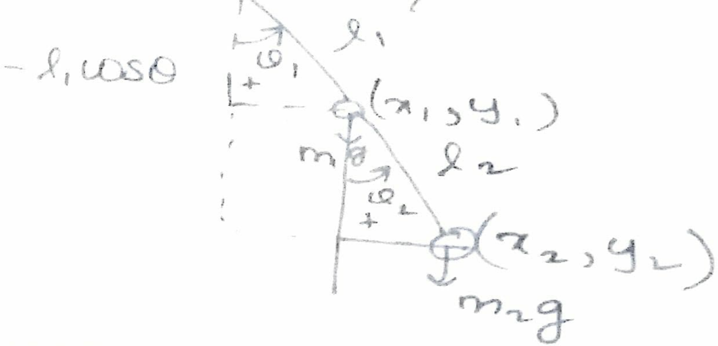

counterclockwise positive

$$y_1 = -l_1 \cos\theta_1$$
$$x_1 = l_1 \sin\theta_1$$

$$y_2 = -(l_1 \cos\theta_1 + l_2 \cos\theta_2)$$
$$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2$$

$$\dot{x}_1 = (l_1 \cos\theta_1)\,\dot{\theta}_1$$
$$\dot{y}_1 = (+l_1 \sin\theta_1)\,\dot{\theta}_1$$

$$\dot{x}_2 = \cancel{l_1 \sin\theta} \{l_1 \cos\theta\,\dot{\theta}_1 + l_2 \cos\theta_2\}\,\dot{\theta}_2$$
$$\dot{y}_2 = l_1 \sin\theta_1\,\dot{\theta}_1 + l_2 \sin\theta_2\,\dot{\theta}_2$$

$$\dot{x}_1^2 + \dot{y}_1^2 = l_1^2 \cos^2\theta_1\,\dot{\theta}_1^2 + l_1^2 \sin^2\theta_1\,\dot{\theta}_1^2$$
$$= (l_1 \dot{\theta}_1)^2 [\cos^2\theta_1 + \sin^2\theta_1]$$
$$\boxed{v_1^2 = (l_1 \dot{\theta}_1)^2}$$

$$\dot{x}_2^2+\dot{y}_2^2=(l_1\cos\theta_1\dot{\theta}_1)^2+(l_2\cos\theta_2\dot{\theta}_2)^2+2l_1l_2\cos\theta_1\cos\theta_2\dot{\theta}_1\dot{\theta}_2$$
$$+(l_1\sin\theta_1\dot{\theta}_1)^2+(l_2\sin\theta_2\dot{\theta}_2)^2+2l_1l_2\sin\theta_1\sin\theta_2\dot{\theta}_1\dot{\theta}_2$$

$$\boxed{v_2^2=(l_1\dot{\theta}_1)^2+(l_2\dot{\theta}_2)^2+2l_1l_2(\dot{\theta}_1\dot{\theta}_2)\cos(\theta_1-\theta_2)}$$

Kinetic Energy

$$L=T-V$$

Potential Energy

$$T=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$$

$$=\frac{1}{2}m_1(l_1\dot{\theta}_1)^2+\frac{1}{2}m_2\left[(l_1\dot{\theta}_1)^2+(l_2\dot{\theta}_2)^2+2l_1l_2(\dot{\theta}_1\dot{\theta}_2)\cos(\theta_1-\theta_2)\right]$$

$$\cancel{=\frac{1}{2}m_1(l_1\dot{\theta}_1)^2}$$

$$T=\frac{1}{2}m_1l_1^2\dot{\theta}_1^2+\frac{1}{2}m_2l_1^2\dot{\theta}_1^2+\frac{1}{2}m_2l_2^2\dot{\theta}_2^2$$
$$+\frac{1}{2}m_2l_1l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta-\theta_2)$$

$$V=mgy_1+mgy_2$$
$$=-m_1gl_1\cos\theta_1+m_2g(-l_1\cos\theta_1-l_2\cos\theta_2)$$
$$=-m_1gl_1\cos\theta_1-m_2gl_1\cos\theta_1-m_2gl_2\cos\theta_2$$

$L = T - V.$

$$= \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2$$

$$+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$+ m g l_1 \cos\theta_1 + m_2 g l_1 \cos\theta_1 + m_2 g l_2 \cos\theta_2$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = \cancel{\frac{2}{2}} m_1 l_1^2 \dot{\theta}_1 + \cancel{\frac{2}{2}} m_2 l_1^2 \dot{\theta}_1 + 0$$

$$+ m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = \cancel{\frac{2}{2}} m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos\theta - \theta_2)$$

$$\frac{\partial L}{\partial \theta_1} = m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(-\theta_1 + \theta_2) - m g l_1 \sin\theta_1$$

$$- m_2 g l_1 \cancel{\cos\theta_1} \sin\theta_1$$

$$\frac{\partial L}{\partial \theta_2} = m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - m_2 g l_2 \sin\theta_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$+ m_2 l_1 l_2 \dot{\theta}_2 [+\sin(-\theta_1 + \theta_2)] (\dot{\theta}_1 - \dot{\theta}_2)$$

$$(m_1 + m_2) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2)$$

$$+ (m_1 + m_2) g \sin\theta_1 = 0$$