1) Ans

$$A = \begin{vmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{vmatrix}$$

Applying $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 0 \\ 0 & -4 & -11 & 0 \end{bmatrix} \xrightarrow{R_4 \to R_4 - R_3} \begin{vmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 0 \\ 0 & 0 & -3 & 0 \end{vmatrix}$$

$\Rightarrow R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank}(A) = 3}$ Am

2) Ans: find Rank & nullity of T.

Since the max^m degree of polynomial $T = 2$, so $\dim(P_2) = 3$

So. a is subset of kernel T is $T(a) = 0$

$$(a-b) + (b-c)x + (c-a)x^2 = 0$$

$$\boxed{a = b = c = t \text{ (let)}}$$

new Matrix $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$

dimension of kernal is 1. because there are only one independent paratmeter as 't'.

Acc to rank nullity theorem →

rank (T) + nullity (T) = dimention(W)

Rann (T) + 1 = 4

Rann (T) = 3

$\boxed{\text{So rann of T is 3 \& nullity is 1}}$

8 Ans → $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ find A^{-1} & $A+4I$.

* $A^{-1} = \frac{1}{|A|} \text{adj} A$

$|A| = 3$

$\text{adj} A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$= \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} =$

$= \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$

* $A+4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

Now → A^{-1} [Eigen value & Eigen vectors)

$\Rightarrow [A^{-1} - \lambda I] = 0$

$= \left| \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$

$= \begin{vmatrix} 2/3 - \lambda & 1/3 \\ 1/3 & 2/3 - \lambda \end{vmatrix} = 0$

$= (2/3 - \lambda)^2 - (1/3)^2 = 0$

$= \cancel{\frac{4}{9} - \frac{4}{3}\lambda + \lambda^2 - \frac{1}{9} = 0}$

$= (2/3 - \lambda + 1/3)(2/3 - \lambda - 1/3) = 0$

$= (1 - \lambda)(1/3 - \lambda) = 0$

$\boxed{\lambda = 1, 1/3}$

for $\lambda = 1$ Eigen Vector

$A = [A^{-1} - \lambda I][X] = 0$

$= \left(\begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right) (X) = 0$

$= \left(\begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)(x) = 0$

$\begin{pmatrix} -1/3 & 1/3 \\ 1/3 & -1/3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

$R_2 \to R_2 + R_1$

$\begin{pmatrix} -1/3 & 1/3 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

$-1/3 x_1 + 1/3 x_2 = 0$

$x_1 = x_2$

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let $x_1 = k$

$x_2 = k$

$$\boxed{\therefore \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

for $\lambda = 1/3$ <u>Eigon Vector</u>

$$[A - \lambda I](x] = 0$$

$$\left[\begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{pmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1/3 & 1/3 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\frac{x_1}{3} + \frac{x_2}{3} = 0$$

$$\boxed{x_1 = -x_2}$$

Let $x_1 = k$

$x_2 = -k$

$$\boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = k\begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

* Now $A + 4I = \begin{pmatrix} 6 & -1 \\ -1 & 6 \end{pmatrix}$

<u>Eigon vector</u> $\Rightarrow (A - \lambda I) = 0$

$$\begin{pmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{pmatrix} = 0$$

$$(6-\lambda)^2 - (1)^2 = 0$$

$$(6-\lambda-1)(6-\lambda+1) = 0$$

$$\boxed{\lambda = 5, 7}$$

Eigon vector for $\lambda = 5$

$$[A - \lambda I](x) = 0$$

$$\left[\begin{pmatrix} 6 & -1 \\ -1 & 6 \end{pmatrix} - 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$x_1 - x_2 = 0$$

$$\boxed{x_1 = x_2}$$

let $x_1 = k$

$x_2 = k$ $\Rightarrow$ $\boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = k\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

Eigone Vector for $\lambda = 7$

$$\left(\begin{pmatrix} 6 & -1 \\ -1 & 6 \end{pmatrix} - 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$R_2 \to R_2 - R_1$

$$\begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$-x_1 + x_2 = 0$$

$$\boxed{x_1 = -x_2}$$

let $x_1 = k$

$x_2 = -k$

$$\boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = k\begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

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(4) Ans →

$3x - 0.1y - 0.2z = 7.85$
$0.1x + 7y - 0.3z = -19.3$
$0.3x - 0.2y + 10z = 71.4$

$x(0) = 0$
$y(0) = 0$
$z(0) = 0$

It's diagonally dominated system of Equation
Its iteration

$3x - 0.1y - 0.2z = 7.85$
$3x = 0.1y + 0.2z + 7.85$
$x = \frac{0.1y + 0.2z + 7.85}{3}$

Let $y = z = 0$
$x = \frac{7.85}{3} = 2.616$

$\Rightarrow$ $0.1x + 7y - 0.3z = -19.3$

Let $z = 0$ & $x = 2.616$

$0.1 \times 2.616 + 7y = -19.3$
$7y = -19.3 - 0.2616$
$y = \frac{-19.5616}{7}$ $\boxed{y = -2.9745}$

Now $\Rightarrow$ $0.3x - 0.2y + 10z = 71.4$
$0.3 \times 2.616 - 0.2 \times -2.9745 + 10z = 71.4$

$10z = 70.0563$
$\boxed{z = 7.00563}$

2nd Equation

$3x - 0.1y - 0.2z = 7.85$
$3x - 0.1(-2.97) - 0.2 \times 7.0053 = 7.85$

$3x = 8.971676$
$\boxed{x = 2.99056}$

Now → $0.1x + 7y - 0.3z = -19.3$
$0.1 \times 2.99057 + 7y - 0.3 \times 7.00563 = -19.3$
$\boxed{y = 0.26013}$

