Assignment - 2

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1) $y' = \lambda y \qquad \lambda < 0$

$$u_{n+1} = u_n + \frac{h}{4}(k_1 + 3k_1)$$

$$k_1 = f(t_n, u_n)$$
$$k_2 = f\left(t_n + \frac{h}{3}, u_n + \frac{h}{3}(k_1 + k_2)\right)$$

Sol. Test eq^n :- $y' = \lambda y \quad , \quad \lambda < 0$

$$\therefore f(x,y) = \lambda y \Rightarrow f(x,u) = \lambda u$$

from ② $\quad k_1 = \lambda u_n$

from ③ $\quad k_2 = \lambda\left(u_n + \frac{h}{3}(k_1 + k_2)\right)$

$$k_2 = \lambda\left(u_n + \frac{h}{3}(\lambda u_n + k_2)\right)$$

$$k_2\left(1 - \frac{\lambda h}{3}\right) = \lambda u_n + \frac{\lambda^2 h}{3} u_n$$

$$k_2 = \frac{\left[\lambda u_n + \frac{\lambda^2}{3} h u_n\right]}{\left(1 - \frac{\lambda h}{3}\right)}$$

$$u_{n+1} = u_n + \frac{h}{4}\left(\lambda u_n + 3\left(\frac{\lambda u_n + \frac{\lambda^2 h}{3} u_n}{\left(1 - \frac{\lambda h}{3}\right)}\right)\right)$$

$$u_{n+1} = u_n E(\lambda h)$$

$$E(\lambda h) \Rightarrow \frac{\left(1 + \frac{\lambda h}{4}\right)\left(1 - \frac{\lambda h}{3}\right) + \frac{3\lambda h}{4} + \left(\frac{\lambda h}{4}\right)^2}{1 - \frac{\lambda h}{3}}$$

$$E(\lambda h) = \frac{1 + \frac{\lambda h}{4} + \frac{3\lambda h}{4} - \frac{\lambda h}{3} - \frac{(\lambda h)^2}{12} + \frac{(\lambda h)^2}{4}}{1 - \lambda h/3}$$

$$E(\lambda h) = \frac{1 + \frac{2}{3}\lambda h + \frac{(\lambda h)^2}{6}}{1 - \lambda h/3}$$

for Absolute stability, $|E(\lambda h)| \leq 1$

$$\left| \frac{1 + \frac{2\lambda h}{3} + \frac{(\lambda h)^2}{6}}{1 - \lambda h/3} \right| \leq 1$$

$$\left(\frac{1 + \frac{2\lambda h}{3} + \frac{\lambda^2 h^2}{6}}{1 + \frac{\lambda h}{3}} - 1 \right) \left(\frac{1 + \frac{2\lambda h}{3} + \frac{\lambda^2 h^2}{6}}{1 + \frac{\lambda h}{3}} + 1 \right) \leq 0$$

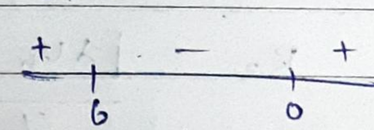
$$\left(\lambda h + \left(\frac{\lambda^2 h^2}{6} \right) \right) \left(2 + \frac{\lambda h}{3} + \frac{(\lambda h)^2}{6} \right) \leq 0$$

$$\lambda h \left(1 + \frac{\lambda h}{6} \right) \left(12 + 2\lambda h + \lambda^2 h^2 \right) \leq 0$$

$D = 4 - 48 < 0 \rightarrow$ always +ve

$a = 1 > 0$

$$\lambda h \left(1 + \frac{\lambda h}{6} \right) \leq 0$$



$\lambda h \in [-6, 0]$ region of stability

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(2) $u_{n+1} = u_n + \frac{1}{4}(3k_1 + k_2)$ — (1)

$k_1 = hf(t_n + h/3,\ u_n + k_1/3)$ — (2)

$k_2 = hf(t_n + h,\ u_n + k_1)$ — (3)

