Lec7: Eckart-Young: The Closest Rank k Matrix to A

$A = U\Sigma V^T = \sigma_1 u_1 v_1^T + \cdots + \sigma_r u_r v_r^T$

$A_k = U_k \Sigma_k V_k = \sigma_1 u_1 v_1^T + \cdots + \sigma_k u_k v_k^T$

If B has rank k, then $\|A-B\| \geq \|A-A_k\|$.   Eckart-Young

ℓ^2 $\|v\|_2 = \sqrt{|v_1|^2 + \cdots + |v_n|^2}$ $\Rightarrow$ sparse

ℓ^1 $\|v\|_1 = |v_1| + \cdots + |v_n|$ $\Rightarrow$ a lot of little numbers.

ℓ_∞ $\|v\|_\infty = \max\{|v_i|\}$

property: $\|cv\| = c\|v\|$.

$\|v+w\| \leq \|v\| + \|w\|$

$\|A\|_2 = \sigma_1$ (largest singular matrix)

$\|A\|_F = \sqrt{|a_{11}|^2 + \cdots + |a_{1n}|^2 + \cdots + |a_{n1}|^2 + \cdots + |a_{nn}|^2}$

Forbenius

$\|A\|_{\text{Nuclear}} = \sigma_1 + \sigma_2 + \cdots + \sigma_n$ Netflix / MRI

$k=2$.

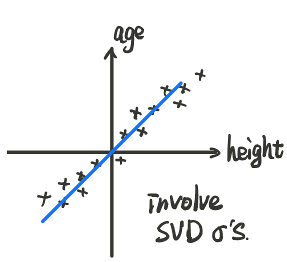
$$\Sigma = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \Sigma_2 = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{rank}=2.$$

$$B = \begin{pmatrix} 3.5 & 3.5 & 0 & 0 \\ 3.5 & 3.5 & 0 & 0 \\ 0 & 0 & 1.5 & 1.5 \\ 0 & 0 & 1.5 & 1.5 \end{pmatrix} \quad \text{rank}=2.$$

$A = U\Sigma V^T$

$\|Qv\|^2 = (Qv)^T(Qv) = v^T Q^T Q v = \|v\|$.

$QA = QU\Sigma V^T = (QU)\Sigma V^T$



involve SVD σ's.

$$A_0 = \begin{bmatrix} h_1 & h_2 & \cdots & h_n \\ a_1 & a_2 & \cdots & a_n \end{bmatrix} \begin{matrix} \leftarrow \text{height} \\ \leftarrow \text{age} \end{matrix}$$

$v_1 \quad v_2 \quad v_n$

$$A = A_0 - \begin{bmatrix} a_h & a_h & \cdots & a_h \\ a_a & a_a & \cdots & a_a \end{bmatrix}$$

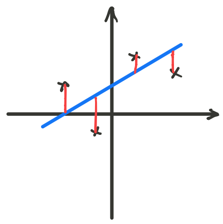
each row now adds to 0

Principle Component Analysis.

Sample.

Covariance matrix 2×2: $\dfrac{AA^T}{N-1}$

computed from samples.



$\min \|b - Ax\|^2 \longrightarrow \underbrace{A^T A \hat{x} = A^T b.}_{\text{normal equations.}}$

regression.

Our problems was not least squares, not the same as least squares

Looking for best c that age = c. height ($c = \sigma_1$).