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Assignment - 4

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① $x^2y'' - 2y + x = 0 \quad y(2) = 0 \quad y(3) = 0$

$y'' = \dfrac{2y - x}{x^2} \qquad h = \dfrac{1}{4} = 0.25$

2nd order taylor's series →

$$y_{j+1} = y_j + hy_j' + \frac{h^2}{2} y_j''$$
$$y_{j+1}' = y_j' + hy_j'' + \frac{h^2}{2} y_j'''$$

$$y''' = \frac{(2y'-1)x^2 - 2x(2y-x)}{x^4} = \frac{2y'x - 4y + x}{x^3}$$

General sol $y(x) = \lambda(y_1(x) + (1-\lambda) y_2(x))$

$x^2 y_1'' - 2y_1 + x = 0 \qquad y_1'(2) = 0 \qquad y_1(2) = 0$
$x^2 y_2'' - 2y_2 + x = 0 \qquad y_2'(2) = 1 \qquad y_2(2) = 0$

Using $y_{j+1} = y_j + hy_j' + \dfrac{h^2}{2}\left(\dfrac{2y_j - x_j}{x_j^2}\right)$

& $y_{j+1}' = y_j' + h\left(\dfrac{2y_j - x_j}{x_j^2}\right) + \dfrac{h^2}{2!}\left(\dfrac{2y_j' x_j - 4y_j + x_j}{x_j^3}\right)$

$j = 0 \Rightarrow x_0 = 2 \quad y_1(2) = 0 \quad y_1'(2) = 0$
$y_1 = y_1(2.25) = -0.015625$
$y_1' = y_1'(2.25) = -0.1171875$

$j = 1 \Rightarrow x_1 = 2.25 \quad y_1(2.25) = -0.015625$
$y_1'(2.25) = -0.1171875$

$y_1(2.5) = -0.05900366$

$y_1'(2.5) = -0.22494427$

$j = 2 \Rightarrow \quad x_2 = 2.5$

$y_1(2.75) = -0.12832977$

$y_1'(2.75) = -0.3264419796$

$j = 3 \Rightarrow \quad x_3 = 2.75$

$y_1(3) = -22.2364477$

$y_1'(3) = -0.4236299912$

(ii) y for y_2

$y_2(2) = 0 \qquad y_2'(2) = 1$

$y_2(2.25) = 0.234375 \qquad y_2'(2.25) = 0.8984375$

$y_2(2.5) = 0.4477890046 \qquad y_2'(2.5) = 0.825167187$

$y(2.75) = 0.6462606899 \qquad y_2'(2.75) = 0.7706740612$

$y_2(3) = 0.8329065663 \qquad y_2'(3) = 0.7291100265$

$$\lambda = \frac{y(3) - y_2(3)}{y(3) - y_2(3)} = 0.7892821201$$

$$y(x) = 0.7892821201\, y_1(x) + (0.2107178)\, y_2(x)$$

$y(2) = 0 \qquad y(2.25) = 0.0370547$

$y(2.5) \simeq 0.0478275$

$y(2.75) \simeq 0.03489028$

$y(3) = 0$

(2) $y'' = 2y - y' \quad y(1) = 2e + \frac{1}{e^2} \quad y(2) = 2e^2 + \frac{1}{e^4}$

$y''' = 2y' - y''$

$h = \frac{1}{3} \quad [y_x = \lambda y_1(x)$

$y^{iv} = 2y'' - y'''$

$y_{j+1} = y_j + h y_j' + \frac{h^2}{2} y_j'' + \frac{h^3}{6} y_j'''$

$y'_{j+1} = y_j' + h y_j'' + \frac{h^2}{2} y_j''' + \frac{h^3}{6} y_j^{iv}$

$y''' = 2y' - (2y - y') = 3y' - 2y$

$y^{iv} = 2(2y - y') - (3y' - 2y) = 6y - 5y'$

$\therefore y_{j+1} = y_j + h y_j' + \frac{h^2}{2}(2y_j - y_j') + \frac{h^3}{6}(3y_j' - 2y_j)$

