Time Series Lab5

Rubrics:

The main purpose of the phase I is to implement the maximum likelihood estimation algorithm called **(Levenberg Marquardt)** using Python program to estimate the coefficient of ARMA model. The main objective of the **phase II is to use statsmodel** package to estimate the parameters of the known ARAM models (8 examples).

In Phase 1, questions from 1 till 7 are coding requirements whose correctness can be seen from the results obtained for examples 1 to 7 in question 8.

Example1: $y(t) - 0.5y(t-1) = e(t)$

Epoch: 0 SSE: 9966.43647179607 MSE: 0.9966436471796071 theta: [-0.48798487]
Epoch: 1 SSE: 9966.43647179415 MSE: 0.996643647179415 theta: [-0.48798528]

Estimated ARMA Coefficients:

a1: -0.4879852831363678 conf_int: [-0.5075116619349954, -0.4684589043377402]

Since the 0 is not included in the confidence interval, the coefficient is statistically significant.

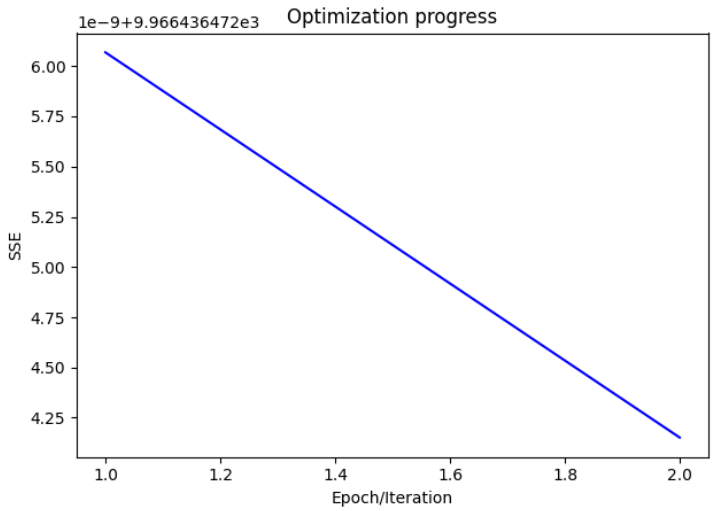
Estimated variance of error:

0.9967433215115662

Covariance of the estimated coefficients:

[[9.53198672e-05]]

Zeros: [0.48798528]



Example2: ARMA (0,1): y(t) = e(t) + 0.5e(t-1)

Epoch: 0 SSE: 10046.44240331044 MSE: 1.004644240331044 theta: [0.4034821]
Epoch: 1 SSE: 9946.903767063792 MSE: 0.9946903767063792 theta: [0.51137286]
Epoch: 2 SSE: 9943.860033618224 MSE: 0.9943860033618225 theta: [0.4986844]
Epoch: 3 SSE: 9943.77756340506 MSE: 0.994377756340506 theta: [0.4958658]
Epoch: 4 SSE: 9943.776306126438 MSE: 0.9943776306126437 theta: [0.49621522]
Epoch: 5 SSE: 9943.776286196107 MSE: 0.9943776286196108 theta: [0.49617127]

Estimated ARMA Coefficients:

b1: 0.4961712658405304 conf_int: [0.4767433690261758, 0.515599162654885]

Since the 0 is not included in the confidence interval, the coefficient is statistically significant.

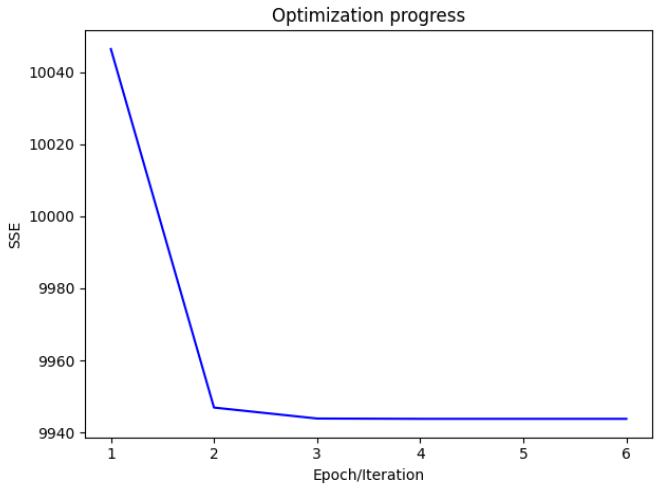
Estimated variance of error:

