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Ref.	WP10153
Title	Feedback-based quantum algorithm inspired by counterdiabatic driving
Authors	Rajesh K. Malla, Hiroki Sukeno, Hongye Yu, Tzu-Chieh Wei, Andreas Weichselbaum, and Robert M. Konik

Dear Editor Juan-Jose Lietor-Santos,

Thank you very much for forwarding the two referee reports on our manuscript. We have revised it to address the critiques and are resubmitting it for further consideration in Physical Review Research.

The main critique from the first referee highlights discrepancies between the experimental data and theoretical simulations, which prevents them from recommending publication. This discrepancy is unavoidable in the present devices and has been observed in prior studies related to feedback-based algorithms (e.g., [1, 2]). However, we have conducted new experimental simulations using a more recent and advanced IBM machine with more measurement samples and found that it performs better than what was reported in the previous version of our manuscript, which uses an older machine. We have replaced the old Figure 11 with a new figure (Now Fig. 12) containing data from these new simulations. In response to the second referee, we have introduced new paragraphs and a new figures to illustrate how measurement is performed and to detail the measurement costs for different protocols. Additionally, we have addressed the other comments and requests for clarification from the referees in our detailed reply below.

We have addressed the main critique of the referee as well as additional comments and clarifications in the revised manuscript. We believe the revised manuscript is ready to be published in Physical Review Research.

Sincerely,
Rajesh K. Malla
Hiroki Sukeno
Hongye Yu
Tzu-Chieh Wei
Andreas Weichselbaum
Robert M. Konik

Summary of major changes

- We have replaced Fig. 11 with a new figure now Fig. 12 with data from quantum simulations performed on a better IBM machine.
- We have added brief discussions on gate noise and statistical errors due to measurement sample size at the end of Sec. VI, and in Appendix C, respectively. These discussions are supported with new figures, Fig. 13 and Fig. 17.
- We have added and refined the discussion regarding measurements in Sec. IV to address the referee's comments. A follow-up paragraph has been included in Sec. V. Additionally, we have introduced a new figure, Fig. 3, to demonstrate how the average energy decreases as a function of the number of measurements.

All major changes are marked in blue color in the resubmitted text.
Please find below a point-by-point reply.

I. DETAILED RESPONSE TO FIRST REFEREE

Referee 1: *In their manuscript, the authors demonstrate the potential of a novel method for preparing a quantum system in its energy ground state. Their protocol relies on a variant of the Quantum Lyapunov Control (QLC), which includes a third layer involving a unitary Hamiltonian evolution in addition to the problem and control Hamiltonians typical of QLC. The authors construct this new Hamiltonian by picking evolution operators from a pool inspired by Counter-diabatic Driving (CD), applied to their system to facilitate faster evolution towards the ground state with respect to a standard QLC. They then apply their CD-inspired, feedback-based quantum algorithm to prepare the ground state of a 1D Ising chain under external magnetic field, exploring different orientations and intensities. Additionally, using a classical simulation, they meticulously study their system by varying all the different parameters characterizing their protocol, demonstrating advantage over traditional feedback-based quantum algorithms. Finally, they exhibit some practical results obtained using IBM superconducting quantum computer through cloud-based simulations. The paper is (mostly) well written, and the authors have conducted a thorough theorical analysis. However, some implementation issues with the protocol prevent me from recommending publication.*

We thank the referee for reviewing our manuscript and providing valuable questions and comments that have helped us improve the manuscript. Below, we address the main critique regarding

Figure 11 (Now Fig. 12) of our manuscript. The referee pointed out a discrepancy, which we argue is unavoidable and has been observed in prior studies related to feedback-based algorithms (e.g., [1, 2]). However, given the rapid evolution of qubit devices, we have conducted new experimental simulations using a more advanced IBM machine and found that it performs better than what was reported in the previous version of our manuscript. We have replaced the old Figure 11 with a new figure (Fig. 12) containing data from these new simulations. Additionally, we have addressed all related questions and comments within the main manuscript. We believe the revised manuscript addresses all the referee's concerns and hope the changes are satisfactory, leading to a recommendation for publication.

