Lec 8: Norms of Vectors and Matrices.

Probability Matching:

Baised coin: Head 75% Tail 25%.

Pay off $1 guess right

-$1 guess wrong.

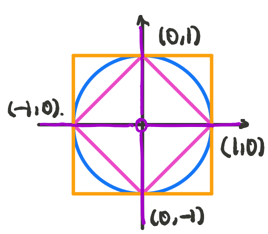
Intelligent	H T H H T H H H H H H H
Actual	H H T H H T ⋯

Vector norm $\|v\|_{p=2,1,\infty,0}$.

p	$\|v\|_p$	
2	$\sqrt{v_1^2+\cdots+v_n^2}$	
1	$\|v_1\|+\cdots+\|v_n\|$	ℓ^1 norm.
∞	$\max \|v_i\|$	
0	number of non-zero component ($\|2v\|_0 = \|v\|_0$).	
$\underset{\text{pos-def / sym}}{S}$	$\sqrt{v^TSv}$	

$\|v\|_p = (|v_1|^p + \cdots + |v_n|^p)^{\frac{1}{p}}$

2D.

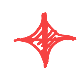


$\|v\|_2 = 1$ $(v_1^2+v_2^2=1)$

$\|v\|_1 = 1$ $(|v_1|+|v_2|=1)$.

$\|v\|_\infty = 1$ $(\max |v_i| = 1)$.

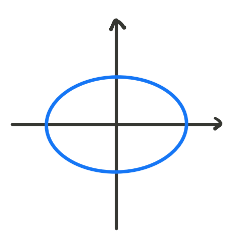
$\|v\|_0 = 1$ (x and y axies without $(0,0)$).

true norm: convex unit ball $\|v\| \le 1$ ($p = \frac{1}{2}$ )

NOT

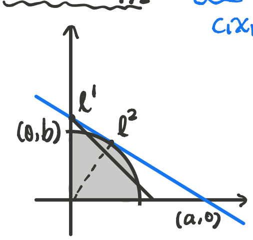
shape of v^TSv

$S = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ $v^TSv = 2v_1^2 + 3v_2^2 = 1$.



min $\|x\|_{1/2}$, $Ax = b$.

$c_1x_1 + c_2x_2 = b$.



norm of matrices.

matrix norm from vector norm

$\|A\|_2 = \max \sigma_i = \sigma_1$ $\qquad \|A\|_2 = \max\limits_{\text{all } x} \frac{\|Ax\|_2}{\|x\|_2} = \max$ blowup.

winner $x = v_1$ (1st right singular vectors).

$$\frac{\|Av_1\|}{\|v_1\|} = \frac{\|\sigma_1 u_1\|}{1} = \sigma_1 \quad (Av_k = \sigma_k u_k)$$

Forbenius:

$$\|A\|_F = \sqrt{\text{add all } |v_{ij}|^2} = \sqrt{\sigma_1^2 + \cdots + \sigma_r^2}$$

$$A = U \Sigma V^T$$

$$\|\Sigma\|_F = \sqrt{\sigma_1^2 + \cdots + \sigma_r^2}$$

Nuclear:

$$\|A\|_N = \sigma_1 + \cdots + \sigma_r$$

(*)