

Segment Tree

5/08/2020

1	3	2	4	3	-2	6
0	1	2	3	4	5	6

Q1 Range minimum query.
minimum element in a range.

e.g.	0-3	1
	2-4	2
	4-6	-2

How we did.

(1)

using linear search. L-R.

$O(n)$ for 1 query.
 $O(n^2)$ for n queries.

$\Theta \uparrow \propto T \uparrow$.

so segment tree will reduce query time.

(2) Pre computation. ($O(n^2)$ time to build)

	0	1	2	3	4	5	6
0							
1	*						
2	*	*	.				
3	*	*	*				
4	*	*	*	*	*		
5	*	*	*	*		*	
6	*	*	*	*	*	*	

query $O(1)$ time

① Queries $O(1)$.

② updates $O(n^2)$.

(In 2nd approach)

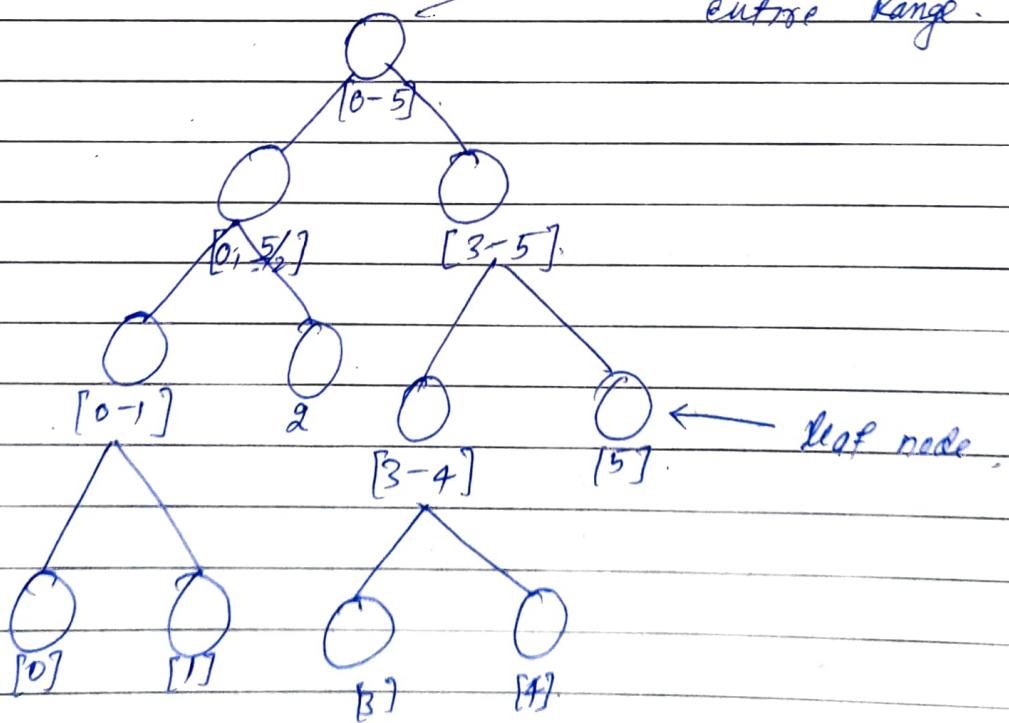
1000 q. 1000 Update.
 $O(1).$ $O(n^2).$

① data structure -

Segment Tree for above problem. (complete b.t.).

