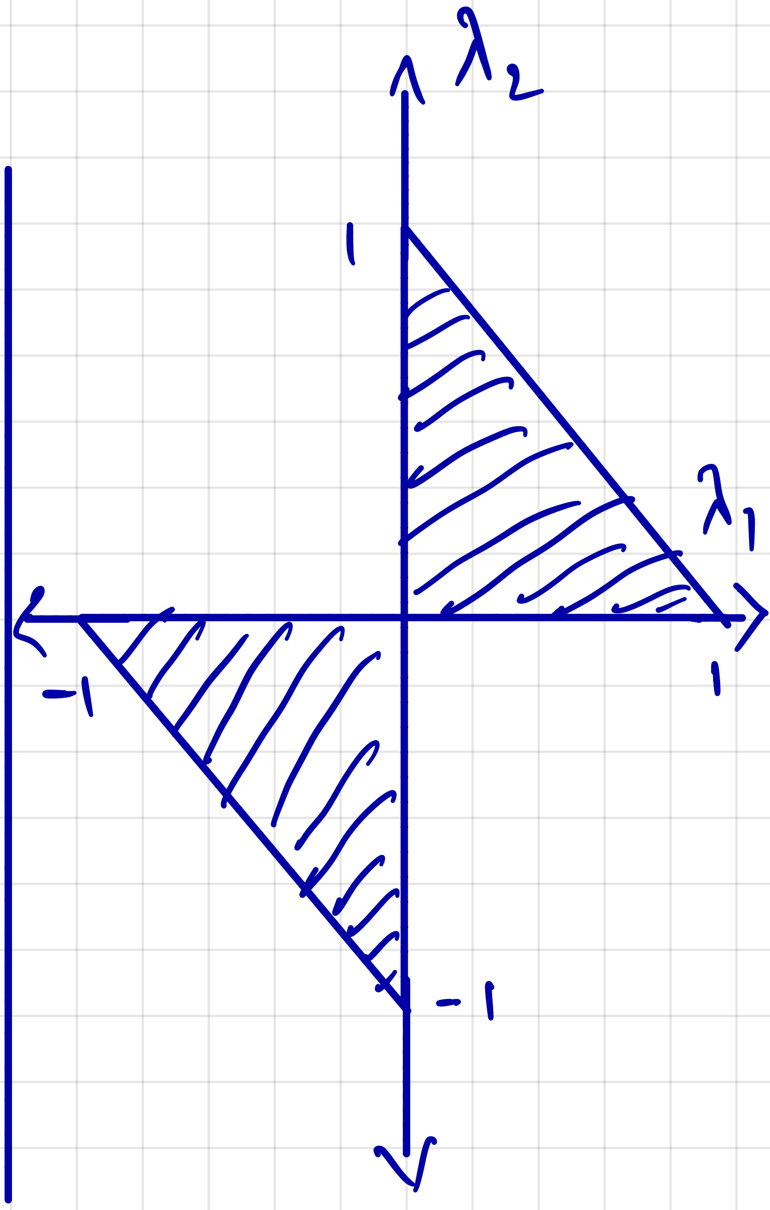
Time Series Analysis Homework 1

2^{nd} Question

2. a.
$$f_{X,Y}(\lambda_1,\lambda_2) = \begin{cases} c \ ; & -1<\lambda_1<1 \text{ and } -1<\lambda_2<1 \\ 0 \ ; & \text{Else} \end{cases}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(\lambda_1,\lambda_2)\, d\lambda_1\, d\lambda_2$$

$$= \int_{-1}^{0}\int_{-1-\lambda_2}^{0} c \, d\lambda_1 \, d\lambda_2 + \underline{2^{nd} \text{ part}}$$



$$= \int_{-1}^{0} c\lambda_1 \Big|_{-1-\lambda_2}^{0} d\lambda_2 = \int_{-1}^{0} -(c(-1-\lambda_2))\, d\lambda_2$$

$$= \int_{-1}^{0} -(-c - c\lambda_2)\, d\lambda_2$$

$$= \int_{-1}^{0} c + c\lambda_2 \; d\lambda_2$$

$$= c\lambda_2 + \frac{c\lambda_2^2}{2} \Big|_{-1}^{0}$$

$$= -\left(-c + \frac{c}{2}\right)$$

$$= -\left(-\frac{c}{2}\right) = \frac{c}{2}$$

Right-side

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(\lambda_1, \lambda_2) \, d\lambda_1 \, d\lambda_2$$

$$= \int_0^1 \int_0^{1-\lambda_2} c \; d\lambda_1 \, d\lambda_2 = \int_0^1 c\lambda_1 \Big|_0^{1-\lambda_2}$$

$$= \int_0^1 c(1-\lambda_2) \, d\lambda_2$$

$$= \int_0^1 c - c\lambda_2 \, d\lambda_2$$

$$= c\lambda_2 - \frac{c\lambda_2^2}{2} \Big|_0^1$$

$$= c - \frac{c}{2} = \frac{c}{2}$$

Left-side + Right-side = 1

$$\frac{c}{2} + \frac{c}{2} = 1$$

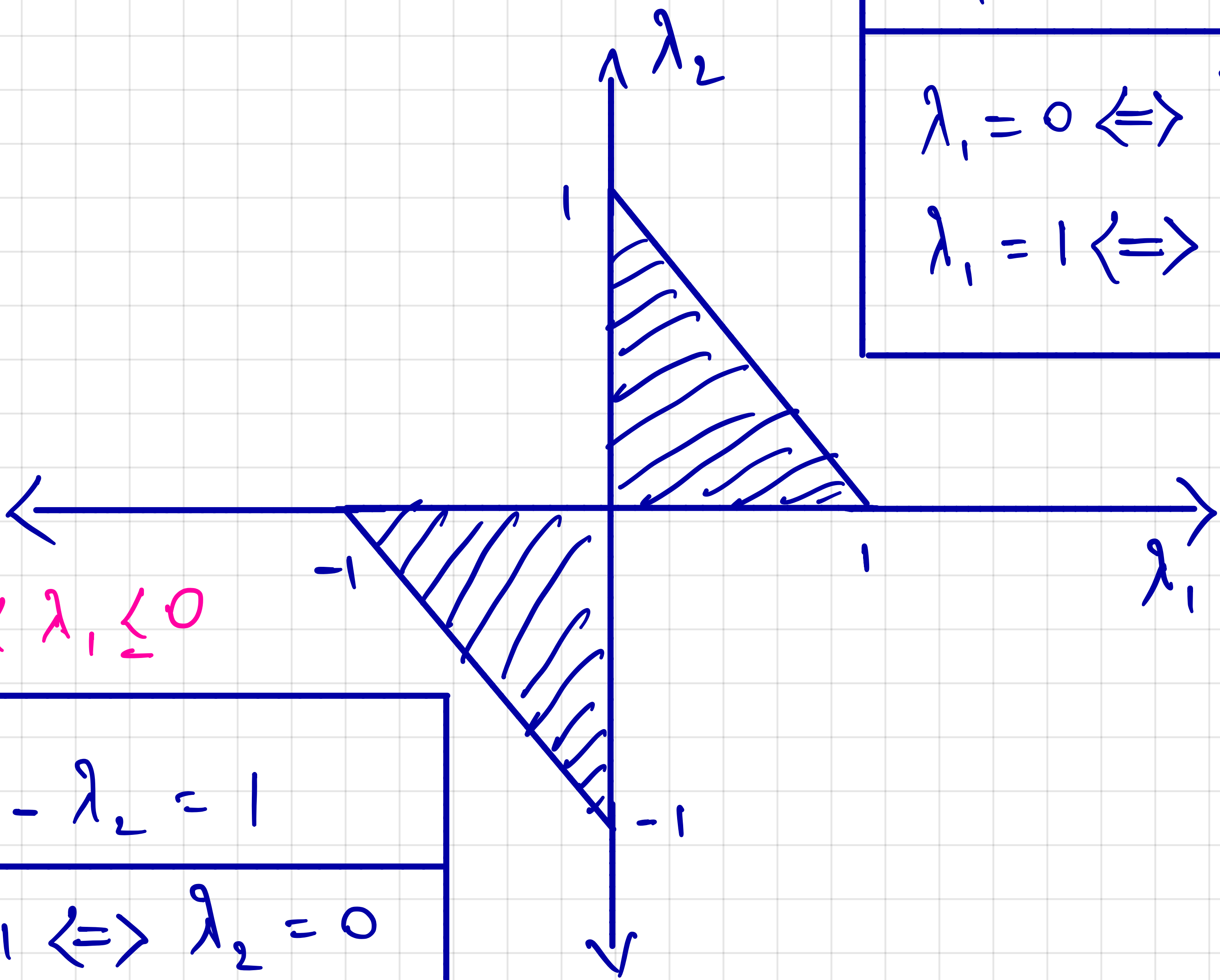
$$2c = 2$$

$$\boxed{c = 1}$$

b. Find the marginal density $f_x(\lambda_1)$ and $f_y(\lambda_2)$

$0 < \lambda_2 \leq 1$

$$\boxed{\begin{array}{c} \lambda_1 + \lambda_2 = 1 \\ \hline \lambda_1 = 0 \Leftrightarrow \lambda_2 = 1 \\ \lambda_1 = 1 \Leftrightarrow \lambda_2 = 0 \end{array}}$$