Referee 1: *The main issue arises from the discrepancies observed between the experimental data points shown in Fig. 11 and the expected outcomes from simulations. While the authors attribute this disparity to the low fidelity of quantum gates and poor accuracy of measurement outcomes, they do not provide a comprehensive study to confirm this hypothesis. Moreover, this discrepancy raises serious concerns about the robustness of their method for real-world applications, since even the qualitative/asymptotic behavior of several experimental curves seems to be remarkably different from the theoretically-expected counterpart.*

The major reason for the discrepancies between the experimental data and classical simulation data stems from gate errors that accumulate exponentially with circuit depth. Calculating energies and parameters β and γ requires measuring expectation values of Pauli operators. When gate noise and errors are excessively large, the expectation values of Pauli operators tend to be zero, causing significant discrepancies. This issue can be mitigated by a machine with less noise and reducing the Trotterization steps.

In the revised version of the manuscript, we performed the experimental simulations using the latest IBM Torino machine, which has noise levels approximately one-fourth of the previous IBM Hanoi machine. Additionally, we reduced the Trotterization step to a single layer, yielding results close to the exact time-evolution operators. As a result, the discrepancy between the new experimental simulation and the classical simulation has been significantly reduced.

We have replaced the experimental figure in the manuscript with data obtained from improved simulations, as discussed at the end of Sec. VI. Additionally, we have introduced two subsections to discuss the effects of gate noise and the statistical errors in each measurement. A new figure has been added to illustrate a comparison between different IBM machines.

The last two paragraphs in Sec VI read:

In Fig. 12, we observe that the experimental data deviates from the classical simulations. The primary reason for such discrepancy stems from the limitations of the current NISQ devices. When the circuit depth grows linearly, the gate errors will accumulate exponentially. In Fig. 13 we have presented the quantum simulations performed in IBM_Sherbrooke and IBM_Torino. The energy curve obtained from the IBM_Sherbrooke for Y-FQA deviates significantly from the theoretical curve for $L > 3$. One can use a less noisy machine to improve this discrepancy or error mitigation

schemes such as zero-noise extrapolation (ZNE)[3, 4]. As is shown in Fig. 13, curves obtained from both machines get closer to the theoretical one after applying ZNE. The average gate error per layer for IBM_Sherbrooke is around 1.7%, while the rate for IBM_Torino is 0.8%. Thus, the curve from IBM_Torino looks better, and is consistent with the theoretical curve up to $L = 5$. In zero-noise extrapolation [4] method folds the circuit according to $UU^\dagger U, UU^\dagger UU^\dagger U, ...$ to amplify the noise strength with $3, 5, ...$ times, and extrapolates the expectation values to the zero-noise limit. The method can achieve a better accuracy of the expectation value, but may also increase the uncertainty of the results, as shown in Fig. 13.

In addition to the gate errors, statistical errors for each measurement may also play a role in the experimental results. We observe that the dynamics does not accumulate statistical errors with circuit depth L, and thus can easily be made negligible compared to the gate errors by increasing the repetition number for each measurement. The reason for the non-accumulation is that, if we obtain β's and γ's with some uncertainties, it effectively changes the parameter α slightly for each layer. When the uncertainties are small, we will obtain another parameter trajectory for β's and γ's close to the exact ones, which likely leads to the same converged energies. Thus, the statistical error will not accumulate during the feedback-based protocol. The overall uncertainty brought by the statistical errors is proportional to $\frac{1}{\sqrt{M}}$, where M is the repetition number for each measurement. To showcase the argument, we did "dry-run" simulations of our protocol, where we performed repetitive measurements on a noiseless simulator to get expectation values. The results for different repetition numbers M demonstrate that the statistical errors do not accumulate and converge to the same energy values. This discussion is in Appendix C, supported by a new figure Fig. 17.

Referee 1: *Furthermore, something seems to be missing in the description of their protocol. The evolution parameters are updated by measuring the expectation value of the commutator between the problem Hamiltonian and the Hamiltonian related to the considered parameter, and this rises two questions. First, how can the expectation value be evaluated by a single measurement? Some comments on how such a measurement can be carried out are needed.*

We do not claim to compute expectation value using a single measurement. We use the commonly used method for the measurements. To find the expectation value of commutator between the problem Hamiltonian and the Hamiltonian related to the considered parameter, we first express the commutator as a linear combination of Pauli basis operators. Then each Pauli operator is measured many times to find the expectation value of that particular Pauli operator. In the end we collect all measurement of the Pauli operators to compute the expectation value of the commutator. We have added these details in the revised manuscript Sec. IV, last two paragraphs.

The expectation value of an operator is obtained by expanding the operator as a linear combination of Pauli basis operators, $\langle\psi_k|\hat{O}|\psi_k\rangle = \sum_i^N \gamma_i \langle\hat{P}_i\rangle$ where γ_i are scalar coefficients and P_i are Pauli basis operators. The measurement of the Pauli operators P_i is repeated to collect statistics. The resulting expectation values are combined to find the expectation value of $\mathcal{O}$ in $|\psi_k\rangle$.

