ILC Project

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1 Equations of Motion

For a 4DOF, a RPRP robot (2 revolute joints and 2 prismatic joints), with link lengths $l_1, r_1(t), l_2, r_2(t)$ and masses m_1, m_2, m_3, m_4.

$KE = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}m_3(\dot{x}_3^2 + \dot{y}_3^2) + \frac{1}{2}m_4(\dot{x}_4^2 + \dot{y}_4^2)$

$PE = 0$

where

$\dot{x}_1 = l_1 \cos(\theta_1)$
$\dot{y}_1 = l_1 \sin(\theta_1)$

$\dot{x}_2 = (l_1 + r_1)\cos(\theta_1)$
$\dot{y}_2 = (l_1 + r_1)\sin(\theta_1)$

$\dot{x}_3 = (l_1 + r_1)\cos(\theta_1) + l_2 \cos(\theta_2)$
$\dot{y}_3 = (l_1 + r_1)\sin(\theta_1) + l_2 \sin(\theta_2)$

$\dot{x}_4 = (l_1 + r_1)\cos(\theta_1) + (l_2 + r_2)\cos(\theta_2)$
$\dot{y}_4 = (l_1 + r_1)\sin(\theta_1) + (l_2 + r_2)\sin(\theta_2)$

$\theta_1(t), \theta_2(t), r_1(t), r_2(t)$ are the joint variables

Lagrangian is given as -
$\mathcal{L} = KE - PE$

The equations of motion can now be expressed in terms of the Lagrangian as follows -
$\tau = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q}$

The equations of motion are given as:

$\tau_1 = M_{11}\ddot{\theta}_1 + M_{12}\ddot{r}_1 + M_{13}\ddot{\theta}_2 + M_{14}\ddot{r}_2 + H_1 + G_1$
$F_1 = M_{21}\ddot{\theta}_1 + M_{22}\ddot{r}_1 + M_{23}\ddot{\theta}_2 + M_{24}\ddot{r}_2 + H_2 + G_2$
$\tau_2 = M_{31}\ddot{\theta}_1 + M_{32}\ddot{r}_1 + M_{33}\ddot{\theta}_2 + M_{34}\ddot{r}_2 + H_3 + G_3$
$F_2 = M_{41}\ddot{\theta}_1 + M_{42}\ddot{r}_1 + M_{43}\ddot{\theta}_2 + M_{44}\ddot{r}_2 + H_4 + G_4$

$$
\begin{bmatrix} \tau_1 \\ F_1 \\ \tau_2 \\ F_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{r}_1 \\ \ddot{\theta}_2 \\ \ddot{r}_2 \end{bmatrix} + \begin{bmatrix} H_1 + G_1 \\ H_2 + G_2 \\ H_3 + G_3 \\ H_4 + G_4 \end{bmatrix}
\tag{1}
$$

$M_{11} = (m_1 + m_2 + m_3 + m_4)l_1^2 + (m_2 + m_3 + m_4)r_1^2 + 2(m_2 + m_3 + m_4)l_1 r_1$
$M_{12} = 0$
$M_{13} = m_4 \cos(\theta_1 - \theta_2)r_1 r_2 + (m_3 + m_4)l_2 l_1 \cos(\theta_1 - \theta_2) + (m_3 + m_4)l_2 r_1 \cos(\theta_1 - \theta_2) + m_4 l_1 r_2 \cos(\theta_1 - \theta_2)$
$M_{14} = -m_4 \sin(\theta_1 - \theta_2)(l_1 + r_1)$

$M_{21} = 0$
$M_{22} = m_2 + m_3 + m_4$
$M_{23} = m_3 l_2 \sin(\theta_1 - \theta_2) + m_4 l_2 \sin(\theta_1 - \theta_2) + m_4 r_2 \sin(\theta_1 - \theta_2)$
$M_{24} = m_4 \cos(\theta_1 - \theta_2)$