$-1 < \lambda_1 \leq 0$

$$\boxed{\begin{array}{c} -\lambda_1 - \lambda_2 = 1 \\ \hline \lambda_1 = -1 \Leftrightarrow \lambda_2 = 0 \\ \lambda_1 = 0 \Leftrightarrow \lambda_2 = -1 \end{array}}$$

b. Find the marginal density $f_x(\lambda_1)$ and $f_y(\lambda_2)$

Solve using the equation $-\lambda_1 - \lambda_2 = 1$

then $-\lambda_2 = 1 + \lambda_1$

$\lambda_2 = -\lambda_1 - 1$

Left-side

$$f_x(\lambda_1) = \int_{-\infty}^{\infty} f_{x,y}(\lambda_1, \lambda_2)\, d\lambda_2$$

$$= \int_{-\lambda_1 - 1}^{0} 1 \quad d\lambda_2 \quad \Rightarrow \lambda_2 \Big|_{-\lambda_1 - 1}^{0}$$

$$= -(-\lambda_1 - 1) = \lambda_1 + 1$$

$$\boxed{f_x(\lambda_1) = \lambda_1 + 1 \quad : \quad -1 \leq \lambda_1 \leq 0}$$

Equation just for the left-side

Solve using the equation $\lambda_1 + \lambda_2 = 1$

$\lambda_2 = 1 - \lambda_1$

Right-side

$$f_x(\lambda_1) = \int_{-\infty}^{\infty} f_{x,y}(\lambda_1, \lambda_2)\, d\lambda_2$$

$$= \int_0^{1-\lambda_1} 1 \quad d\lambda_2$$

$$= \lambda_2 \Big|_0^{1-\lambda_1}$$

Eq. Verification

$\lambda_2 = 1 - \lambda_1$

$\lambda_2 = 1$ ✓

$\lambda_2 = 1 - 1$

$= 0$ ✓

$$\boxed{f_x(\lambda_1) = 1 - \lambda_1 \; : \; 0 \leq \lambda_1 \leq 1}$$

→ Equation just for the right side

$$\boxed{f_x(\lambda_1) = \begin{cases} \lambda_1 + 1 & : \; -1 \leq \lambda_1 \leq 0 \\ 1 - \lambda_1 & : \; 0 \leq \lambda_1 \leq 1 \end{cases}}$$

Find $f_y(\lambda_2)$

$$f_y(\lambda_2) = \int_{-\infty}^{\infty} f_{x,y}(\lambda_1, \lambda_2)\, d\lambda_1$$

$$\boxed{\begin{array}{l} -\lambda_1 - \lambda_2 = 1 \longrightarrow 1^{st} \text{ part} \\ \lambda_1 + \lambda_2 = 1 \longrightarrow 2^{nd} \text{ part} \end{array}}$$

Left-side

$$\lambda_1 = -1 - \lambda_2$$

$$f_y(\lambda_2) = \lambda_1 \Big|_{-1-\lambda_2}^{0}$$

$$= (0) \underbrace{- \quad (-1-\lambda_2)}$$

$$= 1 + \lambda_2$$

Right-side

$$\boxed{\lambda_1 + \lambda_2 = 1 \Rightarrow \lambda_1 = 1 - \lambda_2}$$

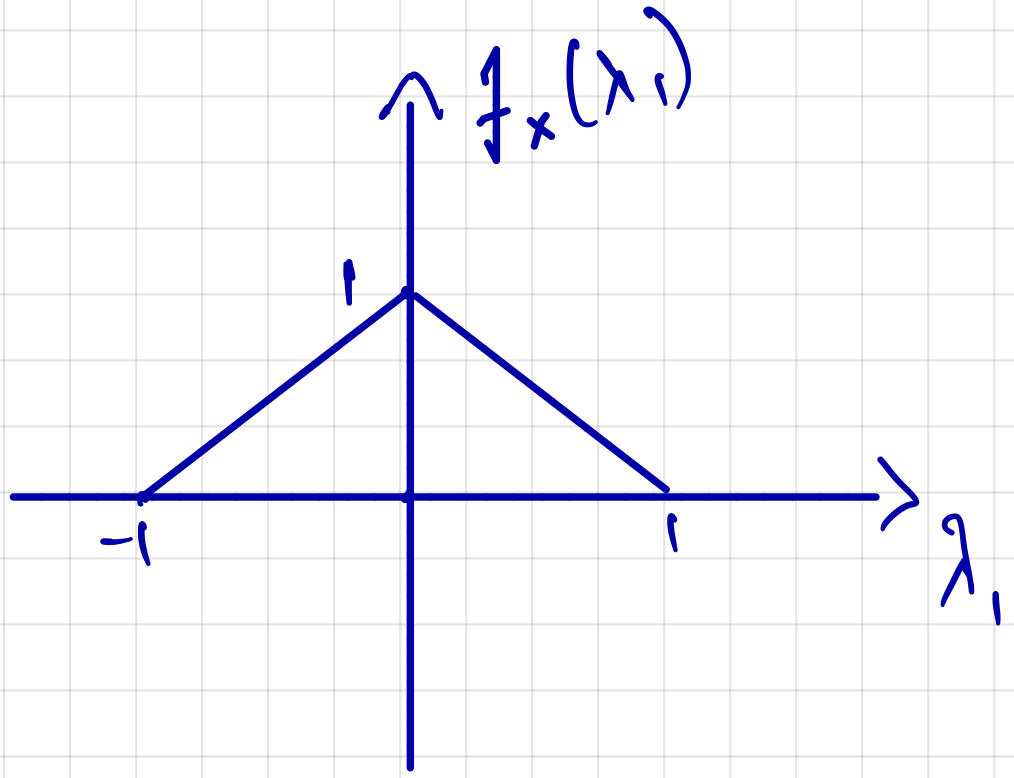
$$= \lambda_1 \Big|_{0}^{1-\lambda_2} \Rightarrow (1-\lambda_2)$$

