ILC Project

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1 Equations of Motion

For a 4DOF, a RPRP robot (2 revolute joints and 2 prismatic joints), with link lengths $l_1, r_1(t), l_2, r_2(t)$ and masses m_1, m_2, m_3, m_4.

$KE = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}m_3(\dot{x}_3^2 + \dot{y}_3^2) + \frac{1}{2}m_4(\dot{x}_4^2 + \dot{y}_4^2)$

$PE = (m_1+m_2+m_3+m_4)gl_1\sin(\theta_1)+(m_2+m_3+m_4)gr_1\sin(\theta_1)+(m_3+m_4)gl_2\sin(\theta_2)+m_4gr_2\sin(\theta_2)$

where

$\dot{x}_1 = l_1\cos(\theta_1)$
$\dot{y}_1 = l_1\sin(\theta_1)$

$\dot{x}_2 = (l_1 + r_1)\cos(\theta_1)$
$\dot{y}_2 = (l_1 + r_1)\sin(\theta_1)$

$\dot{x}_3 = (l_1 + r_1)\cos(\theta_1) + l_2\cos(\theta_2)$
$\dot{y}_3 = (l_1 + r_1)\sin(\theta_1) + l_2\sin(\theta_2)$

$\dot{x}_4 = (l_1 + r_1)\cos(\theta_1) + (l_2 + r_2)\cos(\theta_2)$
$\dot{y}_4 = (l_1 + r_1)\sin(\theta_1) + (l_2 + r_2)\sin(\theta_2)$

$\theta_1(t), \theta_2(t), r_1(t), r_2(t)$ are the joint variables

Lagrangian is given as -
$\mathcal{L} = KE - PE$

The equations of motion can now be expressed in terms of the Lagrangian as follows -
$\tau = \frac{d}{dt}(\frac{\partial\mathcal{L}}{\partial\dot{q}}) - \frac{\partial\mathcal{L}}{\partial q}$

The equations of motion are given as:

$\tau_1 = M_{11}\ddot{\theta}_1 + M_{12}\ddot{r}_1 + M_{13}\ddot{\theta}_2 + M_{14}\ddot{r}_2 + H_1 + G_1$
$F_1 = M_{21}\ddot{\theta}_1 + M_{22}\ddot{r}_1 + M_{23}\ddot{\theta}_2 + M_{24}\ddot{r}_2 + H_2 + G_2$
$\tau_2 = M_{31}\ddot{\theta}_1 + M_{32}\ddot{r}_1 + M_{33}\ddot{\theta}_2 + M_{34}\ddot{r}_2 + H_3 + G_3$
$F_2 = M_{41}\ddot{\theta}_1 + M_{42}\ddot{r}_1 + M_{43}\ddot{\theta}_2 + M_{44}\ddot{r}_2 + H_4 + G_4$

$$\begin{bmatrix}\tau_1\\F_1\\\tau_2\\F_2\end{bmatrix} = \begin{bmatrix}M_{11} & M_{12} & M_{13} & M_{14}\\M_{21} & M_{22} & M_{23} & M_{24}\\M_{31} & M_{32} & M_{33} & M_{34}\\M_{41} & M_{42} & M_{43} & M_{44}\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\\ddot{r}_1\\\ddot{\theta}_2\\\ddot{r}_2\end{bmatrix} + \begin{bmatrix}H_1+G_1\\H_2+G_2\\H_3+G_3\\H_4+G_4\end{bmatrix} \tag{1}$$

$M_{11} = (m_1 + m_2 + m_3 + m_4)l_1^2 + (m_2 + m_3 + m_4)r_1^2 + 2(m_2 + m_3 + m_4)l_1r_1$
$M_{12} = 0$
$M_{13} = m_4\cos(\theta_1-\theta_2)r_1r_2+(m_3+m_4)l_2l_1\cos(\theta_1-\theta_2)+(m_3+m_4)l_2r_1\cos(\theta_1-\theta_2)+m_4l_1r_2\cos(\theta_1-\theta_2)$
$M_{14} = -m_4\sin(\theta_1 - \theta_2)(l_1 + r_1)$

