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Dirac oscillator in perpendicular magnetic and transverse electric fields

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HIGHLIGHTS

- We study Dirac Oscillator with magnetic as well as electric field.
- Exact solutions are found.
- Critical cases have been examined.

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ABSTRACT

We study $(2+1)$ dimensional massless Dirac oscillator in the presence of perpendicular magnetic and transverse electric fields. Exact solutions are obtained and it is shown that there exists a critical magnetic field B_c such that the spectrum is different in the two regions $B > B_c$ and $B < B_c$. The situation is also analyzed for the case $B = B_c$.

1. Introduction

The Dirac oscillator is one of the few interactions for which the Dirac equation is exactly solvable [1]. Apart from the intrinsic interest the Dirac oscillator and various related models have been studied extensively [2] because of their probable applications in many branches of physics, e.g., in optics [3], graphene [4], Jaynes–Cummings model [5] etc. Recently it has also been realized experimentally in microwaves [6]. An interesting model emerges when the Dirac oscillator is subjected to a

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homogeneous perpendicular magnetic field. This system too is exactly solvable [7]. A particularly important feature of this system is that it exhibits quantum chirality phase transition [8]. That is, there exists a critical magnetic field B_c such that the system exhibits a chirality quantum phase transition when $B > B_c$ and the spectrum is different in the two regions $B > B_c$ and $B < B_c$ [8]. In this context we would like to point out that the origin of relativistic Landau problem and the Dirac oscillator is entirely different—in the former case the magnetic field is introduced via minimal coupling while in the latter case the interaction is introduced via non minimal coupling and can be viewed as anomalous magnetic interaction.

It may be noted that the massless $(2+1)$ dimensional Dirac equation is exactly solvable in the presence of a homogeneous perpendicular magnetic and a transverse homogeneous electric field [9–11]. Such a system plays an important role e.g, in the context of Hall effect in graphene [11]. Here we shall examine the same system as in Ref. [11] but in addition we would also incorporate an oscillator type interaction. It will be shown that this model is exactly solvable and there exists a critical magnetic field B_c such that the spectra in the regions $B > B_c$ and $B < B_c$ are different. Furthermore the critical point $B = B_c$ is actually a point of discontinuity of the spectrum. The present problem can also be viewed in a different way. It is known that the Dirac oscillator problem remains exactly solvable in the presence of a homogeneous magnetic field [7]. It will be seen that it remains so even when a homogeneous transverse electric field is introduced. In particular we shall analyze in detail the spectrum for $B > B_c, B < B_c$ as well as for $B = B_c$. It will also be shown that when the magnetic field is turned off the problem (Dirac oscillator in the presence of an electric field) still admits exact bound state solutions provided the oscillator strength (k) is greater than a certain critical value (k_c). The organization of the paper is as follows : in Section 2 we formulate the problem; in Sections 3–5 we obtain solutions in different regimes of the magnetic field; in Section 6 we analyze the case when $c|\nu| = |E|$ where ν and E denote the effective magnetic and electric field strength respectively; finally Section 7 is devoted to a conclusion.

2. Formulation of the problem

The $(2+1)$ dimensional massless Dirac equation in the presence of a perpendicular magnetic and a transverse electric field is given by [11]

$$c\left[\sigma_x P_x + \sigma_y P_y\right]\Psi - \mathbf{e}V\Psi = \epsilon\Psi \tag{1}$$

where c denotes the velocity of light, $\mathbf{e}$ denotes the electric charge and $\Psi = \binom{\Psi_1}{\Psi_2}$ is a two component spinor. The generalized momenta and the electric field are given by

$$P_x = p_x + \mathbf{e}A_x, \qquad P_y = p_y + \mathbf{e}A_y, \qquad A_x = -By, \qquad A_y = 0, \qquad V = Ey. \tag{2}$$

It will be seen that it is possible to introduce an oscillator interaction in Eq. (1), still keeping the resulting equation solvable. Eq. (1) in the presence of an oscillator interaction in the same direction as the electric field is given by

$$c(\sigma_x P_x + \sigma_y P_y - iky\sigma_y\sigma_z)\Psi - \mathbf{e}Ey\Psi = \epsilon\Psi. \tag{3}$$

We shall now obtain exact solutions of Eq. (3) and examine how the spectrum depends on the magnetic and electric fields as well as the oscillator interaction. More precisely it will be seen that there exists a critical value of the magnetic field, namely, $B_c = \frac{k}{\mathbf{e}}$ such that the solutions are different for $B > B_c$ and $B < B_c$.

