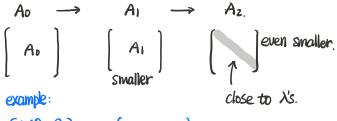
Lec 12: Computing Eigenvalues and Singular Values.

$$A_0 = Q_0 R_0 = R_0 Q_0 = A_1$$

A1 is similar to Ao

$$^{\uparrow}$$
 RoQo = RoAo Ro $^{\uparrow}$



$$\begin{bmatrix}
\cos \theta & ? \\
\sin \theta & ?
\end{bmatrix} \rightarrow \begin{bmatrix}
- \\
(\sin \theta)^3 -
\end{bmatrix}$$

$$A_0 \qquad \text{quickly} \qquad A_1$$
become 0

Introduce shifts.

$$A_0 - SI = Q_0 R_0 = R_0 Q_0 + SI = A_1$$

$$A_1 = R_0 Q_0 + SI = R_0 (A_0 - SI) R_0^{-1} + SI$$

$$= R_0 A_0 R_0^{-1} - SR_0 I R_0^{-1} + SI$$

$$= R_0 A_0 R_0^{-1}$$

If
$$A_0 = \begin{cases} 5 \times 5 \end{cases}$$
 impossible to find formula for λ 's.

Hessenberg Matrix

- 1) Reduce A to Hessenberg form
- @ QR with shifts



(dor't do it!) $\det(A-\lambda^{2})=0.$

 $QSQ^T = QSQ^T = tridiagonal$. same λ 's

SVD:
$$A = U \subseteq V^T$$
. $(Q_1 U) \supseteq (V^T Q_2)$.

$$(Qu)^{-1} = U^{\dagger}Q^{\dagger} = U^{\dagger}Q^{\dagger} = (Qu)^{\dagger} \quad A^{\dagger}A \rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
 tridiagonal.

b,
$$Ab$$
, A^2b , ..., $A^{qq}b$
 $V = C_1b + C_2(Ab) + C_3(A^2b) + ... + C_{100}(A^{qq}b) + error$

ignored.

A is reduced to dimension 100.

 $A_K = in K_{100} + out K_{100}$ ignored.

(100000)