$$x \to \boxed{F} \to y$$

$$y_d(x) = \sin x \qquad x \in (0, 2\pi)$$

$$\hat{y}(x) = F(x) \quad \leftarrow \text{model}$$

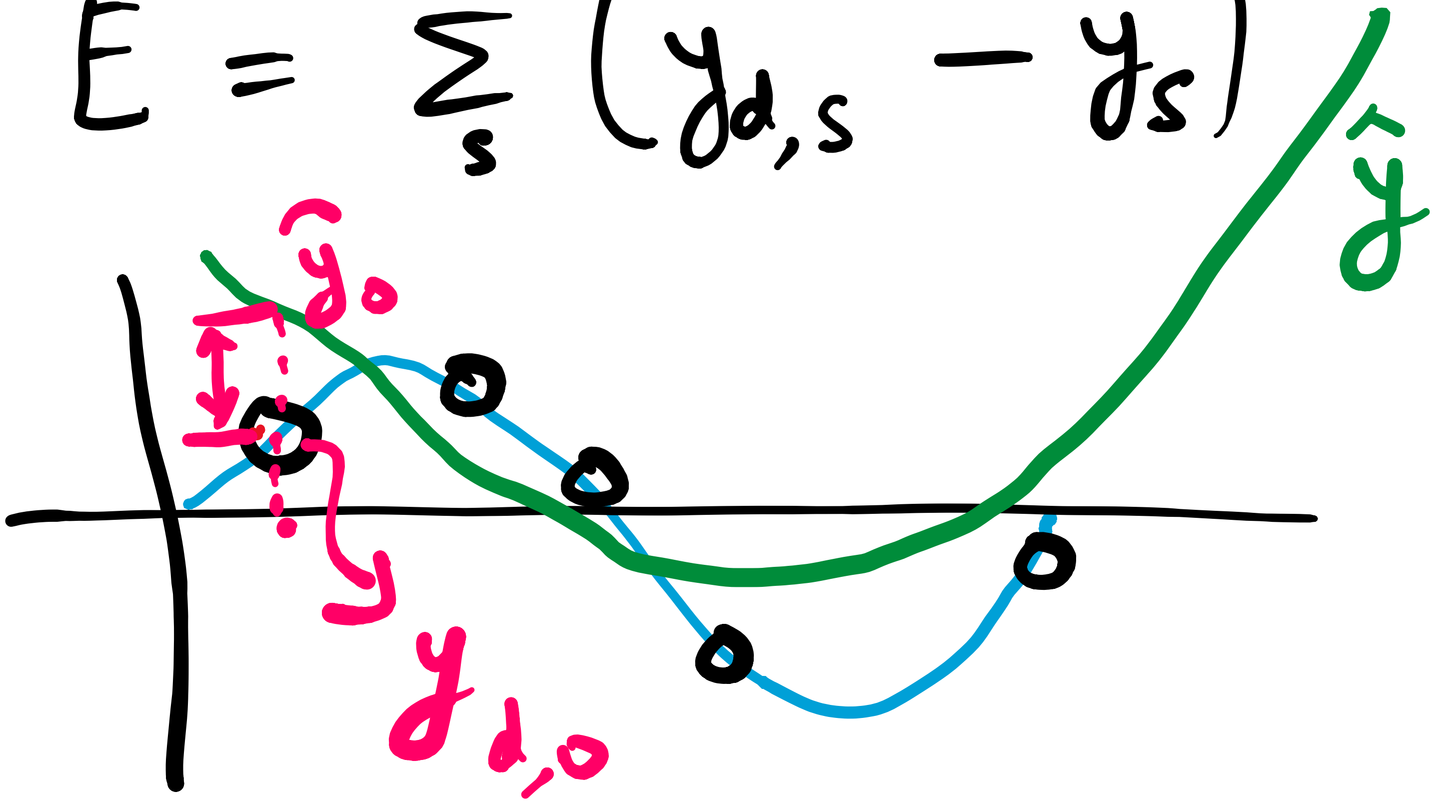
$$= w_0 + w_1 x + w_2 x^2 \; ; \; D = 2$$

