CSE 455-555
Assignment 4-5 Report
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1. (5 points) We will train a neural network to identify the digit on a image in the MNIST data set from a training data set. This neural network has 10 softmax output nodes generating $\log p(t = m|x;w)$ where m=0,1,...,9. Let $x_n \in R^{28 \times 28}$ be the 28×28 images arranged into a vector, t_n be the label of the image x_n, w be the synaptic weights of the neural network, and n be the index of a pattern in the training data set.

Demonstrate that a neural network to maximize the log likelihood of observing the training data is one that has softmax output nodes and minimizes the criterion function of the negative log probability of training data set: $J_0(w) = -\log p(\{(x_n,t_n): n = 1,2,\cdots,\};w) = -\log \prod_n \prod_{m=0}^{9} p(t_n = m|x_n;w)$. Demonstrate that a neural network to maximize the a posterior likelihood of observing the training data given a Gaussian prior of the weight distribution $p(w;\alpha) = N(0,\alpha I)$ is one that minimizes the criterion function with L2 regularization $J(w) = J_0(w) - \log p(w;\alpha^{-1})$.

Ans:

Ans Neural networks with at least 1 hidden layer are universal approximators, which means that they can approximate any (continuous) function. This approximation can be improved by increasing the number of hidden neurons in the network (but increases the risk of overfitting).

→ A key advantage to neural networks is that they are capable of learning features independently without much human involvement.

Softmax function :-
The softmax function (called such because it is like a "softened" maximum function) may be used as the output layer's activation function. It takes the form:
Softmax is usually used for multivariate logistic regression because it produces a categorical distribution by squashing activation values to be between 0 & 1 and sum to 1. We have used it to implement a different type of plenty (entropy-based) on distributions.
→ This function has the properties that it sums to 1 and that all of its outputs are +ve, which are useful for modeling probability distributions.
→ The cost function to use with softmax is the (categorical) cross-entropy loss function. It has the nice property of having a very ~~ve~~ big gradient when the target value is 1 and the output is almost 0.

Negative Log-Likelihood:-

In practice, the softmax function is used in tandem with the -ve log-likelihood. This loss function is very interesting if we interpret it in relation to the behavior of softmax.

First, let's write down our loss function;

This is summed for all the correct classes.

-> Recall, that when training a model, we aspire to find the minima of a loss function given a set of parameters (in a neural network, these are weights and biases). We can interpret the loss as the "unhappiness" of the network with respect to its parameters. The higher the loss, the higher the unhappiness: we don't want that. We want to make our models happy.

For example:

Let us assume that we have N images and y_i is the ~~image of the~~ label of the image i, where y_i contains $R^{C\times 1}$ - a binary vector of length C (no. of classes) $y_{ic} = 1$ when the image is belonging to class c.

Consider the following two log functions

$$L1 = -\sum \sum_{c=1}^{C} y_{ic} \log P(y_{ic}|D))$$

$$L2 = \sum_{i=1} \sum_{c=1}^{C} (y_{ic} - P(y_{ic}|D))^2$$

L2 is not always used with neural networks, indeed for statistical pattern recognition problems the cross-entropy loss (with a softmax activation function for the output layer) is the preferred option.

Ans Neural networks with at least 1 hidden layer are universal approximators, which means that they can approximate any (continuous) function) This approximation can be improved by increasing the number of hidden neurons in the network (but increases the risk of overfitting).

→ A key advantage to neural networks in that they are capable of learning features independently - without much human involvment.

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Other than the provided answer the below claims also support the argument:

The negative log likelihood (eq.80) is also known as the multiclass cross-entropy (ref: Pattern Recognition and Machine Learning Section 4.3.4), as they are in fact two different interpretations of the same formula.

eq.57 is the negative log likelihood of the Bernoulli distribution, whereas eq.80 is the negative log likelihood of the multinomial distribution with one observation (a multiclass version of Bernoulli).

For binary classification problems, the softmax function outputs two values (between 0 and 1 and sum to 1) to give the prediction of each class. While the sigmoid function outputs one value (between 0 and 1) to give the prediction of one class (so the other class is 1-p).
So eq.80 can't be directly applied to the sigmoid output, though it is essentially the same loss as eq.57.

These three definitions are essentially the same.

1) In the Tensorflow introduction,

$$C = -\frac{1}{n}\sum_x \sum_j (y_j \ln a_j)$$

it satisfies that $\sum_j a_j = 1$ and y is the one-hot representation of the label.
2) For binary classifications $j = 2$, it becomes

$$C = -\frac{1}{n}\sum_x (y_1 \ln a_1 + y_2 \ln a_2)$$

and because of the constraints $\sum_j a_j = 1$ and y being one-hot, it can be rewritten as

$$C = -\frac{1}{n}\sum_x (y_1 \ln a_1 + (1 - y_1)\ln(1 - a_1))$$

which is the same as in the 3rd chapter.

3) Moreover, say the non-zero element of a one-hot vector is y_k, then the cross entropy loss of the corresponding sample is

$$C_x = -\sum_j (y_j \ln a_j) = -(0 + 0 + \ldots + y_k \ln a_k) = -\ln a_k.$$

In the cs231 notes, the cross entropy loss of one sample is given together with softmax normalization as

$$C_x = -\ln(a_k) = -\ln(\frac{e^{f_k}}{\sum_j e^{f_j}}).$$

P.s: the following equation has been taken from a blog on neural networks, found online.

2 (a). (5 points) Build a neural network with 1 hidden layer of 30 sigmoid nodes, and an output layer10 softmax nodes from 1000 training images (100 images per digit). Train the network for 30 complete epochs, using mini-batches of 10 training examples at a time, a learning rate η=0.1. Plot the training error, testing error, criterion function on training data set, criterion function on testing data set of a separate 1000 testing images (100 images per digit), and the learning speed of the hidden layer (the average absolute changes of weights divided by the values of the weights).

Soln:
The code for this is the following file:single_hidden_layer
Here are the specifications of this execution:
The following libraries have been used: tensorflow along with keras.
Also the following requirements have been met in the code:

1) The code was run only on 1000 training and 1000 test images.
2) It had only one hidden layer.
3) The hidden layer had sigmoid as it's activation function.
4) There were 30 sigmoid nodes.
5) An output layer was there having 10 softmax nodes.
6) The network was trained for 30 complete epochs.
7) Also a mini batch of 10 training examples was used at a time
8) Learning rate was in initially kept as 0.1
9) In keras the learning rate speed can be changed using the decay parameter of the activation function
10)

Below is the output:

```
Layer (type)              Output Shape           Param #
=================================================================
dense_3 (Dense)            (None, 30)             23550
_________________________________________________________________
dense_4 (Dense)            (None, 10)             310
=================================================================
Total params: 23,860
Trainable params: 23,860
Non-trainable params: 0
_________________________________________________________________
Train on 1000 samples, validate on 1000 samples
Epoch 1/30
/Users/rajivranjan/anaconda3/lib/python3.6/site-packages/keras/models.py:942: UserWarning:
The `nb_epoch` argument in `fit` has been renamed `epochs`.
  warnings.warn('The `nb_epoch` argument in `fit` '
 - 0s - loss: 2.2700 - acc: 0.1830 - val_loss: 2.1821 - val_acc: 0.4040
 - LR: 0.090909

Epoch 2/30
 - 0s - loss: 2.0282 - acc: 0.4160 - val_loss: 1.8824 - val_acc: 0.5900
 - LR: 0.083333

Epoch 3/30
 - 0s - loss: 1.6315 - acc: 0.6300 - val_loss: 1.5300 - val_acc: 0.5910
 - LR: 0.076923

Epoch 4/30
 - 0s - loss: 1.2944 - acc: 0.7160 - val_loss: 1.2702 - val_acc: 0.6580
 - LR: 0.071429

Epoch 5/30
 - 0s - loss: 1.0616 - acc: 0.7660 - val_loss: 1.1003 - val_acc: 0.7110
```

- LR: 0.066667

Epoch 6/30
- 0s - loss: 0.8979 - acc: 0.8050 - val_loss: 0.9914 - val_acc: 0.7340
- LR: 0.062500

Epoch 7/30
- 0s - loss: 0.7893 - acc: 0.8270 - val_loss: 0.8957 - val_acc: 0.7540
- LR: 0.058824

Epoch 8/30
- 0s - loss: 0.7050 - acc: 0.8470 - val_loss: 0.8329 - val_acc: 0.7620
- LR: 0.055556

Epoch 9/30
- 0s - loss: 0.6441 - acc: 0.8520 - val_loss: 0.7924 - val_acc: 0.7790
- LR: 0.052632

Epoch 10/30
- 0s - loss: 0.5927 - acc: 0.8660 - val_loss: 0.7525 - val_acc: 0.7810
- LR: 0.050000

Epoch 11/30
- 0s - loss: 0.5545 - acc: 0.8730 - val_loss: 0.7189 - val_acc: 0.7940
- LR: 0.047619

Epoch 12/30
- 0s - loss: 0.5210 - acc: 0.8760 - val_loss: 0.6961 - val_acc: 0.8000
- LR: 0.045455

Epoch 13/30
- 0s - loss: 0.4930 - acc: 0.8840 - val_loss: 0.6772 - val_acc: 0.8000
- LR: 0.043478

Epoch 14/30
- 0s - loss: 0.4687 - acc: 0.8900 - val_loss: 0.6615 - val_acc: 0.8010
- LR: 0.041667

Epoch 15/30
- 0s - loss: 0.4477 - acc: 0.8970 - val_loss: 0.6465 - val_acc: 0.8100
- LR: 0.040000

Epoch 16/30
- 0s - loss: 0.4291 - acc: 0.8950 - val_loss: 0.6342 - val_acc: 0.8070
- LR: 0.038462

Epoch 17/30
 - 0s - loss: 0.4138 - acc: 0.9060 - val_loss: 0.6201 - val_acc: 0.8110
 - LR: 0.037037

Epoch 18/30
 - 0s - loss: 0.3986 - acc: 0.9070 - val_loss: 0.6111 - val_acc: 0.8130
 - LR: 0.035714

Epoch 19/30
 - 0s - loss: 0.3859 - acc: 0.9140 - val_loss: 0.6063 - val_acc: 0.8110
 - LR: 0.034483

Epoch 20/30
 - 0s - loss: 0.3740 - acc: 0.9140 - val_loss: 0.5941 - val_acc: 0.8130
 - LR: 0.033333

Epoch 21/30
 - 0s - loss: 0.3638 - acc: 0.9170 - val_loss: 0.5847 - val_acc: 0.8210
 - LR: 0.032258

Epoch 22/30
 - 0s - loss: 0.3536 - acc: 0.9220 - val_loss: 0.5796 - val_acc: 0.8200
 - LR: 0.031250

Epoch 23/30
 - 0s - loss: 0.3440 - acc: 0.9270 - val_loss: 0.5730 - val_acc: 0.8280
 - LR: 0.030303

Epoch 24/30
 - 0s - loss: 0.3361 - acc: 0.9250 - val_loss: 0.5663 - val_acc: 0.8250
 - LR: 0.029412

Epoch 25/30
 - 0s - loss: 0.3282 - acc: 0.9280 - val_loss: 0.5633 - val_acc: 0.8280
 - LR: 0.028571

Epoch 26/30
 - 0s - loss: 0.3213 - acc: 0.9280 - val_loss: 0.5594 - val_acc: 0.8270
 - LR: 0.027778

Epoch 27/30
 - 0s - loss: 0.3145 - acc: 0.9290 - val_loss: 0.5534 - val_acc: 0.8280
 - LR: 0.027027

Epoch 28/30
 - 0s - loss: 0.3077 - acc: 0.9340 - val_loss: 0.5493 - val_acc: 0.8310

- LR: 0.026316

Epoch 29/30
- 0s - loss: 0.3022 - acc: 0.9350 - val_loss: 0.5458 - val_acc: 0.8280
- LR: 0.025641

Epoch 30/30
- 0s - loss: 0.2966 - acc: 0.9350 - val_loss: 0.5451 - val_acc: 0.8260
- LR: 0.025000

Baseline Error: 17.40%
dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])

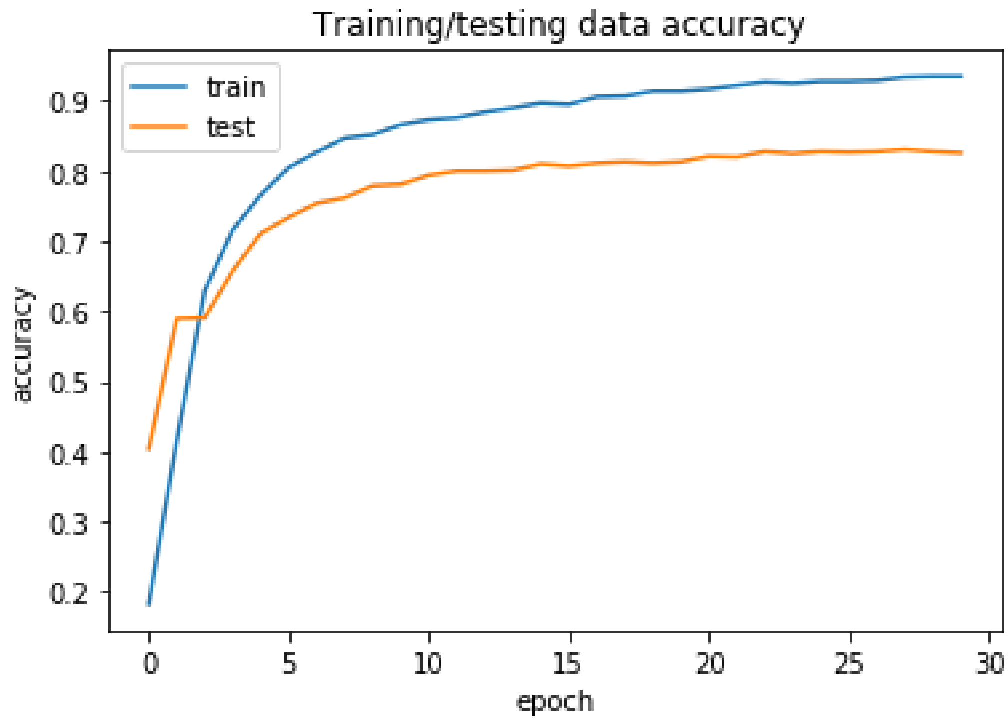
As we can see the accuracy is around 83 percent on test data, 94 on training data and error is around 17 percent.

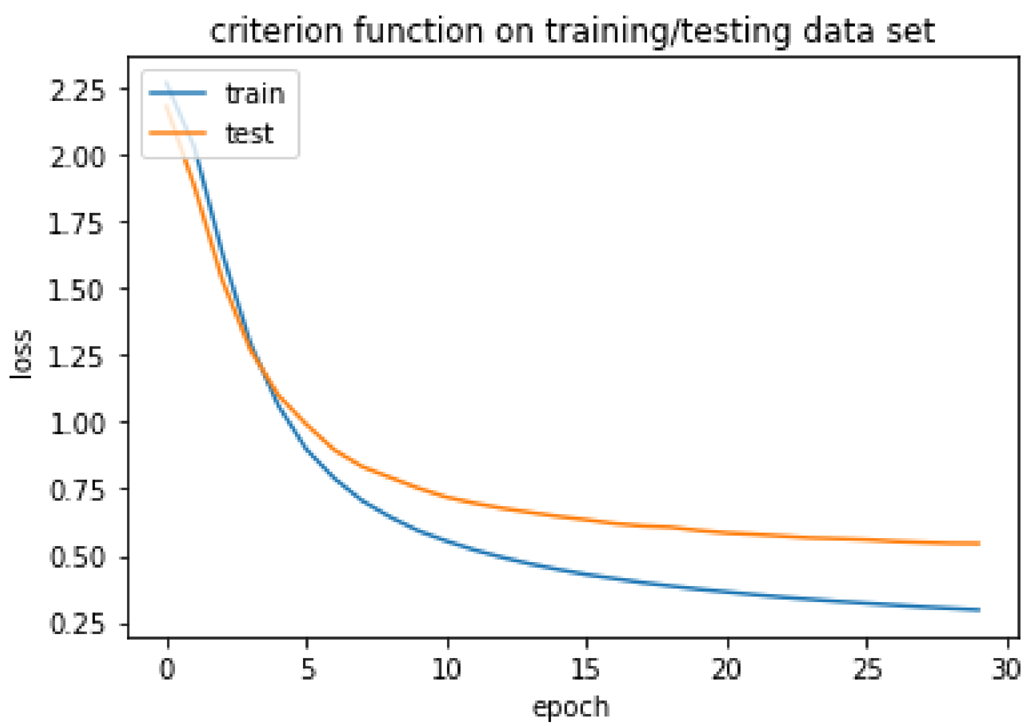
These are good metrics considering we used only 1000 images for our implementation, and it is well known that for neural networks at least we need millions of images to get a good model.