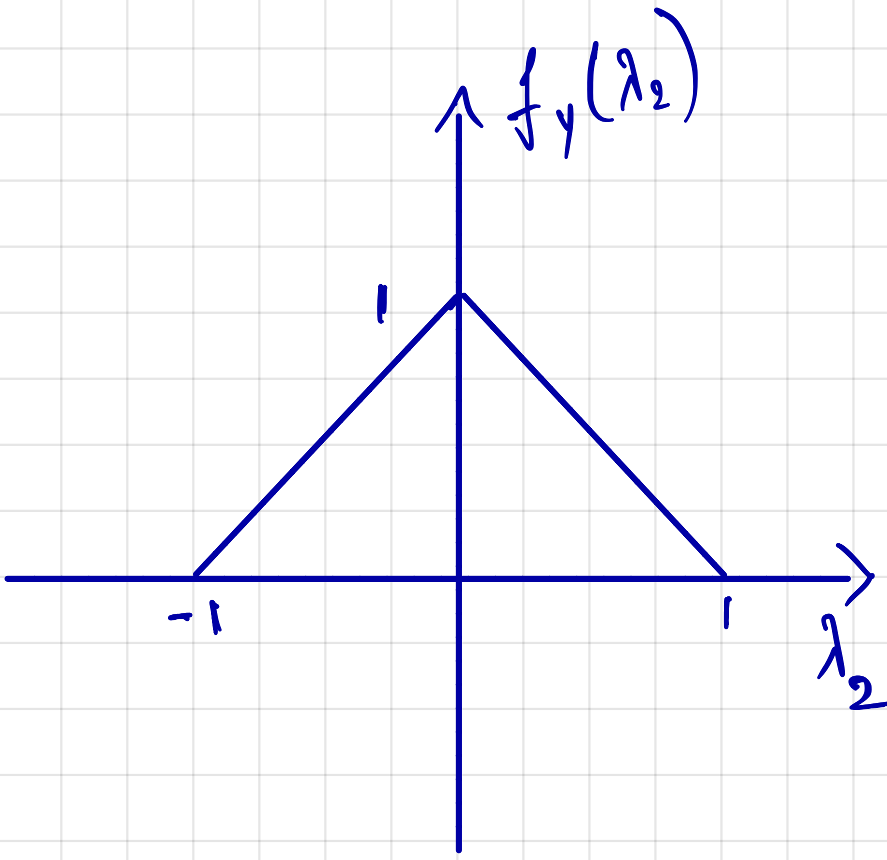
$$\boxed{f_y(\lambda_2) = \begin{cases} 1 + \lambda_2 & : -1 \leq \lambda_2 \leq 0 \\ 1 - \lambda_2 & : 0 \leq \lambda_2 \leq 1 \end{cases}}$$

$$f_Y(\lambda_2) = \begin{cases} 1 + \lambda_2 & : -1 \leq \lambda_2 \leq 0 \\ 1 - \lambda_2 & : 0 \leq \lambda_2 \leq 1 \end{cases}$$

Marginal density Graph

C.





Area of the marginal density

$$\int_{-1}^{1} f_X(\lambda_1)\, d\lambda_1 = 1$$

Area of the marginal density

$$\int_{-1}^{1} f_Y(\lambda_2)\, d\lambda_2 = 1$$

Hence, they both are valid marginal density

d. What is $\mathbb{E}[X]$? and $\mathbb{E}[Y]$?

$$\mathbb{E}[X] = \mathbb{E}[\lambda_1] = \int_{-\infty}^{\infty} \lambda_1 f_X(\lambda_1)\, d\lambda_1$$

$$f_X(\lambda_1) = \begin{cases} \lambda_1 + 1 & : \quad -1 \leq \lambda_1 \leq 0 \\ 1 - \lambda_1 & : \quad 0 \leq \lambda_1 \leq 1 \end{cases}$$

$$\mathbb{E}[\lambda_1] = \int_{-1}^{0} \lambda_1 (\lambda_1 + 1)\, d\lambda_1 + \int_{0}^{1} \lambda_1 (1 - \lambda_1)\, d\lambda_1$$

$$= \int_{-1}^{0} \lambda_1^2 + \lambda_1 \, d\lambda_1 + \int_{0}^{1} \lambda_1 - \lambda_1^2 \, d\lambda_1$$

$$= \frac{\lambda_1^3}{3} + \frac{\lambda_1^2}{2} \Big|_{-1}^{0} + \frac{\lambda_1^2}{2} - \frac{\lambda_1^3}{3} \Big|_{0}^{1}$$

$$= -\left(-\frac{1}{3} + \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{1}{3} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} = 0$$

d. What is $\mathbb{E}[Y]$?

$$\boxed{-1 < \lambda_2 \leq 1}$$

$$\mathbb{E}[Y] = \mathbb{E}[\lambda_2] = \int_{-\infty}^{\infty} \lambda_2 \, f_Y(\lambda_2) \, d\lambda_2$$

$$f_Y(\lambda_2) = \begin{cases} 1+\lambda_2 & : -1 \leq \lambda_2 \leq 0 \\ 1-\lambda_2 & : 0 \leq \lambda_2 \leq 1 \end{cases}$$

$$= \int_{-1}^{0} \lambda_2 \, (1+\lambda_2) \, d\lambda_2 + \int_{0}^{1} \lambda_2 \, (1-\lambda_2) \, d\lambda_2$$

$$= \int_{-1}^{0} \lambda_2 + \lambda_2^2 \, d\lambda_2 + \int_{0}^{1} \lambda_2 - \lambda_2^2 \, d\lambda_2$$

$$= \frac{\lambda_2^2}{2} + \frac{\lambda_2^3}{3} \Bigg|_{-1}^{0} + \frac{\lambda_2^2}{2} - \frac{\lambda_2^3}{3} \Bigg|_{0}^{1}$$

$$= -\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= -\frac{1}{2} + \frac{1}{3} + \frac{1}{2} - \frac{1}{3} = 0$$

e. Find and graph $f_{X|Y}(\lambda_1|\lambda_2)$. What is $f_{X|Y}(\lambda_1|\lambda_2)$ when $\underline{\lambda_2 = 0.5}$

$$f_{X|Y}(\lambda_1|\lambda_2) = \frac{f_{X,Y}(\lambda_1,\lambda_2)}{f_Y(\lambda_2)}$$

$$f_{X|Y}(\lambda_1|\lambda_2=0.5) = \frac{1}{0.5} = 2 \,//$$

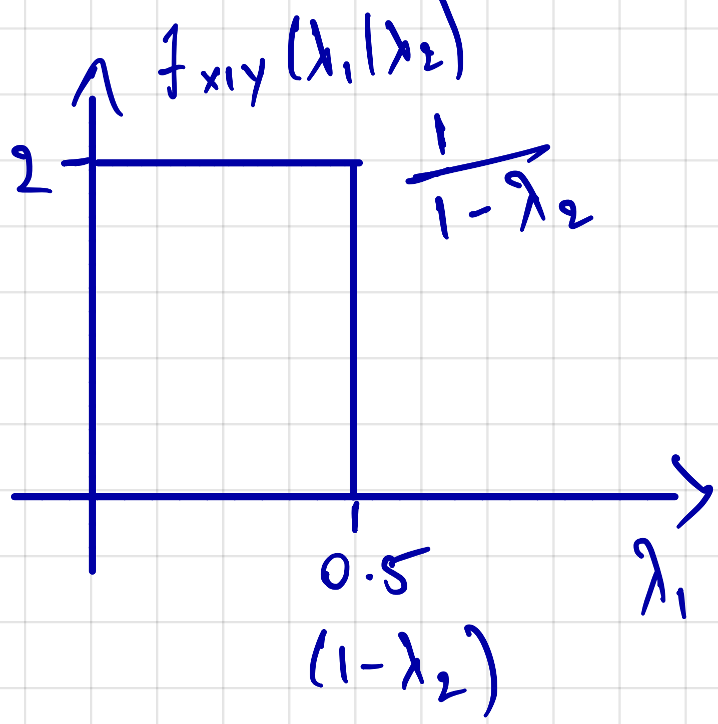
0.5 is the prob for $f_Y(\lambda_2=0.5)$

