

Kaon mixing: chiral and continuum extrapolations

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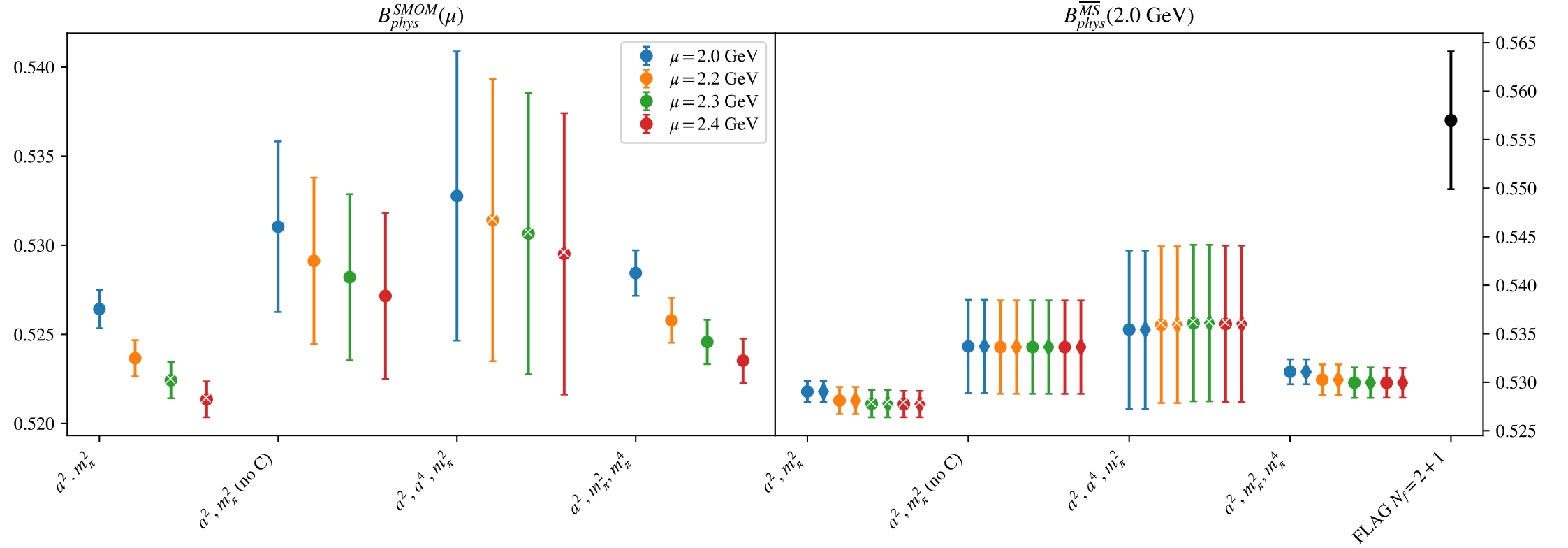


Figure 1: B_1
(left) B_{phys} in RI/SMOM scheme from fit variations (fits with p -value < 0.05 marked with “x”).
(right) B_{phys} in \overline{MS} computed using $B^{\overline{MS}} = R^{\overline{MS} \leftarrow SMOM}(2.0) \sigma_{npt}(2.0, \mu) B^{SMOM}(\mu)$.

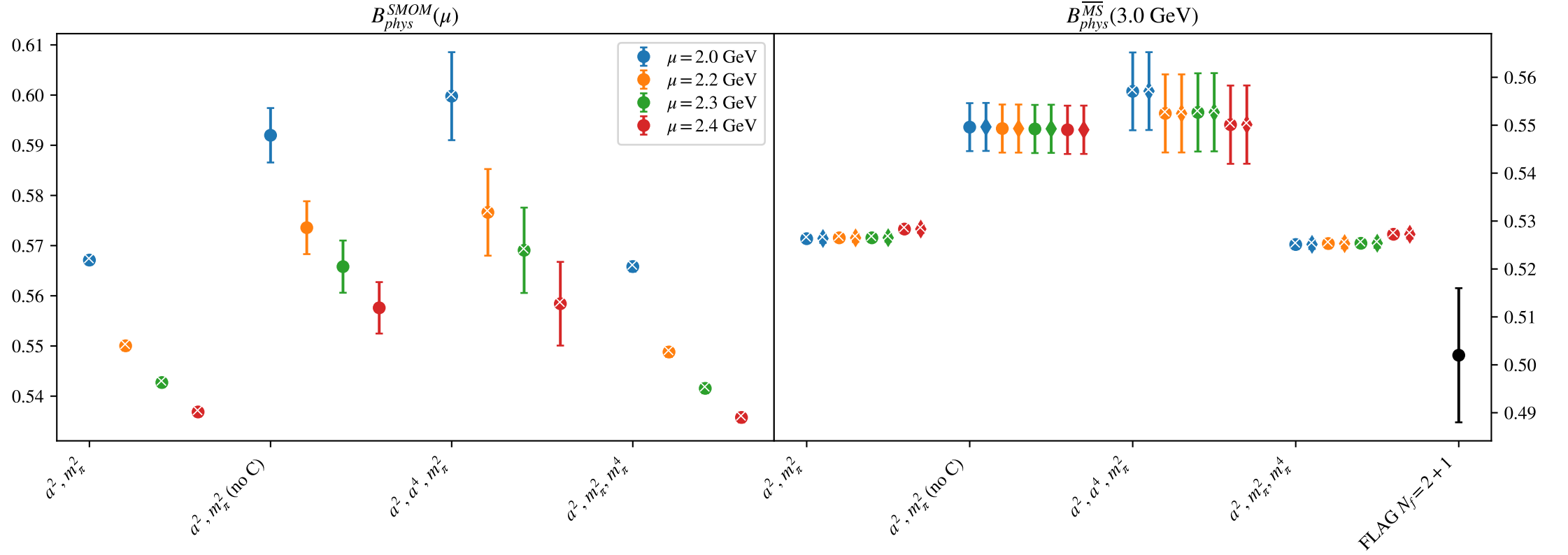


Figure 2: B_2
(left) B_{phys} in RI/SMOM scheme from fit variations (fits with $p\text{-value} < 0.05$ marked with “x”).
(right) B_{phys} in \overline{MS} computed using $B^{\overline{MS}} = R^{\overline{MS} \leftarrow SMOM}(3.0) \sigma_{npt}(3.0, \mu) B^{SMOM}(\mu)$.

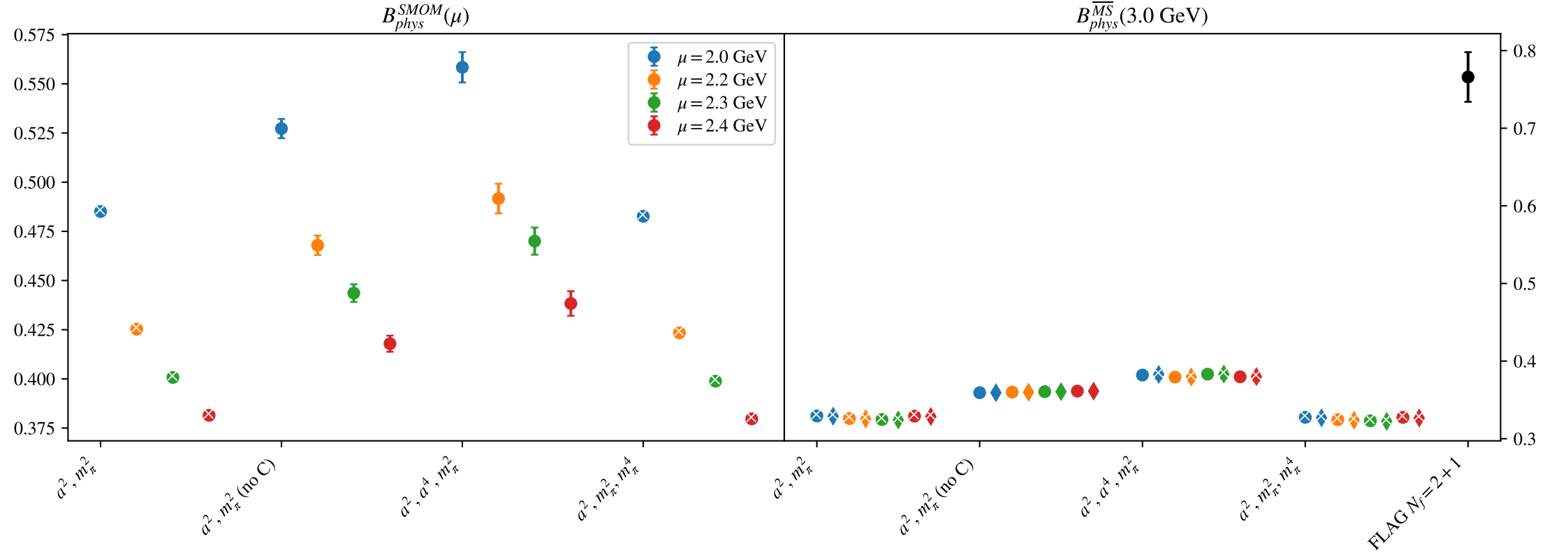


Figure 3: B_3
(left) B_{phys} in RI/SMOM scheme from fit variations (fits with p -value < 0.05 marked with “x”).
(right) B_{phys} in \overline{MS} computed using $B^{\overline{MS}} = R^{\overline{MS} \leftarrow SMOM}(3.0) \sigma_{npt}(3.0, \mu) B^{SMOM}(\mu)$.

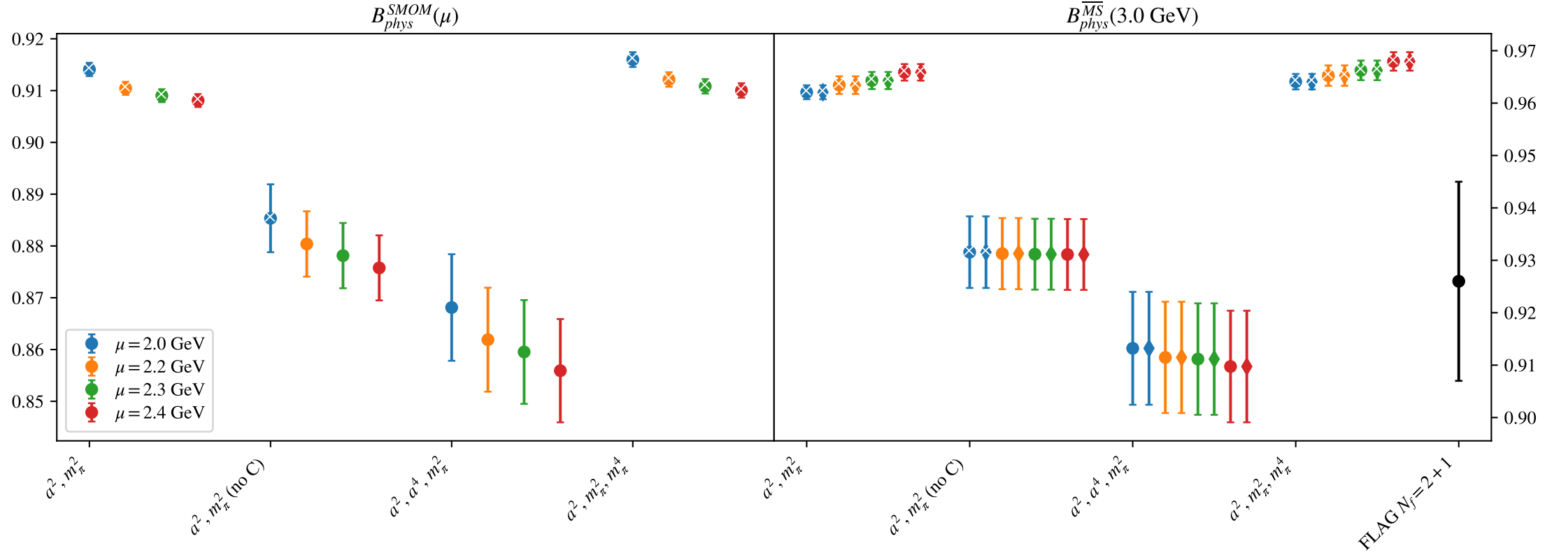


Figure 4: B_4
(left) B_{phys} in RI/SMOM scheme from fit variations (fits with p -value < 0.05 marked with “x”).
(right) B_{phys} in \overline{MS} computed using $B^{\overline{MS}} = R^{\overline{MS} \leftarrow SMOM}(3.0) \sigma_{npt}(3.0, \mu) B^{SMOM}(\mu)$.

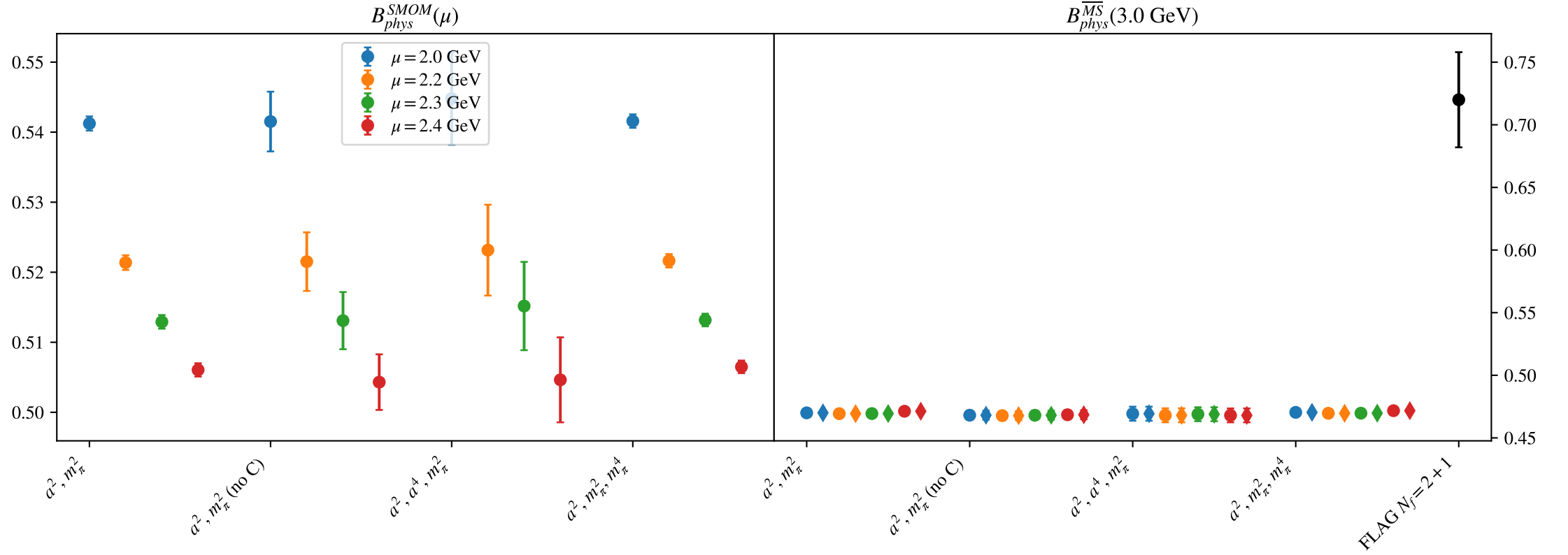


Figure 5: B_5
(left) B_{phys} in RI/SMOM scheme from fit variations (fits with p -value < 0.05 marked with “ \times ”).
(right) B_{phys} in \overline{MS} computed using $B^{\overline{MS}} = R^{\overline{MS} \leftarrow SMOM}(3.0) \sigma_{npt}(3.0, \mu) B^{SMOM}(\mu)$.