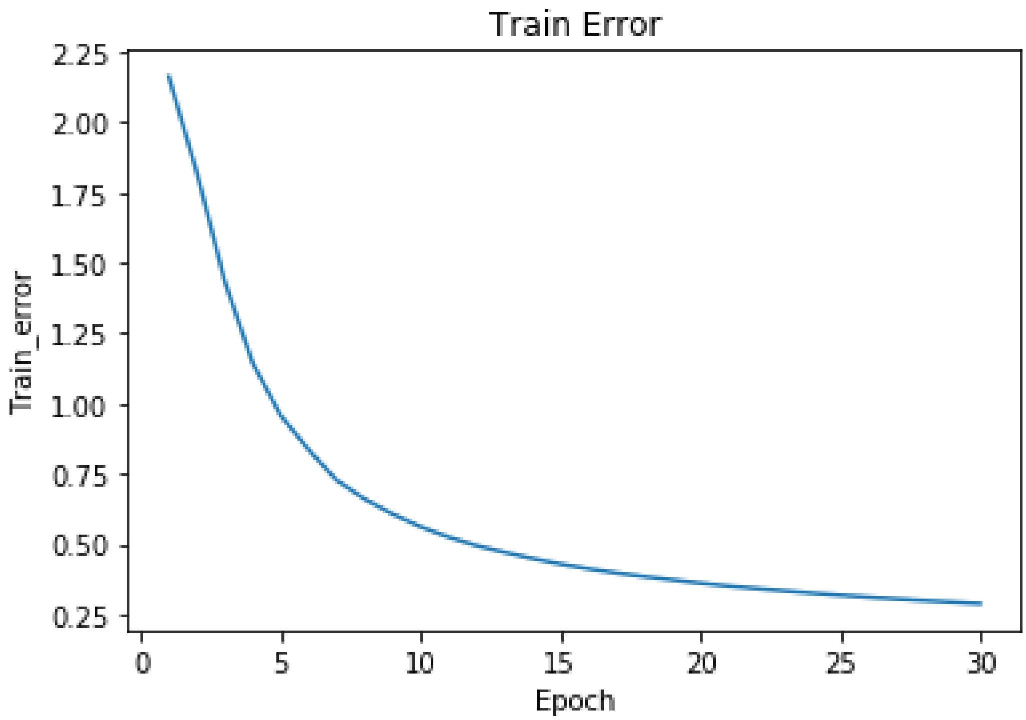
2(a): part 2)

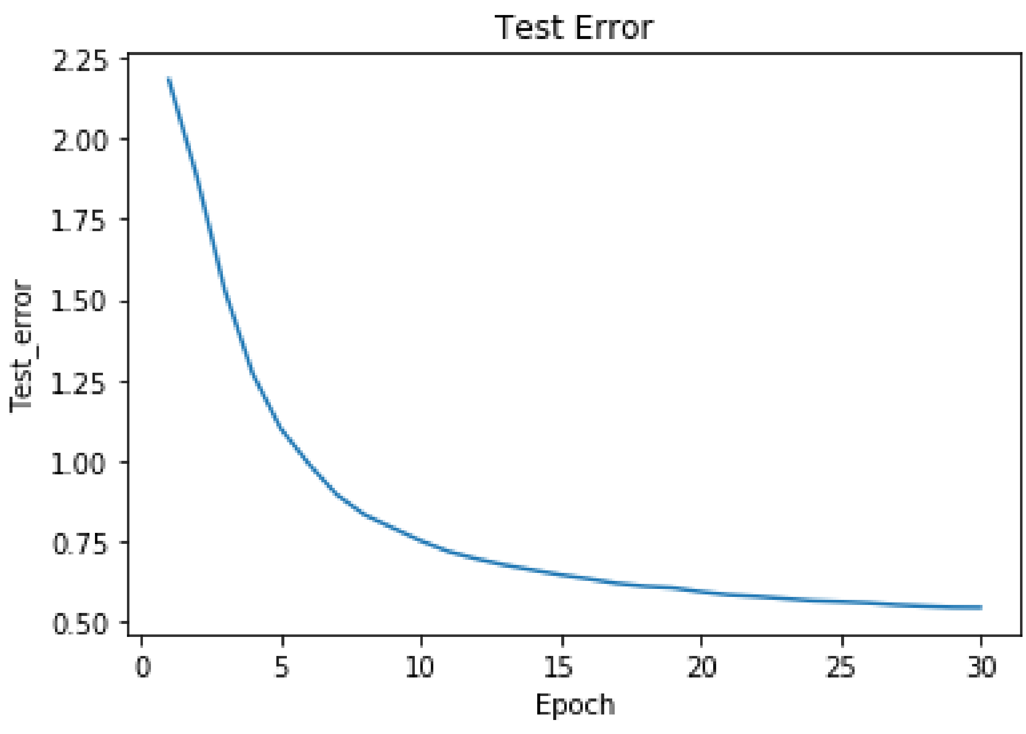
Plot the training error, testing error, criterion function on training data set, criterion function on testing data set of a separate 1000 testing images (100 images per digit), and the learning speed of the hidden layer (the average absolute changes of weights divided by the values of the weights).

Soln:

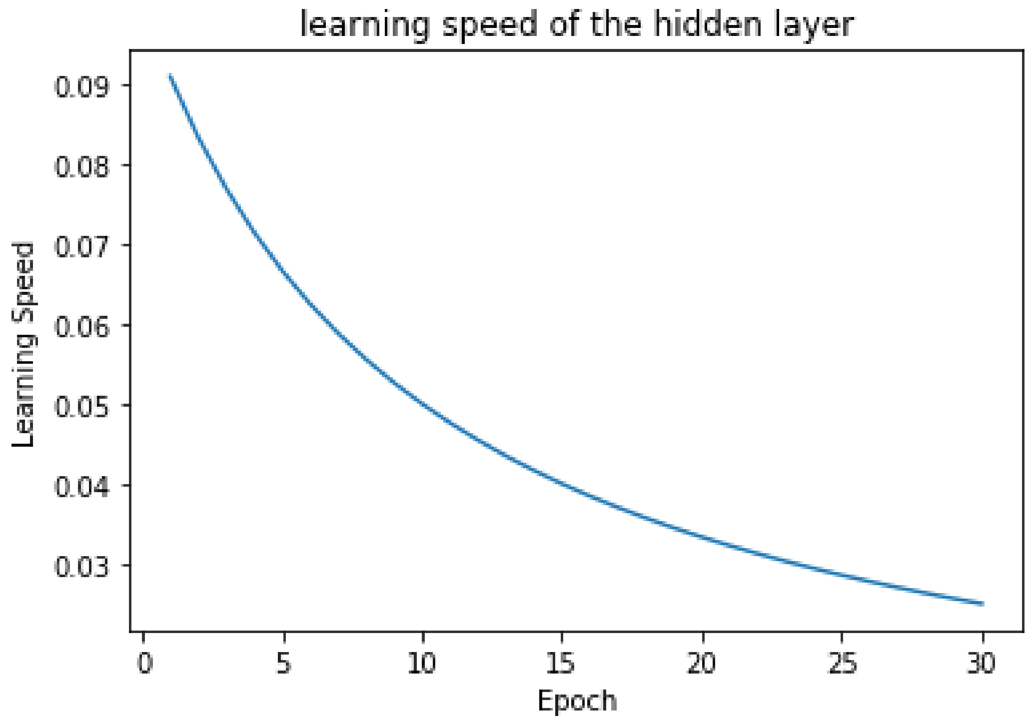

Training/testing data accuracy
train
test
accuracy
epoch


criterion function on training/testing data set
train
test
loss
epoch





For calculating the learning speed the weights were calculated at each step. They are not shown here because they would take a lot of space.



2 (b). (5 points) Repeat 2 (a) with 2 hidden layers of 30 sigmoid nodes each, 3 hidden layers of 30 sigmoid nodes each, and with and without L2 regularization $\lambda\|W\|^2$ and

$\lambda=5$. (You will repeat 2(a) for 5 times: 1 for 2 hidden layer network; 1 for 3 hidden layer network; and 1 times each for 1, 2, 3 hidden layers with regularization.)

Soln:
Below are the five files for this question:
two_hidden_layer.py--- two_hidden_layer without the L2 regularization
three_hidden_layer.py---three hidden layer without the L2 regularization
single_layer_l2.py --- single layer with L2 regularization, lambda=5
two_hidden_l2.py----two hidden layer with L2 regularization, lambda=5
three_hidden_l2.py ---- three hidden layer with L2 regularization, lambda=5

output of two_hidden_layer.py--- two_hidden_layer without the L2 regularization :

Layer (type)	Output Shape	Param #
dense_5 (Dense)	(None, 30)	23550
dense_6 (Dense)	(None, 30)	930
dense_7 (Dense)	(None, 10)	310

===

```
Total params: 24,790
Trainable params: 24,790
Non-trainable params: 0
_________________________________________________________________
/Users/rajivranjan/anaconda3/lib/python3.6/site-packages/keras/models.py:942: UserWarning:
The `nb_epoch` argument in `fit` has been renamed `epochs`.
  warnings.warn('The `nb_epoch` argument in `fit` '
Train on 1000 samples, validate on 1000 samples
Epoch 1/30
 - 0s - loss: 2.3276 - acc: 0.0860 - val_loss: 2.3050 - val_acc: 0.1000
 - LR: 0.099900

Epoch 2/30
 - 0s - loss: 2.3274 - acc: 0.0860 - val_loss: 2.3023 - val_acc: 0.1000
 - LR: 0.099800

Epoch 3/30
 - 0s - loss: 2.3234 - acc: 0.0760 - val_loss: 2.3074 - val_acc: 0.1000
 - LR: 0.099701

Epoch 4/30
 - 0s - loss: 2.3163 - acc: 0.1050 - val_loss: 2.3121 - val_acc: 0.1000
 - LR: 0.099602

Epoch 5/30
 - 0s - loss: 2.3197 - acc: 0.0740 - val_loss: 2.3150 - val_acc: 0.1000
 - LR: 0.099502

Epoch 6/30
 - 0s - loss: 2.3120 - acc: 0.1020 - val_loss: 2.3157 - val_acc: 0.1000
 - LR: 0.099404

Epoch 7/30
 - 0s - loss: 2.3113 - acc: 0.1010 - val_loss: 2.2981 - val_acc: 0.2000
 - LR: 0.099305

Epoch 8/30
 - 0s - loss: 2.2979 - acc: 0.1200 - val_loss: 2.3027 - val_acc: 0.1000
 - LR: 0.099206

Epoch 9/30
 - 0s - loss: 2.2922 - acc: 0.1230 - val_loss: 2.2810 - val_acc: 0.1160
 - LR: 0.099108

Epoch 10/30
 - 0s - loss: 2.2695 - acc: 0.1590 - val_loss: 2.2510 - val_acc: 0.3130
```

- LR: 0.099010

Epoch 11/30
- 0s - loss: 2.2224 - acc: 0.2380 - val_loss: 2.1829 - val_acc: 0.2680
- LR: 0.098912

Epoch 12/30
- 0s - loss: 2.1192 - acc: 0.2810 - val_loss: 2.0821 - val_acc: 0.2720
- LR: 0.098814

Epoch 13/30
- 0s - loss: 1.9850 - acc: 0.3000 - val_loss: 1.9313 - val_acc: 0.3510
- LR: 0.098717

Epoch 14/30
- 0s - loss: 1.8203 - acc: 0.3570 - val_loss: 1.7833 - val_acc: 0.4370
- LR: 0.098619

Epoch 15/30
- 0s - loss: 1.6430 - acc: 0.4190 - val_loss: 1.6186 - val_acc: 0.4290
- LR: 0.098522

Epoch 16/30
- 0s - loss: 1.4836 - acc: 0.4680 - val_loss: 1.4931 - val_acc: 0.4740
- LR: 0.098425

Epoch 17/30
- 0s - loss: 1.3652 - acc: 0.5280 - val_loss: 1.4035 - val_acc: 0.5040
- LR: 0.098328

Epoch 18/30
- 0s - loss: 1.2758 - acc: 0.5620 - val_loss: 1.3373 - val_acc: 0.5580
- LR: 0.098232

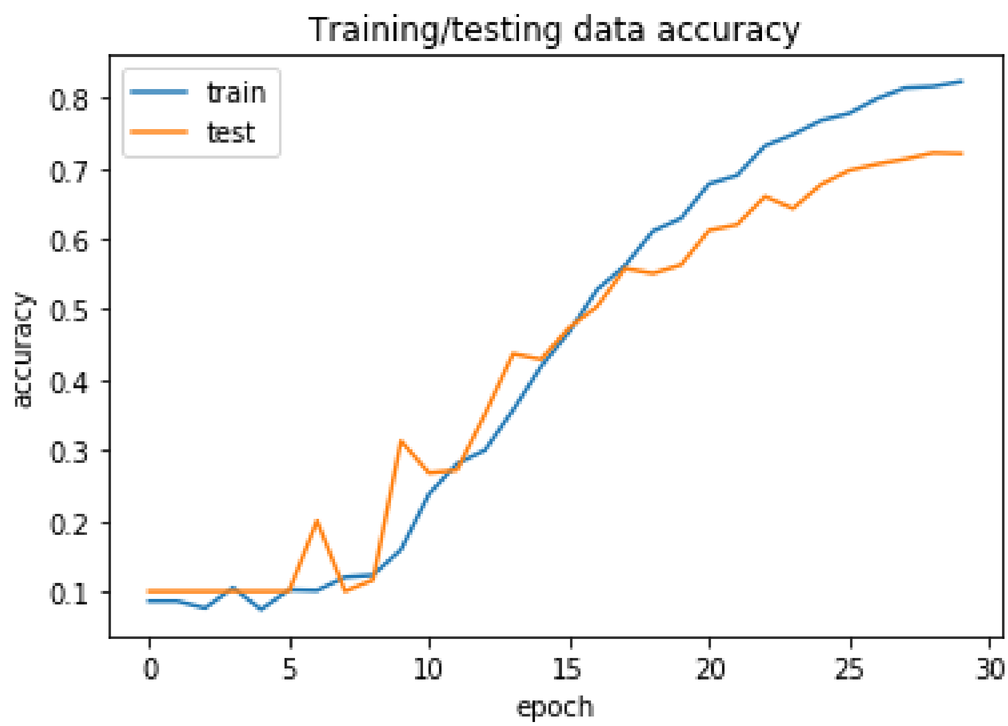
Epoch 19/30
- 0s - loss: 1.1999 - acc: 0.6110 - val_loss: 1.2843 - val_acc: 0.5510
- LR: 0.098135

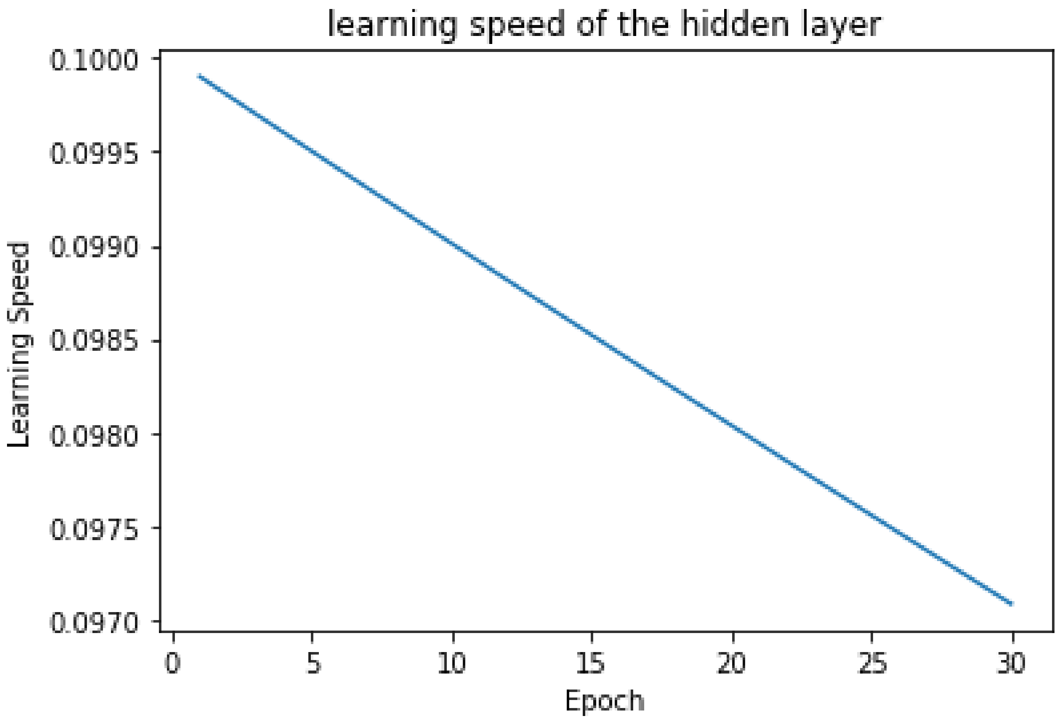
Epoch 20/30
- 0s - loss: 1.1305 - acc: 0.6290 - val_loss: 1.2458 - val_acc: 0.5630
- LR: 0.098039