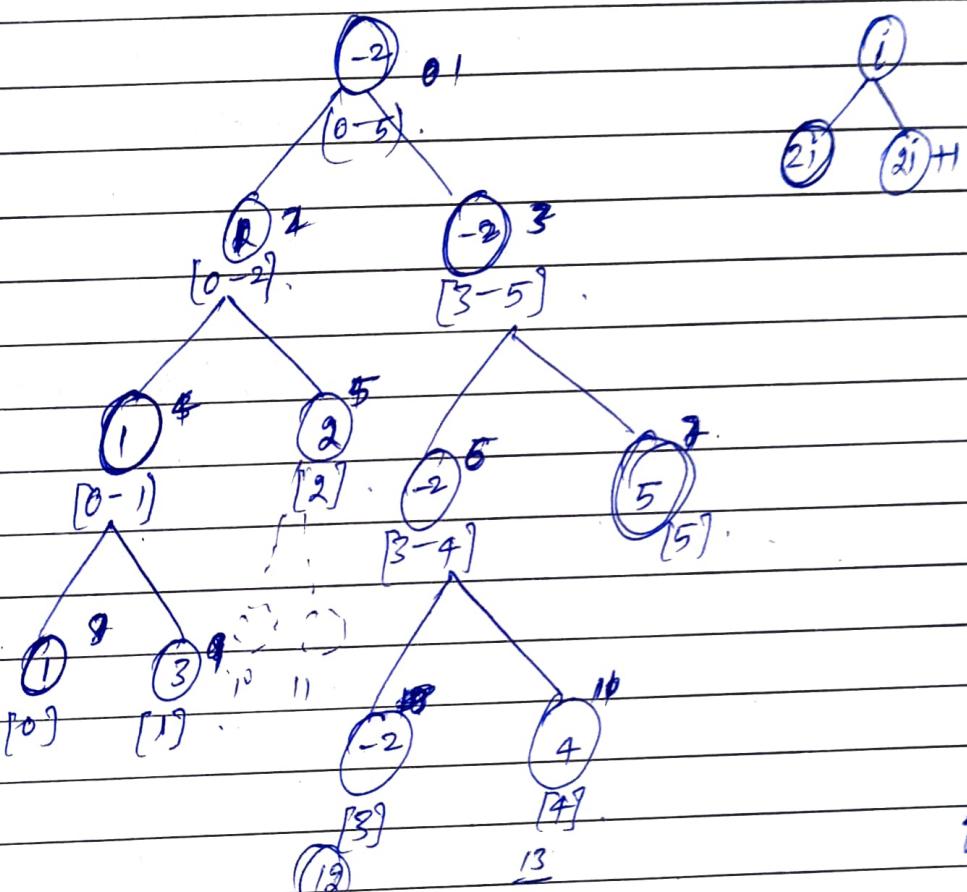
1	3	2	-2	4	5
0	1	2	3	4	5

nodes store min of entire Range.



Segment Tree (based on divide & conquer)

1	3	2	-2	4	5
0	1	2	3	4	5



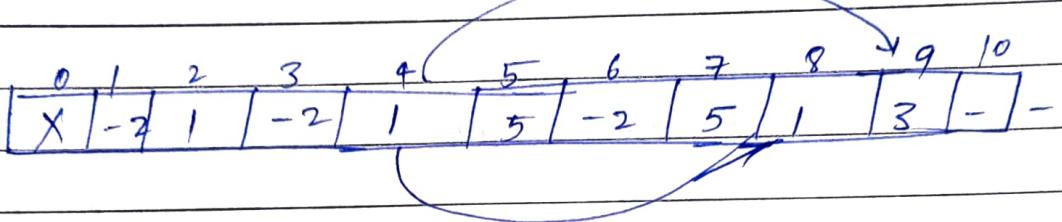
- ① Leaf node fill करना
 ② bottom up way द्वारा रेंज से fill करना

- ① Build
 ② query
 ③ update

① Array to build tree

(Bottom up way
 Recursion)

→ used to store data in hierarchical tree.



Tree → Hierarchical info stored ~~in array~~
 ↓
(Array Relationship)

At no. of elements = $2n - 1$
 = n leaf nodes.
 = $n-1$ internal nodes.

Array size = $2 \times 2^{\lceil \log_2 N \rceil} - 1$

$\underbrace{\hspace{10em}}$

not need to

Max array size $\approx 4N + 1$
 to build a tree of size N

→ we can start numbering from 0 as well.

③ actual:

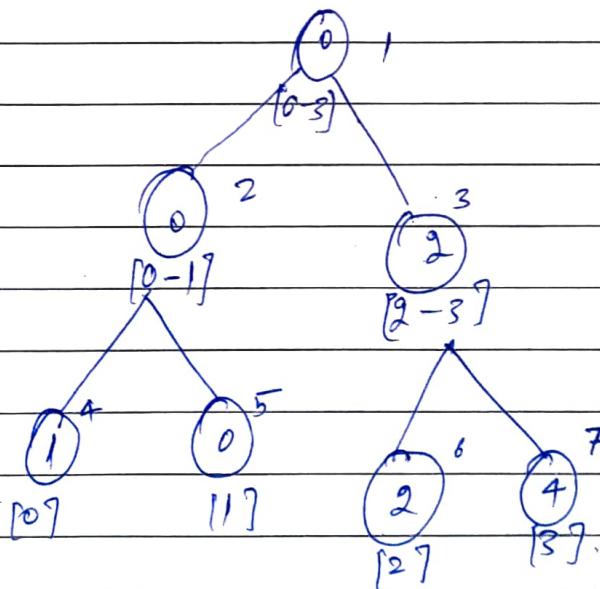
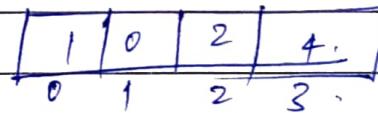
$$\begin{aligned}\text{Segm cut} &\rightarrow 2N - 1 \\ &= 2 \times 6 - 1 \\ &= 11\end{aligned}$$