Test eq^n = $y' = \lambda y$

$f(x,y) = \lambda y \Rightarrow f(t,u) = \lambda u$

$k_1 = h\lambda(u_n + k_1/3)$

$k_1\left(1 - \frac{\lambda h}{3}\right) = \lambda h u_n \Rightarrow k_1 = \frac{\lambda h}{\left(1-\frac{\lambda h}{3}\right)} u_n$

Also, $k_2 = (u_n + k_1)h\lambda$

$k_2 = u_n\left(h\lambda + \frac{(h\lambda)^2}{1-\lambda h/3}\right)$

$k_2 = u_n\left(\frac{h\lambda + 2/3 h^2\lambda^2}{1-\frac{h\lambda}{3}}\right)$

$u_{n+1} = u_n + 1/4\left(\frac{3\lambda h}{1-\lambda h/3}\right)u_n + \frac{1}{4}\left(\frac{\lambda h + 2/3(\lambda h)^2}{1-\lambda h/3}\right)u_n$

for absolute stability $|E(\lambda h)| \leq 1$

$\left(\frac{1 + \frac{2\lambda h}{3} + \frac{\lambda^2h^2}{6}}{1-\lambda h/3}\right)^2 - 1^2 \leq 0$

$\left(\frac{1+\frac{2}{3}\lambda h + \frac{\lambda^2h^2}{6}}{1-\lambda h/3} - 1\right)\left(\frac{1+\frac{2}{3}\lambda h + \frac{\lambda^2h^2}{6}}{1-\frac{\lambda h}{3}} + 1\right) \leq 0$

$\lambda h\left(1 + \frac{\lambda h}{6}\right)(12 + 2\lambda h + \lambda^2h^2) \leq 0$

$D = 4 - 48 < 0$

$a = 1 > 0$

$\lambda h\left(1 + \frac{\lambda h}{6}\right) \leq 0 \qquad \lambda h \in [-6, 0]$

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$y' = t^2 + y^2 \qquad y(1.0) = 2, \ h = 0.1$

$y(1.1) = \ldots$

$y_{n+1} = y_n + \frac{1}{4}(3k_1 + k_2)$

$k_1 = hf(t_n + h/3, \ y_n + k_1/3)$

$k_2 = hf(t_n + h, \ y_n + k_1)$

Iteration - 1 : $n = 0, \ t_0 = 1, \ y_0 = 2$

$y_1 = y_0 + \frac{1}{4}(3k_1 + k_2)$

$k_1 = 0.1 f(t_0 + h/3, \ y_0 + k_1/3)$

$k_1 = 0.1 f(1 + 0.1/3, \ 2 + k_1/3)$

$k_1 = 0.1 \left(\left(1 + \frac{0.1}{3}\right)^2 + \left(2 + \frac{k_1}{3}\right)^2 \right)$

$10 k_1 = 1.067 + 4 + \frac{k_1^2}{9} + \frac{4k_1}{3}$

$\frac{k_1^2}{9} - \frac{26k_1}{3} + 5.067 = 0$

Using N-R method

$g(k_1) = \frac{k_1^2}{9} - \frac{26k_1}{3} + 5.067$

$g'(k_1) = \frac{2}{9}k_1 - \frac{26}{3}$

$(k_1)_{n+1} = (k_1)_n - \frac{g(k_1)_n}{g'(k_1)_n}$

$(k_1)_1 = (k_1)_0 - \frac{g(k_1)_0}{g'(k_1)_0} = 0 - \frac{5.067}{-26/3} = 0.65512819$

$(k_2)_2 = (k_1)_1 - \frac{g(k_1)_1}{g'(k_1)_1} = 0.65512819 - (-0.56231184\ldots)$

