Applications of DSP

Project 3

UNIVERSITY OF CENTRAL FLORIDA
DEPARTMENT OF ELECTRICAL ENGINEERING

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Authored by**: Rajib Dey**

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Project 3

Here in this project, I have designed linear phase FIR using the Fourier Series method. Unless otherwise specified, I have used the minimum number of samples.

Question 1:

Given,

i. f_s=48 KHz
ii. f_l=9.2 KHz
iii. f_p=5.2 KHz
iv. stopband attenuation ≥50dB
v. passband ripple ≤0.3dB

We have to design a low pass filter with odd number of h(n) samples, meeting the above specifications and using the Hamming window if feasible.

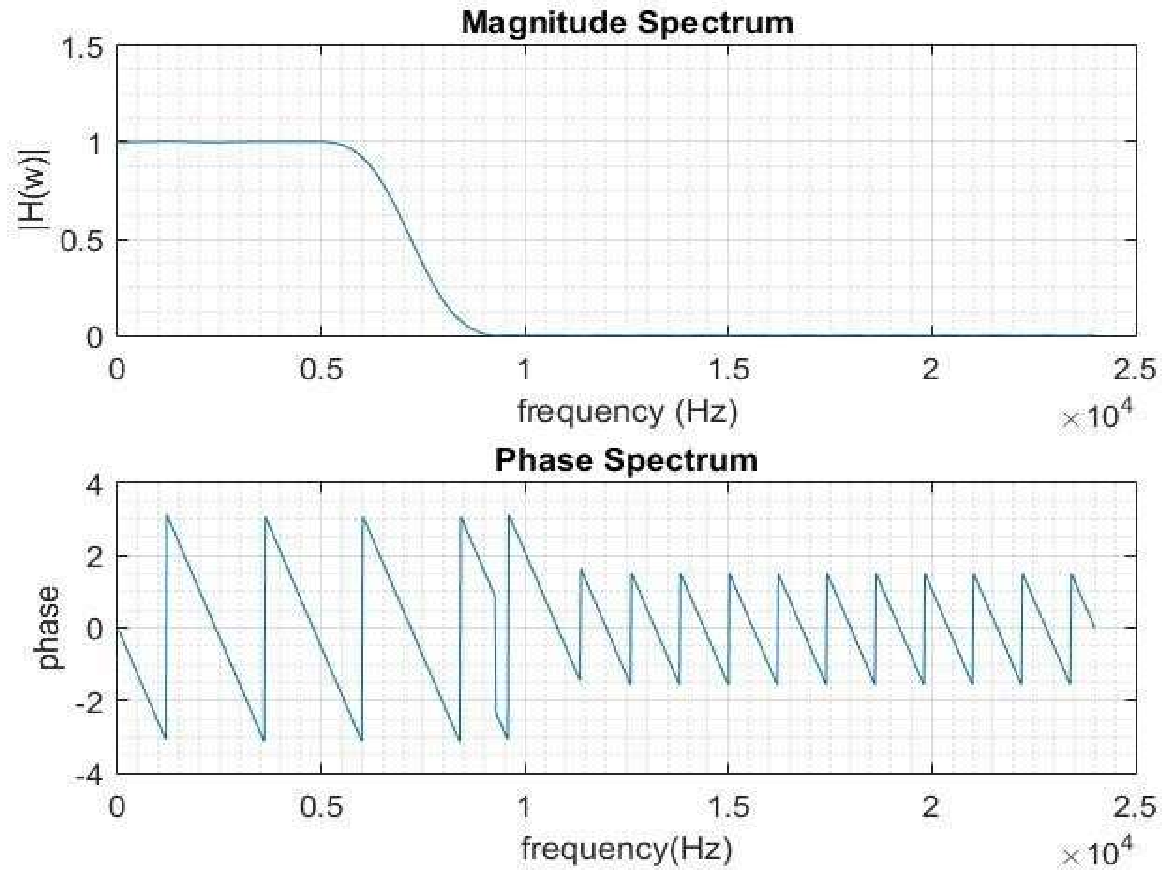
In MATLAB, I have defined the low pass, band pass, Hilbert transformer and digital differentiator using inline functions. Since the stopband attenuation and the passband ripple meets the specs of a Hamming window, it is feasible to use it for this question. As discussed by Prof. Wasfy in the class, I have taken the expression of Hamming window as follows: $\boldsymbol{Whamming} = \mathbf{0.54} + \mathbf{0.46}\boldsymbol{cos}\,(\mathbf{2}\boldsymbol{\pi}/\boldsymbol{n}),\ \boldsymbol{where}\ |\boldsymbol{n}| \leq \boldsymbol{L}/\mathbf{2}$.

