

# Questions and Answers

## 1. Define null production

Ans: A null production is a production rule in a context-free grammar that doesn't produce any new output in terms of the symbols in the language being parsed. It is represented as epsilon ( $\epsilon$ ).

## 2. Define Recursively Enumerable Language

Ans: A recursively enumerable language (RE language) is a language where a "computable" algorithm can list all the strings in the language. Think of it like a language we can "read" through.

## 3. State Church Turing Thesis

Ans: The Church-Turing Thesis is a fundamental assertion in computer science. It states that any problem that can be solved by a Turing Machine (TM) can be solved by a program written in a Turing Machine's language. It essentially equates "computation" with any problem.

## 4. Define instantaneous description of TM. Give Example

Ans: Think of a TM as a 'black box' machine: It has a few key components. It has:

States: Each state is a different configuration of the TM.

Tape: The tape is the area the TM uses to store its information.

Head: The head is the pointer that scans the tape. Instantaneous: It can't remember the past, and the current state of the TM determines how the TM processes a string.

Example

Imagine a TM for checking if a string is a palindrome:

Initial state: The TM starts in a specific state.

Read the string: The head moves across the tape, one character at a time.

Check for matching: The TM looks at the character it is currently reading, and compares it to the character on the other side of the tape.

Move to next state: Depending on the match, the TM will move to the next state.

Repeat steps 2-4: This process continues until the end of the string is reached.

## 5. Define left recursion

Ans: Left recursion is a type of grammar that's often used to define language patterns. The recursive rules can start at the beginning of the expression, and this is a common problem in compilers.

## 6. Design a CFG for the language $L = \{a^3 b^n c^n \mid n \geq 0\}$ and convert it into CNF

### 1. CFG Design

We can build this CFG in a recursive fashion:

$S \rightarrow a^3$

#### Explanation:

- $S$  is our start symbol for the production.
- $a^3$  means we need to produce a sequence of three  $a$ 's.

### 2. CFG for $L$

The grammar for the language  $L$  can be described as follows:

- **Start symbol:**  $S$
- **Non-terminal symbol:**  $a, b, c$
- **Production rules:**
  - $S \rightarrow a^3$
  - $B \rightarrow b^n$  where  $n \geq 0$
  - $C \rightarrow c^n$  where  $n \geq 0$

### 3. Conversion to CNF

- **CNF (Chomsky Normal Form)** represents grammars in a simpler and more concise form, making it easier to analyze.

#### Step 1: Build a chart:

- The chart will represent all possible non-terminals, rules, and how they relate to each other.

#### Step 2: Apply a CFG reduction:

- Reduce each non-terminal to its simpler form.

## 7. Construct PDA for the Language $L = \{a^n b^{3n} \mid n \geq 1\}$ acceptance by empty stack

### Understanding the Language

The language  $L = \{a^n b^{3n} \mid n \geq 1\}$  has a specific structure:

- Each string in the language consists of alternating 'a's and 'b's in the specified pattern.

### Constructing the PDA

The following PDA will accept the strings in  $L = \{a^n b^{3n} \mid n \geq 1\}$ , where strings must be in a specific order:

#### States:

- **S:** Initial state
- **A:** State for accepting a string in the language.
- **E:** State for the final string in the language.

#### Input Symbols:

- **a:** The symbol used to denote an 'a' in the string.
- **b:** The symbol used to denote a 'b' in the string.

#### Transitions:

- **S  $\rightarrow$  a :** Move to the next state with a string that starts with an 'a'
- **S  $\rightarrow$  b :** Move to the next state with a string that starts with a 'b'
- **A  $\rightarrow$  S :** Move to the state S

#### Accept State:

- **A** is the acceptance state. The string will be accepted if it ends in an 'A'.

#### Working of the PDA:

1. **Initialization:** The PDA starts in state S, looking at the first letter of the input string.
2. **Processing:** The PDA reads characters from the input string and performs actions based on the current state.
3. **Determining Acceptance:** After processing the entire input string, the PDA will be either in the accepting state or rejecting state. If it's in the accepting state, the string is accepted; otherwise, it's rejected.

## 8. Design a TM which recognize the language $L = \{a^r c d b^r \mid r > 0\}$

### Understanding the Language

The language  $L$  comprises strings that consist of alternating repetitions of 'a' and 'b' based on the number 'r'.

