INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

APPLIED COMPUTATIONAL METHODS LABORATORY

LAB 8 REPORT

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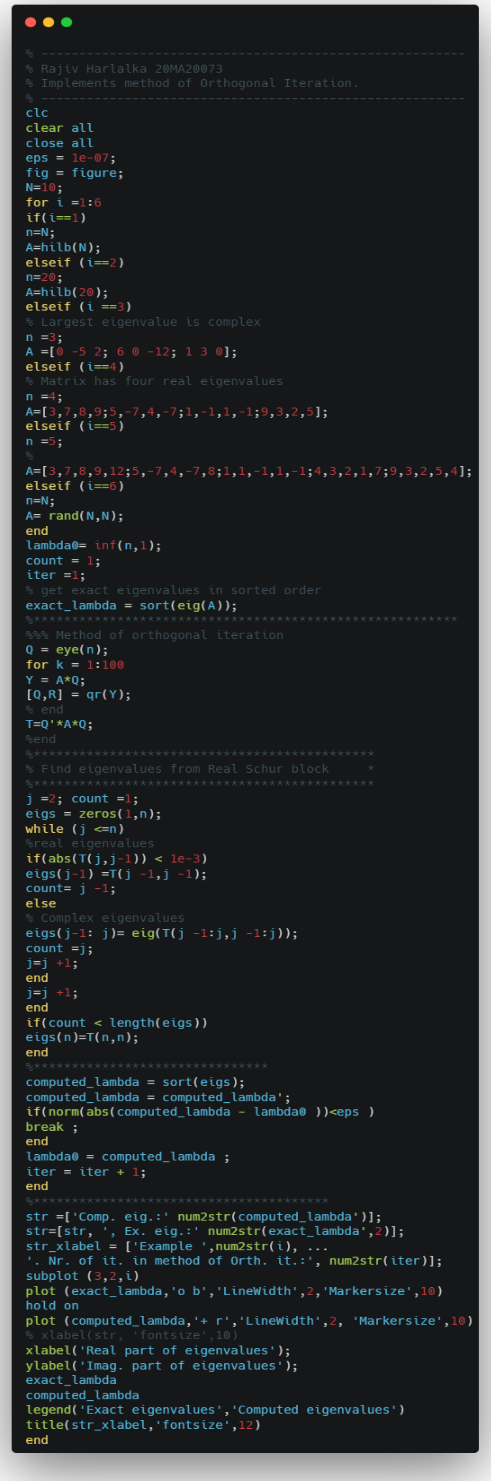
OR ITERATIONS

Codes:



```
% ----------------------------------------------------------
% Rajiv Harlalka 20MA20073
% Implements method of Orthogonal Iteration.
% ----------------------------------------------------------
clc
clear all
close all
eps = 1e-07;
fig = figure;
N=10;
for i =1:6
if(i==1)
n=N;
A=hilb(N);
elseif (i==2)
n=20;
A=hilb(20);
elseif (i ==3)
% Largest eigenvalue is complex
n =3;
A =[0 -5 2; 6 0 -12; 1 3 0];
elseif (i==4)
% Matrix has four real eigenvalues
n =4;
A=[3,7,8,9;5,-7,4,-7;1,-1,1,-1;9,3,2,5];
elseif (i==5)
n =5;
%
A=[3,7,8,9,12;5,-7,4,-7,8;1,1,-1,1,-1;4,3,2,1,7;9,3,2,5,4];
elseif (i==6)
n=N;
A= rand(N,N);
end
lambda0= inf(n,1);
count = 1;
iter =1;
% get exact eigenvalues in sorted order
exact_lambda = sort(eig(A));
%*********************************************************
%%% Method of orthogonal iteration
Q = eye(n);
for k = 1:100
Y = A*Q;
[Q,R] = qr(Y);
% end
T=Q'*A*Q;
%end
%*********************************************
% Find eigenvalues from Real Schur block      *
%*********************************************
j =2; count =1;
eigs = zeros(1,n);
while (j <=n)
%real eigenvalues
if(abs(T(j,j-1)) < 1e-3)
eigs(j-1) =T(j -1,j -1);
count= j -1;
else
% Complex eigenvalues
eigs(j-1: j)= eig(T(j -1:j,j -1:j));
count =j;
j=j +1;
end
j=j +1;
end
if(count < length(eigs))
eigs(n)=T(n,n);
end
%********************************
computed_lambda = sort(eigs);
computed_lambda = computed_lambda';
if(norm(abs(computed_lambda - lambda0 ))<eps )
break ;
end
lambda0 = computed_lambda ;
iter = iter + 1;
end
%****************************************
str =['Comp. eig.:' num2str(computed_lambda')];
str=[str, ', Ex. eig.:' num2str(exact_lambda',2)];
str_xlabel = ['Example ',num2str(i), ...
'. Nr. of it. in method of Orth. it.:', num2str(iter)];
subplot (3,2,i)
plot (exact_lambda,'o b','LineWidth',2,'Markersize',10)
hold on
plot (computed_lambda,'+ r','LineWidth',2, 'Markersize',10)
% xlabel(str, 'fontsize',10)
xlabel('Real part of eigenvalues');
ylabel('Imag. part of eigenvalues');
exact_lambda
computed_lambda
legend('Exact eigenvalues','Computed eigenvalues')
title(str_xlabel,'fontsize',12)
end
```