1 B_1

| μ (GeV) | a^2, m_π^2 | a^2, m_π^2 (no C) | a^2, a^4, m_π^2 | a^2, m_π^2, m_π^4 |
|-------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| 2.0 | 0.5264(10) : 1.858 (0.098) | 0.5310(47) : 0.876 (0.417) | 0.5327(81) : 2.173 (0.069) | 0.5284(12) : 0.661 (0.619) |
| 2.2 | 0.5236(10) : 2.214 (0.05) | 0.5291(46) : 1.143 (0.319) | 0.5314(79) : 2.525 (0.039) | 0.5257(12) : 0.923 (0.449) |
| 2.3 | 0.5224(10) : 2.304 (0.042) | 0.5282(46) : 1.197 (0.302) | 0.5306(78) : 2.605 (0.034) | 0.5245(12) : 0.993 (0.41) |
| 2.4 | 0.5213(10) : 2.348 (0.039) | 0.5271(46) : 1.223 (0.294) | 0.5295(78) : 2.663 (0.031) | 0.5235(12) : 1.005 (0.403) |

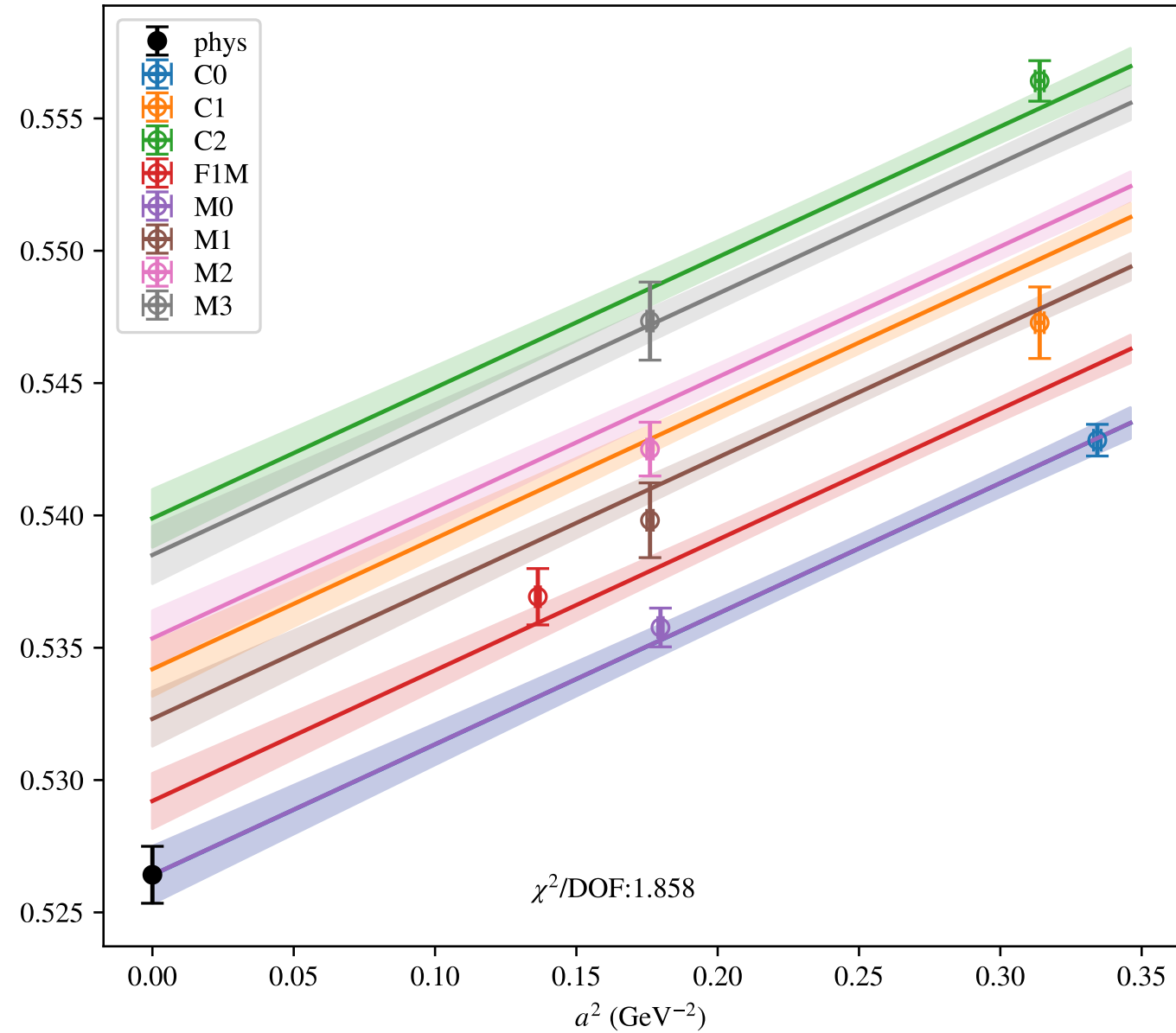
Table 1: Physical point value from chiral and continuum extrapolation at renormalisation scale μ . Entries are **value(error)**: χ^2/DOF (p -value).

| μ (GeV) | | a^2, m_π^2 | a^2, m_π^2 (no C) | a^2, a^4, m_π^2 | a^2, m_π^2, m_π^4 |
|-------------|----------|----------------|-----------------------|---------------------|-------------------------|
| 2.0 | α | 0.0937(71) | 0.047(53) | -0.017 | 0.0813(82) |
| | β | 0.00261(14) | 0.00223(27) | 0.00263(15) | 0.00031(90) |
| 2.2 | α | 0.0977(70) | 0.041(52) | -0.038 | 0.0846(82) |
| | β | 0.00261(14) | 0.00220(27) | 0.00264(14) | 0.00020(89) |
| 2.3 | α | 0.0992(70) | 0.039(52) | -0.045 | 0.0859(82) |
| | β | 0.00262(14) | 0.00220(27) | 0.00265(14) | 0.00018(89) |
| 2.4 | α | 0.0999(70) | 0.040(52) | -0.044 | 0.0864(82) |
| | β | 0.00263(14) | 0.00220(27) | 0.00266(14) | 0.00017(89) |

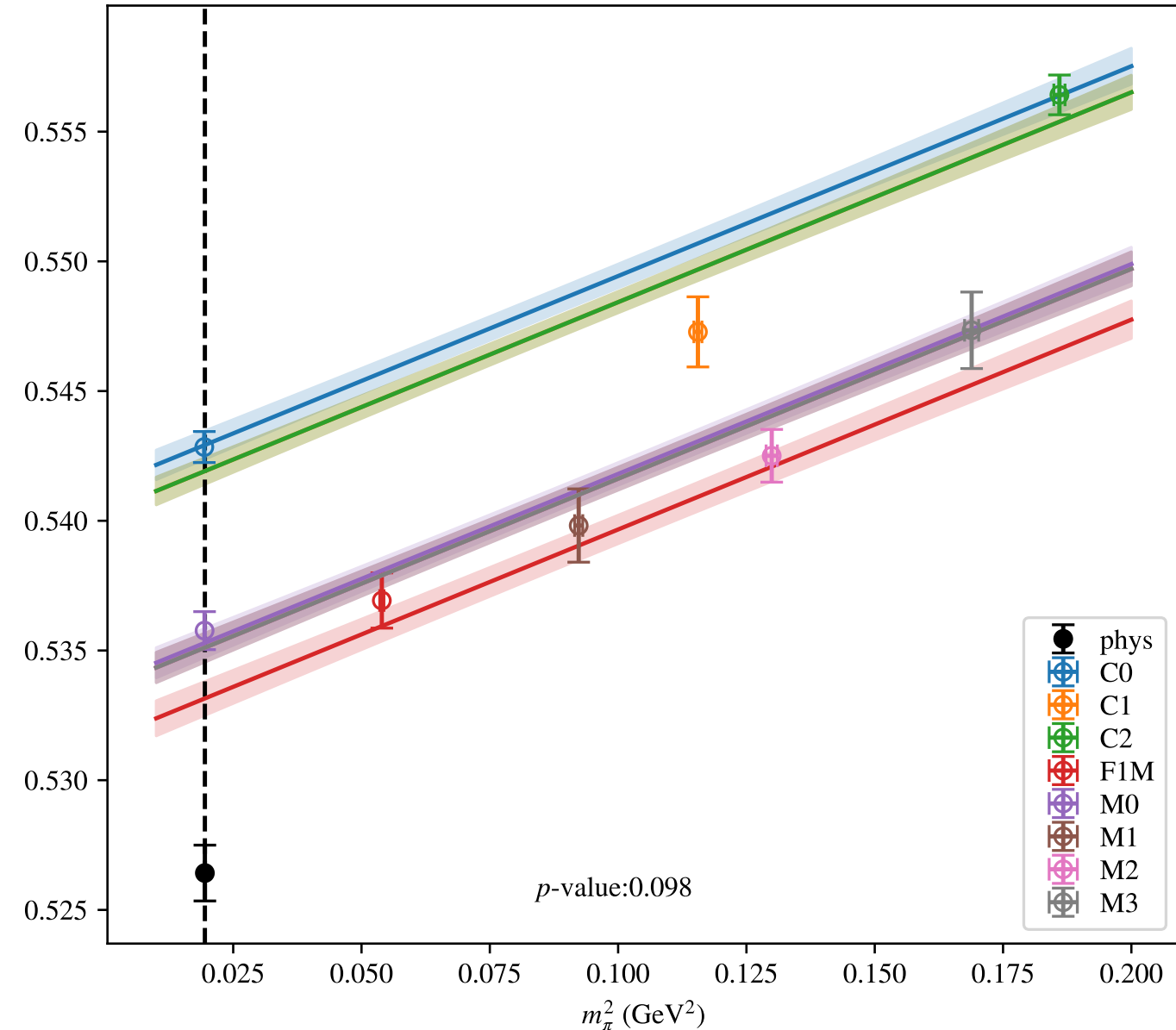
Table 2: Fit values of coefficients in $B = B_{phys} + \alpha a^2 + \beta \left(\frac{m_\pi^2}{f_\pi^2} - \frac{m_{\pi,PDG}^2}{f_\pi^2} \right) + \dots$