0.9944770763272435

Covariance of the estimated coefficients:

[[9.43607937e-05]]

Poles: [-0.49617127]



Example3: ARMA (1,1): y(t) + 0.5y(t-1) = e(t) + 0.25e(t-1)

Epoch: 0 SSE: 10245.182789779494 MSE: 1.0245182789779494 theta: [0.12706323 -0.1328083]
Epoch: 1 SSE: 10161.842553215476 MSE: 1.0161842553215477 theta: [0.42611095 0.17767146]
Epoch: 2 SSE: 10160.30139951194 MSE: 1.016030139951194 theta: [0.47877735 0.22774461]
Epoch: 3 SSE: 10159.536722044406 MSE: 1.0159536722044407 theta: [0.4764703 0.23372182]
Epoch: 4 SSE: 10159.535988190437 MSE: 1.0159535988190438 theta: [0.47757062 0.23475572]
Epoch: 5 SSE: 10159.535649608692 MSE: 1.015953564960869 theta: [0.47754827 0.23490493]
Epoch: 6 SSE: 10159.535649121384 MSE: 1.0159535649121385 theta: [0.47756672 0.23492666]

Estimated ARMA Coefficients:

a1: 0.4775667190551758 conf_int: [0.40564431428019243, 0.5494891238301591]
b1: 0.23492665588855743 conf_int: [0.1667249070387537, 0.30312840473836117]
Since the 0 is not included in the confidence intervals, the coefficients are statistically significant.

Estimated variance of error:

1.0161567962713927

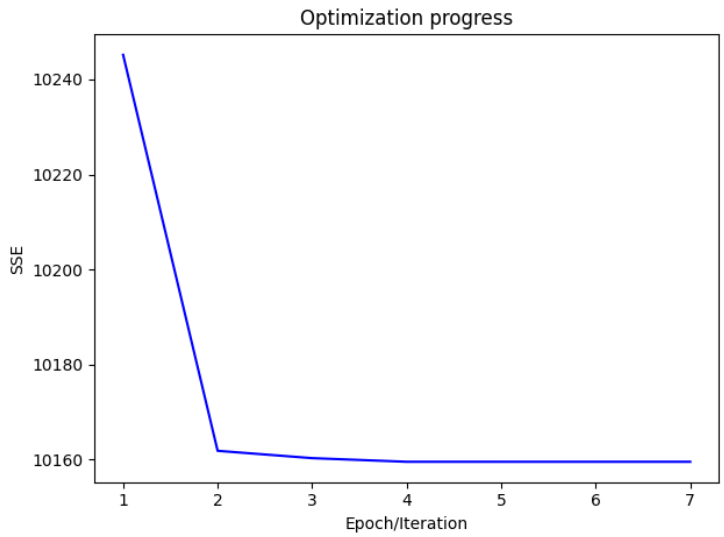
Covariance of the estimated coefficients:

[[0.00129321 0.00117962]

[0.00117962 0.00116287]]

Zeros: [-0.47756672]

Poles: [-0.23492666]



Example4: ARMA (2,0): y(t) + 0.5y(t-1) + 0.2y(t-2) = e(t)

Epoch: 0 SSE: 10064.68045187125 MSE: 1.006468045187125 theta: [0.4887949 0.1844982]
Epoch: 1 SSE: 10064.680451868024 MSE: 1.0064680451868024 theta: [0.4887955 0.18449856]

Estimated ARMA Coefficients:

a1: 0.4887954890727997 conf_int: [0.466806803843973, 0.5107841743016264]
a2: 0.18449856340885162 conf_int: [0.16565092590504937, 0.20334620091265387]
Since the 0 is not included in the confidence intervals, the coefficients are statistically significant.

