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SUPERSYMMETRIC QUANTUM MECHANICS IN ONE, TWO AND THREE DIMENSIONS

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We discuss a few supersymmetric quantum mechanical models in one, two and three dimensions. In the case of one particle in both two and three dimensions we present some examples where supersymmetry is unbroken and the ground state is many-fold degenerate. Further, we consider the supersymmetric generalization of the nearest neighbour XY model and show that supersymmetry remains unbroken in this case.

1. Introduction

Recently supersymmetric quantum mechanical models have attracted a lot of attention (for a recent review see [1]. The main motivation is to understand the mechanism responsible for the dynamical breaking of supersymmetry (SUSY). Witten [2] has conjectured that instantons may break SUSY. One such example has been explicitly worked out by Salomonson and Van Holten [3]. However Abbot and Zarkrzewski [4] have presented a counterexample to this. Further Cooper and Freedman [5] have studied a model where SUSY is broken even though there are no instantons. In a recent paper [6] we have examined these questions carefully and have argued that even though only instantons may not be responsible for SUSY breaking, whenever they are present they will break SUSY unless the potential in question corresponds to a first-order phase transition. All these results have been obtained for models in one-dimensional quantum mechanics.

Recently Crombrugghe and Rittenberg [7] have constructed supersymmetric quantum mechanical models in higher dimensions. Such models may be relevant in nuclear physics [8] and statistical mechanics [9]. The purpose of this paper is to study in detail the question of SUSY breaking in some specific models.

We shall first consider the motion of a single particle in two dimensions. We show that for a specific class of potentials this problem can be reduced to a one-dimensional quantum mechanical problem so that the earlier results on SUSY breaking continue to hold good. In particular for the hamiltonian which gives rise to Landau levels the SUSY remains unbroken.

We also discuss some genuine two- and three-dimensional potentials and show that in these cases the SUSY remains unbroken.

Finally we discuss the interesting case of n particles in one dimension. The structure of such hamiltonians is similar to those corresponding to commensurate–incommensurate phase transition [10]. We consider the supersymmetric generalization of the nearest-neighbour XY model and show that the SUSY remains unbroken in this case.

The plan of the paper is as follows. In sect. 2 we discuss a supersymmetric single particle in two dimensions. In sect. 3 we consider the case of a three-dimensional supermultiplet. In sect. 4 we deal with the problem of n particles in one dimension while sect. 5 is reserved for discussions.

2. Supersymmetry in two dimensions

The supersymmetric hamiltonian for the motion of one particle in two dimensions is given by [7]*

$$2H=(p_x+A_x)^2+(p_y+A_y)^2+(\nabla\times A)_z\sigma_z\,. \tag{2.1}$$

The corresponding supercharges are

$$Q^1=\sqrt{\tfrac{1}{2}}[-(p_y+A_y)\sigma_x+(p_x+A_x)\sigma_y]\,, \tag{2.2a}$$

$$Q^2=\sqrt{\tfrac{1}{2}}[(p_x+A_x)\sigma_x+(p_y+A_y)\sigma_y]\,, \tag{2.2b}$$

which satisfy the superalgebra

$$\{Q^\alpha,Q^\beta\}=2H\delta^{\alpha\beta}\,,\qquad \alpha,\beta=1,2\,, \tag{2.3a}$$

$$[Q^\alpha,H]=0\,. \tag{2.3b}$$

It is interesting to note that eq. (2.1) is the two-dimensional Pauli equation where SUSY has fixed the gyromagnetic ratio to be equal to two. As noted in ref. [7] the hamiltonian of eq. (2.1) has the additional symmetry $O(2)\otimes O(2)$ given by σ_z and an $O(2)$ rotation in the A^1, A^2 plane besides the original $S(2)$ symmetry ($A^1\equiv p_x+A_x$, $A^2\equiv p_y+A_y$).