OUTPUT PLOT:

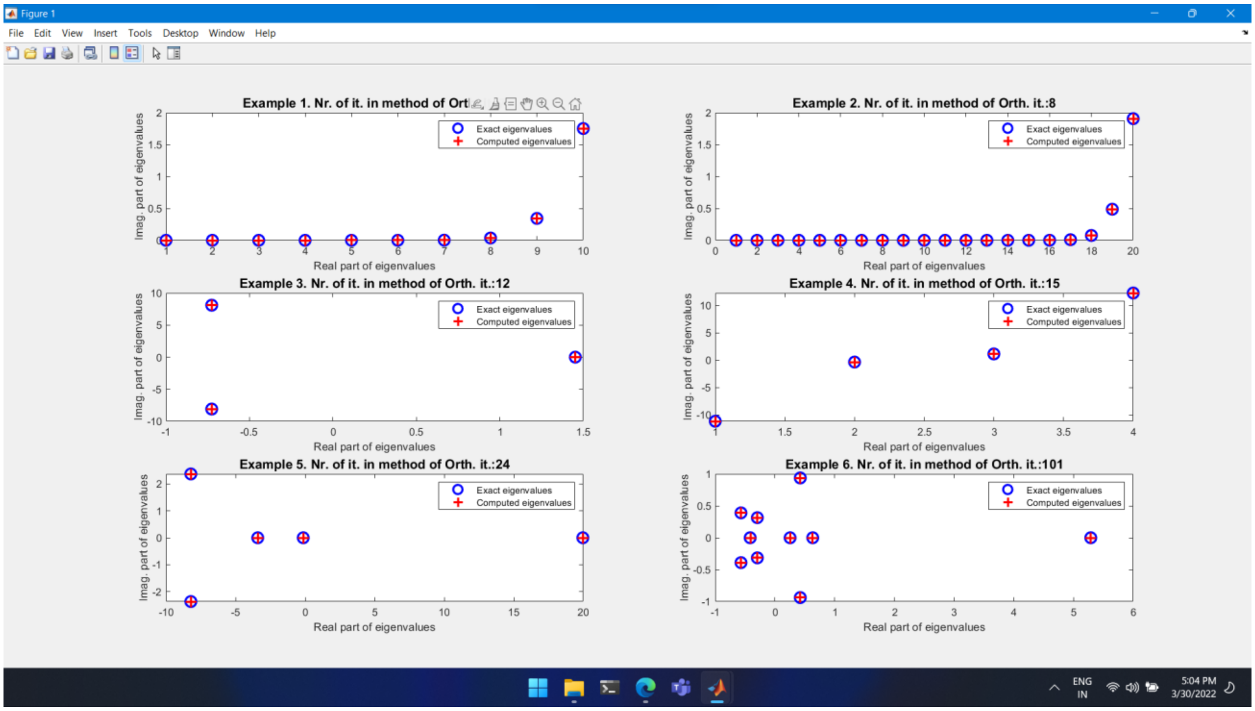


Figure 1
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Example 1. Nr. of it. in method of Ort
Exact eigenvalues
Computed eigenvalues
Imag. part of eigenvalues
Real part of eigenvalues
2
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9 10
Example 2. Nr. of it. in method of Orth. it.:8
Exact eigenvalues
Computed eigenvalues
Imag. part of eigenvalues
Real part of eigenvalues
2
1.5
1
0.5
0
0 2 4 6 8 10 12 14 16 18 20
Example 3. Nr. of it. in method of Orth. it.:12
Exact eigenvalues
Computed eigenvalues
Imag. part of eigenvalues
Real part of eigenvalues
10
5
0
-5
-10