↑ ↑ ↑ parameters (tunable)

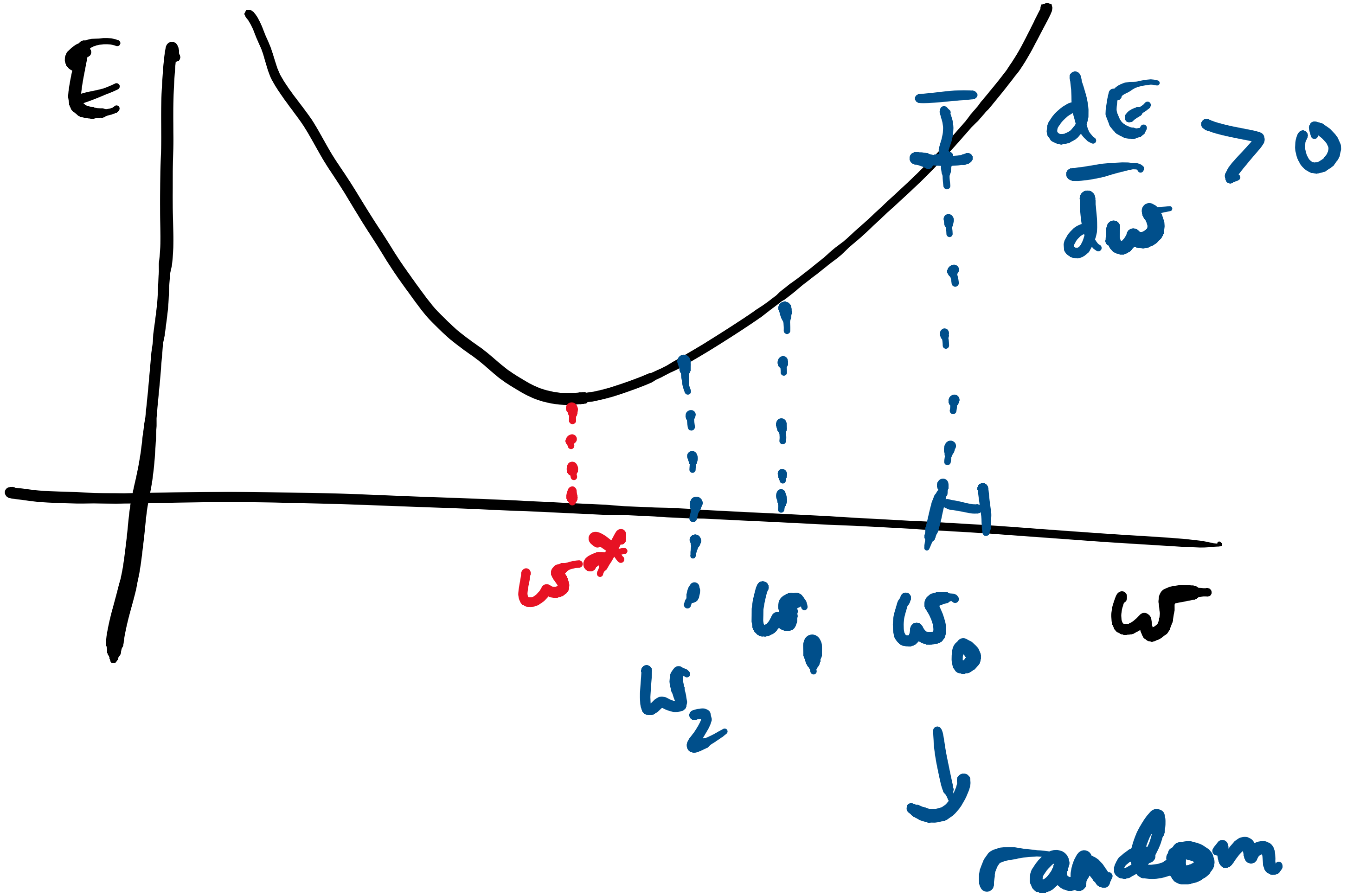
$1+D > N$ then L.S. solution doesn't work