Estimated variance of error:

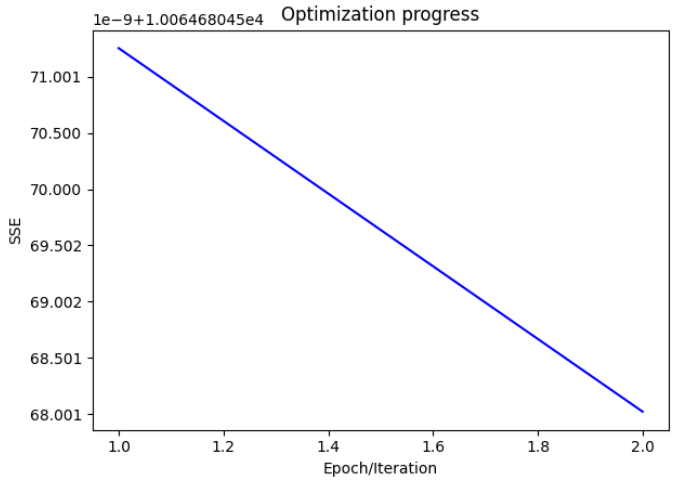
1.0066693790626149

Covariance of the estimated coefficients:

[[1.20875570e-04 4.27580559e-05]

 [4.27580559e-05 8.88083599e-05]]

Zeros: [-0.24439774+0.35322556j -0.24439774-0.35322556j]



Example5: ARMA (2,1): y(t) + 0.5y(t-1) + 0.2y(t-2) = e(t) - 0.5e(t-1)

Epoch: 0 SSE: 11027.559974280757 MSE: 1.1027559974280756 theta: [0.43424162 0.43386403 -0.43422672]
Epoch: 1 SSE: 10139.964228179764 MSE: 1.0139964228179765 theta: [0.49694428 0.1912865 -0.50625604]
Epoch: 2 SSE: 10134.72182018499 MSE: 1.013472182018499 theta: [0.47949532 0.19968885 -0.5069376]
Epoch: 3 SSE: 10134.680266297037 MSE: 1.0134680266297036 theta: [0.4815865 0.20006348 -0.5070276]
Epoch: 4 SSE: 10134.679673300858 MSE: 1.0134679673300857 theta: [0.48134178 0.20008078 -0.50703955]
Epoch: 5 SSE: 10134.679664138157 MSE: 1.0134679664138158 theta: [0.48137116 0.20008174 -0.50704116]

Estimated ARMA Coefficients:

a1: 0.4813711643218994 conf_int: [0.4427583417318094, 0.5199839869119894]
a2: 0.2000817358493805 conf_int: [0.17223022526222778, 0.2279332464365332]
b1: -0.5070411562919617 conf_int: [-0.5337757641223723, -0.4803065484615511]
Since the 0 is not included in the confidence intervals, the coefficients are statistically significant.

Estimated variance of error:

1.0137720980432288

Covariance of the estimated coefficients:

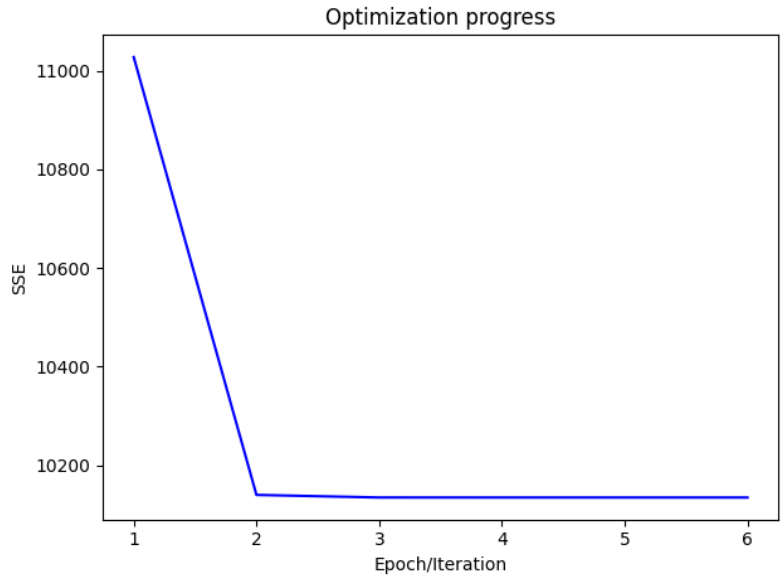
[[0.00037274 0.00020467 0.00021246]