so worst case # numbering will go to,

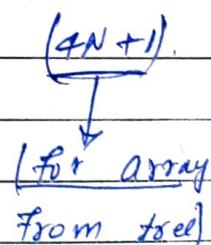
$$\begin{aligned}&4 \times 6 - 1 \\ &\Leftarrow 24 + 1 \rightarrow \text{if start from 1} \\ &\Leftarrow 3\end{aligned}$$

$$= 0.4N * 1 \rightarrow \text{if start from 0.}$$

eg.



$$\begin{aligned}\text{tree node} &= 2N + 1 \\ &= 7.\end{aligned}$$



Divide & Conquer :- problem solving technique.

Segment Tree :- Data Structure.

void buildTree(int *tree, int a, int index, int s, int e)
 {

// Base Case:

if ($s > e$) {
 return;

}

// Base Case for Leaf node.

if ($s == e$) {

tree[index] = a[s];

return;

}

// Recursive Calls.

int mid = $\frac{s+e}{2}$.

// Left Sub Tree.

buildTree (tree, a, 2 * index + 1, mid + 1, e);

right

buildTree (tree, a, 2 * index + 1, mid + 1, e);

int left = tree[2 * index];

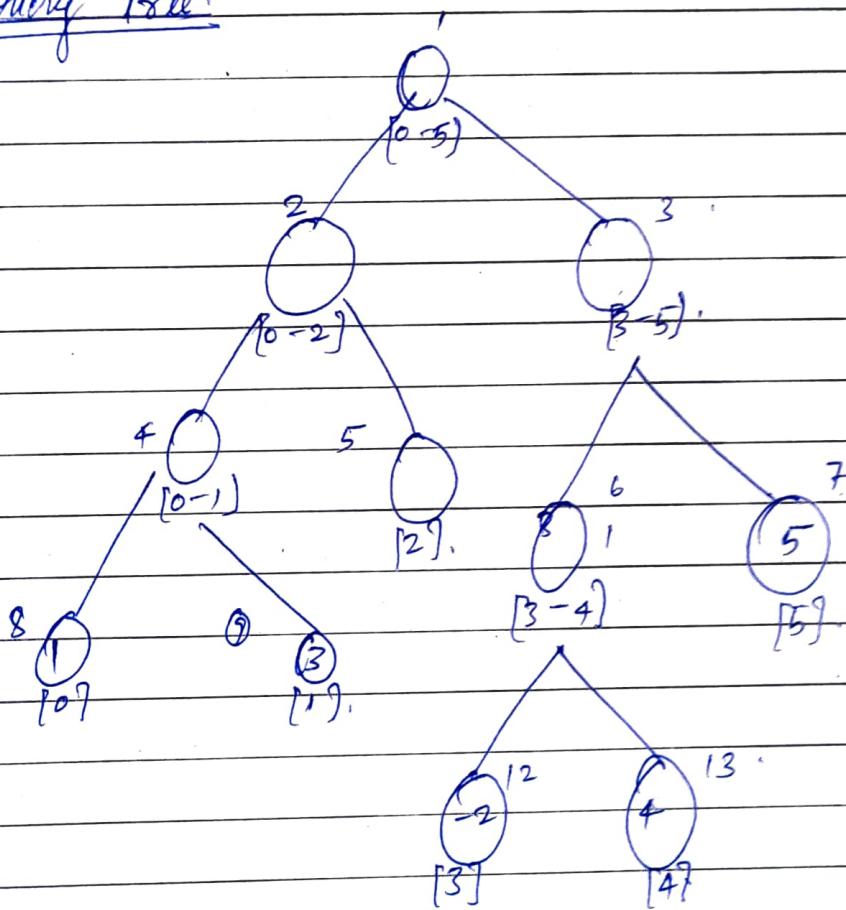
int right = tree[2 * index + 1];

tree[index] = min(left, right);

?