Error fn. or loss fn to be minimized.

$$E = \sum_s \left(y_{d,s} - \hat{y}_s\right)^2$$



$$E = \left(w^3 + \log w + \sin w + e^{w} + |w|\right)^{1/3}$$

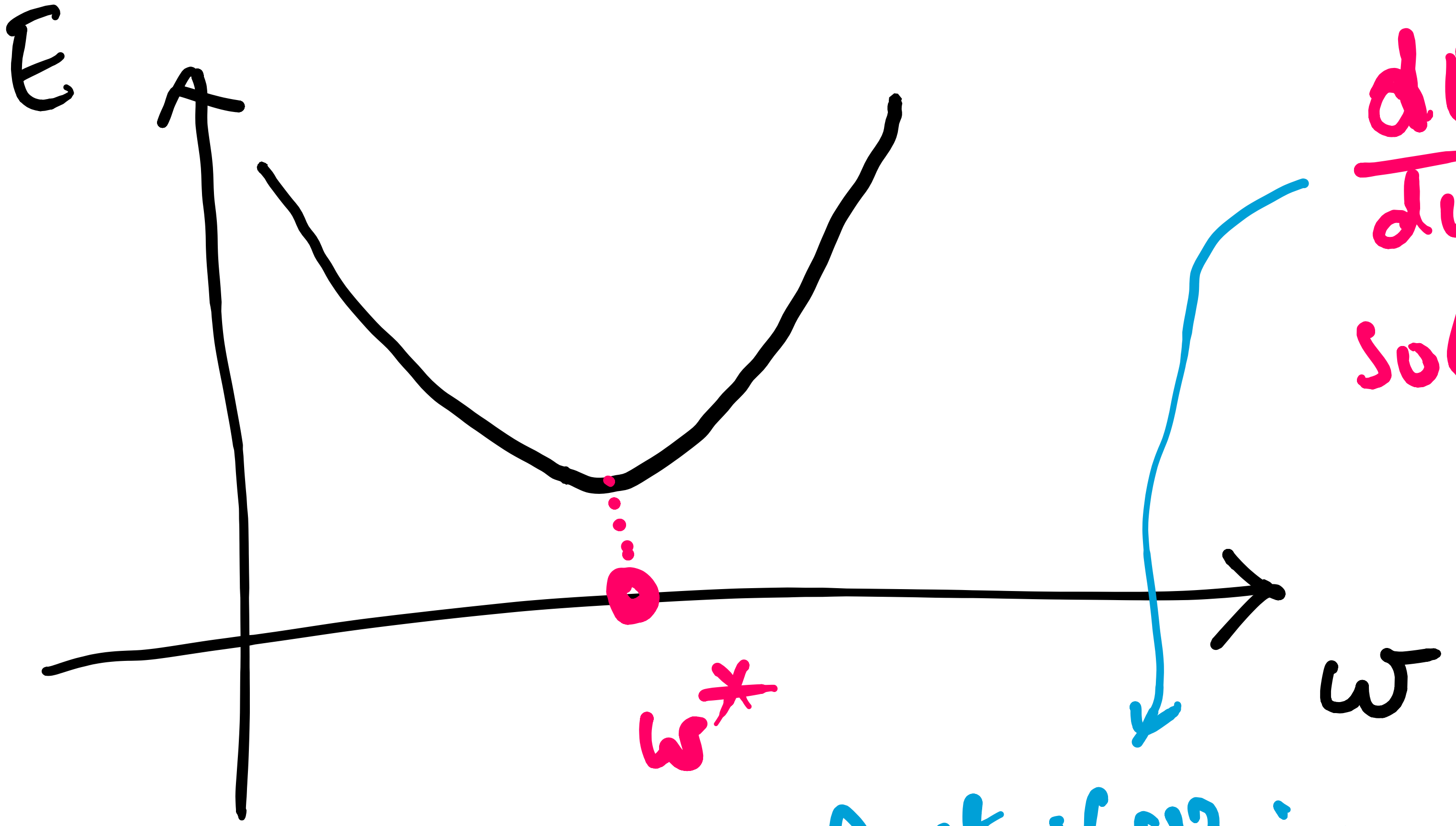


Iterative Methods

$$w_1 = w_0 - \frac{dE}{dw}$$

$$w_2 = w_1 - \frac{dE}{dw}$$

$$\underline{w} = \arg\min_{\underline{w}} E \quad (\text{minimizing } E \text{ w.r.t. } \underline{w})$$