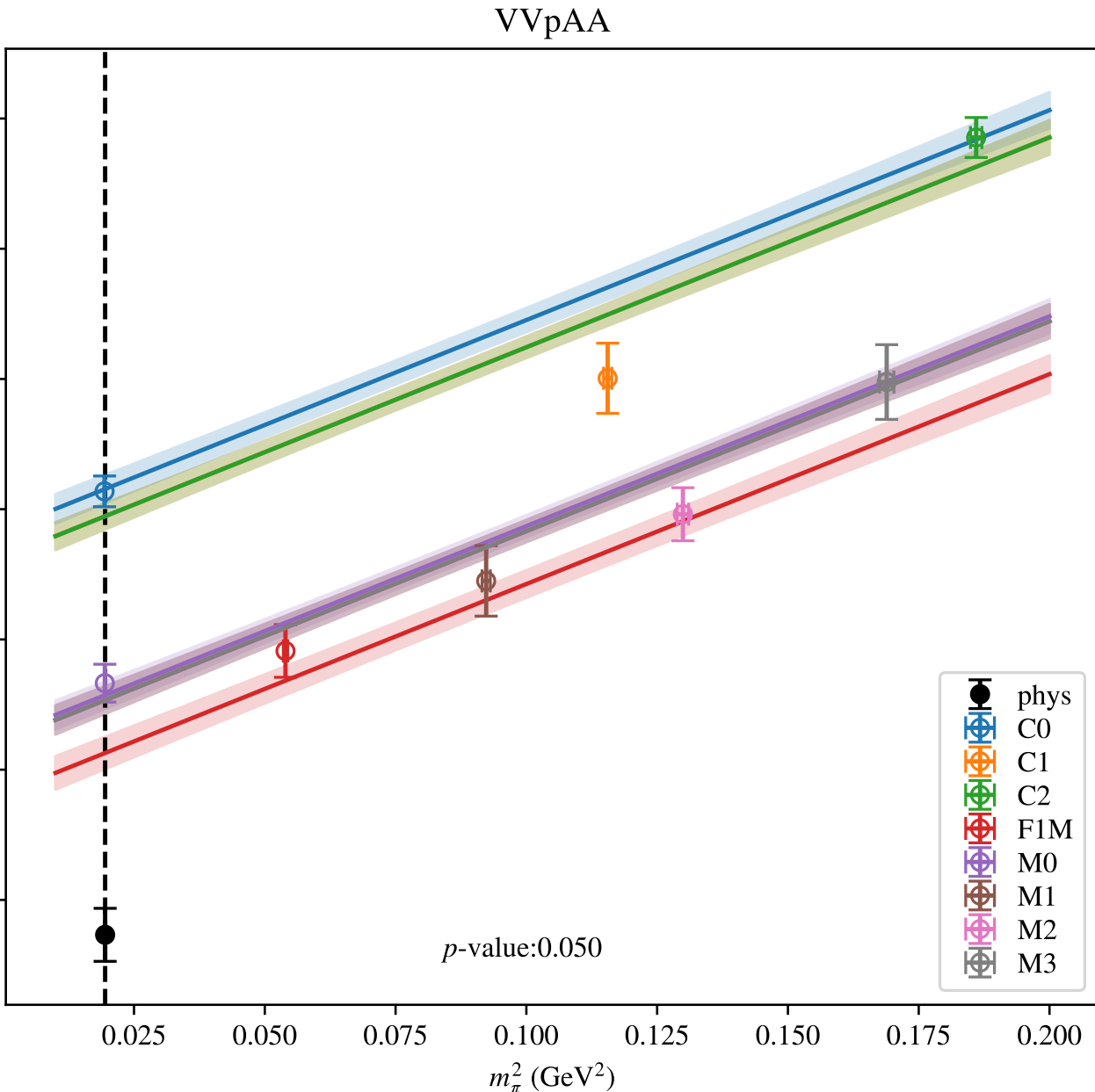
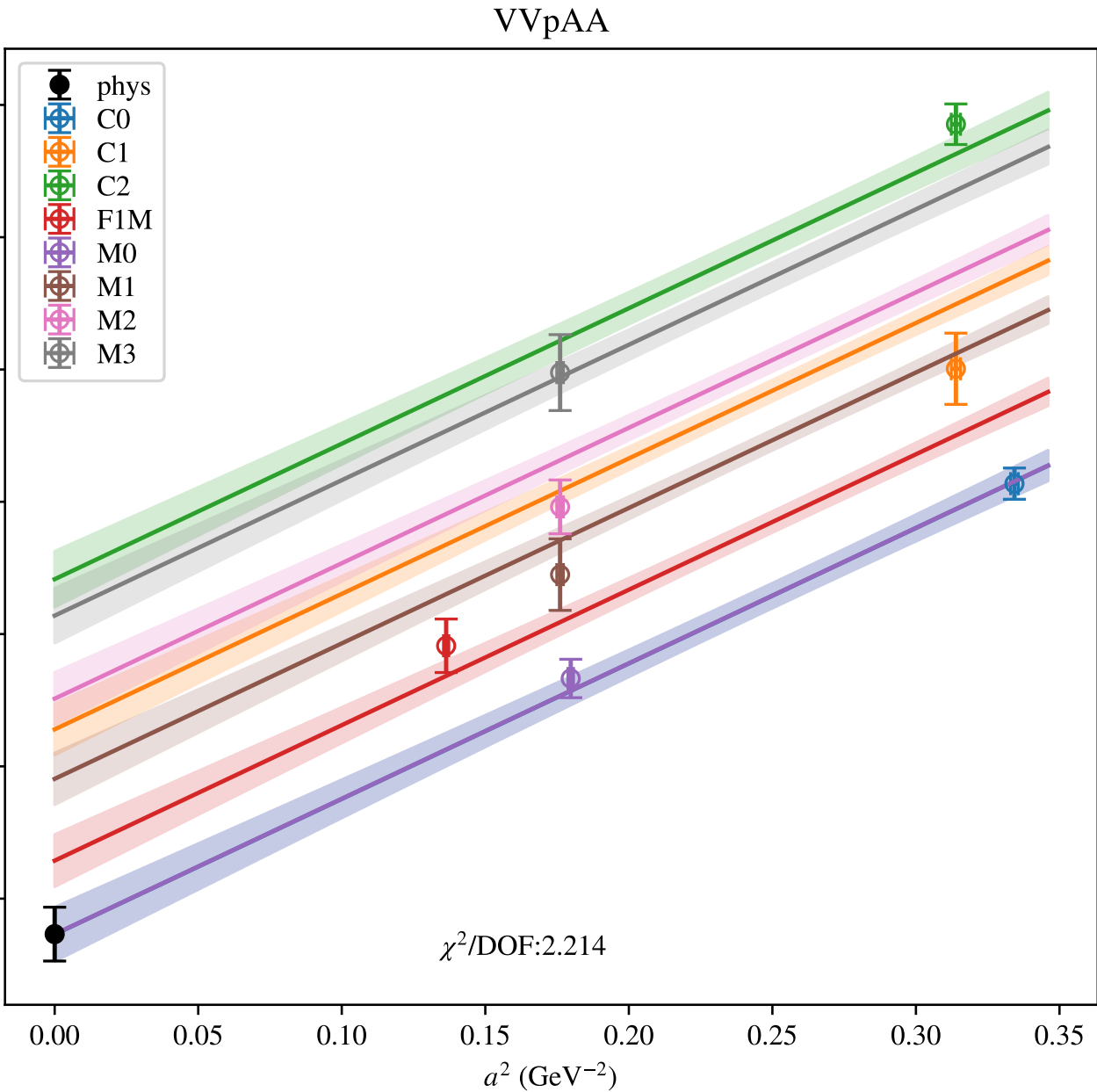
$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$

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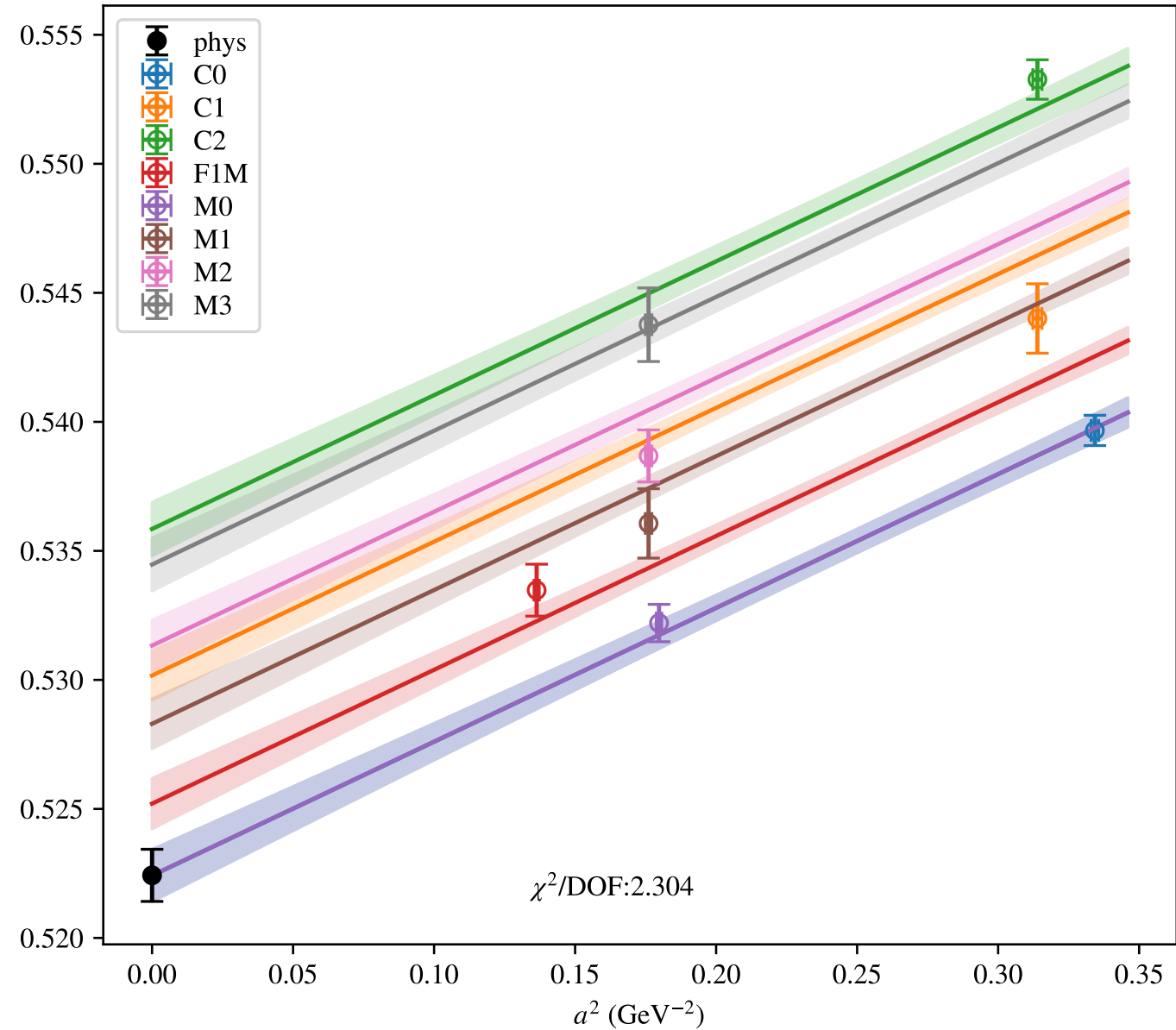
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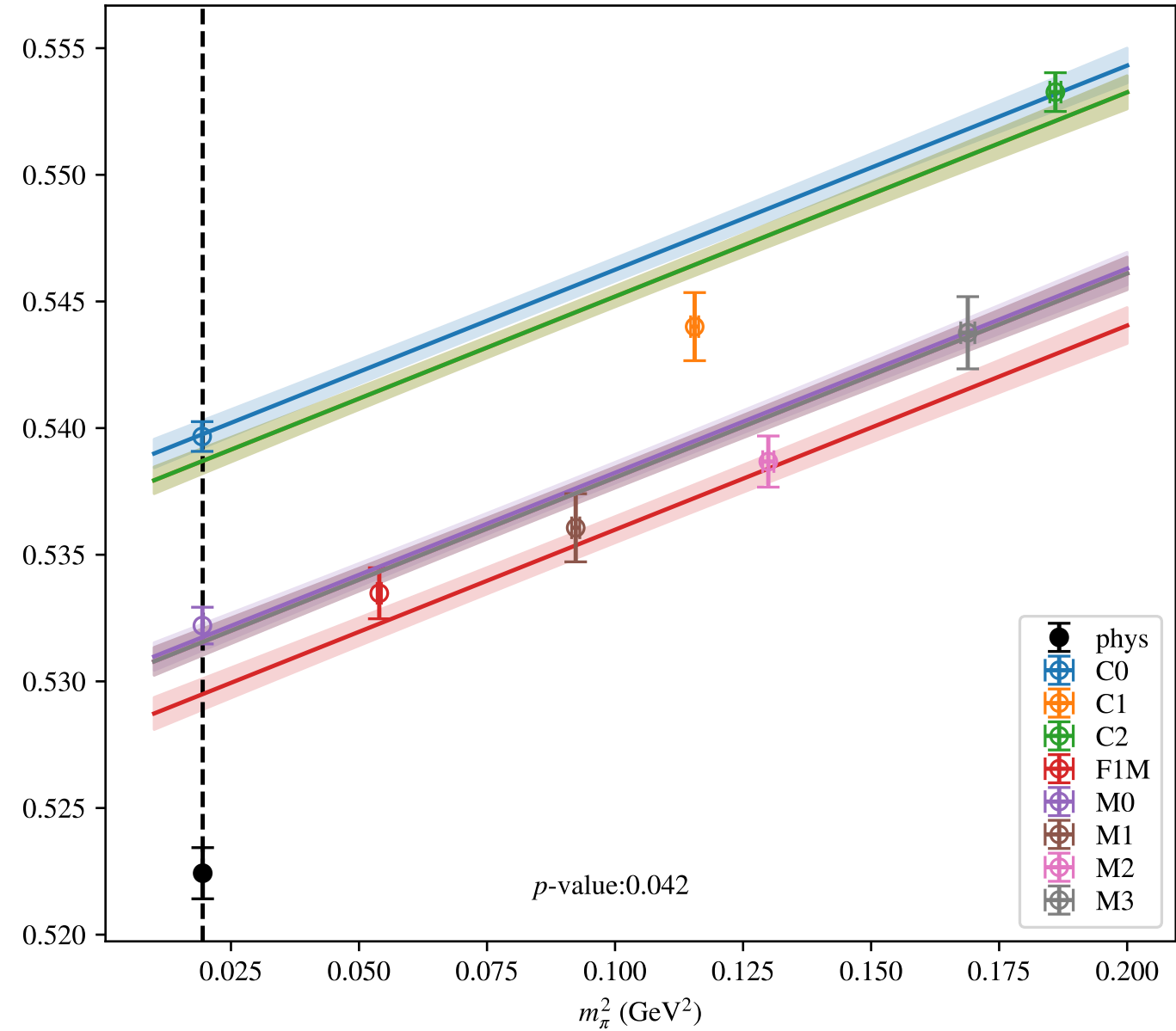
$a^2, m_\pi^2, \mu = 2.2 \text{ GeV}$ 

$a^2, m_\pi^2, \mu = 2.3 \text{ GeV}$

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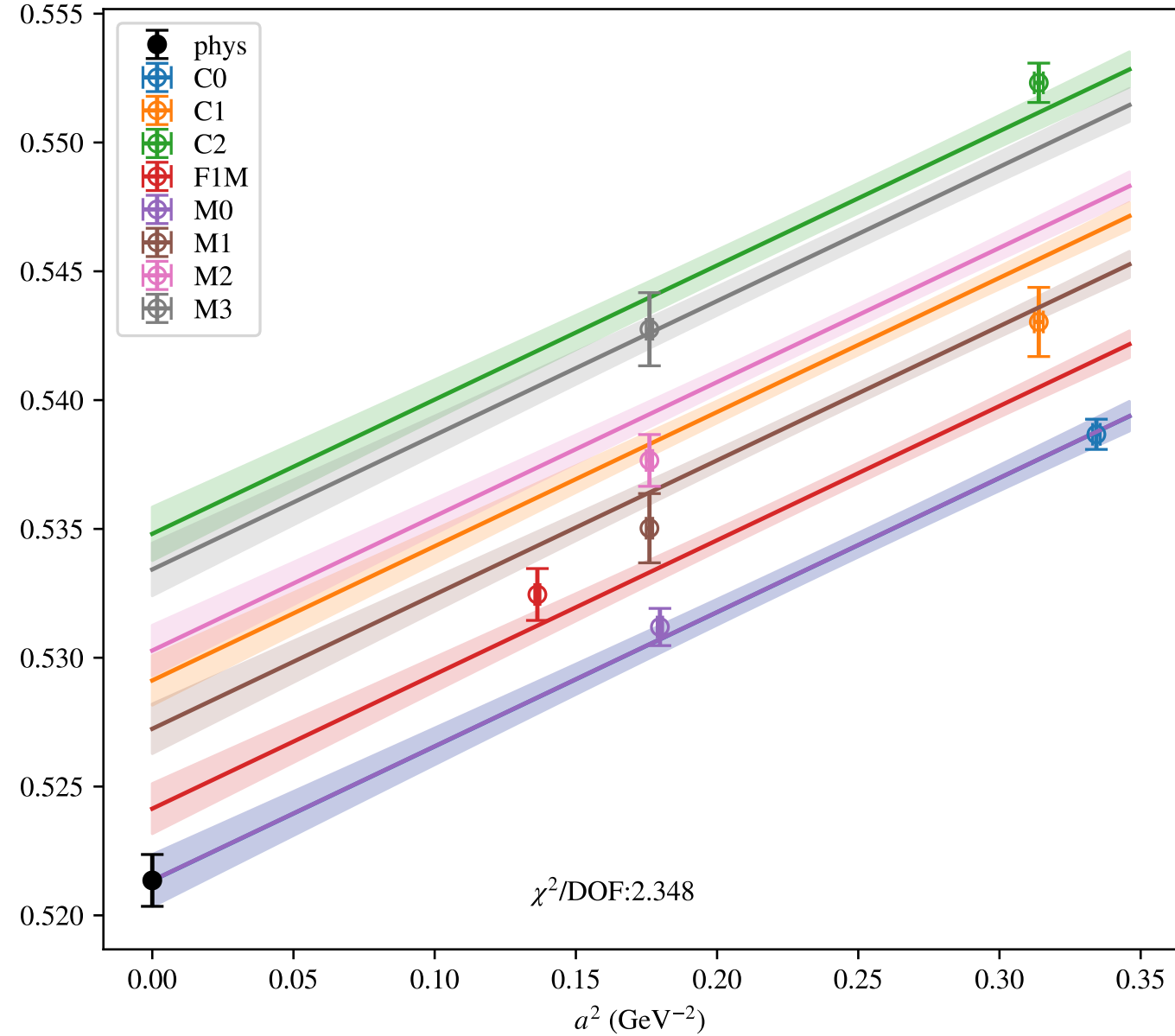