$M_{21} = 0$
$M_{22} = m_2 + m_3 + m_4$
$M_{23} = m_3l_2\sin(\theta_1 - \theta_2) + m_4l_2\sin(\theta_1 - \theta_2) + m_4r_2\sin(\theta_1 - \theta_2)$
$M_{24} = m_4\cos(\theta_1 - \theta_2)$

$M_{31} = m_4\cos(\theta_1 - \theta_2)(l_1 + r_1)(l_2 + r_2) + m_3l_2(l_1 + r_1)\cos(\theta_1 - \theta_2)$
$M_{32} = m_3l_2\sin(\theta_1 - \theta_2) + m_4(l_2 + r_2)\sin(\theta_1 - \theta_2)$
$M_{33} = m_3l_2^2 + m_4l_2^2 + 2m_4r_2(l_2 + \frac{r_2}{2})$
$M_{34} = 0$

$M_{41} = -m_4\sin(\theta_1 - \theta_2)(l_1 + r_1)$
$M_{42} = m_4\cos(\theta_1 - \theta_2)$
$M_{43} = 0$
$M_{44} = m_4$

$H_1+G_1 = (2m_2\dot{r}_1\dot{\theta}_1+2m_3\dot{r}_1\dot{\theta}_1+2m_4\dot{r}_1\dot{\theta}_1+m_3l_2\sin(\theta_1-\theta_2)\dot{\theta}_2^{\,2}+m_4l_2\sin(\theta_1-\theta_2)\dot{\theta}_2^{\,2}+2m_4\dot{r}_2\dot{\theta}_2\cos(\theta_1-\theta_2) + m_4\sin(\theta_1 - \theta_2)r_2\dot{\theta}_2^{\,2})(l_1 + r_1) + (m_1 + m_2 + m_3 + m_4)gl_1\cos(\theta_1) + (m_2 + m_3 + m_4)gr_1\cos(\theta_1)$

$H_2 + G_2 = 2m_4 \sin(\theta_1 - \theta_2)\dot{r_2}\dot{\theta_2} - m_3 l_1 \dot{\theta_1}^2 - m_4 l_1 \dot{\theta_1}^2 - m_2 r_1 \dot{\theta_1}^2 - m_3 r_1 \dot{\theta_1}^2 - m_4 r_1 \dot{\theta_1}^2 - m_3 l_2 \dot{\theta_2}^2 \cos(\theta_1 - \theta_2) - m_4 l_2 \dot{\theta_2}^2 \cos(\theta_1 - \theta_2) - m_2 l_1 \dot{\theta_1}^2 - m_4 r_2 \dot{\theta_2}^2 \cos(\theta_1 - \theta_2) + (m_2 + m_3 + m_4) g \sin(\theta_1)$

$H_3 + G_3 = 2m_4 l_2 \dot{r_2}\dot{\theta_2} + 2m_4 r_2 \dot{r_2}\dot{\theta_2} - m_3 l_2 \sin(\theta_1 - \theta_2) r_1 \dot{\theta_1}^2 - m_4 l_1 \sin(\theta_1 - \theta_2) r_2 \dot{\theta_1}^2 - m_4 l_2 \sin(\theta_1 - \theta_2) r_1 \dot{\theta_1}^2 + 2m_4 r_2 \dot{r_1}\dot{\theta_1} \cos(\theta_1 - \theta_2) - m_4 \sin(\theta_1 - \theta_2) r_1 r_2 \dot{\theta_1}^2 - m_3 l_1 l_2 \sin(\theta_1 - \theta_2) \dot{\theta_1}^2 - m_4 l_1 l_2 \sin(\theta_1 - \theta_2) \dot{\theta_1}^2 + 2m_3 l_2 \dot{r_1}\dot{\theta_1} \cos(\theta_1 - \theta_2) + m_4 l_2 \dot{r_1}\dot{\theta_1} \cos(\theta_1 - \theta_2) + (m_3 + m_4) g l_2 cos(\theta_2) + m_4 g r_2 cos(\theta_2)$

$H_4 + G_4 = -m_4 l_2 \dot{\theta_2}^2 - m_4 r_2 \dot{\theta_2}^2 - m_4 l_1 \dot{\theta_1}^2 \cos(\theta_1 - \theta_2) - 2m_4 \sin(\theta_1 - \theta_2) \dot{r_1}\dot{\theta_1} - m_4 r_1 \dot{\theta_1}^2 \cos(\theta_1 - \theta_2) + m_4 g \sin(\theta_2)$