$y'_{j+1} = y_j' + h(2y_j - y_j') + \frac{h^2}{2}(3y_j' - 2y_j)$
$+ \frac{h^3}{6}(6y_j - 5y_j')$

taking $j = 0, 1, 2, 3 \ldots$ for y_1

$y_1(1) = 5.57189894 \qquad y_1'(1) = 0$

$y_1(4/3) = 6.1222099464 \qquad y_1'(4/3) = 3.3018660385$

$y_1(5/3) = 7.7052033352 \qquad y_1'(5/3) = 6.2776218510$

$y_1(2) = 10.3262441760 \qquad y_1'(2) = 9.6036442035$

taking $j = 0, 1, 2 \ldots$

$y_2(1) = 5.57189 \qquad y_2'(1) = 0$

$y_2(4/3) = 6.4185067481 \qquad y_2'(4/3) = 4.1043354469$

$y_2(5/3) = 8.2685327322 \qquad y_2'(5/3) = 7.0971620735$

$y_2(2) = 11.1880407769 \qquad y_2'(2) = 10.5951297645$

$$r = \frac{y(2) - y_2(2)}{y_1(2) - y_2(2)} = -4.187066438$$

$$y(x) = -(-4.187066438)\, y_1(x) + (5.187066438)\, y_2(x)$$

$$y(1) = 5.5718989679$$

$$y(4/3) = 7.65958373993$$

$$y(5/3) = 10.6272303439$$

$$y(2) = 14.796427897$$

(5) $y'' = y \quad y'(0) = 3 \quad y'(1) = e + \frac{2}{e} \quad h = 1/4$

$y''' = y' \quad y^{iv} = y'' = y$

$y(x) = \lambda y_1(\alpha) + (1-\alpha) y_2(x)$ general solution

$y_1'' = y_1 \;,\; y_1'(0) = 3 \quad y_1(0) = 0$
$y_2'' = y_1 \;,\; y_2'(0) = 3 \quad y_2(0) = 1$

3rd order taylor's series

$y_{j+1} = y_j + h y_j' + \frac{h^2}{2} y_j'' + \frac{h^3}{6} y_j'''$

$y_{j+1}' = y_j' + h y_j'' + \frac{h^2}{2} y_j''' + \frac{h^2}{6} y_j^{iv}$

$\therefore y_{j+1} = y_j + h y_j' + \frac{h^2}{2} y_j + \frac{h^3}{6} y_j'$

$= \left(1 + \frac{h^2}{2}\right) y_j + \left(h + \frac{h^3}{6}\right) y_j'$

$y'_{j+1} = y_j' + h y_j + \frac{h^2}{2} y_j' + \frac{h^3}{6} y_j$
$= \left(h + \frac{h^3}{6}\right) y_j + \left(1 + \frac{h^2}{2}\right) y_j'$

$$y_{j+1}' = y_j' + h y_j + \frac{h^2}{2} y_j' + \frac{h^3}{6} y_j$$
$$= \left(h + \frac{h^3}{6}\right) y_j + \left(1 + \frac{h^2}{2}\right) y_j'$$

using ① and ②

$y_1(0) = 0$ $\quad y_1'(0) = 3$

$y_1(0.25) = 0.7578125$ $\quad y_1'(0.25) = 3.09375$

$y_1(0.5) = 1.362988 2813$ $\quad y_1'(0.5) = 3.3818562826$

$y_1(0.75) = 2.466102653)$ $\quad y_1'(0.75) = 3.8823566437$

$y_1(1) = 3.52 3867825 7$ $\quad y_1'(1) = 4.6266280964$

111^{rd}

$y_2(0) = 1$ $\quad y_2'(0) = 3$

$y_2(0.25) = 1.7890625$ $\quad y_2'(0.25) = 3.3463541667$

$y_2(0.5) = 2.6902737088$ $\quad y_2'(0.5) = 3.9028523763$

$y_2(0.75) = 3.7602 2153$ $\quad y_2'(0.75) = 4.704390861 4$

$y_2(1) = 5.0660771905$ $\quad y_2'(1) = 5.8012507029$

$$\lambda = \frac{y'(1) - y_2'(1)}{y_1'(1) - y_2'(1)} = 1.998267332$$

$y(x) = \cancel{0.00} \ -0.99826733$

$y(0.25) = -0.2716306916$

$y(0.5) = 0.4376560590$

$y(0.75) = 1.1742260431$

$y(1) = 1.98433058870$

(7) $y'' = xy+1 \quad y(0)+y'(0) = 1$

$y''' = xy' + y \quad h = 0.25$

$y^{iv} = xy'' + 2y' = 2y' + x^2y + x$

general soln $y(x) = \lambda y_1(x) + (1-\lambda) y_2 x$

$y_1'' = xy_1 + 1 \quad y_1(0) = 0 \quad y_1'(0) = 1$

$y_2'' = xy_2 + 1, \quad y_2(0) = 1 \quad y_2'(0) = 0$

3rd order taylor's series

$y_{j+1} = y_j + hy_j' + \frac{h^2}{2} y_j'' + \frac{h^3}{6} y_j'''$

$y_{j+1}' = y_j' + hy_j'' + \frac{h^2}{2} y_j''' + \frac{h^3}{6} y_j^{iv}$

$y_{j+1} = y_j + hy_j' + \frac{h^2}{2}(x_j y_j + 1) + \frac{h^3}{6}(x_j y_j' + y_j)$

