Prombola: A Probabilistic War of Tambola

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Abstract
Prombola is a game inspired from a very old traditional Indian game called Tambola. It is played with a uniform distribution of natural numbers. ***We did statistical analysis of the winning strategies of the game when played with N-1 other players and extended it further to come up with a maximizing house advantage so it can be played at fundraisers or charity events to raise money.***

1. Introduction

As an applied statistician you get in touch with many challenging problems in need of a statistical solution. Often we have a working model ready to explain tangible solutions, but often we do not have the vision that a very basic problem could have insightful statistical revelations. Our intuitions might suggest that an occurrence may be just a random event, but in fact we might just have a statistical model waiting to be explored. In this project, we have tried to understand and explain the statistical analysis of the game Tambola.

The original game play consists of multiple players playing with one set of tickets containing 15 numbers randomly chosen between 1 to 90, where every number is equiprobable and the tickets are handed to participating people for a cost. The numbers on tickets are arranged in a seemingly random order in a 3X10 grid. The dealer picks up random numbers from 1 to 90 without replacement and announces them to the players. The players circle out the numbers that are announced and the first one to circle out all the numbers in the first row/column or the last row/column wins a prize. The grand prize goes to the person whose numbers are all called out.

In order to *redesign and renovate* this game to be played at charity events and fundraisers, we will be analyzing the game for N players and based on the expectations of the game winning probabilities, the rewards for the winner(s) will be set so as to maximize the house advantage.

In this project, we have attempted to observe and understand the patterns of winning based on strategies, number of players and the distribution of sequences using a probabilistic and statistical approach. It is important to simulate the game unbiased to conclude on our observations. This report will discuss what to expect when one is playing to win a particular strategy against many other players. Since the game doesn't involve a human approach to strategizing it, we will look at the probabilistic model of the game. *We have strategized the house policies on deciding the game prize money keeping in mind that most of the player money has to be donated to a charity whilst keeping the participating players content as well.*

2. Game Rules

2.1 Game Play Rules

1. There's a *Caller* in the game, who calls numbers ranging from [1,90] sequentially, not in any order, it is usually randomly.
2. There can be *N players*, no limit on the number of players.
3. Each player gets a ticket which is composed of a 3X9 grid, where the cells are meant to be filled with numbers. A ticket, consists of 15 numbers drawn uniformly without repetition from [1,90], and they are scattered over the grid with the given constraints:
    a) Every column has at least one number.
    b) Every row can have at most 5 numbers
    c) Column 1 has values between 1–10
    d) Column 2 has values from 11–20
    e) Column 3 has values from 21–30 and so on till column 8 with values from 71–80
    f) Column 9 has values from 81–90

1	10	27	31		59			
9	13		37			63		80
	14		38	44		68	79	

Fig 1: Example of a Tambola ticket

4. In every turn, until a player wins, the caller calls out one number. The players who have that number present on their tickets cross it out.
5. The player to complete a particular winning strategy wins the reward for that strategy. There's a strategy priority order among all the strategies, and the rewards are decided accordingly. The strategies are as follows in the decreasing order of importance/rewards:

Full House
The first player to cross out all 15 numbers on their ticket wins

1	10	27	31	0	59	0	0	0
9	13	0	37	0	0	63	0	80
0	14	0	38	44	0	68	79	0

Fig 2: Example of a Tambola ticket

Bottom Line
The first player to cross out all numbers on the bottom row of the ticket wins