The number of Pauli operators and the number of measurements will depend on the structure of H_P, H_1, and $H_{\rm CD}$. Here, we consider Hamiltonians with nearest-neighbor hopping. Therefore the number of Pauli operators is $O(N)$.

The measurement of many Pauli operators can be efficiently parallelized [5–9]. Consider, for example, the measurement of a spin Hamiltonian $\sum_i^N \sigma_i^a \sigma_{i+1}^b$ for $a \neq b$. The total number of two-qubit Pauli strings required to be measured per layer is N. The terms $\sigma_i^a \sigma_{i+1}^b$ and $\sigma_j^a \sigma_{j+1}^b$ can be measured simultaneously if they commute. This holds trivially if the Pauli strings do not overlap, i.e., for $|i-j|>1$. On the other hand, overlapping, yet commuting Pauli strings share the Pauli basis operators. In the example above the number of Pauli strings can be divided into two sets containing commuting Pauli strings that act either on even or odd bonds. All Pauli strings within a set can be measured simultaneously. Consequently, the number of parallel measurements required per layer is 2. This number corresponds to the two noncommuting terms in the Hamiltonian that act on any given spin. For any counterdiabatic Hamiltonian $H_{\rm CD}$, the number measurements can be obtained from the number of noncommuting terms in the commutator $[H_P, H_{\rm CD}]$ that act on any given spin. Later, we show, in detail, the number of measurements needed for the LFI model with different counterdiabatic Hamiltonians and compare it with the standard FQA.

Referee 1: *Second, after the single-run measurement one would expect the quantum state of the system to collapse; do you use any ancillary system to conduct measurements? Do you need to somehow re-prepare the system on its k-th state? If so, how would you know the state to be re-prepared? I find those points quite confusing.*

Indeed, the referee is right. The system collapses after each measurement and therefore we need to rebuild the state to acquire statistics. When determining the parameters β_{k+1} and γ_{k+1}, the parameters $\{\beta\}_k$ and $\{\gamma\}_k$ up to k-th layer are known. This allows us to prepare the state up to k-th layer deterministically. The circuit is built in an iterative process. Given the wave function up to the k-th layer, we perform measurements to obtain the parameters β and γ for the k+1-th layer.

We have added this discussion related to measurement in Sec. IV third paragraph:

The quantum circuit in CD-FQA is constructed iteratively. The unitaries U_1 and $U_{\rm CD}$ for the $(k+1)^{\rm th}$ layer depend on the respective parameters β_{k+1} and γ_{k+1}. To determine these parameters, we compute the commutators $i\langle[H_1, H_P]\rangle$ and $i\langle[H_{\rm CD}, H_P]\rangle$ using a quantum circuit for the state $|\psi_k\rangle$, i.e., a state that is built up to the $k^{\rm th}$ layer. Following the conventional choice for the application of QLC, we set the control fields to the following expectation values (cf. Fig. 1):

$$\begin{aligned} \beta_{k+1} &= \tfrac{i\alpha}{N} \langle\psi_k|[H_P, H_1\ ]|\psi_k\rangle, \\ \gamma_{k+1} &= \tfrac{i\alpha}{N} \langle\psi_k|[H_P, H_{\rm CD}]|\psi_k\rangle\ . \end{aligned} \tag{R1}$$

Once these parameters are determined, the procedure is repeated iteratively to construct the $(k+1)^{\rm th}$ layer. Since parameters β and γ enter as prefactors to Hamiltonians, they need to scale independent of system size. This necessitates the scale factor $1/N$.

Referee 1: *Finally, some theoretical points, such as the heuristic choice of the evolution Hamiltonian described in Section III, require a more detailed explanation.*

We have considered the standard heuristic choices inspired from quantum annealing and QAOA. Since our Hamiltonian consists of mostly σ_z operators, we consider the first control Hamiltonian to be $\sum \sigma_x$. We have added this statement in Sec. III. The statement reads:

The first control Hamiltonian, H_1, is chosen heuristically. It is inspired from quantum annealing or QAOA. The operator H_1 is also called a mixer, i.e., that mixes the eigenstates of the problem Hamiltonian. Here we will consider the H_1 to be a sum of operators that act only on local qubits. For example, for the applications on the Ising model below, we will choose H_1 as a sum over Pauli-x operators which mixes across different S_z sectors.