We first choose

$$A_y(x,y)=0\,,\qquad A_x(x,y)=-v'(y)\,, \tag{2.4}$$

where $v(y)$ is an arbitrary function of y and prime denotes the derivative of the function with respect to its argument. In such a case the hamiltonian (2.1) takes the form

$$2H=(p_x-v'(y))^2+p_y^2+v''(y)\sigma_z\,. \tag{2.5}$$

Since this hamiltonian does not depend on x the wave function $\psi(x,y)$ can be factorized as

$$\psi(x,y)=e^{ikx}\phi(y)\,, \tag{2.6}$$

* We follow the notation of ref. [7] throughout this paper and also put $\hbar=m=1$.

where k is the eigenvalue of the operator p_x. The Schrödinger equation for $\phi(y)$ then takes the form

$$\left[-\frac{d^2}{dy^2}+(v'(y)-k)^2+v''(y)\sigma_z\right]\phi(y)=2E\phi(y)\,. \tag{2.7}$$

On comparing eq. (2.7) with eq. (2.4) of ref. [6] (see also ref. [3]) we notice that

$$V_B(y)=v'(y)-k\,. \tag{2.8}$$

Thus we have reduced the problem to that of SUSY in one dimension so that all the well-known results about SUSY breaking will continue to hold good.

As an illustration, consider the case when

$$v'(y)=By\,, \tag{2.9}$$

in which case the energy eigenvalues known as Landau levels are given by [11]

$$E_n=(n+\tfrac{1}{2}+\tfrac{1}{2}\sigma)B\,, \tag{2.10}$$

where σ is the eigenvalue ($\sigma=\pm1$) of the operator σ_z. Thus SUSY remains unbroken since $E_0=0$. It is worth noticing that E_n does not depend on the quantity k which assumes a continuous sequence of values, i.e. the energy levels (including the ground state) are continuously degenerate. Clearly this is going to be true for any choice of the potential $v'(y)$.

Let us now consider genuine two-dimensional problem. We choose*

$$A_x=-Byf(\rho)\,,\qquad A_y=Bxf(\rho)\,, \tag{2.11}$$

where $\rho=\sqrt{x^2+y^2}$ and B is a constant. In that case the hamiltonian given by eq. (2.1) takes the form

$$2H=-\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}\right)+B^2\rho^2f^2+2BfL_z+(2Bf+B\rho f'(\rho))\sigma_z\,, \tag{2.12}$$

where L_z is the z-component of the orbital angular momentum operator. the corresponding Schrödinger equation can be solved in cylindrical coordinates ρ, ϕ. In this case the wave function $\psi(\rho,\phi)$ can be factorized as

$$\psi(\rho,\phi)=R(\rho)\,e^{im\phi}\,, \tag{2.13}$$

where $m=0,\pm1,\pm2,\dots$ is the eigenvalue of L_z. In that case the Schrödinger equation for $R(\rho)$ takes the form

$$R''(\rho)+\frac{1}{\rho}R'(\rho)-\left[B^2\rho^2f^2+\frac{m^2}{\rho^2}+2Bmf\right.$$
$$\left.+(2Bf+B\rho f'(\rho))\sigma_z\right]R(\rho)=-2ER(\rho)\,. \tag{2.14}$$

* We assume that $\rho f(\rho)\to\infty$ as $\rho\to\infty$ and has good behaviour near $\rho=0$ so that the spectrum is only discrete.

On substituting

$$R(\rho)=\rho^{-1/2}A(\rho)\,, \tag{2.15}$$

eq. (2.13) takes the form (choosing the lower eigenvalue of σ_z)

$$\left\{-\frac{d^2}{d\rho^2}+\left[B^2\rho^2f^2-2Bf+2Bmf-B\rho f'(\rho)+\frac{m^2-\frac{1}{4}}{\rho^2}\right]\right\}A(\rho)$$

$$=2EA(\rho)\,. \tag{2.16}$$

Since the left-hand side operator can be written as a^+a, where

$$a=\frac{d}{d\rho}+B\rho f-\frac{|m|+\frac{1}{2}}{\rho} \tag{2.17}$$

(for $m\leqslant 0$), hence $E_0\geqslant 0$. In fact $E_0=0$ is possible if and only if the solution $\psi_0(\rho)$ of

$$\left[\frac{d}{d\rho}+B\rho f-\frac{|m|+\frac{1}{2}}{\rho}\right]\psi_0(\rho)=0 \tag{2.18}$$

is square integrable, clearly one can choose different forms of $f(\rho)$ and see if SUSY is broken or unbroken. As an illustration we shall consider the following two forms of $f(\rho)$.