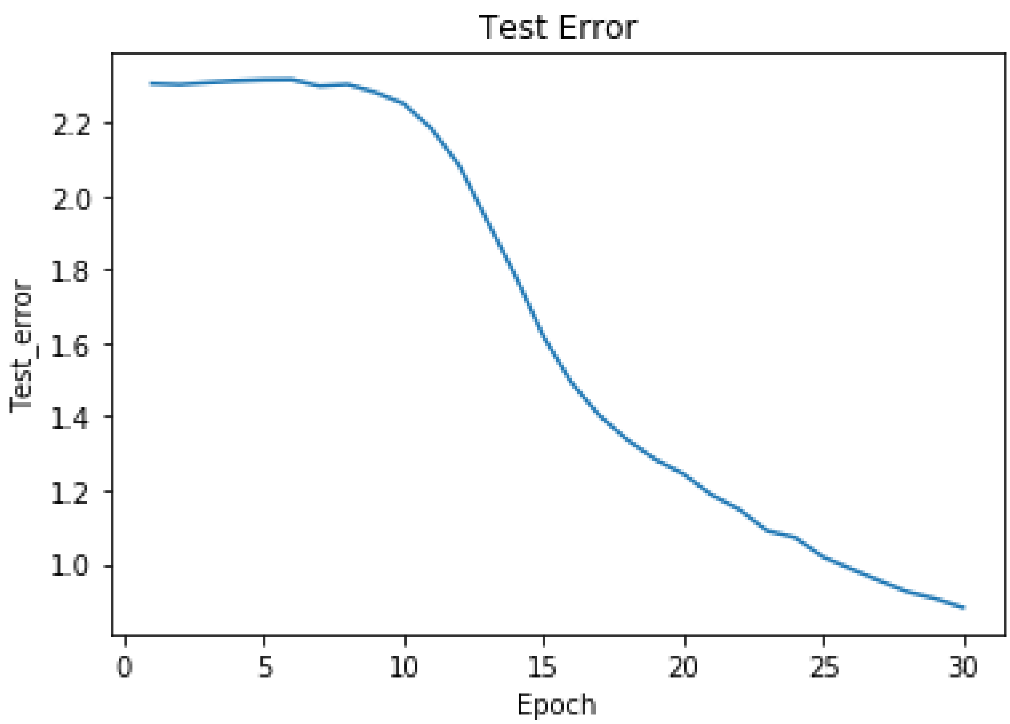
Epoch 21/30
- 0s - loss: 1.0697 - acc: 0.6780 - val_loss: 1.1888 - val_acc: 0.6120
- LR: 0.097943

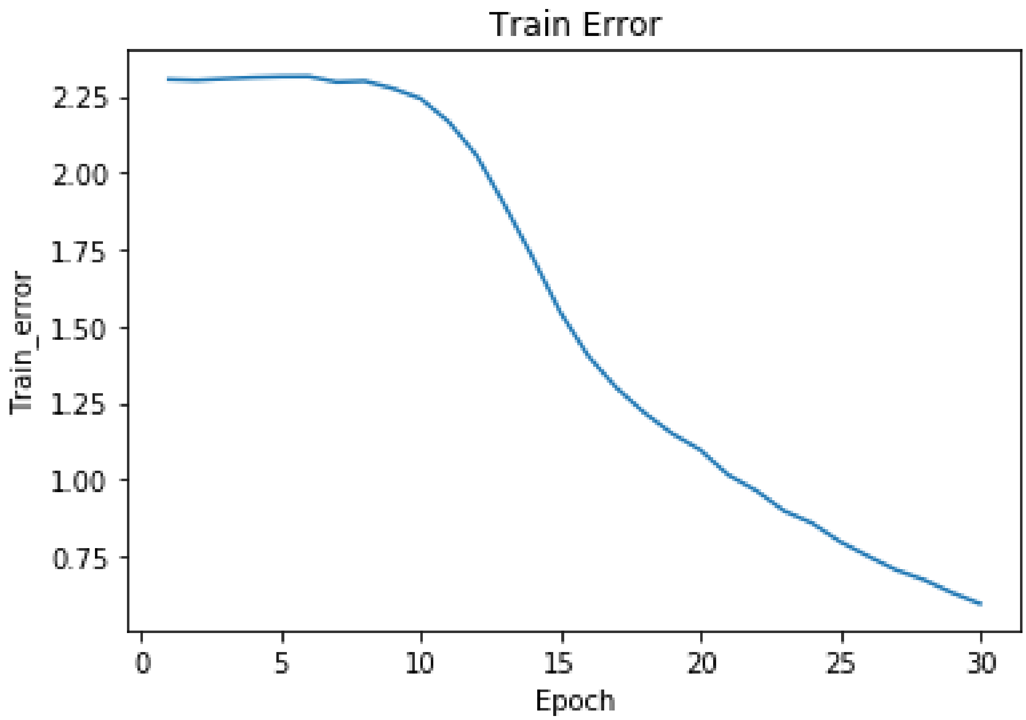
```
Epoch 22/30
 - 0s - loss: 1.0111 - acc: 0.6900 - val_loss: 1.1489 - val_acc: 0.6200
 - LR: 0.097847

Epoch 23/30
 - 0s - loss: 0.9568 - acc: 0.7320 - val_loss: 1.0907 - val_acc: 0.6600
 - LR: 0.097752

Epoch 24/30
 - 0s - loss: 0.8968 - acc: 0.7480 - val_loss: 1.0723 - val_acc: 0.6430
 - LR: 0.097656

Epoch 25/30
 - 0s - loss: 0.8435 - acc: 0.7680 - val_loss: 1.0196 - val_acc: 0.6770
 - LR: 0.097561

Epoch 26/30
 - 0s - loss: 0.7967 - acc: 0.7780 - val_loss: 0.9871 - val_acc: 0.6970
 - LR: 0.097466

Epoch 27/30
 - 0s - loss: 0.7526 - acc: 0.7990 - val_loss: 0.9554 - val_acc: 0.7060
 - LR: 0.097371

Epoch 28/30
 - 0s - loss: 0.7097 - acc: 0.8140 - val_loss: 0.9253 - val_acc: 0.7130
 - LR: 0.097276

Epoch 29/30
 - 0s - loss: 0.6722 - acc: 0.8160 - val_loss: 0.9065 - val_acc: 0.7220
 - LR: 0.097182

Epoch 30/30
 - 0s - loss: 0.6397 - acc: 0.8230 - val_loss: 0.8820 - val_acc: 0.7210
 - LR: 0.097087

Baseline Error: 27.90%
dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
```


criterion function on training/testing data set
train
test
loss
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0
5
10
15
20
25
30
epoch


Training/testing data accuracy
train
test
accuracy
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
15
20
25
30
epoch


learning speed of the hidden layer
0.1000
0.0995
0.0990
0.0985
0.0980
0.0975
0.0970
Learning Speed
0
5
10
15
20
25
30
Epoch


Test Error
2.2
2.0
1.8
1.6
1.4
1.2
1.0
Test_error
0
5
10
15
20
25
30
Epoch



output of three_hidden_layer.py--- three_hidden_layer without the L2 regularization:

Layer (type) Output Shape Param #
===
dense_36 (Dense) (None, 30) 23550

---

dense_37 (Dense) (None, 30) 930

---

dense_38 (Dense) (None, 30) 930

---

dense_39 (Dense) (None, 10) 310
===
Total params: 25,720
Trainable params: 25,720
Non-trainable params: 0

/Users/rajivranjan/anaconda3/lib/python3.6/site-packages/keras/models.py:942: UserWarning: The `nb_epoch` argument in `fit` has been renamed `epochs`.
 warnings.warn('The `nb_epoch` argument in `fit` '
Train on 1000 samples, validate on 1000 samples
Epoch 1/30
 - 1s - loss: 2.3134 - acc: 0.0830 - val_loss: 2.3027 - val_acc: 0.1000
 - LR: 0.009091

```
Epoch 2/30
 - 0s - loss: 2.3053 - acc: 0.0720 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.004762

Epoch 3/30
 - 0s - loss: 2.3041 - acc: 0.0760 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.003226

Epoch 4/30
 - 0s - loss: 2.3037 - acc: 0.0930 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.002439

Epoch 5/30
 - 0s - loss: 2.3035 - acc: 0.0840 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.001961

Epoch 6/30
 - 0s - loss: 2.3033 - acc: 0.0860 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.001639

Epoch 7/30
 - 0s - loss: 2.3032 - acc: 0.0880 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.001408

Epoch 8/30
 - 0s - loss: 2.3031 - acc: 0.0850 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.001235

Epoch 9/30
 - 0s - loss: 2.3030 - acc: 0.0790 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.001099

Epoch 10/30
 - 0s - loss: 2.3030 - acc: 0.0760 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.000990

Epoch 11/30
 - 0s - loss: 2.3029 - acc: 0.0830 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.000901

Epoch 12/30
 - 0s - loss: 2.3029 - acc: 0.0870 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.000826

Epoch 13/30
 - 0s - loss: 2.3029 - acc: 0.0780 - val_loss: 2.3026 - val_acc: 0.1000
```

- LR: 0.000763

Epoch 14/30
- 0s - loss: 2.3029 - acc: 0.0830 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000709

Epoch 15/30
- 0s - loss: 2.3028 - acc: 0.0970 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000662

Epoch 16/30
- 0s - loss: 2.3028 - acc: 0.0980 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000621

Epoch 17/30
- 0s - loss: 2.3028 - acc: 0.1000 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000585

Epoch 18/30
- 0s - loss: 2.3028 - acc: 0.0900 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000552

Epoch 19/30
- 0s - loss: 2.3028 - acc: 0.0810 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000524

Epoch 20/30
- 0s - loss: 2.3028 - acc: 0.0840 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000498

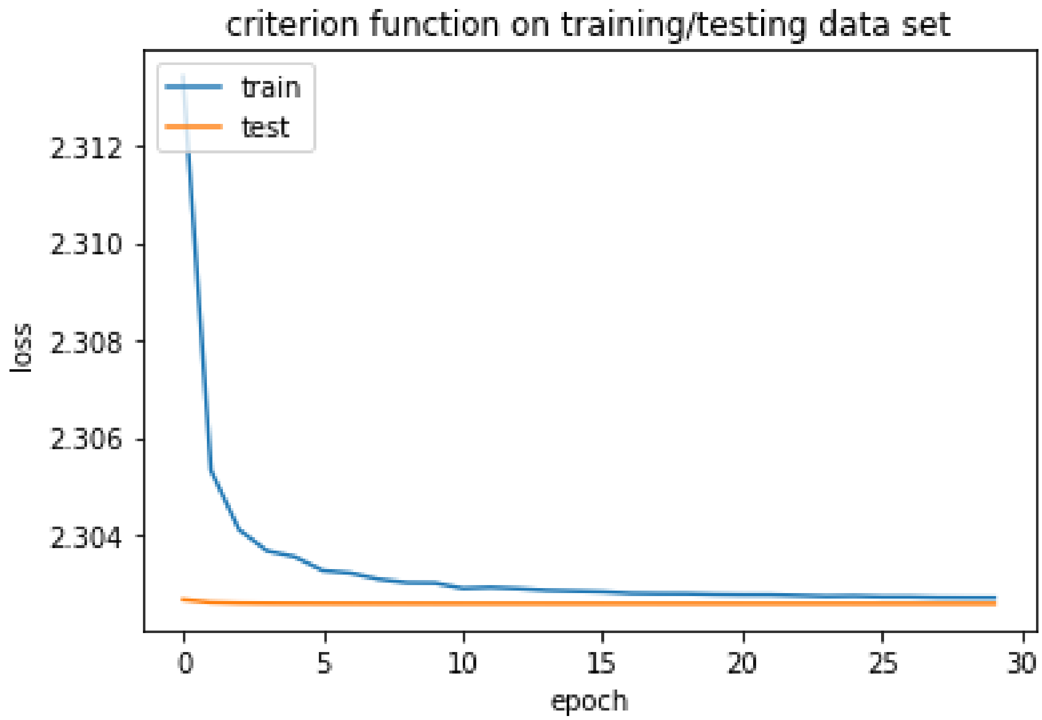
Epoch 21/30
- 0s - loss: 2.3028 - acc: 0.0720 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000474

Epoch 22/30
- 0s - loss: 2.3028 - acc: 0.0890 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000452

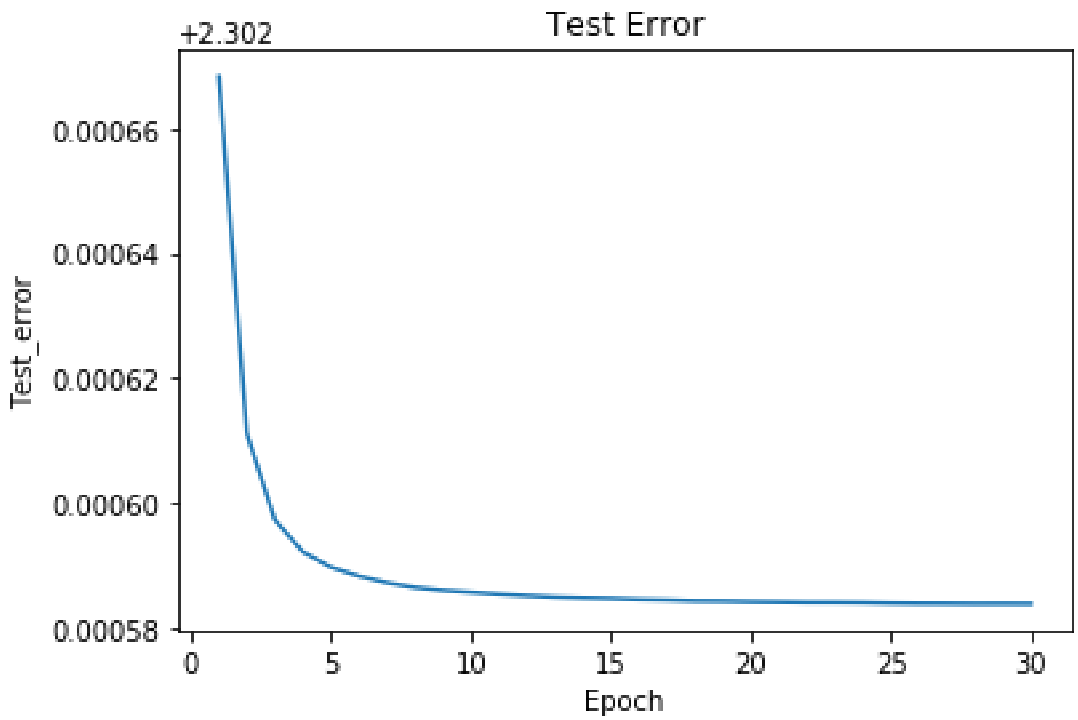
Epoch 23/30
- 0s - loss: 2.3027 - acc: 0.0940 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000433

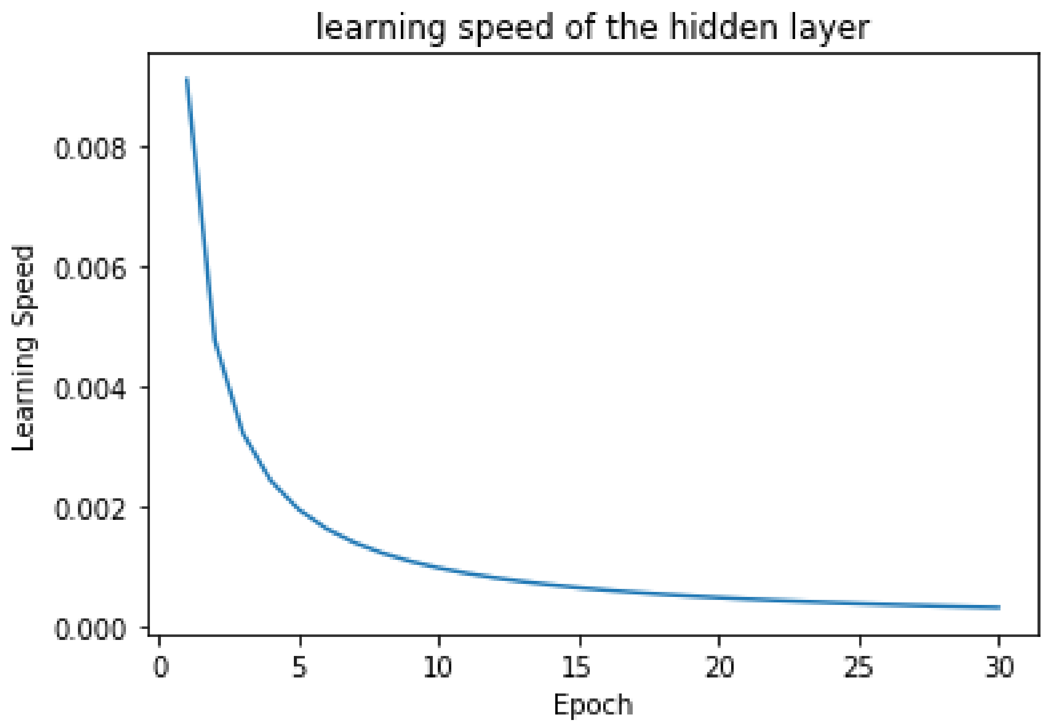
Epoch 24/30
- 0s - loss: 2.3027 - acc: 0.0870 - val_loss: 2.3026 - val_acc: 0.1000
- LR: 0.000415

