

### Assignment 18.2 :

In one state, 52% of the voters are Republicans, and 48% are Democrats. In a second state, 47% of the voters are Republicans, and 53% are Democrats. Suppose a simple random sample of 100 voters are surveyed from each state.

What is the probability that the survey will show a greater percentage of Republican voters in the second state than in the first state?

### Solution

For this analysis,

Let  $P_1$  = proportion of Republican voters in the first state

$P_2$  = proportion of Republican voters in the second state

$p_1$  = proportion of Republican voters in the sample from the first state

$p_2$  = proportion of Republican voters in the sample from the second state.

The number of voters sampled from the first state  $(n_1) = 100$ ,

and The number of voters sampled from the second state  $(n_2) = 100$ .

**1 :** To check if the sample size is big enough to model differences with a normal population.

Because  $n_1P_1 = 100 * 0.52 = 52$ ,

$$n_1(1 - P_1) = 100 * 0.48 = 48,$$

$$n_2P_2 = 100 * 0.47 = 47, \text{ and}$$

$$n_2(1 - P_2) = 100 * 0.53 = 53$$

all above values are each greater than 10, the sample size is large enough.

**2:** the mean of the difference in sample proportions:

$$\mu_{p_1 - p_2} = P_1 - P_2 = 0.52 - 0.47 = 0.05.$$

**4:** Find the standard deviation of the difference.

$$\sigma_d = \sqrt{[P_1(1 - P_1) / n_1] + [P_2(1 - P_2) / n_2]}$$

$$\sigma_d = \sqrt{[(0.52)(0.48) / 100] + [(0.47)(0.53) / 100]}$$

$$\sigma_d = \sqrt{0.002496 + 0.002491} = \sqrt{0.004987} = 0.0706$$

**3:** Find the probability.

This problem requires us to find the probability that  $p_1$  is less than  $p_2$ . This is equivalent to finding the probability that  $p_1 - p_2$  is less than zero. To find this probability, we need to transform the random variable  $(p_1 - p_2)$  into a z-score.

That transformation appears below :

$$z_{p_1 - p_2} = (x - \mu_{p_1 - p_2}) / \sigma_d = (0 - 0.05) / 0.0706 = -0.7082 = \text{approx to 2 decimals} = -0.71$$

Using Table for Standard Normal Distribution, we find

Table of Standard Normal Probabilities,  $P(Z < z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451

$$P(z \leq -0.7082) = 0.2389 = 0.24 \text{ (upto 2 decimals)}$$

the probability of a z-score being  $-0.7082 = 0.71$  (upto 2 decimals) or less is 0.24

Therefore, the probability that the survey will show a greater percentage of Republican voters in the second state than in the first state is 0.24.