Brookhaven National Laboratory

U.S. DEPARTMENT OF ENERGY

Quantum algorithms to find the low-energy states

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Feedback-Based Quantum Optimization

- PRL 129, 250502 (2022)

- Goal: Find approximate solutions to the combinatorial optimization problem without the need for any classical optimization effort

- Effectively the goal is to find the ground state

Main idea

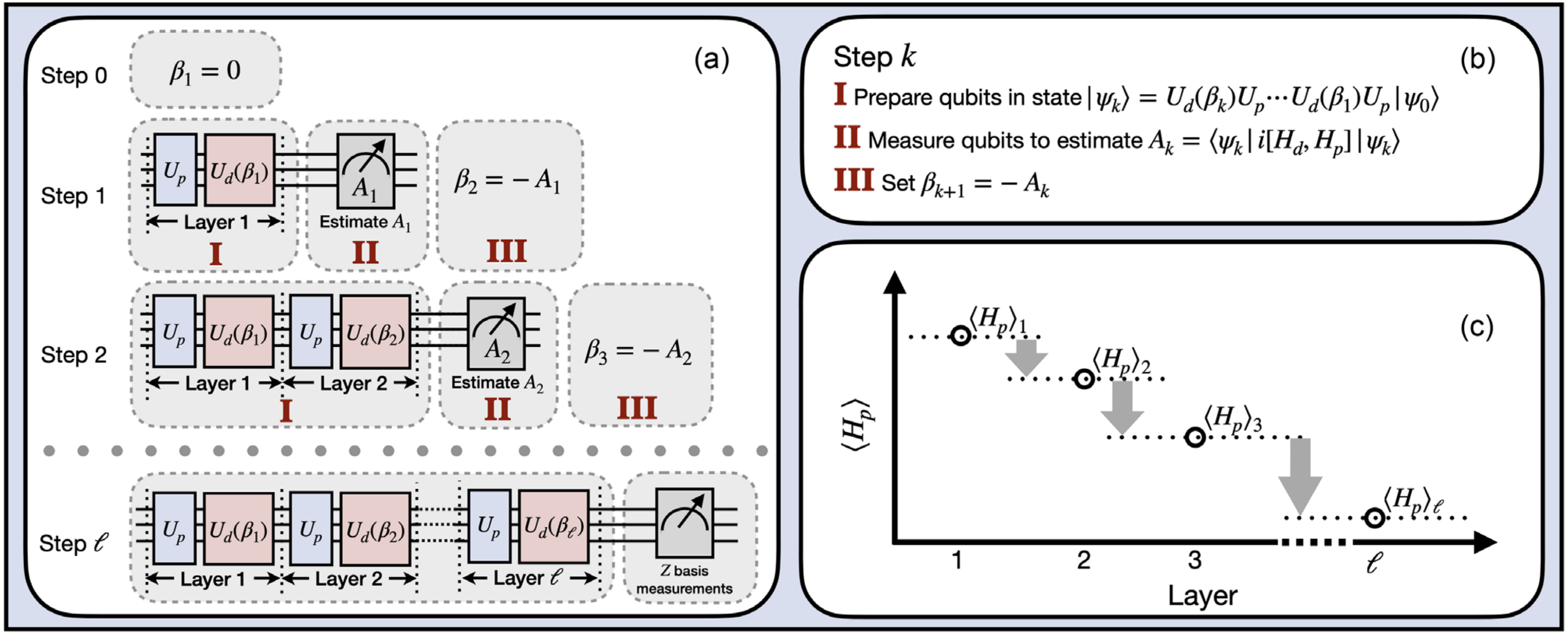
Schrodinger equation: $i\frac{d}{dt}|\psi(t)\rangle = [H_p + H_d\beta(t)]|\psi(t)\rangle$

H_P - Problem Hamiltonian
H_D - drift Hamiltonian
$\beta(t)$ – time-dependent control

$$\text{minimize } \langle H_p\rangle = \langle\psi(t)|H_p|\psi(t)\rangle$$

$$\frac{d}{dt}\langle\psi(t)|H_p|\psi(t)\rangle(t) \leq 0, \quad \forall \ t \geq 0.$$