To calculate the cut-of frequency f_c, I have used the formula $\boldsymbol{f_c} = (\boldsymbol{fp}+\boldsymbol{f1})/\mathbf{2}$. Using the equation $\mathbf{6.6}\boldsymbol{\pi}/(\boldsymbol{L}+\mathbf{1}) = \mathbf{2}\boldsymbol{\pi} * (\boldsymbol{f1}-\boldsymbol{fp})/\boldsymbol{fs}$, we get L=40. It is to be noted here that for hamming window, $\boldsymbol{\Delta\omega} = \mathbf{6.6}\boldsymbol{\pi}/(\boldsymbol{L}+\mathbf{1})$ **.** The coefficient values for h(n) are as follows:

```
H =

  Columns 1 through 15

   -0.0000   -0.0012   -0.0017   -0.0008    0.0020    0.0046    0.0036   -0.0025   -0.0100   -0.0110    0.0000    0.0175    0.0258    0.0105   -0.0253

  Columns 16 through 30

   -0.0551   -0.0427    0.0311    0.1480    0.2561    0.3000    0.2561    0.1480    0.0311   -0.0427   -0.0551   -0.0253    0.0105    0.0258    0.0175

  Columns 31 through 41

    0.0000   -0.0110   -0.0100   -0.0025    0.0036    0.0046    0.0020   -0.0008   -0.0017   -0.0012   -0.0000
```

After this, I have used the equation: $\mathbf{H(z)=}\boldsymbol{z^{-(L/2)}} * \Sigma \sum_{n=-\left(\frac{L}{2}\right)}^{L/2} ((\mathbf{sin}\ \boldsymbol{\omega}\mathbf{c}\ \mathbf{Tn})/(\ \boldsymbol{\pi}\mathbf{n})$ $*(\boldsymbol{Whamming(n)}) * \boldsymbol{z^{-n}}$ to plot the magnitude and phase spectrum of the filter.

The plot for part 1 is as follows:



Question 2:

Given,

i. Passband: 300→ 500 Hz
ii. Transition band: 100 Hz
iii. Passband ripple 0.1 dB
iv. Stopband attenuation 60dB
v. f_s=2khz

We have to design a band pass filter which meets the above specifications using the Blackman Window if feasible.

The expression for the Blackman window is: $\mathbf{W_{Black}=0.42+0.5\cos(2\pi n/ L)} + \mathbf{0.08\cos\,(4\pi n/L)}$ **, where $|n|\leq L/2$**. Since the passband ripples and stopband attenuation meet the pecs of a Blackman window, it is feasible to use Blackman window.

Now I have calculated the $\Delta\omega$ for Blackman as follows:

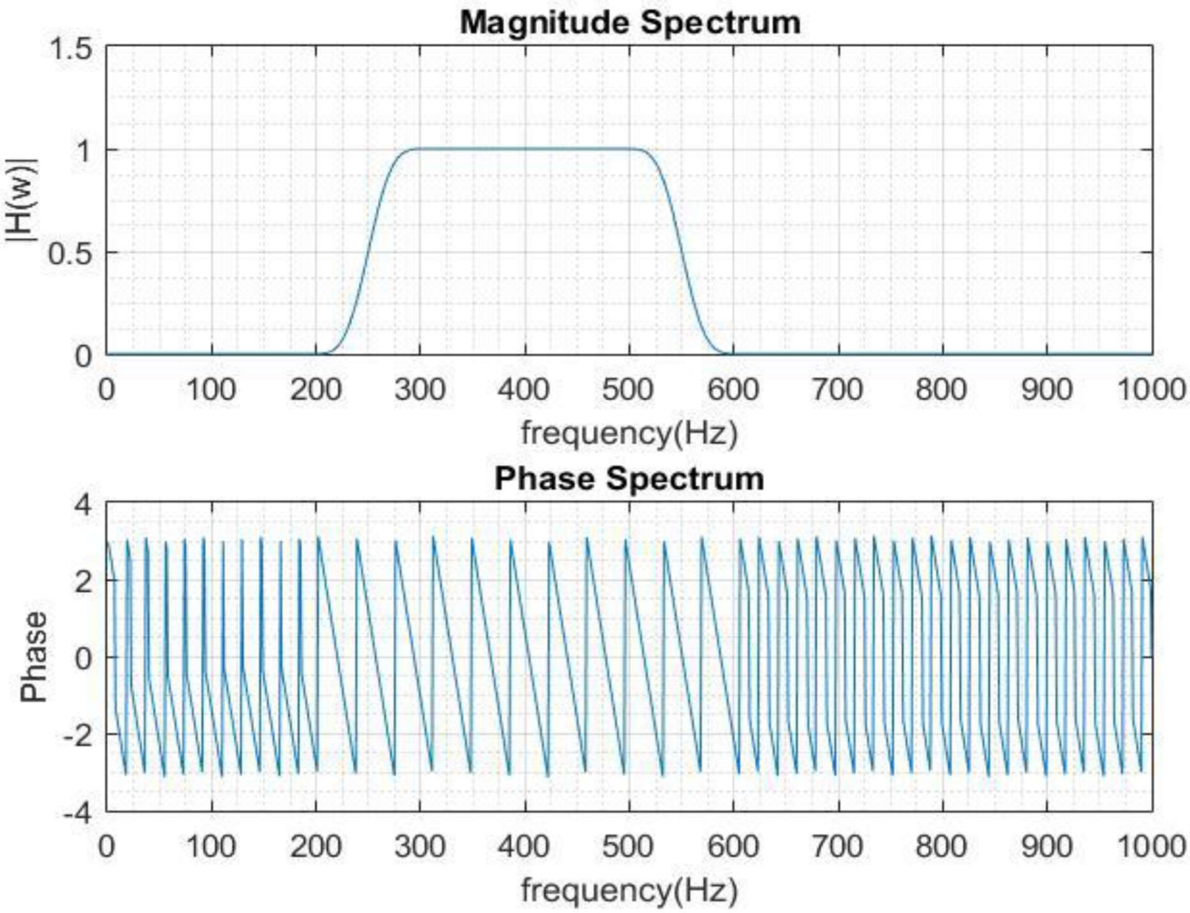
$$\mathbf{11\pi\,/(L+1) = 2\pi\,((300-200)/\,(2*10^{3})\,) = 2\pi(\,(600-500)/\,(2*10^{3}\,))}$$