Range QR with respect to the segment tree is used.

Every tree:

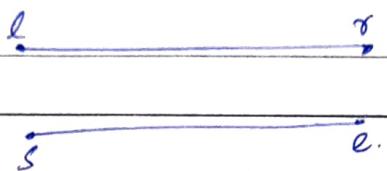
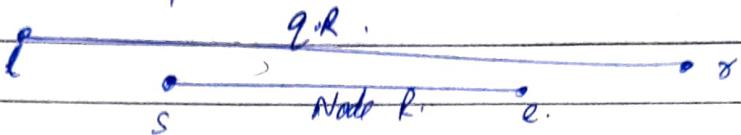


3 Case:

we are finding
query \rightarrow min
but can be
anything.

query Range, node Range

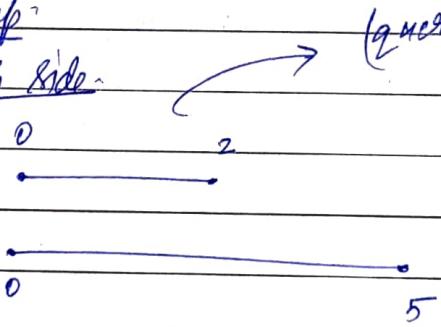
① Complete overlap



In that case return
tree [index] $\text{ट्री } \text{पर } \text{इ } \text{स } \text{ू } \text{फ } \text{ू } \text{र }$

② Partial overlap

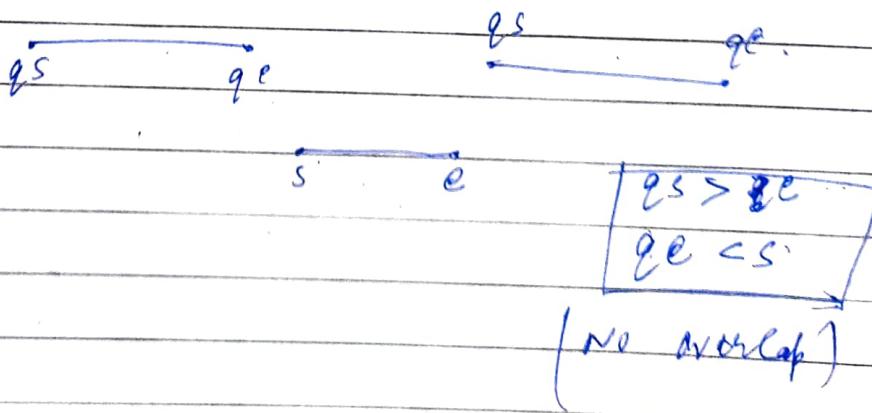
call on both side



(query) Range.

③

No overlap
return '0'

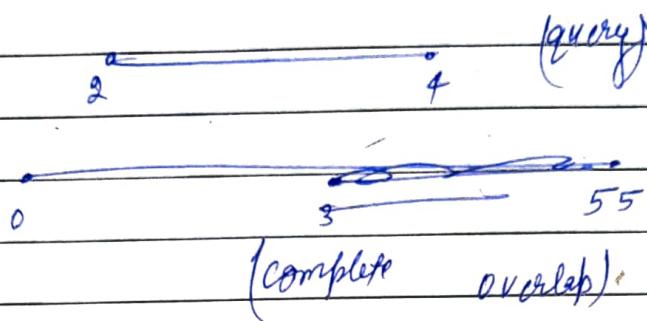


segment tree [index]

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Date : _____

query & range partially lies on node & range.

(3)



// return a min element from the tree lying in range q8 and qe.

int query(int *tree, int index, int s, int e, int q8, int qe)

// 3 cases .

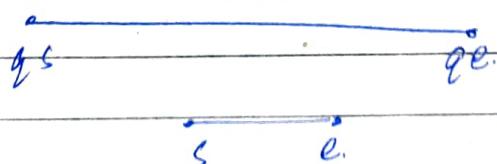
// 1. No overlap.

if (q8 > e || qe < s) {

return INT_MAX;

}

// 2. Complete Overlap.



