Project:P3

Ding Zhao
24-677 Special Topics: Linear Control Systems

Due: Nov 26, 2019, 5:00 pm

- You need to upload your solution to Gradescope (https://www.gradescope.com/) to be graded. The link is on the panel of CANVAS. If you are not familiar about the tool, post your questions on Piazza or ask during the office hours. We will use the online submission time as the timestamp.
- Submit **[andrew_id]_controller.py** and **BuggyStates.npz** to Gradescope under **Programming_P3** and your solutions in **.pdf** format to **Project-P3**. Insert the Buggy Simulator performance plot image in the .pdf. We will test your controller.py and manually check all answers.
- You are recommended to test your codes in Google Colab before submission, to ensure it executes with standard python compilers. Please refer to (http://bit.ly/2rtnrcy) for documentation on how to use Colab.
- You can also post your questions on the Piazza. We will try our best to give feedback within 24 hours during the workday and within 48 hours during the weekend. We will keep answering questions until 8:00 pm, Monday.
- **Note:** use **python3** for coding the executing the controller scripts.

1 Introduction

In this project, you will complete the following goals:

1. Design an optimal controller to complete the Buggy Track within the Baseline time.

[Remember to submit the write-up and codes on the Gradescope.]

2 Model

The error-based linearized state-space for the lateral dynamics:
e_1: distance of the c.g. of the vehicle from the reference trajectory
e_2: the orientation error of the vehicle with respect to the reference trajectory

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{4C_\alpha}{mV_x} & \frac{4C_\alpha}{m} & -\frac{2C_\alpha(l_f-l_r)}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_\alpha(l_f-l_r}{I_zV_x} & \frac{2C_\alpha(l_f-l_r}{I_z} & -\frac{2C_\alpha(l_f^2+l_r^2}{I_zV_x} \end{bmatrix} \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2C_\alpha l_f}{I_z} & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix} + \begin{bmatrix} o \\ -\frac{2C_\alpha(l_f-l)r)}{mV_x} - V_x \\ 0 \\ -\frac{2C_\alpha(l_f^2+l_r^2}{I_zV_x} \end{bmatrix} \dot{\psi}_{des}$$

In lateral vehicle dynamics, $\dot{\psi}_{des}$ is a time varying disturbance in the state space equation. Its value is proportional to the longitudinal speed. When deriving the error-based state space model for controller design, $\dot{\psi}_{des}$ can be safely assumed to be zero.

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{4C_\alpha}{mV_x} & \frac{4C_\alpha}{m} & -\frac{2C_\alpha(l_f-l_r)}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_\alpha(l_f-l_r}{I_zV_x} & \frac{2C_\alpha(l_f-l_r}{I_z} & -\frac{2C_\alpha(l_f^2+l_r^2}{I_zV_x} \end{bmatrix} \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2C_\alpha l_f}{I_z} & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

For the longitudinal control:

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\psi}\dot{y} - fg \end{bmatrix}$$

Assuming $\dot{\psi} = 0$:

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

3 Resources

3.1 Buggy Simulator

A Buggy Simulator designed in python has been provided along with the assignment. The simulator takes the control command[steering, longitudinal Force] and then outputs the buggy state after the given fixed time step (fixed fps). Additional script `util.py` contains functions to help you design and execute the controller. Please design your controller in `controller.py`. After the complete run, a response plot is generated by the simulator. This plot contains visualization of the buggy trajectory and variation of states with respect to time.

3.2 Trajectory Data

The trajectory is given in `buggyTrace.csv`. It contains the coordinates of the trajectory: (x, y). The satellite map of the track is shown in Figure 1.

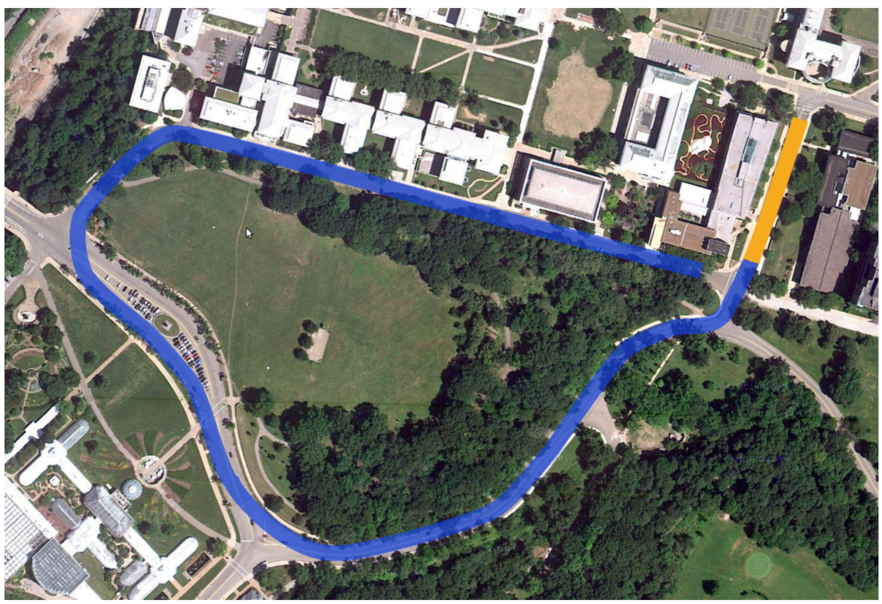

Figure 1: Buggy track[3]