[0.00020467 0.00019393 0.0001374]

[0.00021246 0.0001374 0.00017868]]

Zeros: [-0.24068558+0.37703076j -0.24068558-0.37703076j]

Poles: [0.50704116]



Example6: ARMA (1,2): y(t) + 0.5y(t-1) = e(t) + 0.5e(t-1) - 0.4e(t-2)

Epoch: 0 SSE: 10401.412159541647 MSE: 1.0401412159541648 theta: [0.02746432 -0.0274671 0.35585755]
Epoch: 1 SSE: 9939.599897434546 MSE: 0.9939599897434546 theta: [0.48649952 0.46359074 0.32999605]
Epoch: 2 SSE: 9904.700487146478 MSE: 0.9904700487146478 theta: [0.49857247 0.4996535 0.3930772]
Epoch: 3 SSE: 9903.775647429486 MSE: 0.9903775647429486 theta: [0.49639964 0.49345875 0.3833642]
Epoch: 4 SSE: 9903.757698126758 MSE: 0.9903757698126757 theta: [0.49652705 0.49417064 0.3847632]
Epoch: 5 SSE: 9903.757321289064 MSE: 0.9903757321289064 theta: [0.496514   0.49407002 0.38456008]
Epoch: 6 SSE: 9903.75731344665 MSE: 0.990375731344665 theta: [0.4965153 0.49408433 0.38458946]

Estimated ARMA Coefficients:

a1: 0.49651530385017395 conf_int: [0.45375064903700746, 0.5392799586633404]
b1: 0.49408432841300964 conf_int: [0.45318927038302925, 0.53497938644299]
b2: 0.38458946347236633 conf_int: [0.3630851412590835, 0.4060937856856492]

Since the 0 is not included in the confidence intervals, the coefficients are statistically significant.

Estimated variance of error:

0.9906729332246323

Covariance of the estimated coefficients:

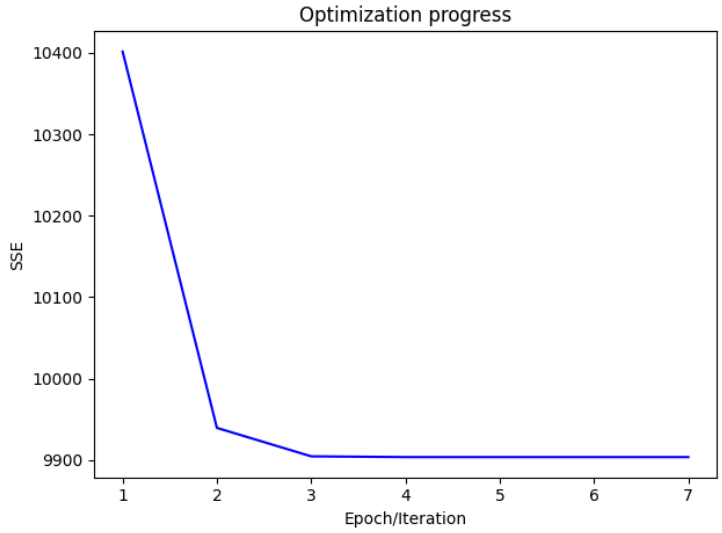
[[4.57203925e-04 3.77364722e-04 -6.40561578e-05]

 [3.77364722e-04 4.18101443e-04 -1.48189448e-05]

 [-6.40561578e-05 -1.48189448e-05 1.15608968e-04]]

Zeros: [-0.4965153]

Poles: [-0.24704216+0.56882304j -0.24704216-0.56882304j]



Example7: ARMA (0,2): y(t) = e(t) + 0.5e(t-1) - 0.4e(t-2)