$$\therefore \mathbf{110 = L+1}$$
L=109

The coefficient values of h(n) are as follows:

```
H =

  Columns 1 through 15

   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0001    0.0000    0.0001   -0.0000    0.0002    0.0003   -0.0002   -0.0007   -0.0002    0.0002

  Columns 16 through 30

   -0.0003    0.0003    0.0017    0.0007   -0.0018   -0.0013    0.0001   -0.0015   -0.0011    0.0041    0.0047   -0.0016   -0.0029    0.0002   -0.0041

  Columns 31 through 45

   -0.0076    0.0038    0.0127    0.0031   -0.0032    0.0036   -0.0043   -0.0219   -0.0081    0.0203    0.0139   -0.0009    0.0157    0.0108   -0.0410

  Columns 46 through 60

   -0.0466    0.0159    0.0301   -0.0022    0.0469    0.0949   -0.0551   -0.2333   -0.0849    0.2404    0.2404   -0.0849   -0.2333   -0.0551    0.0949

  Columns 61 through 75

    0.0469   -0.0022    0.0301    0.0159   -0.0466   -0.0410    0.0108    0.0157   -0.0009    0.0139    0.0203   -0.0081   -0.0219   -0.0043    0.0036

  Columns 76 through 90

   -0.0032    0.0031    0.0127    0.0038   -0.0076   -0.0041    0.0002   -0.0029   -0.0016    0.0047    0.0041   -0.0011   -0.0015    0.0001   -0.0013

  Columns 91 through 105

   -0.0018    0.0007    0.0017    0.0003   -0.0003    0.0002   -0.0002   -0.0007   -0.0002    0.0003    0.0002   -0.0000    0.0001    0.0000   -0.0001

  Columns 106 through 110

   -0.0000    0.0000    0.0000   -0.0000   -0.0000
```

The magnitude and phase response are as follows:



<u>Question 3:</u>

Here in this part, we are required to design a Hilbert transformer h(n), n=0→40 using a Kaiser window with $\alpha = 3$ and ω_s=10rad/sec.

I defined, a Kaiser window as follows:

$$W_k(n)=\begin{cases} \frac{Io(\beta)}{Io(\alpha)}, & |n| \le \frac{L}{2} \\ 0, & otherwise \end{cases} \text{ , where } \beta=\alpha\sqrt{1-\left(\frac{2n}{L}\right)^2}$$

$I_0(x)$ is the zeroth order modifies Bessel function of the first kind. $\mathbf{I_0(x)=1} + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$.

I have calculated $\mathbf{A_a=-20log\ (min\ \delta_1,\delta_2)}$.

Passband ripple in dB=$\boldsymbol{Ap'}$=$\mathbf{20log\frac{1+\delta 2}{1-\delta 2}}$.

B_t=Transition Bandwidth, $\mathbf{BW=\omega_a\text{-}\omega_p}$, $\boldsymbol{\omega_c=\frac{\omega p+\omega a}{2}}$, $\mathbf{h(nT)=\frac{\sin\omega cTn}{\pi n}}$, $\boldsymbol{\delta 1 = 10^{-0.05Aa'}}$, $\boldsymbol{\delta 2 = \frac{10^{\frac{Ap'}{20}}-1}{10^{\frac{Ap'}{20}}+1}}$

I have chosen α as follows:

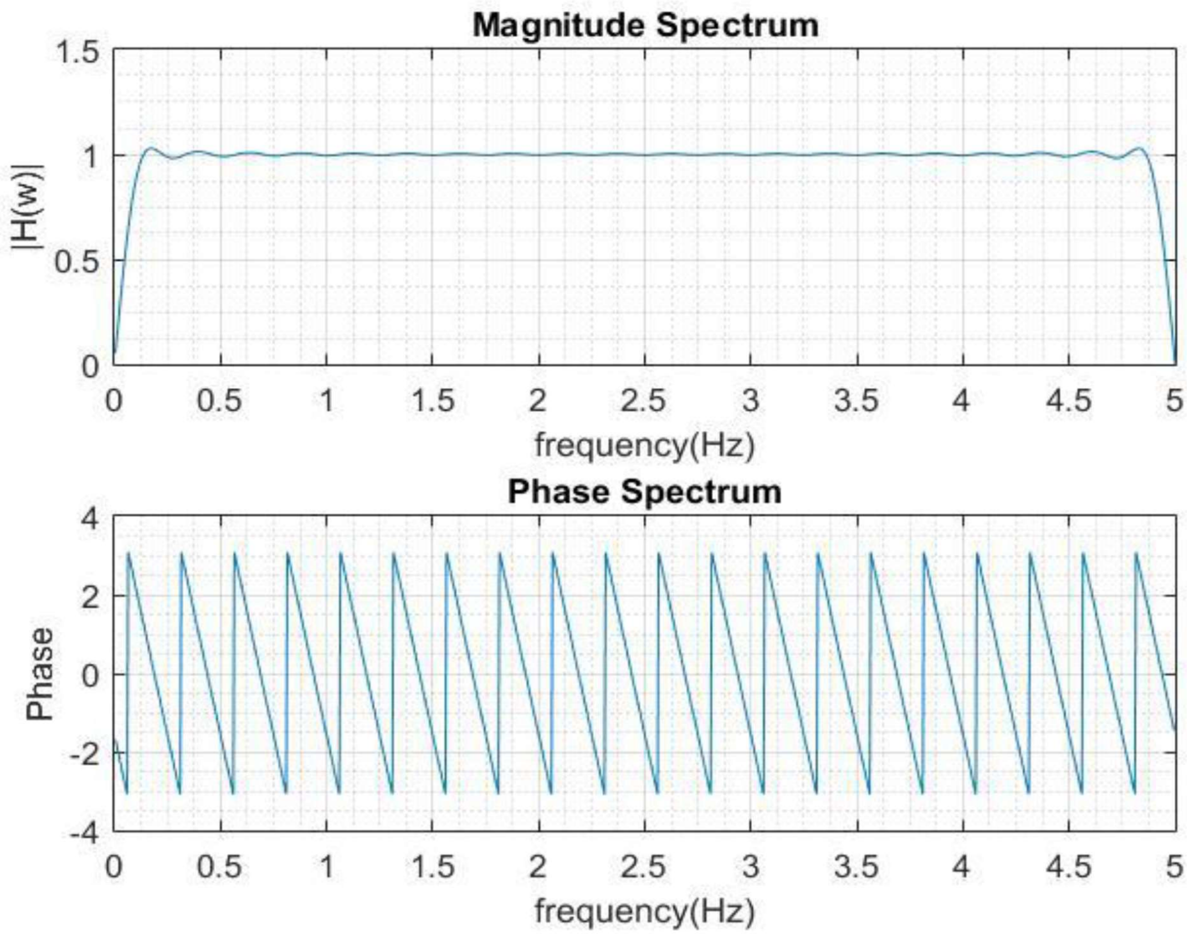
$$\boldsymbol{\alpha = \begin{cases} 0\ for\ Aa \leq 21 \\ 0.5842(Aa-21)for\ 21 < Aa \leq 50 \\ 0.1102(Aa-8.7)for\ Aa > 50 \end{cases}}$$

I have chosen D as follows:

$$\boldsymbol{D = \begin{cases} 0.9222\ for\ Aa \leq 21 \\ \frac{Aa-7.95}{14.36}\ for\ Aa > 21 \end{cases}}$$

Finally, I formed $W_k(nT)$ with the help of equation: $\mathbf{H(z)=z^{-\frac{L}{2}}.\sum_{n=-(\frac{L}{2})}^{\frac{L}{2}}} \boldsymbol{WknT.h(nT).z^{-n}}$.

The magnitude and phase response of $H_D(w)$ for w from 0 to $\omega s/2$ are as follows:



Question 4:

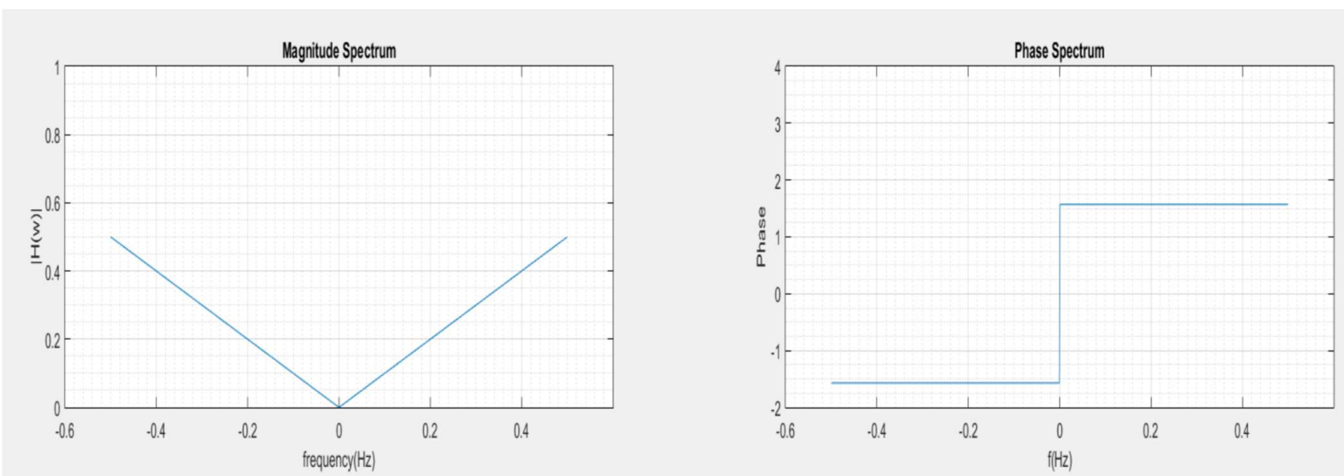
Here in this part, we are required to design a digital differentiator (0 to $\omega s/2$), L=20 and f_s=1KHz.

The equation for the digital differentiator is: $\mathbf{h(nT)}=\frac{1}{2\pi n}[\omega s.\cos(n\pi) - \left(\frac{2.\sin(n\pi)}{nT}\right)]$.

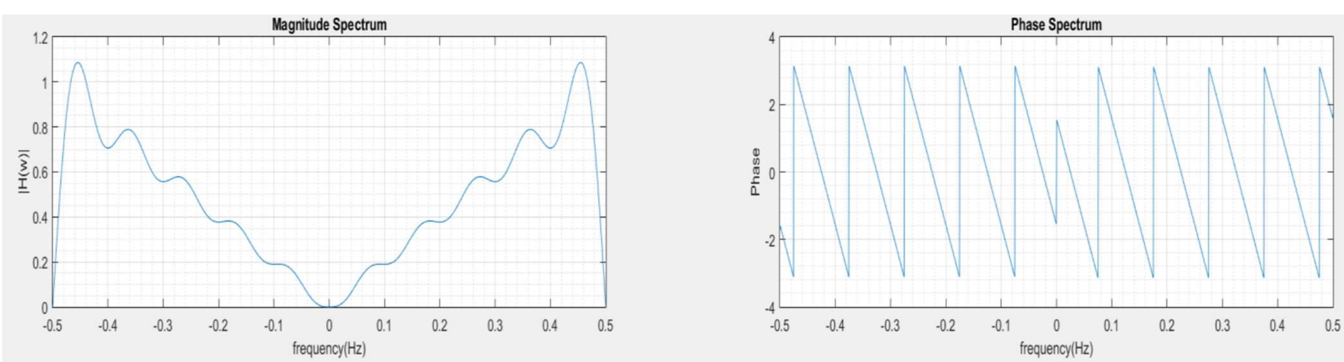
The weights for rectangular and Hamming windows are as follows:

```
weights using rectangular window are:
hr =

 Columns 1 through 10:

  -0.03183   0.03537  -0.03979   0.04547  -0.05305   0.06366  -0.07958   0.10610  -0.15915   0.31831

 Columns 11 through 20:

   0.00000  -0.31831   0.15915  -0.10610   0.07958  -0.06366   0.05305  -0.04547   0.03979  -0.03537

 Column 21:

   0.03183

weights using hamming window are:
hm =

 Columns 1 through 10:

  -0.00255   0.00363  -0.00668   0.01226  -0.02111   0.03438  -0.05428   0.08598  -0.14517   0.31114

 Columns 11 through 20:

   0.00000  -0.31114   0.14517  -0.08598   0.05428  -0.03438   0.02111  -0.01226   0.00668  -0.00363

 Column 21:

   0.00255
```

