Optimization

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Overview

Problem Statement

Express the following problem in matrix form

$$\min_{\mathbf{x}} \; x11 + x12$$

with constrains $x11 + x22 = 1$ and $\mathbf{X} \geq 0$ ($\geq$ means positive definite)
where

$$\mathbf{X} = \begin{bmatrix} x11 & x12 \\ x12 & x22 \end{bmatrix}$$

Solution

We can write

$$x11 + x12 = \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{X^T} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and

$$x11 + x22 = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} & 0 \\ 0 & \mathbf{X} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Solution

Hence, the problem can be reformulated as

$$\min_{\mathbf{x}} \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{X}^{\mathbf{T}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad s.t$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} & 0 \\ 0 & \mathbf{X} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 1 \, , \mathbf{X} \geq 0$$

The End