3. Weak magnetic field : $B < B_c$

In this case Eq. (3) can be written as

$$c\begin{pmatrix} -\dfrac{\mathbf{e}}{c}Ey & p_x - ip_y + \mathbf{e}\nu y \\ p_x + ip_y + \mathbf{e}\nu y & -\dfrac{\mathbf{e}}{c}Ey \end{pmatrix}\Psi = \epsilon\Psi \tag{4}$$

where $\nu = \frac{k}{\mathbf{e}} - B$. It may be noted that the effect of the oscillator interaction is to produce an *effective* magnetic field of magnitude ν pointed along the negative z direction.

Since the motion in the x direction is a free one, we take the solution as

$$\Psi = e^{-ik_x x}\psi \tag{5}$$

and obtain from Eq. (4)

$$c\begin{pmatrix} -\frac{\mathbf{e}}{c}Ey & -k_x\hbar - ip_y + \mathbf{e}\nu y \\ -k_x\hbar + ip_y + \mathbf{e}\nu y & -\frac{\mathbf{e}}{c}Ey \end{pmatrix}\psi = \epsilon\psi. \tag{6}$$

Now performing a change of variable

$$y = l_1\bar{y} + l_1^2 k_x, \qquad l_1 = \sqrt{\frac{\hbar}{\mathbf{e}\nu}} \tag{7}$$

we obtain from Eq. (4)

$$\mathcal{H}\psi = \epsilon_0\psi \tag{8}$$

where

$$\mathcal{H} = \begin{pmatrix} \mathbf{e}El_1\bar{y} & -\frac{\hbar c}{l_1}\left(-\frac{\partial}{\partial\bar{y}} + \bar{y}\right) \\ -\frac{\hbar c}{l_1}\left(\frac{\partial}{\partial\bar{y}} + \bar{y}\right) & \mathbf{e}El_1\bar{y} \end{pmatrix} \tag{9}$$

and

$$\epsilon_0 = -(\epsilon + \mathbf{e}El_1^2 k_x). \tag{10}$$

We note that the form of $\mathcal{H}$ is still quite complicated and the eigenvalue equation (8) cannot be solved directly. In order to achieve further simplification we now consider [11]

$$\mathcal{H}_1 = \sigma_z\mathcal{H}\sigma_z = \begin{pmatrix} \mathbf{e}El_1\bar{y} & \frac{\hbar c}{l_1}\left(-\frac{\partial}{\partial\bar{y}} + \bar{y}\right) \\ \frac{\hbar c}{l_1}\left(\frac{\partial}{\partial\bar{y}} + \bar{y}\right) & \mathbf{e}El_1\bar{y} \end{pmatrix}. \tag{11}$$

Then it follows that

$$\widehat{\mathcal{H}}\psi = \epsilon_0^2\psi \tag{12}$$

where $\psi = \binom{u}{v}$ is a two component spinor and

$$\widehat{\mathcal{H}} = [\epsilon_0(\mathcal{H}_1 + \mathcal{H}) - \mathcal{H}_1\mathcal{H}] = \begin{pmatrix} J - \frac{c^2\hbar^2}{l_1^2} & \mathbf{e}Ec\hbar \\ -\mathbf{e}Ec\hbar & J + \frac{c^2\hbar^2}{l_1^2} \end{pmatrix}. \tag{13}$$