(i) Choose

$$f(\rho)=1\,, \tag{2.19}$$

in which case $\psi_0(\rho)$ given by

$$\psi_0(\rho)=N\rho^{(|m|+1/2)}\exp\left(-\tfrac{1}{2}B\rho^2\right) \tag{2.20}$$

is clearly square integrable. Hence in this case $E_0=0$, i.e. SUSY remains unbroken.

It may be noted that the choice $f(\rho)=1$ corresponds to the motion of a particle in a uniform magnetic field. The dynamics correspond to that of a particle in a harmonic oscillator potential in the $x-y$ plane subjected to constant external magnetic field B along the z-direction which interacts with the orbital and the spin angular momentum of the particle. This hamiltonian provides a quantal description of the diamagnetism of free electrons and other phenomena such as the de Haas–van Alphen effect.

For this system the exact eigenvalues and eigenfunctions are well known. For example the eigenvalues are [12]

$$E_n=(n+m+|m|)B\,,\qquad n=0,1,2,\ldots \tag{2.21}$$

It is interesting to note that E_n and hence the ground state is many-fold degenerage. This is because, irrespective of whether $m=0,-1,-2,\ldots$, E_0 is always zero. Clearly this feature will persist for a wide class of functions $f(\rho)$.

(ii)
$$f(\rho)=(\rho-a)(\rho-b)/\rho^2\,, \tag{2.22}$$

in which case using eq. (2.18) we find that

$$\psi_0(\rho) = N\rho^{|m|+1/2-abB} \exp\left(-\tfrac{1}{2}B\rho^2 + (a+b)B\rho\right). \tag{2.23}$$

The ground state wave function is always square integrable if

$$\tfrac{1}{2} \geqslant abB. \tag{2.24}$$

Hence if $abB \leqslant \frac{1}{2}$ then SUSY remains unbroken and irrespective of whether $m = 0, -1, -2, \ldots$, the ground state energy is zero. Now as the field strength B is increased, $\psi_0(\rho)$ will be normalizable only for those m satisfying

$$|m| + \tfrac{1}{2} \geqslant abB. \tag{2.25}$$

In other words, as abB becomes more than $\frac{1}{2}$ SUSY is still unbroken but now only for those negative integer values of m which satisfy eq. (2.25) is $E_0 = 0$.

It is worthwhile pointing out that there is a magnetic flux $2\pi abB$ along the z-axis. Indeed the appearance of nonrenormalizable states is related to the existence of this singularity.

The fact that in both of our examples there is an infinite number of degenerate ground states with zero energy can be understood from the Aharonov–Casher theorem [16]. For our examples the magnetic field is given by

$$B_z = 2Bf(\rho) + B\rho f'(\rho), \tag{2.26}$$

so that the total magnetic flux given by $\int B_z \, dx \, dy$ (and hence zero energy normalizable states) are infinite since we have chosen examples where $\rho f(\rho) \to \infty$ as $\rho \to \infty$.

3. Supersymmetry in three dimensions

The supersymmetric hamiltonian for the motion of one particle in three dimensions is given by [7]

$$2H = (\boldsymbol{p} + \boldsymbol{A})^2 + K^2 + 4\boldsymbol{S} \cdot (\boldsymbol{\nabla} \times \boldsymbol{A}), \tag{3.1}$$

with the constraint

$$\boldsymbol{\nabla} \times \boldsymbol{A} = -\boldsymbol{\nabla} K. \tag{3.2}$$

Thus SUSY in this case has imposed that the supermultiplet is in the self-dual external electromagnetic field. Further, in this case the SUSY has fixed the gyromagnetic ratio to be four. The hamiltonian (3.1) has the additional SU(2)⊗SU(2) symmetry besides the original S(4) symmetry [7].

The self-duality condition (3.2) puts severe restrictions on the possible choices of $\boldsymbol{A}$ and K. In fact we have found only one example where we could obtain exact eigenvalues of (3.1) and in that case we find that SUSY is unbroken.