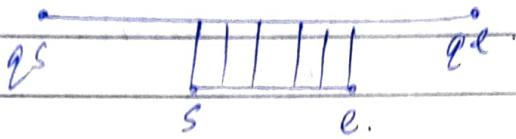
$s \geq q8 \text{ and } e \leq qe$.

index = shows current array index

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Date:

3. If Node at range lies within query range.
then only will be complete overlap.



$$s \geq q_s \text{ and } e \leq q_e$$

if ($s \geq q_s$ and $e \leq q_e$){

return tree[index];

}

3. Partial overlap. - Call both side.

$$\text{int mid} = \left\lfloor \frac{s+e}{2} \right\rfloor;$$

$$\text{int left mid} = \left\lfloor \frac{s+e}{2} \right\rfloor;$$

$$\text{int leftAns} = \text{query}(\text{tree}, 2 * \text{index}, s, \text{mid}, q_s, q_e);$$

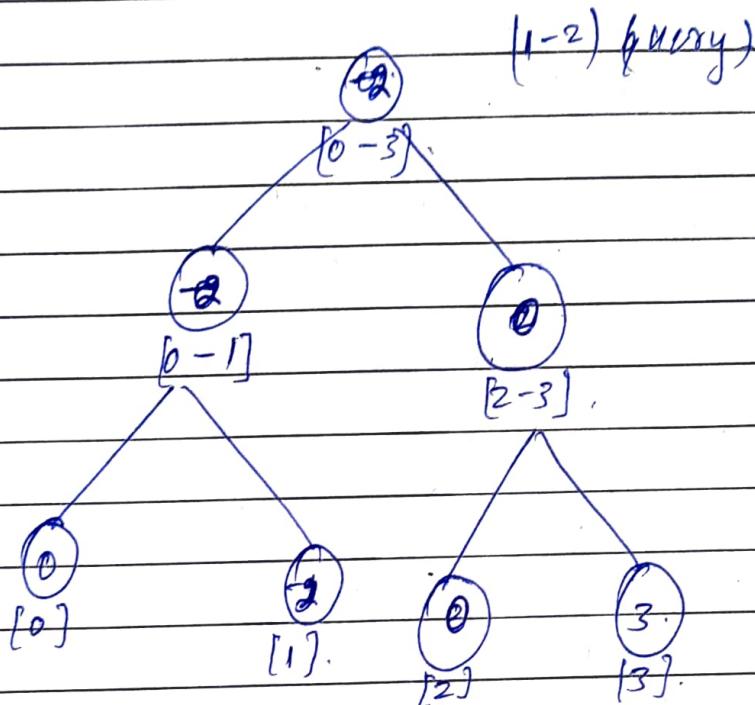
$$\text{int rightAns} = \text{query}(\text{tree}, 2 * \text{index} + 1, \text{mid} + 1, e, q_s, q_e);$$

$$\text{return min(leftAns, rightAns);}$$

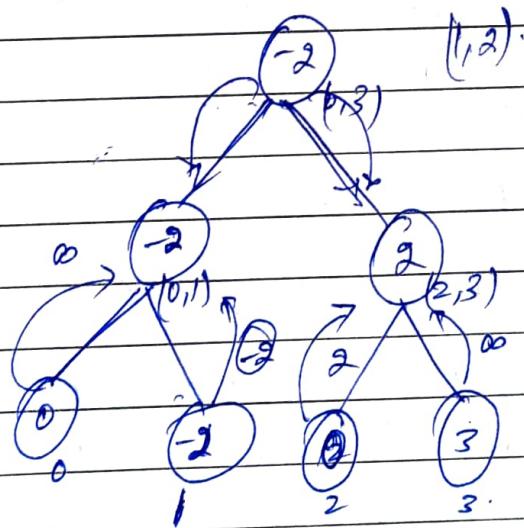
};

dry run

1	-2	0	3.
0	1	2	3.



Partial overlap



if query range out of helper range.

check wth mt helper range & through

query(tree, l, s, e, qs,qe);

