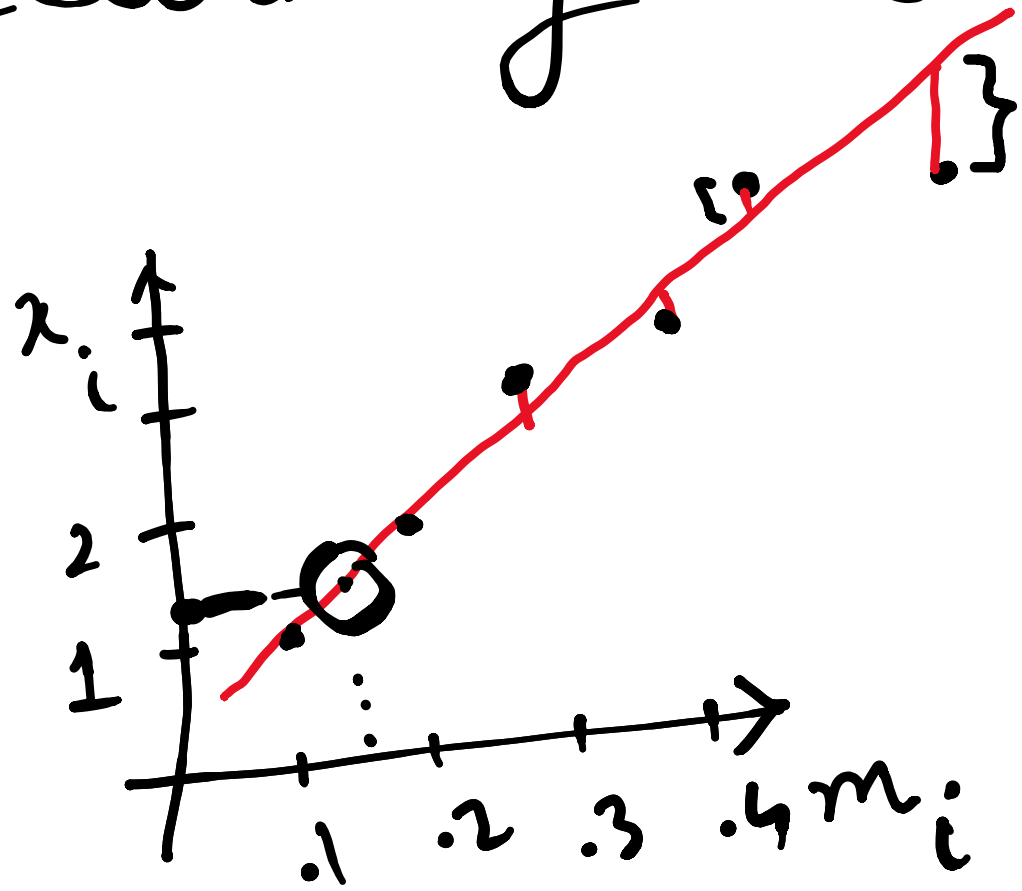
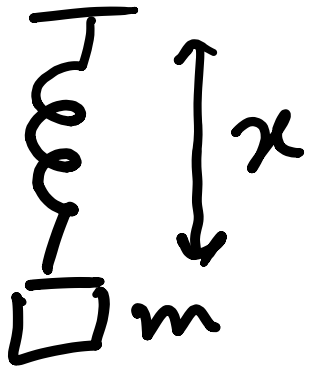
Learning a linear model from data