From Eqs. (12) and (13) it follows that

$$\left(J - \frac{c^2\hbar^2}{l_1^2} - \epsilon_0^2\right)u = -\mathbf{e}Ec\hbar\ v \tag{14}$$

$$\left(J + \frac{c^2\hbar^2}{l_1^2} - \epsilon_0^2\right)v = \mathbf{e}Ec\hbar\, u \tag{15}$$

where the operator J is given by

$$J = -\frac{c^2\hbar^2}{l_1^2}\frac{\partial^2}{\partial\bar{y}^2} + \left(\frac{c^2\hbar^2}{l_1^2} - \mathbf{e}^2E^2l_1^2\right)\bar{y}^2 + 2\mathbf{e}E\epsilon_0 l_1\bar{y}. \tag{16}$$

The above equations can be easily disentangled and the resulting (fourth order) equations for the components are given by

$$\begin{aligned}\left(J + \frac{c^2\hbar^2}{l_1^2} - \epsilon_0^2\right)\left(J - \frac{c^2\hbar^2}{l_1^2} - \epsilon_0^2\right) u &= -\mathbf{e}^2E^2c^2\hbar^2\ u\\ \left(J - \frac{c^2\hbar^2}{l_1^2} - \epsilon_0^2\right)\left(J + \frac{c^2\hbar^2}{l_1^2} - \epsilon_0^2\right) v &= -\mathbf{e}^2E^2c^2\hbar^2\ v.\end{aligned} \tag{17}$$

It may be noted that J can be identified with the Hamiltonian of a shifted harmonic oscillator *provided* $c|\nu| > |E|$. On the other hand if $c|\nu| < |E|$, the operator J would correspond to an inverted shifted oscillator which does not possess normalizable discrete energy states [12]. Thus even though the magnetic field may be very weak and its magnitude less than that of the electric field the oscillator frequency must be large enough such that the *effective* magnetic field satisfies the above condition. Now in order to solve the fourth order equations in Eqs. (17) we take v as a normalized eigenfunction of the shifted oscillator satisfying [13]

$$\begin{aligned} &Jv_n = \Omega_{Jn}v_n\\ &v_n(z) = \left(\frac{\sqrt{\lambda_1}}{2^n n!\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\frac{1}{2}\lambda_1 z^2}H_n(\sqrt{\lambda_1}\,z), \quad z = \bar{y} + \frac{2\sqrt{2}B_1\epsilon_0}{F_1^2\lambda_1^2}\\ &\Omega_{Jn} = \omega_1\left(n+\frac{1}{2}\right) - \frac{4\epsilon_0^2B_1^2F_1^2}{\omega_1^2}, \quad n = 0, 1, 2, \ldots\end{aligned} \tag{18}$$

where

$$B_1 = \frac{\mathbf{e}El_1}{\sqrt{2}}, \qquad F_1 = \frac{\sqrt{2}c\hbar}{l_1}, \qquad \omega_1 = \sqrt{F_1^4 - 4B_1^2F_1^2}, \qquad \lambda_1 = \frac{\omega_1}{F_1^2}. \tag{19}$$

Then from Eqs. (17) and (18) we obtain

$$\epsilon_0^2(n,\pm) = \frac{\omega_1^3}{F_1^4}\left[n + \frac{1}{2}(1\pm 1)\right]. \tag{20}$$

These are however not the eigenvalues of the original problem. Making use of Eq. (10) we finally get

$$\begin{aligned}\epsilon(n,+) &= -\mathbf{e}El_1^2k_x - \frac{(F_1^2 - 4B_1^2)^{\frac{3}{4}}}{F_1^{\frac{1}{2}}}\sqrt{n+1}\\ \epsilon(n,-) &= -\mathbf{e}El_1^2k_x + \frac{(F_1^2 - 4B_1^2)^{\frac{3}{4}}}{F_1^{\frac{1}{2}}}\sqrt{n}.\end{aligned} \tag{21}$$

Next using Eq. (15) we find that

$$\begin{aligned}u_n &= C_{1\mp}\left(\frac{\sqrt{\lambda_1}}{2^n n!\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\frac{1}{2}\lambda_1 z^2}H_n(\sqrt{\lambda_1}\,z),\\ C_{1\mp} &= \frac{\left(\Omega_{Jn} + \frac{c^2\hbar^2}{l_1^2} - \epsilon_0^2\right)}{\mathbf{e}Ec\hbar} = \frac{1}{2}\left(\frac{F_1}{B_1} \mp \frac{\omega_1}{B_1F_1}\right).\end{aligned} \tag{22}$$