$= 0.8521028$

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$(k_1)_2 = 0.58913763$

$(k_1)_3 = 0.58913763 - \frac{g(k_1)_2}{g'(k_1)_2} = 0.589194227$

$|(k_1)_3 - (k_1)_2| \leq 10^{-5}$

$k_1 = 0.589194227.$

Now,

$k_2 = 0.1 f(1+0.1,\ 2+0.589194227) = 0.1(1.1^2 + 2.589194227^2)$

$k_2 = 0.791392267.$

$y_1 = y_0 + \frac{1}{4}(3k_1 + k_2) = 2.639743839$

$y(1.1) \simeq y_1 = 2.6397439$

(3) $y' = x + y \quad f(x,y) = x + y$

$x(0) = 0, \quad h = 0.1 \quad y(0) = 1$

Classical RK method

$$y_{n_1} = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$k_1 = 0.1(x_0 + y_0) = 0.1(0+1) = 0.1$

$k_2 = 0.2(0 + 0.1/2, 1 + 0.1/2) = 0.11$

$k_3 = 0.1(0 + 0.1/2, 1 + 0.11/2) = 0.1105$

$k_4 = 0.1(0 + 0.1, 1 + k_3) = 0.12105$

$y_1 = y_0 + 1/6(0.1 + 2\times0.11 + 2\times0.1105 + 0.12105)$

$y_1 = 1.1103416667 \quad x_1 = 0.1$

Iteration-2

$$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$k_1 = 0.1(x_1 + y_1) = 0.121034167$

$k_2 = 0.1\left(0.1 + 0.1/2 + 1.110341667 + \frac{0.121034167}{2}\right) = 0.13208587 5$

$k_3 = 0.1\left(0.1 + 0.1/2 + 1.110341667 + \frac{0.132085875}{2}\right)$

$= 0.132638460417$

$k_4 = 0.1(0.1 + 0.1 + 1.110341667 + 0.132638460417)$

$k_4 = 0.1442980127$

$y_2 = y_1 + \frac{1}{6}(0.121034167 + 2(0.132085875) + 0.1442980127$
$+ 2(0.132638460417))$

$y_2 = 1.24280514176$, $x_2 = 0.2$

Iteration - 3 $n=2$

$y_3 = y_2 + 1/6(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = 0.1(x_2 + y_2) = 0.144280514176$

$k_2 = 0.1\left(0.2 + \frac{0.1}{2} + 1.24280514176 + \frac{0.144280514176}{2}\right)$

$k_2 = 0.15649453985$

$k_3 = 0.1\left(0.2 + \frac{0.1}{2} + 1.24280514176 + \frac{0.15649453985}{2}\right)$

$k_3 = 0.157105241 17$

$k_4 = 0.1(0.2 + 0.1 + 1.24280514176 + 0.15710524117)$

$k_4 = 0.1699910382 93$

$y_3 = y_2 + 1/6(0.144280514176 + 2(0.15694539885) +$
$0.169991038293 + 2(0.15710524117))$

$y_3 = 1.39971699419$, $x_3 = 0.3$

	x_i	y_i	$f_i = x_i + y_i$
$i=0$	0	1	1
$i=1$	0.1	1.110341667	1.210341667
$i=2$	0.2	1.24280574176	1.44280514176
$i=3$	0.3	1.39971699419	1.69971699419

Predictor $y_4^P = y_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3)$

$$y_4^P = 1 + \frac{4}{3}(0.1)\left(2(1.210341667) - 1.44280514176 + 2(1.6997169919)\right)$$

$$y_4^P = 1.5836416 2408$$

Corrector $y_4^C = y_2 + \frac{h}{3}(f_2 + 4f_3 + f_4)$

$$f_4 = x_4 + y_4^P = 0.4 + 1.58364162408$$

$$= 1.98364162408$$

$$y_4^C = 1.24280514176 + \frac{0.1}{3}\left(1.44280514176 + 4(1.699716994) + 1.98364162408\right)$$