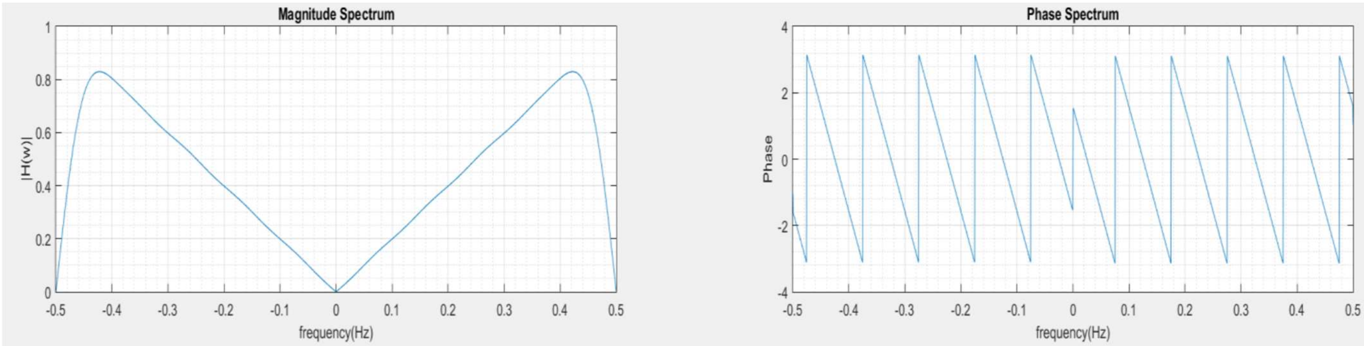
Magnitude and phase for the ideal case:



Magnitude and phase for the rectangular window:



Magnitude and phase for the Hamming window:



The following is the codes and plots of the entire project in MATLAB.