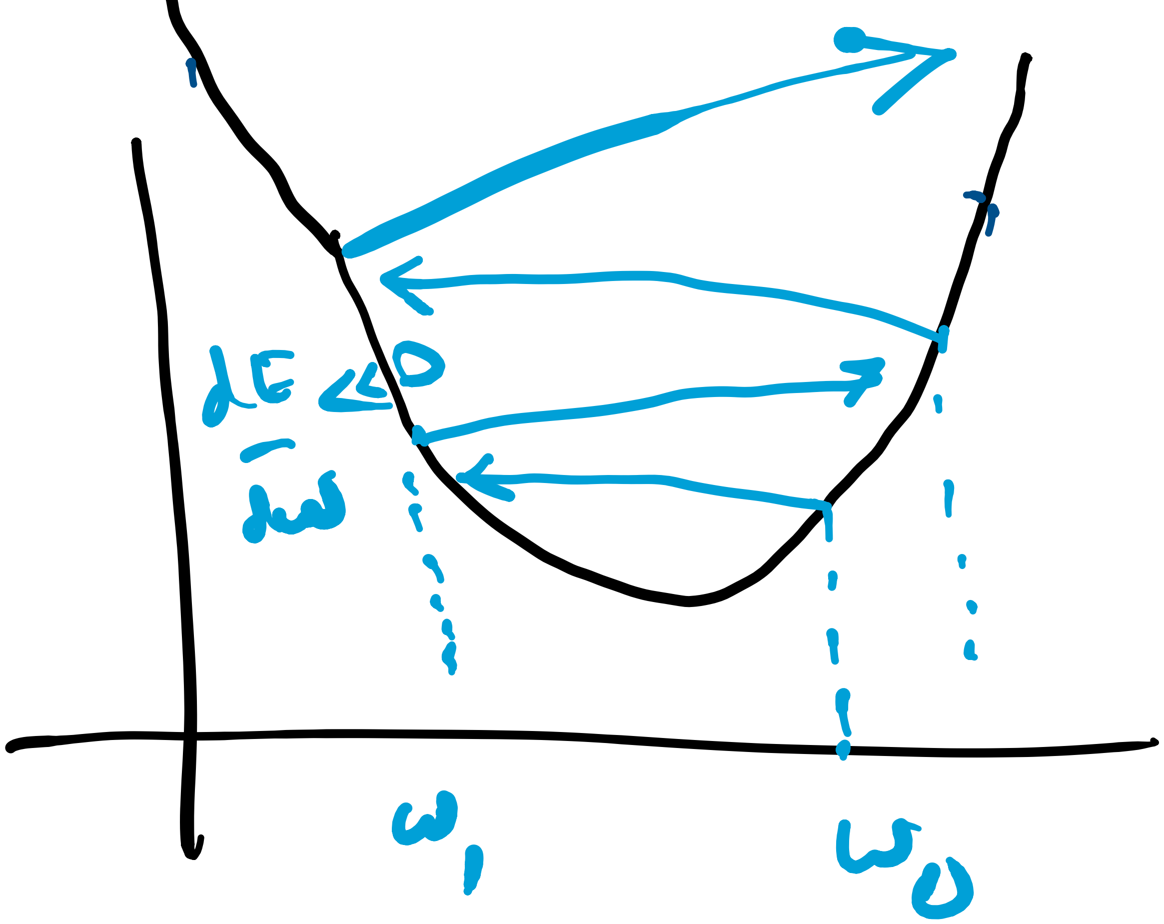
$$\frac{dE}{dw} = 0 \quad \dots (1)$$

solve for w

$$\frac{d^2E}{dw^2} > 0$$

for min E

what if (1) is not solvable



$$\frac{dE}{dw} \gg 0$$

$$w_1 = w_0 - \frac{dE}{dw} \cdot \eta$$

step size

$$w_{i+1} = w_i - \eta \left.\frac{dE}{dw}\right|_{w=w_i}$$

	S.	N.S.
S	TP $\uparrow$	FN $\downarrow$
N.S.	FP $\downarrow$	TN $\uparrow$

$C_{00}\uparrow$	$C_{11}\uparrow$	$C_{01}\downarrow$	$C_{10}\downarrow$
TP	TN	FN	FP