$0.3x - 0.2y + 10z = 71.4$
$0.3 \times (2.99057) - 0.2 \times (0.26013) + \cancel{0.2(\ldots)}$ $10z = 71.4$
$\boxed{z = 7.05548}$

3rd Equation

$3x - 0.1y - 0.2z = 7.85$
$3x - 0.1 \times (0.2601) - 0.2 \times (7.05549) = 7.85$
$\boxed{x = 3.0957038}$

$0.1x + 7y - 0.3z = -19.3$
$0.1 \times (3.0957) + 7y - 0.3(7.05549) = -19$
$\boxed{y = -3.015296}$

$0.3x - 0.2y + 10z = 71.4$
$0.3(3.0957038) - 0.2(-3.015296) + 10z = 71.4$
$\boxed{z = 6.9868}$

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$$x+3y+2z=0$$
$$2x-y+3z=0$$
$$3x-5y+4z=0$$
$$x+17y+4z=0$$

We have following equation $\leftrightarrow$

$$[A][x]=0$$

$$\begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{vmatrix} [x]=0$$

Auguwented Matrix $\rightarrow$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix} \quad \begin{array}{l} R_4 \to R_4 + R_3 \\ R_3 \to R_3 - 2R_2 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A)=2$$
$$\rho(A:B)=2$$

no. of unknown $=3$

Infinite sol^n

$$-7x_2 - x_3 = 0$$
$$-7x_2 = x_3$$
let $x_3 = k$
$$\boxed{x_2 = -k/7}$$

$$x_1 + 3x_2 + 2x_2 = 0$$
$$x_1 + 3\left(-\frac{k}{7}\right) + 2k = 0$$
$$x_1 - \frac{3k}{7} + 2k = 0$$
$$\boxed{x_1 = \frac{-11k}{7}}$$

Sol^n $\Rightarrow$ $k \begin{bmatrix} -11/7 \\ -1/k \\ k \end{bmatrix}$

Ans:-

6) $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

i) Additive ~~Additive~~ →

$T(u+v) \Rightarrow T(u)+T(v)$

$u = a_1+b_1x+c_1x^2$

$v = a_2+b_2x+c_2x^2$

$T(u+v) \Rightarrow T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)x^2)$

$\Rightarrow (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$

$\Rightarrow (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x+(c_2+1)x^2$

$\Rightarrow \boxed{T(u)+T(v)}$ proved

(ii) Homogenity →

$T(ku) \Rightarrow kT(u)$

$T(k(a+bx+cx^2))$

$T(ka+kbx+kcx^2)$

$\Rightarrow (ka+kb+kc+1)+(ka+kb+kc+1)x+(ka+kb+kc+1)x^2$

$\Rightarrow k(a+1)+k(b+1)x+k(c+1)x^2$

$\boxed{kT(u)}$ proved

7) Ans:-

$a(1,2,3)+b(3,1,0)+c(-2,1,3) = (0,0,0)$

$a+3b-2c=0$

$2a+b+c=0$

$3a+3c=0$

$c=-a, b=-a$

8) Ans:-

$3x-6y+2z=23$

$-4x+y-z=-15$

$x-3y+7z=16$

So. we can use Jacobi method for the above set of system of equation

$x_1 = \frac{23+6y-2z}{3}$

$y_2 = \frac{-15+4x+z}{\cancel{-4}}$

$z_3 = \frac{16-x+3y}{7}$

$x_0=1,\ y_0=1\ z_0=1$

1st iteratn →

$x = \frac{23+6\times1-2\times1}{3} = 9$

$y = -15*4+1 = -10$

$z = \frac{16-1+3}{7} = \frac{18}{7} = 2.57$

2nd Iteration :- $x=9, y=-10, z=2.57$

$x = \frac{23+6y-2z}{3} =$

$= \frac{23-60-5.14}{3} = \frac{42.14}{\cancel{3}}\ 14.05$

$\boxed{x = 14.05}$

$y = -15*4x+z = -15*36+2.57$

$= 23.57$

$z = \frac{16-x+3y}{7}$

$= \frac{16-9+3\times-10}{7}$

$= \frac{-23}{7} = -3.286$

3rd iteratn $x=14.05,\ y=23.57,\ z=-3.28$

$x = \frac{23+6y-2z}{3} = \frac{23*6\times23.57+6.56}{3}$

$= \frac{170.98}{3} = \boxed{56.99}$

$y = -15+4\times14.05+(-3.28)$

$= -15+56.2-3.28$

$\boxed{y = 38.92}$

$z = \frac{16-14.05+(-9.84)}{7}$

$= \frac{16-23.89}{7} = \frac{7.89}{\cancel{7}} = 1.12$

$\boxed{z = 1.127}$

(9) Ans Suppose we have a 2-D image represented as grid or pixels we can use RT matrix to rotate around centre.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ rotatⁿ of image by θ to rotate it around centre

1) Translation to origin → Translate the image so that it centre alligns with origin

2) Rotation → Apply rotation matrix.

3) Translation back :- translate it back will its original position by adding coordinate of centre.

(10) Ans :- Linear Transformation for rotation 2-D images involves applying a rotation matrix to each pixel coordinate. This matrix rotates points counterclockwise by an angle θ around the origin. It presaves geometric properties like parallelism and distance. Rotation is essential in tasks like image allignment and object detection in computer vision.

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