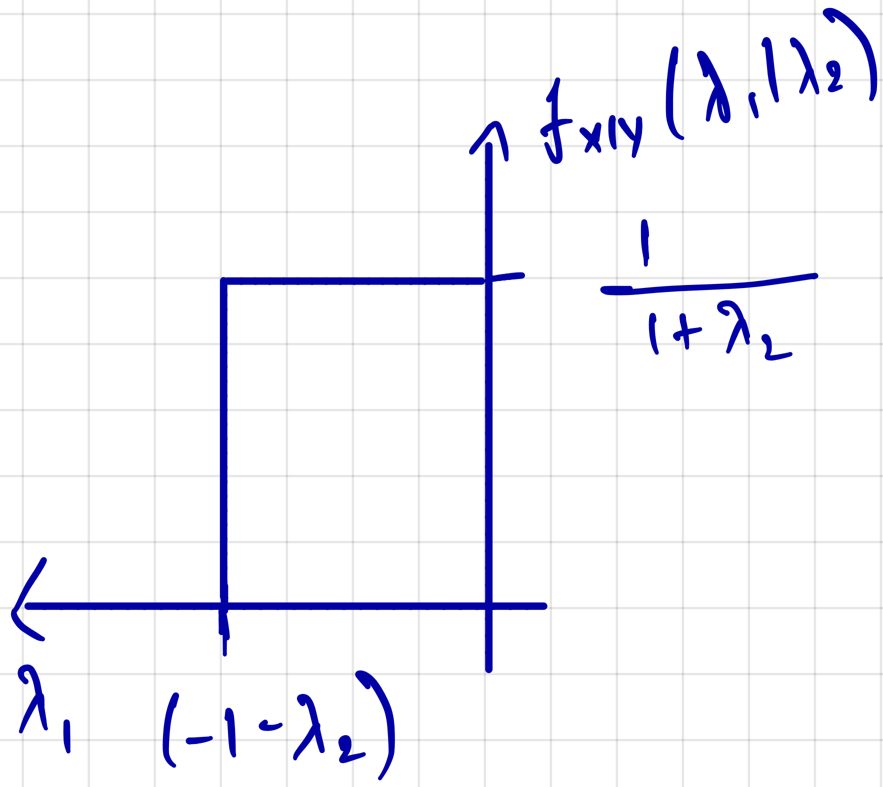
f. $$f_{Y|X}(\lambda_2|\lambda_1=0.5) = \frac{f_{X,Y}(\lambda_1,\lambda_2)}{f_X(\lambda_1=0.5)}$$

$$= \frac{1}{0.5} = 2 \,//$$

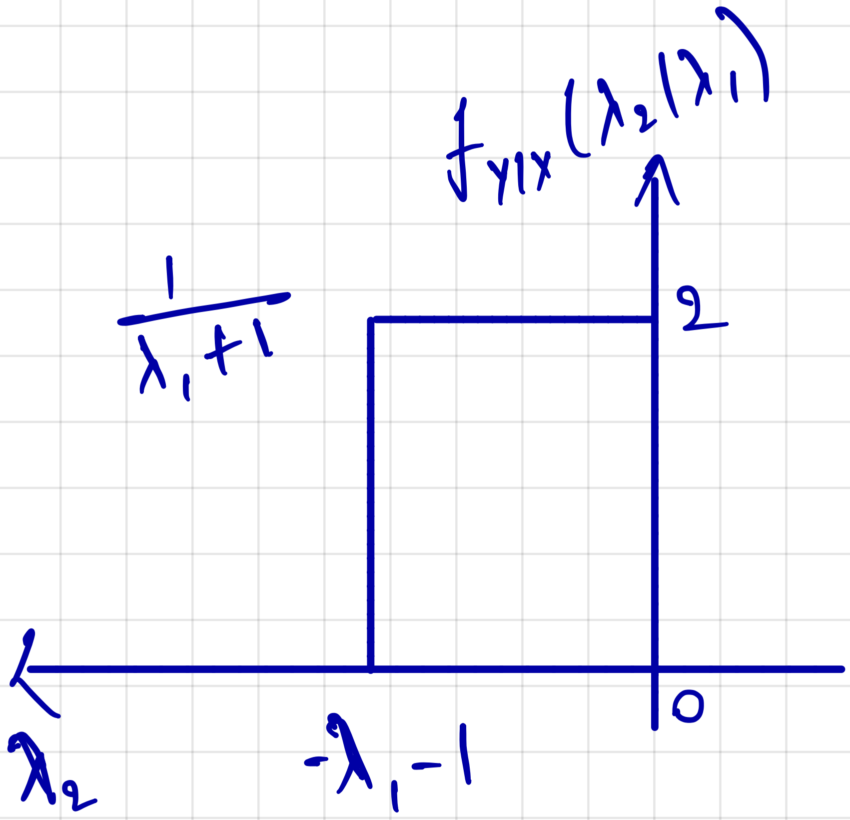
Graph of Conditional density

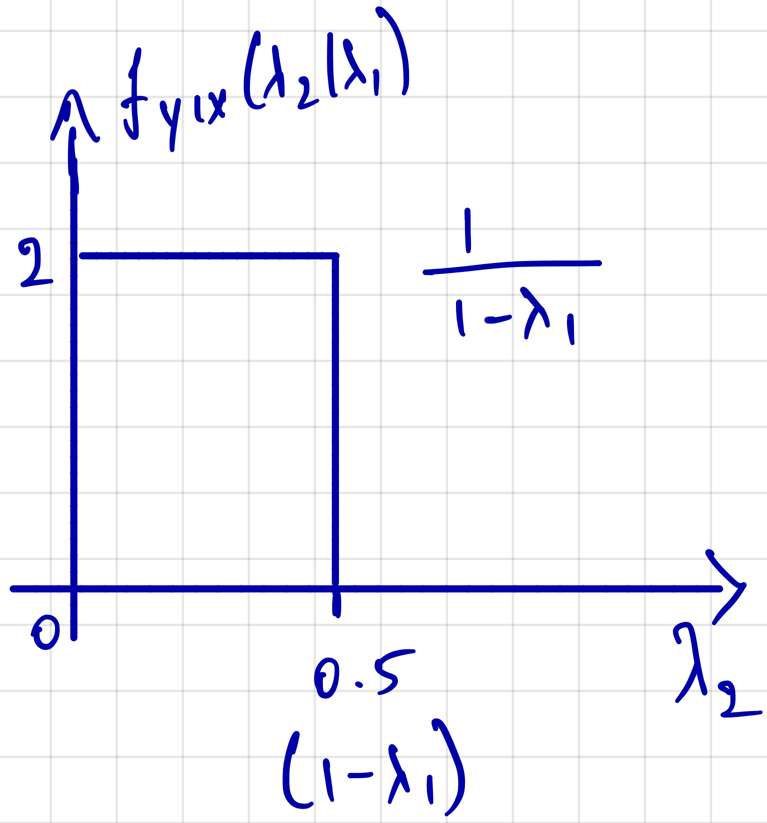
e).





f).





g. Are random variables X and Y independent?

Independence property:-

$$f_{X,Y}(\lambda_1, \lambda_2) = f_X(\lambda_1) \times f_Y(\lambda_2)$$

$$f_{X,Y}(\lambda_1, \lambda_2) = 1$$

$$f_X(\lambda_1 = -1) = 0$$

$$f_Y(\lambda_2 = -1) = 0$$

$$1 \neq 0$$

No, X and Y are not independent

3.

a). $$f_{X,Y}(\lambda_1,\lambda_2) = \begin{cases} c\lambda_1^2 + \dfrac{\lambda_1\lambda_2}{3} & : \; 0<\lambda_1<1 \text{ and } 0<\lambda_2<2 \\ 0 & : \; \text{Else} \end{cases}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c\lambda_1^2 + \frac{\lambda_1\lambda_2}{3} \, d\lambda_1 \, d\lambda_2$$

$$= \int_0^2 \int_0^1 c\lambda_1^2 + \frac{\lambda_1\lambda_2}{3} \, d\lambda_1 \, d\lambda_2$$

$$= \int_0^2 \frac{c\lambda_1^3}{3} + \frac{\lambda_1^2\lambda_2}{6} \Big|_0^1 \, d\lambda_2$$

$$= \int_0^2 \left(\frac{c}{3} + \frac{\lambda_2}{6}\right) d\lambda_2 = 1$$

$$= \frac{c\lambda_2}{3} + \frac{\lambda_2^2}{12} \Big|_0^2 = 1$$

$$\Rightarrow \frac{2c}{3} + \frac{4}{12} = 1$$

$$\frac{2c+1}{3} = 1 \Rightarrow 2c+1 = 3$$

$$2c = 3-1$$

$$2c = 2$$

$$c = 2/2 \Rightarrow c = 1 \; //$$

d). Find $P(x+y \geq 1)$

Solve using equation $\lambda_1 + \lambda_2 = 1$

$$P(\lambda_1 + \lambda_2 \geq 1) = 1 - P(\lambda_1 + \lambda_2 \leq 1)$$

$$= 1 - \int_0^1 \int_0^{1-\lambda_1} \lambda_1^2 + \frac{\lambda_1 \lambda_2}{3} \, d\lambda_2 \, d\lambda_1$$

$$= 1 - \int_0^1 \lambda_1^2 \lambda_2 + \frac{\lambda_1 \lambda_2^2}{6} \Big|_0^{1-\lambda_1} d\lambda_1$$

$$= 1 - \int_0^1 \lambda_1^2 (1-\lambda_1) + \frac{\lambda_1 (1-\lambda_1)^2}{6} \, d\lambda_1$$

