance, we use a simplifying assumption that is fixed. That is, and .
* We denote the risk-free rate .

## Rate of Return without Insurance

When a firm retains risk, it must allocate capital for that risk. The firm may allocate that capital with or without a formal analysis as to the amount of capital that it must set aside. In this section, we present an example of actions that a firm with an ERM strategy may take when faced with risk. We recognize that not all firms will allocate capital using this type of analysis.

The firm’s ERM strategy will require that it reserve capital for the possibility of that it experiences a capital depletion event. We denote this reserve as to indicate that the reserve is a segregation of capital to absorb realizations of . We present that capital allocation approach in Figure .



Capital Deployment

Using the naught superscript to represent this base case, the rate of return is then as follows:

The numerator is the sum of the return on working capital and the return on the reserve, less the value of the claim amount. The inclusion of the random variable indicates that the firm’s return is a function of the realized value of the loss event.

We note, however, that the expected value of in high excess layers is nearly 0. That is, . As s result, taking expectations, we have:

## Rate of Return with Insurance

The firm may alternatively elect to purchase insurance to cover a portion of the cost of the loss event. We develop that rate of return in this section. We use the following notation in our rate of return equation:

* represents the portion of the distribution of claims values that the firm retains. We allow to vary between 0 (equal to the no insurance case) and (in which case, the firm fully transfers the risk). We observe that if retention will not reduce capital below , i.e., , then no capital need to be set aside to cover loss events, i.e., .
* We use superscript to represent the “insurance case.” and for premium (in the insurance case). This is consistent with the notation that we use for capital. (They both represent approaches to fund risk.)

The rate of return in the insurance case is as follows:

The rational firm (where [[4]](#footnote-33)) purchases insurance such that it need not set aside any capital. Therefore, and .

As we noted above, . Therefore, and . We can then simplify our rate of return equation to:

We recognize that is the sum of the expected loss and an insurance charge. As noted, the expected loss is approximately . The insurance charge compensates the insurer for underwriting the exposure and provides for a return on its capital. Therefore we expect the insurance charge to be a of .[[5]](#footnote-34) In our construct, the prospective insured only observes premiums quoted under various options. As such, it is not concerned with the function the insurer uses to develop its risk charge, but it is aware that the premium is almost entirely comprised of the risk charge.

## Capital Deployment Value Creation Equation

The insurance transaction creates value when the expected return for the insurance buyer exceeds the expected return without insurance. Specifically, we compare the expected returns from equation () and equation (). We observe that the purchase of insurance creates value when:

We include the complete algebraic derivation of equation in Appendix . Equation () has a straightforward, intuitive interpretation that the insurance transaction creates value when the premium is less than the excess return on capital that would be earned on the reserve reduced for the return if the premium amount were also deployed as capital.

# Risk Adjustment to Required Return

The second source of value created by insurance results from the reduction in the volatility of returns for insurance buyers. The value creation results from the lower required return for firms with reduced earning volatility.

The Sharpe Ratio relates the risk premium required (numerator) to return volatility (denominator). More specifically, the Sharpe Ratio indicates the risk premium required for every unit of volatility. As we did in measuring the value through capital deployment, we first calculate the Sharpe Ratio without insurance (i.e., the base case) and then compare to the Sharpe Ratio for the insurance buyer.

As we are concerned with the marginal value created by the insurance transaction, we can calculate , in this section under the following assumptions:

* The risk-free rate, , is fixed and
* and are independent.

## Sharpe Ratio without insurance

As with the discussion in Section , we use the naught superscript to represent the base case.

Recall that we developed the expected rate of return in Section . We rewrite equation as presented below.

We start with equation () and develop the variance of returns under the simplifying conditions described at the beginning of this section.

We can now calculate the Sharpe Ratio without insurance.

## Sharpe Ratio with Insurance

We can also calculate the Sharpe Ratio for the insurance buyer, using the superscript to represent this case. As with the no insurance case, we start with equation () from Section:

We start with equation () and develop the variance under the simplifying conditions described at the beginning of this section.

Now we calculate the Sharpe Ratio under the insurance case:

## Complete Value Creation Equation

The insurance purchase created value when it results in an increase in the Sharpe Ratio. We can use equation () and equation () to calculate the maximum premium that results in value creation. We present that equation below. We include the complete algebraic derivation in Appendix .

The interpretation of equation () is slightly more difficult than that posed by equation (). However, we can recognize that the first term on the righthand side is a measure of the value of capital deployment from equation (). Then the second term is the value provided by the reduction in risk volatility, and we intuitively understand that is a representation of that reduction. Further, it is intuitive that the reduced reduction in risk would apply to the excess returns on working capital, i.e., .

We refer to equation () as the “Complete Value Creation Equation” since it includes sources of value. We should not find this surprising since the Sharpe Ratio includes expected returns in its numerator and risk/volatility in the denominator.

# Conclusion and Summary

Through the analysis presented in the paper, we have developed a measurement of two sources of value from the insurance transaction.

