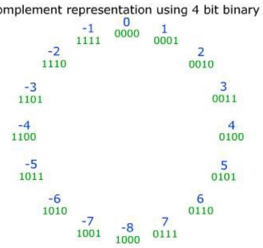
TABLE 1 Set Identities.

Identity	*Name*
$A \cup \emptyset = A$ $A \cap U = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

Two's Complement representation using 4 bit binary strings



$\neg$ Not

$\wedge$ AND

$\vee$ OR

Number	0	1	2	3	4	5	6	7
Binary	0000	0001	0010	0011	0100	0101	0110	0111
Hexadecimal	0	1	2	3	4	5	6	7

Number	8	9	10	11	12	13	14	15
Binary	1000	1001	1010	1011	1100	1101	1110	1111
Hexadecimal	8	9	A	B	C	D	E	F

Permutation

- Permutation is the arrangement of items in which **order matters**
- Number of ways of **selection and arrangement of items** in which Order Matters

$$^nP_r = \frac{n!}{(n-r)!}$$

Combination

- Combination is the selection of items in which **order does not matters**.
- Number of ways of **selection of items** in which Order does not Matters

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

The Generalized Pigeonhole Principle

The Generalized Pigeonhole Principle: If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

Proof: We use a proof by contraposition. Suppose that none of the boxes contains more than $\lceil N/k \rceil - 1$ objects. Then the total number of objects is at most

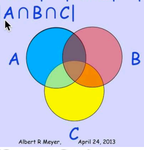
$$k\left(\left\lceil \frac{N}{k} \right\rceil - 1\right) < k\left(\left(\frac{N}{k} + 1\right) - 1\right) = N,$$

where the inequality $\lceil N/k \rceil < \lceil N/k \rceil + 1$ has been used. This is a contradiction because there are a total of n objects. ◀

Example: Among 100 people there are at least $\lceil 100/12 \rceil = 9$ who were born in the same month.

Inclusion-Exclusion (3 Sets)

$|A \cup B \cup C| =$
$|A| + |B| + |C|$
$- |A \cap B| - |A \cap C| - |B \cap C|$
$+ |A \cap B \cap C|$



Albert R Meyer, April 24, 2013

Balls into Bins Problem

- Placing n balls into m distinguishable bins
- Possible restrictions
  - If there are at most one ball per bin, then m ≥ n
  - If the same number of balls must be placed in each bin, then m must evenly divide n

	No restrictions	At most one ball per bin	Same number of balls in each bin
	(any positive m and n)	(m must be at least n)	(m must evenly divide n)
Indistinguishable balls	C(n+m-1, m-1)	C(m, n)	1
Distinguishable balls	m^n	P(m, n)	$\frac{n!}{((n/m)!)^m}$

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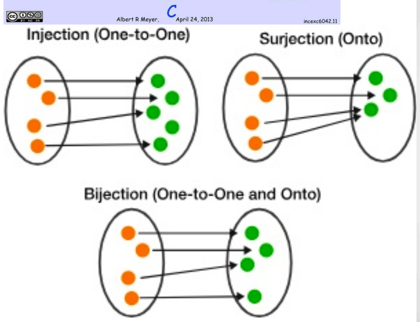
Two's Complement Ranges

Range = $-2^{(n-1)}$ to $+(2^{(n-1)} - 1)$

where **n** is the number of bits used to store the two-s complement signed integer.

Numbers with values GREATER than **$+(2^{(n-1)} - 1)$** and values LESS than **$-2^{(n-1)}$** would require more bits. If you try to store too large/too small a value without using more bits, OVERFLOW will occur.



8-bit Two's Complement

need to fix # of bits

Positive Integers (and zero) -> same as signed magnitude

Example: $(13)_{10}$ → $(00001101)_2$

Negative Integers ->

1. Compute binary representation of magnitude
2. Flip all the bits (1's become 0's, 0's become 1's)
3. Add 1

magnitude
00001101 → flip → 11110010
+ 1

Example: $(-13)_{10}$ → $(11110011)_2$

The ∃ symbol is used to represent *"there exists"*

The ∀ symbol is used to represent *"for all"*

∃ x big(x) ∧ blue(x)

ℕ - Natural Numbers {0,1,2,3,4,...} (

ℤ - Integers {...,-2,-1,0,1,2,...}

Ø - Empty Set { }

ℝ - Real Numbers (pretty much any think of)

A = {x ∈ ℕ | x > 5 }

"that are"

"is set A is equal to..."

"the set of"

"elements in ℕ"

condition on x

A = { 6, 7, 8, ... }

Associative Laws

$(A \cup B) \cup C = A \cup (B \cup C)$
$(A \cap B) \cap C = A \cap (B \cap C)$

Double Negation

$(A^C)^C = A$

DeMorgan's Laws

$(A \cup B)^C = A^C \cap B^C$
$(A \cap B)^C = A^C \cup B^C$

Distributive Laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Absorption Laws

$A \cap (A \cup B) = A$
$A \cup (A \cap B) = A$

Complement Laws

$A \cup A^C = U$
$A \cap A^C = \emptyset$

Idempotent Laws

$A \cup A = A$
$A \cap A = A$

Identity

$\emptyset \cup A = A$
$U \cap A = A$

Domination:

$U \cup A = U$
$\emptyset \cap A = \emptyset$

How many passwords can be made from lowercase letters if the length of the password is 6, 7, 8, 9, or 10?

$|P_6| = |L \times L \times L \times L \times L \times L| = 26^6$ (product rule)

$i \in \{6,7,8,9,10\}$ $|P_i| = 26^i$

$|P| = |P_6 \cup P_7 \cup P_8 \cup P_9 \cup P_{10}|$ (partition)

$= |P_6| + |P_7| + |P_8| + |P_9| + |P_{10}|$ (sum rule)

$\boxed{= 26^6 + 26^7 + 26^8 + 26^9 + 26^{10}}$ (product rule)

A: There are 10 different CS elective courses, and you need to take 3 of them. How many different combinations of electives are there?

↳ order doesn't matter ↳ combinations

{1, 2, 3} = {2, 3, 1}

B: There are 10 different CS elective courses, and you need to take 3 of them. How many different sequences of electives are there?

1→2→3 ≠ 2→3→1

↳ order matters → Permutations

Counting Combinations

"n choose k"

If we want to count the number of ways, $\binom{n}{k}$, to choose **k** *unordered* objects from a set of **n** total objects:

$$\binom{n}{k} = \frac{n!}{(n-k)!\;k!}$$