-1 -0.5 0 0.5 1 1.5
Example 4. Nr. of it. in method of Orth. it.:15
Exact eigenvalues
Computed eigenvalues
Imag. part of eigenvalues
Real part of eigenvalues
10
5
0
-5
-10
1 1.5 2 2.5 3 3.5 4
Example 5. Nr. of it. in method of Orth. it.:24
Exact eigenvalues
Computed eigenvalues
Imag. part of eigenvalues
Real part of eigenvalues
2
1
0
-1
-2
-10 -5 0 5 10 15 20
Example 6. Nr. of it. in method of Orth. it.:101
Exact eigenvalues
Computed eigenvalues
Imag. part of eigenvalues
Real part of eigenvalues
1
0.5
0
-0.5
-1
-1 0 1 2 3 4 5 6
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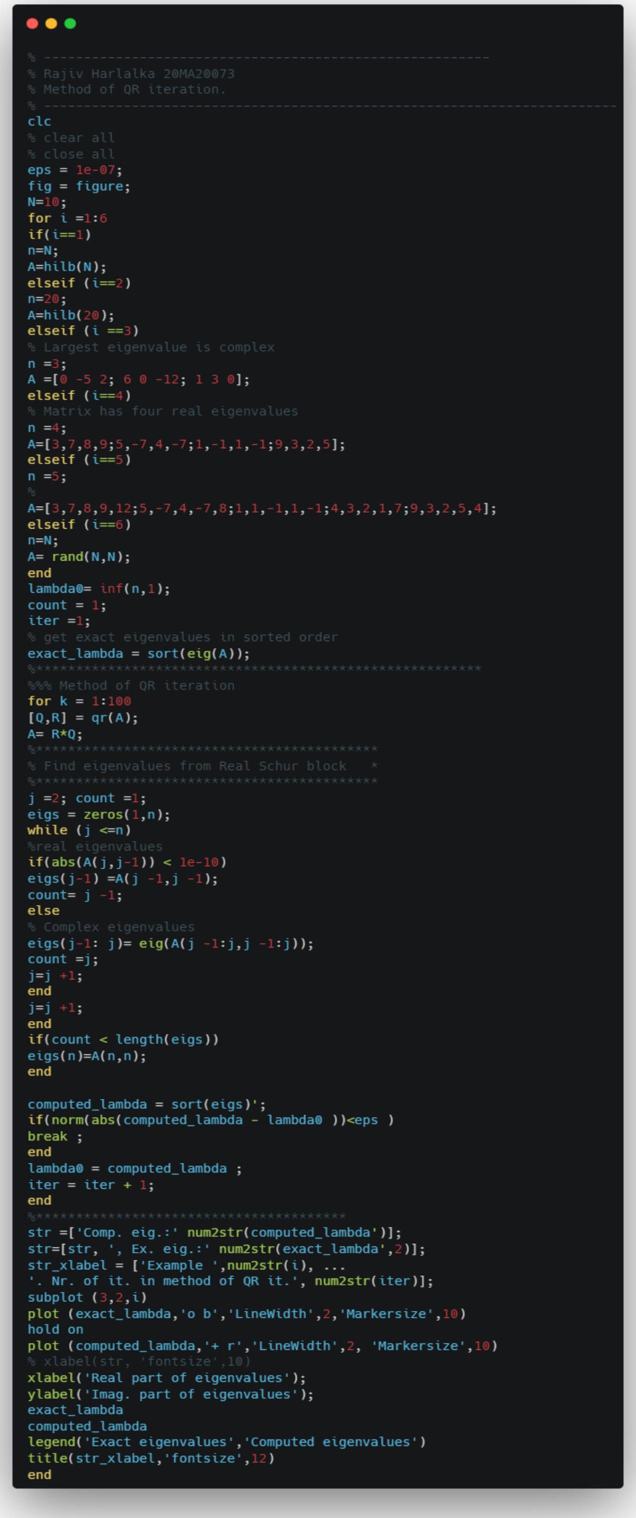
QR ITERATIONS