Let us choose

$$A_x = -B_y, \qquad A_y = B_x, \qquad A_z = 0. \tag{3.3}$$

The constraint (3.2) then requires that

$$k(x, y, z) = -2BZ + c, \tag{3.4}$$

where c is an arbitrary constant. With this choice the hamiltonian (3.1) takes the form

$$\begin{aligned} 2H &= p_x^2 + p_y^2 + B^2(x^2 + y^2) + p_z^2 + (c - 2BZ)^2 \\ &\quad + 2B(L_z + 2\sigma_z) . \end{aligned} \tag{3.5}$$

This eigenvalue problem can be exactly solved in the cylindrical coordinate system by following the procedure of sect. 2. We find that

$$E_n = (n + m + |m| + 2 + 2\sigma)B , \qquad n = 0, 1, 2, \ldots , \tag{3.6}$$

where $\sigma(=\pm 1)$ is the eigenvalue of σ_z. Thus $E_0 = 0$ for any nonpositive m and consequently the ground state is many-fold degenerate. Further SUSY remains unbroken.

The hamiltonian (3.5) corresponds to the motion of a particle in an unisotropic harmonic oscillator potential subjected to a constant external magnetic field B along the z-direction which interacts with the orbital and the spin angular momentum of the particle.

For our choice of the potential given by eqs. (3.3) and (3.4) the supercharges take a very simple form [7]:

$$Q_{1/2} = (ib_x^+ - b_y^+)a_2^+ + ib_z^+ a_1^+ , \tag{3.7a}$$

$$Q_{-1/2} = (-ib_x - b_y)a_1^+ + ib_z a_2^+ , \tag{3.7b}$$

where

$$\begin{aligned} b_x &= \sqrt{\tfrac{1}{2}}(Bx + ip_x) , \qquad b_y = \sqrt{\tfrac{1}{2}}(By + ip_y) , \\ b_z &= \sqrt{\tfrac{1}{2}}(2Bz - c + ip_z) , \end{aligned} \tag{3.8}$$

i.e. they could be interpreted as bosonic annihilation operators. On the other hand $a_k (k = 1, 2)$ are the fermionic annihilation operators satisfying the algebra

$$\begin{aligned} \{a_k, a_l^+\} &= \delta_{kl} , \qquad k, l = 1, 2 , \\ \{a_k, a_l\} &= \{a_k^+, a_l^+\} = 0 . \end{aligned} \tag{3.9}$$

The supercharges satisfy the algebra

$$\begin{aligned} &\{Q_\alpha, Q_\beta^+\} = \delta_{\alpha\beta} H , \\ &\{Q_\alpha, Q_\beta\} = 0 , \qquad [Q_\alpha, H] = 0 , \qquad \alpha, \beta = \pm\tfrac{1}{2} . \end{aligned} \tag{3.10}$$

4. n particles in one dimension

The supersymmetric hamiltonian for the motion of n particles in one dimension is given by [7]

$$2H = \sum_{k=1}^{n} \left[\left(p_k + \frac{\partial F}{\partial x_k}\right)^2 + \left(\frac{\partial G}{\partial x_k}\right)^2 \right]$$
$$- 2 \sum_{k,l=1}^{n} \frac{\partial^2 G}{\partial x_k \, \partial x_l} F_{(1,k),(2,l)} \, , \tag{4.1}$$

with

$$F_{(1,k),(2,l)} = \tfrac{1}{4} i [C_k(1), C_l(2)] \, . \tag{4.2}$$

Here the Clifford generator $C_k(i)$ satisfy the algebra

$$\{C_k(i), C_l(j)\} = 2\delta_{ij}\delta_{lk} \, , \qquad i, j = 1, 2 \, . \tag{4.3}$$