1	10	27	31	0	59	0	0	0
9	13	0	37	0	0	63	0	80
0	14	0	38	44	0	68	79	0

Fig 3: Example of a Tambola ticket

<table>
<tr><td>Middle Line
The first player to cross out all numbers on the middle row of the ticket wins</td><td><table><tr><td>1</td><td>10</td><td>27</td><td>31</td><td>0</td><td>59</td><td>0</td><td>0</td><td>0</td></tr><tr><td>9</td><td>13</td><td>0</td><td>37</td><td>0</td><td>0</td><td>63</td><td>0</td><td>80</td></tr><tr><td>0</td><td>14</td><td>0</td><td>38</td><td>44</td><td>0</td><td>68</td><td>79</td><td>0</td></tr></table>Fig 4: Example of a Tambola ticket</td></tr>
<tr><td>Top Line
The first player to cross out all numbers on the top row of the ticket wins</td><td><table><tr><td>1</td><td>10</td><td>27</td><td>31</td><td>0</td><td>59</td><td>0</td><td>0</td><td>0</td></tr><tr><td>9</td><td>13</td><td>0</td><td>37</td><td>0</td><td>0</td><td>63</td><td>0</td><td>80</td></tr><tr><td>0</td><td>14</td><td>0</td><td>38</td><td>44</td><td>0</td><td>68</td><td>79</td><td>0</td></tr></table>Fig 5: Example of a Tambola ticket</td></tr>
<tr><td>Four Corners
The first player to cross out the following four numbers win: First and last number on their first row, First and last number on their bottom row</td><td><table><tr><td>1</td><td>10</td><td>27</td><td>31</td><td>0</td><td>59</td><td>0</td><td>0</td><td>0</td></tr><tr><td>9</td><td>13</td><td>0</td><td>37</td><td>0</td><td>0</td><td>63</td><td>0</td><td>80</td></tr><tr><td>0</td><td>14</td><td>0</td><td>38</td><td>44</td><td>0</td><td>68</td><td>79</td><td>0</td></tr></table>Fig 6: Example of a Tambola ticket</td></tr>
<tr><td>Early Five
The first player to cross out any five numbers on the ticket</td><td><table><tr><td>1</td><td>10</td><td>27</td><td>31</td><td>0</td><td>59</td><td>0</td><td>0</td><td>0</td></tr><tr><td>9</td><td>13</td><td>0</td><td>37</td><td>0</td><td>0</td><td>63</td><td>0</td><td>80</td></tr><tr><td>0</td><td>14</td><td>0</td><td>38</td><td>44</td><td>0</td><td>68</td><td>79</td><td>0</td></tr></table>Fig 7: Example of a Tambola ticket</td></tr>
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Table 1: Winning Strategies

6. Since there are multiple players and multiple strategies, the game is about completing as many game strategies as one can to accumulate more money/rewards. No particular strategy guarantees a win. At the end the total rewards of every player is calculated and the person with highest rewards wins, while other players with reward points also win their earned money.
7. A person can win one or more of these winning strategies.
8. The game continues till all the game winning strategies have been completed.

2.2 Game Reward Policies

1. Further in this report, we will go on to find the expectations of the number of turns it takes for 1 out of N participating players to win a particular strategy. To maintain a fair game, we will set the expectations of the number of turns as a constraint for winning the rewards for that strategy. i.e. If the expectation of the number of turns it takes for a player when competing against N-1 players is 23, a player is awarded the reward only if they win the strategy before the 23rd number is called. This way we save more money to give away for the fundraising.

2. Strategy Priorities and rewards:
   The following rewards were decided for each of the strategies based on their probability of occurring at every turn, which has been discussed in the following sections. More probable events have lower rewards assigned, and vice versa.

Strategy	**Reward(in $)**
Full House	60
Top Line	40
Bottom Line	40
Middle Line	40
Four Corners	20
Early Five	10

Table 2: Strategy-Rewards

3. Exploration and Analysis:

In this project, we have simulated the game for one player. The tickets of the player contain numbers drawn randomly, and the caller calls numbers in the range [1, 90] all drawn uniformly. We have simulated the game 1000 times on the random sample, to draw statistical inference from our observations, which have been discussed in this report.

3.1 Choice of parameters

After experimenting with a huge variations of parameters and having observed the game simulated for a a various parameters, we have chose the following parameters to provide a minimalistic understand of our analysis approaches:

a) Maximum range of numbers in tickets: 90
b) Number of simulations: 10^5
c) Number of players: 1
d) Price for playing: 10$

3.2 Game Analysis

3.2.1 Analysis of the simulation on 1 Player with a fixed caller sequence every run

In a real life Tambola game, we will always have more than one player. But we do this experiment with just one player to simulate and understand the trend of winning all the 6 strategies. This experiment sets a basis of our further exploration in this project.