Code:



```
% -------------------------------------------------------
% Rajiv Harlalka 20MA20073
% Method of QR iteration.
% -----------------------------------------------------------------------
clc
% clear all
% close all
eps = 1e-07;
fig = figure;
N=10;
for i =1:6
if(i==1)
n=N;
A=hilb(N);
elseif (i==2)
n=20;
A=hilb(20);
elseif (i ==3)
% Largest eigenvalue is complex
n =3;
A =[0 -5 2; 6 0 -12; 1 3 0];
elseif (i==4)
% Matrix has four real eigenvalues
n =4;
A=[3,7,8,9;5,-7,4,-7;1,-1,1,-1;9,3,2,5];
elseif (i==5)
n =5;
%
A=[3,7,8,9,12;5,-7,4,-7,8;1,1,-1,1,-1;4,3,2,1,7;9,3,2,5,4];
elseif (i==6)
n=N;
A= rand(N,N);
end
lambda0= inf(n,1);
count = 1;
iter =1;
% get exact eigenvalues in sorted order
exact_lambda = sort(eig(A));
%*******************************************************
%%% Method of QR iteration
for k = 1:100
[Q,R] = qr(A);
A= R*Q;
%*******************************************
% Find eigenvalues from Real Schur block   *
%*******************************************
j =2; count =1;
eigs = zeros(1,n);
while (j <=n)
%real eigenvalues
if(abs(A(j,j-1)) < 1e-10)
eigs(j-1) =A(j -1,j -1);
count= j -1;
else
% Complex eigenvalues
eigs(j-1: j)= eig(A(j -1:j,j -1:j));
count =j;
j=j +1;
end
j=j +1;
end
if(count < length(eigs))
eigs(n)=A(n,n);
end

computed_lambda = sort(eigs)';
if(norm(abs(computed_lambda - lambda0 ))<eps )
break ;
end
lambda0 = computed_lambda ;
iter = iter + 1;
end
%***************************************
str =['Comp. eig.:' num2str(computed_lambda')];
str=[str, ', Ex. eig.:' num2str(exact_lambda',2)];
str_xlabel = ['Example ',num2str(i), ...
'. Nr. of it. in method of QR it.', num2str(iter)];
subplot (3,2,i)
plot (exact_lambda,'o b','LineWidth',2,'Markersize',10)
hold on
plot (computed_lambda,'+ r','LineWidth',2, 'Markersize',10)
% xlabel(str, 'fontsize',10)
xlabel('Real part of eigenvalues');
ylabel('Imag. part of eigenvalues');
exact_lambda
computed_lambda
legend('Exact eigenvalues','Computed eigenvalues')
title(str_xlabel,'fontsize',12)
end
```