Epoch: 0 SSE: 10737.204360909205 MSE: 1.0737204360909205 theta: [0.2893326 -0.34378678]
Epoch: 1 SSE: 10720.696548909833 MSE: 1.0720696548909834 theta: [0.5131059 -0.45792857]
Epoch: 2 SSE: 10128.068364507246 MSE: 1.0128068364507246 theta: [0.50403255 -0.4417194]
Epoch: 3 SSE: 9953.38054807323 MSE: 0.995338054807323 theta: [0.50085175 -0.40902165]
Epoch: 4 SSE: 9943.990581131184 MSE: 0.9943990581131184 theta: [0.49639818 -0.39861256]
Epoch: 5 SSE: 9943.981511881837 MSE: 0.9943981511881836 theta: [0.49544248 -0.39950937]
Epoch: 6 SSE: 9943.981427641218 MSE: 0.9943981427641218 theta: [0.4955351 -0.3994244]
Epoch: 7 SSE: 9943.981426862425 MSE: 0.9943981426862425 theta: [0.49552608 -0.3994323]

Estimated ARMA Coefficients:

b1: 0.4955260753631592 conf_int: [0.475015531752883, 0.5160366189734353]
b2: -0.39943230152130127 conf_int: [-0.419942878650591, -0.37892172439201155]
Since the 0 is not included in the confidence intervals, the coefficients are statistically significant.

Estimated variance of error:

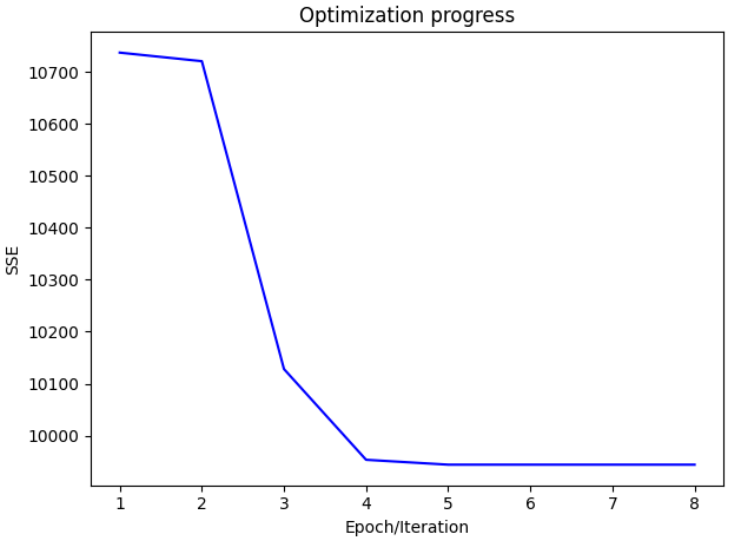
0.9945970620986623

Covariance of the estimated coefficients:

[[1.05170600e-04 8.67756652e-05]

[8.67756652e-05 1.05170944e-04]]

Poles: [-0.9265994 0.43107334]



Example8: ARMA (2,2): y(t)+0.5y(t-1) +0.2y(t-2) = e(t)+0.5e(t-1) - 0.4e(t-2)

Epoch: 0 SSE: 11968.502884068514 MSE: 1.1968502884068513 theta: [0.10391562 0.23437996 -0.10390227 -0.20925467]
Epoch: 1 SSE: nan MSE: nan theta: [0.734856   0.00361862 0.6445144 -0.64141434]

Estimated ARMA Coefficients:

a1: 0.7348560094833374 conf_int: [nan, nan]
a2: 0.0036186212673783302 conf_int: [nan, nan]
b1: 0.6445143818855286 conf_int: [nan, nan]
b2: -0.6414143443107605 conf_int: [nan, nan]

Estimated variance of error:

nan

Covariance of the estimated coefficients:

[[nan nan nan nan]

 [nan nan nan nan]

 [nan nan nan nan]

 [nan nan nan nan]]

Zeros: [-0.7298983 -0.00495771]

Poles: [-1.185544   0.5410296]