In this experiment, with the above parameters, we tried to simulate the game where there is a caller and just a single player was playing the game. We simulate 10^5 games and for every game, a caller sequence is randomized and so are the player tickets. We do this in simulation so as to accommodate for the uncertainties or probabilities of getting a particular sequence which favors the winning of one of the strategies first. And it was observed that simulation over 10^3 are fairly unbiased.

Demonstrated below are plots of the probabilities of the player winning on a game winning strategy at the T^{th} turn of the called numbers:

$$Probability\ of\ winning\ strategy\ S\ at\ turn\ T\ =\ \frac{Number\ of\ times\ a\ player\ wins\ at\ turn\ T}{Total\ number\ of\ times\ they\ played}$$

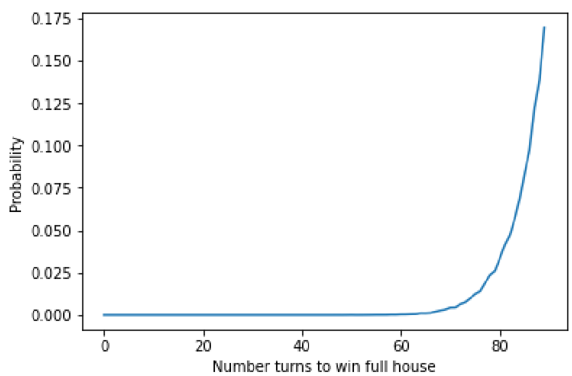

Fig 8: Probability of winning a **full house** with every turn
Expected number of turns: 85.36

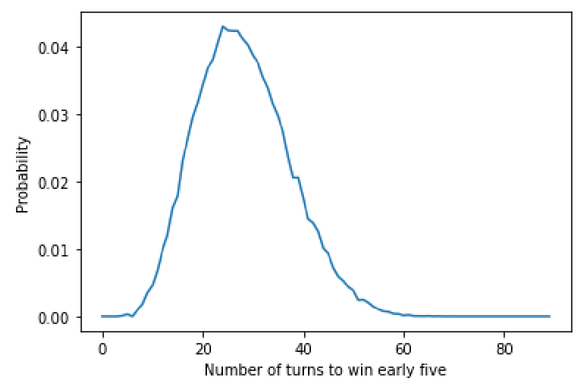

Fig 12: Probability of winning a **early five** with every turn
Expected number of turns: 29.08

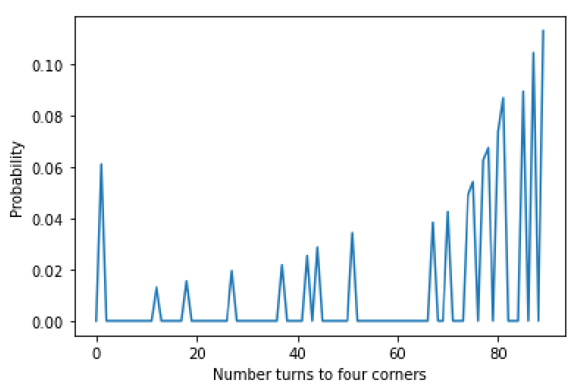

Fig 10: Probability of winning a **four corner** with every turn
Expected number of turns: 69.83

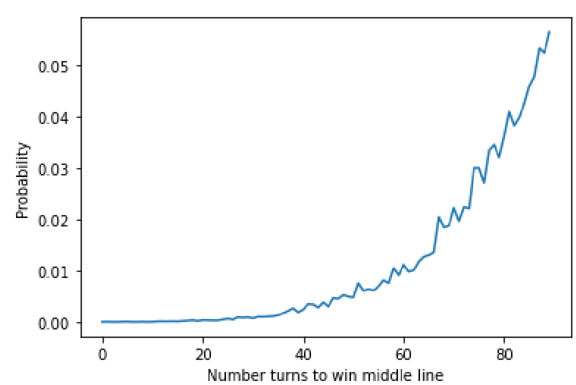

Fig 9: Probability of winning a **middle line** with every turn
Expected number of turns: 75.51