4 P3:Problems [Due 5:00 PM, November 19]

Exercise 1. Design a Discrete Time, Infinite Horizon LQR controller for the lateral control of the Vehicle. For the longitudinal control, use a PID controller.
Design the controllers in **controller.py**.
[You have to edit only the controller.py python script]
Execute the `main.py` python script to check your controller. It generates a performance plot and saves the vehicle states in a .npz file. Submit the Buggy states in .npz file, the response plots in pdf file, and your controller in the **[andrew_id]_controller.py** script.
Your controller is required to achieve the following performance criteria:

1. Time to complete the loop = 250 s
2. Maximum deviation from the reference trajectory = 6.0 m
3. Average deviation from the reference trajectory = 3.0 m

[10% Bonus]: Complete the loop within 130 s. The maximum deviation and the average deviation should be within in the allowable performance criteria mentioned above.

5 Appendix

(Already covered in P1)

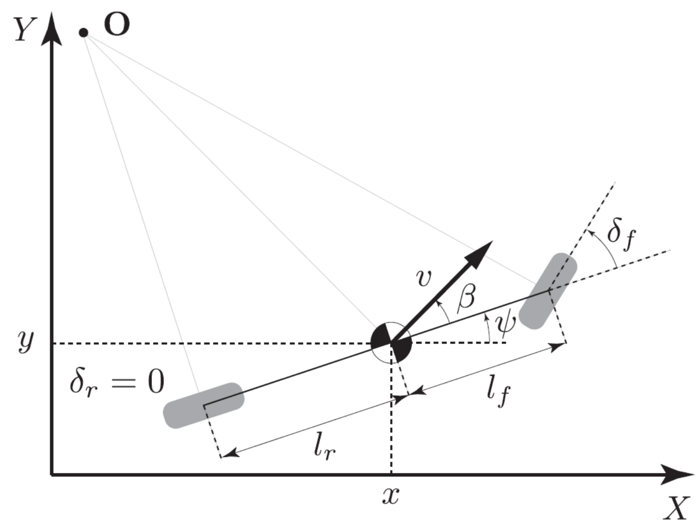

Figure 2: Bicycle model [2]

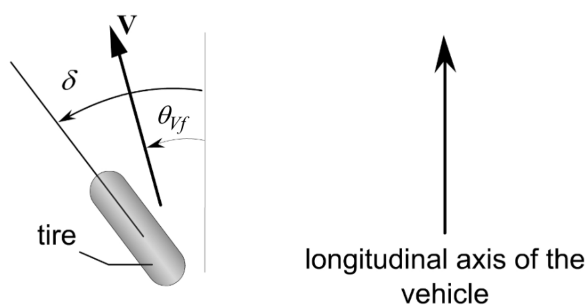

Figure 3: Tire slip-angle [2]

Here you will use the same bicycle model introduced in P1. We will work with the linearized version of the model for all the questions that you formulated for P1. Shown in Figure 2, the car is modeled as a two-wheel vehicle in two degree of freedom, described in longitudinal and lateral dynamics separately. The model parameters are defined in Table 1.

5.1 Lateral dynamics

Ignoring road bank angle and applying Newtons second law for motion along the y axis

$$ma_y = F_{yf} \cos \delta_f + F_{yr}$$

where $a_y = \left(\dfrac{d^2y}{dt^2}\right)_{inertial}$ is the inertial acceleration of the vehicle at the center of geometry in the direction of the y axis, F_{yf} and F_{yr} are the lateral tire forces of the front and rear wheels respectively and δ_f is the front wheel angle whichi will be denoted as δ later. Two terms contribute to a_y: the acceleration $\ddot{y}$ which is due to motion along the y axis and the centripetal acceleration . Hence

$$a_y = \ddot{y} + \dot{\psi}\dot{x}$$

Combining the two equations, the equation for the lateral transnational motion of the vehicle is obtained as