Phase 2

Example1: $y(t) - 0.5y(t-1) = e(t)$

The AR coefficient a1 is -0.508

```
The AR coefficient a1 is -0.508

                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                10000
Model:                 ARIMA(1, 0, 0)   Log Likelihood              -14124.546
Date:                Mon, 05 Dec 2022   AIC                          28257.092
Time:                        18:01:45   BIC                          28285.933
Sample:                             0   HQIC                         28266.855
                                - 10000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0268      0.013     -2.046      0.041      -0.052      -0.001
x1          3.198e-06   2.27e-06      1.409      0.159   -1.25e-06    7.65e-06
ar.L1         -0.5079      0.009    -59.575      0.000      -0.525      -0.491
sigma2         0.9872      0.014     70.484      0.000       0.960       1.015
===================================================================================
Ljung-Box (L1) (Q):                   0.10   Jarque-Bera (JB):                 4.34
Prob(Q):                              0.75   Prob(JB):                         0.11
Heteroskedasticity (H):               0.99   Skew:                             0.05
Prob(H) (two-sided):                  0.80   Kurtosis:                         2.99
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Example2: ARMA (0,1): y(t) = e(t) + 0.5e(t-1)

The MA coefficient b1 is 0.483

SARIMAX Results

Dep. Variable:	y	No. Observations:	10000
Model:	ARIMA(0, 0, 1)	Log Likelihood	-14125.112
Date:	Mon, 05 Dec 2022	AIC	28258.223
Time:	18:03:13	BIC	28287.065
Sample:	0	HQIC	28267.986
	- 10000		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
const	-0.0602	0.029	-2.052	0.040	-0.118	-0.003
x1	7.197e-06	5.09e-06	1.413	0.158	-2.79e-06	1.72e-05
ma.L1	0.4827	0.009	55.211	0.000	0.466	0.500
sigma2	0.9908	0.014	70.227	0.000	0.963	1.018

Ljung-Box (L1) (Q):	0.38	Jarque-Bera (JB):	4.30
Prob(Q):	0.54	Prob(JB):	0.12
Heteroskedasticity (H):	0.99	Skew:	0.05
Prob(H) (two-sided):	0.80	Kurtosis:	2.99

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

Example3: ARMA (1,1): y(t) + 0.5y(t-1) = e(t) + 0.25e(t-1)

The AR coefficient a1 is 0.492
The MA coefficient b1 is 0.246

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                10000
Model:                 ARIMA(1, 0, 1)   Log Likelihood              -14124.403
Date:                Mon, 05 Dec 2022   AIC                          28258.805
Time:                        18:04:03   BIC                          28294.857
Sample:                             0   HQIC                         28271.008
                                - 10000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.1005      0.048     -2.076      0.038      -0.195      -0.006
x1          1.202e-05   8.41e-06      1.430      0.153   -4.46e-06    2.85e-05
ar.L1          0.4923      0.013     36.950      0.000       0.466       0.518
ma.L1          0.2461      0.015     16.764      0.000       0.217       0.275
sigma2         0.9875      0.014     70.462      0.000       0.960       1.015
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):                 4.34
Prob(Q):                              0.96   Prob(JB):                         0.11
Heteroskedasticity (H):               0.99   Skew:                             0.05
Prob(H) (two-sided):                  0.78   Kurtosis:                         2.99
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Example4: ARMA (2,0): y(t) + 0.5y(t-1) + 0.2y(t-2) = e(t)

The AR coefficient a1 is 0.489
The AR coefficient a2 is 0.204

```
The AR coefficient a1 is 0.489
The AR coefficient a2 is 0.204

                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                10000
Model:                 ARIMA(2, 0, 0)   Log Likelihood              -14124.376
Date:                Mon, 05 Dec 2022   AIC                          28258.753
Time:                        18:05:37   BIC                          28294.805
Sample:                             0   HQIC                         28270.956
                                - 10000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.1342      0.064     -2.088      0.037      -0.260      -0.008
x1          1.604e-05   1.11e-05      1.438      0.150   -5.82e-06    3.79e-05
ar.L1          0.4890      0.010     50.435      0.000       0.470       0.508
ar.L2          0.2039      0.010     20.994      0.000       0.185       0.223
sigma2         0.9872      0.014     70.483      0.000       0.960       1.015
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):                 4.33
Prob(Q):                              0.99   Prob(JB):                         0.11
Heteroskedasticity (H):               0.99   Skew:                             0.05
Prob(H) (two-sided):                  0.78   Kurtosis:                         2.99
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Example5: ARMA (2,1): y(t) + 0.5y(t-1) + 0.2y(t-2) = e(t) - 0.5e(t-1)