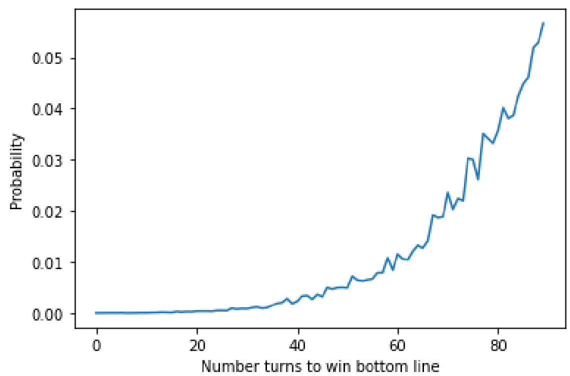

Fig 11: Probability of winning a **bottom line** with every turn
Expected number of turns: 75.55

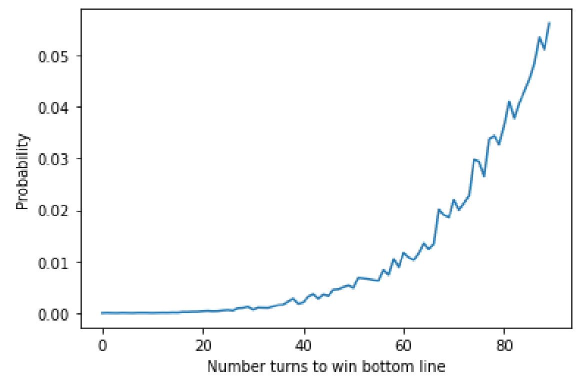

Fig 13: Probability of winning a **top line** corner with every turn
Expected number of turns: 75.45

The probability plots lay down the probability of a single player playing the game, winning the strategy at a given turn of the game. The turn is not the same as the actual number being called by the caller. The plots are probability mass functions

Full House (Fig 8): We observe that a full house has the highest expectation in terms of the number of turns taken to win one. This makes sense because it takes all the numbers of the player to be called out to win one.

Early Five (Fig 12): The probability of the early five is of particular interest to our findings. The curve is very close to the normal or Gaussian curve. And this makes sense because, early five requires any five numbers of the player to be crossed out. Hence, we see that the expectation for this distribution is 29.08. So, we can expect that a player wins five by the time ~29 numbers have been called out. It decreases from there on because, in all our simulations, the maximum game plays had the player win the strategy by 29 turns and hence there are a lesser number of gameplays distributed afterwards.

Four corners (Fig 10): This is important to note that four corners is very similar to the early five strategy except that we have constraints on where the numbers are located on a players' tickets.
We observe that there's no continuity in the probability patterns in this plot, but the probability does increase with the number of turns. Since, this strategy requires crossing out numbers at the four corners of the player's ticket, the probability of the corners being crossed out increases as more and more numbers are called out. But there's no pattern to the whole trend, which makes sense for four corners of the ticket, as they could be any number.

Top, Middle and Bottom Line (Fig 9, 11 and 13): It is important to note that these 3 strategies are essentially the same. The constraint difference being in, which row we want to cross out, which is identical in probabilistic terms. This strategy is an extension of the four corners but, with a location constraint of a row and the number of entries to be crossed out is more. The probability distribution of these three strategies are expectedly similar and it increases with the number of turns.

3.2.2 Analysis of the simulation on more than 1 Player with a random caller sequence every run

Having observed the patterns and trends in the probability distribution of the 6 strategies mentioned, we are now ready to extend the discussion to the general real-life case where we will be playing the game in charity/fundraising events with N players, where N >1.