$M_{31} = m_4 \cos(\theta_1 - \theta_2)(l_1 + r_1)(l_2 + r_2) + m_3 l_2(l_1 + r_1)\cos(\theta_1 - \theta_2)$
$M_{32} = m_3 l_2 \sin(\theta_1 - \theta_2) + m_4(l_2 + r_2)\sin(\theta_1 - \theta_2)$
$M_{33} = m_3 l_2^2 + m_4 l_2^2 + 2m_4 r_2(l_2 + \frac{r_2}{2})$
$M_{34} = 0$

$M_{41} = -m_4 \sin(\theta_1 - \theta_2)(l_1 + r_1)$
$M_{42} = m_4 \cos(\theta_1 - \theta_2)$
$M_{43} = 0$
$M_{44} = m_4$

$H_1 + G_1 = (2m_2 \dot{r}_1 \dot{\theta}_1 + 2m_3 \dot{r}_1 \dot{\theta}_1 + 2m_4 \dot{r}_1 \dot{\theta}_1 + m_3 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_2^2 + m_4 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_2^2 + 2m_4 \dot{r}_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + m_4 \sin(\theta_1 - \theta_2)r_2 \dot{\theta}_2^2)(l_1 + r_1)$

$H_2 + G_2 = 2m_4 \sin(\theta_1 - \theta_2)\dot{r}_2\dot{\theta}_2 - m_3 l_1 \dot{\theta}_1^{\,2} - m_4 l_1 \dot{\theta}_1^{\,2} - m_2 r_1 \dot{\theta}_1^{\,2} - m_3 r_1 \dot{\theta}_1^{\,2} - m_4 r_1 \dot{\theta}_1^{\,2} - m_3 l_2 \dot{\theta}_2^{\,2} \cos(\theta_1 - \theta_2) - m_4 l_2 \dot{\theta}_2^{\,2} \cos(\theta_1 - \theta_2) - m_2 l_1 \dot{\theta}_1^{\,2} - m_4 r_2 \dot{\theta}_2^{\,2} \cos(\theta_1 - \theta_2)$

$H_3 + G_3 = 2m_4 l_2 \dot{r}_2 \dot{\theta}_2 + 2m_4 r_2 \dot{r}_2 \dot{\theta}_2 - m_3 l_2 \sin(\theta_1 - \theta_2) r_1 \dot{\theta}_1^{\,2} - m_4 l_1 \sin(\theta_1 - \theta_2) r_2 \dot{\theta}_1^{\,2} - m_4 l_2 \sin(\theta_1 - \theta_2) r_1 \dot{\theta}_1^{\,2} + 2m_4 r_2 \dot{r}_1 \dot{\theta}_1 \cos(\theta_1 - \theta_2) - m_4 \sin(\theta_1 - \theta_2) r_1 r_2 \dot{\theta}_1^{\,2} - m_3 l_1 l_2 \sin(\theta_1 - \theta_2) \dot{\theta}_1^{\,2} - m_4 l_1 l_2 \sin(\theta_1 - \theta_2) \dot{\theta}_1^{\,2} + 2m_3 l_2 \dot{r}_1 \dot{\theta}_1 \cos(\theta_1 - \theta_2) + m_4 l_2 \dot{r}_1 \dot{\theta}_1 \cos(\theta_1 - \theta_2)$