By applying the Jordan–Wigner transformation the hamiltonian (4.1) can be transformed to

$$2H = \sum_{k=1}^{n} \left[\left(p_k + \frac{\partial F}{\partial x_k}\right)^2 + \left(\frac{\partial G}{\partial x_k}\right)^2 \right] + \sum_{k=1}^{n} \frac{\partial^2 G}{\partial x_k^2} \sigma_z(k)$$
$$- \tfrac{1}{2} \sum_{k \neq l} \frac{\partial^2 G}{\partial x_k \, \partial x_l} [\sigma_x(k) \otimes \sigma_x(l) + \sigma_y(k) \otimes \sigma_y(l)] \, , \tag{4.4}$$

where the following representations have been adopted:

$$F_{(1,k),(2,k)} = -\tfrac{1}{2} \sigma_z(k) \, , \tag{4.5a}$$

$$F_{(1,k),(2,l)} + F_{(1,l),(2,k)} = \tfrac{1}{2} [\sigma_x(k) \otimes \sigma_x(l) + \sigma_y(k) \otimes \sigma_y(l)] \, . \tag{4.5b}$$

By making suitable choices for the functions F and G we will obtain generalized XY models. As an illustration we shall discuss two examples explicitly. In both cases we find that SUSY remains unbroken.

(i) Choose

$$F = 0 \, , \qquad G = \eta \sum_{k=1}^{n} x_k x_{k+1} - \lambda \sum_{k=1}^{n} x_k^2 \, , \tag{4.6}$$

in which case the hamiltonian (4.4) takes the form

$$2H = 2H_{\mathrm{B}} + 2H_{\mathrm{F}} \, , \tag{4.7}$$

where

$$2H_{\mathrm{B}} = \sum_{k=1}^{n} [p_k^2 + (4\lambda^2 + 2\eta^2) x_k^2] + (2\eta^2) \sum_{k=1}^{n} x_k x_{k+2}$$
$$- 8\lambda\eta \sum_{k=1}^{n} x_k x_{k+1} \, , \tag{4.8}$$

$$2H_F = -2\lambda \sum_{k=1}^{n} \sigma_z(k) - \eta \sum_{k=1}^{n} [\sigma_x(k)\sigma_x(k+1) + \sigma_y(k)\sigma_y(k+1)] . \tag{4.9}$$

Systems with a structure as given by eq. (4.7) are of interest in statistical mechanics. In particular the fermionic hamiltonian given by eq. (4.9) corresponds to the XY model with the nearest-neighbour interaction experiencing a constant magnetic field in the z-direction [7]. It may also be noted that in H_B, whereas the interaction between the nearest neighbour is ferromagnetic-like, it is antiferromagnetic-like between the next-to-nearest neighbours.

The ground state energy and also the statistical mechanics of the nearest neighbour XY model has already been discussed before [13]. It has been shown that the ground state energy of H_F is given by*

$$E_0^F = -\sum_{k=1}^{n} \left[\lambda - \eta \cos\frac{2\pi k}{n}\right] . \tag{4.10}$$

The hamiltonian H_B can be dizgonalised by going over to the k-space and then it is easy to show that [14]

$$E_0^B = +\sum_{k=1}^{n} \left[\lambda - \eta \cos\frac{2\pi k}{n}\right] . \tag{4.11}$$

Thus

$$E_0^{\text{total}} = E_0^B + E_0^F = 0 , \tag{4.12}$$

i.e. SUSY remains unbroken.

(ii) This case is simply obtained from (i) by putting $\lambda = 0$ everywhere so that SUSY is again unbroken. In this case H_F corresponds to the XY model with a nearest-neighbour interaction which has also been extensively studied in the literature [13]. From these calculations also one can independently verify that SUSY remains unbroken.

Let us now consider the choice

$$F = 0 , \qquad G = -\lambda \sum_{k=1}^{n} x_k^2 + \eta \sum_{k\times=1}^{n} x_k x_{k+r} . \tag{4.13}$$

With this choice of F and G

$$2H_B = \sum_{k=1}^{n} [p_k^2 + (4\lambda^2 + 2\eta^2)x_k^2] + (2\eta^2) \sum_{k=1}^{n} x_k x_{k+2r} - 8\lambda\eta \sum_{k=1}^{n} x_k x_{k+r} , \tag{4.14}$$

* A cyclic boundary condition ($n + l = l$) has been used in this section.