What we expect to see in this case is, how the expected number of turns changes as we play with more players in a game. In this experiment we simulate the games for a varying number of players in the range [1,100]. In every game we keep a track of how many turns it took for any one of the players to win one of the strategies. Using these data from the simulation we calculate the probability and in turn the expectation of the number of turns taken in a game for any one player to win a strategy. It is important We have again used a simulation of 10^5 games to arrive at the following analysis figure.

$$\begin{aligned}&Expected\ number\ of\ turns\ to\ win\ strategy\ S\ while\ playing\ among\ N\ Players\\&=\ (Turn\ value\ T)\ *\ (Probability\ of\ winning\ strategy\ S\ at\ turn\ T\ by\ any\ one\ of\ N\ Players)\end{aligned}$$

The probability of winning strategy S at turn T for one player was calculated in the previous experiment. We maintain the winning states of all players, and observe at which turn one of the N players wins a strategy. Eventually we take a sample average of the number of turns taken by N players to win a strategy S to plot each of the points below:

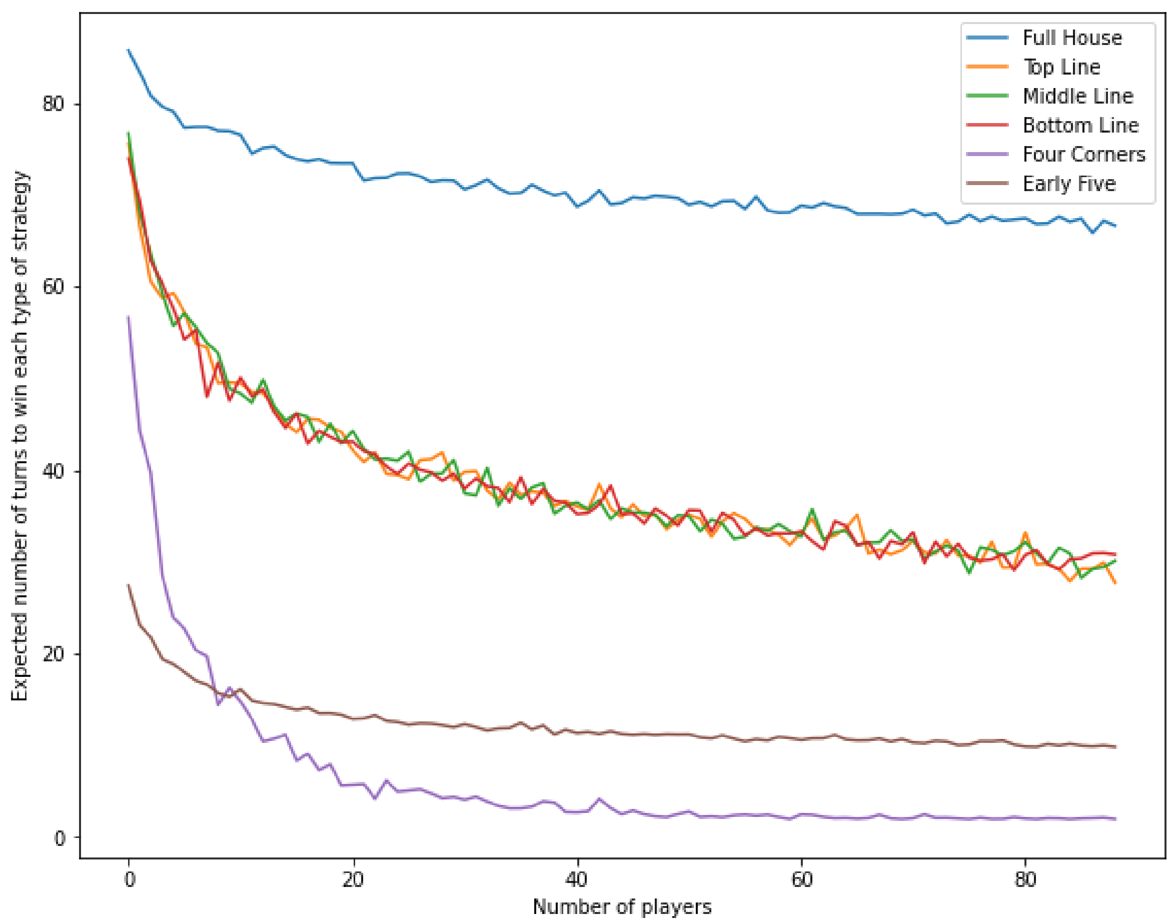

Fig 14: Expected number of turns to win vs Number of players for different strategies