$$\boxed{y(0.4) \approx y(4) = 1.58364896651}$$

(4) $y' = x^3 - y^2 - 2 \quad x = 0, y = 1$

Taylor series

$$y(x+h) = y(x) + hy'(x) + \frac{h^2}{2!}(y''(x)) + \frac{h^3}{3!}(y'''(x))$$

$$y'(x) = x^3 - y^2 - 2$$

$$y''(x) = 3x^2 - 2yy' \qquad y'''(x) = 6x - 2(y'^2 + yy'')$$

$$y_0' = -3, \; y_0'' = 6 \qquad y_0''' = -30$$

$$y_1 = 1 + 0.1(-3) + \frac{0.1^2}{2}(6) + \frac{(0.1)^3}{6}(-30) = 0.725$$

$$y_1' = -2.524625$$

$$y_1'' = 3.69070625 \qquad y_1''' = -17.0498968$$

$$y_2 = 0.725 + (0.1)y_1' + \frac{(0.1)^2}{2}y_1'' + \frac{(0.1)^3}{6}y_1''' = 0.488074533 \text{ at } x_2 = 0.2$$

$$y_{-1} = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' \quad (\text{where } h = 0.1)$$

$$y_{-1} = 1.335 \quad \text{at} \quad x_{-1} = -0.1$$

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i	x_i	y_i	f_i
-1	-0.1	1.335	-3.783225
0	0	1	-3
1	0.1	0.725	-2.524625
2	0.2	0.488074533	-2.23021675

Predictor $y_3^P = y_{-1} + \frac{4(0.1)}{3}(2f_0 - f_1 + 2f_2) = 0.2768922$

$f_3^P = -2.048713735$

Corrector : $y_3^c = y_1 + h/3(f_1 + 4f_2 + f_3)$

$= 0.2768921 = 0.27516129$

$y(0.3) \simeq y_3 = 0.275193142 12$

Predictor $= y_4^P = y_0 + 4h/3(2f_1 - f_2 + 2f_3) = 0.0778000626$

Corrector :- $y_4 = y_1 + h/3(f_1 + 4f_2 + f_3)$

$y_4 = 0.0758342 1$; $y_4 = 0.075844776$

$y(0.4) \simeq y_4 = 0.075844776$

Predictor : $y_5^P = y_1 + 4h/3(2f_2 - f_3 + 2f_4) = 0.114360946$

Corrector : $y_5 = y_2 + h/2(f_2 + 4f_3 + f_4)$

$y_5 = -0.114934172$ $y_5 = -0.114938553$

$y(0.5) \simeq y_5 = -0.114938552$

Predictor : $y_6 = y_2 + 4h/3(2f_3 - f_4 + 2f_5) = -0.3028763789$

Corrector : $y_6^{(c)} = y_3 + h/3(f_3 + 4f_4 + f_5)$

$y_6^{c(1)} = -0.303166224$, $y_6^{ci} = -0.30317207951$

$y(0.6) \simeq y_6 = -0.30317207951$

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(5) $\frac{dy}{dx} = y + x, \quad y(0) = 1 \qquad h = 0.1$

Milne Simpson method of order 4.

$y_{i+1} = y_{i-1} + \frac{h}{3}(f_{i+1} + 4f_i + f_{(-1)})$

Fourth Order taylor series

$y_1 = y_0 + hy_0' + \frac{h^2}{2}y''(0) + \frac{h^3}{6}y'''0 + \frac{h^4}{4}y''''0$