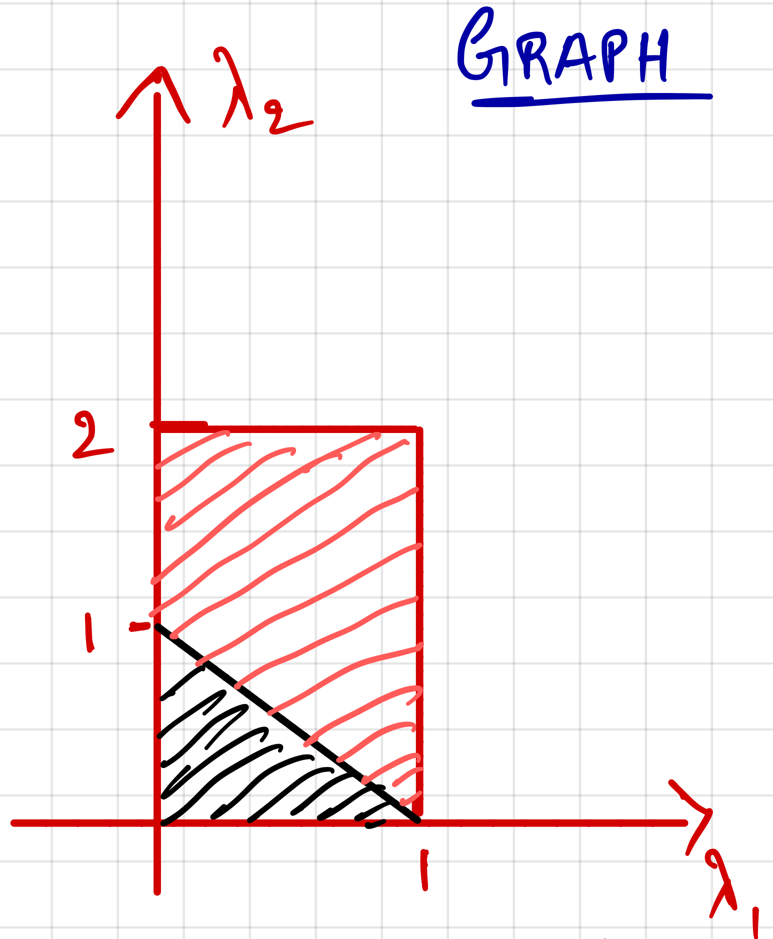
$$= 1 - \int_0^1 \lambda_1^2 - \lambda_1^3 + \frac{\lambda_1 (1 - 2\lambda_1 + \lambda_1^2)}{6} \, d\lambda_1$$

$$= 1 - \int_0^1 \lambda_1^2 - \lambda_1^3 + \frac{\lambda_1}{6} - \frac{2\lambda_1^2}{6} + \frac{\lambda_1^3}{6} \, d\lambda_1$$

$$= 1 - \left[\frac{\lambda_1^3}{3} - \frac{\lambda_1^4}{4} + \frac{\lambda_1^2}{12} - \frac{2\lambda_1^3}{18} + \frac{\lambda_1^4}{24} \right]_0^1$$

$$= 1 - \left[\frac{1}{3} - \frac{1}{4} + \frac{1}{12} - \frac{\cancel{2}^{1}}{\cancel{18}_{9}} + \frac{1}{24} \right]$$

GRAPH



Black line corresponds to equation $x + y = 1$
when $x=0, y=1 \mid x=1, y=0$

$$= 1 - \left[\frac{1}{3} - \frac{1}{4} + \frac{1}{12} - \frac{\cancel{2}^{\,1}}{\cancel{18}_{\,9}} + \frac{1}{24} \right]$$

$$= 1 - \left[-\frac{1}{4} - \frac{1}{9} + \frac{1}{3} + \frac{1}{12} + \frac{1}{24} \right]$$

$$= 1 - \left[\frac{-9-4}{36} + \frac{8+2+1}{24} \right]$$

$$= 1 - \left[-\frac{13}{36} + \frac{11}{24} \right] = 0.9 \quad //$$

b). Find the marginal density $f_X(\lambda_1)$ and $f_Y(\lambda_2)$

$$f_X(\lambda_1) = \int_{-\infty}^{\infty} \lambda_1^2 + \frac{\lambda_1 \lambda_2}{3} \, d\lambda_2$$

$$= \lambda_1^2 \lambda_2 + \frac{\lambda_1 \lambda_2^2}{6} \Big|_0^2$$

$$= 2\lambda_1^2 + \frac{4\lambda_1}{6}$$

$$\boxed{f_X(\lambda_1) = 2\lambda_1^2 + \frac{2}{3}\lambda_1 \qquad 0 \le \lambda_1 \le 1}$$

Is this a valid density?

$$\int_0^1 2\lambda_1^2 + \frac{2}{3}\lambda_1 \, d\lambda_1$$

$$= \frac{2\lambda_1^3}{3} + \frac{2}{3}\frac{\lambda_1^2}{2} \Big|_0^1$$

$$= \frac{2\lambda_1^3}{3} + \frac{2}{6}\lambda_1^2 \Big|_0^1$$

$$= \frac{2}{3} + \frac{\cancel{2}^{1}}{\cancel{6}_{3}} = \frac{3}{3} = 1 \,//$$

Find the marginal density $f_y(\lambda_2)$

$$f_y(\lambda_2) = \int_{-\infty}^{\infty} \lambda_1^2 + \frac{\lambda_1 \lambda_2}{3} \, d\lambda_1$$

$$= \int_0^1 \lambda_1^2 + \frac{\lambda_1 \lambda_2}{3} \, d\lambda_1$$

$$= \frac{\lambda_1^3}{3} + \frac{\lambda_1^2 \lambda_2}{6} \Big|_0^1$$

$$\boxed{f_y(\lambda_2) = \frac{1}{3} + \frac{\lambda_2}{6} \quad : \quad 0 \leq \lambda_2 \leq 2}$$

Is this a valid density :-

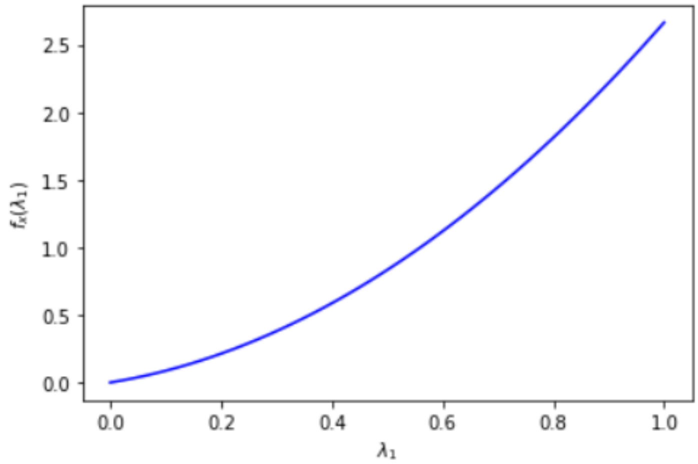
$$\int_0^2 \frac{1}{3} + \frac{\lambda_2}{6} \, d\lambda_2$$