Now making use of the fact that $\epsilon_0^2(n+1,-)=\epsilon_0^2(n,+)$ the eigenvectors of Eq. (12) can be written as

$$\psi_{n+1}=\sqrt{\frac{1}{2}\frac{B_1}{F_1}}\begin{pmatrix}\sqrt{C_{1-}}\,v_n \pm \sqrt{C_{1+}}\,v_{n+1}\\ \dfrac{1}{\sqrt{C_{1-}}}v_n \pm \dfrac{1}{\sqrt{C_{1+}}}v_{n+1}\end{pmatrix}, \quad n=0,1,2,3,\ldots. \tag{23}$$

The ground state is

$$\psi_0=\sqrt{\frac{B_1}{F_1}}\begin{pmatrix}\sqrt{C_{1+}}\,v_0\\ \dfrac{1}{\sqrt{C_{1+}}}v_0\end{pmatrix}, \qquad \epsilon_0^2(0,-)=0 \tag{24}$$

which gives $\epsilon(0,-)=-\mathbf{e}El_1^2k_x$. Finally we note that when the magnetic field is turned off i.e., we have the Dirac oscillator in the presence of an electric field bound states exist when the oscillator strength exceeds a critical value k_c i.e., $k>k_c$ where $k_c=\frac{\mathbf{e}|E|}{c}$. In this case the analysis remains the same and the spectrum and the corresponding solutions can be obtained by replacing ν by $\frac{k}{\mathbf{e}}$.

4. Strong magnetic field : $B > B_c$

In this case Eq. (3) can be written as

$$c\begin{pmatrix}-\dfrac{\mathbf{e}}{c}Ey & p_x-ip_y-\mathbf{e}\mu y\\ p_x+ip_y-\mathbf{e}\mu y & -\dfrac{\mathbf{e}}{c}Ey\end{pmatrix}\Psi=\epsilon\Psi \tag{25}$$

where $\mu=B-\frac{k}{\mathbf{e}}$. Note that here the *effective* magnetic field is in the positive z direction.

The equation corresponding to (12) reads

$$\widehat{\mathcal{H}}\psi=\epsilon_0^2\psi \tag{26}$$

where

$$\widehat{\mathcal{H}}=[\epsilon_0(\mathcal{H}_1+\mathcal{H})-\mathcal{H}_1\mathcal{H}]=\begin{pmatrix}J+\dfrac{c^2\hbar^2}{l_2^2} & \mathbf{e}Ec\hbar\\ -\mathbf{e}Ec\hbar & J-\dfrac{c^2\hbar^2}{l_2^2}\end{pmatrix} \tag{27}$$

$$\epsilon_0=-(\epsilon+\mathbf{e}El_2^2k_x), \quad l_2=\sqrt{\frac{\hbar}{\mathbf{e}\mu}}. \tag{28}$$

The operator J in Eq. (27) is given by

$$J=-\frac{c^2\hbar^2}{l_2^2}\frac{\partial^2}{\partial\bar{y}^2}+\left(\frac{c^2\hbar^2}{l_2^2}-\mathbf{e}^2E^2l_2^2\right)\bar{y}^2+2\mathbf{e}E\epsilon_0l_2\bar{y}. \tag{29}$$

The equations for the components read

$$\begin{aligned}\left(J-\frac{c^2\hbar^2}{l_2^2}-\epsilon_0^2\right)\left(J+\frac{c^2\hbar^2}{l_2^2}-\epsilon_0^2\right)\,u&=-\mathbf{e}^2E^2c^2\hbar^2\,u\\ \left(J+\frac{c^2\hbar^2}{l_2^2}-\epsilon_0^2\right)\left(J-\frac{c^2\hbar^2}{l_2^2}-\epsilon_0^2\right)\,v&=-\mathbf{e}^2E^2c^2\hbar^2\,v.\end{aligned} \tag{30}$$

