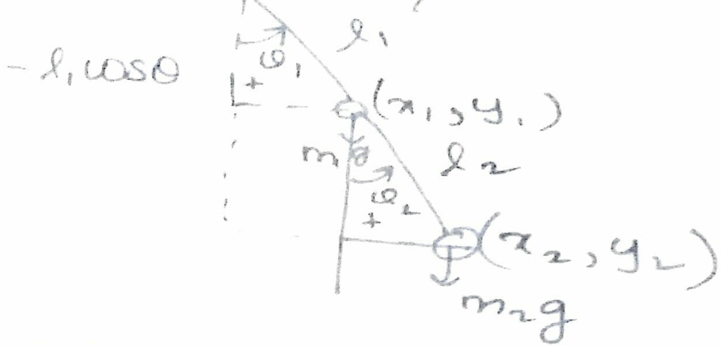

counterclockwise positive

$$y_1 = -l_1\cos\theta_1$$

$$x_1 = l_1\sin\theta_1$$

$$y_2 = -(l_1\cos\theta_1 + l_2\cos\theta_2)$$

$$x_2 = l_1\sin\theta_1 + l_2\sin\theta_2$$

$$\dot{x}_1 = (l_1\cos\theta_1)\,\dot{\theta}_1$$

$$\dot{y}_1 = (+l_1\sin\theta_1)\,\dot{\theta}_1$$

$$\dot{x}_2 = \cancel{l_1\sin\theta}\,\{l_1\cos\theta\,\dot{\theta}_1 + l_2\cos\theta_2\}\,\dot{\theta}_2$$

$$\dot{y}_2 = l_1\sin\theta_1\,\dot{\theta}_1 + l_2\sin\theta_2\,\dot{\theta}_2$$

$$\dot{x}_1^2 + \dot{y}_1^2 = l_1^2\cos^2\theta_1\,\dot{\theta}_1^2 + l_1^2\sin^2\theta_1\,\dot{\theta}_1^2$$

$$= (l_1\dot{\theta}_1)^2\left[\cos^2\theta_1 + \sin\theta_1^2\right]$$

$$\boxed{v_1^2 = (l_1\dot{\theta}_1)^2}$$

$$\dot{x}_2^2+\dot{y}_2^2 = (l_1\cos\theta_1\dot{\theta}_1)^2 + (l_2\cos\theta_2\dot{\theta}_2)^2 + 2l_1l_2\cos\theta_1\cos\theta_2\dot{\theta}_1\dot{\theta}_2$$

$$+ (l_1\sin\theta_1\dot{\theta}_1)^2 + (l_2\sin\theta_2\dot{\theta}_2)^2 + 2l_1l_2\sin\theta_1\sin\theta_2\dot{\theta}_1\dot{\theta}_2$$

$$\boxed{v_2^2 = (l_1\dot{\theta}_1)^2 + (l_2\dot{\theta}_2)^2 + 2l_1l_2(\dot{\theta}_1\dot{\theta}_2)\cos(\theta_1-\theta_2)}$$

Kinetic Energy

$$L = T - V$$

Potential Energy

$$T = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}m_1(l_1\dot{\theta}_1)^2 + \frac{1}{2}m_2\left[(l_1\dot{\theta}_1)^2 + (l_2\dot{\theta}_2)^2 + 2l_1l_2(\dot{\theta}_1\dot{\theta}_2)\cos(\theta_1-\theta_2)\right]$$

$$\cancel{= \frac{1}{2}m_1(l_1\dot{\theta}_1)^2}$$

$$T = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2l_2^2\dot{\theta}_2^2$$

$$+ \frac{1}{\cancel{2}}m_2l_1l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta-\theta_2)$$

$$V = mgy_1 + mgy_2$$

$$= -m_1gl_1\cos\theta_1 + m_2g(-l_1\cos\theta_1 - l_2\cos\theta_2)$$

$$= -m_1gl_1\cos\theta_1 - m_2gl_1\cos\theta_1 - m_2gl_2\cos\theta_2$$

$L = T - V$

$= \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2$

$+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$

$+ m g l_1 \cos\theta_1 + m_2 g l_1 \cos\theta_1 + m_2 g l_2 \cos\theta_2$

$\frac{\partial L}{\partial \dot{\theta}_1} = \cancel{\frac{2}{2}} m_1 l_1^2 \dot{\theta}_1 + \cancel{\frac{2}{2}} m_2 l_1^2 \dot{\theta}_1 + 0$

$+ m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$

$\frac{\partial L}{\partial \dot{\theta}_2} = \cancel{\frac{2}{2}} m_2 l_2^2 \theta_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos\theta - \theta_2)$

$\frac{\partial L}{\partial \theta_1} = m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(-\theta_1 + \theta_2) - m g l_1 \sin\theta_1$

$- m_2 g l_1 \cancel{\cos\theta_1} \sin\theta_1$

$\frac{\partial L}{\partial \theta_2} = m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - m_2 g l_2 \sin\theta_2$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2)$

$+ m_2 l_1 l_2 \dot{\theta}_2 [-\sin(-\theta_1 + \theta_2)$
$(\dot{\theta}_1 - \dot{\theta}_2)$

$(m_1 + m_2) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2)$

$+ (m_1 + m_2) g \sin\theta_1 = 0$