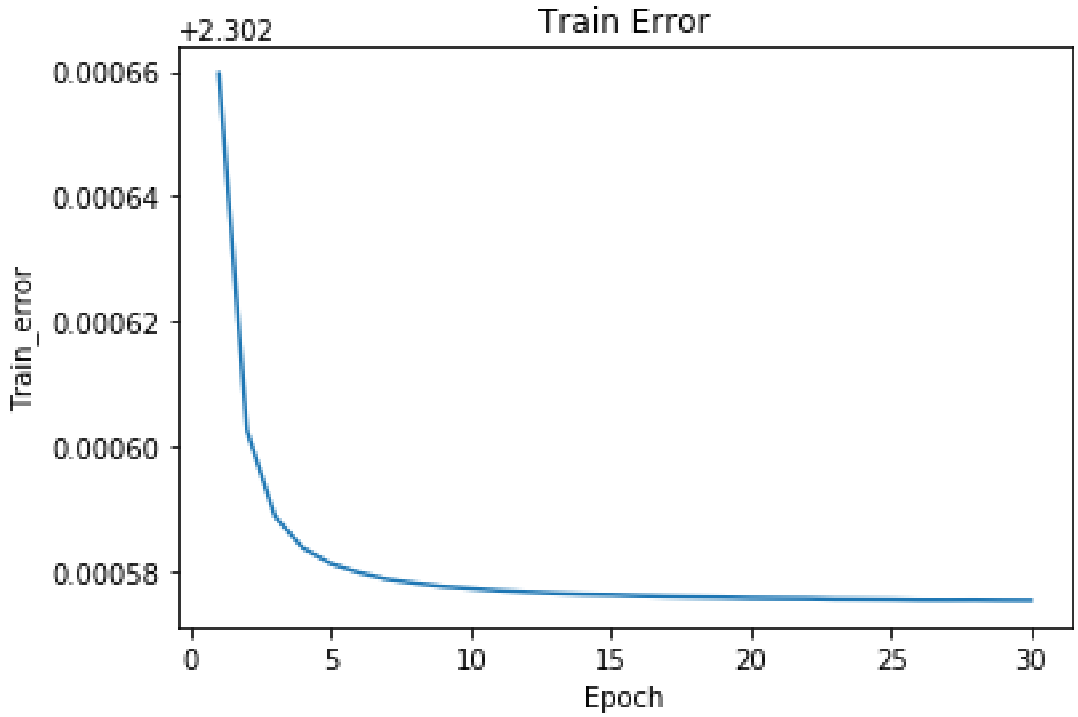
```
Epoch 25/30
 - 0s - loss: 2.3027 - acc: 0.0760 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.000398

Epoch 26/30
 - 0s - loss: 2.3027 - acc: 0.0920 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.000383

Epoch 27/30
 - 0s - loss: 2.3027 - acc: 0.0950 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.000369

Epoch 28/30
 - 0s - loss: 2.3027 - acc: 0.0890 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.000356

Epoch 29/30
 - 0s - loss: 2.3027 - acc: 0.0980 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.000344

Epoch 30/30
 - 0s - loss: 2.3027 - acc: 0.0920 - val_loss: 2.3026 - val_acc: 0.1000
 - LR: 0.000332

Baseline Error: 10.00%
dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
```


criterion function on training/testing data set
train
test
loss
epoch


Training/testing data accuracy
train
test
accuracy
epoch


Test Error
+2.302
0.00066
0.00064
0.00062
0.00060
0.00058
Test_error
0
5
10
15
20
25
30
Epoch


learning speed of the hidden layer
0.008
0.006
0.004
0.002
0.000
Learning Speed
0
5
10
15
20
25
30
Epoch



single_layer_l2.py --- single layer with L2 regularization, lambda=5
output:

Layer (type)	Output Shape	Param #
dense_40 (Dense)	(None, 30)	23550
dense_41 (Dense)	(None, 10)	310

===

Total params: 23,860
Trainable params: 23,860
Non-trainable params: 0

Train on 1000 samples, validate on 1000 samples
Epoch 1/30
 - 1s - loss: 2.2759 - acc: 0.1830 - val_loss: 2.1882 - val_acc: 0.4040
 - LR: 0.090909

Epoch 2/30
 - 0s - loss: 2.0348 - acc: 0.4160 - val_loss: 1.8895 - val_acc: 0.5900
 - LR: 0.083333

Epoch 3/30
 - 0s - loss: 1.6393 - acc: 0.6300 - val_loss: 1.5381 - val_acc: 0.5910
 - LR: 0.076923

Epoch 4/30
 - 0s - loss: 1.3030 - acc: 0.7150 - val_loss: 1.2791 - val_acc: 0.6580
 - LR: 0.071429

Epoch 5/30
 - 0s - loss: 1.0709 - acc: 0.7660 - val_loss: 1.1096 - val_acc: 0.7110
 - LR: 0.066667

Epoch 6/30
 - 0s - loss: 0.9075 - acc: 0.8050 - val_loss: 1.0011 - val_acc: 0.7350
 - LR: 0.062500

Epoch 7/30
 - 0s - loss: 0.7993 - acc: 0.8270 - val_loss: 0.9057 - val_acc: 0.7530
 - LR: 0.058824

Epoch 8/30
 - 0s - loss: 0.7153 - acc: 0.8480 - val_loss: 0.8433 - val_acc: 0.7610
 - LR: 0.055556

Epoch 9/30
 - 0s - loss: 0.6547 - acc: 0.8530 - val_loss: 0.8031 - val_acc: 0.7790
 - LR: 0.052632

Epoch 10/30
 - 0s - loss: 0.6037 - acc: 0.8640 - val_loss: 0.7634 - val_acc: 0.7810
 - LR: 0.050000

Epoch 11/30
 - 0s - loss: 0.5657 - acc: 0.8730 - val_loss: 0.7301 - val_acc: 0.7930
 - LR: 0.047619

Epoch 12/30
 - 0s - loss: 0.5325 - acc: 0.8760 - val_loss: 0.7075 - val_acc: 0.8000
 - LR: 0.045455

Epoch 13/30
 - 0s - loss: 0.5047 - acc: 0.8840 - val_loss: 0.6888 - val_acc: 0.8000
 - LR: 0.043478

Epoch 14/30

- 0s - loss: 0.4806 - acc: 0.8890 - val_loss: 0.6733 - val_acc: 0.8010
- LR: 0.041667

Epoch 15/30
- 0s - loss: 0.4597 - acc: 0.8970 - val_loss: 0.6584 - val_acc: 0.8110
- LR: 0.040000

Epoch 16/30
- 0s - loss: 0.4413 - acc: 0.8950 - val_loss: 0.6463 - val_acc: 0.8080
- LR: 0.038462

Epoch 17/30
- 0s - loss: 0.4262 - acc: 0.9060 - val_loss: 0.6324 - val_acc: 0.8130
- LR: 0.037037

Epoch 18/30
- 0s - loss: 0.4112 - acc: 0.9060 - val_loss: 0.6235 - val_acc: 0.8130
- LR: 0.035714

Epoch 19/30
- 0s - loss: 0.3987 - acc: 0.9130 - val_loss: 0.6189 - val_acc: 0.8110
- LR: 0.034483

Epoch 20/30
- 0s - loss: 0.3869 - acc: 0.9150 - val_loss: 0.6067 - val_acc: 0.8130
- LR: 0.033333

Epoch 21/30
- 0s - loss: 0.3769 - acc: 0.9170 - val_loss: 0.5975 - val_acc: 0.8210
- LR: 0.032258

Epoch 22/30
- 0s - loss: 0.3668 - acc: 0.9210 - val_loss: 0.5924 - val_acc: 0.8200
- LR: 0.031250

Epoch 23/30
- 0s - loss: 0.3574 - acc: 0.9270 - val_loss: 0.5860 - val_acc: 0.8270
- LR: 0.030303

Epoch 24/30
- 0s - loss: 0.3496 - acc: 0.9250 - val_loss: 0.5794 - val_acc: 0.8260
- LR: 0.029412

Epoch 25/30
- 0s - loss: 0.3418 - acc: 0.9280 - val_loss: 0.5764 - val_acc: 0.8270
- LR: 0.028571

Epoch 26/30
 - 0s - loss: 0.3350 - acc: 0.9280 - val_loss: 0.5727 - val_acc: 0.8270
 - LR: 0.027778

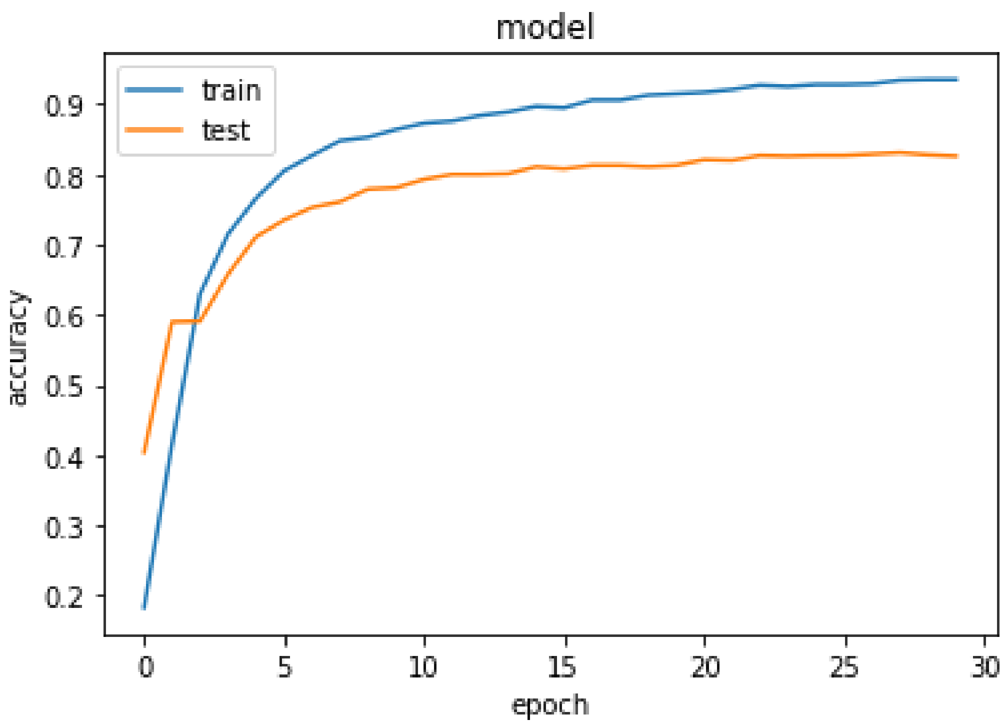
Epoch 27/30
 - 0s - loss: 0.3283 - acc: 0.9290 - val_loss: 0.5667 - val_acc: 0.8290
 - LR: 0.027027

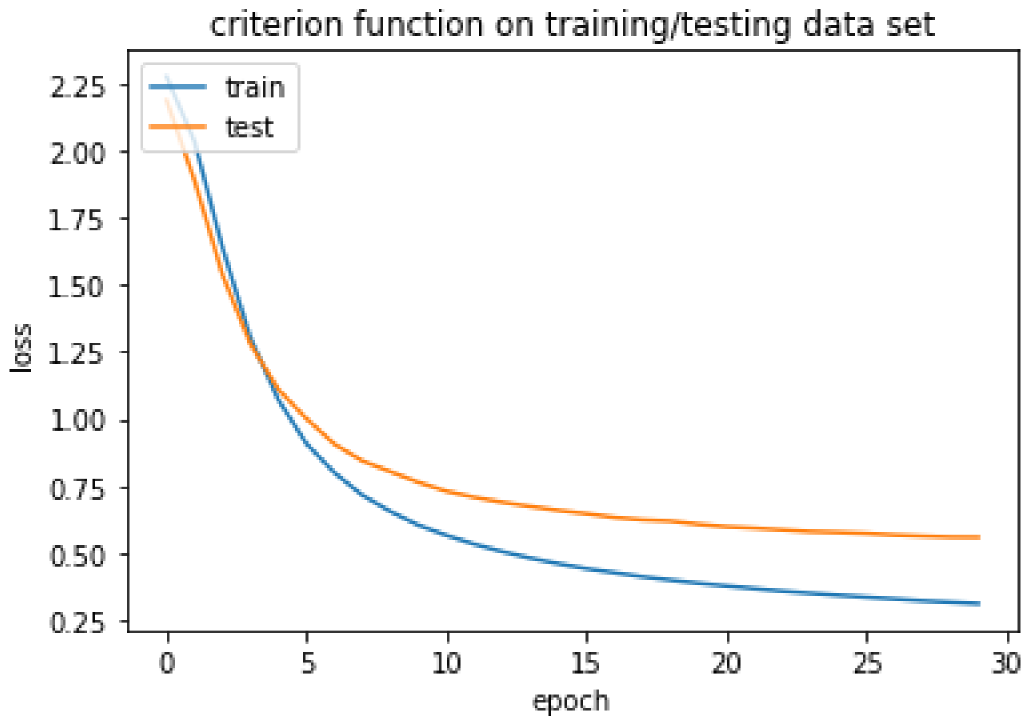
Epoch 28/30
 - 0s - loss: 0.3216 - acc: 0.9340 - val_loss: 0.5628 - val_acc: 0.8310
 - LR: 0.026316

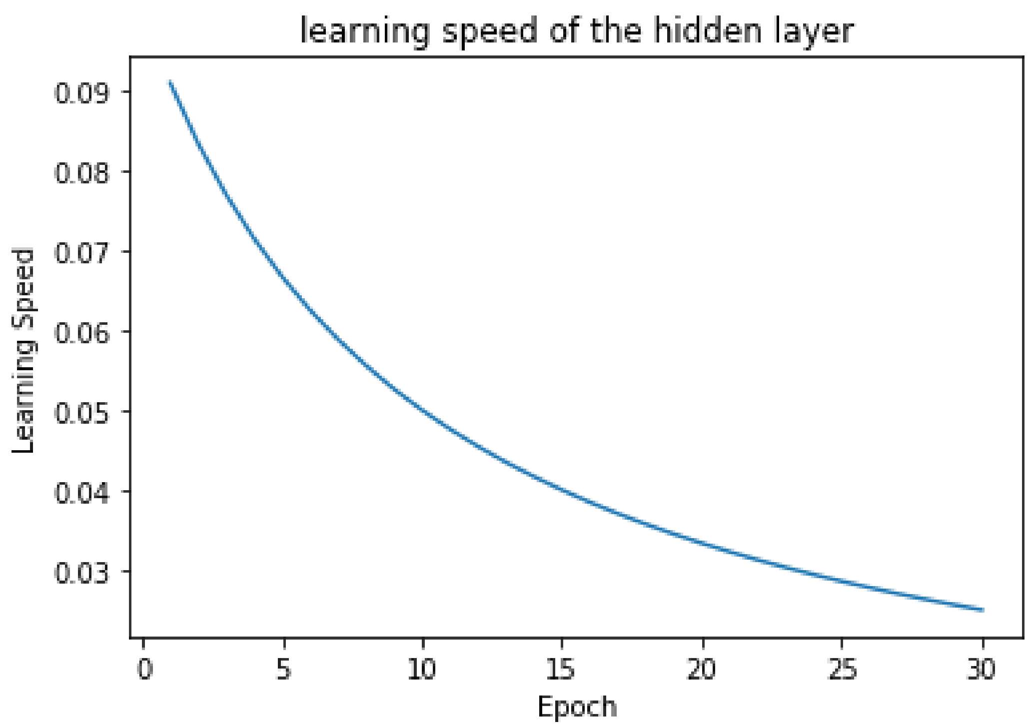
Epoch 29/30
 - 0s - loss: 0.3162 - acc: 0.9350 - val_loss: 0.5594 - val_acc: 0.8280
 - LR: 0.025641

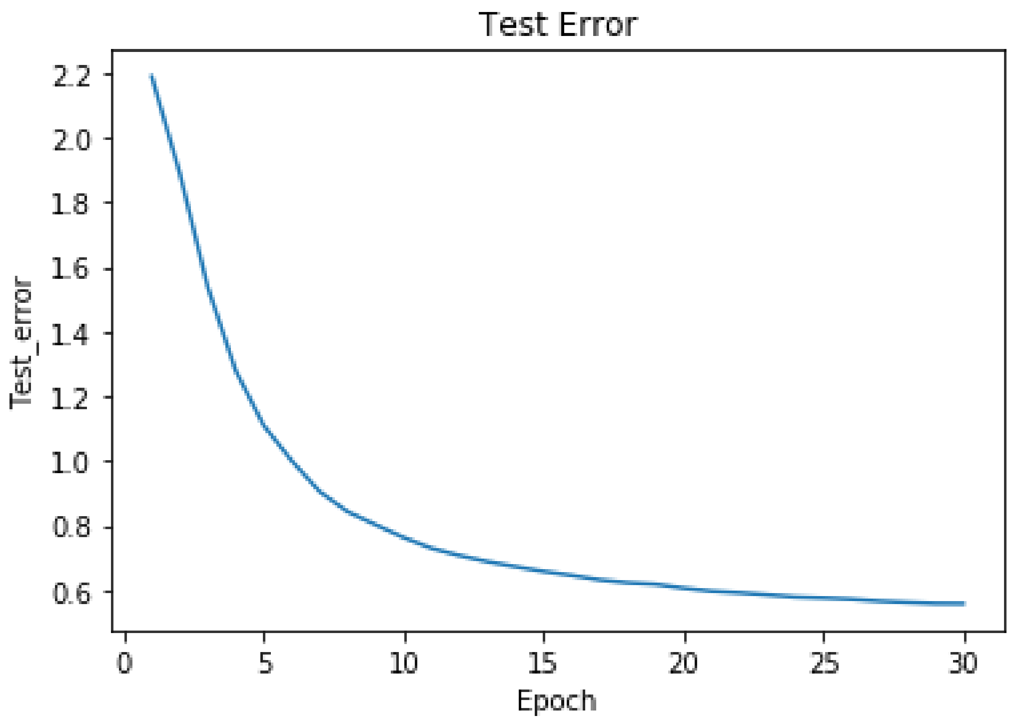
Epoch 30/30
 - 0s - loss: 0.3107 - acc: 0.9350 - val_loss: 0.5588 - val_acc: 0.8260
 - LR: 0.025000