$y_{j+1}' = y_j' + h(x_j y_j + 1) + \frac{h^2}{2}(x_j y_j' + y_j) + \frac{h^3}{6}(2y_j' + x_j^2 y_j + x_j)$

by taking $j = 0, 1, 2, 3$

$y_1(0) = 0 \quad y_1'(0) = 1$

$y_1(0.25) = 0.28125 \quad y_1'(0.25) = 1.255208333$

$y_1(0.5) = 0.6300489638 \quad y_1'(0.5) = 1.5486161974$

$y_1(0.75) = 1.0619347080 \quad y_1'(0.75) = 1.9310364569$

$y_1(1) = 1.6073904482 \quad y_1'(1) = 2.4721639205$

III^{rd} for y_2

$y_2(0) = 1 \quad y_2'(0) = 0$

$y_2(0.25) = 1.0338541667 \quad y_2'(0.25) = 0.28125$

$y_2(0.5) = 1.1463690864 \quad y_2(0.5) = 0.6328655249$

$y_2(0.75) = 1.35750218 \quad y_2(0.75) = 1.0770041545$

$y_2(1) = 1.6954600514 \quad y_2(1) \quad 1.6582514971$

$$\lambda = \frac{(b_0 y(b) + b_1 y'(b)) \quad (b_0 y_2(b) + b_1 y_2'(b))}{(b_0 y_1(b) + b_1 y_1'(b)) - b_0 y_2(b) + b_1 y_2'(b))}$$

$$\lambda = 7.89669361$$

$$y(x) = (7.89669361) y_1(x) + 6.89669361 \, y_2(x)$$

$$y(0) = -6.8966936$$
$$y(0.25) = -4.90923034$$
$$y(0.5) = -2.930852726$$
$$y(0.75) = -0.97635835$$
$$y(1) = 1$$

5. $y'' = 6y^2 \quad y(0) = 1 \quad y\left(\frac{3}{10}\right) = \frac{100}{169} \quad h = 0.1$

Non-linear BVP

$y_1'' = 6y_1^2 \quad y_1(0) = 1 \quad y_1'(0) = s^0 = -1.8 = \frac{-9}{5}$

$y_2'' = 6y_2^2 \quad y_2(0) = 1 \quad y_2'(0) = s' = -1.9 = \frac{-19}{10}$

3rd order taylor series

$y_{j+1} = y_j + hy_j' + \frac{h^2}{2} y_j'' + \frac{h^3}{6} y_j'''$

$y_{j+1}' = y_j' + hy_j'' + \frac{h^2}{2} (y_j)''' + \frac{h^3}{6} y_j^{iv}$

using the above

for $j = 0, 1, 2$

$y_1(0.1) \cong 0.8464 \quad y_1'(0.1) = -1.308$

$y_1(0.2) \cong 0.734879 \quad y_1'(0.2) = -0.94459$

$y_1(0.3) = 0.655239 \quad y_1'(0.3) = -0.528913$

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$\phi(s) = y_1(s, x) - y(x)$

$\phi(s) = y_1(s, 0.3) - y(0.3)$

$\phi(s^{(0)}) = 0.063516$

II^{nd} for y_2

$y_2(0.1) = 0.8362$ $\quad y_2'(0.1) = -1.414$

$y_2(0.2) = 0.7132412$ $\quad y_2'(0.2) = -1.065405$

$y_2(0.3) = 0.620620$ $\quad y_2'(0.3) = -0.805635$

$\phi(s') = y_2(s, 0.3) - y(0.3)$

$\Rightarrow \phi(s') = 0.02804$

$s^2 = -1.983504$

III^{rd} for y_3, $y_3'' = 6y_3^2$ $\quad y_3(0) = 1$ $\quad y_3'(0) = s^2$
$= 1.983504$

