Feedback-based quantum algorithm inspired by Counterdiabatic Driving

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Scientific background

- The feedback-based quantum algorithm is a "fully quantum" algorithm that uses a quantum control method to design quantum circuits.
- This can be used to prepare the ground state of a quantum many-body system as well as solve combinatorial optimization problems.
- The equation of motion: $i\frac{d|\psi(t)\rangle}{dt} = (H_P + H_C(t))\,|\psi(t)\rangle,$

$$H_C(t) = \sum_{m=1}^{M} \beta_m(t) H_m$$

H_m are the time-independent Hermitian operators

β_m(t) are control parameters

- The goal is to find a time-evolution that ensures that the average energy decreases with increasing time

$$\frac{dE(t)}{dt} = i\langle\psi(t)|[H_C, H_P]|\psi(t)\rangle \leq 0$$

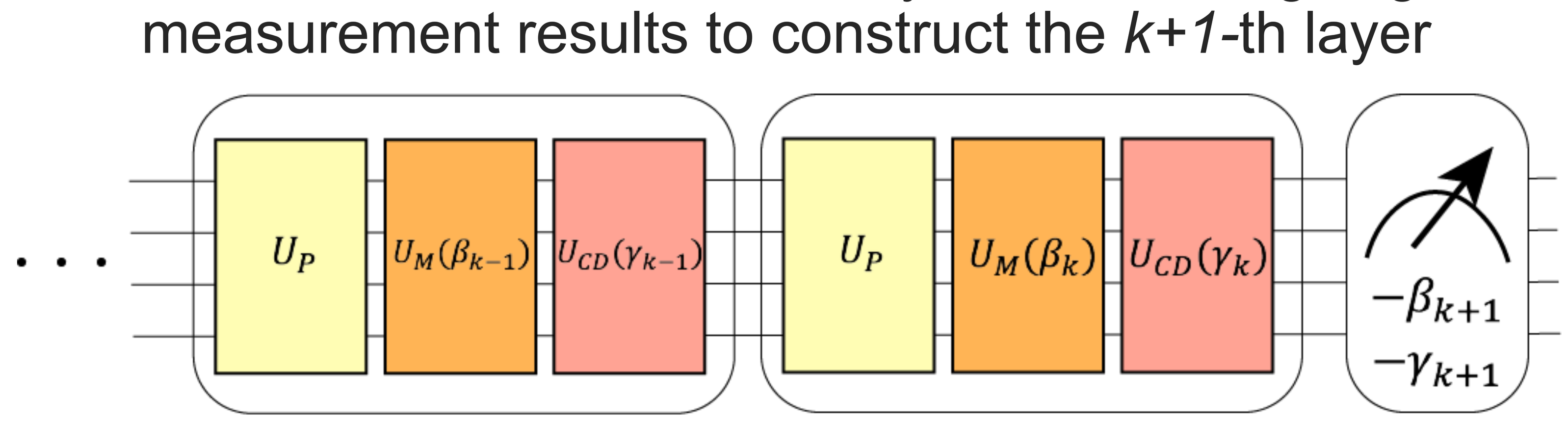
- By assigning $\beta_m(t) = -i < \psi(t)|[H_m, H_P]|\psi(t) >$, one can ensure that the energy decreases.
- In Ref. [1], it was demonstrated for combinatorial optimization problem with a single control Hamiltonian.

Motivation and the algorithm

- The feedback-based algorithm is slow and requires a deep quantum circuit to implement.
- Here, we argue that the feedback-based algorithm can be accelerated by adding another control parameter inspired from counterdiabatic drive.
- The counterdiabatic drive is applied to compensate nonadiabatic excitations that take place under a fast drive. The approximate form of the drive has the form

$$\mathcal{A}_\lambda^{(\ell)} = i\sum_{k=1}^{\ell} \alpha_k \underbrace{[\mathcal{H},[\mathcal{H},\ldots[\mathcal{H}}_{2k-1},\partial_\lambda \mathcal{H}]]], \qquad \mathcal{H}_{\mathrm{CD}}(t) = \mathcal{H}(\lambda) + \dot{\lambda}\mathcal{A}_\lambda.$$

- We choose a single operator heuristically from an operator pool to demonstrate that such an addition can indeed accelerate the algorithm.
- The quantum circuit for such an algorithm is built iteratively by computing expectation value via measurement after *k*-th layer and assigning the measurement results to construct the *k+1*-th layer

