Infimum and Supremum

One of the math concepts that continue to confound me are that of the *supremum* and *infimum*. Why do we have to complicate math discourse when we have the conventional *maxima* and *minima* that have been adsorbed into the minds of every high schooler ever (Hardy 1992)? Is it because mathematicians are inherently elitist, egotistical, painfully do-hard blue-bloods? Well, no.

Set theoretic definition

Both the supremum and infimum talk about a boundary condition. An explicitly vanilla definition is as follows. Let's say both of them are defined on a bounded set $\mathcal{A}$.

Supremum. This is the minimum of all possible upper bounds. Note that there can be many upper bounds for a given set.

Infimum. This is the maximum of all possible minimums. Again there can be many minimums for a given bounded set.

For example, consider the set in $\mathcal{Z}$, $\mathcal{A} = \{5, 10, 15\}$. Its upper bounds are $15, 16, ...\infty$ and its lower bounds are $-\infty, ..., 3, 4, 5$. (Note that the equality holds). In this case, the supremum (minimum of the upper bounds) is 15 and the infimum (maximum of the lower bounds) is 5.

An alternate characterisation

If $\mathcal{A} \subset \mathbb{R}$

Web references

- The supremum and infimum in the context of a Reimann Integral

Bibliography

Hardy, Godfrey Harold. 1992. *A Mathematician's Apology*. Cambridge University Press.