$$F = -kx$$

$$F = -mg$$

$$kx = mg$$

$$x = \frac{g}{k} m$$

$$x = a_0 + a_1 m$$

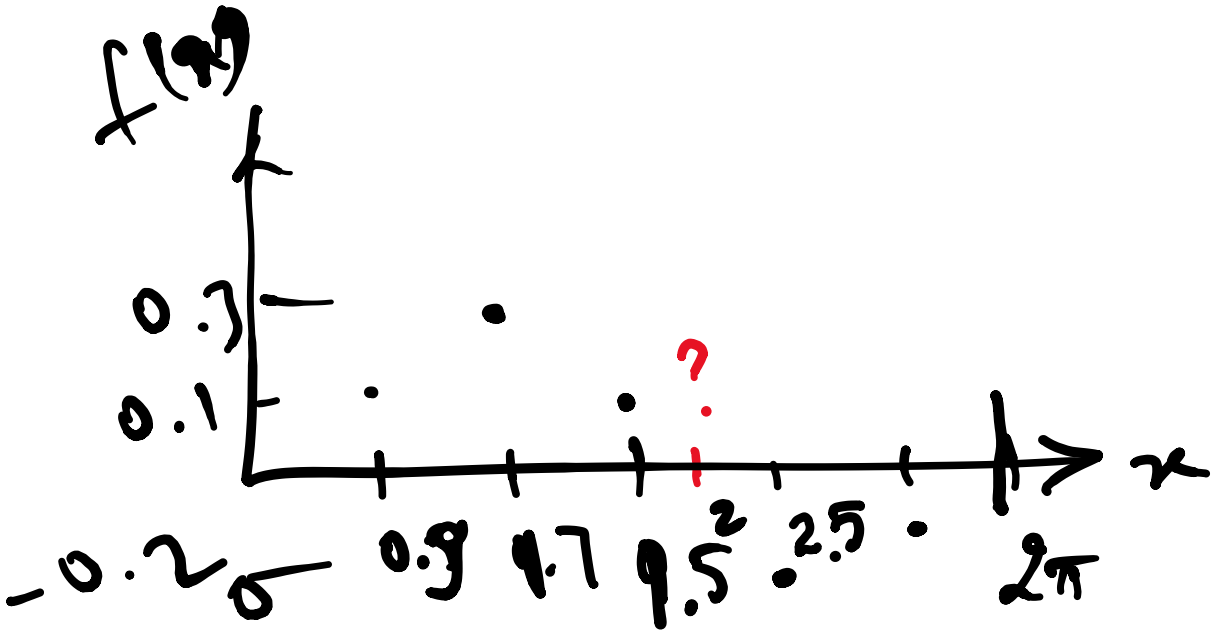
$$\left.\begin{aligned} 1 &= a_0 + a_1\ 0.1 \\ 1.5 &= a_0 + a_1\ 0.2 \end{aligned}\right\}$$

$$\begin{bmatrix} 1 & 0.1 \\ 1 & 0.2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$$