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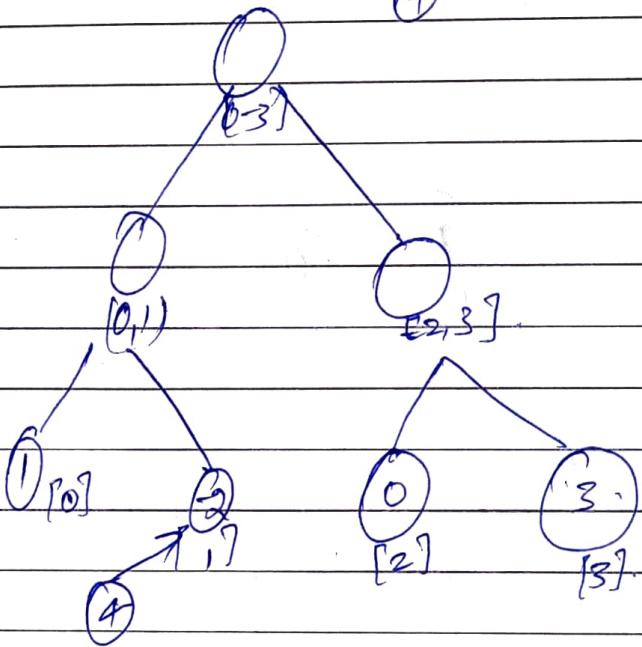
Update Function

1	1	0	3	-2	.
---	---	---	---	----	---

how to update

- ① Node update
- ② Range update.

1	1	-2	0	3	.
---	---	----	---	---	---



i=1, v=4.

i = 1, j = 1, Index update at 0, 1, 2, (value = $\frac{\text{old value} + \text{new value}}{2}$)
Void UpdateNode (int *tree, int index, int s, int e,
int i, int value).

// no overlap

```
if (i < s || i > e) {
    return;
```

// Reached leafNode.

```
if (s == e) {
```

```
    tree[index] = value;
    return;
```

}

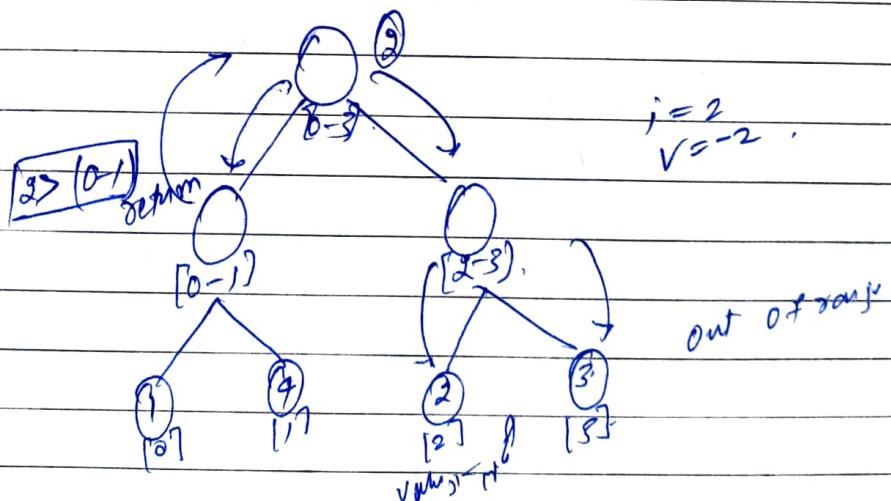
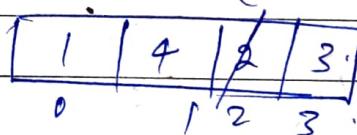
// Partial overlap: 'i' is being in b/w (s, e) range.

```
else int mid = (s + e) / 2;
```

```
UpdateNode(tree, 2 * index, s, mid, i, value);
```

```
UpdateNode(tree, 2 * index + 1, mid + 1, e, i, value);
```

?



* Range Update

↑ment every value in a range by a Value 'V'.

Operations supported

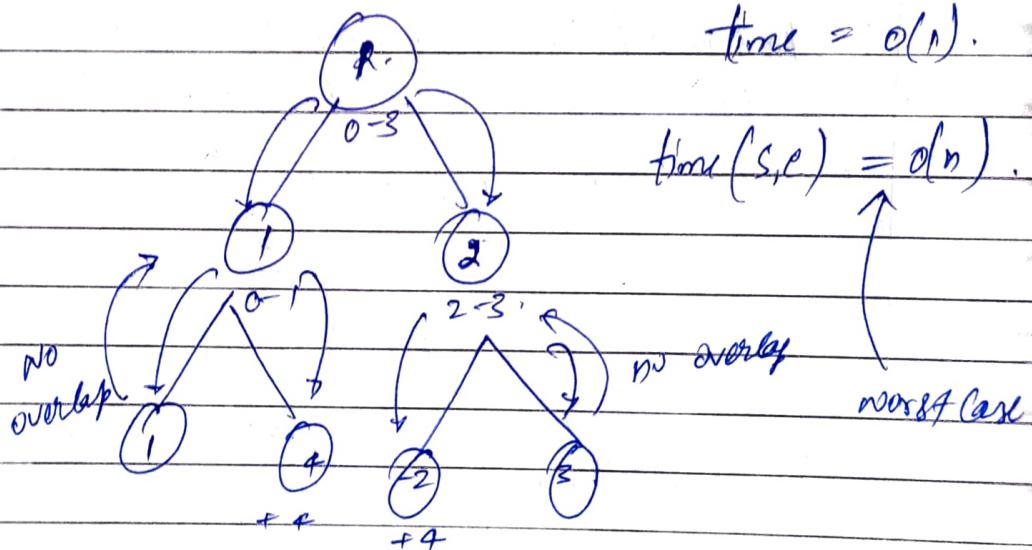
- ① Range update
- ② query
- ③ Build a segment tree.
- ④ Node update.