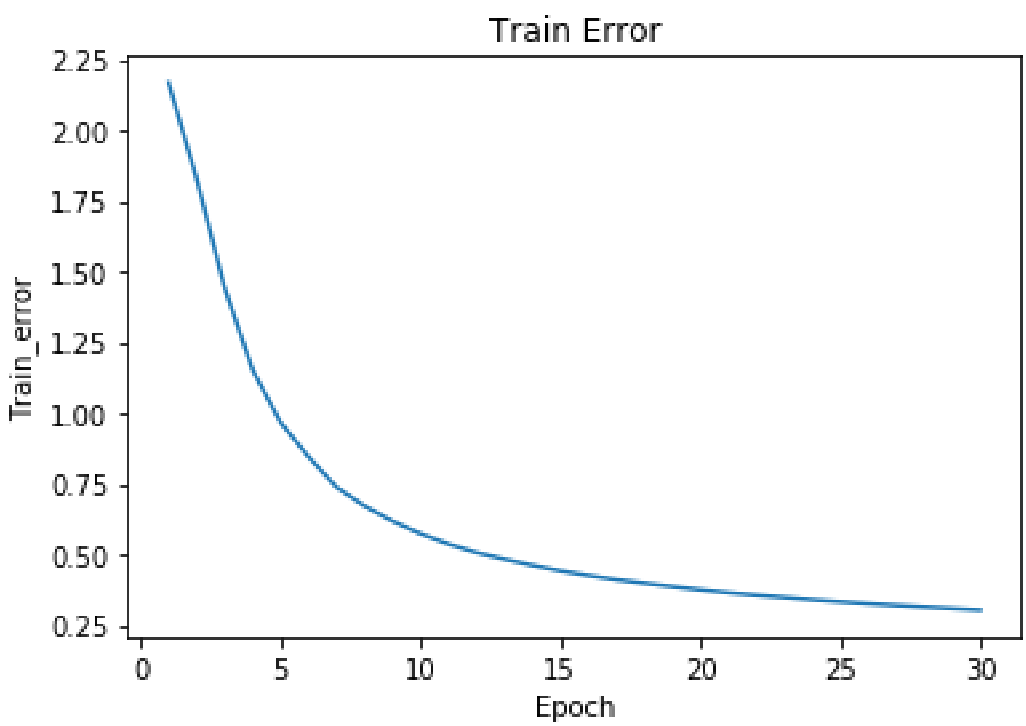
Baseline Error: 17.40%
dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])




criterion function on training/testing data set
train
test
loss
epoch
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0
5
10
15
20
25
30


learning speed of the hidden layer
Learning Speed
Epoch
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0
5
10
15
20
25
30





two_hidden_l2.py----two hidden layer with L2 regularization, lambda=5

Layer (type)	Output Shape	Param #

```
=================================================================
dense_42 (Dense)             (None, 30)               23550
_________________________________________________________________
dense_43 (Dense)             (None, 30)               930
_________________________________________________________________
dense_44 (Dense)             (None, 10)               310
=================================================================
Total params: 24,790
Trainable params: 24,790
Non-trainable params: 0
_________________________________________________________________
Train on 1000 samples, validate on 1000 samples
Epoch 1/30
 - 1s - loss: 2.3337 - acc: 0.0860 - val_loss: 2.3111 - val_acc: 0.1000
 - LR: 0.099900

Epoch 2/30
 - 0s - loss: 2.3334 - acc: 0.0860 - val_loss: 2.3083 - val_acc: 0.1000
 - LR: 0.099800

Epoch 3/30
 - 0s - loss: 2.3294 - acc: 0.0760 - val_loss: 2.3134 - val_acc: 0.1000
 - LR: 0.099701

Epoch 4/30
 - 0s - loss: 2.3223 - acc: 0.1050 - val_loss: 2.3180 - val_acc: 0.1000
 - LR: 0.099602

Epoch 5/30
 - 0s - loss: 2.3257 - acc: 0.0740 - val_loss: 2.3209 - val_acc: 0.1000
 - LR: 0.099502

Epoch 6/30
 - 0s - loss: 2.3181 - acc: 0.1010 - val_loss: 2.3217 - val_acc: 0.1000
 - LR: 0.099404

Epoch 7/30
 - 0s - loss: 2.3176 - acc: 0.1010 - val_loss: 2.3045 - val_acc: 0.2000
 - LR: 0.099305

Epoch 8/30
 - 0s - loss: 2.3046 - acc: 0.1190 - val_loss: 2.3095 - val_acc: 0.1000
 - LR: 0.099206

Epoch 9/30
 - 0s - loss: 2.2997 - acc: 0.1180 - val_loss: 2.2889 - val_acc: 0.1100
```

- LR: 0.099108

Epoch 10/30
- 0s - loss: 2.2789 - acc: 0.1570 - val_loss: 2.2616 - val_acc: 0.3160
- LR: 0.099010

Epoch 11/30
- 0s - loss: 2.2363 - acc: 0.2250 - val_loss: 2.1993 - val_acc: 0.2680
- LR: 0.098912

Epoch 12/30
- 0s - loss: 2.1403 - acc: 0.2780 - val_loss: 2.1048 - val_acc: 0.2720
- LR: 0.098814

Epoch 13/30
- 0s - loss: 2.0113 - acc: 0.2960 - val_loss: 1.9582 - val_acc: 0.3480
- LR: 0.098717

Epoch 14/30
- 0s - loss: 1.8519 - acc: 0.3470 - val_loss: 1.8153 - val_acc: 0.4220
- LR: 0.098619

Epoch 15/30
- 0s - loss: 1.6786 - acc: 0.4090 - val_loss: 1.6510 - val_acc: 0.4230
- LR: 0.098522

Epoch 16/30
- 0s - loss: 1.5159 - acc: 0.4600 - val_loss: 1.5206 - val_acc: 0.4700
- LR: 0.098425

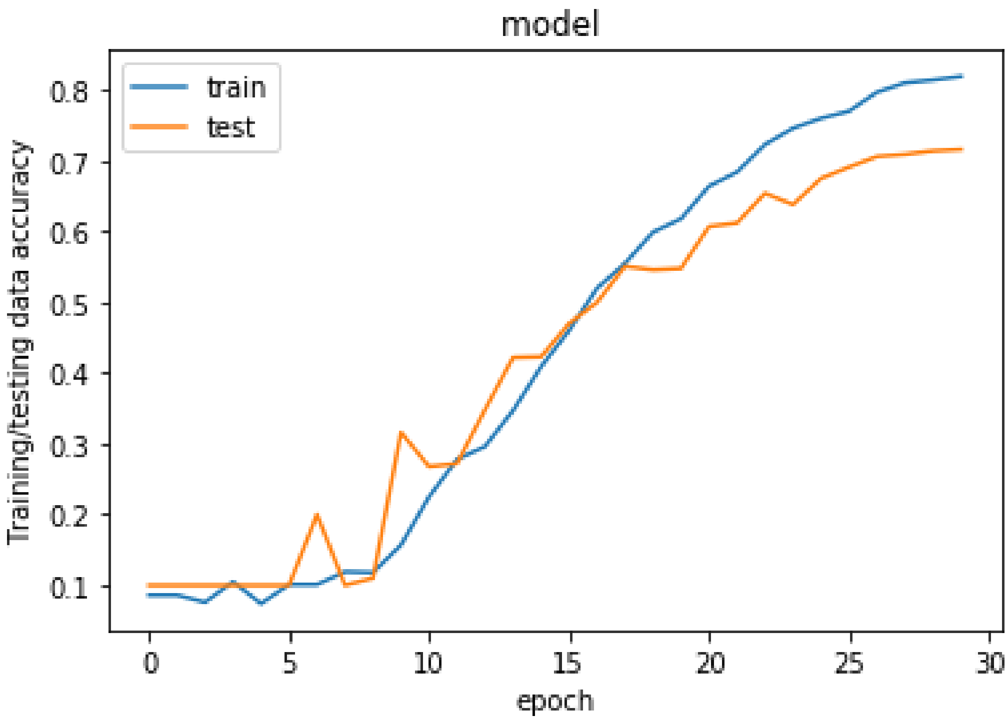
Epoch 17/30
- 0s - loss: 1.3933 - acc: 0.5200 - val_loss: 1.4278 - val_acc: 0.5000
- LR: 0.098328

Epoch 18/30
- 0s - loss: 1.3023 - acc: 0.5550 - val_loss: 1.3611 - val_acc: 0.5510
- LR: 0.098232

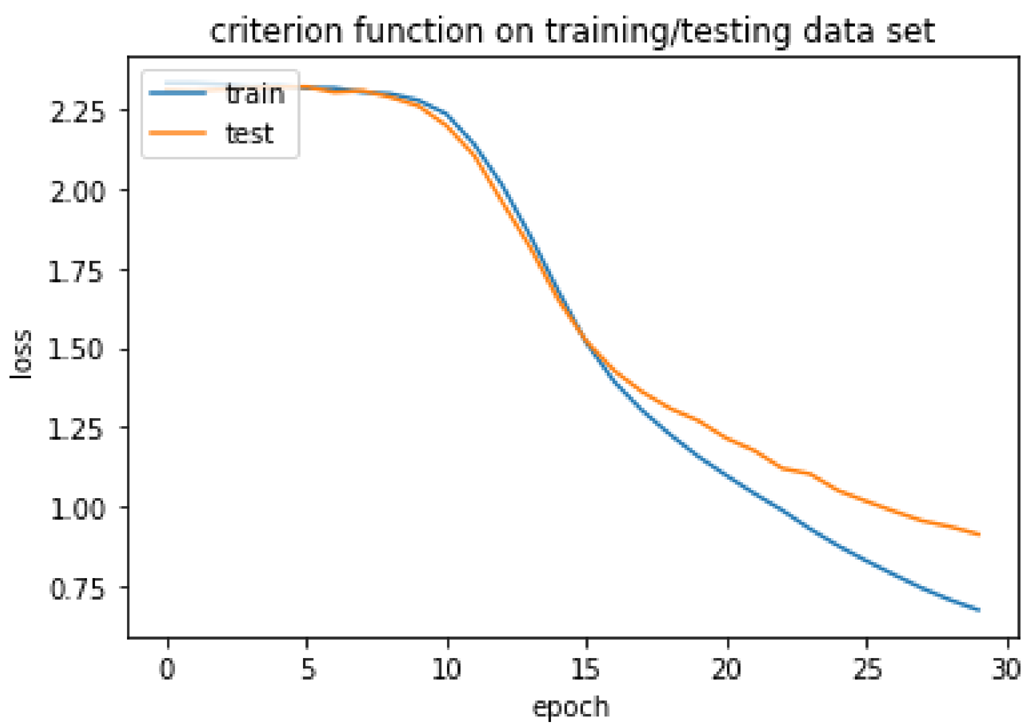
Epoch 19/30
- 0s - loss: 1.2265 - acc: 0.5990 - val_loss: 1.3087 - val_acc: 0.5460
- LR: 0.098135

Epoch 20/30
- 0s - loss: 1.1578 - acc: 0.6180 - val_loss: 1.2706 - val_acc: 0.5480
- LR: 0.098039

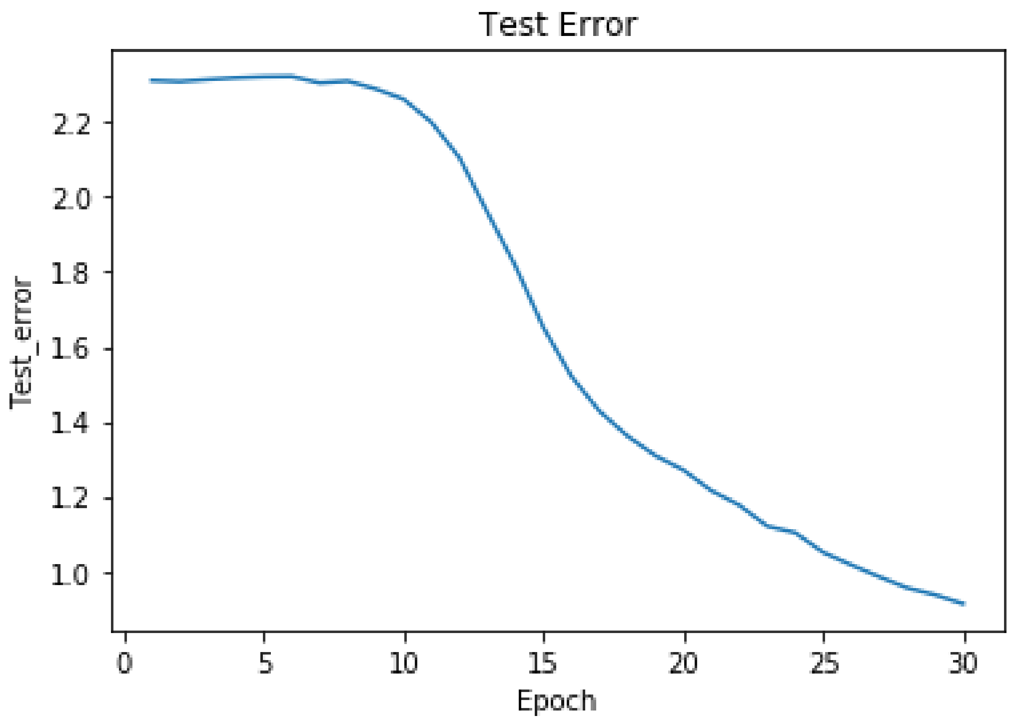
```
Epoch 21/30
 - 0s - loss: 1.0983 - acc: 0.6640 - val_loss: 1.2153 - val_acc: 0.6070
 - LR: 0.097943

Epoch 22/30
 - 0s - loss: 1.0413 - acc: 0.6840 - val_loss: 1.1766 - val_acc: 0.6120
 - LR: 0.097847

Epoch 23/30
 - 0s - loss: 0.9885 - acc: 0.7230 - val_loss: 1.1203 - val_acc: 0.6540
 - LR: 0.097752

Epoch 24/30
 - 0s - loss: 0.9293 - acc: 0.7460 - val_loss: 1.1028 - val_acc: 0.6380
 - LR: 0.097656

Epoch 25/30
 - 0s - loss: 0.8762 - acc: 0.7600 - val_loss: 1.0502 - val_acc: 0.6750
 - LR: 0.097561

Epoch 26/30
 - 0s - loss: 0.8294 - acc: 0.7700 - val_loss: 1.0168 - val_acc: 0.6910
 - LR: 0.097466

Epoch 27/30
 - 0s - loss: 0.7853 - acc: 0.7970 - val_loss: 0.9851 - val_acc: 0.7060
 - LR: 0.097371

Epoch 28/30
 - 0s - loss: 0.7427 - acc: 0.8100 - val_loss: 0.9552 - val_acc: 0.7090
 - LR: 0.097276

Epoch 29/30
 - 0s - loss: 0.7059 - acc: 0.8140 - val_loss: 0.9366 - val_acc: 0.7140
 - LR: 0.097182

Epoch 30/30
 - 0s - loss: 0.6746 - acc: 0.8190 - val_loss: 0.9129 - val_acc: 0.7160
 - LR: 0.097087

Baseline Error: 28.40%
dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
```

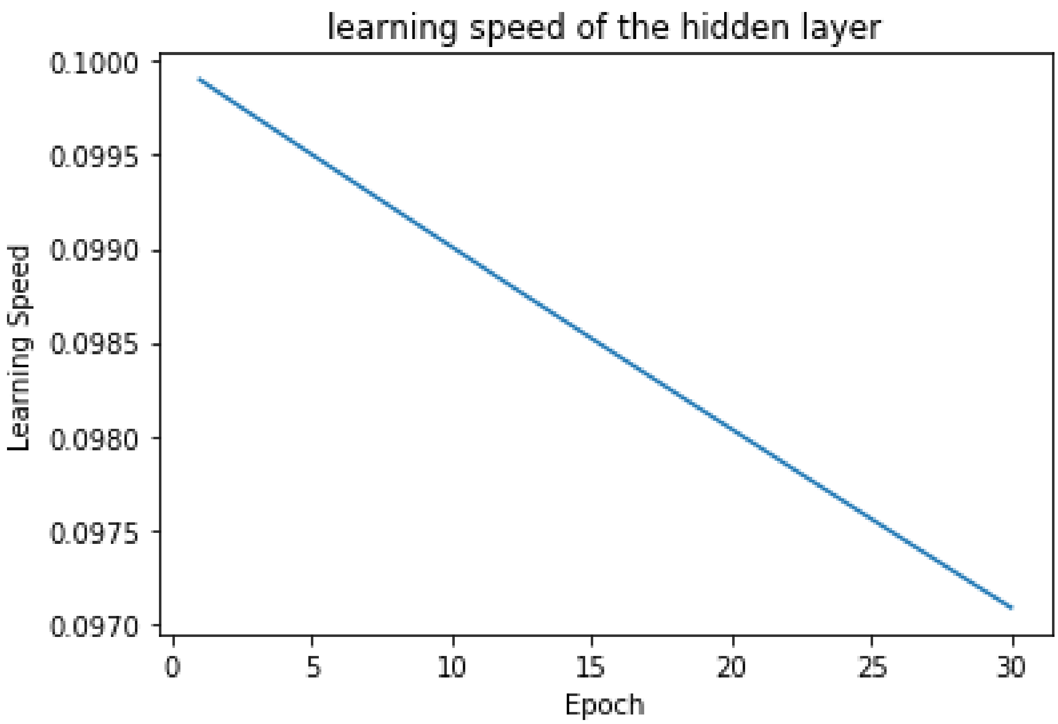


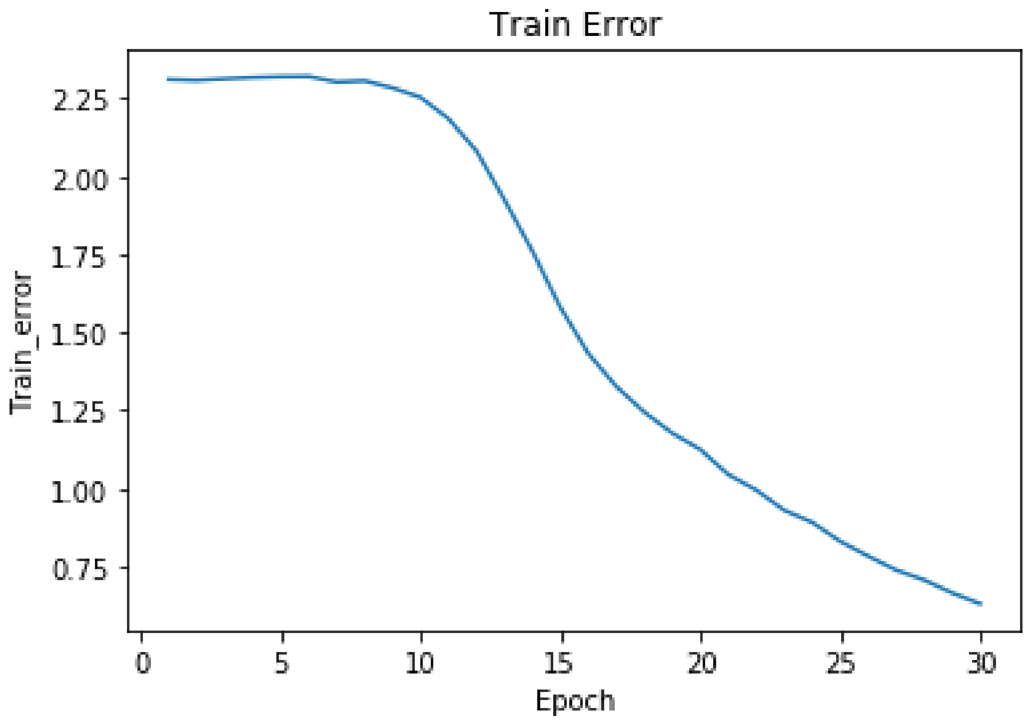
model
train
test
Training/testing data accuracy
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
5
10
15
20
25
30
epoch



criterion function on training/testing data set
train
test
loss
0.75
1.00
1.25
1.50
1.75
2.00
2.25
0
5
10
15
20
25
30
epoch