OUTPUT PLOT

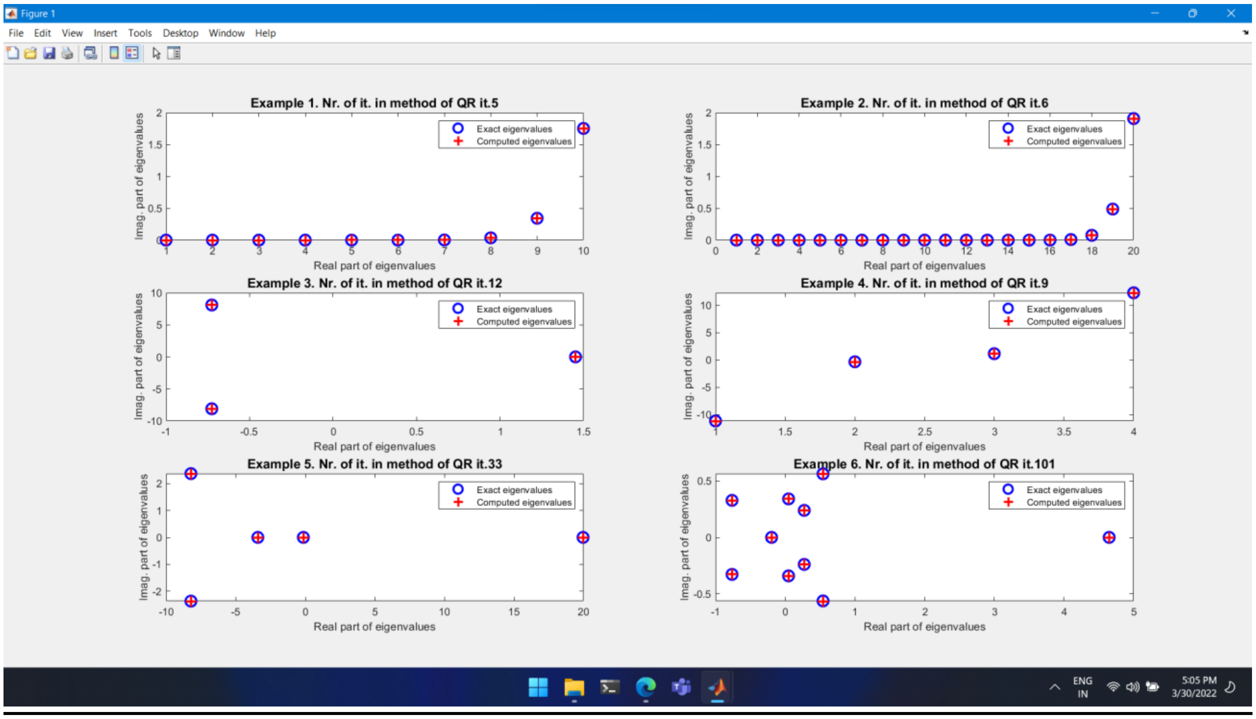


Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Example 1. Nr. of it. in method of QR it.5
Example 2. Nr. of it. in method of QR it.6
Example 3. Nr. of it. in method of QR it.12
Example 4. Nr. of it. in method of QR it.9
Example 5. Nr. of it. in method of QR it.33
Example 6. Nr. of it. in method of QR it.101
Exact eigenvalues
