Superconducting Quantum Interference Device

Project Report

PHY 416: Experimental Methods in Physics

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Contents

1 Introduction

Have you ever wondered how are magnetic fields of very small order measured? How is a particle detected? Well, that's when SQUIDs come into the picture. **The Superconducting Quantum Interference Device**. A tiny superconducting device built with magical superconducting materials that operate at low temperatures to detect feeble magnetic fields. Sounds interesting, doesn't it? SQUIDs are ultra-sensitive magnetometers built using superconducting materials that operate at cryogenic temperatures. They're capable of detecting extremely weak magnetic fields, making them essential tools in both fundamental research and practical applications

Superconductivity is a macroscopic quantum phenomenon when there is no resistance to electron transport in a material below its critical temperature. This forms the basis for studying and performing unique measurements and interesting experiments. Based on this phenomenon of superconductivity a Superconducting Quantum Interference Device(SQUID) is built. An extremely sensitive device useful for measuring weak magnetic fields or fluxes with incredible high precision. It is based on loops that are superconducting with one or more Josephson Junctions, which are able to measure magnetic fields of the order of 10^{-14} Tesla accurately. The Josephson Effect in Josephson Junctions, which forms the basic design of SQUIDs, is instrumental in studies of experimental Condensed Matter Physics.SQUIDs are recognized for their precision, accuracy, and versatility in measurements [4]. The interference of supercurrents in the loop leads to a periodic modulation of the total current as a function of magnetic flux—hence the "quantum interference" in their name.

1.1 History

Brian Josephson was the first physicist to theoretically predict the tunneling of Cooper pairs between two superconductors—a phenomenon now known as the Josephson effect, for which he was awarded the Nobel Prize in Physics in 1973. Philip Anderson later provided key insights that helped interpret the broader implications of this effect in condensed matter systems. The first SQUID (Superconducting Quantum Interference Device) was built in 1964 and has since become an essential tool in experimental physics [6].

2 Design and Working

Superconducting materials like niobium and aluminum are commonly used in the fabrication of SQUIDs and Josephson junctions. Aluminum, in particular, is widely used for making Josephson junctions due to its ease of fabrication and well-controlled native oxide layer, which forms a reliable tunneling barrier. These devices operate at ultra-low temperatures to maintain superconductivity and suppress thermal noise.
To understand the operation and design of SQUIDs, it's essential to study the Josephson effect and the dynamics of Josephson junctions, which form the heart of these quantum interference devices.

2.1 Josephson Junction

Josephson Junction is a foundational basic building block for SQUIDS. The Josephson effect is observed across a Josephson junction, which induces the production of a current, called the supercurrent. It flows continuously without the presence of an applied voltage. The Josephson Junctions are made of certain materials that show superconductivity at lower temperatures. Their design consists of a thin layer of non-superconducting material that is sandwiched between two superconducting materials, see Figure 1. So, when the Josephson devices are brought to very low temperatures phase transitions occurs. The temperature at which this takes place is the critical temperature. At this temperature the metal or alloy we are using changes from it normal state which has finite resistance to a superconducting state wherein there is no resistance to flow of electric current. The electrons comprising our current pair up to form what are called the cooper pairs, which are attractively bound and have equal and opposite spins. Tunneling of cooper pairs of electrons takes place across the insulating junction. They behave like that of bosons. Cooper pairs that are formed on either side of the junction can be depicted by wavefunctions comparable to a free particle wavefunction. Let's now understand the physics behind Josephson Junctions [5].

$$\Psi(\vec{r}) = \sqrt{n_s} \cdot e^{i\phi(\vec{r})} \tag{1}$$

(1) describes the wave function of cooper pairs that are in phase. . Here $n_s = \Psi \cdot \Psi^*$ is the cooper pair density and $\phi(\vec{r})$ is the phase. The width of the insulator is considered to be around 100 nm. Here we assume that the

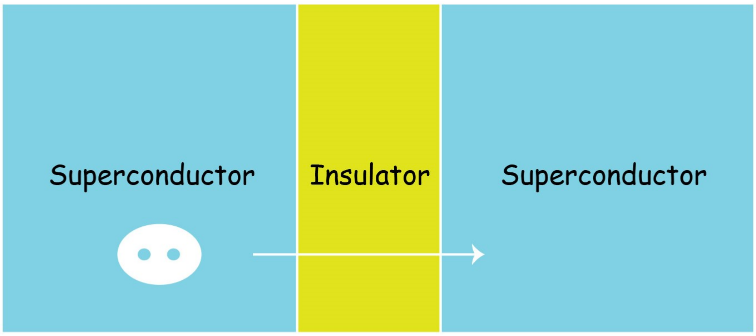

Figure 1: Josephson Junction

superconductors are weakly coupled. The wavefunction of two superconductors is thus given by