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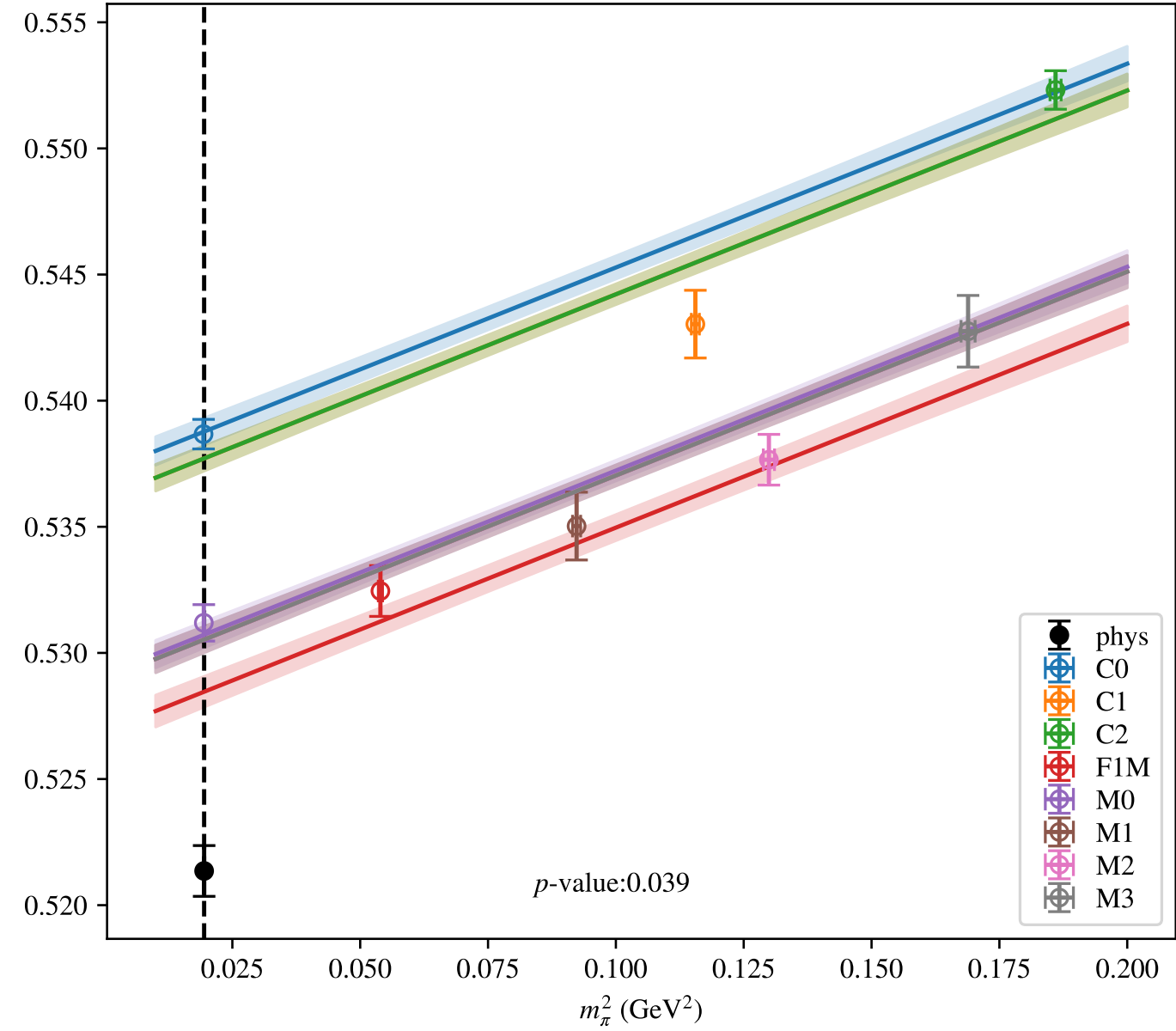


$a^2, m_\pi^2, \mu = 2.4 \text{ GeV}$

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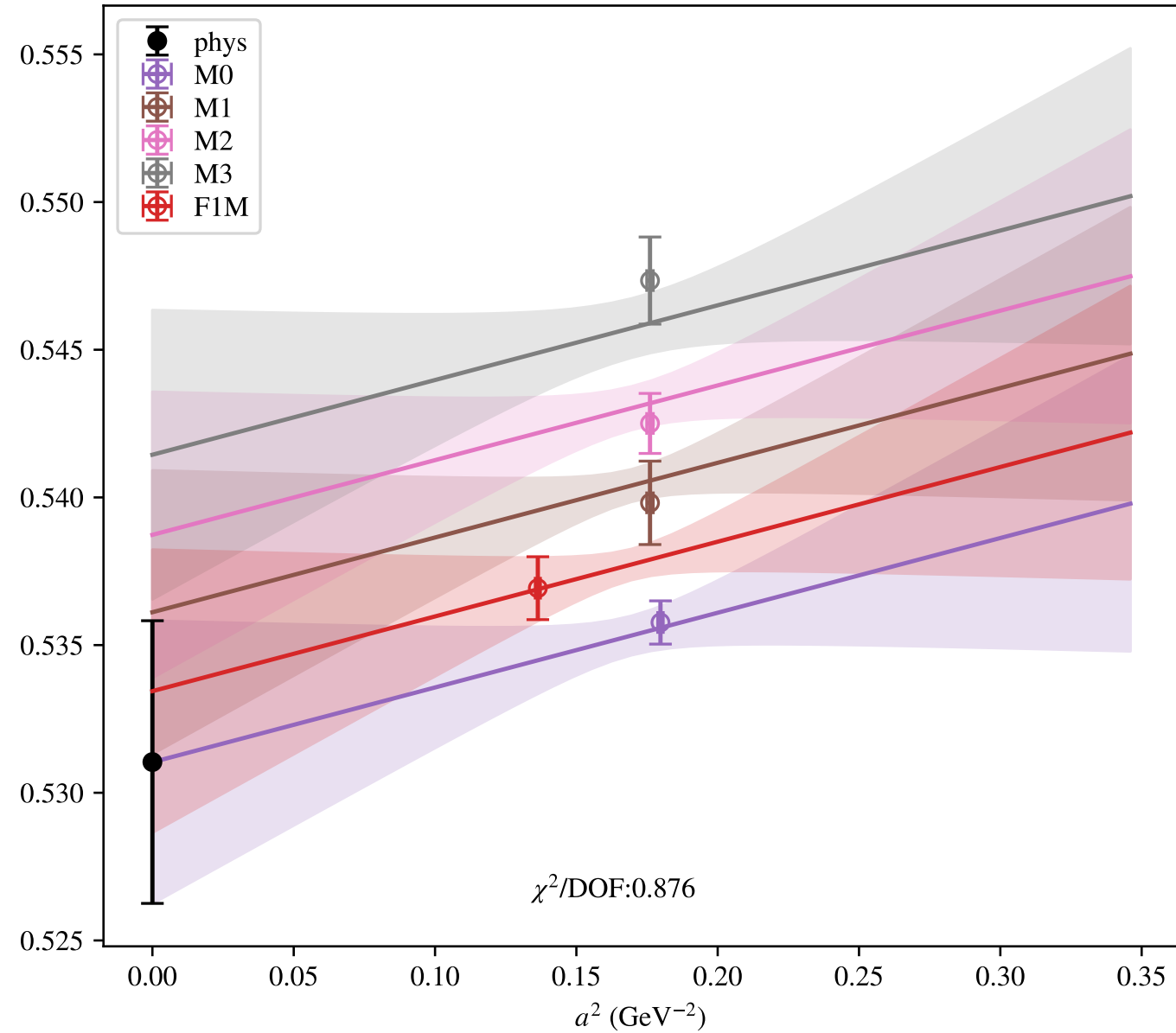


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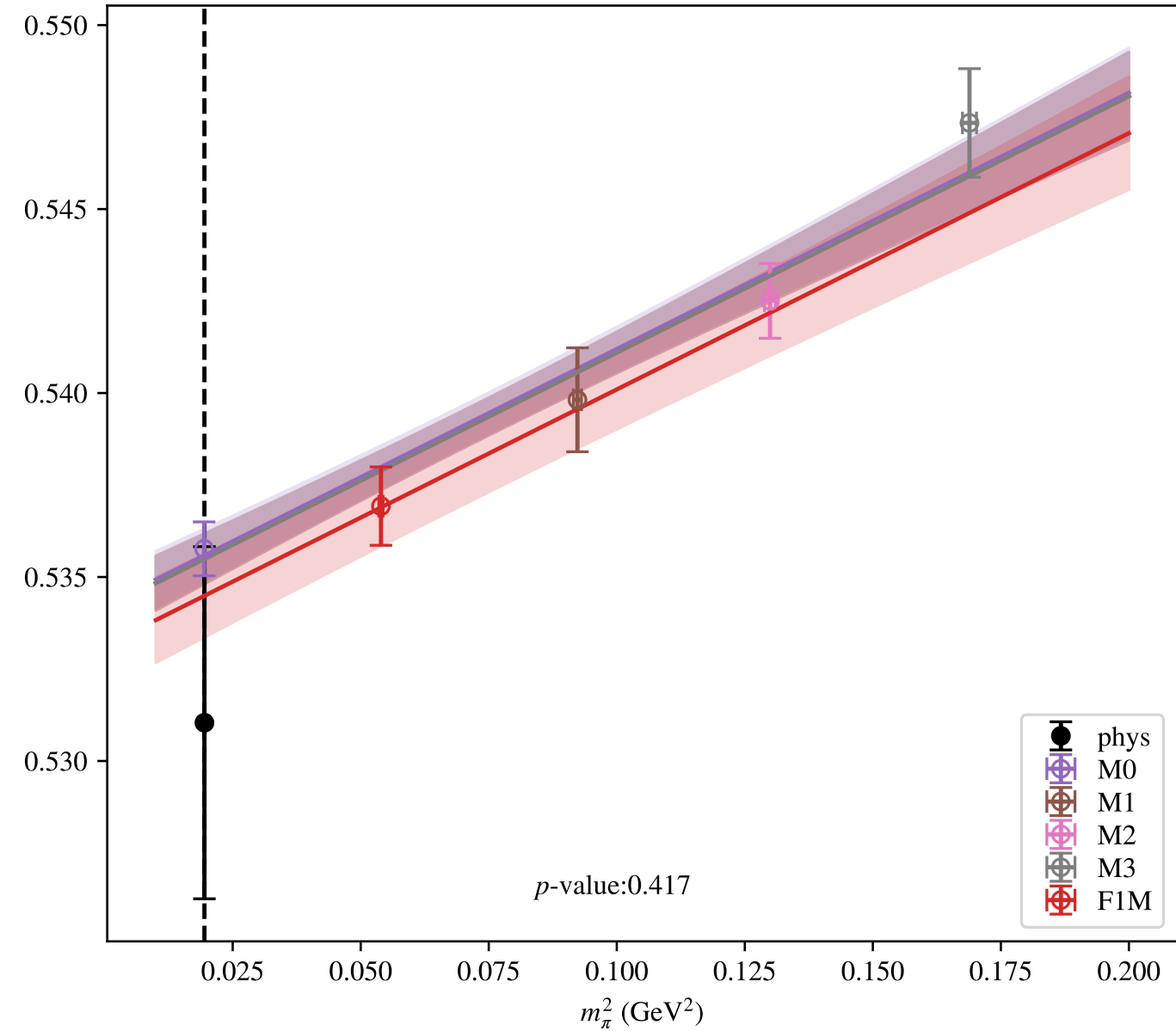


a^2, m_π^2 (no C), $\mu = 2.0$ GeV

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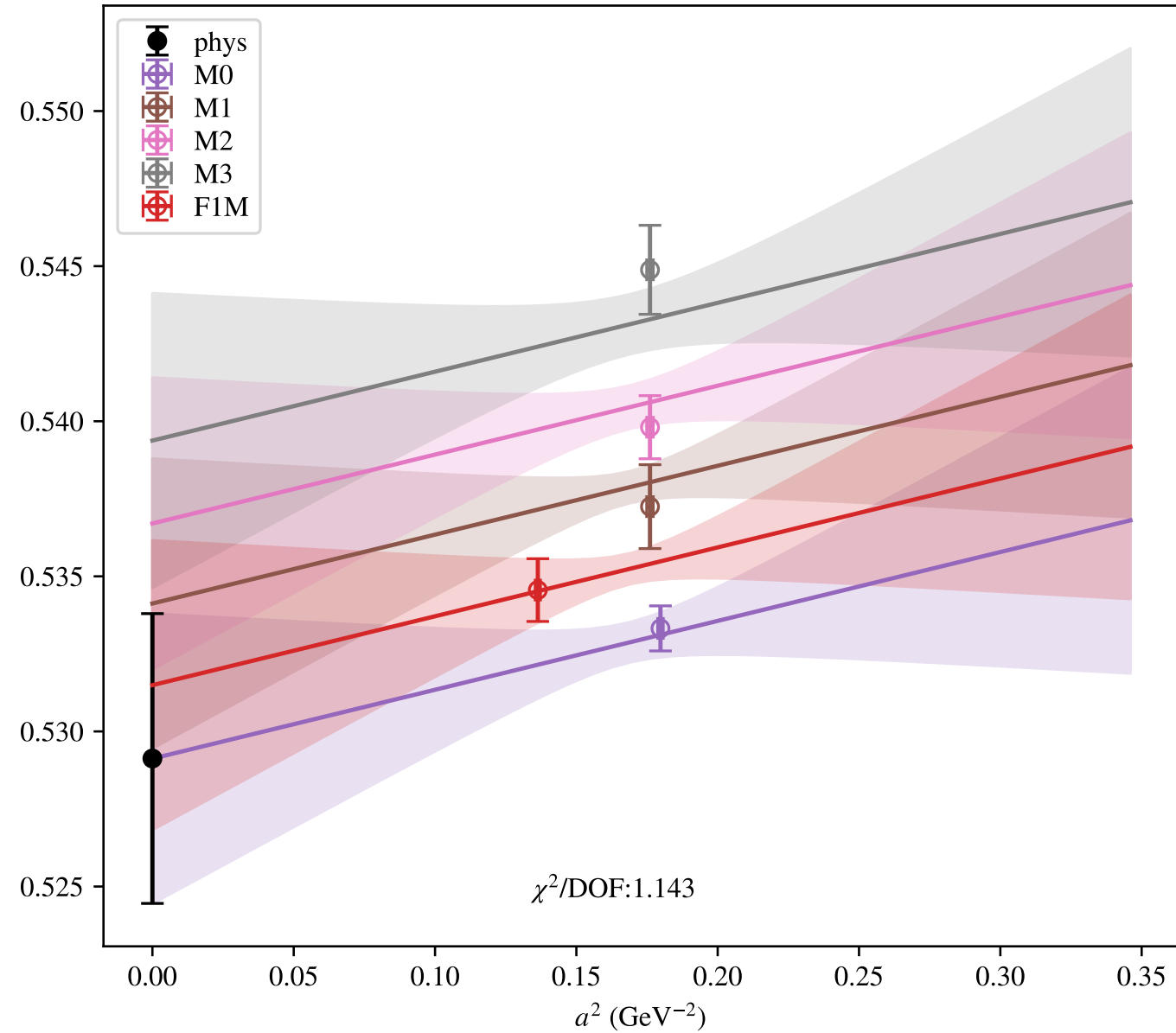