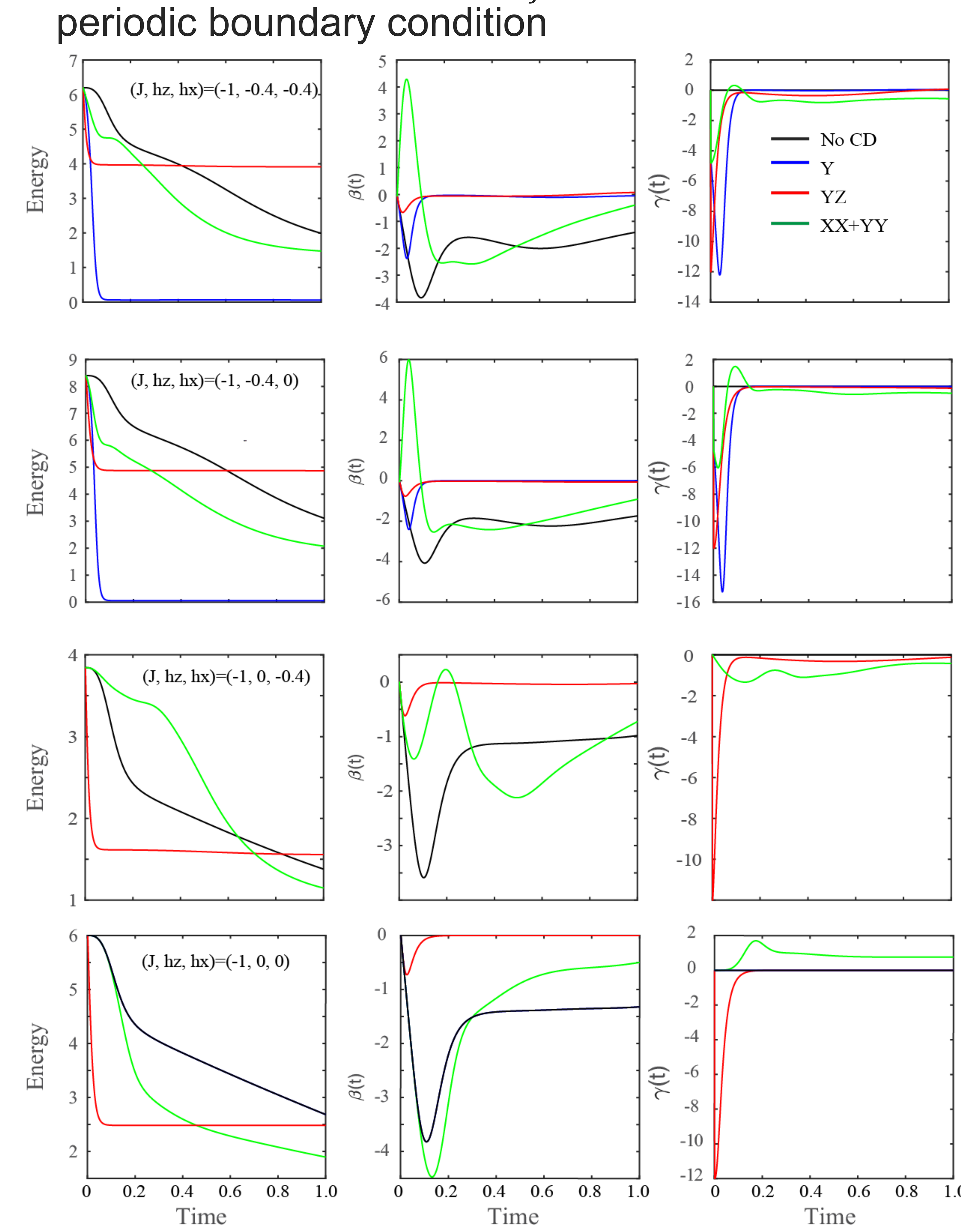


$U_P = e^{-iH_P\Delta t}$ $\quad U_{CD} = e^{-i\gamma_k H_{CD}\Delta t}$

$U_M = e^{-i\beta_k H_M \Delta t}$ $\quad \Delta t$ is the free parameter

Results

- Hamiltonian: $H_{\mathrm{prob}}(\sigma) = -\sum_{\langle i,j\rangle} J_{ij}\sigma_i^z\sigma_j^z - \sum_i h_i\sigma_i^z - \sum_i k_i\sigma_i^x$
- For simplicity we choose $J_{ij} = J, h_i = h_z, k_i = h_x$, and periodic boundary condition



Conclusions

- We observe that indeed adding an additional control parameter inspired by the counterdiabatic drive can accelerate the feedback-based algorithm.
- However, for certain Hamiltonian this comes at a cost of not reaching the ground state. Such problems can be avoided using proper perturbations.
- Nevertheless, for small enough system size the addition of CD operator increases the ground state probability for small circuit depth.

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References

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