Test Error
Test_error
Epoch
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
5
10
15
20
25
30


learning speed of the hidden layer
Learning Speed
Epoch
0.1000
0.0995
0.0990
0.0985
0.0980
0.0975
0.0970
0
5
10
15
20
25
30



output of three_hidden_l2.py ---- three hidden layer with L2 regularization, lambda=5

____

Layer (type)   Output Shape   Param #
===
dense_46 (Dense)   (None, 30)   23550
____
dense_47 (Dense)   (None, 30)   930
____
dense_48 (Dense)   (None, 30)   930
____
dense_49 (Dense)   (None, 10)   310
===
Total params: 25,720
Trainable params: 25,720
Non-trainable params: 0
____
Train on 1000 samples, validate on 1000 samples
Epoch 1/30
 - 1s - loss: 2.3351 - acc: 0.0830 - val_loss: 2.3130 - val_acc: 0.1000
 - LR: 0.099010

Epoch 2/30
 - 0s - loss: 2.3376 - acc: 0.0820 - val_loss: 2.3154 - val_acc: 0.1000
 - LR: 0.098039

Epoch 3/30
 - 0s - loss: 2.3357 - acc: 0.0790 - val_loss: 2.3136 - val_acc: 0.1000
 - LR: 0.097087

Epoch 4/30
 - 0s - loss: 2.3282 - acc: 0.1010 - val_loss: 2.3226 - val_acc: 0.1000
 - LR: 0.096154

Epoch 5/30
 - 0s - loss: 2.3324 - acc: 0.0940 - val_loss: 2.3248 - val_acc: 0.1000
 - LR: 0.095238

Epoch 6/30
 - 0s - loss: 2.3249 - acc: 0.0990 - val_loss: 2.3303 - val_acc: 0.1000
 - LR: 0.094340

Epoch 7/30
 - 0s - loss: 2.3307 - acc: 0.0990 - val_loss: 2.3114 - val_acc: 0.1000
 - LR: 0.093458

Epoch 8/30
 - 0s - loss: 2.3277 - acc: 0.0800 - val_loss: 2.3146 - val_acc: 0.1000
 - LR: 0.092593

Epoch 9/30
 - 0s - loss: 2.3229 - acc: 0.0880 - val_loss: 2.3205 - val_acc: 0.1000
 - LR: 0.091743

Epoch 10/30
 - 0s - loss: 2.3272 - acc: 0.0820 - val_loss: 2.3128 - val_acc: 0.1000
 - LR: 0.090909

Epoch 11/30
 - 0s - loss: 2.3247 - acc: 0.0780 - val_loss: 2.3104 - val_acc: 0.1000
 - LR: 0.090090

Epoch 12/30
 - 0s - loss: 2.3230 - acc: 0.1000 - val_loss: 2.3212 - val_acc: 0.1000
 - LR: 0.089286

Epoch 13/30
 - 0s - loss: 2.3268 - acc: 0.0810 - val_loss: 2.3152 - val_acc: 0.1000
 - LR: 0.088496

Epoch 14/30

- 0s - loss: 2.3223 - acc: 0.0930 - val_loss: 2.3130 - val_acc: 0.1000
- LR: 0.087719

Epoch 15/30
- 0s - loss: 2.3205 - acc: 0.0910 - val_loss: 2.3186 - val_acc: 0.1000
- LR: 0.086957

Epoch 16/30
- 0s - loss: 2.3255 - acc: 0.0880 - val_loss: 2.3134 - val_acc: 0.1000
- LR: 0.086207

Epoch 17/30
- 0s - loss: 2.3211 - acc: 0.0960 - val_loss: 2.3135 - val_acc: 0.1000
- LR: 0.085470

Epoch 18/30
- 0s - loss: 2.3220 - acc: 0.0840 - val_loss: 2.3113 - val_acc: 0.1000
- LR: 0.084746

Epoch 19/30
- 0s - loss: 2.3224 - acc: 0.0850 - val_loss: 2.3125 - val_acc: 0.1000
- LR: 0.084034

Epoch 20/30
- 0s - loss: 2.3218 - acc: 0.0860 - val_loss: 2.3126 - val_acc: 0.1000
- LR: 0.083333

Epoch 21/30
- 0s - loss: 2.3226 - acc: 0.0780 - val_loss: 2.3107 - val_acc: 0.1000
- LR: 0.082645

Epoch 22/30
- 0s - loss: 2.3174 - acc: 0.1070 - val_loss: 2.3152 - val_acc: 0.1000
- LR: 0.081967

Epoch 23/30
- 0s - loss: 2.3196 - acc: 0.1080 - val_loss: 2.3120 - val_acc: 0.1000
- LR: 0.081301

Epoch 24/30
- 0s - loss: 2.3153 - acc: 0.0970 - val_loss: 2.3198 - val_acc: 0.1000
- LR: 0.080645

Epoch 25/30
- 0s - loss: 2.3225 - acc: 0.0930 - val_loss: 2.3123 - val_acc: 0.1000
- LR: 0.080000

Epoch 26/30
 - 0s - loss: 2.3181 - acc: 0.1140 - val_loss: 2.3118 - val_acc: 0.1000
 - LR: 0.079365

Epoch 27/30
 - 0s - loss: 2.3184 - acc: 0.0880 - val_loss: 2.3116 - val_acc: 0.1000
 - LR: 0.078740

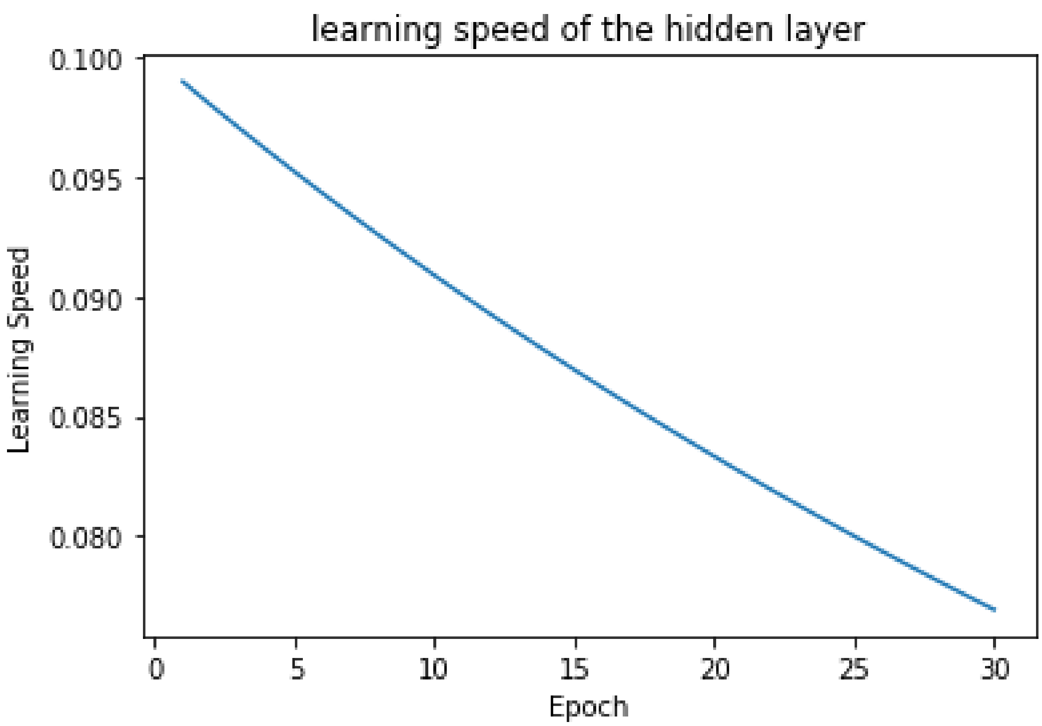
Epoch 28/30
 - 0s - loss: 2.3185 - acc: 0.0920 - val_loss: 2.3106 - val_acc: 0.1000
 - LR: 0.078125

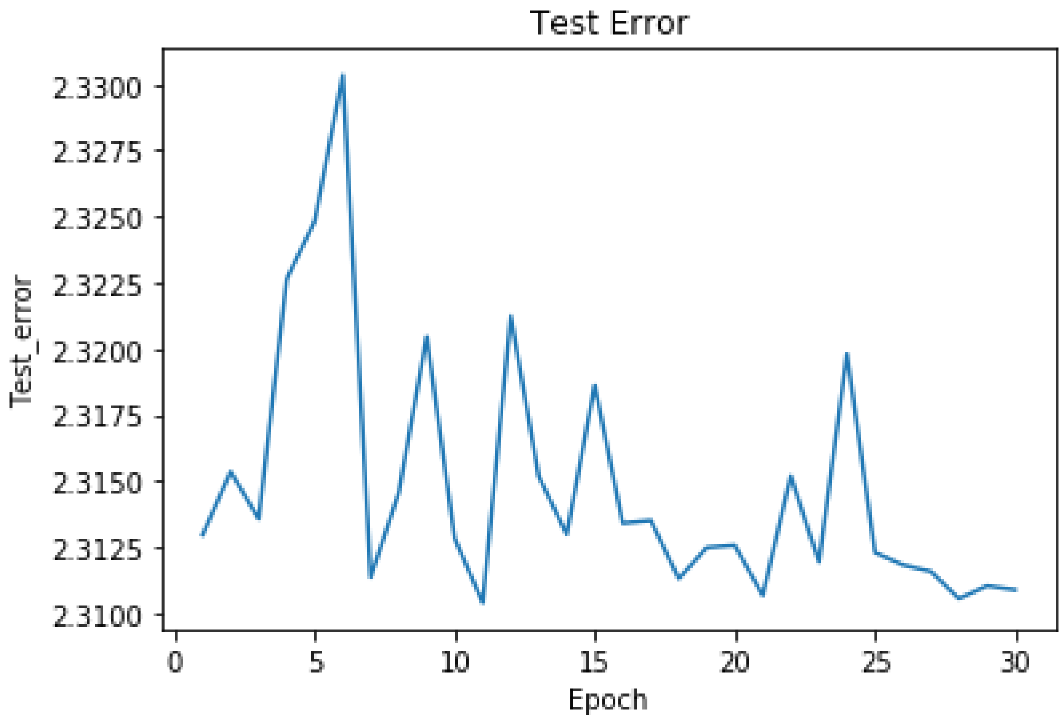
Epoch 29/30
 - 0s - loss: 2.3185 - acc: 0.0900 - val_loss: 2.3110 - val_acc: 0.1000
 - LR: 0.077519

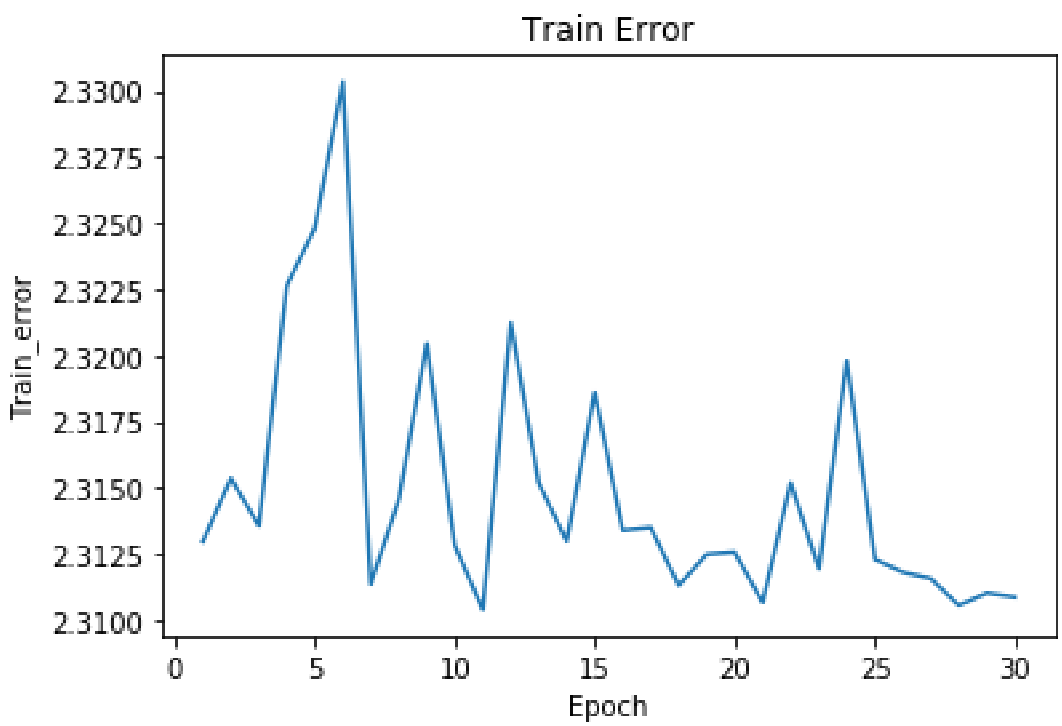
Epoch 30/30
 - 0s - loss: 2.3189 - acc: 0.0860 - val_loss: 2.3109 - val_acc: 0.1000
 - LR: 0.076923

Baseline Error: 10.00%
dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])


criterion function on training/testing data set
train
test
loss
epoch
2.335
2.330
2.325
2.320
2.315
2.310
0
5
10
15
20
25
30


learning speed of the hidden layer
Learning Speed
Epoch
0.100
0.095
0.090
0.085
0.080
0
5
10
15
20
25
30


Test Error
Test_error
2.3100
2.3125
2.3150
2.3175
2.3200
2.3225
2.3250
2.3275
2.3300
0
5
10
15
20
25
30
Epoch


Train Error
Train_error
2.3100
2.3125
2.3150
2.3175
2.3200
2.3225
2.3250
2.3275
2.3300
0
5
10
15
20
25
30
Epoch

2 (c). (5 points) Construct and train convolutional neural network for MNIST classification. Regularize the training of the neural network through dropout. Regularize the training of neural network through augment your selection of 1000 images by rotating them for 1-3 degrees clockwise and counter clockwise, and shifting them for 3 pixels in 8 different directions. You can find many tutorials on those techniques, and our emphasize is that we understand those techniques.

Soln:
Construct and train convolutional neural network for MNIST classification.
This is without dropout.
Filename: **MNIST_convo.py**
This was run on the whole 60000 training data and the 10000 testing data
It was run for 10 epochs. The output is below:
Train on 60000 samples, validate on 10000 samples
Epoch 1/10
 - 171s - loss: 0.2331 - acc: 0.9334 - val_loss: 0.0800 - val_acc: 0.9756
Epoch 2/10
 - 163s - loss: 0.0661 - acc: 0.9805 - val_loss: 0.0640 - val_acc: 0.9785
Epoch 3/10
 - 162s - loss: 0.0452 - acc: 0.9864 - val_loss: 0.0422 - val_acc: 0.9856
Epoch 4/10
 - 160s - loss: 0.0338 - acc: 0.9895 - val_loss: 0.0395 - val_acc: 0.9859
Epoch 5/10
 - 163s - loss: 0.0273 - acc: 0.9913 - val_loss: 0.0385 - val_acc: 0.9869
Epoch 6/10
 - 185s - loss: 0.0213 - acc: 0.9936 - val_loss: 0.0376 - val_acc: 0.9875
Epoch 7/10
 - 234s - loss: 0.0163 - acc: 0.9954 - val_loss: 0.0396 - val_acc: 0.9874
Epoch 8/10
 - 214s - loss: 0.0124 - acc: 0.9962 - val_loss: 0.0377 - val_acc: 0.9887
Epoch 9/10
 - 212s - loss: 0.0097 - acc: 0.9972 - val_loss: 0.0359 - val_acc: 0.9887
Epoch 10/10
 - 176s - loss: 0.0088 - acc: 0.9973 - val_loss: 0.0369 - val_acc: 0.9887
CNN Error: 1.13%
THE accuracy is 98.87 percent and the error is 1.13%.

