Lec 14: Low Rank Changes in A and Its Inverse

Chapter IV Low Rank Matrices.

IV.1 $(I-uv^T)^{-1}$ $\quad$ $(I-UV^T)\ (A-UV^T)$

$\uparrow$ rank 1 $\qquad$ $\nwarrow$ rank K $\nearrow$

perturbations. Matrix Inversion Formula

$$(I-uv^T)^{-1} = I + \frac{uv^T}{1-v^Tu}$$ $\leftarrow$ number / 1×1 matrix

check

$$(I-uv^T)^{-1}\left(I+\frac{uv^T}{1-v^Tu}\right) = I-uv^T + \frac{(I-uv^T)\,uv^T}{1-v^Tu}$$

$uv^T - u(v^Tu)v^T = u(1-v^Tu)v^T$

$k=1$

$n\times n$ matrix to invert.

$$(I_n-UV^T)^{-1} = I_n + U\,(I_k-V^TU)^{-1}V^T$$

Sherman-Morrison-Woodbury.

$(n\times k)(k\times n)$. rank $=k$. $\qquad$ $k\times k$ inverse.

$U-UV^TU = U(\cancel{I-V^TU})$

$$(I_n-UV^T)\left[I_n+U(I_k-V^TU)^{-1}V^T\right] = I_n-UV^T + (I-UV^T)U\cancel{(I-V^TU)^{-1}}V^T$$

$$= I_n - UV^T + UV^T = I_n.$$

$$(A-UV^T)^{-1} = A^{-1} - A^{-1}U(I_k - V^TA^{-1}U)^{-1}V^TA^{-1}$$

$n\times n$ $\quad$ $(n\times k)(k\times n)$ rank k.

To use the formula.

Use 1: Solve $(A-UV^T)^{-1}x = b$.

Use 2: New measurement in least square.

OLD $\quad Ax=b \xrightarrow{\text{normal}} A^TA\hat{x} = A^Tb.$

NEW $\quad \begin{bmatrix} A \\ v^T \end{bmatrix}\begin{bmatrix} x_{new} \end{bmatrix} = \begin{bmatrix} b \\ b_{new} \end{bmatrix} \xrightarrow{\text{new normal eqn}} [A^T\ v]\begin{bmatrix} A \\ v^T \end{bmatrix}\hat{x}_{new} = [A^T\ v]\begin{bmatrix} b \\ b_{new} \end{bmatrix}$

$$(A^TA + vv^T)\,\hat{x}_{new} = A^Tb + v^Tb_{new}.$$

least² standard.

① covariance $= I$.

②. state equation.

$\uparrow$ position of satellite.

recursive least square.

Kalman filter. (dynamic least²).

To discover the formula.

suppose $Aw=b$ is solved for w.

now <u>solve</u> $(A-uv^T)x=b$ quickly.

<u>solve</u> $Az=u$.

changes.

$$\Rightarrow \quad x = w + \frac{wv^Tz}{1-v^Tz}$$

$A[w\ z]=[b\ u]$

$Ax=b$	$x=w$
$(A-uv^T)x=b$	$x=w+\dfrac{wv^Tz}{1-v^Tz}$