The AR coefficient a1 is 0.494
The AR coefficient a2 is 0.204
The MA coefficient b1 is -0.497

```
The AR coefficient a1 is 0.494
The AR coefficient a2 is 0.204
The MA coefficient b1 is -0.497

                               SARIMAX Results
==========================================================================================
Dep. Variable:                          y   No. Observations:                10000
Model:                 ARIMA(2, 0, 1)   Log Likelihood              -14124.560
Date:                Mon, 05 Dec 2022   AIC                           28261.119
Time:                        18:06:31   BIC                           28304.381
Sample:                             0   HQIC                          28275.763
                                - 10000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0670      0.033     -2.038      0.042      -0.131      -0.003
x1          7.999e-06   5.71e-06      1.402      0.161   -3.19e-06    1.92e-05
ar.L1          0.4938      0.031     16.123      0.000       0.434       0.554
ar.L2          0.2036      0.011     18.471      0.000       0.182       0.225
ma.L1         -0.4975      0.031    -16.151      0.000      -0.558      -0.437
sigma2         0.9909      0.014     70.217      0.000       0.963       1.019
===================================================================================
Ljung-Box (L1) (Q):                   0.54   Jarque-Bera (JB):                 4.36
Prob(Q):                              0.46   Prob(JB):                         0.11
Heteroskedasticity (H):               0.99   Skew:                             0.05
Prob(H) (two-sided):                  0.80   Kurtosis:                         2.99
===================================================================================
```

Example6: ARMA (1,2): y(t) + 0.5y(t-1) = e(t) + 0.5e(t-1) - 0.4e(t-2)

The AR coefficient a1 is 0.766
The MA coefficient b1 is 0.141
The MA coefficient b2 is -0.602

```
The AR coefficient a1 is 0.766
The MA coefficient b1 is 0.141
The MA coefficient b2 is -0.602

                               SARIMAX Results
==========================================================================================
Dep. Variable:                          y   No. Observations:                10000
Model:                 ARIMA(1, 0, 2)   Log Likelihood              -14261.831
Date:                Mon, 05 Dec 2022   AIC                           28535.662
Time:                        18:07:22   BIC                           28578.924
Sample:                             0   HQIC                          28550.306
                                - 10000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0884      0.048     -1.833      0.067      -0.183       0.006
x1          1.055e-05   8.36e-06      1.261      0.207   -5.84e-06    2.69e-05
ar.L1          0.7661      0.055     13.822      0.000       0.657       0.875
ma.L1          0.1406      0.059      2.370      0.018       0.024       0.257
ma.L2         -0.6022      0.053    -11.309      0.000      -0.707      -0.498
sigma2         1.0610      0.016     66.104      0.000       1.030       1.092
===================================================================================
Ljung-Box (L1) (Q):                  36.22   Jarque-Bera (JB):                 3.43
Prob(Q):                              0.00   Prob(JB):                         0.18
Heteroskedasticity (H):               0.99   Skew:                             0.05
Prob(H) (two-sided):                  0.85   Kurtosis:                         3.00
===================================================================================
```

This example is a peculiar case where the coefficients are subjected to zero pole cancellation, and may have a reduced ARMA expression post the zero pole cancellation.