Referee 1: *MINOR ISSUES: Some quantities in Eq. (5) are not defined. A double check of all the formulas might be needed.*

We thank the referee for noticing this. We have defined all the terms of Eq. 5 below the equation.

II. DETAILED RESPONSE TO SECOND REFEREE

Referee 2: *The authors present a derivative of feedback-based quantum optimization, where they add terms inspired by counterdiabatic protocols. Specifically, they compute terms through nested commutators. They demonstrate the performance of their algorithm by preparing the ground state of Ising models and also show demonstrations on IBM cloud quantum computers. The work is interesting, but some clarification is needed before accepting the paper.*

We sincerely thank the referee for their supportive review and constructive feedback on our manuscript. We have carefully addressed all the comments and questions and have revised the manuscript accordingly.

Referee 2: *The authors mention that their goal is not to make the system depict counterdiabatic driving; rather, their goal is to find suitable operators from the nested commutators expansion. However, in the abstract, they mention that they are integrating quantum Lyapunov control with counterdiabatic driving. This could sound misleading since this method is inspired by counterdiabatic driving, as correctly mentioned in the title. I would suggest rephrasing that sentence for better clarity for the audience.*

We have rephrased the sentence in the abstract of the revised manuscript. The modified line now reads:

Here, we propose a substantial enhancement by implementing a protocol that uses ideas from quantum Lyapunov control and the counterdiabatic driving protocol, a key concept from quantum

adiabaticity.

Referee 2: *The number of measurements is a key resource in feedback-based quantum algorithms, and this method doubles the measurements required to reach the solution. The authors claim that this is compensated by faster convergence to the ground state. However, the commutators mentioned in Equations 12 and 13 can yield operators whose expectation values cannot be measured simultaneously since there will be many non-commuting terms. It would be nice to see a comparison of the measurements required versus convergence to see the advantage of this method more precisely.*

Indeed, the number of measurements is a key resource in our protocol. Adding additional control Hamiltonians increases the measurement cost. Although it may appear that the measurement cost doubles, in reality, this number depends on the structure of both the problem Hamiltonian and the CD-operator. In the revised version, we have discussed how the measurement is performed in Sec. IV and the application on the LFI model in Sec. V, Subsec. A. We have also added a new figure, Fig. 3, where the average energy is plotted as a function of measurement required.

Below, we have copied the exact response from the manuscript for convenience.

The expectation value of an operator is obtained by expanding the operator as a linear combination of Pauli basis operators, $\langle\psi_k|\hat{O}|\psi_k\rangle = \sum_i^N \gamma_i \langle\hat{P}_i\rangle$ where γ_i are scalar coefficients and P_i are Pauli basis operators. The measurement of the Pauli operators P_i is repeated to collect statistics. The resulting expectation values are combined to find the expectation value of $\mathcal{O}$ in $|\psi_k\rangle$. The number of Pauli operators and the number of measurements will depend on the structure of H_P, H_1, and $H_{\rm CD}$. Here, we consider Hamiltonians with nearest-neighbor hopping. Therefore the number of Pauli operators is $O(N)$.

The measurement of many Pauli operators can be efficiently parallelized [5–9]. Consider, for example, the measurement of a spin Hamiltonian $\sum_i^N \sigma_i^a \sigma_{i+1}^b$ for $a \neq b$. The total number of two-qubit Pauli strings required to be measured per layer is N. The terms $\sigma_i^a\sigma_{i+1}^b$ and $\sigma_j^a\sigma_{j+1}^b$ can be measured simultaneously if they commute. This holds trivially if the Pauli strings do not overlap, i.e., for $|i-j|>1$. On the other hand, overlapping, yet commuting Pauli strings share the Pauli basis operators. In the example above the number of Pauli strings can be divided into two sets containing commuting Pauli strings that act either on even or odd bonds. All Pauli strings within a set can be measured simultaneously. Consequently, the number of parallel measurements required per layer is 2. This number corresponds to the two noncommuting terms in the Hamiltonian that act on any given spin. For any counterdiabatic Hamiltonian $H_{\rm CD}$, the number measurements can be obtained from the number of noncommuting terms in the commutator $[H_P, H_{\rm CD}]$ that act on any given spin. Later, we show, in detail, the number of measurements needed for the LFI model with different counterdiabatic Hamiltonians and compare it with the standard FQA.