The eigenvectors and eigenvalues of Eq. (26) can be found as

$$\begin{aligned}
\psi_{n+1} &= \sqrt{\frac{1}{2}\frac{B_2}{F_2}} \begin{pmatrix} -\sqrt{C_{2+}}\, v_n \mp \sqrt{C_{2-}}\, v_{n+1} \\ \frac{1}{\sqrt{C_{2+}}} v_n \pm \frac{1}{\sqrt{C_{2-}}} v_{n+1} \end{pmatrix}, \\
\epsilon_0^2(n, \pm) &= \frac{\omega_2^3}{F_2^4}\left[n + \frac{1}{2}(1 \pm 1)\right], \quad n = 0, 1, 2, \ldots
\end{aligned} \tag{31}$$

where

$$\begin{aligned}
v_n(z) &= \left(\frac{\sqrt{\lambda_2}}{2^n n! \sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\frac{1}{2}\lambda_2 z^2} H_n(\sqrt{\lambda_2}\, z), \quad z = \bar{y} + \frac{2\sqrt{2} B_2 \epsilon_0}{F_2^2 \lambda_2^2} \\
B_2 &= \frac{\mathbf{e} E l_2}{\sqrt{2}}, \qquad F_2 = \frac{\sqrt{2}\hbar c}{l_2}, \qquad \omega_2 = \sqrt{F_2^4 - 4B_2^2 F_2^2}, \\
\lambda_2 &= \frac{\omega_2}{F_2}, \qquad C_{2\pm} = \frac{1}{2}\left(\frac{F_2}{B_2} \pm \frac{\omega_2}{B_2 F_2}\right).
\end{aligned} \tag{32}$$

Finally the energy levels of the Dirac equation can be found as

$$\begin{aligned}
\epsilon(n, +) &= -\mathbf{e} E l_2^2 k_x - \frac{(F_2^2 - 4B_2^2)^{\frac{3}{4}}}{F_2^{\frac{1}{2}}} \sqrt{n+1} \\
\epsilon(n, -) &= -\mathbf{e} E l_2^2 k_x + \frac{(F_2^2 - 4B_2^2)^{\frac{3}{4}}}{F_2^{\frac{1}{2}}} \sqrt{n}.
\end{aligned} \tag{33}$$

We would like to note that the energy levels in Eq. (33) agree with those of Ref. [11]. Finally the ground state can be obtained as

$$\psi_0 = \sqrt{\frac{B_2}{F_2}} \begin{pmatrix} -\sqrt{C_{2-}}\, v_0 \\ \frac{1}{\sqrt{C_{2-}}} v_0 \end{pmatrix}, \qquad \epsilon_0^2(0, -) = 0 \tag{34}$$

which gives $\epsilon(0, -) = -\mathbf{e} E l_2^2 k_x$.

Let us now analyze the spectra in the regions $B > B_c$ and $B < B_c$. From Eqs. (21) and (33) it is not difficult to see that the spectrum in these two regions has a discontinuity at the critical point $B = B_c$. In Fig. 1 we have presented a plot of the positive branches of the energies in Eqs. (21) and (33) which clearly shows the discontinuity at $B = B_c$.

5. Critical magnetic field : $B = B_c$

We shall now analyze the model at the point of discontinuity i.e., for $B = B_c$. In this case the *effective* magnetic field vanishes and one is left with only the electric field. Then the equation analogous to Eq. (6) can be written as

$$c \begin{pmatrix} -\frac{\mathbf{e}}{c} E y & -k_x \hbar - i p_y \\ -k_x \hbar + i p_y & -\frac{\mathbf{e}}{c} E y \end{pmatrix} \psi = \epsilon \psi. \tag{35}$$

