

Renormalisation of meson mixing operators on the lattice

P. Boyle, F. Erben, J. Flynn, N. Garron, R. Mukherjee, T. Tsang for the RBC-UKQCD Collaborations

Introduction

Charge-parity violation (CPV) in the Standard Model (SM) has been connected to matter-antimatter asymmetry in the universe. Studying meson mixing in and beyond the SM helps look at CPV and also probe new physics. As mesons interact strongly at low energies, we use the lattice to simulate these interactions non-perturbatively. We use High Performance Computing to evaluate multidimensional integrals with large degrees of freedom. In this study we look at four-quark operators corresponding to meson mixing, and the procedure of renormalising them non-perturbatively.

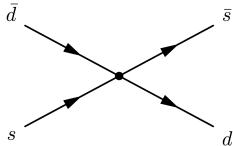


Fig 1: four quark operator contribution to kaon mixing

Meson Mixing

- 1980: Nobel Prize in Physics to Cronin & Fitch for discovery of CPV in neutral Kaon decays involving Kaon mixing → now also known in charm and b -meson systems.
- Meson mixing: 4-quark interaction. Different interactions correspond to different operators. The interactions allowed within SM is given by the SM four-quark operator.
- BSM four-quark interactions/operators are interesting in the search for new physics - so we also study them.

Lattice QCD

- Meson interactions are mediated by the strong force, and cannot be described using perturbation theory at the relevant (low) energy scales.
- Lattice QCD is a non-perturbative formulation of QCD where fields are simulated on a grid of finite spacing and volume. Expectations values of operators can be computed using Euclidean path integrals on the lattice.

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \bar{\psi}, U] O e^{-S_E[\psi, \bar{\psi}, U]}$$

- We numerically evaluate path integrals using Monte Carlo simulations → generate a large number of field configurations in a finite box → measure our desired operators → extrapolate results to the continuum.

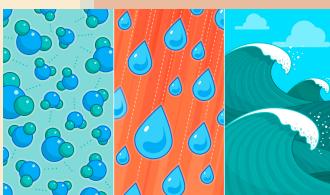


Fig 2: physics is scale-dependent: waves can be studied without studying droplets or molecules of water
Source: Quanta Magazine

Renormalisation

- Physics of a system is scale-dependent → renormalisation helps connect theoretical calculations to real life measurements by demanding that a set of physical quantities at a given scale are equal to their experimental values.
- To make reliable theoretical predictions about interactions given by operators, we must renormalise them.

$$O_i^{\text{ren}}(q) = Z_O(q) O_i^{\text{lattice}}(q)$$

Renormalisation Factors $Z(q)$ of Four-quark Operators

Simulation Details

Expectation values of our operators (1 SM and 4 BSM) are computed on 9 lattices varying in

- lattice spacing: 0.07-0.11 fm
- lattice sizes: 24^3 , 32^3 , 48^3
- action: variants of 2+1 flavour Domain Wall Fermion action
- simulated value of quark masses

Computing Z on multiple lattices helps extrapolate results to zero lattice spacing, infinite volume and physical quark masses.

$$Z(p) = \lim_{a \rightarrow 0} Z(p, a)$$

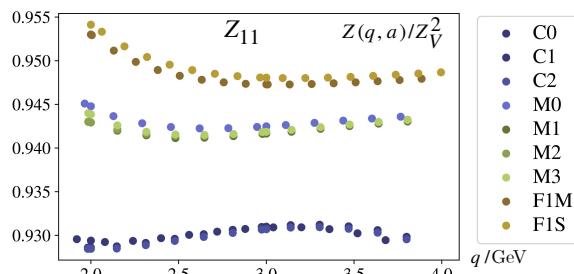


Fig 3: Momentum-dependent renormalisation factors of the SM fourquark operator on various lattices.

- We study 5 operators: some have the same representation under chiral transformation ⇒ their Z -factors mix during renormalisation

$$O_i^{\text{ren}}(q) = Z_{ij}(q) O_j^{\text{lattice}}(q)$$

- Block structure: 1x1 block for the SM operator, and two 2x2 blocks for the BSM operators, shown in figs 3-5.
- We also compare the scaling of the Z -factors calculated on the lattice (non-perturbative) to the scaling predicted by perturbation theory.

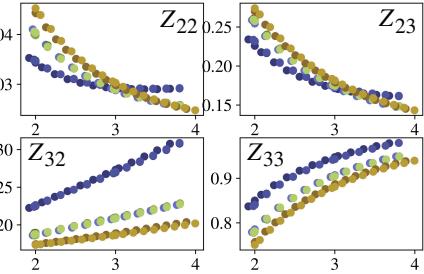


Fig 4: Renormalisation factors of the BSM operators that transform as (8,8) under chiral symmetry.

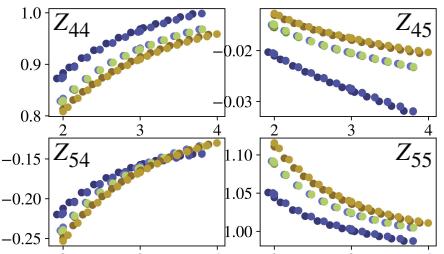


Fig 5: Renormalisation factors of the BSM operators that transform as (6,6) under chiral symmetry.

Summary and Outlook

- We have calculated renormalisation factors for SM and BSM operators for Kaon mixing and B -meson mixing.
- Next, we use these to renormalise matrix elements in mixing processes → related to bag parameters in and beyond the SM → related to CPV parameters.

Acknowledgements

This work used the DiRAC Extreme Scaling service at the University of Edinburgh, operated by the Edinburgh Parallel Computing Centre on behalf of the STFC DiRAC HPC Facility (www.dirac.ac.uk) under project codes dp008 and dp207. This equipment was funded by BEIS capital funding via STFC capital grant ST/R00238X/1 and STFC DiRAC Operations grant ST/R001006/1. DiRAC is part of the National e-Infrastructure.