2022 (November)

BCA 1st Semester(New Course)

BCASE123A (Mathematical Foundation)

Full Marks: 75
Pass Mark: 30
Time: 3 hours

PART-A (All questions are compulsory) **[10X1]**

1. (i) Evaluate $\log_3 8$ by changing the base.
   (ii) Find the coefficient of x^5 in $(x+3)^8$
   (iii) Find the equation of the line passing through the point (-1,1) and (2, -4)
   (iv) Define Skew-symmetric matrix with example.
   (v) Find $\frac{dy}{dx}$ of $y = \log\sqrt{x}$
   (vi) When is a function f(x) said to be differentiable at a given point?
   (vii) What is the condition for a system of homogenous linear equations to have infinite number of non-trivial solutions?
   (viii) Evaluate $\int \frac{1}{4+x^4}\,dx$
   (ix) Define Geometric Progression.
   (x) What is the value of $cosec\ (-1410^o)$?

PART-B (Answer any five questions) **[5x2]**

2. (i) Using binomial theorem, expand $(\frac{x}{3} - \frac{1}{x})^5$
   (ii) Derive the equation of a circle when the circle touches both the axes.
   (iii) If A and B are non-singular matrices of the same order, the prove that
   $$(AB)^{-1} = B^{-1}A^{-1}$$
   (iv) Evaluate $\tan 105^o$
   (v) Evaluate $\int x\ sec^2 x\,dx$

(vi) Differentiate $\cos x^3 . \sin^2(x^5)$ w.r.t x.

(vii) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

(viii) Insert five numbers between 8 and 26 such that the resulting sequence is an Arithmetic Progression.

PART-C (Answer any five questions) [5x5]

3. (i) Find the co-efficient of a^5 in the product $(1+2a)^4(2-a)^5$ using binomial theorem.

(ii) A line perpendicular to the line segment joining the points (1,0) and (2,3) divides it in the ratio 1:3. Find the equation of the line

(iii) Find the condition of k such that the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & k & 6 \\ -1 & 5 & 1 \end{bmatrix}$ has an inverse.

Also find A^{-1} for $k = 1$.

(iv) If $f(x) = \begin{cases} ax^2 - b, & if\ |x| < 1 \\ \frac{1}{|x|}, & if\ |x| \geq 1 \end{cases}$ is differentiable at $x = 1$, find a and b.

(v) Prove that $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$

(vi) Find the rank of the matrix by reducing it into echelon form

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

(vii) Verify Cayley Hamilton theorem and find the inverse of

$$\begin{bmatrix} 5 & -1 & 5 \\ 0 & 2 & 0 \\ -5 & 3 & -15 \end{bmatrix}$$

(viii) Find (a) $\frac{dy}{dx}$ of $y = x^{x^x}$ and (b) $\int x \tan^{-1} x\, dx$ $[2\frac{1}{2} + 2\frac{1}{2}]$

PART-D (Answer any three questions) [3x10]

4. (i) Find the equation of the circle which has its centre at the point (3,4) and touches the straight line $5x + 12y - 1 = 0$. [4]

(ii) If $\sin A = \frac{1}{2}, \cos B = \frac{12}{13}$, where $\frac{\pi}{2} < A < \pi$ and $\frac{3\pi}{2} < B < 2\pi$, find
(a) $\sin(A+B)$ (b) $\cos(A-B)$ (c) $\tan(A-B)$ (d) $\cot(A-B)$ [4]

(iii) Check the continuity of the function defined by $f(x) = x - [x]$ at all integral points. [2]

5. (i) Find the rank of the matrix by reducing it into normal form

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$ [4]

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)$ [4]

(iii) If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ [2]

6. (i) Find all point of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} |x| + 3, if\ x \geq -3 \\ -2x, if\ -3 < x < 3 \\ 6x + 2, if\ x \geq 3 \end{cases}$$ [4]

(ii) Find for what values of a and b the system of linear equations
$3x - 2y + z = b\,; 5x - 8y + 9z = 3\,; 2x + y + az = -1$
has (i) a unique solution, (ii) no solution, (iii) infinitely many solutions. [3]

(iii) Evaluate $\int \frac{1}{\sqrt{(x-a)(x-b)}} dx$ [3]

7. (i) Evaluate $\int \frac{x}{(x-1)^2(2x+1)} dx$ [4]

(ii) Find the coordinates of the points which trisects the line segment joining the points A (4,2,-6) and B (10,-16,6) [4]

(iii) If $\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$, find the value of (i) $\sin\frac{x}{2}$, (ii) $\cos\frac{x}{2}$ [2]

8. (i) Evaluate $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$ [4]

(ii) Evaluate $\sin(-420^o)\cos(390^o) + \cos(-660^o)\sin(330^o)$ [2]

(iii) Find $\frac{dy}{dx}$ of $x^y + y^x = 1$ [4]