$$\ddot{y} = -\dot{\psi}\dot{x} + \frac{1}{m}(F_{yf}\cos\delta + F_{yr})$$

Moment balance about the axis yields the equation for the yaw dynamics as

$$\ddot{\psi}I_z = l_f F_{yf} - l_r F_{yr}$$

The next step is to model the lateral tire forces F_{yf} and F_{yr}. Experimental results show that the lateral tire force of a tire is proportional to the slip-angle for small slip-angles **when vehicle's speed is large enough**, let's say when $\dot{x} \geq 0.5$ m/s. The slip angle of a tire is defined as the angle between the orientation of the tire and the orientation of the velocity vector of the wheel. the slip angle of the front and rear wheel is

$$\alpha_f = \delta - \theta_{Vf}$$
$$\alpha_r = -\theta_{Vr}$$

where θ_{Vp} is the angle that the velocity vector makes with the longitudinal axis of the vehicle for$p \in \{f, r\}$. A linear approximation of the tire forces are given by

$$F_{yf} = 2C_\alpha \left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right)$$
$$F_{yr} = 2C_\alpha \left(-\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right)$$

where C_α is called the cornering stiffness of tires. If $\dot{x} < 0.5$ m/s, we just set F_{yf} and F_{yr} both to zeros.

5.2 Longitudinal dynamics

Similarly, a force balance along the vehicle longitudinal axis yields

$$\ddot{x} = \dot{\psi}\dot{y} + a_x$$
$$ma_x = F - sign(\dot{x})F_f$$
$$F_f = fmg$$

where F is the total tire force along x axis, F_f is the force due to rolling resistance at the tires, and f is the friction coefficient. $sign$ function returns +1 when $\dot{x} \geq 1$ otherwise -1.

5.3 Global coordinates

In the global frame we have

$$\dot{X} = \dot{x}\cos\psi - \dot{y}\sin\psi$$
$$\dot{Y} = \dot{x}\sin\psi + \dot{y}\cos\psi$$

5.4 System equation

Gathering all the equations, if $\dot{x} \geq 0.5$ m/s we have:

$$\begin{aligned}
\ddot{y} &= -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}(\cos\delta\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}) \\
\ddot{x} &= \dot{\psi}\dot{y} + \frac{1}{m}(F - fmg) \\
\ddot{\psi} &= \frac{2l_fC_\alpha}{I_z}\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_rC_\alpha}{I_z}\left(-\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \\
\dot{X} &= \dot{x}\cos\psi - \dot{y}\sin\psi \\
\dot{Y} &= \dot{x}\sin\psi + \dot{y}\cos\psi
\end{aligned}$$

otherwise since the lateral tire forces are zeros we only consider the longitudinal model.

5.5 Measurements

The observable states are with some Gaussian noise $\epsilon = N(0, \sigma)$, where

$$y = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ X \\ Y \\ \psi \end{bmatrix} + \epsilon,\ \sigma = \begin{bmatrix} 0.5 & & & \cdots & & 0 \\ & 0.5 & & & & \\ & & 0.05 & & & \\ \vdots & & & 0.05 & & \\ & & & & 1 & \\ 0 & & & & & 0.5 \end{bmatrix}$$

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5.6 Physical constraints

The system satisfies the constraints that:

$$\begin{aligned}
&|\delta| \leqslant \tfrac{\pi}{6}\ rad/s \\
&|\dot{\delta}| \leqslant \tfrac{\pi}{6}\ rad/s \\
&|F| \leqslant 10000\ N \\
&0 \text{ m/s} \leqslant \dot{x} \leqslant 100 \text{ m/s} \\
&|\dot{y}| \leqslant 10 m/s
\end{aligned}$$

Table 1: Model parameters.

Name	Description	Unit	Value
$(\dot{x}, \dot{y})$	Vehicle's velocity along the direction of vehicle frame	m/s	State
(X, Y)	Vehicle's coordinates in the world frame	m	State
ψ, $\dot{\psi}$	Body yaw angle, angular speed	rad	State
δ or δ_f	Front wheel angle	rad	State
$\dot{\delta}$	Steering Rate	rad	Input
F	Total input force	N	Input
m	Vehicle mass	kg	1000
l_r	Length from front tire to the center of mass	m	1.7
l_f	Length from front tire to the center of mass	m	1.1
C_α	Cornering stiffness of each tire	N	15000
I_z	Yaw intertia	kg m^2	3344
F_{pq}	Tire force, $p \in \{x, y\}$,$q \in \{f, r\}$	N	Depend on input force
m	vehicle mass	Kg	2000
f	Friction coefficient	1	0.01

6 Reference

1. Rajamani Rajesh. Vehicle dynamics and control. Springer Science & Business Media, 2011.

2. Kong Jason, et al. "Kinematic and dynamic vehicle models for autonomous driving control design." Intelligent Vehicles Symposium, 2015.

3. cmubuggy.org, `https://cmubuggy.org/reference/File:Course_hill1.png`