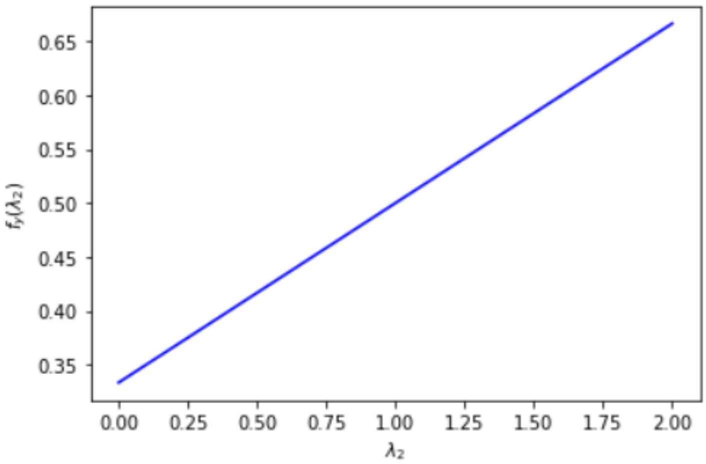
$$= \frac{1}{3}\lambda_2 + \frac{\lambda_2^2}{12} \Big|_0^2$$

$$= \frac{2}{3} + \frac{\cancel{4}^{\,1}}{\cancel{12}_{\,3}}$$

$$= \frac{3}{3} = 1 \,//$$

e. Graph the marginal density for . R·V X and Y





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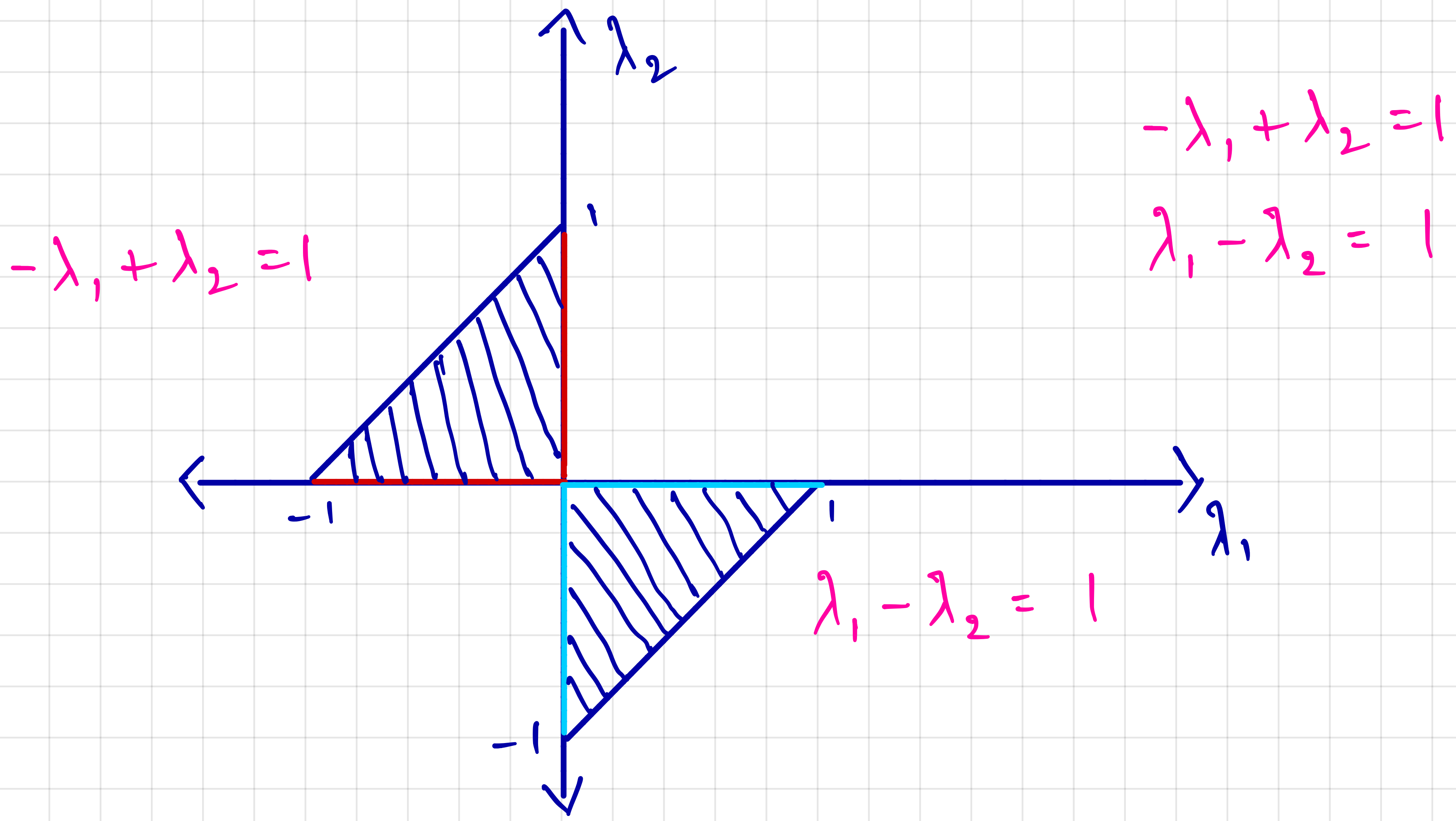
1. $$f_{X,Y}(\lambda_1,\lambda_2)=\begin{cases} c\,; & -1<\lambda_1<1 \text{ and } -1<\lambda_2<1 \\ 0\,; & \text{Else} \end{cases}$$



a. Find constant c

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(\lambda_1,\lambda_2)\, d\lambda_2\, d\lambda_1 = 1$$

$$\Rightarrow \int_{-1}^{0}\int_{0}^{1+\lambda_1} c \; d\lambda_2\, d\lambda_1 + \int_{0}^{1}\int_{\lambda_1-1}^{0} c \; d\lambda_2\, d\lambda_1 = 1$$

$$\Rightarrow \int_{-1}^{0} c\lambda_2 \Big|_{0}^{1+\lambda_1} d\lambda_1 + \int_{0}^{1} c\lambda_2 \Big|_{\lambda_1-1}^{0} d\lambda_1 = 1$$

$$\Rightarrow \int_{-1}^{0} c(1+\lambda_1)\, d\lambda_1 + \int_{0}^{1} -c(\lambda_1-1)\, d\lambda_1 = 1$$

$$\Rightarrow \int_{-1}^{0} c(1+\lambda_1)\, d\lambda_1 + \int_{0}^{1} -c(\lambda_1 - 1)\, d\lambda_1 = 1$$

$$\Rightarrow c\lambda_1 + \frac{c\lambda_1^2}{2} \Big|_{-1}^{0} + \left(-\frac{c\lambda_1^2}{2} + c\lambda_1\right) \Big|_{0}^{1} = 1$$

$$\Rightarrow -\left(-c + \frac{c}{2}\right) + \left(-\frac{c}{2} + c\right) = 1$$

$$\Rightarrow c - \frac{c}{2} - \frac{c}{2} + c$$

$$\Rightarrow 2c - c = 1$$

$$\boxed{\therefore c = 1}$$

b. <u>Marginal density $f_X(\lambda_1)$</u>

$$f_X(\lambda_1) = \int_0^{\lambda_1+1} f_{X,Y}(\lambda_1,\lambda_2)\, d\lambda_2$$

$$= \lambda_2 \Big|_0^{\lambda_1+1} = \lambda_1 + 1$$

$$\boxed{f_X(\lambda_1) = \lambda_1 + 1 \quad : \quad -1 < \lambda_1 \leq 0}$$ Left-side

$$f_X(\lambda_1) = \int_{\lambda_1-1}^{0} f_{X,Y}(\lambda_1,\lambda_2)\, d\lambda_2$$

$$= \lambda_2 \Big|_{\lambda_1-1}^{0} = \left(0 - (\lambda_1 - 1)\right)$$

$$= -\lambda_1 + 1$$

$$\boxed{f_X(\lambda_1) = -\lambda_1 + 1 \quad : \quad 0 < \lambda_2 \leq 1}$$ Right-side

$$f_X(\lambda_1) = \begin{cases} \lambda_1 + 1 & : \; -1 < \lambda_1 \leq 0 \\ -\lambda_1 + 1 & : \; 0 < \lambda_2 \leq 1 \end{cases}$$

$$f_x(\lambda_1) = \begin{cases} \lambda_1 + 1 & : -1 < \lambda_1 \leq 0 \\ -\lambda_1 + 1 & : 0 < \lambda_1 \leq 1 \end{cases}$$

Is this marginal density valid?