An evident observation from the plot is the **decreasing trend in the expectation of the number of turns taken to win a strategy as we increase the number of players** in a game of *Tambola*. This can be explained by the fact that, the more players we have in a game, there's a wide range of distribution of the sets of 15 numbers in everyone's tickets. Hence, keeping the randomness in mind, it only makes sense to conclude that the more the number of players, the sooner someone will win one of the strategies.

3.3.3 Analysis of the simulation for N=30 players with random caller sequence every run

We can now extend our knowledge to a fixed number of players, with the caller calling a sequence of random numbers. In this experiment, we stimulate 10^5 games with 30 players and every game has a different randomized caller sequence. We chose the number 30 to stimulate this game on, as that is usually the number in which people usually play the original game of Tambola, which we are trying to improvise and redesign here.

$$Probability\ of\ winning\ strategy\ S\ at\ turn\ T\ = \frac{Number\ of\ times\ a\ player\ wins\ at\ turn\ T}{Total\ number\ of\ times\ they\ played}$$

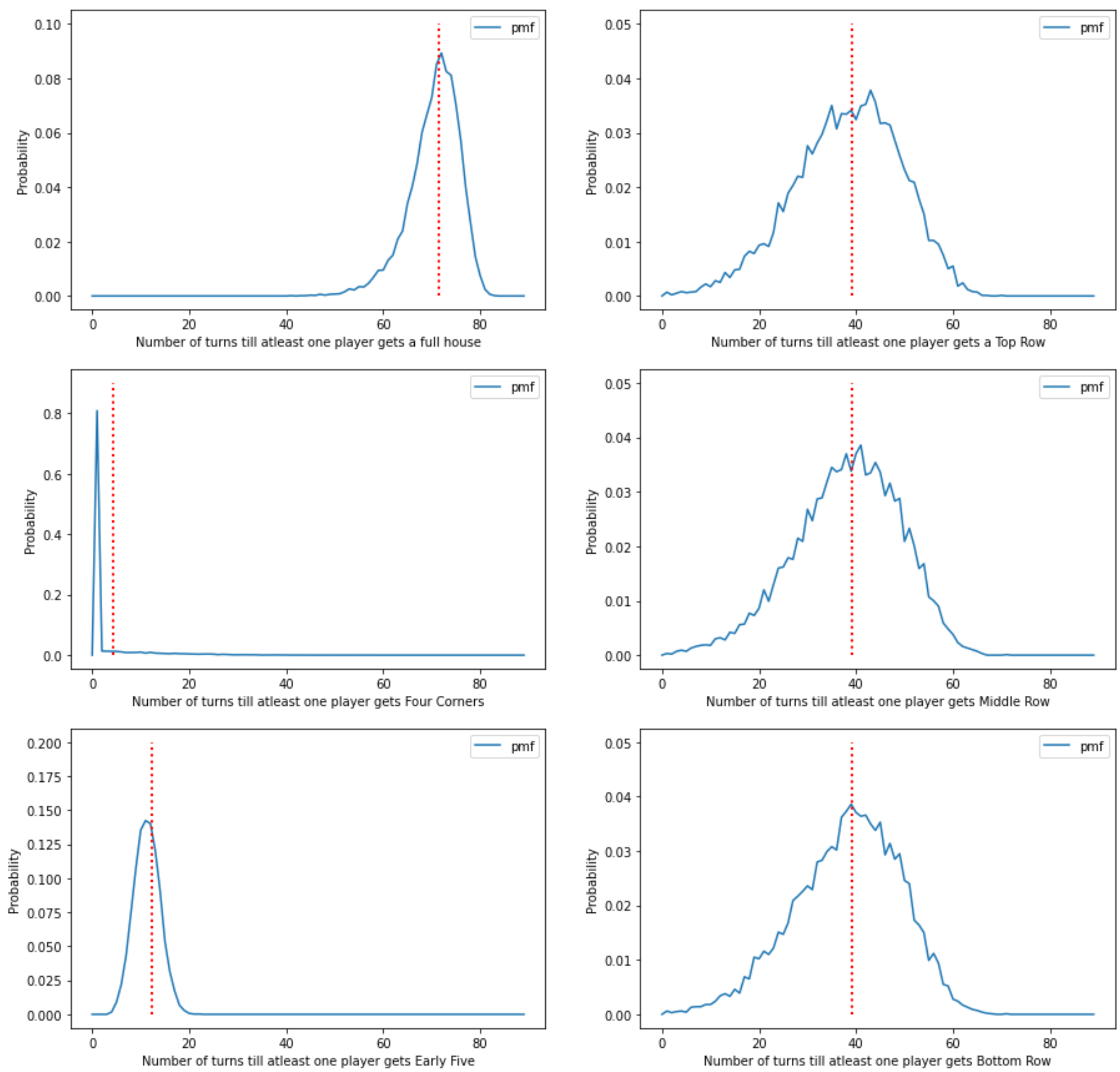

Fig 15: Plots of Probability of winning different strategies at every turn for a game of 30 players
(Marked in red are the expected values at which any one player wins the given strategy)

We try to connect the above observations to the observations from Section 3.2.1, as the only difference between these two simulations is in the caller sequence. In the Section 3.2.1, the caller sequence was fixed, i.e all simulations were on the same sequence. In this section, apart from increasing the number of players, we randomized the caller sequence for each game of the sequence. We observe that the probability mass function approaches very close to a **Gaussian distribution,** *like everything should be!*

4. Conclusion