$$E = C_{01}$$

$$E = C_{10}$$

$$E = C_{10} + C_{01}$$

E should be differentiable

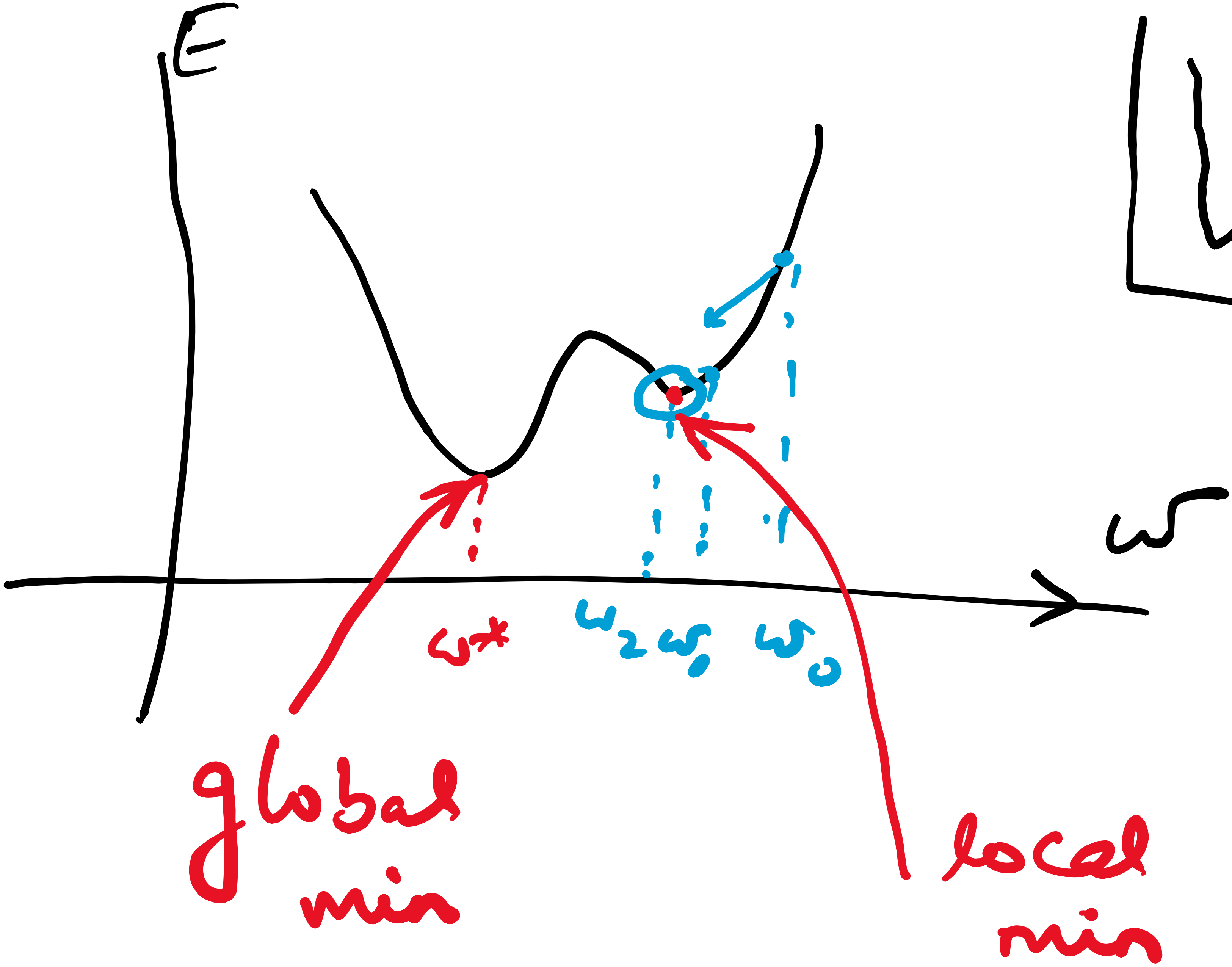
$$w_0^{(i+1)} = w_0^{(i)} - \eta \left.\frac{\partial E}{\partial w_0}\right|_{w_0 = w_0^{(i)}}$$