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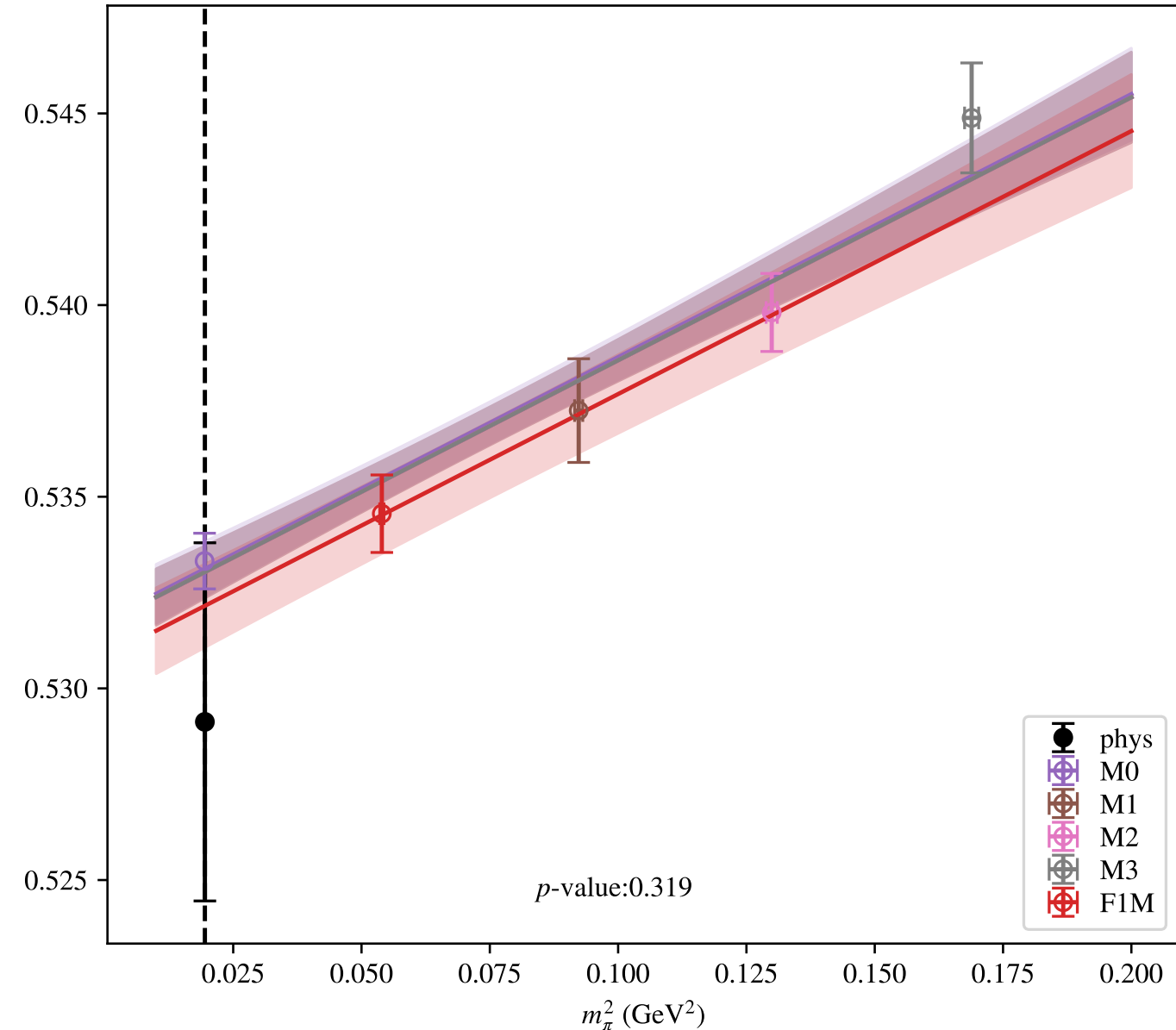


a^2, m_π^2 (no C), $\mu = 2.2$ GeV

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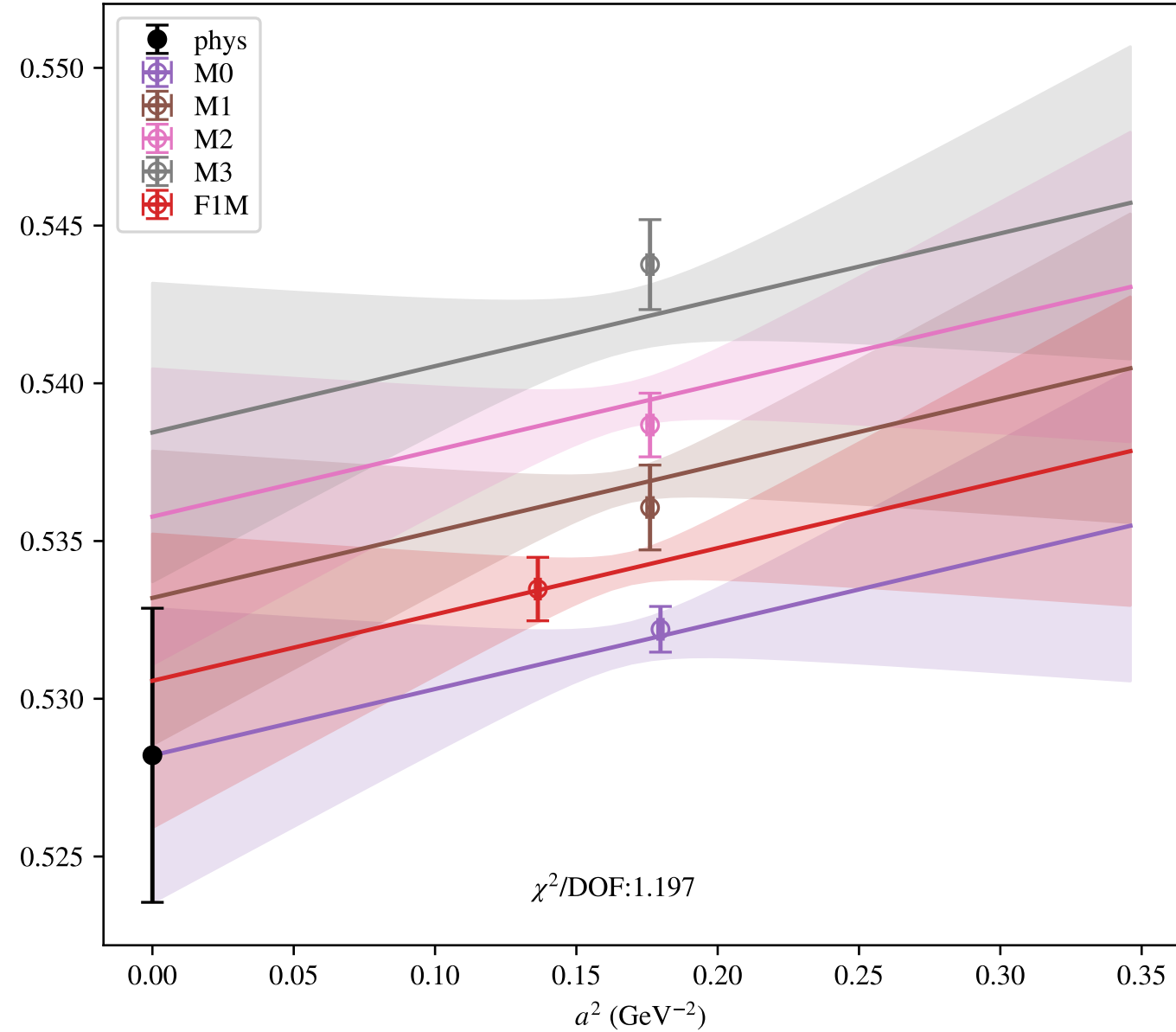


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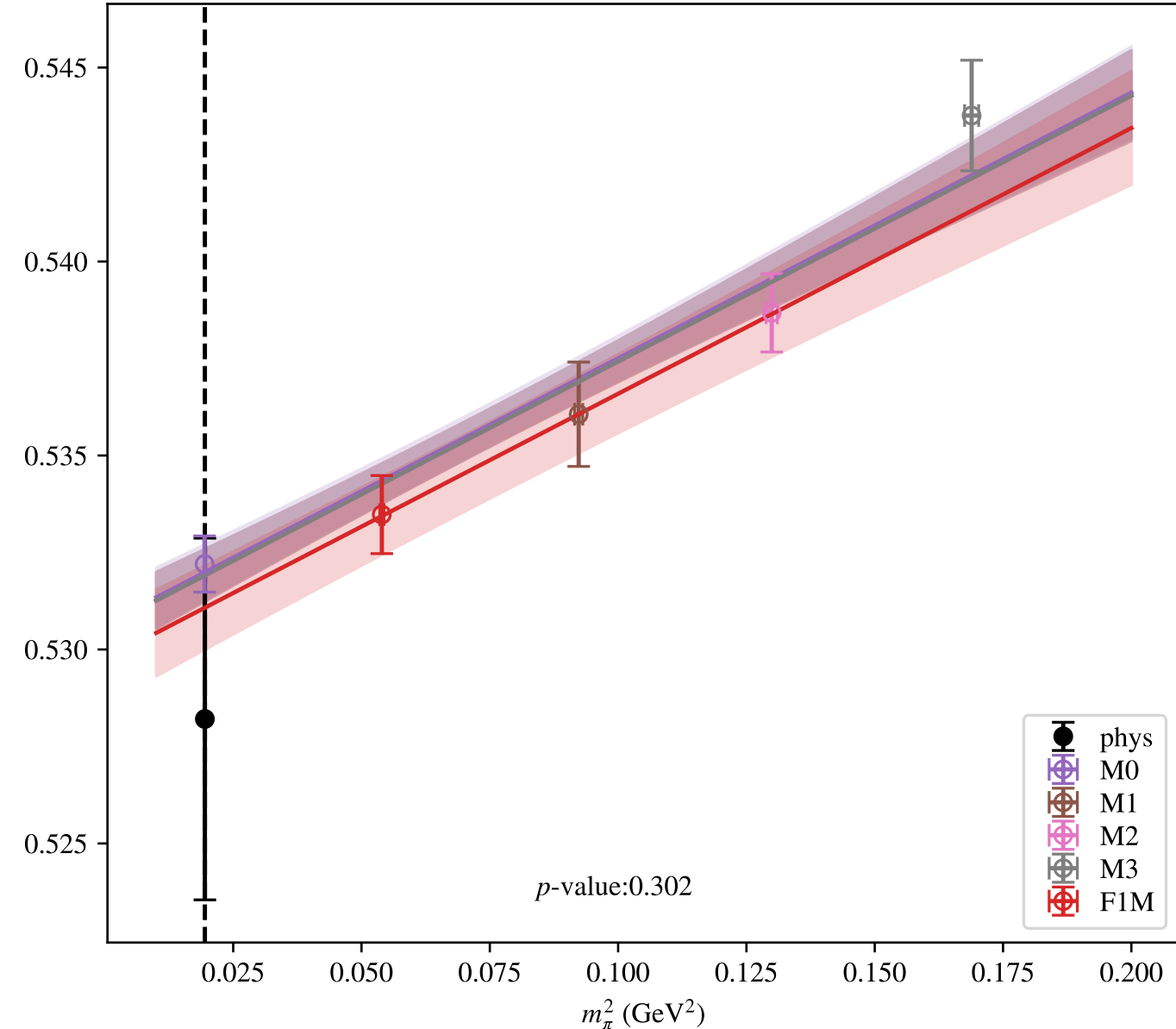


a^2, m_π^2 (no C), $\mu = 2.3$ GeV

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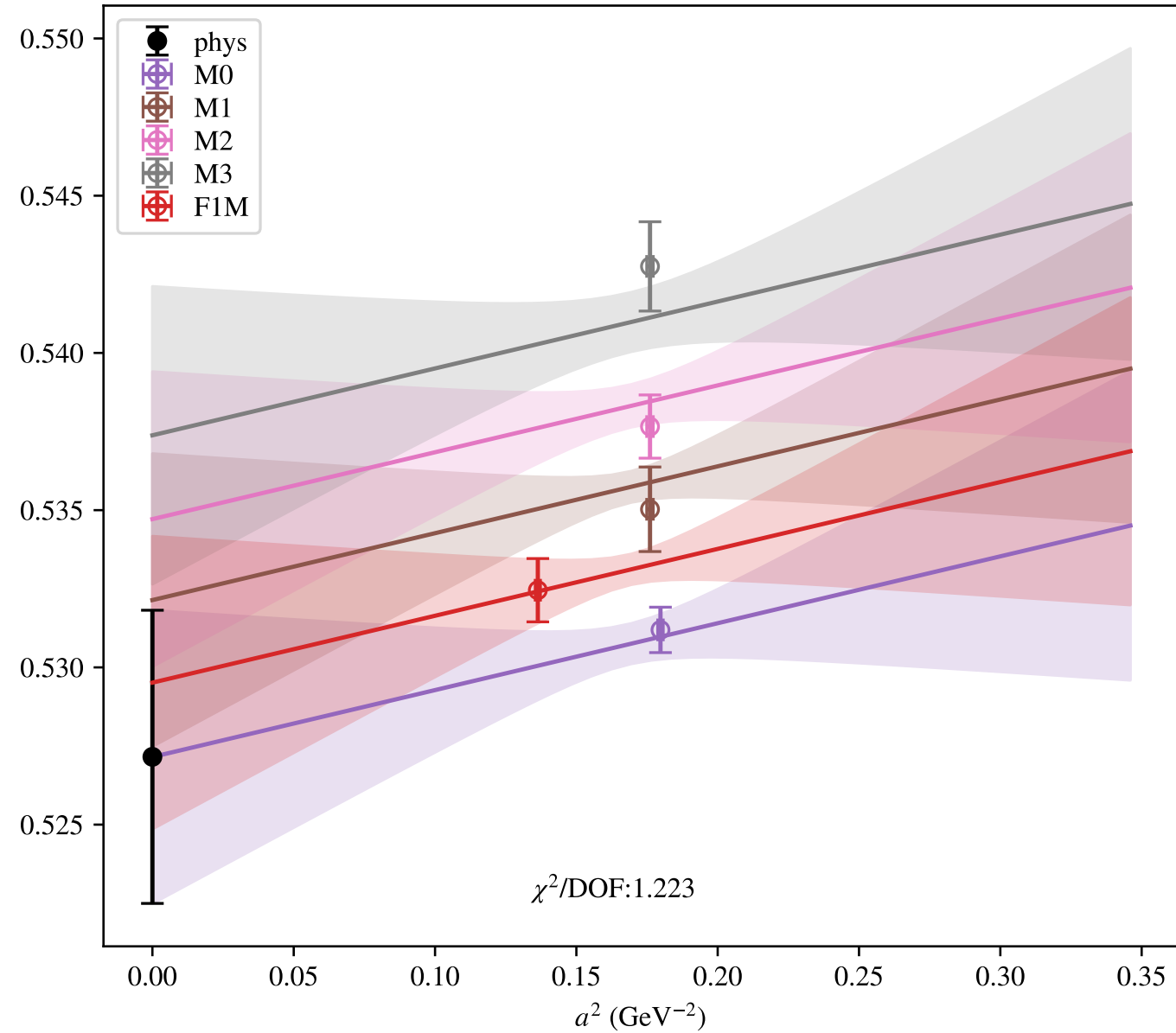