$y_3(0.1) = 0.835616$ $\quad y_3'(0.1) = -1.502520$

$y_3(0.2) = 0.703801$ $\quad y_3'(0.2) = -1.158899$

$y_3(0.3) = 0.601140$ $\quad y_3'(0.3) = -0.910626$

$\phi(s^2) = y_3(s, 0.3) - y(0.3) = 0.009424$

$$s^3 = s^2 - \left[\frac{s^2 - s'}{\phi(s^2) - \phi(s)}\right]\phi(s^2) = -2.023909$$

exact value of $y'(0) = -2$

finally value of $y'(0) = -2$

by using 3^{rd} order taylor's series

$y(0.1) = 0.823561$

$y(0.2) = 0.686829$

$y(0.3) = 0.577837$

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~~(6) Iteratively~~

(6) $u'' = 2uu'$ $\quad 0<x<1$ $\quad u(0) = 1/2$ $\quad u(1) = 0.5$ $\quad h = 0.25$

$u'(0) = s^0 = 0.09$

using taylor's series of 2^{nd} order

$u_1(0.25) = 0.5253125$ $\quad u'(0.25) = 0.11581873$

$u_1(0.5) = 0.558069752$ $\quad u'(0.5) = 0.151072712$

$u_1(0.75) = 0.6011072498$ $\quad u_1'(0.75) = 0.200534998$

$u(1) = 0.658774939$ $\quad u'(1) = 0.2723773241$

$\phi(s^{(0)}) = 0.6587749395 - 0.5 = 0.1587749395$

here $a_1 = 0$ $\quad a_0 = 1$ $\quad b_1 = 0$ $\quad b_0 = 1$

1^{st} Variational eqn

$v'' = (2u')v + 2uv'$ $\quad v(0) = a_1/a_0 = 0$ $\quad v'(0) = 1$

$v''' = 2u''v + 2u'v' + 2u'v' + 2u(2u'v + 2uv')$

$= 4u'v(2u+1) + 4u^2v'$

$v(0.25) = 0.28125$ $\quad v'(0.25) = 1.28125$

$v(0.5) = 0.5661179077$ $\quad v'(0.5) = 1.32478705$

$v(0.75) = 0.9480113067 2$ $\quad v'(0.75) = 1.807454544$

$v(1) = 1.479663193$ $\quad v'(1) = 2.579715205$

$\phi'(s^{(0)}) = 2.57 9715205$

$$s^{(1)} = s^{(0)} - \frac{\phi(s^{(0)})}{\phi'(s^{(0)})} = 0.028452 53182$$

$s^{(1)} = 0.028452$ $\quad u(0) = 1/2$ $\quad u'(0) = s^{(1)}$

after 1 iteration

$u(0.25) = 0.5080022746$ $\quad u'(0.25) = 0.0365054030 6$

$u(0.5) = 0.5180177767$ $\quad u'(0.5) = 0.0448794+2813$
$u(0.75) = 0.5306906454$ $\quad u'(0.75) = 0.0581358656\,61$
$u(1) = 0.5471525886$ $\quad u'(1) = 0.0751919472$

⑦ $y'' = \frac{3}{2} y^2$ $\quad y(0) = 1$ $\quad y(1) = 4$ $\quad h = 0.25$ $\quad s^0 = 0.9$

IVPS $\Rightarrow$ $u'' = \frac{3u^2}{2}$ $\quad u(0) = 1$ $\quad u'(0) = 0.9$

$v'' = 3uv$ $\quad v(0) = 0$ $\quad v'(0) = 1$

2nd order RK method $\Rightarrow$ $\bar{u}_{j+1} = \bar{u}_j + \frac{h}{2}[\bar{k}_1 + \bar{k}_2]$

$k_{11} = f_1(t_j, u_j, u_{2j})$
$k_{12} = f_1(t_j + h, u_j + h k_{11}, u_j + k_{21})$

$f_1 = u_2$ $\quad f_2 = \frac{3u_1^2}{2}$ $\quad u_1 = u$ $\quad u_2 = u'$

by using the above eqn

$u(0.25) = 1.069375$ $\quad u_1(0.25) = 0.47353324219$
$u(0.5) = 1.2413622616$ $\quad u_1(0.5) = 0.9524671808$
$u(0.75) = 1.551713535$ $\quad u_1(0.75) = 1.65181515$
$u(1) = 2.07753364\,6$ $\quad u_1(1) = 2.827015009$

$\phi(s^{(0)}) = 2.07753364$ $\quad u_1 = v$ $\quad f_1 = u_2$
$u_2 = v'$ $\quad f_2 = 3uv,$