2 Graphs

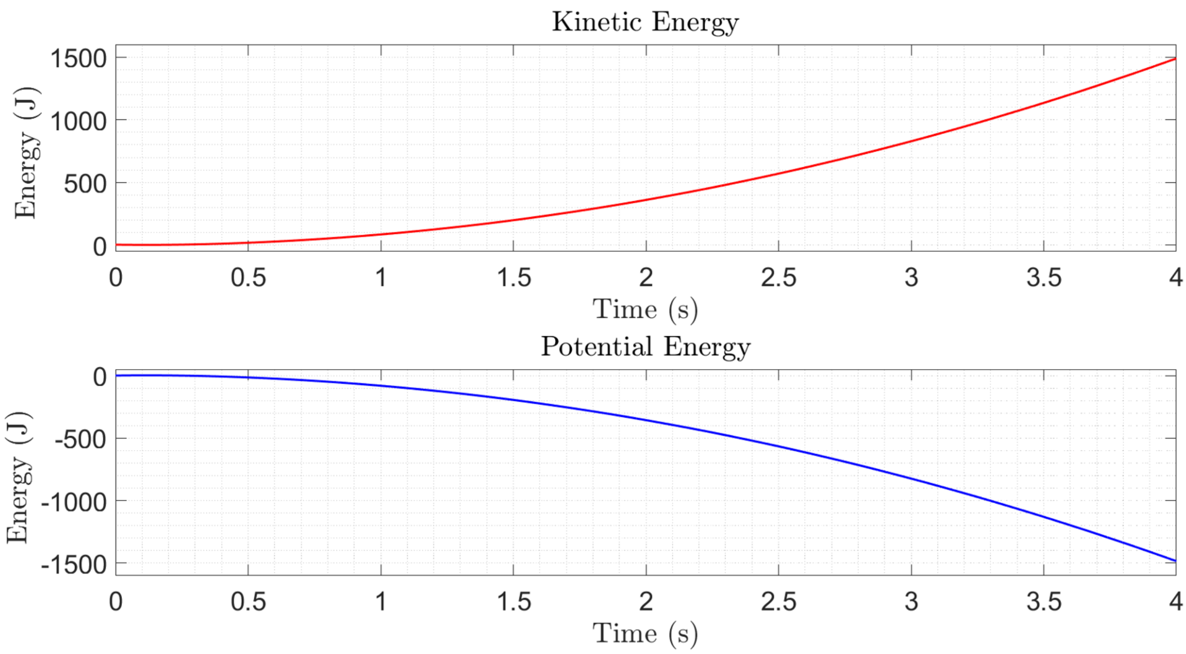

Figure 1: Graphs of Kinetic Energy and Potential Energy, respectively.

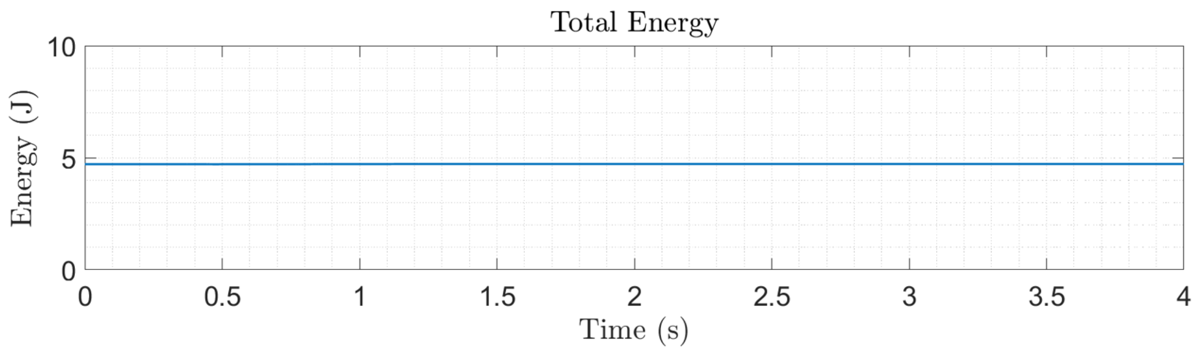

Figure 2: Graph of Total Energy.