### Building the TM

To design a TM for  $L$ , we'll focus on:

1. **States:** We need a state to handle both 'a' and 'b' and transitions to handle their processing.
2. **Tape:** A fixed length tape that holds the input.
3. **Head:** A pointer that scans the tape.
4. **Accepting:** The TM will stop when the pattern is complete.

### Steps

#### 1. Defining States

- **S:** The start state.
- **A:** State representing "a"
- **B:** State representing "b"
- **E:** The accepting state.

#### 2. Transition Table (T)

Input	Head	S	A	B	E
'a'	'a'	S	A		
'b'	'b'	S	A	B	
'c'	'c'		A	B	E

#### 3. State Changes:

- The TM moves its head to the next symbol in the tape according to its input.

4. **Acceptance:** The TM is said to accept the input if the head reaches the accepting state.

### Explanation:

- **Initialization:** The TM starts at the initial state 'S' and scans the first symbol of the input.
- **Processing:** The TM processes the input symbol and transitions to a new state.
- **Final State:** When it reaches the accepting state, the TM accepts the string.

### Key Concepts:

- **TM:** This TM is designed to handle strings with alternating 'a', 'b', and 'c' symbols.
- **States:** States represent different parts of the TM's internal representation.
- **Transitions:** Transitions represent how the TM moves from one state to another based on input and current state.

## Design a TM to compute subtraction of two numbers

**The Problem:** We aim to design a TM that takes two numbers as input (represented by binary strings) and outputs the difference between them. The TM should also ensure that the input is valid.

### Challenges:

- **Limited Memory:** TMs generally have a finite number of states.
- **Integer Representation:** How do we represent and store the numbers for subtraction?
- **Subtraction Logic:** We need a way to implement the standard subtraction logic.

### TM Design

- **States:**
  - **S0:** Initial state
  - **S1:** State to handle the first number
  - **S2:** State to handle the second number
  - **S3:** State for the result.
- **Input Symbols:**
  - **0 and 1:** To represent digits of the numbers.

### Steps:

1. **Initialization:** The TM begins in state S0 with the input numbers as strings.
2. **Processing:**
  - The TM reads the input symbols one by one from the tape.
  - Based on the current input and the states, the TM will move to a new state.
  - It then processes the current input symbol.
  - The TM compares the current numbers.
3. **Subtraction Logic:**
  - The TM uses a series of steps and transitions to find the difference between the numbers,
  - It will have rules for handling both positive and negative values.

**Explanation of the TM** \* The TM will handle a binary representation of a number. \* The TM will check to ensure that both inputs are valid.

## Key Considerations:

- **Efficiency:** How to minimize the number of transitions required to compute the difference between two numbers.

## 10. Write a short note on Chomsky Hierarchy. Differentiate between Recognizer and Decider. Discuss TM as enumerator

### The Chomsky Hierarchy

The Chomsky Hierarchy is a classification of languages based on their computational power and how easily we can determine if a language belongs to it.

- **The Hierarchy:** This hierarchy arranges languages according to their complexity, with the “strongest” languages at the bottom, and the weakest at the top.
- **Key Levels:**
  - **L(1):** Languages that can be recognized by a Turing Machine.
  - **L(2):** Languages that can be recognized by a Deterministic Finite Automaton (DFA).
  - **L(3):** Languages that can be recognized by a Context-Free Grammar (CFG)
  - **L(4):** Languages that can be recognized by a Context-Sensitive Grammar.
  - ...and so on.

### Recognizing vs. Deciding

- **Recognizable Language (recognizer):** A language can be recognized if we can determine, in a deterministic way, if a string belongs to that language (without any ambiguity).
- **Decidability Language (decipher):** A language is decidable if we can determine whether a string belongs to that language, but we don't need to determine if it's valid.

### Turing Machines as Enumerators

Turing Machines (TMs) can be considered enumerators. They can enumerate the strings within a language.

- **The Concept:** A TM, by default, accepts the string by scanning it. Turing machines are able to do this in a very systematic way.
- **Strengths:** They are capable of recognizing strings within a language in a systematic way, and they don't need to determine if the string is valid.

### Key Differences

- **Recognition vs. Decipherability:** TMs can recognize (classify) languages, but not necessarily decipher them in a more strict sense.
- **Enumerability vs. Decidability:** TMs are enumerators by definition, but not necessarily decidable.