$v(0.25) = 0.25$ $\quad v'(0.25) = 1.09375$
$v(0.5) = 0.5485009766$ $\quad v'(0.5) = 1.40391052\,2$
$v(0.75) = 0.963391\,41$ $\quad v'(0.75) = 2.077960919$
$v(1) = 1.622938157$ $\quad v'(1) = 3.497482172$

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$\phi'(s^{(0)}) = 3.4974821\,72$

$s^{(1)} = s^{(0)} - \dfrac{\phi(s^{(0)})}{\phi'(s^{(0)})} = -0.5040083591$

$s^{(1)} = u'(0) \cdot \quad u(0) = 1$

$u(0.25) = 0.9208729102 \qquad u'(0.25) = -0.173282294$

$u(0.5) = 0.9761258906 \qquad u'(0.5) = 0.4570416432$

$u(1) = 1.120161981 \qquad u'(1) = 0.8586224008$

(8)(1) $y'' = y + x \qquad y(0) = 0 \qquad y(1) = 0$

at $x = 4$

$$\frac{y(x_{j+1}) - 2y(x_j) + y(x_{j-1})}{h^2} = y(x_j) + x_j$$

$$y(x_{j-1}) - (2+h^2)\,y(x_j) + y(x_{j+1}) = h^2 x_j$$

(a) $h = 1/2$

$$y(0) - (2 + 1/4)\cdot y(1/2) + y(1) = 1/4 \times 1/2$$

$$y(1/2) = -1/18$$

(b) $h = 1/3$

$$y(0) - \frac{19}{9} y(1/3) + y(2/3) = 1/27$$

$$y(1/3) - \frac{19}{9} y(2/3) + y(1) = 2/27$$

$$y(1/3) = -\frac{39}{840} = -0.04404761903$$

(c) $h = 1/4$

$$y(0) - \frac{33}{16} y(1/4) + y(1/2) = 1/64$$

$$y(1/4) - \frac{33}{16} y(1/2) + y(3/4) = 1/32$$

$$y(1/2) - \frac{33}{16} y\left(\frac{3}{4}\right) + y(1) = 3/64$$

$y(1/4) = -2657/76164 = -0.03488524762$

$y(1/2) = -65/1154 = -0.05632582322$

$y(3/4) = -3811/76164 = -0.05003676298$

(ii) $x^2y'' = 2y - x \quad y(2) = 0 \quad y(3) = 0$

$$\frac{y(x_{j+1}) - 2y(x_j) + y(x_{j-1})}{h^2} = \frac{2y(x_j) - x_j}{x_j^2}$$

$$y(x_{j+1}) - y(x_j)\left[2 + \frac{2h^2}{x_j^2}\right] = \frac{-h^2}{x_j} + y(x_{j-1})$$

(a) $h = 1/2$

$$y(3) - y(5/2)\left(2 + 4/_{2\times2}\right) + y(2) = -\frac{2}{4\times5} = -1/10$$

$$y(5/2) = 1/10$$

$$y(5/2) = 0.04807692308$$

(b) $h = 1/3$

$$y(2) - y(7/3)\left(2 + \frac{2\times9}{9\times4}\right) + y(8/3) = \frac{-3}{9\times7}$$

$$y(7/3) - y(8/3)\left(2 + \frac{2\times9}{9\times6}\right) + y(3) = \frac{-3}{9\times8}$$

$$y(7/3) = 217/4932 = 0.04399837$$

$$y(8/3) = 52/1233 = 0.04217356042$$

(iii) $y'' - 3y' + 2y = 0 \quad 2y(0) - y'(0) = 1 \quad h = 0.5$

$y(1) + y'(1) = 2e + 3e^2$

$$\frac{y(x_{j+1}) - 2y(x_j) + y(x_{j-1})}{h^2} - 3\frac{y(x_{j+1}) - y(x_{j-1})}{2h} + 2y_j = 0$$

$-y_0' - 1 - 2y_0 = \frac{-(y_1 - y_{-1})}{2 \times 1/2} = 1 - 2y_0$

$\Rightarrow \quad y_{-1} = 1 - 2y_0 + y_1$

$y_2 + y_2' = 2e + 3e^2$

$y_3 = y_1 - y_2 + 2e + 3e^2$

$4y_{j+1} - 8y_j + 4y_{j-1} - 3y_{j+1} + 3y_{j-1} + 2y_j = 0$

$y_{j+1} - 6y_j + 7y_{j-1} = 0$

$j = 0, 1, 2 \Rightarrow y_1 - 6y_0 + 7y_{-1} = y_1 - 6y_0 + 7 - 14y_0 + 7y_1 = 0$