which can be easily diagonalized by going over to the K-space. The energy eigenvalues are given by

$$E_N^{\mathrm{B}} = \sum_i (m_i + \tfrac{1}{2})|\omega_i| , \tag{4.15}$$

where $m_1, m_2 \dots m_n$ are n integers and

$$\omega_k = 2\left[\lambda - \eta \cos \frac{2\pi k r}{n}\right] . \tag{4.16}$$

The ground state energy is obtained by setting $m_1 = m_2 = \cdots m_n = 0$, i.e.

$$E_0^{\mathrm{B}} = \sum_{k=1}^{n} \left[\lambda - \eta \cos \frac{2\pi k r}{n}\right] . \tag{4.17}$$

The fermionic part of the hamiltonian can also be diagonalized in a similar way. In the hamiltonian (4.1) if we identify

$$C_k(1) = a_k + a_k^+ , \tag{4.18a}$$

$$iC_k(2) = a_k - a_k^+ , \tag{4.18b}$$

then the fermionic part of the hamiltonian can be written as

$$\begin{aligned} 2H_{\mathrm{F}} = -2\lambda &\sum_{k=1}^{n} (a_k a_k^+ - a_k^+ a_k) \\ &+ 2\eta \sum_{k=1}^{n} (a_{k+r} a_k^+ - a_{k+r}^+ a_k) . \end{aligned} \tag{4.19}$$

This H_{F} can be diagonalized by going over to the momentum space and one find that

$$E_N^{\mathrm{F}} = \sum_i (M_i - \tfrac{1}{2})|\omega_i| , \qquad i = 1, 2, \dots n , \tag{4.20}$$

where ω_i is precisely given by eq. (4.16), while M_i are integers which can only take values 0 or 1 due to Fermi–Dirac statistics. Hence

$$E_0^{\mathrm{F}} = -\tfrac{1}{2}\sum_i |\omega_i| , \tag{4.21}$$

so that

$$E_0^{\mathrm{total}} = E_0^{\mathrm{B}} + E_0^{\mathrm{F}} = 0 , \tag{4.22}$$

i.e. SUSY remains unbroken.

Finally we shall consider the choice

$$F = 0 , \qquad G = -\lambda \sum_{k=1}^{n} x_k^2 + \sum_{k=1}^{n} x_k x_{k+1} - \mu \sum_{k=1}^{n} x_k x_{k+2} , \tag{4.23}$$

in which case we have

$$2H = 2H_{\mathrm{B}} + 2H_{\mathrm{F}} , \tag{4.24}$$

$$2H_{\rm B} = \sum_{k=1}^{n} [p_k + (4\lambda^2 + 2\mu^2 + 2)x_k^2] - (8\lambda + 4\mu)$$
$$\times \sum_{k=1}^{n} x_k x_{k+1} + (8\lambda\mu + 2) \sum_{k=1}^{n} x_k x_{k+2}$$
$$-4\mu \sum_{k=1}^{n} x_k x_{k+3} + 2\mu^2 \sum_{k=1}^{n} x_k x_{k+4} , \qquad (4.25)$$

$$2H_{\rm F} = -2\lambda \sum_{k=1}^{n} \sigma_z(k)$$
$$- \sum_{k=1}^{n} [\sigma_x(k)\sigma_x(k+1) + \sigma_y(k)\sigma_y(k+1)]$$
$$+ \mu \sum_{k=1}^{n} [\sigma_x(k)\sigma_x(k+2) + \sigma_y(k)\sigma_y(k+2)] . \qquad (4.26)$$

Here $H_{\rm F}$ corresponds to the XY model with the nearest and the next-to-nearest neighbour interaction experiencing a constant magnetic field in the z-direction. It is then not very difficult to show that

$$E_N^{\rm B} = \sum_k (m_k + \tfrac{1}{2})|\omega_k| , \qquad (4.27)$$

with

$$\omega_k = 2\left[\lambda - \cos\frac{2\pi k}{n} + \mu \cos\frac{4\pi k}{n}\right] . \qquad (4.28)$$

Thus the bosonic ground state energy is

$$E_0^{\rm B} = \sum_{k=1}^{n} \left[\lambda - \cos\frac{2\pi k}{n} + \mu \cos\frac{4\pi k}{n}\right] . \qquad (4.29)$$