$$\sum_i \frac{1}{2}\left(x_i^d - x_i(m_i)\right)^2$$

$$x = a_0 + a_1 m + a_2 m^2 + \dots$$

$$f(m) = a_0 + a_1 m + \dots\dots + a_N m^N \qquad N >> 1$$



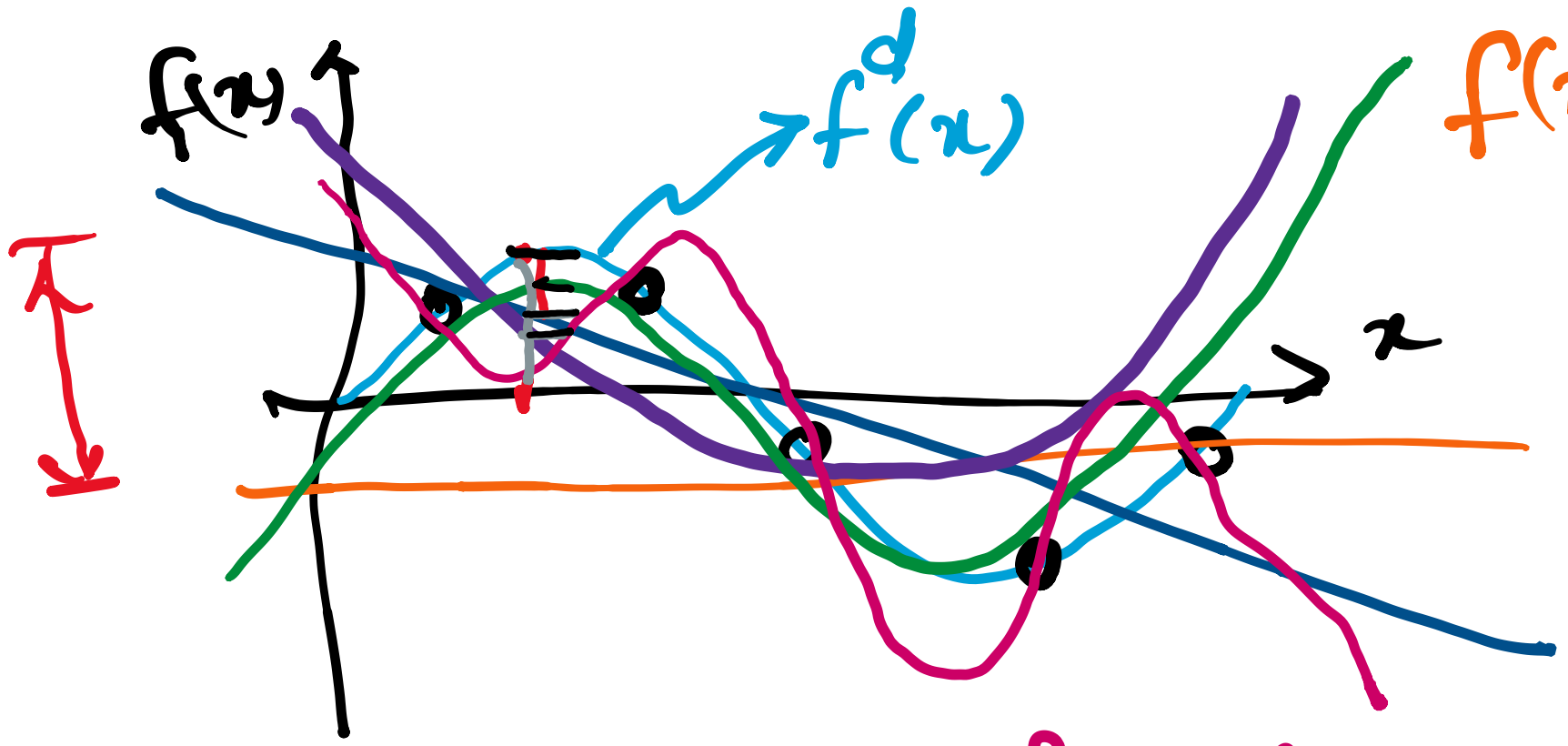
parabola

$\rightarrow f(x) = w_0 + w_1 x + w_2 x^2 \longrightarrow$ degree 2

$$\begin{bmatrix} 0.1 \\ 0.3 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 0.9 & (0.9)^2 \\ 1 & (1.7) & (1.7)^2 \\ & \vdots & \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix}$$

Find w_0, w_1, w_2 ; then

$$f(2) = w_0 + w_1 2 + w_2 \cdot 2^2$$



$f(x) = a_0$

$f(x) = a_0 + a_1$

$f(x) = a_0 + a_1 x$

$+ a_2$

$f(x) = a_0 + a_1 x$

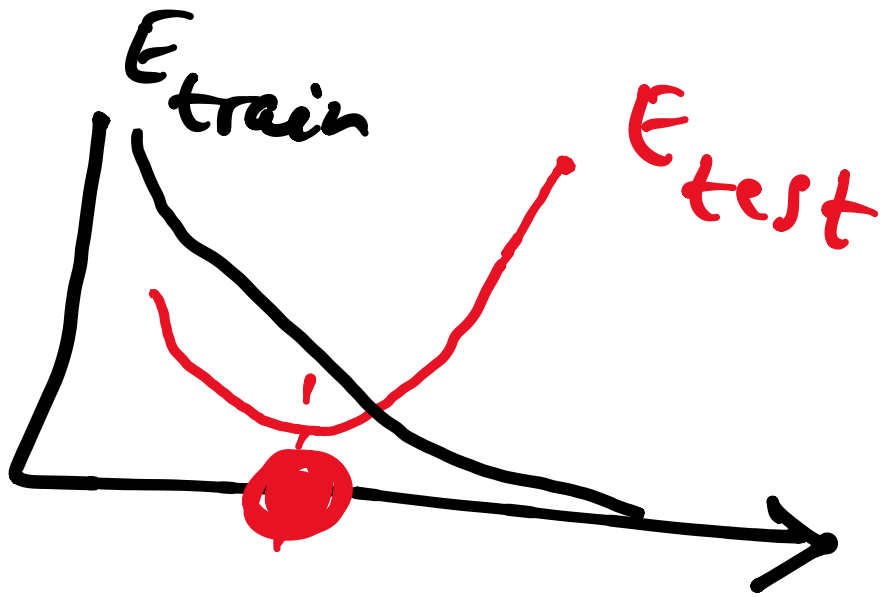
$+ a_2 x^2 +$

$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 +$

d	$E_{train\ data}$	$E_{test\ pt.}$
0	10	1.5
1	5	0.5
2	3	0.6
3	1	0.3
5	0 over fit	0.9

w → trainable parameter

d → hyper parameter



Finding the best d:

|| Hyperparameter Tuning

random variable

$x \in (0,1)$

how to get $y \in (-2\pi, 2\pi)$

$y = g(x)$, what is $g(\cdot)$?

$$y = A w$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_5 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ & \vdots & \\ 1 & x_5 & x_5^2 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix}$$

$$y = a_0 + a_1 x + a_2 x^2$$

$$\uparrow \quad \uparrow \quad \uparrow$$

$$1 \quad 2 \quad -3$$

← training found this

$$y = 1 + 2x - 3x^2$$ ← Model

what is y at $x = 0.5$? ← testing

$$y = 1 + 2 \times 0.5 - 3 \times (0.5)^2$$

$$= 1.25$$