* The value that results from the ability of the firm to deploy additional capital:

When premiums amounts satisfy equation (), the insurance transaction will be accretive to the rate of return.

* The value created through the reduced earnings expectations that result from the reduction in volatility:

When premiums amounts do not satisfy equation () but satisfy equation (), the insurance transaction will will not be accretive to the rate of return. However, the reduced rate of return will be less than the reduction in the rate of return demanded by the shareholder.

When premiums amounts do not satisfy either equation () or equation (), then the insurance transaction does not add financial value in excess of premiums. However, it may still provide value in supporting a restful night’s sleep.

# Algebraic Derivation of Value Creation Equations

## Derivation of Capital Deployment Value Creation Equation

## Derivation of Complete Value Creation Equation

# Literature Review

We identified and reviewed the following papers in our literature review. As noted, these papers focused on the return on the insuer’s capital rather than the value of capital preservation for the insured.

[Macalaster, Spencer. Insurance as a Form of Capital. January 12, 2015.](https://www.insurancejournal.com/magazines/mag-closingquote/2015/01/12/353262.htm) Described the issue conceptually but without measurement and does not mention the role of the actuary in establishing reserves.

[Arrow, K.J. Essays in the Theory of Risk Bearing; Markham: Chicago, IL, USA, 1971.](https://content.ebscohost.com/ContentServer.asp?T=P&P=AN&K=5054063&S=R&D=bth&EbscoContent=dGJyMNXb4kSeqa440dvuOLCmsEieqK5Ss664TbOWxWXS&ContentCustomer=dGJyMPGut1C0rLNRuePfgeyx83vt5OyF39%2Fs) Arrow’s theorem is the cornerstone of the application of expected-ulitily theory in insurance

[Raviv, A. The design of optimal insurance policy. Am. Econ. Rev. 1979, 69, 84–96.](https://www.jstor.org/stable/1802499) Used expected utility theory to measure value for an insured.

[Gollier, C. Optimum insurance of approximate losses. J. Risk Insur. 1996, 63, 369–380.](https://www.jstor.org/stable/253617) Also used expected utility theory to measure optimize the insured retentions.

[Yaari, M. The dual theory of choice under risk. Econometrica 1987, 55, 95–115.](https://www.jstor.org/stable/1911158) The modification of expected utility theory

[Kai A. Konrad; Stegrios Skaperds. Self-Insurance and Self-Protection: A Nonexpected Utility Analysis. The Geneva Papers on Risk and Insurance Theory, 1993-12, Vol.18 (2), p.131-146.](https://www.jstor.org/stable/41953283) Used rank-dependent expected utility preferences to measure value for an insured

[Christophe Courbage. Self-Insurance, Self-Protection and Market Insurance within the Dual Theory of Choice. The Geneva papers on risk and insurance theory, 2001-06-01, Vol.26 (1), p.43-56.](https://www.researchgate.net/publication/5221351_Self-Insurance_Self-Protection_and_Market_Insurance_within_the_Dual_Theory_of_Choice_In_The_Geneva_Papers_on_Risk_and_Insurance_Theory_26_1_43-56) Used dual theory to measure value for an insured

[Michael Merz; Mario V. Wüthrich, Demand of Insurance under the Cost-of-Capital Premium Calculation Principle. Risks, 2014, Vol.2(2), 226-248.](https://www.mdpi.com/2227-9091/2/2/226/htm) Used risk premium approach to measure value for an insured

[Bernard, C.; He, X.D.; Yan, J.-A.; Zhou, X.Y. Optimal insurance design under rank-dependent expected utility. In Mathematical Finance; SSRN Preprint: Waterloo, Canada, 2014.](https://www.researchgate.net/publication/228293026_Optimal_Insurance_Design_Under_Rank-Dependent_Expected_Utility) Used rank-dependent expected utility theory to measure value for an insured too

[Laury, S.; M. Mcinnes, and J.Swarthout(2009). Insurance Decisions for Low-Probability Losses. Journal of Risk and Uncertainty, 39, 17–44.](https://www.researchgate.net/publication/225581818_Insurance_Decisions_for_Low_Probability_Losses) Used experimental evidence to show individual’s under-insure decision for low-probability high-loss events

[Kunreuther, H.; Pauly, M. (2004). Neglecting disaster: Why don’t people insure against large losses? Journal of Risk and Uncertainty, 28, 5–21.](https://www.jstor.org/stable/41761127) Used utility theory to show the reason why individuals don’t insure low-probability and high-loss events

1. The insured may realize value though specialized risk or claims management services provided by the insurer. [↑](#footnote-ref-20)
2. That is, the value beyond the restful night of sleep. [↑](#footnote-ref-23)
3. This is an illustrative claim distribution that we present to support further development of value measurement. [↑](#footnote-ref-28)
4. Without this condition there is no economic value to the firm’s existence. [↑](#footnote-ref-33)
5. We recognize that the insurer has the ability to diversify this volatility across risks. As a result the function referenced can result in a premiums that creates value. [↑](#footnote-ref-34)