$= 1 + 0.1 + 0.01 + 0.0003333 + 0.0000083$

$y(0.1) \cong y_1 = 1.1103416$

Now, $y_{i+2} = y_i + \frac{h}{3}(f_{i+2} + 4f_{i+1} + f_i)$

$\frac{29}{30}y_{i+2} = y_i + \frac{1}{30}(x_{i+2} + 4x_{i+1} + 4f_{i+1} + y_i)$

Iteration-1 $\quad i = 0$

$\frac{29}{30}y_2 = y_0 + \frac{1}{30}(x_2 + 4x_1 + x_0 + 4y_1 + y_0)$

$= 1 + \frac{1}{30}(0.6 + 4(1.1103416) + 1) = 1.2013789$

$y(0.2) \cong 4\,y_2 = 1.2428057$

Iteration-2 $\quad i = 1$

$\frac{29}{30}y_3 = y_1 + \frac{1}{30}(x_3 + 4x_2 + x_1 + 4y_2 + y_1)$

$= 1.3530604$

$y(0.3) \cong y_3 = 1.399717655$

(6) $y' = 2 - xy^2$, $y(0) = 10$, $h = 0.2$

Classical R-K method

$y_{n+1} = y_n + \frac{1}{6}(K_1 + 2k_2 + 2k_3 + k_4)$

$K_1 = hf(x_n, y_n)$

$K_2 = hf(x_n + h/2, y_n + k_1/2)$

$K_3 = hf(x_n + h/2, y_n + k_2/2)$

$K_4 = hf(x_n + h, y_n + k_3)$

Iteration 1 :- $n = 0$

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = 0.4$

$k_2 = -1.6808$

$k_3 = (0.2)(2 - (0.1)(10 - \frac{1.608}{2})^2) = -1.2779654432$

$K_4 = (0.2)(2 - (0.2)(10 - 1.2779654432)) = -2642935432$

$y_1 = y_0 + 1/6(k_1 + 2k_2 + 2k_3 + k_4) = 8.6399189402$

Iteration 2 :- $n = 1$

$k_1 = 0.2(2 - 0.2(y_1)^2) = -2.2859279138$

$k_2 = 0.2(2 - 0.3(y_1 + k_1/2)^2) - 2.835664826$

$k_3 = 2.7283121873$

$k_4 = -2.395862138$

$y_2 = y_1 + 1/6(k_1 + 2k_2 + 2k_3 + k_4) = 5953994917332$

Iteration 3 :- $n = 2$

$K_1 = 0.2(2 - 0.4(y_2)^2) = -2.43600448804$

$k_2 = 0.2(2 - 0.5(y_2 + k_1/2)^2) = -1.842862683$

$k_3 = 0.2(2 - 0.5(y_2 + k_3)^2) = -1.3523494$

$y_3 = y_2 + \frac{1}{6}(k + 2k_2 + 2k_3 + k_4) = 3.867363637$

at $x_3 = 0.6$

Iteration - 4. $n = 3$

$k_1 = 0.2(2 - (0.6)\, y_3^2) = -1.394780 18167$

$k_2 = 0.2(2 - 0.7(y_3 + k_1/2)^2) = -1.0082257$

$k_3 = 0.2(2 - 0.7(y_3 + k_2/2)^2) = 1.842645$

$k_4 = -0.75184328$

$y_4 = y_3 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 2.779230 69231$

Predictor : $y_5^{(P)} = y_1 + \frac{4h}{3}(2f_2 - f_3 + 2f_4)$

$= 8.639918 9402 + 0.8/3 \,(2\times(-12.180022\,1102) - (6.97390090836) + 2(-4.179298592\ldots))$

$y_5^P = 1.77465476 \Rightarrow f_5 = -1.1493995\,3423$

Corrector : $y_5^P = y_1 + \frac{4h}{3}(2f_2 - f_3 + 2f_4)$

$= 8.63991894 02 + 0.8/3\,(-6.97390090836)$

$+ 4(-4.179298542 86) - 1.1493995 3423$

$y_5^{(c)} = 2.21133064608 \qquad f_5^{(c)} = -2.88$

$y_5' = 2.09529 1733 \qquad f_5' = -2.390247$

$y_5^{2c} = 2.12807451 \qquad f_c^2 = -2.530969$

$y(1) \approx y_5 = 2.11922596942$

(7) $y' = -y \qquad y(0) = 1 \qquad x_0 = 0 \qquad y_0 = 1 \qquad h = 0.1$

Classical RK method

$y_{n+1} = y_n + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = h f(x_n, y_n)$