Algorithm



$$U_p = e^{-iH_p \Delta t},$$

$$U_d(\beta_k) = e^{-i\beta_k H_d \Delta t}$$

$$\frac{d}{dt}\langle\psi(t)|H_p|\psi(t)\rangle = A(t)\beta(t).$$

$$A(t) \equiv \langle\psi(t)|i[H_d, H_p]|\psi(t)\rangle$$

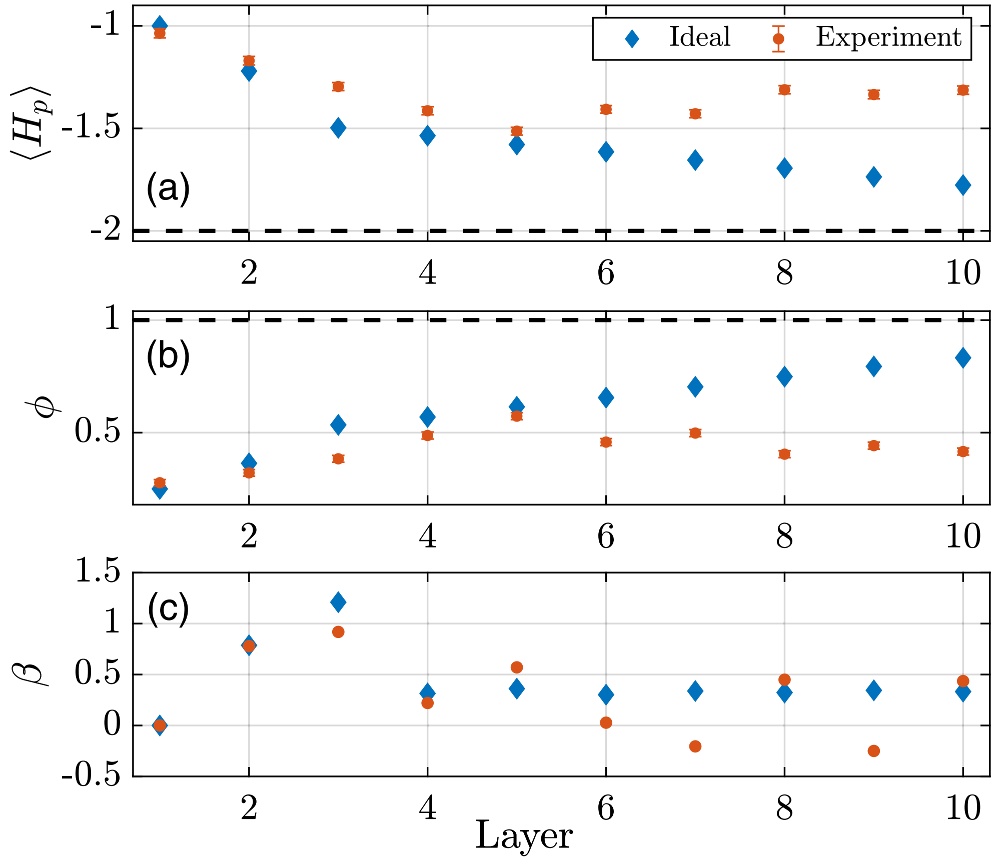
One can choose $\beta(t) = -A(t)$

In practice $\beta(t) = -A(t-\tau)$

Example

$$H_p = -\sum_{i,j\in\mathcal{E}} \frac{1}{2}(1 - Z_i Z_j) \qquad H_d = \sum_{j=1}^{n} X_j.$$

$$i[H_d, H_p] = \sum_{i,j\in\mathcal{E}} Y_i Z_j + Z_i Y_j$$



the qubits were initialized in the ground state of H_d.

Past layer 5, it is evident that FALQON is no longer able to decrement $\langle H_p \rangle$ using this hardware platform, despite exhibiting a continued monotonic decrease in associated noise-free numerical simulations (blue point markers). This reveals the limitations that hardware noise presents for this problem instance. Looking ahead, we are optimistic that continuous improvements to quantum hardware will pave the way toward applications of FALQON to increasingly complex combinatorial optimization problems.