Computed eigenvalues
Real part of eigenvalues
Imag. part of eigenvalues
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12
14
16
18
20
2
1.5
1
0.5
0
10
5
0
-5
-10
-1
-0.5
0
0.5
1
1.5
10
5
0
-5
-10
1
1.5
2
2.5
3
3.5
4
2
1
0
-1
-2
-10
-5
0
5
10
15
20
0.5
0
-0.5
-1
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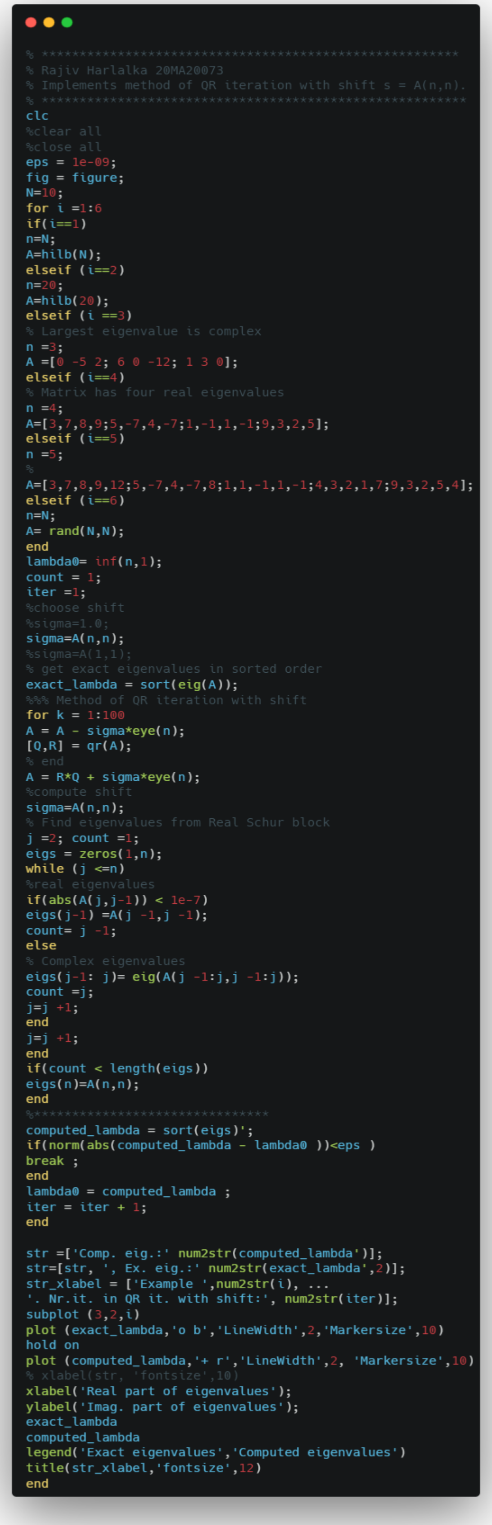
QR ITERATIOSN WITH SHIFTS