Now proceeding exactly as in the previous two cases we find

$$\widehat{\mathcal{H}} \psi = \epsilon^2 \psi \tag{36}$$

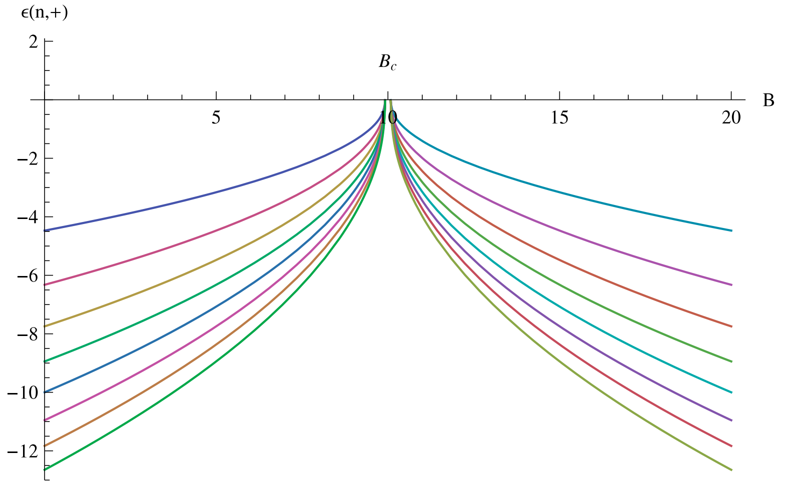

Fig. 1. Plot $\epsilon(n,+)$ as given by Eqs. (21) and (33) as a function of B with $k_x = 0$, $c = \hbar = e = 1$, $E = 0.08$, $k = 10$ and $n = 0, 1, 2, 3, 4, 5, 6, 7$.

where $\widehat{\mathcal{H}}$ is given by

$$\widehat{\mathcal{H}} = [\epsilon(\mathcal{H}_1 + \mathcal{H}) - \mathcal{H}_1\mathcal{H}] = \begin{pmatrix} J & \mathbf{e}E\hbar c \\ -\mathbf{e}E\hbar c & J \end{pmatrix} \tag{37}$$

while the operator J is given by

$$J = -\hbar^2 c^2 \frac{\partial^2}{\partial y^2} - \mathbf{e}^2E^2y^2 - 2\mathbf{e}E\epsilon_0 y + k^2\hbar^2 c^2. \tag{38}$$

The equations for the components read

$$(J - \epsilon^2)^2 u = -\mathbf{e}^2E^2v_F^2\hbar^2 u \tag{39}$$

$$(J - \epsilon^2)^2 v = -\mathbf{e}^2E^2v_F^2\hbar^2 v. \tag{40}$$

It is easy to recognize that the operator J in Eq. (38) represents the Hamiltonian of an *inverted* shifted oscillator for which only continuous energy states with non square integrable wave functions exist [12]. In other words normalizable bound states do not exist for the inverted shifted oscillator. Thus neither of Eqs. (39) and (40) admit bound state solutions. Thus the Dirac equation (35) admits no bound state solutions when the effective magnetic field vanishes.

6. $c|\nu| = |E|$ or $c|\mu| = |E|$

Before analyzing the case $c|\nu| = |E|$, we first examine the behavior of the energy levels as $|E| \to c|\nu|$ through values less than $c|\nu|$. In this case, let us for example, rewrite the energy levels in Eq. (21) as

$$\begin{aligned} \epsilon(n,+) &= \left[-\sqrt{(n+1)}\,\epsilon_c\,\left(1-\beta^2\right)^{\frac{3}{4}} - \beta\hbar k_x c\right] \\ \epsilon(n,-) &= \left[\sqrt{n}\,\epsilon_c\,\left(1-\beta^2\right)^{\frac{3}{4}} - \beta\hbar k_x c\right], \quad n = 0, 1, 2, \ldots \end{aligned} \tag{41}$$

where $\beta = \frac{E}{\nu c}$, and $\epsilon_c = c\sqrt{2\mathbf{e}\hbar\nu}$. From Eq. (41) it is seen that as $\beta \to 1$ the energy levels tend to coalesce. Fig. 2 describes the collapse of the energy levels in Eq. (41). Needless to say that one may obtain identical results for the energy levels in Case 2.

