SM20MTECH12003

Assignment 1

Question - 1 - (a)

Given $I = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

Filter $F = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

For convolution we will first flip the filter.

→ After flipping

Filter $F = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$

Given I require some necessary padding

$I' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix}$

Now Result Height

$= \left[\text{Image Height} - \text{Filter Height} + 1 \right]$ (as stride=1)

$= \left[3 - 2 + 1 \right]$

$= \left[2 \right]$

Result width

$= [\text{Image width} - \text{filter width} + 1]$

$= [4 - 2 + 1]$

$= [3]$

So output size after convolving I'f will be 2×3.

$$F * I' = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & 4 \end{bmatrix}$$

Question - 1 → b →

As per the Question F given in part a is seperable, that is, it can be written as a product of two filters.:

$F = F_1 \times F_2$

Let us assume $F_1 = \begin{bmatrix} a \\ b \end{bmatrix}$

$F_2 = \begin{bmatrix} c & d \end{bmatrix}$

$F_1 \cdot F_2 = F$

$$\begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} c & d \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

$ac = -1$
$ad = +1$
$bc = -1$
$bd = +1$

Solving these we can get
$a = 1$
$b = 1$
$c = -1$
$d = 1$

Now $F_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad F_2 = \begin{bmatrix} -1 & 1 \end{bmatrix}$

$\rightarrow$ $F_1 * I' = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix}$$

$\rightarrow$ $F_2 * (F_1 * I) = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & 4 \end{bmatrix} = (F * I)$$ Hence prove

Ans→ 1→ c

Given

$$(F * I)[i, J] = \sum_{K,l} I(i-K, J-l) \cdot F(K, l) \quad \text{—(1)}$$

where F is seperable filter & I is our input.

$F = f_1 f_2$

where f_1 is column of size $(K \times 1)$

f_2 is raw of size $(1 \times l)$

putting F in eq (1)

$$(f_1 f_2 * I)[i, J] = \sum_{k,l} I(i-k, J-l) \cdot (f_1(k) \cdot f_2(l))$$

$$= \sum_{l} \sum_{k} \left[I(i-k, J-l) \cdot f_1(k) \right] \cdot f_2(l)$$

$$= \sum_{l} f_2(l) \cdot \sum_{k} I(i-k, J-l) \cdot f_1(k)$$

$$= \sum_{l} f_2(l) \cdot (f_1 * I)$$

$$= \left[f_2 * (f_1 * I) \right] = F * I$$ Hence prove.

Ans → 1 → d →

For part 1 → a our output size is 2×3 & for each output element we multiply kernel with the input image at different location.

So $2\times3 \times [\text{kernel size}]$

$2\times3 \times [2\times2]$

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For part 1 - b we have seperated the filter F in F_1 & F_2 so we are doing convolution operation twice.

For $F_1 * I$

$[(2\times1)\times(2\times4)] = 16$

For $F_2 * (F_1 * I)$

$[(1\times2)\times(2\times3)] = 12$

Total multiplication for 1 - b = 16 + 12 = 28

Part a require fewer operation.

Ans → 1 (e) →

for Image of size $M_1 \times N_1$
Filter size $M_2 \times N_2$

(i) → Multiplication for direct 2D convolution →

$$\rightarrow \underbrace{(M_2 \times N_2)}_{\text{filter size}} \times \underbrace{(M_1 - M_2 + 1) \times (N_1 - N_2 + 1)}_{\text{output size assuming zero padding \& stride of 1}}$$

$$\rightarrow M_2 \times N_2 \times (M_1 - M_2 + 1) \times (N_1 - N_2 + 1)$$

(ii) multiplication to do 1 D convolution on rows & column

$$(M_2 \times (M_1 - M_2 + 1) \times N_1) + (N_2 \times (M_1 - M_2 + 1) \times (N_1 - N_2 + 1))$$

(iii) As per Big-O notation, seperable convolution are more efficient.
Because as the size of Image & filter increase direct 2D convolution will take significantly higher time than seperable convolution

Direct 2D

$O(M_1 \times N_1 \times M_2 \times N_2)$

$O(K^4)$

Seperable

$O((M_2 + N_2) \times M_1 \times N_1)$

$O(K^3)$

Ans → 2 →

Let us first understand how canny edge detector work.

(i) Find grd along y. → [Horizontal sobel]
(ii) find grad along x → [vertical sobel]

(iii) At particular point find magnitude of the resultant edge.