Another example

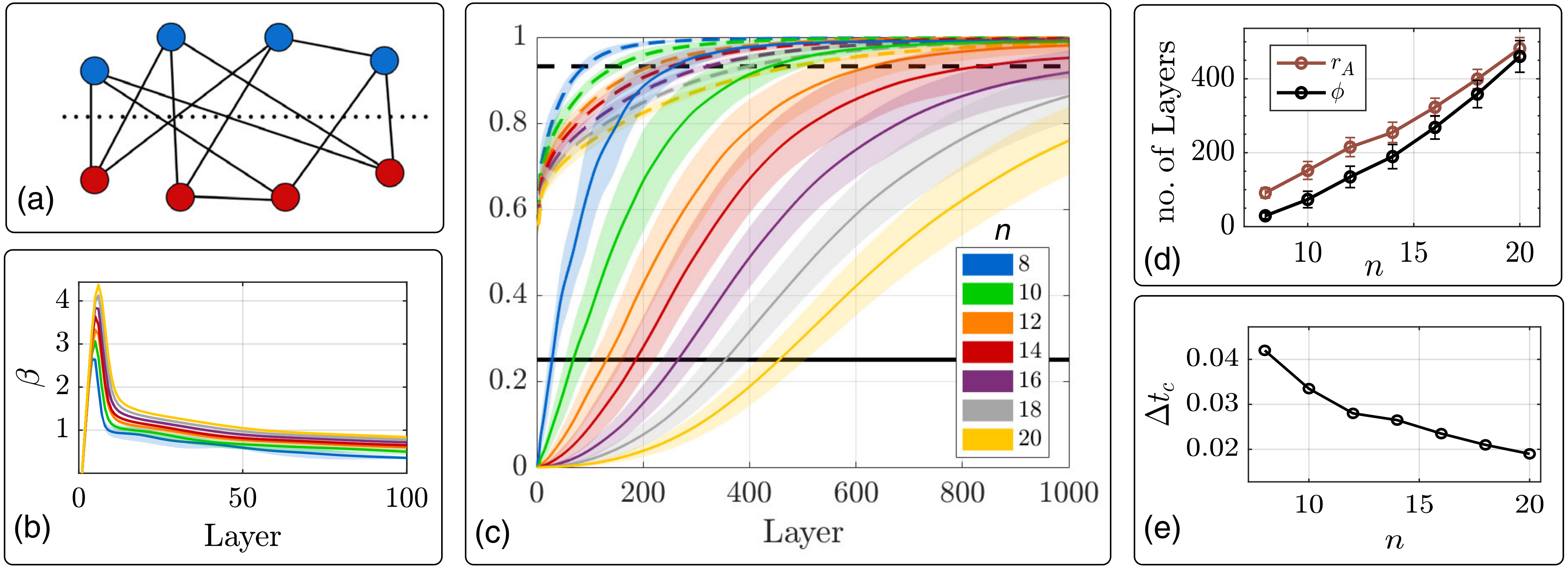


FIG. 3. (a) Pictorial representation of MaxCut on a three-regular graph with 8 vertices. (b) Mean β values are plotted as a function of layer for different n values, with shading showing the standard deviations. (c) The performance of FALQON, as quantified by the approximation ratio (dashed curves) and the success probability of measuring the degenerate ground state (solid curves), is shown for different values of n. (d) The mean number of layers needed to achieve the reference values of $r_A = 0.932$ (dashed curve) and $\phi = 0.25$ (solid curve) is shown; error bars report the associated standard deviation. (e) The critical Δt values for different problem sizes are plotted.

$r_A = \langle H_p \rangle / \langle H_p \rangle_{\min}$ $\quad r_A = 0.932$

classical approximation algorithm

$\phi = \sum_i |\langle \psi | q_{0,i} \rangle|^2$ $\quad \phi = 0.25$

Brainstorming

$i\frac{d}{dt}|\psi(t)\rangle = [H_p + H_d\beta(t)]|\psi(t)\rangle$

- My Proposal: What if we know the exact form of $\beta(t)$, and the form of H_d, such that

$$\frac{d}{dt}\langle\psi(t)|H_p|\psi(t)\rangle = A(t)\beta(t) < 0 \qquad A(t) \equiv \langle\psi(t)|i[H_d, H_p]|\psi(t)\rangle$$

- Let H_D=$|\psi_0\rangle\langle\psi_0|$, where $|\psi_0>$ is the initial state and $\beta(t) = -\frac{g}{t}$, then the solution can be extracted from an exact formula.
- By doing so we can avoid the iterative measurements since $\beta(t)$ is analytical.

- Questions remain: How do one prepare a unitary operator of Hamiltonian H_D=. $|\psi_0\rangle\langle\psi_0|$?

- Can one control the projective measurements discussed in the previous discussion via a time-dependent control.