Delete is not function of segment tree.

$$RU(1,2) = 4 \uparrow \text{ment}$$

1	4	-2	3
0	1	2	3

1	8	2	3
---	---	---	---



void updateRange (int *tree, int index, int s, int e, int rs, int re, int inc) {

// No overlap

if (re < s, rs > e) {
 return;

}

// Reached to leaf node.

if (s == e) {
 tree[index] += inc;
 return;
}

// lying in range call both side.

int mid = (s+e)/2;

updateRange (tree, 2*index, s, mid, rs, re, inc);

updateRange (tree, 2*index + 1, mid + 1, e, rs, re, inc);

tree[index] = min (tree[2*index], tree[2*index + 1]);

return;

?

```
int main() {
```

```
    int a[] = {1, 4, -2, 3};  
    int n = 4;
```

// Build the tree array.

```
    int *tree = new int[4 * n + 1];  
    int index = 1;  
    int s = 0;  
    int e = n - 1;
```

```
    buildtree(tree, a, index, s, e);
```

```
    int no_of_q;
```

```
cin >> no_of_q;
```

```
    updateNode(tree, 1, s, e, 1);
```

```
    while (no_of_q--) {
```

```
        int qs, qe;
```

```
        cin >> qs >> qe;
```

```
        cout << "Min value b/w range is";
```

```
        cout << query(tree, 1, s, e, qs, qe) << endl;
```

```
}
```

```
return 0;
```

① Build = $O(N)$

because $4N + 1$ nodes are build.

So,

$O(N)$.

② query = $\log(N)$.

③ update = $\log(N)$.

④ Range update = $O(N)$.

$O(b, e) = O(N)$
In worst case

Pointers

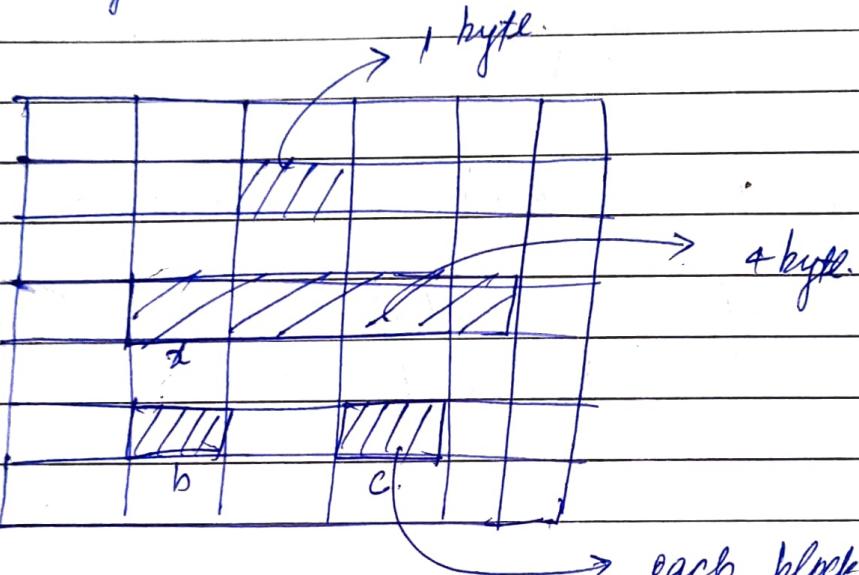
int 4 byte.

bool 1 byte.

char 1 byte.

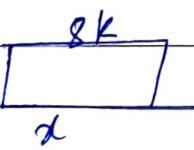
float 4 byte.