$k_2 = h f(x_n + h/2,\ y_n + k_1/2)$

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$k_3 = hf(x_n + h/2,\ y_n + k_2/2)$
$k_4 = hf(x_n + h,\ y_n + k_3)$

Iteration - 1 $n=0$

$y_{n+1} = y_n + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$
$k_1 = hf(x_n, y_n)$
$k_2 = hf(x_n + h/2,\ y_n + k_1/2)$
$k_3 = hf(x_n + h/2,\ y_n + k_2/2)$
$k_4 = hf(x_n + h,\ y_n + k_3)$

Iteration - 1 $n = 0.1$
$x_1 = 0.1(-1) = -0.1$
$x_2 = 0.1\left(-\left(1 - \frac{0.1}{2}\right)\right) = -0.095$
$x_3 = 0.1\,(-(1 - 0.095/2)) = -0.09525$
$x_4 = 0.1\,(-(1 - 0.09525)) = -0.090475$
$y_1 = y_0 + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4) = 0.9048375$

Iteration - 2 $n = 1$
$k_1 = (0.1)(-y_1) = -0.09048375$
$k_2 = 0.1\,(-(y_1 + k_1/2)) = -0.08618577$
$k_4 = 0.1\,(-(y_1 + k_3)) = -0.081865772$

Iteration - 3, $n = 2$
$k_1 = 0.1(-y_2) = -0.081873023$
$k_2 = 0.1\,(-(y_2 + k_1/2)) = -0.07777942$
$k_3 = 0.1\,(-(y_2 + k_2/2)) = -0.07798412$
$k_4 = 0.1\,(-(y_2 + k_3)) = -0.07407468$
$y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 0.74081844154$
$x_3 = 0.1$

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Iteration - 4 , $n=3$

$$k_1 = 0.1(-y_3) = -0.0740818 4415$$
$$k_2 = 0.1(-(y_3 + k_1/2)) = -0.0703177$$
$$k_3 = 0.1(-(y_3 + k_2/2)) = -0.07056295$$
$$k_4 = 0.1(-(y_3 + k_3)) = -0.0670255$$
$$y_4 = y_3 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 0.670320306597$$

Predictor : $y_5 = y_1 + \frac{4h}{3}(2f_2 - f_3 + 2f_4)$
$y_5 = 0.60653296586 7$, $f_5^{p} = -0.606532965$

Corrector :- $y_5^{c} = y_3 + h/3 (f_3 + 4f_4 + f_5^{p})$
$y(0.5) \simeq y_5 = 0.60653006864 31$

Predictor $y_6(p) = y_2 + 4h/3 (2f_3 - f_4 + 2f_5)$
$= 0.548813862608$ $f_6^{P} = 0.548813862608$

Corrector :- $y_6^{(c)} = y_4 + h/2 (f_4 + 4f_5 + f_6^{P})$
$y(0.6) \simeq y_6 = 0.548811736384$

Predictor $y_7^{P} = y_3 + 4h/3 (2f_4 - f_5 + 2f_6)$
$y_7^{P} = 0.49658732182 7$, $f_7^{P} = -0.49658732182$

Corrector $y_7^{c} = y_5 + h/3 (f_5 + 4f_6 + f_7^{P})$
$y(0.7) \simeq y_7 = 0.4965851880 1$

Predictor :- $y_8^{P} = y_4 + 4h/3 (2f_5 - f_6 + 2f_7)$
$y_8^{P} = 0.441330 16499$, $f_8^{P} = -0.449330 96499$

Corrector :- $y_8^{c} = y_6 + h/3 (f_6 + 4f_7 + f_8^{P}) \Rightarrow y_8 = 0.449328954604$

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(8) $\frac{dy}{dx} = x - y^2$ $y(0) = 1$, $y(0.1) = 0.9117$,
$y(0.2) = 0.8494$, $y(0.3) = 0.8061$