$$\int_{-1}^{0} \lambda_1 + 1 \, d\lambda_1 + \int_{0}^{1} -\lambda_1 + 1 \, d\lambda_1$$

$$= \frac{\lambda_1^2}{2} + \lambda_1 \Big|_{-1}^{0} + \frac{-\lambda_1^2}{2} + \lambda_1 \Big|_{0}^{1}$$

$$= -\left(\frac{1}{2} - 1\right) + \left(-\frac{1}{2} + 1\right)$$

$$= -\frac{1}{2} + 1 - \frac{1}{2} + 1$$

$$= 2 - 1 = 1$$

It appears valid since the area under density is 1 //

Marginal density $f_y(\lambda_2)$

$-\lambda_1 + \lambda_2 = 1$ $\qquad$ $\lambda_2 - 1 = \lambda_1$

$\lambda_1 - \lambda_2 = 1$ $\qquad$ $\lambda_2 + 1 = \lambda_1$

$$f_y(\lambda_2) = \int_{-\infty}^{\infty} f_{x,y}(\lambda_1, \lambda_2)\, d\lambda_1$$

Left-side

$$= \int_{\lambda_2 - 1}^{0} 1 \; d\lambda_1 = \lambda_1 \Big|_{\lambda_2 - 1}^{0} = -(\lambda_2 - 1)$$

$$= 1 - \lambda_2$$

Right-side

$$= \int_{0}^{\lambda_2 + 1} 1 \; d\lambda_1 = \lambda_1 \Big|_{0}^{\lambda_2 + 1} = \lambda_2 + 1$$

$$\boxed{f_y(\lambda_2) = \begin{cases} 1 - \lambda_2 & : \; 0 \le \lambda_2 \le 1 \\ \lambda_2 + 1 & : \; -1 \le \lambda_2 \le 0 \end{cases}}$$

Is this a valid density

$$\int_0^1 1 - \lambda_2 \; d\lambda_2 + \int_{-1}^{0} \lambda_2 + 1 \; d\lambda_2$$

$$= \lambda_2 - \frac{\lambda_2^2}{2} \Big|_0^1 + \frac{\lambda_2^2}{2} + \lambda_2 \Big|_{-1}^{0}$$

$$= 1 - \frac{1}{2} + 0 - \left(\frac{1}{2} - 1\right)$$

$$= \cancel{\tfrac{1}{2}} + -\cancel{\tfrac{1}{2}} + 1 = 1 \quad //$$

Density appears valid since the area under it is 1 //

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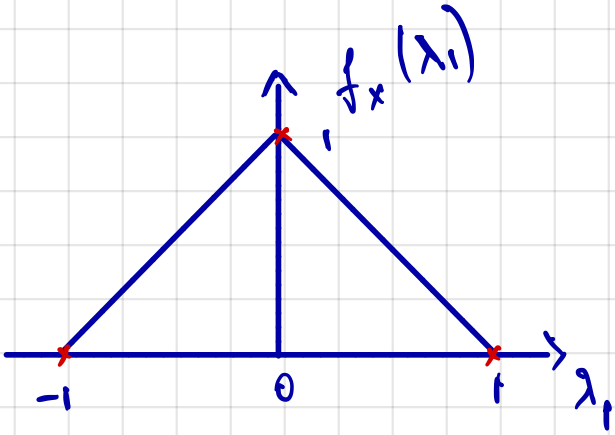
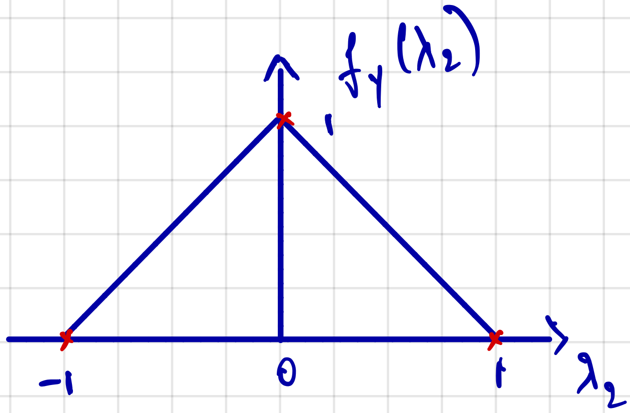
c. <u>Graph of marginal distribution</u>





Area under each curve is unity.

d). $\mathbb{E}[\lambda_1] = \int_{-\infty}^{\infty} \lambda_1 \, f_X(\lambda_1) \, d\lambda_1$

$$f_X(\lambda_1) = \begin{cases} \lambda_1 + 1 & : -1 < \lambda_1 \leq 0 \\ -\lambda_1 + 1 & : 0 < \lambda_2 \leq 1 \end{cases}$$

$$= \int_{-1}^{0} \lambda_1 (\lambda_1 + 1) \, d\lambda_1 + \int_{0}^{1} \lambda_1 (-\lambda_1 + 1) \, d\lambda_1$$

$$= \int_{-1}^{0} \lambda_1^2 + \lambda_1 \, d\lambda_1 + \int_{0}^{1} -\lambda_1^2 + \lambda_1 \, d\lambda_1$$

$$= \frac{\lambda_1^3}{3} + \frac{\lambda_1^2}{2} \Big|_{-1}^{0} + \frac{-\lambda_1^3}{3} + \frac{\lambda_1^2}{2} \Big|_{0}^{1}$$

$$= -\left(-\frac{1}{3} + \frac{1}{2}\right) + \left(-\frac{1}{3} + \frac{1}{2}\right)$$

$$= \frac{1}{3} - \frac{1}{2} - \frac{1}{3} + \frac{1}{2} = 0 \quad //$$

$$\mathbb{E}[\lambda_2] = \int_{-\infty}^{\infty} \lambda_2 \, f_y(\lambda_2) \, d\lambda_2$$

$$f_y(\lambda_2) = \begin{cases} 1-\lambda_2 & : -1 \leq \lambda_2 \leq 0 \\ \lambda_2 + 1 & : 0 \leq \lambda_2 \leq 1 \end{cases}$$

$$= \int_{-1}^{0} \lambda_2 (1-\lambda_2) \, d\lambda_2 + \int_{0}^{1} \lambda_2 (\lambda_2 + 1) \, d\lambda_2$$

$$= \int_{-1}^{0} \lambda_2 - \lambda_2^2 \, d\lambda_2 + \int_{0}^{1} \lambda_2^2 + \lambda_2 \, d\lambda_2$$

$$= \frac{\lambda_2^2}{2} - \frac{\lambda_2^3}{3} \Bigg|_{-1}^{0} + \frac{\lambda_2^3}{3} + \frac{\lambda_2^2}{2} \Bigg|_{0}^{1}$$

$$= -\left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{3} + \frac{1}{2}\right)$$

$$= -\frac{1}{2} - \frac{1}{3} + \frac{1}{3} + \frac{1}{2}$$

$$= 0 \;//$$

e). Find and graph $f_{X|Y}(\lambda_1|\lambda_2)$. What is $f_{X|Y}(\lambda_1|\lambda_2)$ when $\lambda_2 = 0.5$

$$f_{X|Y}(\lambda_1|\lambda_2 = 0.5) = \frac{f_{X,Y}(\lambda_1,\lambda_2)}{f_Y(\lambda_2 = 0.5)}$$

$$= 1 \times \frac{2}{1} = 2 \;//$$

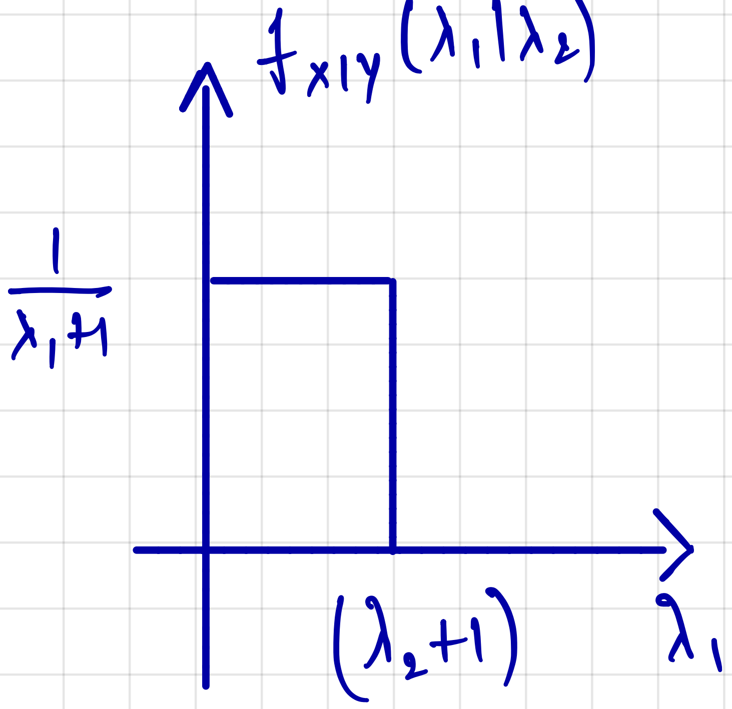
f). Find and graph $f_{Y|X}(\lambda_2|\lambda_1)$. What is $f_{Y|X}(\lambda_2|\lambda_1)$ when $\lambda_1 = 0.5$

