Progress Seminar: Autumn 2024

Area of research:

by

Name (Roll Number)

Under the supervision of Dr. Name of the supervisor

Department of Mathematical Sciences

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Outline

Introduction

- A **graph** G consists of a finite nonempty set V of objects called vertices and a set E of 2-element subsets of V called edges. The sets V and E are the vertex set and edge set of G, respectively.

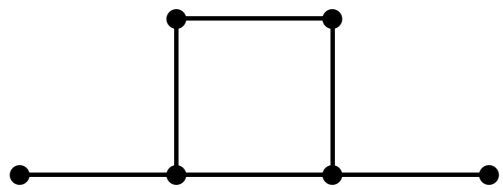

Figure 1: A graph on six vertices and six edges.

Introduction

- There are several matrices associated with a graph, namely, the Adjacency matrix, Incidence matrix, Laplacian matrix and Distance matrix.
- Let G be a connected graph on n vertices with $V(G) = \{1, 2, \ldots, n\}$. Then, the *adjacency matrix* $A(G) = [a_{ij}]$ of G is defined to be the $n \times n$ matrix with

$$a_{ij} = \begin{cases} 1 & \text{if } i \sim j \\ 0 & \text{otherwise.} \end{cases}$$

Irreducible Matrix

- A nonnegative square matrix of order n, $n \geq 2$, is called reducible if there exists a permutation matrix P such that

$$X = P^T \begin{pmatrix} B & C \\ 0 & D \end{pmatrix} P,$$

where B and D are square submatrices. A nonnegative square matrix of order n, $n \geq 2$, is called **irreducible** if it is not reducible.

Example

The given matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

is irreducible.

[0] **minc1988nonnegative**

Properties of Irreducible matrix

- If A is a $n \times n$ irreducible matrix then $(I_n + A)^{n-1} > 0$.
- If $A = [a_{ij}]$ is an irreducible matrix then for each (i, j) there exists an integer k such that $a_{ij}^{(k)} > 0$.
- The spectral radius of an irreducible matrix A, $\rho(A)$ is a simple eigenvalue of it. Also there is a positive eigenvector corresponding to $\rho(A)$.
- An irreducible matrix has exactly one eigenvector in the set

$$\mathbb{E}^n = \left\{ (x_1, x_2, \ldots, x_n) \in \mathbb{P}^n \middle| \sum_{i=1}^{n} x_i = 1 \right\},$$

where $\mathbb{P}$ denote the set of all nonnegative real numbers.

[0] **minc1988nonnegative**

Example

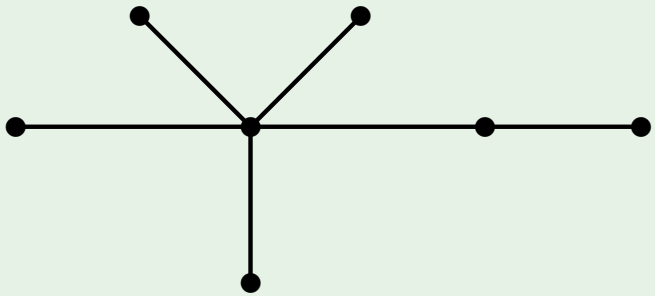

Figure 2: T

Future Plans

References I

Thank You!