Tambola as a game is played very commonly in Indian communities in most social gatherings, as it is an excellent engaging game, when coupled with money prizes. The original game is often played with players buying in more than one ticket for themselves to improvise their chances of winning, similar to the lottery mindset. But their probabilistic chances of winning are not very well researched. In this project we tried to improvise the game to suit modern day charity and fundraising events and analyzed the winning probability distributions, which turned out to be Gaussian. We are extending that knowledge for the fundraising houses to have leverage over the players.

4.1 Inferences about the winning distributions

Using the analysis of this game, we can predict and bet on the winning turn. This can also be used to optimize and design the strategy for giving away award money to the winners, while keeping in mind that most of the money collected from the players during an event has to be saved for the cause of charity.

4.2 Strategizing Rewards

Using the statistics explained above, we have tried to strategize the rewards system for this game. The caller can decide to set a number T, and at T^{th} turn they stop calling out numbers. This way, if they do not go on to call out all the numbers, they may not have to award the prize money for all 6 strategies, hence saving the fundraisers more money.

In this experiment, we have simulated the game for 30 players. Since this is designed for a fundraiser event, we simulated the game for 30 players again, and tried to see at what turn the caller should stop calling numbers out of the sequence to maximize the house profit while still keeping the game fair.

We simulated the game for 1000 iterations to see at each turn on average how much money will be given away to the winning players.

$$\text{Amount given away by nth turn} = \text{Amount given away in (n-1)th turn} + 60*s_1 + 40*s_2 + 40*s_3 + 40*s_4 + 20*s_5 + 10*s_6$$

Where s_1, s_2, s_3, s_4, s_5, s_6 are indicator variables indicating if the respective strategies are reached in that step. The coefficients are the respective rewards mentioned in Table-2. Plotted below are the results from our experiment:

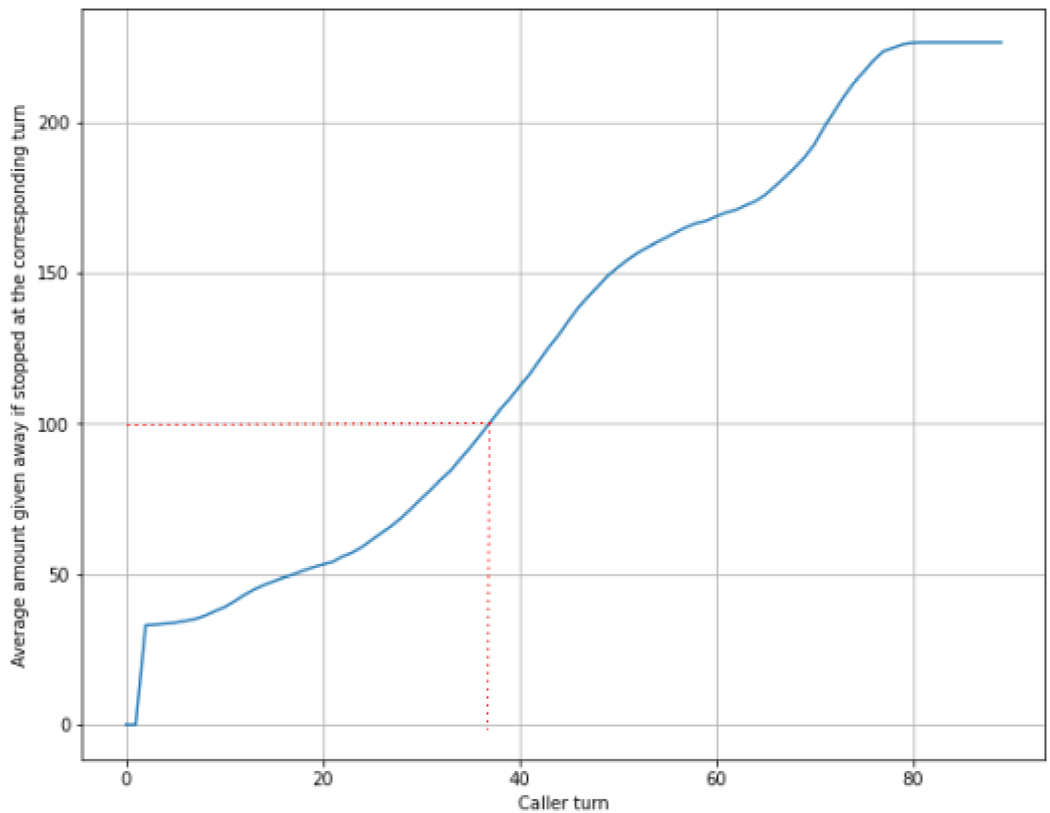

Figure 15: Expected giveaway amount in rewards at each turn of the game

Strategy:
In this experiment of 30 players, since we are charging the players a 10$ per ticket.
Total money collected = 30*10$ = 300$
If we were to target donating 200$ to the charitable cause, we can give away 300-200 = 100$ in rewards.
To achieve this, stop the caller at around the 38th turn to achieve our goal.

The above experiment was performed for N=30 players and the goal was to target gaining an average of 200$ of the event for charity. This can easily be translated into a general case of any N number of players to save any desired amount of money for the cause.

5. Future Work

This study can also be extended to strategizing and optimizing the tickets to be chosen by the players, so that their chances of winning award money is maximized and so is the amount they can win. Given a set of printed tickets at a game, we can provide a player their best chance at the game from the remaining tickets that are left, after a fixed number of players have chosen their best shots.
Since our project aimed at maximizing profits for the fundraising companies, we have chosen not to do that in this current study, but it looks like a promising project topic to be explored.

6. Code Repository

The working implementation of this game design can be found at our Github repository.

7. References

[1] https://en.wikipedia.org/wiki/Tombola_(raffle)
[2] https://www.wikihow.com/Play-Tambola