The number of measurements is a key resource in our protocol. As we have established, the number of measurements per layer equals the number of noncommuting Pauli strings acting on any site. For the LFI model the commutator $[H_P, H_1]$ yields terms $YZ + ZY$ and Y. The term $\sigma_i^y\sigma_{i+1}^z$

commutes with $\sigma_j^z \sigma_{j+1}^y$ for $j = i \pm 1$. Therefore they can be measured simultaneously. Therefore, the number of parallel measurements needed to find expectation values of $YZ + ZY$ and Y is 2, since measuring YZ is enough to extract information about Y measurement. Similarly in CD-FQA protocol, for $H_{\rm CD} = Y$, the commutator $[H_P, H_{\rm CD}]$ yields terms $\{X, XZ, ZX\}$ and requires 2 parallel measurements and a total of 4 measurements per layer. For $H_{\rm CD} = YZ$, the commutator $[H_P, H_{\rm CD}]$ yields $\{X, ZXZ, XZ\}$ and requires 2 parallel measurements per layer, and the protocol requires a total of 4 parallel measurements. For $H_{\rm CD} = YX$, the commutator $[H_P, H_{\rm CD}]$ yields $\{XY, YYZ, ZXX, XX, YY\}$ and requires 8 parallel measurements per layer. Since the terms $YZ + ZY$ and Y can be measured in parallel with the set $\{XY, YYZ, ZXX, XX, YY\}$ the protocol requires a total of 8 parallel measurements per layer. In Fig. 3, we plot the average energy per site vs measurement required for different CD-FQA protocols. The number of measurements in the protocol with Y as a CD operator converges much faster than other protocols. The protocol with YX as a CD operator requires more measurements per layer compared to standard FQA. Nevertheless, the number of measurements is independent of the system size and the measurement cost between different protocols depends on the degree of locality in the system.

Referee 2: *It has been observed in previous works on counterdiabatic driving that Y+YZ+ZY terms perform better than just Y terms (arxiv.org/abs/2201.00790). However, in this work, in most cases, the addition of the Y term leads to the best performance. Is there any physical intuition behind it? Can this behavior be explained with counterdiabatic theory?*

We thank the referee for pointing out this paper. Indeed, the term "Y+YZ+ZY" performs better than just "Y" in the counterdiabatic drive scenario when the coefficients of each operator are variationally optimized. This is because "Y+YZ+ZY" is closer to an exact counterdiabatic operator than "Y" alone.

In the context of the present manuscript, we have not included the term "Y+YZ+ZY" due to the additional computational cost associated with the extra parameters. Including all three terms would result in a circuit where each unit contains five layers, and the number of measurements would increase fourfold. Therefore, we refrained from using the linear combination of operators in general (except for one example).

In the revised manuscript, we have cited the suggested paper and added a line:

Similar linear combination of operators have been utilized as a counterdiabatic term in Ref. [10] in the digitalized counterdiabatic quantum optimization for MFI model.

Referee 2: *The authors mention that a monotonic decrease is guaranteed if the Δt is infinitesimally small. When we scale this algorithm to higher qubits, this might prove to be a bottleneck since a small Δt essentially means that we will require large circuit depths.*

For small time step, Δt, the quantum circuit simulates a continuous Schrödinger equation, and the CD-FQA protocol will require a deep quantum circuit, which indeed is a bottleneck of our protocol. However, this is true for all the protocols based on quantum control approaches.

Quantum control approaches require simulating time-dependent Schrödinger equations. Therefore we emphasize that our protocol is more suitable for quantum computers that are close to fault-tolerant as pointed out in other feedback-based algorithms [1].

Referee 2: *Is there any specific reason why the authors skip Trotterization for simulations in the main text? If an algorithm is presented that is for quantum computers, trotterization will always be required. For instance, in the part where a linear combination of operators is chosen, Trotterization will play a crucial role in L=1000 layers since operators are non-commuting.*

We agree with the referee that Trotterization is essential for implementing quantum simulations on a quantum computer. In our study, we use first-order Trotterization for this purpose. While our classical simulations are performed without Trotterization, using a small time-step to approximate continuous time-evolution, we supplement these with simulations using Trotterization, implemented via PennyLane (see Appendix).

Quantum control methods in experiments typically involve continuous time-evolution. Therefore, our classical simulations are designed to be comparable to continuous time-evolution when the time-step is sufficiently small. Additionally, we include a tunable parameter, α, which determines the rate of energy decay. To better understand the relationship between various quantum control approaches, we present plots derived from protocols that closely mimic continuous time-evolution.

Referee 2: *In Fig. 12c, the plot has some unnecessary misprints on the top right. As a general comment, I would like to suggest that the authors make the graphs more tidy and coherent throughout the manuscript*

We thank the referee for noticing the misprint in Fig. 12c. We have corrected it in the revised manuscript.

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