Magnitude of Resultant Edge $= \sqrt{(\text{Grad } x)^2 + (\text{Grad } y)^2}$

(iv) Direction of resultant Edge

$$= \tan^{-1}\left(\frac{\text{Grad } y}{\text{Grad } x}\right)$$

Ans → 2 → (a) In Question first edge was along Horizontal let us say of magnitude D_{xx} than

Before

Magnitude $= \sqrt{(\text{Grad } x)^2 + (\text{Grad } y)^2}$

$= \sqrt{D_{xx}^2 + 0}$

Edge of magnitude D detected $= \sqrt{D_{xx}^2}$

Now when edge is rotated such that
$x' = x\cos\theta$ & $y' = x\sin\theta$

$D_{x'x'} = D_x \cos\theta$ $D_{y'y'} = D_{xx}\sin\theta$

New magnitude

$$= \sqrt{D^2_{x'x'} + D^2_{y'y'}}$$

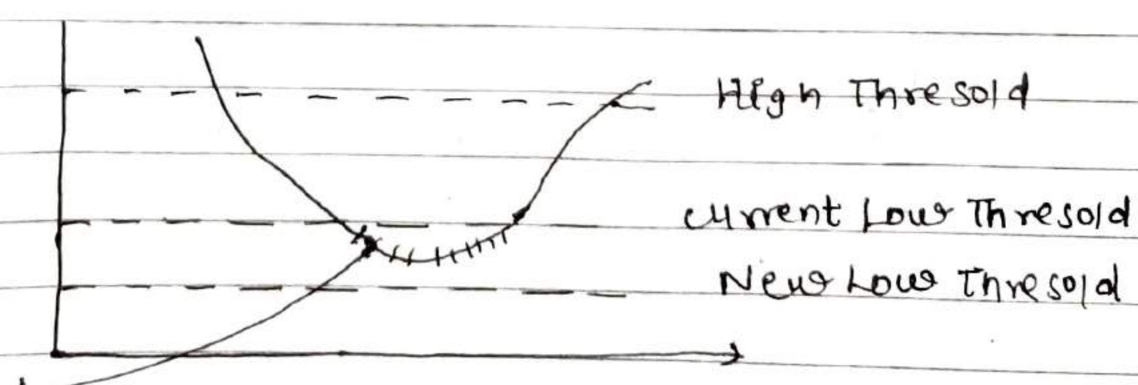
$$= \sqrt{D^2_{xx}\cos^2\theta + D^2_{xx}\sin^2\theta}$$

$$= \sqrt{D^2_{xx}(\cos^2\theta + \sin^2\theta)}$$

$$= D_{xx}$$

Which is same as before. So canny edge detector will be able to find edge.

Ans → 2 → b →



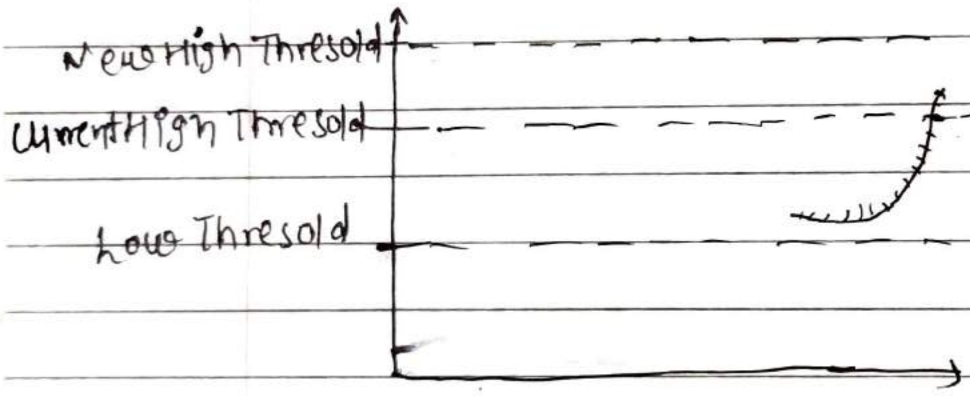
Edge Gap (As this is lower than Lower thresold value)

Only Edge between High Thresold & current Low Thresold will be detected, in addition to edge above

High thresold.

But if we lower down the ~~eta~~ Lower thresold value, longer edges will not break into shorter segment.

→ Some spurious edges may appear if High Thresold is Lower.



→ With current High Thresold this spurious edge will be detected.

But if we increase High Thresold, these spurious edges will disappear.