Lec 2: Multiplying and Factoring Matrices.

$A = LU$ elimination

$A = QR$ Gram-Schmidt.

$S = Q\Lambda Q^T = [q_1 \cdots q_n] \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} \begin{bmatrix} q_1^T \\ \vdots \\ q_n^T \end{bmatrix}$

(symmetric) eigenvectors (orthognal) eigenvalues(real)

$$Q\Lambda Q^T = (Q\Lambda)(Q^T) = \text{sum of rank 1} = (\text{cols of } Q\Lambda)(\text{rows of } Q^T)$$

$$= \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T + \cdots + \lambda_n q_n q_n^T = S$$

$$\text{col 1 of } Q\Lambda = [q_1 \cdots q_n]\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} = q_1\lambda_1$$

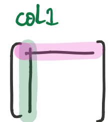
check: Look at $Sq_1 = \lambda_1 q_1 \underbrace{q_1^T q_1}_{\|q_1\|^2=1} + \lambda_2 q_2 \underbrace{q_2^T q_1}_{0} + \cdots + \lambda_n q_n \underbrace{q_n^T q_1}_{0}$ (orthognal)

$= \lambda_1 q_1$

$A = X\Lambda X^{-1}$

$A = U\Sigma V^T =$ (orth) · (diag) · (orth) SVD.

$$\underset{A}{\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}}$$



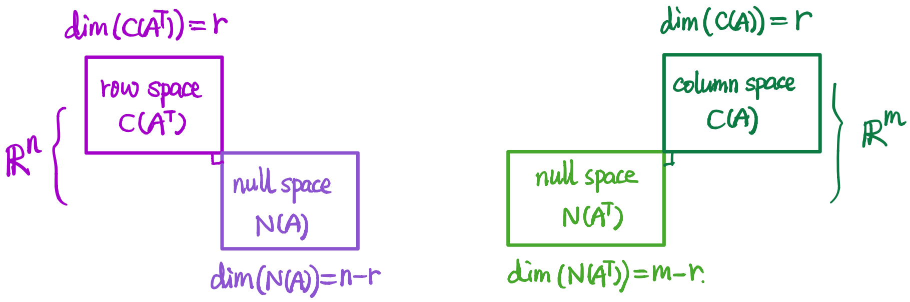
how to do LU? split A into (col1)·(row1) + $\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & A_2 & \\ 0 & & \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}}_{\text{rank1.}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{rank1.}} = [l_1][u_1^T] + [l_2][u_2^T] = LU.$$

4 foundamental subspace. (A $m\times n$ rank: r).

- column space $C(A)$ dim $= r$
- row space $C(A^T)$ dim $= r$.
- null space $N(A)$ dim $= n-r$
- null space $N(A^T)$ dim $= n-r$

null space = all solutions to $\left.\begin{matrix} Ax = 0 \\ Ay = 0 \end{matrix}\right\} \Rightarrow A(x+y) = 0$ space.



dim $\underbrace{r}_{\text{row space}}$ $\underbrace{n-r}_{\text{null space}}$ r $m-r$.

$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ $m=2$, $n=3$, $r=1$.

null space $N(A)$: $Ax=0$. $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$C(A^T)$ and $N(A)$, these two space are at 90 degree angles.