$$\Psi(\vec{r}) = \sqrt{n_1} \cdot e^{i\phi_1(\vec{r})} \tag{2}$$

and

$$\Psi(\vec{r}) = \sqrt{n_1} \cdot e^{i\phi_2(\vec{r})} \tag{3}$$

where n_1 and n_2 are cooper pair densities of superconductors 1 and 2. ϕ_1 and ϕ_2 are the respective phases. The Schrodinger equation for the two superconductors as in figure 1 $(i; j \in \{1, 2\} and i \neq j)$:

$$i\hbar \frac{\partial \dot{U}_i(\vec{t})}{\partial t} = E_i \Psi_2 + K\Psi_j \tag{4}$$

K is the coupling constant between cooper pairs that tunnel between the two superconductors though they are still different enough to be described by different wavefunctions. It is assumed that the same type of superconductor exists across the insulating barrier. When we connect this Josephson junction to a battery, we get a potential difference $E_1 - E_2 = eV$.Thus Schrodinger equation can be rewritten as.

$$i\hbar \frac{\partial \Psi_i(\vec{r})}{\partial t} = (-1)^{i+1} \frac{qV}{2} \Psi_i + K\Psi_i \tag{5}$$

Plugging in (2) and (3) in (5) we can get the dynamics of cooper pairs tunneling across the insulating junction.

$$\frac{dn_1}{dt} = \frac{2K}{\hbar} \sqrt{n_1 \cdot n_2} \cdot \sin(\phi_2 - \phi_1) \tag{6}$$

$$\frac{dn_2}{dt} = -\frac{2K}{\hbar} \sqrt{n_2 \cdot n_1} \cdot \sin(\phi_2 - \phi_1) \tag{7}$$

$$\frac{d\phi_1}{dt} = -\frac{K}{\hbar} \sqrt{n_2/n_1} \cdot \cos(\phi_2 - \phi_1) - \frac{qV}{2\hbar} \tag{8}$$

$$\frac{d\phi_2}{dt} = -\frac{K}{\hbar} \sqrt{n_1/n_2} \cdot \cos(\phi_2 - \phi_1) + \frac{qV}{2\hbar} \tag{9}$$

As the current is

$$I = nAq \frac{dx}{dt} \tag{10}$$

with cross-section A, the charge equation can be written as:

$$I_s = I_0 \cdot \sin(\phi_2 - \phi_1) \tag{11}$$

It is assumed that $I_0 = \frac{2K}{\hbar}2\text{q}\Omega$ and $n_1 = n_2 = n_n$ where Ω is the volume of the two superconductors. (11) is the **First Josephson Equation** which describes that the tunneling current depends on the phase difference between the two superconductors. Another assumption $n_1 = n_2 = n$, yields our **Second Josephson Equation**. To get this we subtract (8) from (9).

$$\frac{d(\phi_2 - \phi_1)}{dt} = \frac{qV}{h} \tag{12}$$

This equation showcases the time evolution of the phase difference given an external voltage. If the applied voltage is constant, then according to (12) phase difference would evolve linearly with time. On substituting (12) in (11), we can see that there is a AC current. This describes our AC Josephson effect. In absence of external voltage, the phase difference is constant now. We can mathematically see a constant current flowing despite zero external voltage. These unconventional results arise from the phase coherence of the cooper pairs, which is also known as the DC Josephson effect. In the **DC Josephson effect**, a current equivalent to the phase difference of the wavefunctions flows through the junction in the absence of voltage. The DC voltage applied across the Josephson junction corresponds to an oscillating frequency. The relation between voltage and frequency involves only fundamental constants and since frequency measurement can be done with extreme accuracy, the Josephson junction is a standard for voltage measurement. The relation is given by

$$f_{\text{Josephson}} = \frac{2e\Delta V}{h} \tag{13}$$

Whereas in the **AC Josephson effect**, the junction oscillates with a frequency that is proportional to the voltage across the junction [3][4].

2.2 SQUIDs

The measurement of magnetic flux by a SQUID (Superconducting Quantum Interference Device) relies on two fundamental superconducting principles: the Josephson effect and flux quantization. In a superconducting loop, the total magnetic flux is quantized in units of the magnetic flux quantum, leading to extremely sensitive flux detection. There are two main types of SQUIDs, namely RF (Radio-Frequency) and DC. The RF SQUID contains a single Josephson junction and operates using a resonant tank circuit. It's simpler and more cost-effective to build, but typically has lower sensitivity. The DC SQUID, on the other hand, incorporates two or more Josephson junctions and directly measures voltage as a function of applied flux. It offers higher sensitivity and precision, making it the preferred choice in most advanced physics experiments. [1].