$H_4 + G_4 = -m_4 l_2 \dot{\theta}_2^{\,2} - m_4 r_2 \dot{\theta}_2^{\,2} - m_4 l_1 \dot{\theta}_1^{\,2} \cos(\theta_1 - \theta_2) - 2m_4 \sin(\theta_1 - \theta_2) \dot{r}_1 \dot{\theta}_1 - m_4 r_1 \dot{\theta}_1^{\,2} \cos(\theta_1 - \theta_2)$
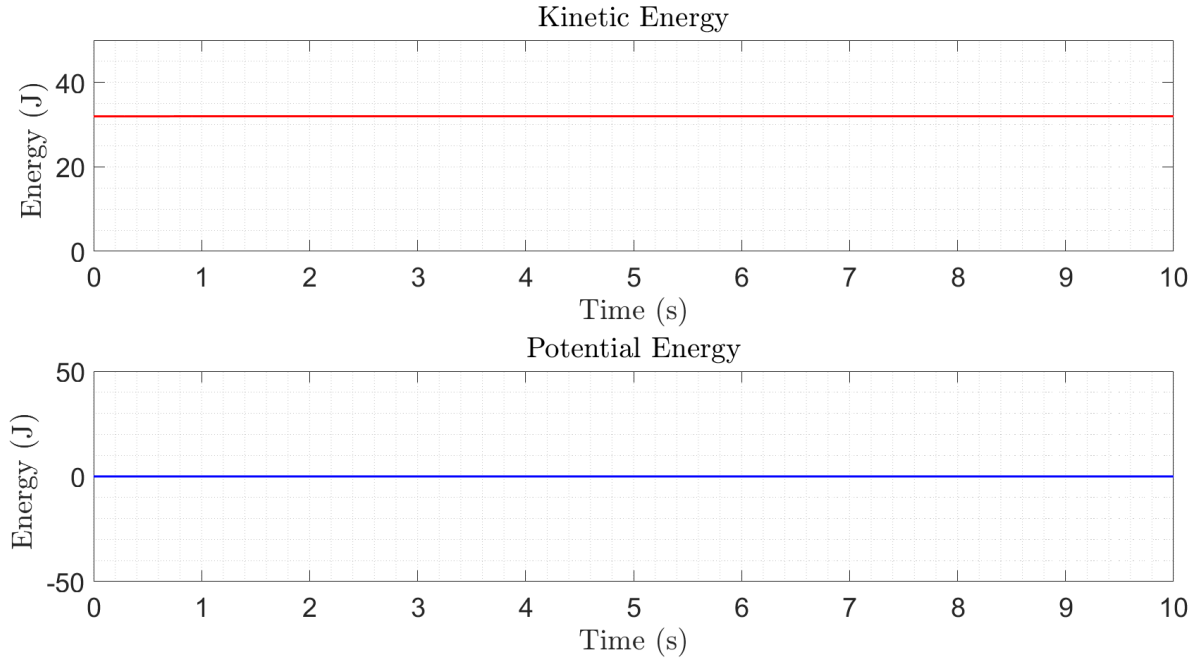
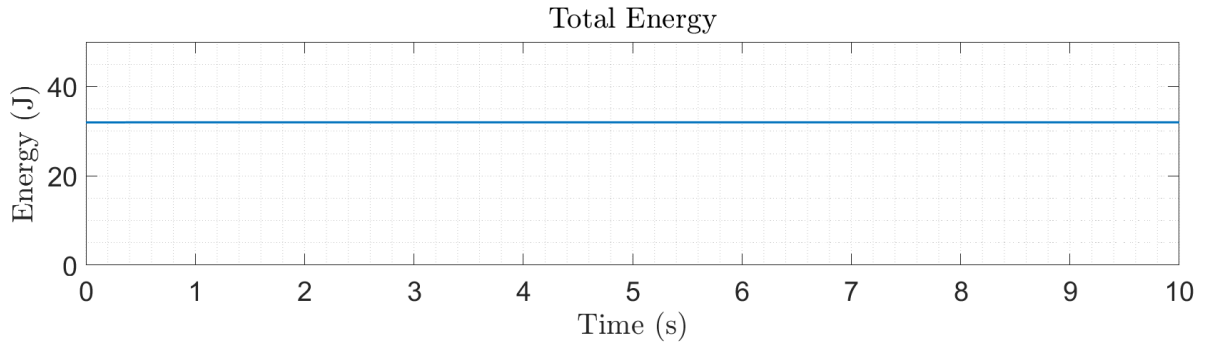## 2 Graphs

Figure 1: Graphs of Kinetic Energy and Potential Energy, respectively.

Figure 2: Graph of Total Energy.