MATLAB Code

```
% ------- Applications of DSP ----------------
% ------------Project 3 ----------------------
%------------Submitted by--------------------
%-------------Rajib Dey ----------------------

clear;
close all;
clc;

% Definition of the ideal filter

H_low=inline('sinc(2*fc*ts.*n)*2*fc*ts','fc','ts','n');
H_band=inline('sinc(2*fh*ts.*n)*2*fh*ts-
sinc(2*fl*ts.*n)*2*fl*ts','fh','fl','ts','n');
H_hil=inline('(1-cos(n*pi))./(n*pi)','n');
H_dif=inline('cos(n.*pi)./(n.*pi)','n');
w_hamming=inline('0.54+0.46*cos(2*pi.*n/L)','L','n');
w_blackman=inline('0.42+0.5*cos(2*pi.*n/L)+0.08*cos(4*pi.*n/L)',
'n','L');
w_kaiser=inline('besseli(0,b)./besseli(0,a)','a','b');

% -------------------Code for Problem 1------------------

fs=48000;
f1=9200;
```

```
fp=5200;
L=40;
ts=1/fs;
fc=(f1+fp)/2;

n=-1*L/2:L/2;
H_n=H_low(fc,ts,n);
H=H_low(fc,ts,n).*w_hamming(L,n)

% -------------------Plot for the spectrum of the filter--------
-----------

for i=1:1000
w(i)=pi*fs*i/1000;
z=exp(j*w(i)*ts);
H_final(i)=z^(-1*L/2)*sum(H.*z.^(-n));
end
figure(1);
subplot(2,1,1);
plot(w/(2*pi),abs(H_final));
xlabel('frequency (Hz)');
ylabel('|H(w)|');
title('Magnitude Spectrum');
grid on;
grid minor;
subplot(2,1,2);
plot(w/(2*pi),angle(H_final));
xlabel('frequency(Hz)');
ylabel('phase');
title('Phase Spectrum');
grid on;
grid minor;
% -------------------Code for Problem 2-------------------

fs=2000;
fh=550;
fl=250;
L=109;
ts=1/fs;
n=-1*L/2:L/2;
H_n=H_band(fh,fl,ts,n);
H=H_band(fh,fl,ts,n).*w_blackman(n,L)

% -------------------Plot for the spectrum of the filter--------
-----------

for i=1:1000
```

```
w(i)=pi*fs*i/1000;
z=exp(j*w(i)*ts);
H_final(i)=z^(-1*L/2)*sum(H.*z.^(-n));
end
figure(2);
subplot(2,1,1);
plot(w/(2*pi),abs(H_final));
xlabel('frequency(Hz)');
ylabel('|H(w)|');
title('Magnitude Spectrum');
grid on;
grid minor;
subplot(2,1,2);
plot(w/(2*pi),angle(H_final));
xlabel('frequency(Hz)');
ylabel('Phase');
title('Phase Spectrum');

grid on;
grid minor;

% -------------------Code for Problem 3-------------------

fs=10/(2*pi);
a=3;
L=80;
ts=1/fs;
n1=-L/2:-1;
b1=a.*(1-(2.*n1./L).^2).^0.5;
H1=H_hil(n1).*w_kaiser(a,b1);
n2=1:L/2;
b2=a.*(1-(2.*n2./L).^2).^0.5;
H2=H_hil(n2).*w_kaiser(a,b2);

%-------------------- Plot for the spectrum of the filter--------
-----------

for i=1:1000
w(i)=pi*fs*i/1000;
z=exp(j*w(i)*ts);
H_final(i)=z^(-1*L/2)*(sum(H1.*z.^(-n1))+sum(H2.*z.^(-n2)));
end
figure(3);
subplot(2,1,1);
plot(w,abs(H_final));
xlabel('frequency(Hz)');
ylabel('|H(w)|');
```

```
title('Magnitude Spectrum');
grid on;
grid minor;
subplot(2,1,2);
plot(w,angle(H_final));
xlabel('frequency(Hz)');
ylabel('Phase');
title('Phase Spectrum');
grid on;
grid minor;

% -------------------Code for Problem 4-------------------

fs=1;
ws=2*pi;
L=20;
ts=1/fs;
n1=-L/2:-1;
H1=H_dif(n1)
n2=1:L/2;
H2=H_dif(n2)

% -------------------Plot the spectrum of the filter------------
-------

for i=1:2001
w(i)=pi*fs*(i-1001)/1000;
z=exp(j*w(i)*ts);
H_i(i)=j*w(i)./ws;
H_final(i)=z.^(-1*L/2).*(sum(H1.*z.^(-n1))+sum(H2.*z.^(-n2)));
H_ham(i)=z.^(-1*L/2).*(sum(H1.*w_hamming(L,n1).*z.^(-
n1))+sum(H2.*w_hamming(L,n2).*z.^(-n2)));
end
figure(4);
subplot(3,2,1);
plot(w/(2*pi),abs(H_i));
axis([-0.6 0.6 0 1]);
xlabel('frequency(Hz)');
ylabel('|H(w)|');
title('Magnitude Spectrum');
grid on;
grid minor;
subplot(3,2,2);
plot(w/(2*pi),angle(H_i));
axis([-0.6 0.6 -2 4]);
xlabel('f(Hz)');
ylabel('Phase');
```

```
title('Phase Spectrum');
grid on;
grid minor;
subplot(3,2,3);
plot(w/(2*pi),abs(H_final));
xlabel('frequency(Hz)');
ylabel('|H(w)|');
title('Magnitude Spectrum');
grid on;
grid minor;
subplot(3,2,4);
plot(w/(2*pi),angle(H_final));
xlabel('frequency(Hz)');
ylabel('Phase');
title('Phase Spectrum');
grid on;
grid minor;
subplot(3,2,5);
plot(w/(2*pi),abs(H_ham));
xlabel('frequency(Hz)');
ylabel('|H(w)|');
title('Magnitude Spectrum');
grid on;
grid minor;
subplot(3,2,6);
plot(w/(2*pi),angle(H_ham));
xlabel('frequency(Hz)');
ylabel('Phase');
title('Phase Spectrum');
grid on;
grid minor;

%-------------------End of the Program-------------------%
```