The fermionic hamiltonian can be diagonalized by exactly following the procedure of the last example. One can show that the hamiltonian (4.26) can be written as

$$2H_{\rm F} = -2\lambda \sum_{k=1}^{n} (a_k a_k^+ - a_k^+ a_k) + 2 \sum_{k=1}^{n} (a_{k+1} a_k^+ - a_{k+1}^+ a_k)$$
$$-2\mu \sum_{k=1}^{n} (a_{k+2} a_k^+ - a_{k+2}^+ a_k) , \qquad (4.30)$$

so that

$$E_N^{\rm F} = \sum_k (M_k - \tfrac{1}{2})|\omega_k| . \qquad (4.31)$$

Here ω_k is precisely the same as in eq. (4.28) while M_k can take values 0 or 1. Hence

$$E_0^{\rm F} = -E_0^{\rm B} ,$$

so that even in this example SUSY is unbroken. It may be noted that in all the examples discussed here G is quadratic in x_k.

5. Discussion

In this paper we have studied the question of SUSY breaking in some model hamiltonian in one-, two- and three-dimensional quantum mechanics. In the two-dimensional case we have shown that for a class of potentials the problem can be reduced to that of SUSY in one dimension except for the fact that the ground state (and all other states too) is degenerate, being independent of k which assumes a continuous sequence of values. We have also studied a class of genuine two-dimensional models. We have shown that in those cases where SUSY is unbroken the ground state is m-fold degenerate. Note that in these cases m can take any non-positive integer value. In a sense one has multiple, degenerate, orthogonal vaccua (ground states) which are somewhat analogous to the θ-vaccua of the QCD. Note however that whereas in QCD the ground state energy is a function of θ, in this case it is independent of m. We have also presented a two-dimensional model where SUSY remains unbroken, but as the strength of the external magnetic field is increased the ground state degeneracy decreases. In the case of three dimensions we have found one example when SUSY remains unbroken and the ground state has a degeneracy similar to that of the unbroken two-dimensional SUSY models.

Finally we have considered four examples concerning n particles in one dimension. In all these examples we could explicitly show that SUSY remains unbroken. The fermionic part of these models correspond to the XY model with or without external magnetic field. It may be noted that in all four cases the bosonic part of the hamiltonian could be diagonalized since the potential also was quadratic in x_k. It would be interesting if some of these models could find some application in condensed matter physics so that hopefully one could test if SUSY has anything to do with nature at the microscopic level.

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References

[1] E. Witten, Princeton preprint based on lectures given at ICTP, Trieste (1981)
[2] E. Witten, Nucl. Phys. B188 (1981) 513
[3] P. Salomonson and J.W. Van Holten, Nucl. Phys. B196 (1982) 509
[4] R.B. Abbott and W.J. Zakrzewski, Z. Phys. 20 (1983) 227
[5] F. Cooper and B. Freedman, Ann. of Phys. 146 (1983) 262
[6] A. Khare and J. Maharana, Z. Phys. in press
[7] M. de Crombrugghe and V. Rittenberg, Ann. of Phys. 151 (1983) 99
[8] A.B. Balantekin, I. Bars and F. Iachellow, Nucl. Phys. A370 (1981) 284
[9] V. Rittenberg, Weizmann-Institute preprint

[10] S. Redner and H.E. Stanley, J. Phys. C10 (1977) 4765
[11] L.D. Landau and E.M. Lifshitz, Non-relativistic quantum mechanics (Pergamon, New York, 1976)
[12] M. Alouso and H. Valk, Quantum mechanics (Addison-Wesley, New York, 1973)
[13] Y. Nambu, Prog. Theor. Phys. 5 (1950) 1;
E. Lieb, T. Schultz and D. Mattis, Ann. of Phys. 16 (1961) 407;
S. Katsura, Phys. Rev. 127 (1962) 1508
[14] N.W. Ashcroft and N.D. Mermin, Solid state physics (Holt Rimehart and Winston, New York, 1976)
[15] C.J. Thompson, in Critical phenomena and phase transitions, vol. 1, ed. C. Domb and M.S. Green (Academic Press, New York, 1970)
[16] Y. Aharonov and A. Casher, Phys. Rev. A19 (1979) 2461