2.2.1 DC SQUID

A DC SQUID consists of 2 superconductors separated by absolutely thin insulating layers forming 2 parallel Josephson junctions. Figure 1 shows the schematic diagram of a DC SQUID with 2 Josephson Junctions, namely a and b. A direct current enters through port A, which is divided into 2 currents I_1 and I_2. These 2 currents pass through Josephson Junctions a and b respectivel,y which leads to a phase shift between them. The supercurrents I_1' and I_2' then interfere at port B. Cooper pairs form the cause of the electric current in superconductors. Phase shift occurs due to the presence of magnetic fields. In absence of a magnetic field, there is no phase shift and phase difference. The current at port B oscillates between a maximum and minimum value. When magnetic flux increases by 1 quantum, the maximum value occurs. Given by

$$\varphi_o = \frac{h}{2e} = 2.06 \times 10^{-15} wb$$

SQUID with small loops of superconductors employs Josephson junctions to achieve superposition wherein each electron moves simultaneously in both directions. As current move in two opposite directions, the electrons can perform as qubits. However, in practice, voltage is measured across the SQUI,D which oscillates with changing magnetic fields. Hence SQUID is referred to as a flux-to-voltage transducer as it converts small changes in magnetic flux to voltage. [6] Thus Change in Magnetic Flux $\Delta\Phi$ can be estimated as the function of ΔV

$$\Delta V = R \cdot \Delta I$$

$2 \cdot I = 2 \cdot \frac{\Delta\Phi}{L}$, where L is the self inductance of the superconducting ring

$$\Delta V = \frac{R}{I} \cdot \Delta\Phi$$

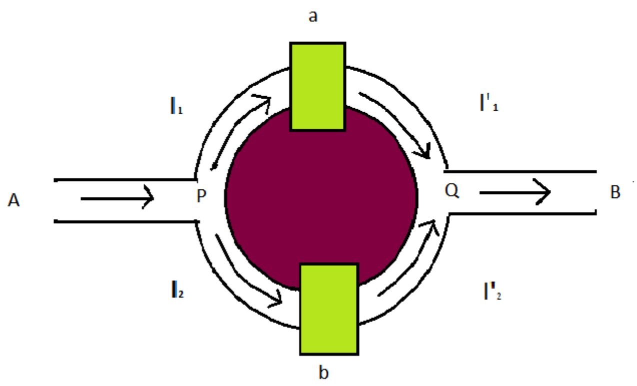

Figure 2: DC SQUID made of 2 Josephson junctions C and D

The general case is evaluated by introducing a parameter.

$$\lambda = \frac{i_e L}{\Phi_0}$$

Where i_c is the critical current of the SQUID. In most cases, λ is of order one.
We studied a Josephson Junction. Now we are ready to apply it to a circuit with 2 Josephson Junctions as in Fig 2. As we all know the probability current in the Electromagnetic field is given as:

$$\mathbf{J} = \frac{1}{2}\left\{\Psi^* \left[\frac{\widehat{P} - q\mathbf{A}}{m}\right]\Psi + \Psi\left[\frac{\widehat{P} - q\mathbf{A}}{m}\right]^* \Psi^*\right\}$$

Substituting (1) in this we get:

$$\mathbf{J} = \frac{\hbar}{m}\left(\nabla\Phi - \frac{q}{\hbar}\mathbf{A}\right) n_s \tag{14}$$

There exists a path where J = 0 around the entire loop. This allows us to simplify (14) significantly by integrating over that path from points 1 to 2 where J = 0.

$$\int_1^2 \nabla\phi ds = \phi_2 - \phi_1 = \frac{2q_e}{\hbar}\int_1^2 \mathbf{A}\mathbf{ds}$$

where q_e is the charge of the electron. Because the wavefunction is single-valued, the change in the phase between the points P and Q in Fig. 2 is:

$$\phi_Q - \phi_P = \delta_a + \frac{2q_e}{\hbar}\int_P^Q \mathbf{A}ds = \delta_b + \frac{2q_e}{\hbar}\int_P^Q \mathbf{A}ds$$

where δ_a and δ_b are the additional phase differences of the superconductors. By using Stokes theorem we get:

$$\delta_b - \delta_a = \frac{2q_e}{\hbar}\oint \mathbf{A}\mathbf{ds} = \frac{2q_e}{\hbar}\int \mathbf{B}d\mathbf{A} = \frac{2q_e}{\hbar}\Phi$$