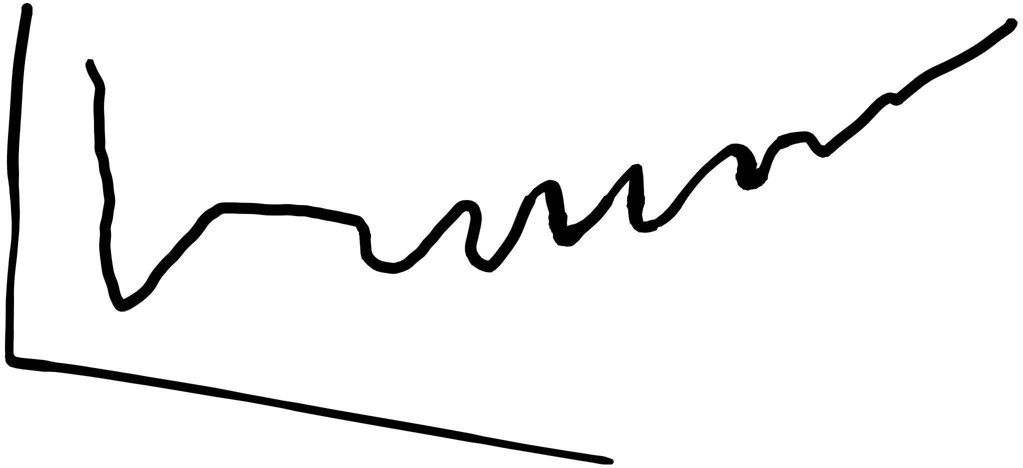
$$w_1^{(i+1)} = w_1^{(i)} - \eta \left.\frac{\partial E}{\partial w_1}\right|_{w_1 = w_1^{(i)}}$$

$$\vdots$$

$$\underline{w} = [w_0, w_1, \ldots, w_D]$$

Gradient Descent Algo.


E
w
w*
w2
w1
w0
global min
local min



$$y_d = f(x)$$

$$\hat{y} = w_0 + w_1 x + w_2 x^2 + \dots + w_D x^D$$

$$E = \frac{1}{N} \sum_{s=0}^{N-1} \left(y_{d,s} - \hat{y}_s \right)^2$$

update $\{w_0, \dots\}$ using

$$w = w - \eta \underbrace{\frac{\partial E}{\partial w}}$$

$$\hat{y} = w_0 + w_1 x + \ldots$$

$$\frac{\partial E}{\partial w_0} = \frac{\partial}{\partial w_0}\left(\sum_s \left(y_s - \hat{y}_s\right)^2\right)$$

$$= \sum_s \frac{\partial}{\partial w_0}\left(\left(y_s - \hat{y}_s\right)^2\right)$$

$$= \sum_s 2\left(y_s - \hat{y}_s\right)\left(-\frac{\partial \hat{y}_s}{\partial w_0}\right)$$

$$= \sum_s 2\left(y_s - \hat{y}_s\right)(-1)$$

$$x = [0.1, 0.5, 1.7, \ldots, 0.01, 2.3]_N \ldots$$

$$y = [\sin 0.1, \sin 0.5, \ldots, \ldots]_N$$

i) initialize w ~~r~~ randomly

w.shape $= (D+1,)$

ii) for this w, compute E and $\frac{\partial E}{\partial w}$

iii) update w

iv) goto ii until $\frac{\partial E}{\partial w} \to 0$; i.e. w does not change much.