With Dropout of 20 percent: File name is ***MNIST_convo_dropout.py***

This was run on the whole 60000 training data and the 10000 testing data
It was run for 10 epochs. The output is below:
Train on 60000 samples, validate on 10000 samples
Train on 60000 samples, validate on 10000 samples
Epoch 1/10
 - 223s - loss: 0.2315 - acc: 0.9343 - val_loss: 0.0815 - val_acc: 0.9743
Epoch 2/10
 - 198s - loss: 0.0738 - acc: 0.9781 - val_loss: 0.0469 - val_acc: 0.9839
Epoch 3/10
 - 178s - loss: 0.0532 - acc: 0.9839 - val_loss: 0.0425 - val_acc: 0.9862
Epoch 4/10
 - 180s - loss: 0.0403 - acc: 0.9879 - val_loss: 0.0402 - val_acc: 0.9869
Epoch 5/10
 - 186s - loss: 0.0336 - acc: 0.9894 - val_loss: 0.0341 - val_acc: 0.9883
Epoch 6/10
 - 185s - loss: 0.0273 - acc: 0.9915 - val_loss: 0.0301 - val_acc: 0.9899
Epoch 7/10
 - 171s - loss: 0.0233 - acc: 0.9927 - val_loss: 0.0342 - val_acc: 0.9886
Epoch 8/10
 - 166s - loss: 0.0202 - acc: 0.9938 - val_loss: 0.0324 - val_acc: 0.9882
Epoch 9/10
 - 167s - loss: 0.0169 - acc: 0.9944 - val_loss: 0.0297 - val_acc: 0.9901
Epoch 10/10
 - 164s - loss: 0.0142 - acc: 0.9960 - val_loss: 0.0316 - val_acc: 0.9910
CNN Error: 0.90%

As we see that with a dropout of 20 percent the accuracy increases:
Now the accuracy is 99.10 percent and the error is just 0.90 percent.

c) Regularize the training of neural network through augment your selection of 1000 images by rotating them for 1-3 degrees clockwise and counter clockwise, and shifting them for 3 pixels in 8 different directions. You can find many tutorials on those techniques, and our emphasize is that we understand those techniques.
Soln: The output is in file:**rotated_neural.py**

(4)

(c) Regularize the training of neural network through augment your selection of 1000 images by rotating them for L-3 degrees clockwise and counter clockwise and shifting them for 3 pixels in 8 different directions.

Soln:- In python, we can rotate an image by using the function

out = rotate(input_img, ~~image~~degree_rd, reshape = False)

→ degree_rot = 3 (counter-clockwise rotation of 3 degree

degree_rot = 357 (360 - 3) clockwise rotation of 3 degree

shifting in 8 directions: for each of 1000 images:

matrice of image
$$\begin{bmatrix} 0,0 & \cdots & 0,28 \\ \vdots & & \\ 28,0 & \cdots & 28,28 \end{bmatrix}$$

28,

The shifting is as follows:-

0,0 0,28

28,0 28,28

left shift rows, 14, 15, 16

shift in this direction

right shift rows 22, 23, 24

(direction)

top shift → ↑ → columns 17, 18, 19

bottom shift → ↓ columns 14, 15, 16

· 4 directions,

+

4 directions

"for this firstly both the diagonals were found out & then they were rotated in both the directions. (2+2) = 4 directions

→ Hence the image was counterclockwise rotated by 3 degree & 3 pixels shifted in 8 direction; right, left, top, bottom, & top left, top/right, bottom left, bottom right → Hence there were implemented

left, top, right, bottom

The output of the execution is below:

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_58 (Dense)             (None, 30)                23550
_________________________________________________________________
dense_59 (Dense)             (None, 10)                310
=================================================================
Total params: 23,860
Trainable params: 23,860
Non-trainable params: 0
_________________________________________________________________
Train on 1000 samples, validate on 1000 samples
Epoch 1/30
/Users/rajivranjan/anaconda3/lib/python3.6/site-packages/keras/models.py:942:
UserWarning: The `nb_epoch` argument in `fit` has been renamed `epochs`.
  warnings.warn('The `nb_epoch` argument in `fit` '
 - 1s - loss: 2.2702 - acc: 0.1840 - val_loss: 2.1824 - val_acc: 0.4030
 - LR: 0.090909

Epoch 2/30
 - 0s - loss: 2.0296 - acc: 0.4150 - val_loss: 1.8840 - val_acc: 0.5890
 - LR: 0.083333

Epoch 3/30
 - 0s - loss: 1.6351 - acc: 0.6260 - val_loss: 1.5323 - val_acc: 0.5910
 - LR: 0.076923

Epoch 4/30
 - 0s - loss: 1.2986 - acc: 0.7150 - val_loss: 1.2722 - val_acc: 0.6600
 - LR: 0.071429

Epoch 5/30
 - 0s - loss: 1.0656 - acc: 0.7660 - val_loss: 1.1014 - val_acc: 0.7120
 - LR: 0.066667