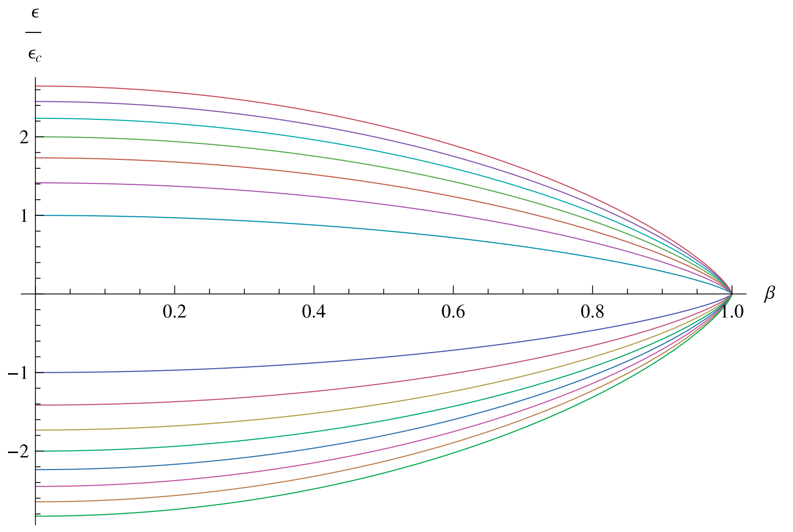

Fig. 2. Plot of $\frac{\epsilon(n)}{\epsilon_c}$ as a function of $\beta = \frac{E}{\nu c}$ with $\epsilon_c = c\sqrt{2\mathbf{e}\hbar\nu}$, $k_x = 0$ and $n = 0, 1, 2, 3, 4, 5, 6, 7$.

We now consider the case when the equality holds i.e., $c|\nu| = |E|$ or $c|\mu| = |E|$. The Hamiltonians corresponding to (13) and (27) read

$$\widehat{\mathcal{H}} = \begin{pmatrix} J - \mathbf{e}^2\nu^2 l_1^2 c^2 & \mathbf{e}^2\nu^2 l_1^2 c^2 \\ -\mathbf{e}^2\nu^2 l_1^2 c^2 & J + \mathbf{e}^2\nu^2 l_1^2 c^2 \end{pmatrix} \tag{42}$$

$$J = -\mathbf{e}^2\nu^2 l_1^2 c^2 \frac{\partial^2}{\partial \bar{y}^2} + 2\mathbf{e}\nu l_1 c\epsilon_0 \bar{y} \tag{43}$$

and

$$\widehat{\mathcal{H}} = \begin{pmatrix} J + \mathbf{e}^2\mu^2 l_2^2 c^2 & \mathbf{e}^2\mu^2 l_2^2 c^2 \\ -\mathbf{e}^2\mu^2 l_2^2 c^2 & J - \mathbf{e}^2\mu^2 l_2^2 c^2 \end{pmatrix} \tag{44}$$

$$J = -\mathbf{e}^2\mu^2 l_2^2 c^2 \frac{\partial^2}{\partial \bar{y}^2} + 2\mathbf{e}\mu l_2 c\epsilon_0 \bar{y}. \tag{45}$$

Both the operators in (43) and (45) are actually non relativistic Hamiltonians for a linear potential whose solutions are given in terms of Airy functions. However these solutions are not normalizable over the whole real line admit bound states only on the half line [13]. Therefore the Dirac equation possesses no bound states when $c|\nu| = |E|$ or $c|\mu| = |E|$.

7. Conclusion

Here we have obtained the spectrum of $(2 + 1)$ dimensional Dirac oscillator in the presence of a perpendicular magnetic and transverse electric fields. In particular we have analyzed in detail the cases for which $B > B_c$, $B < B_c$ and $B = B_c$, B_c being the critical magnetic field. It has also been shown that the critical point is also a point of discontinuity of the spectrum. The situation when $c|\nu| = |E|$ has also been examined. It may be noted that when the magnetic field is turned off the problem becomes a pure Dirac oscillator in the presence of an electric field. Bound state solutions for this problem exist when the oscillator strength k exceeds a critical value k_c and in this case exact solutions can be found by replacing ν by $\frac{k}{\mathbf{e}}$.

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