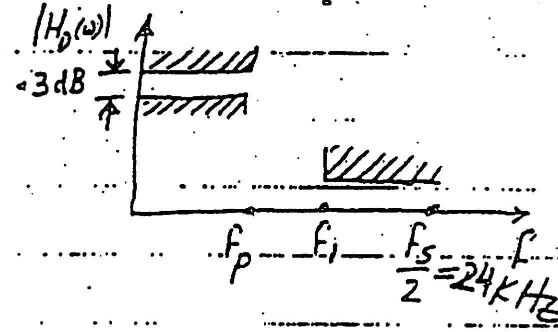
EEL5513 — Project # 3 — Spring, 2017 WBM

Design F.I.R, linear phase, using the Fourier series method. Center ω_c of the ideal filter, where applicable, in the center of the transition period to account for the smearing of the window.

Use the minimum # of samples, unless otherwise specified.

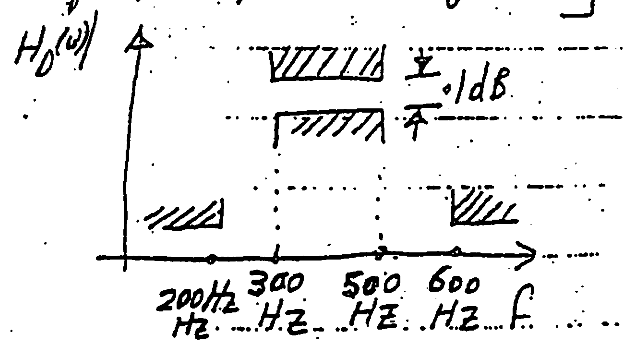
(1) Design a LPF, # of $h(n)$ samples odd, to meet the following specs



$f_s = 48$ KHz
$f_1 = 9.2$ KHz
$f_p = 5.2$ KHz
Stopband atten ≥ 50 dB
passband ripple ≤ -3 dB

Use the Hamming window if feasible
[Print coefficient values $h(n)$, plot freq. response, mag & phase]

(2) Design a BP filter



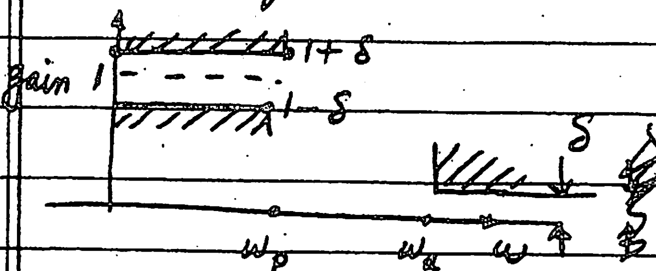
passband 300 → 500 Hz
transition band 100 Hz
passband ripple .1 dB
stopband atten 60 dB
$f_s = 2$ KHz

Use the Blackman window if feasible
[print the coefficient values $h(n)$, $n = 0 \to L$, plot freq response, Mag. only]

(3) Design a Hilbert transformer $h(n)$, $n \equiv 0$ to 40. Use a Kaiser window, $\alpha = 3$, $\omega_s = 10$ rad/sec
Plot mag & phase of $H_D(\omega)$ for ω from 0 to $\frac{\omega_s}{2}$

$\boxed{N = L+1}$

Kaiser Design Procedure



$$w_k(nT) = \begin{cases} \dfrac{I_0(\beta)}{I_0(\alpha)} & |n| \leq \dfrac{N-1}{2} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

α is an indep param &

$$\beta = \alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}$$

$$I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2 \quad \text{fast converging}$$

passband ripple in dB $= A_p = 20 \log \dfrac{1+\delta_2}{1-\delta_2}$ (given) $\qquad A_a = -20 \log \delta_1$ = stopband atten in dB (given)

~~B_t = Transition BW $= \omega_a - \omega_p$~~

$$\omega_c = \frac{\omega_p + \omega_a}{2}, \quad h(nT) = \frac{\sin \omega_c T n}{\pi n}$$

$$\delta_1 = 10^{-.05 A_a}, \quad \delta_2 = \frac{10^{\frac{A_p}{20}} - 1}{10^{\frac{A_p}{20}} + 1}$$

- From specs choose $\delta = \min(\delta_1, \delta_2)$
- Calculate $A_a = -20 \log(\min \delta_1, \delta_2)$
- Choose α as follows

$$\alpha = \begin{cases} 0 & \text{for } A_a \leq 21 \\ .5842\,(A_a - 21)^{.4} + .07886\,(A_a - 21) & \text{for } 21 < A_a \leq 50 \\ .1102\,(A_a - 8.7) & \text{for } A_a > 50 \end{cases} \quad (2)$$

- Choose D as follows

$$D = \begin{cases} .9222 & \text{for } A_a \leq 21 \\ \dfrac{A_a - 7.95}{14.36} & \text{for } A_a > 21 \end{cases} \quad (3)$$

- Select the lowest odd # N satisfying $\qquad N = L+1$

$$N \geq \frac{\omega_s D}{B_t} + 1 \quad (4)$$

- Form $w_k(nT)$ using (1)
- $$H(z) = z^{-\frac{L}{2}} \sum_{n=-\frac{L}{2}}^{\frac{L}{2}} w_k(nT) \cdot h(nT)\, z^{-n}$$

Kaiser window

$$w_k(n) = \begin{cases} \dfrac{I_0(\beta)}{I_0(\alpha)} & |n| \le \dfrac{L}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$\beta = \alpha \sqrt{1 - \left(\frac{2n}{L}\right)^2}$$

$I_0(x)$ is the zeroeth-order modified Bessel function of the first kind.

$$I_0(x) = 1 + \sum_{k=1}^{\infty} \left[\frac{1}{k!} \left(\frac{x}{2}\right)^k \right]^2$$

4- Design a digital differentiator (0 to $\frac{\omega_s}{2}$), $L = 20$, $f_s = 1$ Hz

a- using rectangular window

b- using a Hamming window

- Obtain the weights for ⓐ and ⓑ
- Plot magnitude and phase from $\omega = 0$ to $\omega = \frac{\omega_s}{2}$ for a, b, and ideal.