Epoch 6/30
```

- 0s - loss: 0.9016 - acc: 0.8070 - val_loss: 0.9920 - val_acc: 0.7340
- LR: 0.062500

Epoch 7/30
- 0s - loss: 0.7928 - acc: 0.8260 - val_loss: 0.8961 - val_acc: 0.7540
- LR: 0.058824

Epoch 8/30
- 0s - loss: 0.7084 - acc: 0.8480 - val_loss: 0.8332 - val_acc: 0.7620
- LR: 0.055556

Epoch 9/30
- 0s - loss: 0.6473 - acc: 0.8520 - val_loss: 0.7925 - val_acc: 0.7800
- LR: 0.052632

Epoch 10/30
- 0s - loss: 0.5960 - acc: 0.8650 - val_loss: 0.7525 - val_acc: 0.7820
- LR: 0.050000

Epoch 11/30
- 0s - loss: 0.5576 - acc: 0.8720 - val_loss: 0.7189 - val_acc: 0.7960
- LR: 0.047619

Epoch 12/30
- 0s - loss: 0.5241 - acc: 0.8750 - val_loss: 0.6959 - val_acc: 0.8020
- LR: 0.045455

Epoch 13/30
- 0s - loss: 0.4960 - acc: 0.8830 - val_loss: 0.6770 - val_acc: 0.7990
- LR: 0.043478

Epoch 14/30
- 0s - loss: 0.4717 - acc: 0.8890 - val_loss: 0.6613 - val_acc: 0.8000
- LR: 0.041667

Epoch 15/30
- 0s - loss: 0.4506 - acc: 0.8960 - val_loss: 0.6461 - val_acc: 0.8100
- LR: 0.040000

Epoch 16/30

- 0s - loss: 0.4319 - acc: 0.8950 - val_loss: 0.6338 - val_acc: 0.8090
- LR: 0.038462

Epoch 17/30
- 0s - loss: 0.4166 - acc: 0.9040 - val_loss: 0.6197 - val_acc: 0.8110
- LR: 0.037037

Epoch 18/30
- 0s - loss: 0.4014 - acc: 0.9050 - val_loss: 0.6106 - val_acc: 0.8120
- LR: 0.035714

Epoch 19/30
- 0s - loss: 0.3887 - acc: 0.9140 - val_loss: 0.6057 - val_acc: 0.8120
- LR: 0.034483

Epoch 20/30
- 0s - loss: 0.3768 - acc: 0.9140 - val_loss: 0.5936 - val_acc: 0.8150
- LR: 0.033333

Epoch 21/30
- 0s - loss: 0.3665 - acc: 0.9160 - val_loss: 0.5843 - val_acc: 0.8230
- LR: 0.032258

Epoch 22/30
- 0s - loss: 0.3563 - acc: 0.9200 - val_loss: 0.5790 - val_acc: 0.8220
- LR: 0.031250

Epoch 23/30
- 0s - loss: 0.3467 - acc: 0.9250 - val_loss: 0.5726 - val_acc: 0.8290
- LR: 0.030303

Epoch 24/30
- 0s - loss: 0.3388 - acc: 0.9240 - val_loss: 0.5658 - val_acc: 0.8290
- LR: 0.029412

Epoch 25/30
- 0s - loss: 0.3309 - acc: 0.9270 - val_loss: 0.5629 - val_acc: 0.8270
- LR: 0.028571

Epoch 26/30

- 0s - loss: 0.3240 - acc: 0.9280 - val_loss: 0.5590 - val_acc: 0.8270
- LR: 0.027778

Epoch 27/30
- 0s - loss: 0.3172 - acc: 0.9280 - val_loss: 0.5529 - val_acc: 0.8290
- LR: 0.027027

Epoch 28/30
- 0s - loss: 0.3104 - acc: 0.9330 - val_loss: 0.5489 - val_acc: 0.8310
- LR: 0.026316

Epoch 29/30
- 0s - loss: 0.3048 - acc: 0.9340 - val_loss: 0.5454 - val_acc: 0.8300
- LR: 0.025641

Epoch 30/30
- 0s - loss: 0.2992 - acc: 0.9340 - val_loss: 0.5447 - val_acc: 0.8260
- LR: 0.025000

Baseline Error: 17.40%
dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])

The accuracy is 82.60 percent and loss is 17.40 percent.

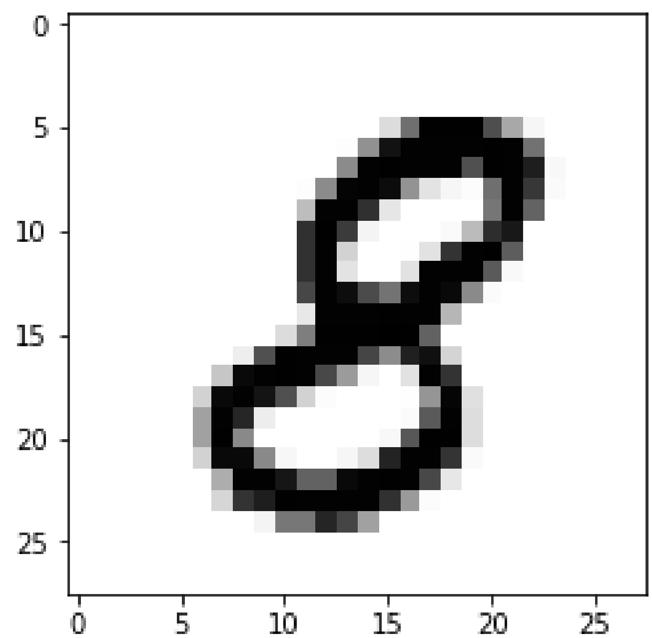
3. (Optional) Train GAN to generate the images for the 10 digits from random noise. Train autoencoder network with linear and sigmoid activation functions for principle component analysis. Train recurrent neural network to accept the 28 rows and output the digit of the image.

Soln:

a) Train GAN to generate the images for the 10 digits from random noise.
Soln: Filename: GANquestion.py
Here "z" is used is the random noise generator
Output is below:
It runs for 10 epochs:
Extracting ../MNIST_data/train-images-idx3-ubyte.gz
Extracting ../MNIST_data/train-labels-idx1-ubyte.gz
Extracting ../MNIST_data/t10k-images-idx3-ubyte.gz
Extracting ../MNIST_data/t10k-labels-idx1-ubyte.gz

['dis/dense/kernel:0', 'dis/dense/bias:0', 'dis/dense_1/kernel:0', 'dis/dense_1/bias:0', 'dis/dense_2/kernel:0', 'dis/dense_2/bias:0']
['gen/dense/kernel:0', 'gen/dense/bias:0', 'gen/dense_1/kernel:0', 'gen/dense_1/bias:0', 'gen/dense_2/kernel:0', 'gen/dense_2/bias:0']
Currently on Epoch 1 of 10 total...
Currently on Epoch 2 of 10 total...
Currently on Epoch 3 of 10 total...
Currently on Epoch 4 of 10 total...
Currently on Epoch 5 of 10 total...
Currently on Epoch 6 of 10 total...
Currently on Epoch 7 of 10 total...
Currently on Epoch 8 of 10 total...
Currently on Epoch 9 of 10 total...
Currently on Epoch 10 of 10 total...
INFO:tensorflow:Restoring parameters from ./models/500_epoch_model.ckpt

We see one of the samples created as follows:It's a digit 8.

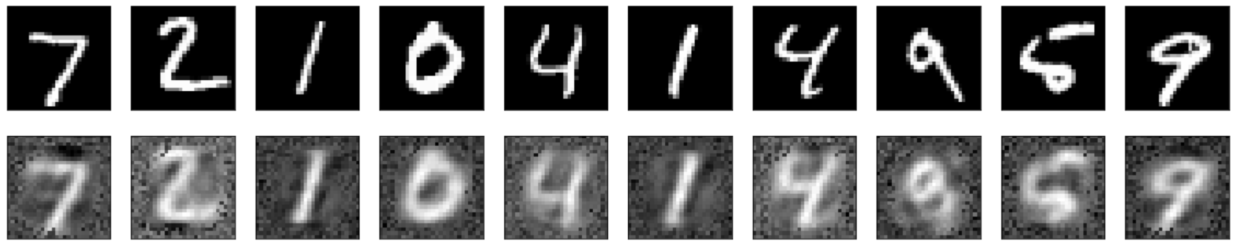


b) Train autoencoder network with linear and sigmoid activation functions for principle component analysis.
Soln:
Linear_autoencoder:filename is autoencoder_linear.py

The output is below:
It runs for 10 epochs:

Using TensorFlow backend.
(60000, 784)
(10000, 784)
Train on 60000 samples, validate on 10000 samples
Epoch 1/10
60000/60000 [==============================] - 7s 115us/step - loss: 0.3381 - val_loss: 0.2420
Epoch 2/10
60000/60000 [==============================] - 6s 101us/step - loss: 0.2178 - val_loss: 0.1997
Epoch 3/10
60000/60000 [==============================] - 7s 110us/step - loss: 0.1877 - val_loss: 0.1762
Epoch 4/10
60000/60000 [==============================] - 7s 112us/step - loss: 0.1727 - val_loss: 0.1659
Epoch 5/10
60000/60000 [==============================] - 7s 109us/step - loss: 0.1630 - val_loss: 0.1574
Epoch 6/10
60000/60000 [==============================] - 7s 111us/step - loss: 0.1519 - val_loss: 0.1468
Epoch 7/10
60000/60000 [==============================] - 7s 111us/step - loss: 0.1457 - val_loss: 0.1426
Epoch 8/10
60000/60000 [==============================] - 7s 112us/step - loss: 0.1416 - val_loss: 0.1375
Epoch 9/10
60000/60000 [==============================] - 7s 117us/step - loss: 0.1383 - val_loss: 0.1347
Epoch 10/10
60000/60000 [==============================] - 6s 108us/step - loss: 0.1358 - val_loss: 0.1328

The accuracy is 88 percent and the loss is 12 percent.
Below is the output of the linear encoder used for PCA.

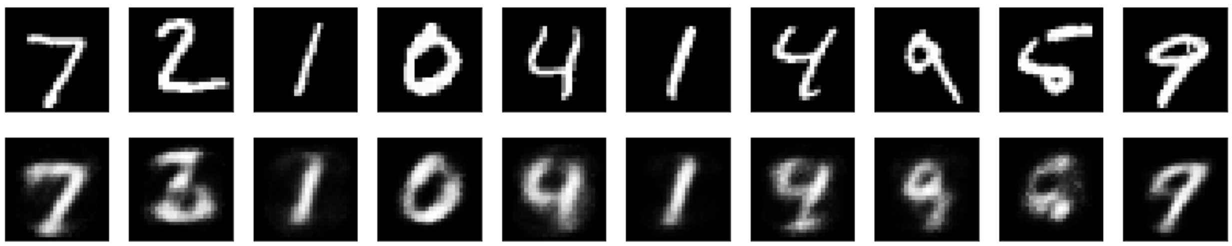


Sigmoid_autoencoder:filename is autoencoder_sigmoid.py

The output is below:
It runs for 10 epochs:
(60000, 784)
(10000, 784)
Train on 60000 samples, validate on 10000 samples
Epoch 1/10
60000/60000 [==============================] - 7s 115us/step - loss: 0.3627 - val_loss: 0.2712
Epoch 2/10
60000/60000 [==============================] - 7s 111us/step - loss: 0.2637 - val_loss: 0.2524
Epoch 3/10
60000/60000 [==============================] - 6s 103us/step - loss: 0.2414 - val_loss: 0.2284
Epoch 4/10
60000/60000 [==============================] - 6s 104us/step - loss: 0.2210 - val_loss: 0.2113
Epoch 5/10
60000/60000 [==============================] - 6s 108us/step - loss: 0.2067 - val_loss: 0.1995
Epoch 6/10
60000/60000 [==============================] - 7s 108us/step - loss: 0.1961 - val_loss: 0.1900
Epoch 7/10
60000/60000 [==============================] - 7s 115us/step - loss: 0.1875 - val_loss: 0.1821
Epoch 8/10
60000/60000 [==============================] - 6s 108us/step - loss: 0.1803 - val_loss: 0.1757
Epoch 9/10

60000/60000 [==============================] - 6s 104us/step - loss: 0.1743 - val_loss: 0.1701
Epoch 10/10
60000/60000 [==============================] - 7s 120us/step - loss: 0.1691 - val_loss: 0.1652

The accuracy is 84 percent and the loss is 16 percent.
Below is the output of the linear encoder used for PCA.
A sample of 10 digits is shown below:



Inference:
Sigmoid performs much better than the linear autoencoder as we can see clearly from the outputs generated by the execution of both the two types of autoencoders.
Also dimensionality reduction was done as follows:
encoding_dim = 32
# 32 floats -> compression of factor 24.5, assuming the input is 784 floats

c)Train recurrent neural network to accept the 28 rows and output the digit of the image.

Soln: The output is in file **Rnn_ques.py**

The output is below:
It was run for 800 iterations with a batch size of 128.
The accuracy is 98.43 percent and 1.5 percent is the error rate.

The o/p is:
Extracting /tmp/data/train-images-idx3-ubyte.gz
Extracting /tmp/data/train-labels-idx1-ubyte.gz
Extracting /tmp/data/t10k-images-idx3-ubyte.gz
Extracting /tmp/data/t10k-labels-idx1-ubyte.gz

WARNING:tensorflow:From /Users/rajivranjan/Desktop/Rnn_ques.py:45: softmax_cross_entropy_with_logits (from tensorflow.python.ops.nn_ops) is deprecated and will be removed in a future version.
Instructions for updating:

Future major versions of TensorFlow will allow gradients to flow
into the labels input on backprop by default.

See tf.nn.softmax_cross_entropy_with_logits_v2.

For iter 10
Accuracy 0.320312
Loss 1.97964

____

For iter 20
Accuracy 0.59375
Loss 1.3016

____

For iter 30
Accuracy 0.609375
Loss 1.18073

____

For iter 40
Accuracy 0.648438
Loss 1.06685

____

For iter 50
Accuracy 0.695312
Loss 0.84303

____

For iter 60
Accuracy 0.703125
Loss 0.959818

____

For iter 70
Accuracy 0.789062
Loss 0.665865

____

For iter 80
Accuracy 0.851562

Loss 0.454843

For iter 90
Accuracy 0.789062
Loss 0.513934

For iter 100
Accuracy 0.867188
Loss 0.63647

For iter 110
Accuracy 0.820312
Loss 0.505687

For iter 120
Accuracy 0.867188
Loss 0.467315

For iter 130
Accuracy 0.914062
Loss 0.286141

For iter 140
Accuracy 0.882812
Loss 0.401626

For iter 150
Accuracy 0.90625
Loss 0.358282

For iter 160
Accuracy 0.882812
Loss 0.317586

For iter 170
Accuracy 0.921875
Loss 0.2495

For iter 180
Accuracy 0.875

Loss 0.351491

For iter 190
Accuracy 0.914062
Loss 0.264019

For iter 200
Accuracy 0.921875
Loss 0.238182

For iter 210
Accuracy 0.914062
Loss 0.264488

For iter 220
Accuracy 0.960938
Loss 0.185066

For iter 230
Accuracy 0.898438
Loss 0.25824

For iter 240
Accuracy 0.9375
Loss 0.223796

For iter 250
Accuracy 0.929688
Loss 0.284195

For iter 260
Accuracy 0.929688
Loss 0.254908

For iter 270
Accuracy 0.921875
Loss 0.20947

For iter 280
Accuracy 0.914062

Loss 0.241348

For iter 290
Accuracy 0.945312
Loss 0.190349

For iter 300
Accuracy 0.9375
Loss 0.198062

For iter 310
Accuracy 0.921875
Loss 0.213394

For iter 320
Accuracy 0.921875
Loss 0.228114

For iter 330
Accuracy 0.914062
Loss 0.150239

For iter 340
Accuracy 0.960938
Loss 0.132705

For iter 350
Accuracy 0.914062
Loss 0.310001

For iter 360
Accuracy 0.953125
Loss 0.127136

For iter 370
Accuracy 0.921875
Loss 0.27158

For iter 380
Accuracy 0.960938

Loss 0.130141

For iter 390
Accuracy 0.953125
Loss 0.167404

For iter 400
Accuracy 0.914062
Loss 0.22073

For iter 410
Accuracy 0.929688
Loss 0.180388

For iter 420
Accuracy 0.945312
Loss 0.181368

For iter 430
Accuracy 0.96875
Loss 0.123066

For iter 440
Accuracy 0.96875
Loss 0.0826505

For iter 450
Accuracy 0.945312
Loss 0.14263

For iter 460
Accuracy 0.984375
Loss 0.0508476

For iter 470
Accuracy 0.984375
Loss 0.0804394

For iter 480
Accuracy 0.976562

Loss 0.129626

For iter 490
Accuracy 0.945312
Loss 0.180093

For iter 500
Accuracy 0.945312
Loss 0.160532

For iter 510
Accuracy 0.992188
Loss 0.0616836

For iter 520
Accuracy 0.960938
Loss 0.130104

For iter 530
Accuracy 0.96875
Loss 0.110087

For iter 540
Accuracy 0.960938
Loss 0.087577

For iter 550
Accuracy 0.992188
Loss 0.0458895

For iter 560
Accuracy 0.9375
Loss 0.169928

For iter 570
Accuracy 0.976562
Loss 0.16944

For iter 580
Accuracy 0.929688

Loss 0.203101

For iter 590
Accuracy 0.960938
Loss 0.130237

For iter 600
Accuracy 0.929688
Loss 0.16592

For iter 610
Accuracy 0.96875
Loss 0.157914

For iter 620
Accuracy 0.945312
Loss 0.174224

For iter 630
Accuracy 0.953125
Loss 0.13913

For iter 640
Accuracy 0.984375
Loss 0.0620078

For iter 650
Accuracy 0.976562
Loss 0.0557024

For iter 660
Accuracy 0.96875
Loss 0.13908

For iter 670
Accuracy 0.984375
Loss 0.0445744

For iter 680
Accuracy 0.953125

Loss 0.22088

For iter 690
Accuracy 0.960938
Loss 0.118377

For iter 700
Accuracy 0.921875
Loss 0.169052

For iter 710
Accuracy 0.945312
Loss 0.155573

For iter 720
Accuracy 0.960938
Loss 0.0951964

For iter 730
Accuracy 0.96875
Loss 0.104905

For iter 740
Accuracy 0.984375
Loss 0.0558364

For iter 750
Accuracy 0.984375
Loss 0.0479834

For iter 760
Accuracy 0.960938
Loss 0.116536

For iter 770
Accuracy 0.96875
Loss 0.111931

For iter 780
Accuracy 0.960938

Loss 0.149431

____

For iter 790
Accuracy 0.960938
Loss 0.129065

____

Testing Accuracy: 0.984375

4. (Optional) Train Bayesian neural network with variational and sampling based method using Edward and Tensorflow. We will cover Bayesian neural network in the lecture.
Soln:
The filename is: variational_sampling.py
The output is below: The code runs for around 3 hours.
The code also runs only for the Tensorflow version 1.2.0 .

I have constructed a simple Bayesian statistical model for MNIST image classification using TensorFlow and Edward. Understanding uncertainty in statistical inference is very important for a variety of applications and we have explored some basic methods for visualising this problem.

Traditional approaches to training neural networks typically produce a point estimate by optimising the weights and biases to minimize a loss function, such as a cross-entropy loss in the case of a classification problem. From the probabilistic viewpoint, this is equivalent to maximising the log likelihood of the observed data $P(D \mid \omega)$ to find the maximum likelihood estimate (MLE), following [Blundell et. al. 2015](#)

$$\begin{aligned}\omega^{\text{MLE}} &= \underset{\omega}{\operatorname{argmax}} \ \log P(D \mid \omega) \\ &= \underset{\omega}{\operatorname{argmax}} \ \sum_{i=1}^{N} \log P(y_i \mid x_i, \omega).\end{aligned}$$

This optimisation is typically carried out using some form of gradient descent (e.g., backpropagation), and then with the weights and biases fixed we can predict a new output $y^* = f(x^*; \omega)$ for a given input x^*.

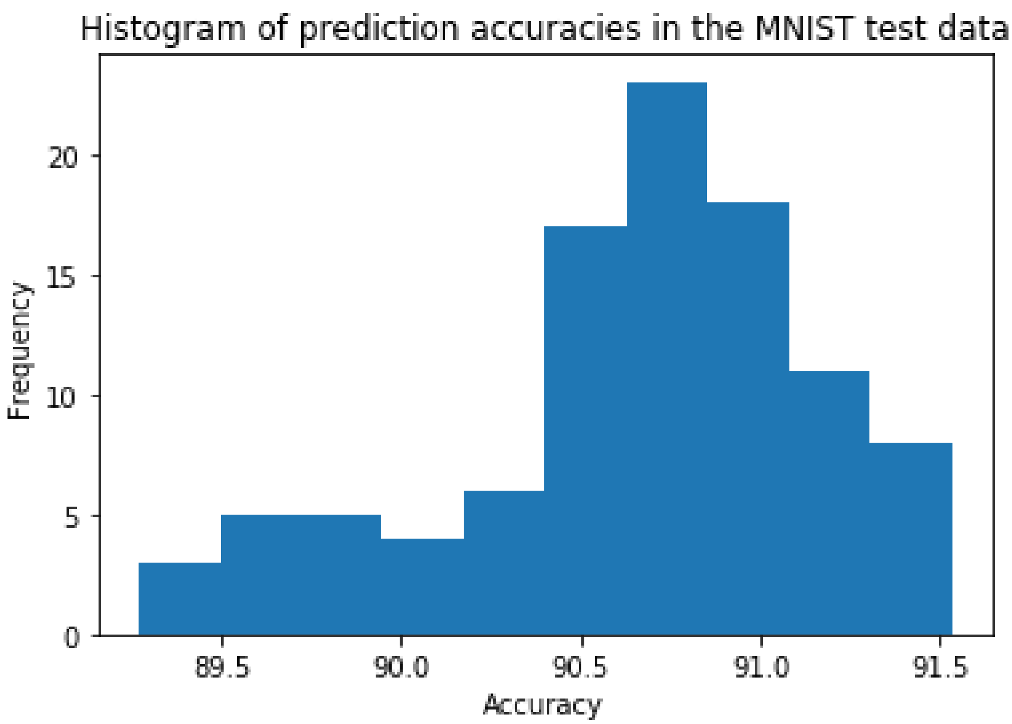
Training a neural network in this way is well known to be prone to overfitting and so often we introduce regularisation term such as an L_2 norm of the weights. One can show that placing L_2 regularization of the weights is equivalent to placing a normal Gaussian prior $P(\omega) \sim (0, I)$ on the weights and maximising the posterior estimate $p(\omega \mid D)$. This gives us the Maximum a-Posteriori estimate (MAP) of the parameters (see chapter 41 of MacKay's [book](#) for details):

$$\begin{aligned}\omega^{\text{MAP}} &= \underset{\omega}{\operatorname{argmax}} \ \log P(\omega \mid D) \\ &= \underset{\omega}{\operatorname{argmax}} \ \log P(D \mid \omega) + \log P(\omega).\end{aligned}$$

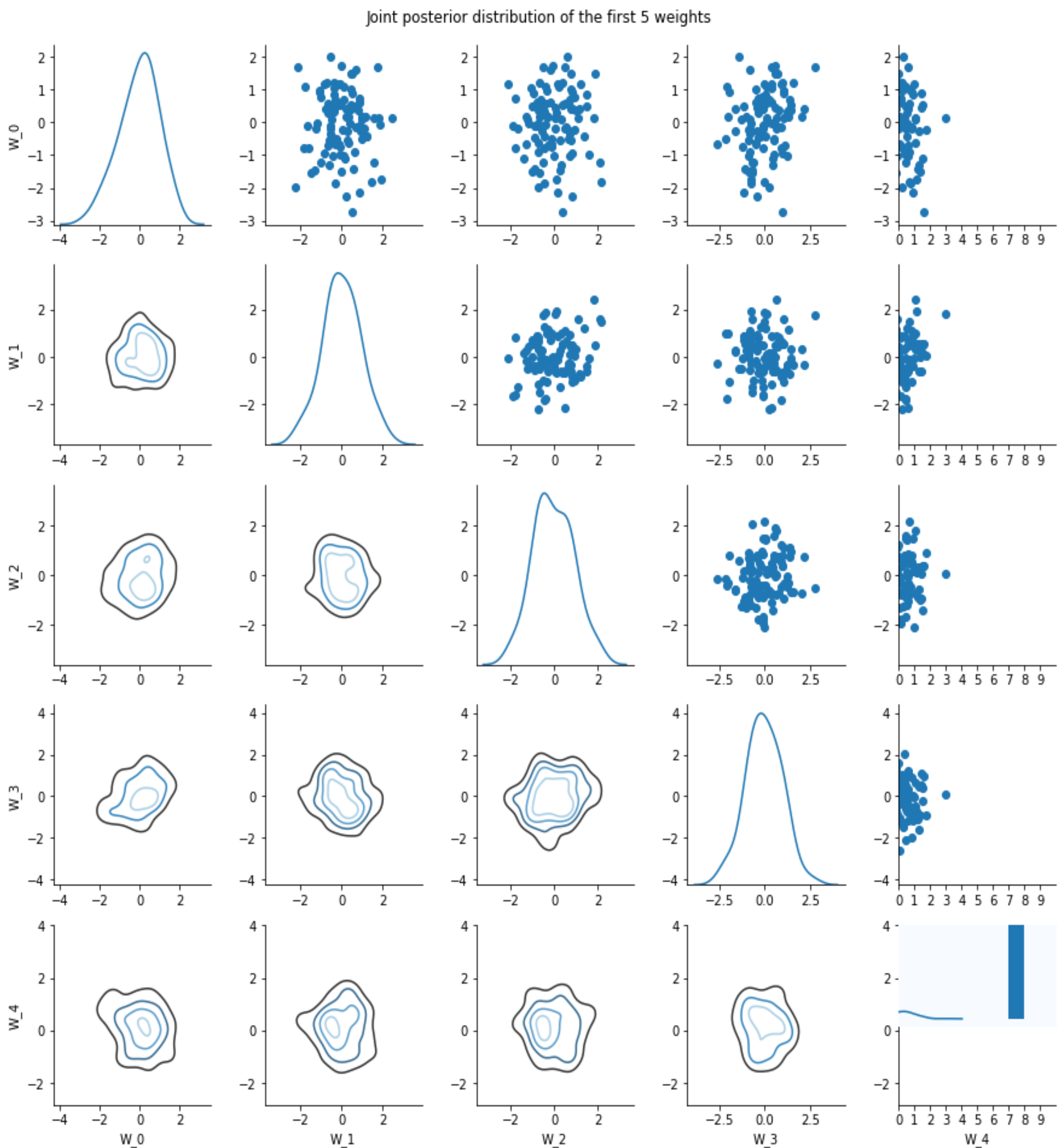
From this we can see that traditional approaches to neural network training and regularisation can be placed within the framework of performing inference using Bayes' rule. Bayesian Neural Networks go one step further by trying to approximate the entire posterior distribution $P(\omega \mid D)$ using either Monte Carlo or Variational Inference techniques. In the rest of the tutorial we will show you how to do this using Tensorflow and [Edward](#).

The output is below:

```
In [4]: runfile('/Users/rajivranjan/Desktop/variational_sampling.py', wdir='/Users/rajivranjan/Desktop')
Reloaded modules: edward, edward.criticisms, edward.criticisms.evaluate, edward.models, edward.models.dirichlet_process, edward.models.random_variable,
edward.models.random_variables, edward.models.empirical, edward.models.param_mixture, edward.models.point_mass, edward.util, edward.util.graphs,
edward.util.metrics, edward.util.progbar, edward.util.random_variables, edward.util.tensorflow, edward.criticisms.ppc, edward.criticisms.ppc_plots,
edward.inferences, edward.inferences.bigan_inference, edward.inferences.gan_inference, edward.inferences.variational_inference,
edward.inferences.inference, edward.inferences.conjugacy, edward.inferences.conjugacy.conjugacy, edward.inferences.conjugacy.conjugate_log_probs,
edward.inferences.conjugacy.simplify, edward.inferences.gibbs, edward.inferences.monte_carlo, edward.inferences.hmc, edward.inferences.implicit_klqp,
edward.inferences.klpq, edward.inferences.klqp, edward.inferences.laplace, edward.inferences.map, edward.inferences.metropolis_hastings,
edward.inferences.replica_exchange_mc, edward.inferences.sgld, edward.inferences.sghmc, edward.inferences.wake_sleep, edward.inferences.wgan_inference,
edward.version
Extracting MNIST_data/train-images-idx3-ubyte.gz
Extracting MNIST_data/train-labels-idx1-ubyte.gz
Extracting MNIST_data/t10k-images-idx3-ubyte.gz
Extracting MNIST_data/t10k-labels-idx1-ubyte.gz
5000/5000 [100%] ██████████████████████████████ Elapsed: 17s | Loss: 34901.473
accuracy in predicting the test data =  92.39
/Users/rajivranjan/anaconda3/lib/python3.6/site-packages/matplotlib/contour.py:967: UserWarning: The following kwargs were not used by contour: 'label',
'color'
  s)
truth =  7
```



We should also look at the posterior distribution. Unfortunately, the number of dimensions is quite large even for a small problem like this and so visualising them is tricky! We look at the first 5 dimensions and produce a triangle plot of the correlations.



Joint posterior distribution of the first 5 weights
W_0
W_1
W_2
W_3
W_4