$f(x,y) = x - y^2$

Adam Moulton Method of order 5:

$$y_{n+4} = y_{n+3} + h\left(\frac{251}{720} f_{n+4} + \frac{646}{720} f_{n+3} + \frac{264}{720} f_{n+2} + \frac{106}{720} f_{n+1} - \frac{19}{720} f_n\right)$$

Iteration - 1 $n = 0$

$f_4 = f(0.4, y_4) = 0.4 - y_4^2$
$f_3 = f(0.3, 0.8061) = -0.3497972$
$f_2 = f(0.2, 0.8494) = -0.05214803$
$f_1 = f(0.1, 0.9117) = -0.7311962$
$f_0 = f(0,1) = -1$

$$y_4 = y_3 + 0.1\left(\frac{251}{720} f_4 + \frac{646}{720} f_3 - \frac{264}{720} f_2 \frac{+106}{720} f_1 \frac{-19}{720}\right)$$

Iteration, $n = 0$
$f_4 = f(0.4, y_4) = 0.4 - y_y^2$
$f_2 = f(0.3, 0.8061) = 0.3 - (0.8061)^2 = -0.349772$

$25.1 y_4^2 + 720 y_y - 575.75149535 = 0$
Using NR method
$g(y_4) = 25.1 y_4^2 + 720 y_n - 575.751$
$(y_4) = (y_4)_0 - \frac{g(y_4)_0}{g'(y_4)_0} = 0.799654855$

$$(y_4)_2 = (y_4)_1 - \frac{g(y_4)_1}{g'(y_4)_1} = 0.7785254688$$

$$(y_4)_3 = (y_4)_2 - g(y_4)_1 / g'(y_4)_2 = 0.7785254688$$

since $|(y_4)_3 - (y_4)_2| \leq 10^{-5}$

$$y(0.4) = y_4 = 0.7785254688$$

(9) $\frac{dy}{dx} = \frac{xy}{2}$ $y(0) = 1$, $y(0.1) = 1.0025$ $y(0.2) = 1.0101$

$f(x,y) = \frac{xy}{2}$ $y(0.3) = 1.0228$ $h = 0.1$

Adam Bashforth of order 4

$$y_{n+1} = y_n + \frac{h}{24}(55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3})$$

Iteration -1 :- $n = 3$

$y_4 = ?$ $f(0) = f(0, 1) = 0$

$f(1) = f(0.1, 1.0025) = 0.05012$

$f(2) = f(0.2, 1.0101) = 0.10101$

$f(3) = f(0.3, 1.0228) = 0.15342$

$$y_4 = y_3 + \frac{h}{24}(55 f_3 - 59 f_2 + 37 f_1 - 9 f_0)$$

$$y(0.4) \cong y_4 = 1.040854729$$

(10) $\alpha = 2$ for consistancy of multistep method

$$u_{n+1} = u_n + h/2 (5u'_n + \alpha u'_{n-1})$$

$$u_{n+1} - u_n = h\left(\frac{5}{2}u'_n + \frac{\alpha}{2}u'_{n-1}\right)$$

$\alpha_1 = 1$ $\alpha_0 = -1$, $\alpha_{-1} = 0$ | $\beta_0 = 5/2$ $\beta_{-1} = \alpha/2$ $\beta_1 = 0$

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$C_0 = \sum \alpha_j = 1 + (-1) + 0 = 0$

$C_1 = \sum j\alpha_j - \beta_j = \cancel{2}\ 1 + 0 + 0 - \left(\frac{5}{2} + \frac{\alpha}{2} + 0\right) = 0$

$\boxed{\alpha = -3}$

We can say that numerical method is consistant with diff eq^n if $\tau_j \to 0$ as $n \to 0$

$$T_j = \frac{1}{\beta h} \sum_{p=0}^{\infty} C_p h^p y^p(jh)$$

$$C_p = \sum \left(\frac{j^p}{p!} \alpha_j - \frac{j^{p-1}}{(p-1)!} \beta_j \right)$$

We have

$C_0 = 0$

hence $C_1 = 0$ has to be true for consistancy

Thus $\boxed{\alpha = -3}$