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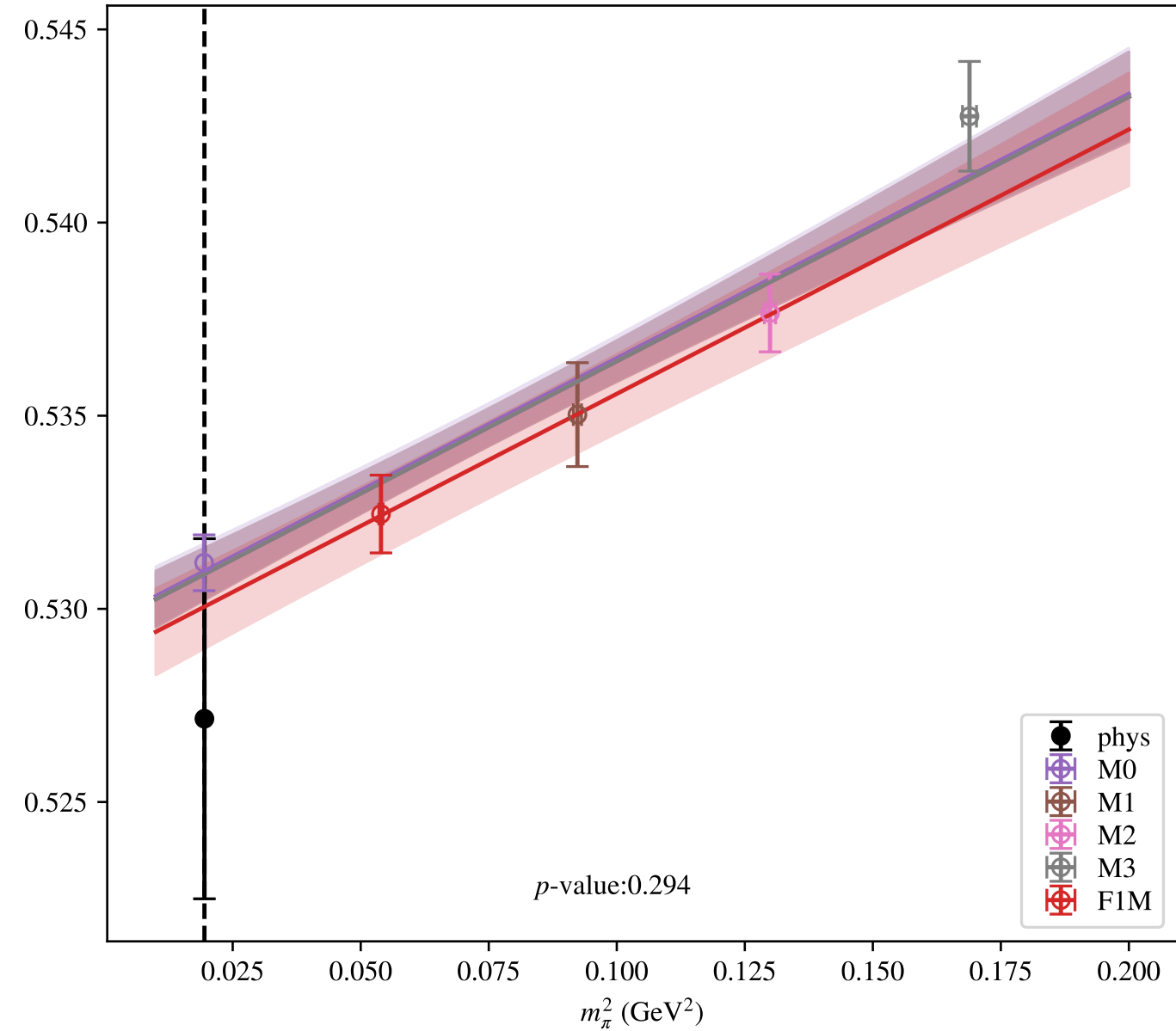


a^2, m_π^2 (no C), $\mu = 2.4$ GeV

VVpAA

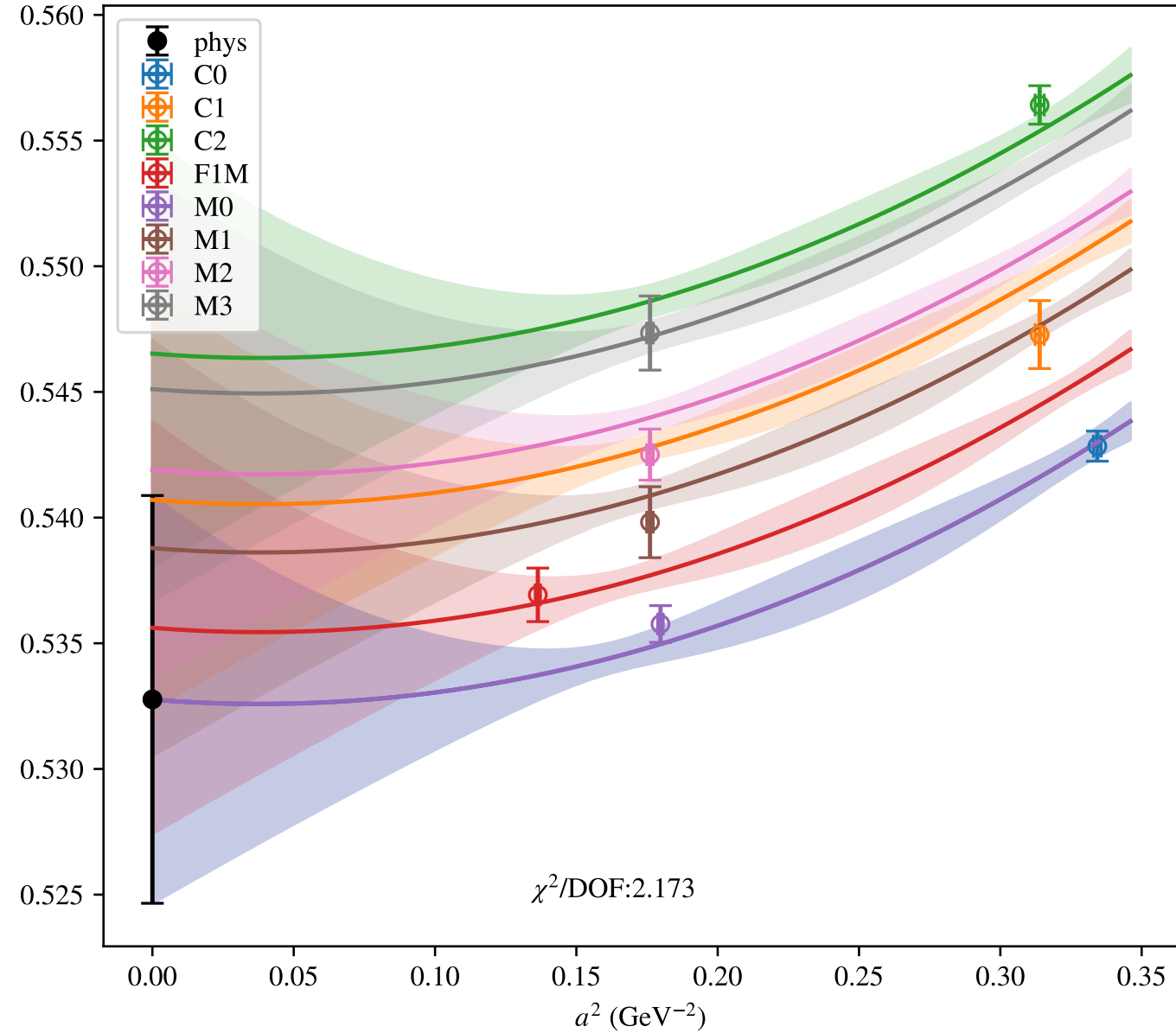


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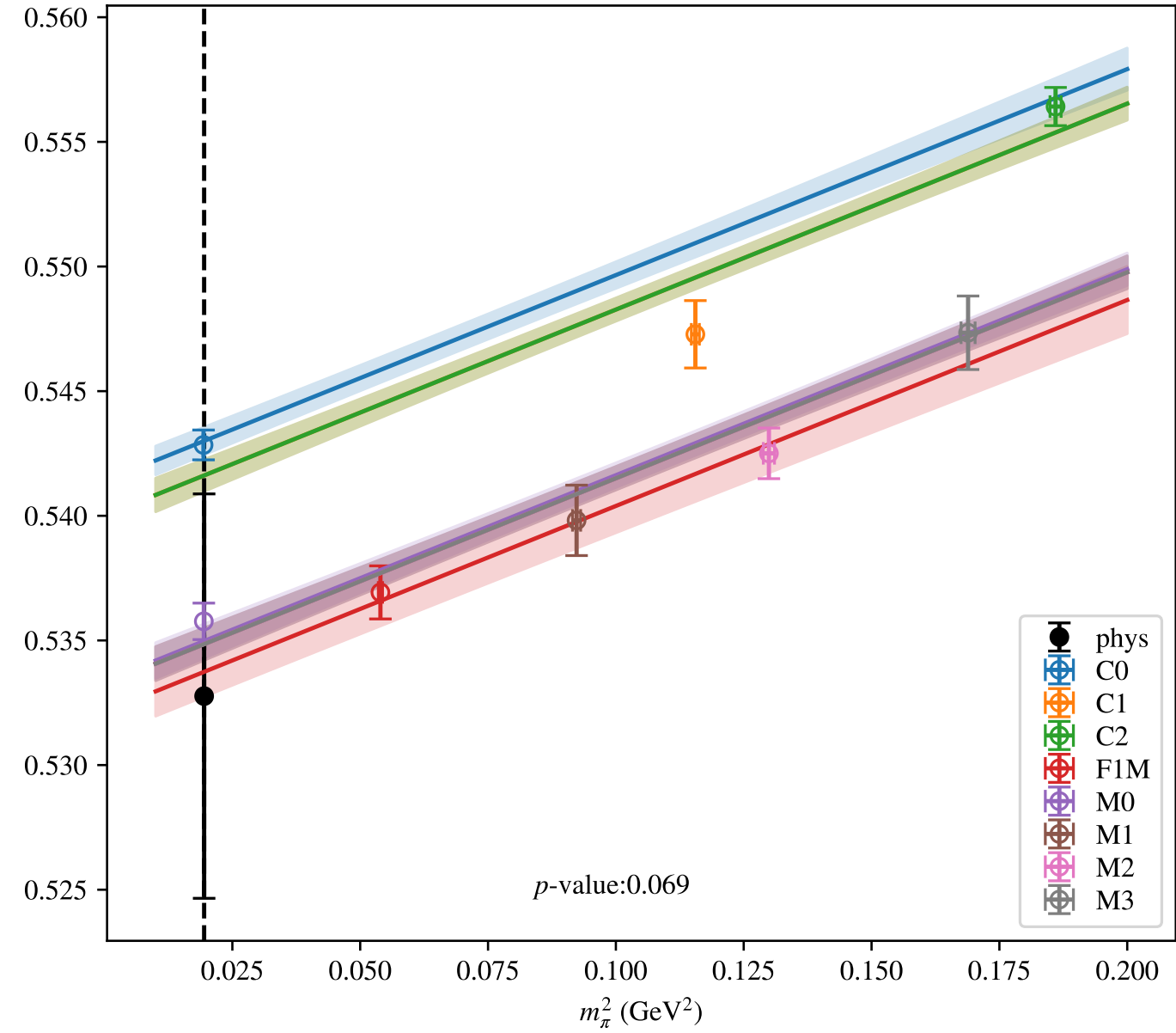