Another methodology considers the gauge invariant phase change and also reaches the same conclusion. The current from the two branches sum up at B given by I_{ges} . Using this, the first Josephson equation is

$$I_{ges} = I_0\left\{\sin\left(\delta_0 + \frac{q_e}{\hbar}\Phi\right) + \sin\left(\delta_0 - \frac{q_e}{\hbar}\Phi\right)\right\}$$

which upon rewriting looks like:

$$I_{ges} = 2I_0\sin(\delta_0)\cos(\frac{q_e}{\hbar}\Phi) \tag{15}$$

This shows that we can determine an unknown magnetic field by measuring the current. $2I_0\sin(\delta_0)$ can first be measured by exposing the DC SQUID to some known magnetic flux $\Phi = \Phi_(known)$, like $\Phi_(known)$ = 0 or the earth's magnetic field . Now the only independent variable would be Φ in the above equation, which can be used to measure the unknown magnetic flux from the current. This is how a DC SQUID is used for measuring unknown magnetic fluxes [2][4].

2.2.2 RF SQUID

Radio frequency SQUIDs work on the principle of the AC Josephson effect. They have one Josephson junction placed in a superconducting loop which is subjected to an external flux Φ_{ex} as shown in Fig. 3. When an external magnetic flux is applied, the dynamics of an RF SQUID can be described by a double-well potential.
For determining the dynamics of magnetic flux we take into account the inductance of the circuit.

$$\Phi = \Phi_{ex} - LI \tag{16}$$

I is the current through the loop. δ represents the phase difference across the Josephson junction. This change in phase is

$$\begin{aligned} \delta + \oint \vec{\nabla}\phi d\vec{s} &= \delta + \frac{2q_c}{\hbar}\oint \vec{A}d\vec{s} \quad (\because eq(6)) \\ &= \delta + \frac{2q_e}{\hbar}\Phi \end{aligned}$$

This phase difference must be equated to $2\pi n$ so there is no change in physical observables of the wave function.

$$\delta = 2\pi n - \frac{2q_e}{\hbar}\Phi$$

Substituting in (11) we get the current in the loop as

$$\begin{aligned} I &= I_0 \sin\left(2\pi n - \frac{2q_e}{\hbar}\Phi\right) \\ &= -I_0 \sin\left(\frac{2q_e}{\hbar}\Phi\right) \end{aligned}$$

Substituting this current in(14) to get the total flux

$$\Phi_{ex} - LI_0 \sin\left(\frac{2\pi}{\Phi_0}\Phi\right)$$

where $\Phi_0 = \frac{h}{2q_c}$

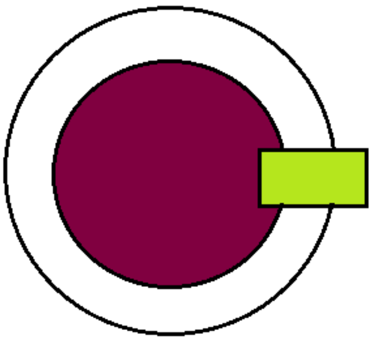

Figure 3: RF SQUID which consists of 1 Josephson Junction

3 Applications

SQUIDs (Superconducting Quantum Interference Devices) are indispensable tools in the field of quantum electronics and superconducting circuits. Their exceptional sensitivity to magnetic flux makes them a cornerstone in the design and operation of superconducting quantum circuits, where they are often used as flux sensors, qubit readout elements, or tunable inductors. In particular, SQUIDs play a crucial role in Josephson Parametric Amplifiers (JPAs), where their nonlinear inductance enables the amplification of weak microwave signals near the quantum noise limit—essential for reading out superconducting qubits and other cryogenic quantum devices. Moreover, SQUID-based circuits are widely employed in the development of quantum-limited amplifiers, flux qubit control, and as elements in quantum-limited magnetometers and gradiometers. Their ability to detect minute changes in magnetic flux with extreme precision continues to drive advancements in quantum computation, sensing, and fundamental physics.

4 Conclusion

SQUIDs are thus resourceful, versatile devices useful for the measurement of magnetic flux in areas of modern physics and research in experimental condensed matter physics. We studied the basic principles and theory behind the working of Josephson Junctions and their implementation in RF and DC SQUIDs. We also studied a few applications though SQUIDs have myriad uses, right from particle detection in CERN to manufacturing quantum computer hardware. All the figures used in this paper were made using paint tools which made the study of Josephson Junction visually as well as mathematically clear to understand.

5 Acknowledgement

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