After removing the 0.5 from both the AR, and the MA process, the resulting ARMA order is: (0,1) with coefficients:

```
ma = [1, -0.4]
ar = [1, 0]
```

The MA coefficient b1 is -0.409

```
The MA coefficient b1 is -0.409

                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                10000
Model:                 ARIMA(0, 0, 1)   Log Likelihood              -14124.256
Date:                Mon, 05 Dec 2022   AIC                          28256.511
Time:                        18:10:00   BIC                          28285.353
Sample:                             0   HQIC                         28266.274
                                - 10000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0241      0.012     -2.064      0.039      -0.047      -0.001
x1          2.875e-06   2.02e-06      1.420      0.156   -1.09e-06    6.84e-06
ma.L1         -0.4091      0.009    -44.850      0.000      -0.427      -0.391
sigma2         0.9883      0.014     70.403      0.000       0.961       1.016
===================================================================================
Ljung-Box (L1) (Q):                   0.04   Jarque-Bera (JB):                 4.36
Prob(Q):                              0.84   Prob(JB):                         0.11
Heteroskedasticity (H):               0.99   Skew:                             0.05
Prob(H) (two-sided):                  0.78   Kurtosis:                         2.99
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Example7: ARMA (0,2): y(t) = e(t) + 0.5e(t-1) - 0.4e(t-2)

The MA coefficient b1 is 0.414
The MA coefficient b2 is -0.379

```
The MA coefficient b1 is 0.414
The MA coefficient b2 is -0.379

                               SARIMAX Results
==========================================================================================
Dep. Variable:                          y   No. Observations:                10000
Model:                 ARIMA(0, 0, 2)   Log Likelihood              -14236.804
Date:                Mon, 05 Dec 2022   AIC                           28483.608
Time:                        18:11:29   BIC                           28519.659
Sample:                             0   HQIC                          28495.811
                                - 10000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0441      0.022     -2.046      0.041      -0.086      -0.002
x1          5.26e-06   3.74e-06      1.405      0.160   -2.08e-06    1.26e-05
ma.L1          0.4137      0.010     41.785      0.000       0.394       0.433
ma.L2         -0.3790      0.010    -37.810      0.000      -0.399      -0.359
sigma2         1.0538      0.016     66.483      0.000       1.023       1.085
===================================================================================
Ljung-Box (L1) (Q):                  30.59   Jarque-Bera (JB):                 3.40
Prob(Q):                              0.00   Prob(JB):                         0.18
Heteroskedasticity (H):               0.99   Skew:                             0.05
Prob(H) (two-sided):                  0.78   Kurtosis:                         3.00
===================================================================================
```

Example 8: ARMA (2,2): y(t)+0.5y(t-1) +0.2y(t-2) = e(t)+0.5e(t-1) - 0.4e(t-2)

The AR coefficient a1 is 0.517
The AR coefficient a2 is 0.202
The MA coefficient b1 is 0.391
The MA coefficient b2 is -0.394

```
The AR coefficient a1 is 0.517
The AR coefficient a2 is 0.202
The MA coefficient b1 is 0.391
The MA coefficient b2 is -0.394

                               SARIMAX Results
==========================================================================================
Dep. Variable:                          y   No. Observations:                10000
Model:                 ARIMA(2, 0, 2)   Log Likelihood              -14245.985
Date:                Mon, 05 Dec 2022   AIC                           28505.970
Time:                        18:12:31   BIC                           28556.443
Sample:                             0   HQIC                          28523.055
                                - 10000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.1475      0.074     -1.990      0.047      -0.293      -0.002
x1          1.763e-05   1.29e-05      1.370      0.171   -7.59e-06    4.28e-05
ar.L1          0.5174      0.035     14.747      0.000       0.449       0.586
ar.L2          0.2018      0.017     11.551      0.000       0.168       0.236
ma.L1          0.3909      0.035     11.160      0.000       0.322       0.460
ma.L2         -0.3939      0.025    -15.795      0.000      -0.443      -0.345
sigma2         1.0565      0.016     66.293      0.000       1.025       1.088
===================================================================================
Ljung-Box (L1) (Q):                  35.29   Jarque-Bera (JB):                 3.39
Prob(Q):                              0.00   Prob(JB):                         0.18
Heteroskedasticity (H):               0.99   Skew:                             0.05
Prob(H) (two-sided):                  0.80   Kurtosis:                         3.00
===================================================================================
```