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

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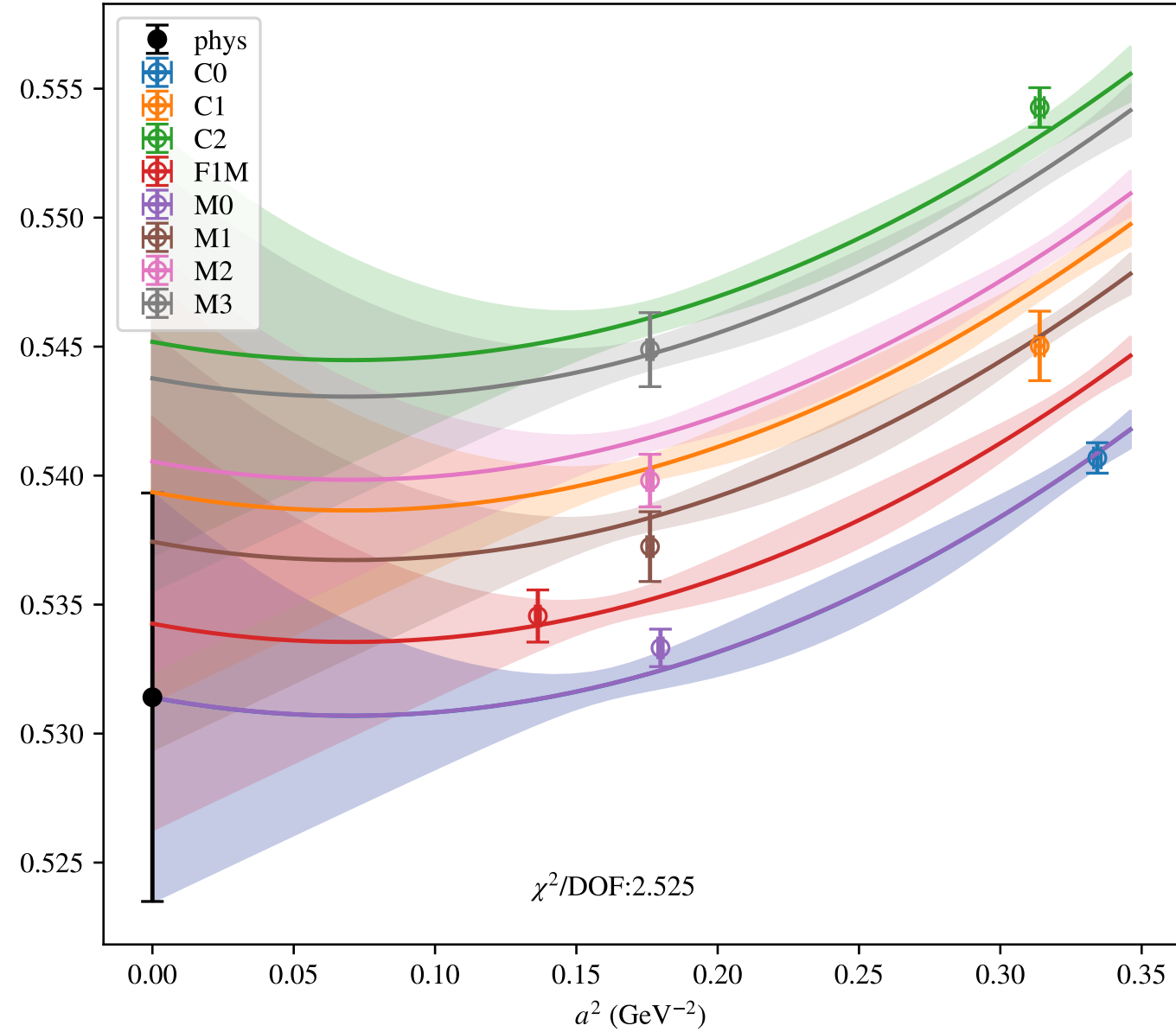


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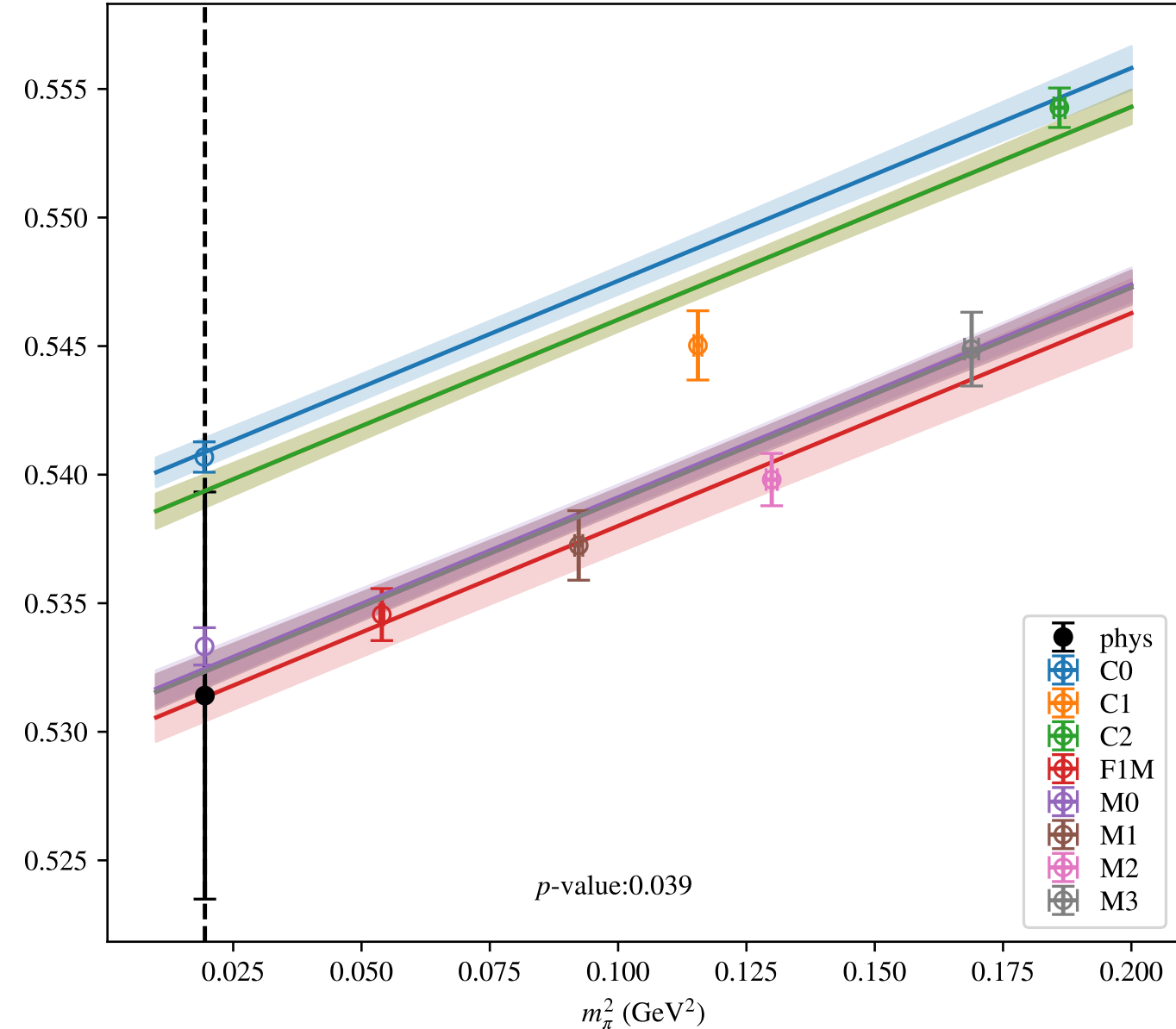


$$a^2, a^4, m_\pi^2, \mu = 2.2 \text{ GeV}$$

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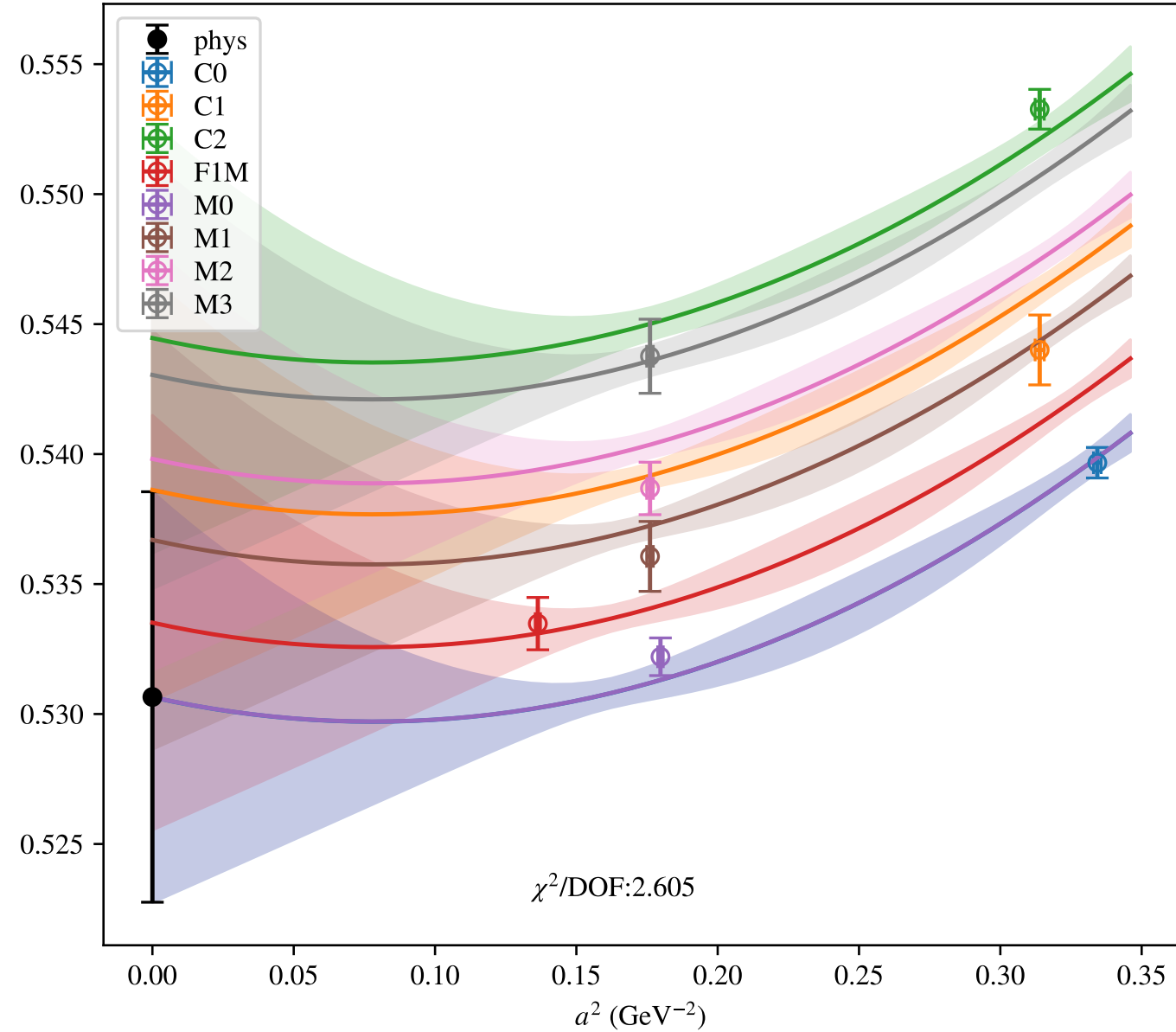


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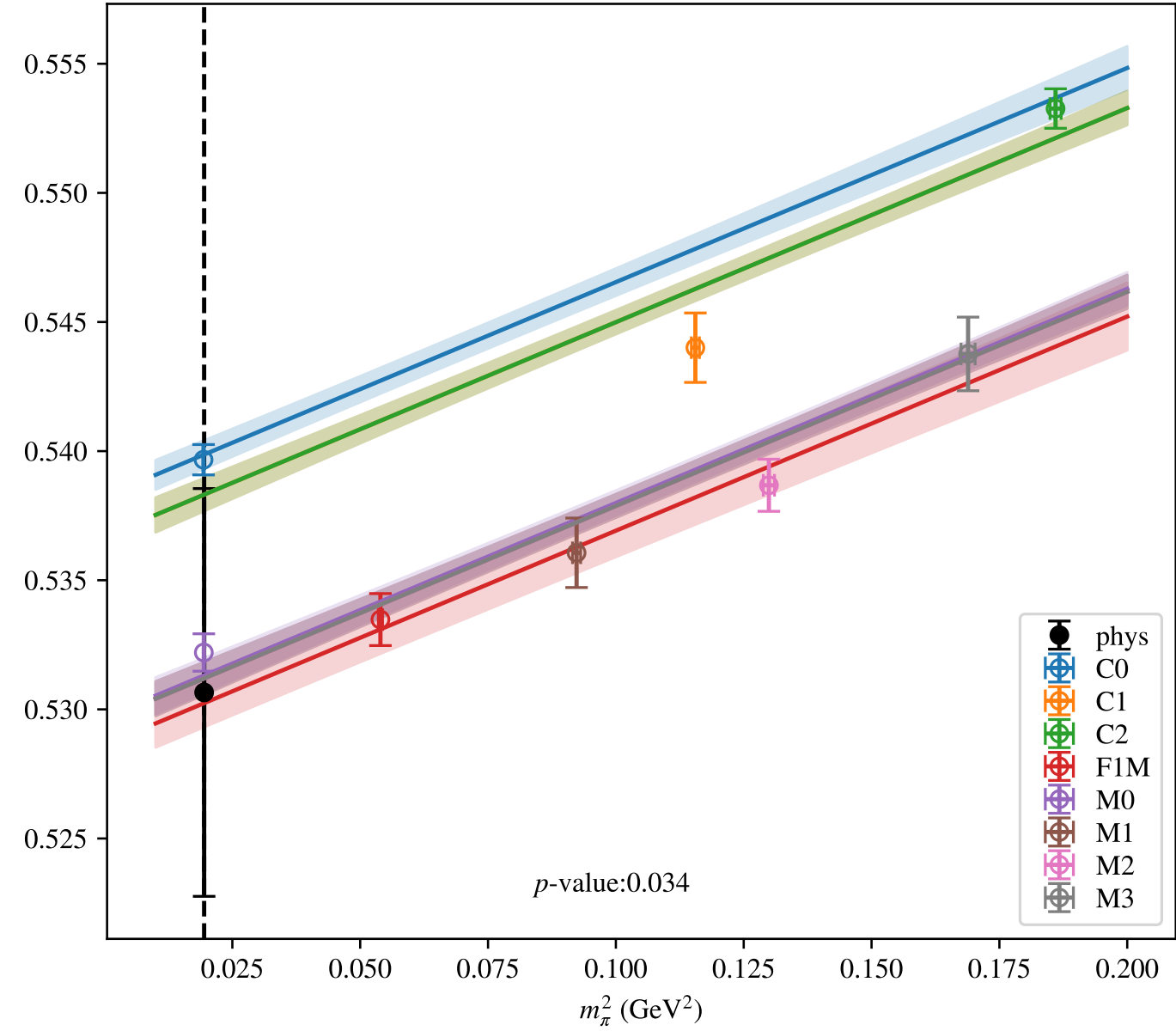