1 byte = 8 bits.

Memoryint $x = 10$.

add in terms of hexa decimal (0-9 and a-f).

& operator called address of operator.



if we write

& operator gives all

whole goes and find
Address of x in terms of hexadecimal format

Pointer :- Pointer is a method by which we stores int these address value.

int * aptr ;

→ says we are going to make pointer 'aptr' which is going to stores address of integer type variable.

int * aptr = &x .

① To store integer we write 'int'

② For boolean type we write 'bool'

③ similarly to store address we writes '*'

int * aptr



int * aptr = (&x) ; or int * aptr
return 8k ;

↑

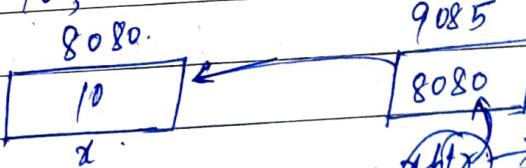
both

same .

aptr stores 8k .

int x = 10 ;

8080 .



9085

8080

aptr

gives add of
value at which it
was pointing .

Same for float & boolean as well.

generic pointer :-

T^* Var
→ Data type

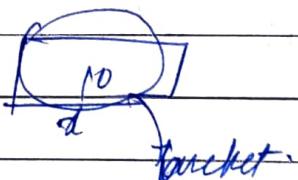
$T^* \text{ Varptr} = k \text{ var}$
→
Stores add of generic data type

Dereference operator :-

$k \rightarrow$ add of variable.

$*k$ denotes the variable not address.

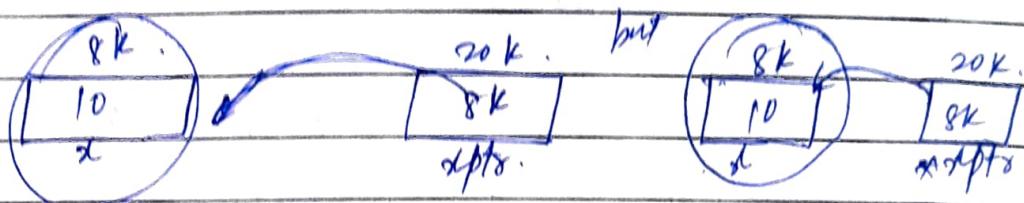
Var. | bucket (\downarrow) → address.



address → \uparrow bucket

dereference. can get bucket.

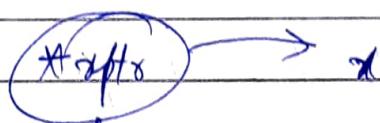
`Print(*xptr)` // gives value.



(extract the variable)
 $xptr \rightarrow$ access memory 8K & by * we

Fetch value from that address.

and using it



This denotes the variable value x .

`cout << (*xptr) << endl;` // print value.

eg.

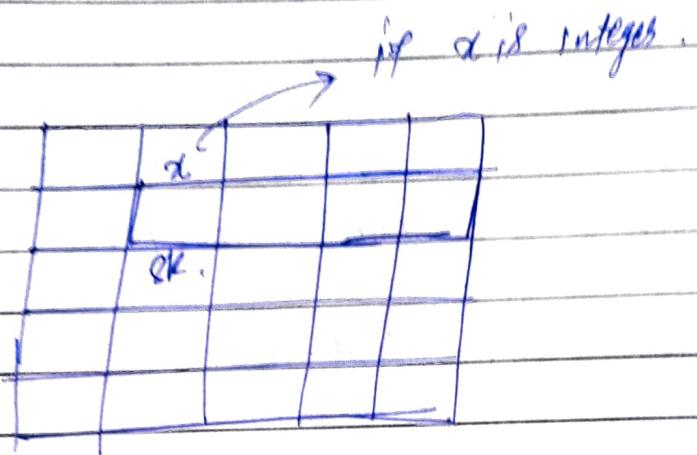
$$\text{(*xptr)} = \text{*xptr + 1}$$

is equivalent to

$$d = x + 1$$

QUESTION

$$T^* \text{ } *x = k \text{ } v_{AB}.$$



so, $\text{int } * \text{aptr} = &x$.

extract $*x$ + byte.

but if it is char type then only 1 memory add is allocated.

`char x = 'a';`

`char *xptr = &x;`

~~$\star \star$~~ $\text{sizeof}(*\text{ptx})$ 32 bit system = 4 byte.

64 " " = 8 byte.