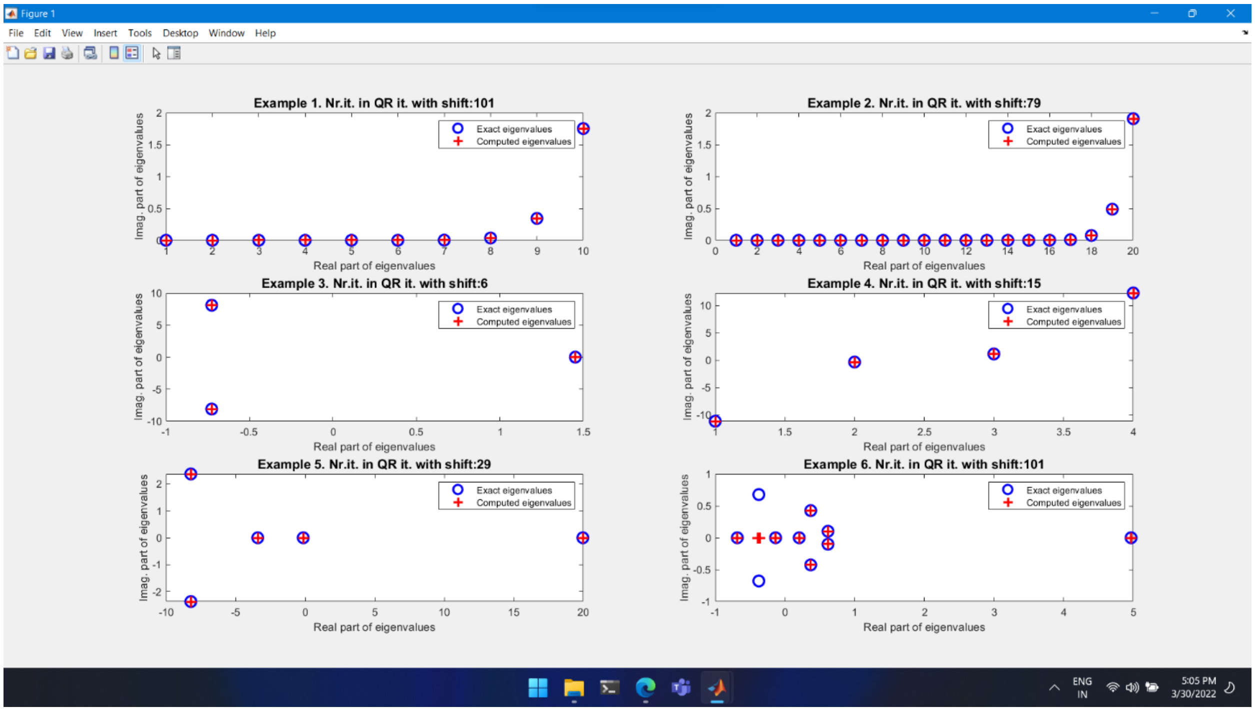
Codes:



```
% *******************************************************
% Rajiv Harlalka 20MA20073
% Implements method of QR iteration with shift s = A(n,n).
% ********************************************************
clc
%clear all
%close all
eps = 1e-09;
fig = figure;
N=10;
for i =1:6
if(i==1)
n=N;
A=hilb(N);
elseif (i==2)
n=20;
A=hilb(20);
elseif (i ==3)
% Largest eigenvalue is complex
n =3;
A =[0 -5 2; 6 0 -12; 1 3 0];
elseif (i==4)
% Matrix has four real eigenvalues
n =4;
A=[3,7,8,9;5,-7,4,-7;1,-1,1,-1;9,3,2,5];
elseif (i==5)
n =5;
%
A=[3,7,8,9,12;5,-7,4,-7,8;1,1,-1,1,-1;4,3,2,1,7;9,3,2,5,4];
elseif (i==6)
n=N;
A= rand(N,N);
end
lambda0= inf(n,1);
count = 1;
iter =1;
%choose shift
%sigma=1.0;
sigma=A(n,n);
%sigma=A(1,1);
% get exact eigenvalues in sorted order
exact_lambda = sort(eig(A));
%%% Method of QR iteration with shift
for k = 1:100
A = A - sigma*eye(n);
[Q,R] = qr(A);
% end
A = R*Q + sigma*eye(n);
%compute shift
sigma=A(n,n);
% Find eigenvalues from Real Schur block
j =2; count =1;
eigs = zeros(1,n);
while (j <=n)
%real eigenvalues
if(abs(A(j,j-1)) < 1e-7)
eigs(j-1) =A(j -1,j -1);
count= j -1;
else
% Complex eigenvalues
eigs(j-1: j)= eig(A(j -1:j,j -1:j));
count =j;
j=j +1;
end
j=j +1;
end
if(count < length(eigs))
eigs(n)=A(n,n);
end
%*******************************
computed_lambda = sort(eigs)';
if(norm(abs(computed_lambda - lambda0 ))<eps )
break ;
end
lambda0 = computed_lambda ;
iter = iter + 1;
end

str =['Comp. eig.:' num2str(computed_lambda')];
str=[str, ', Ex. eig.:' num2str(exact_lambda',2)];
str_xlabel = ['Example ',num2str(i), ...
'. Nr.it. in QR it. with shift:', num2str(iter)];
subplot (3,2,i)
plot (exact_lambda,'o b','LineWidth',2,'Markersize',10)
hold on
plot (computed_lambda,'+ r','LineWidth',2, 'Markersize',10)
% xlabel(str, 'fontsize',10)
xlabel('Real part of eigenvalues');
ylabel('Imag. part of eigenvalues');
exact_lambda
computed_lambda
legend('Exact eigenvalues','Computed eigenvalues')
title(str_xlabel,'fontsize',12)
end
```

OUTPUT PLOTS



Example 1. Nr.it. in QR it. with shift:101

Example 2. Nr.it. in QR it. with shift:79

Example 3. Nr.it. in QR it. with shift:6

Example 4. Nr.it. in QR it. with shift:15

Example 5. Nr.it. in QR it. with shift:29

Example 6. Nr.it. in QR it. with shift:101