$8y_1 - 20y_0 = -7$

$y_2 - 6y_1 + 7y_0 = 0$

$y_1 - y_2 + 2e + 3e^2 - 6y_1 + 7y_1 = 0$

$8y_1 - 7y_2 = -2e - 3e^2$

$y_0(0) = 1.59315922$

$y(0.5) = 3.10789805$

$y(1) = 7.49527337$

(iv) $y'' = 2yy' \quad y(0) = 1/2 \quad y(1) = 1$

at x_j $\frac{y(x_{j+1}) - 2y(x_j) + y(x_{j-1})}{h^2} - 2y\,x_j \left(\frac{y(x_{j+1}) - y(x_{j-1})}{2h}\right) = 0$

(a) $h = 1/2 \quad y_1(0) = 1/4$

$y_2 - 2y_1 + y_0 = hy_1(y_2 - y_0)$

$1 - 2y_1 + 1/2 = 1/2\, y_1 \times 1/2$

$y_1 = 2/3$

(5) $h = 1/3 \quad y_1(0) = 4/5 \quad y_2(0) = 3/5$

$$y_2 - 2y_1 + y_0 = \frac{1}{3} y_1 (y_2 - y_0)$$

$$2y_1 y_2 + 11 y_1 - 6y_2 - 3 = 0 = f_1$$

$$y_3 - 2y_2 + y_1 = 1/3\, y_2 (y_3 - y_1)$$

$$y_1 y_2 + 3y_1 - 7y_2 + 3 = 0 = f_2$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial y_1} & \frac{\partial f_2}{\partial y_2} \\ \frac{\partial f_2}{\partial y_1} & \frac{\partial f_2}{\partial y_2} \end{bmatrix} = \begin{bmatrix} 2y_2 + 11 & 2y_1 - 6 \\ y_2 + 3 & y_1 - 7 \end{bmatrix}$$

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} y_1^{(0)} \\ y_2^{(0)} \end{bmatrix} - \left[J_{y_1^{(0)}} \; y_2^{(0)} \right]^{-1} \begin{bmatrix} f(y_1^{(0)}, y_2^{(0)}) \\ f_2(y_1^{(0)}, y_2^{(0)}) \end{bmatrix}$$

by solving

$$y_1^{(1)} = y(1/3) = 231/389 = 0.5938303342$$

$$y_2^{(1)} = y(2/3) = 1452/1945 = 0.746529563$$

(b) $y'' = \frac{3}{2} y^2 \quad y(0) = 4 \quad y(1) = 1$

at $x_j \Rightarrow \dfrac{y(x_{j+1}) - 2y(x_j) + y(x_{j-1})}{h^2} = \frac{3}{2} y^2(x_j)$

(a) $h = 1/2 \quad y_1^{(0)} = 7/2$

$$y_2 - 2y_1 + y_0 = 3/8\, y_1^2$$

$$f \Rightarrow 3/8\, y_1^2 + 2y_1 - 5 = 0 \quad f' = \frac{3}{4} y_1 + 2$$

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Newton Raphon $\Rightarrow$

$$y_1^{(1)} = y_1^{(0)} - \frac{f(y_1^{(0)})}{f'(y_1^{(0)})} = 3.5 - \frac{6.59375}{4.625}$$

$$y_1^{(1)} = 2.07432432$$

(5) $h = 1/3$ $\quad y_1^{(0)} = 2 \quad y_2^{(0)} = 3$

$$y_2 - 2y + y_0 = \frac{y_1^2}{6} \Rightarrow y_1^2 + 12y_1 - 6y_2 - 24 = 0 \Rightarrow f_1$$

$$y_3 - 2y_2 + y_1 = \frac{y_2^2}{6} \Rightarrow y_2^2 - 6y_2 + 12y_1 - 6 = 0 \Rightarrow f_2$$

$$J = \begin{bmatrix} 2y_1 + 12 & -6 \\ -6 & 2y_2 - 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 16 & -6 \\ -6 & 0 \end{bmatrix}$$

$$\begin{bmatrix} y_1^{(1)} \\ y_2^{(1)} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 16 & -6 \\ -6 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -14 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 & 6/36 \\ 6/36 & 16/8 \end{bmatrix} \begin{bmatrix} -14 \\ 9 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 4/3 \end{bmatrix}$$

$$y_1^{(1)} = y(1/3) = 0.5$$

$$y_2^{(1)} = y(2/3) = 4/3 = 1.333$$