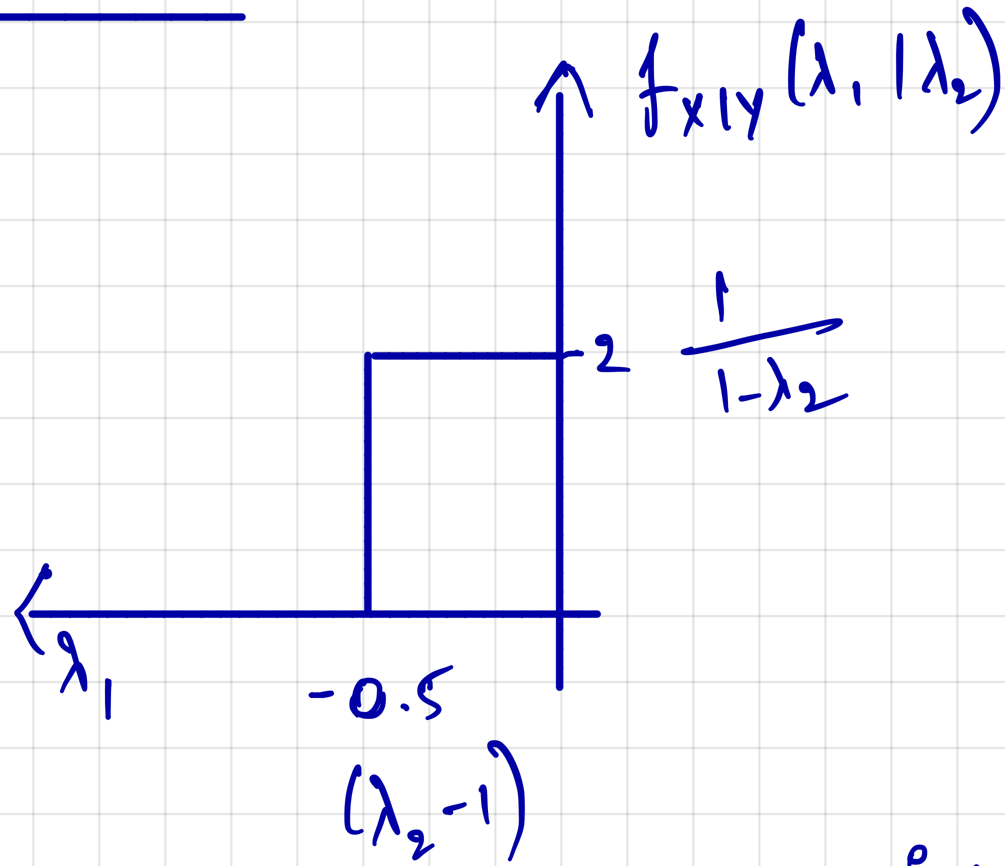
$$f_{Y|X}(\lambda_2|\lambda_1 = 0.5) = \frac{f_{X,Y}(\lambda_1,\lambda_2)}{f_X(\lambda_1 = 0.5)}$$

$$= 1 \times 2 = 2 \;//$$

<u>Graph of Conditional density</u>

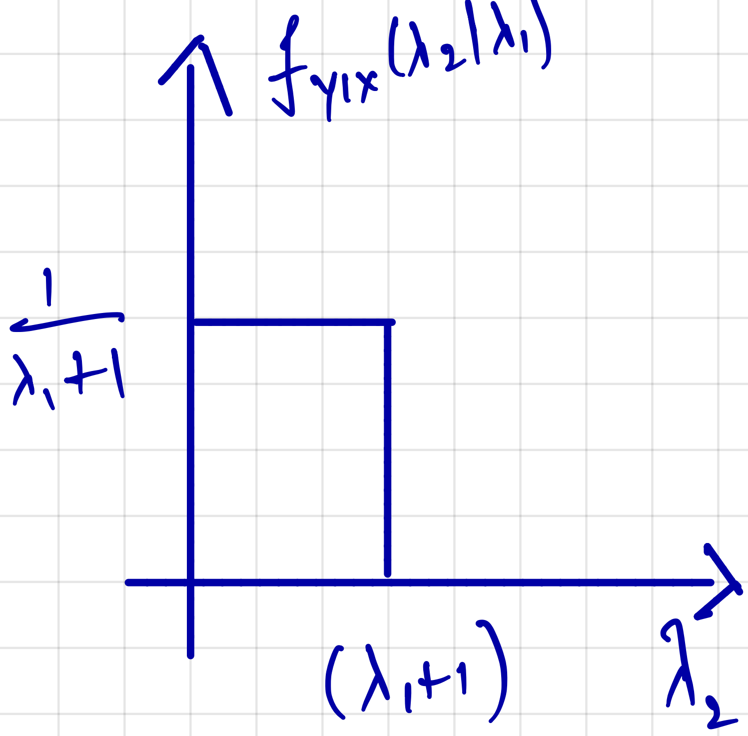
e).

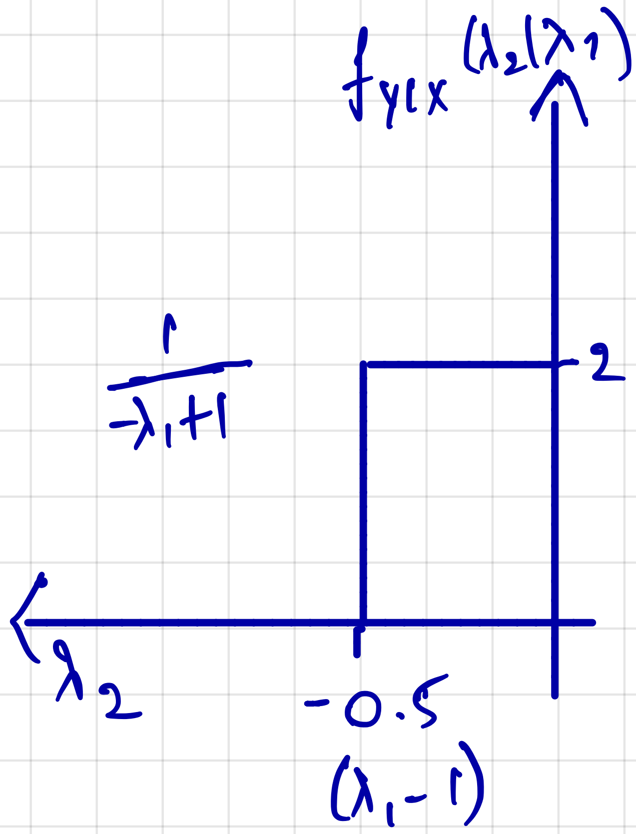




The density is valid since the area is 1

f. $f_{Y|X}(\lambda_2|\lambda_1)$





The density is valid since the area is $0.5 \times 2 = 1$

g). Are random variables X and Y independent?

INDEPENDENCE PROPERTY
$f_{X,Y}(\lambda_1, \lambda_2) = f_X(\lambda_1) \cdot f_Y(\lambda_2)$

where $f_{X,Y}(\lambda_1, \lambda_2) = 1$ and there exists values : $\lambda_1 = 0$, and $\lambda_2 = 0$ that doesn't satisfy the condition. This means X and Y are not independent.