$$a^2, a^4, m_\pi^2, \mu = 2.3 \text{ GeV}$$

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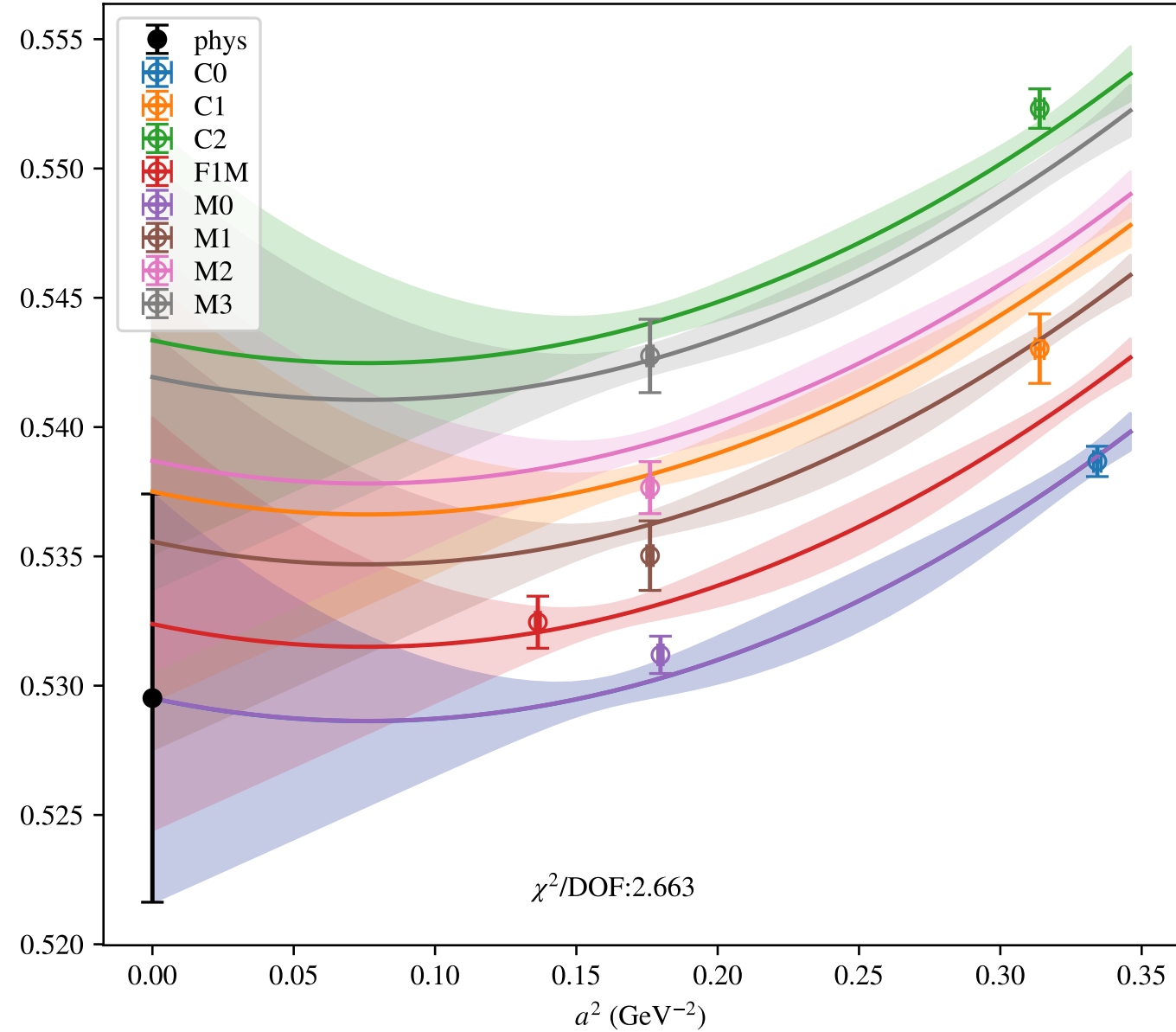


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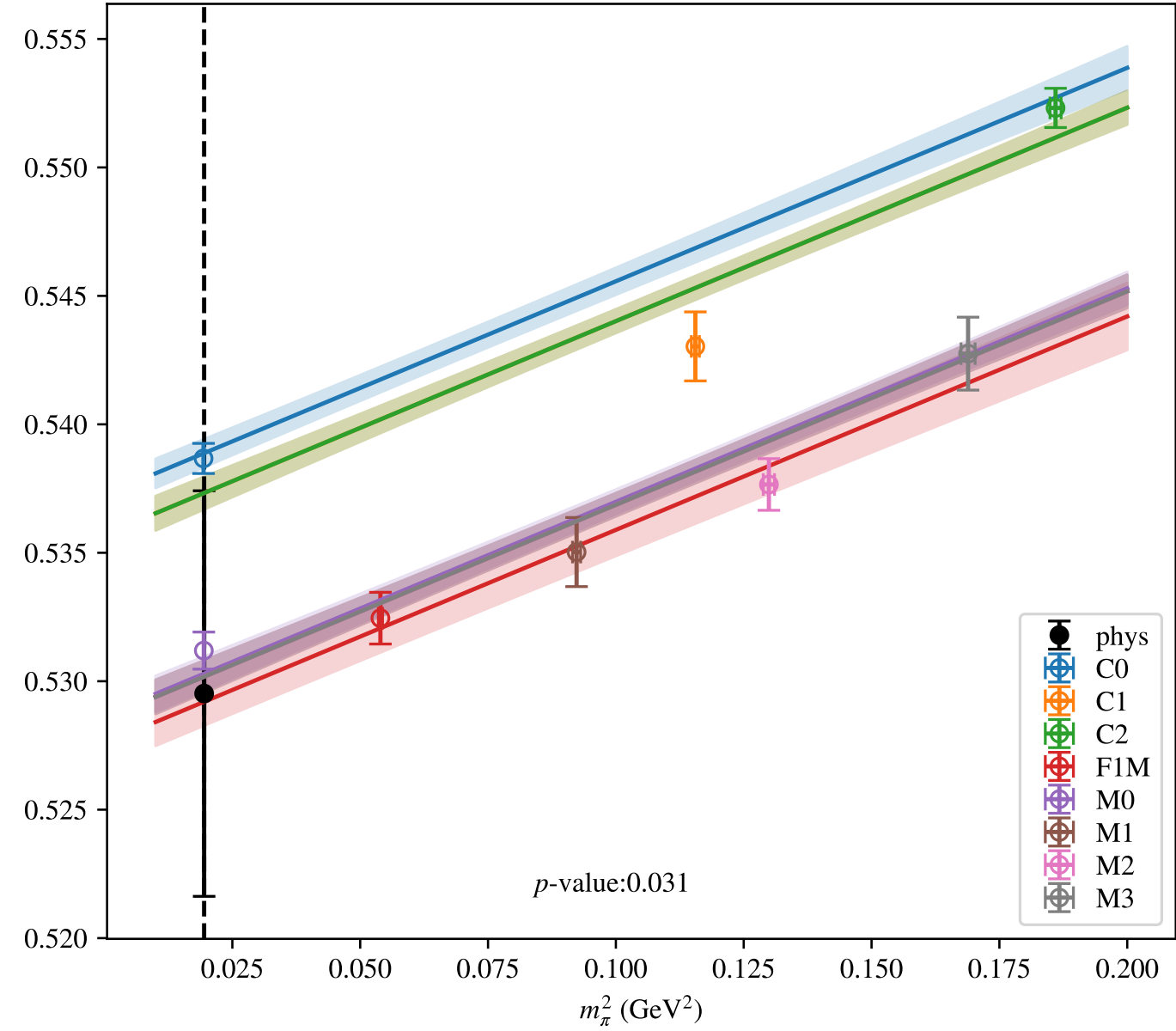


$$a^2, a^4, m_\pi^2, \mu = 2.4 \text{ GeV}$$

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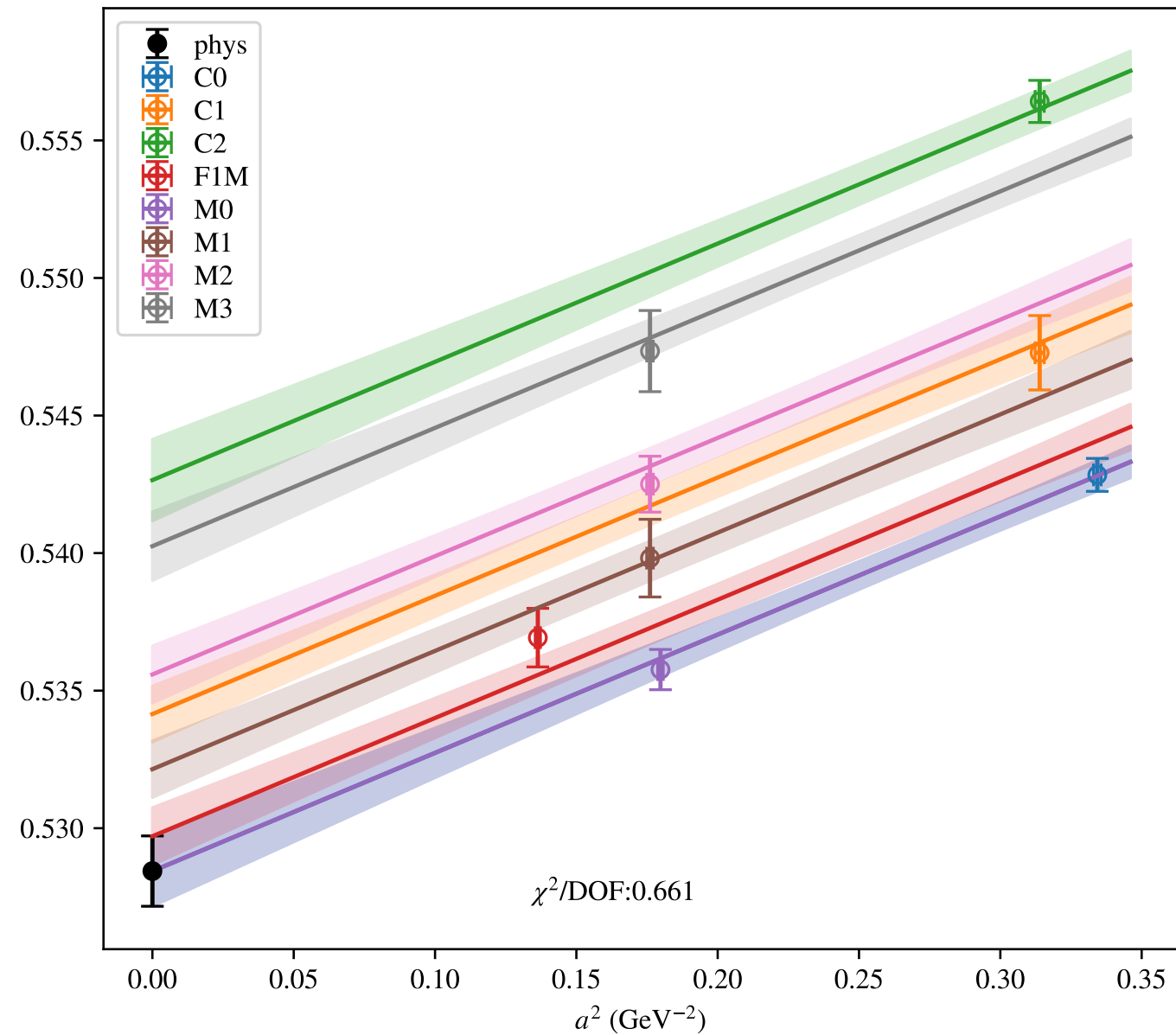


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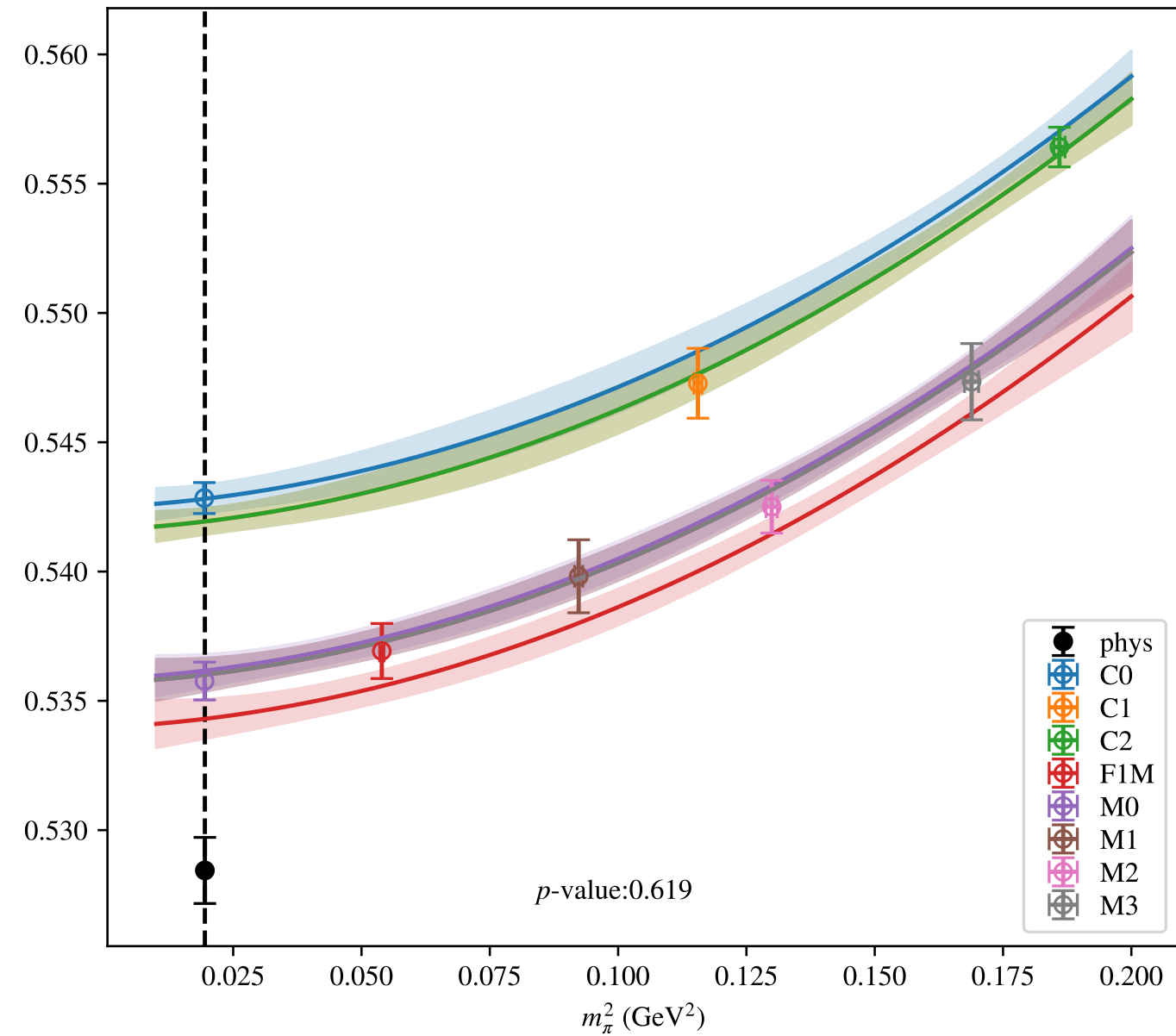


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

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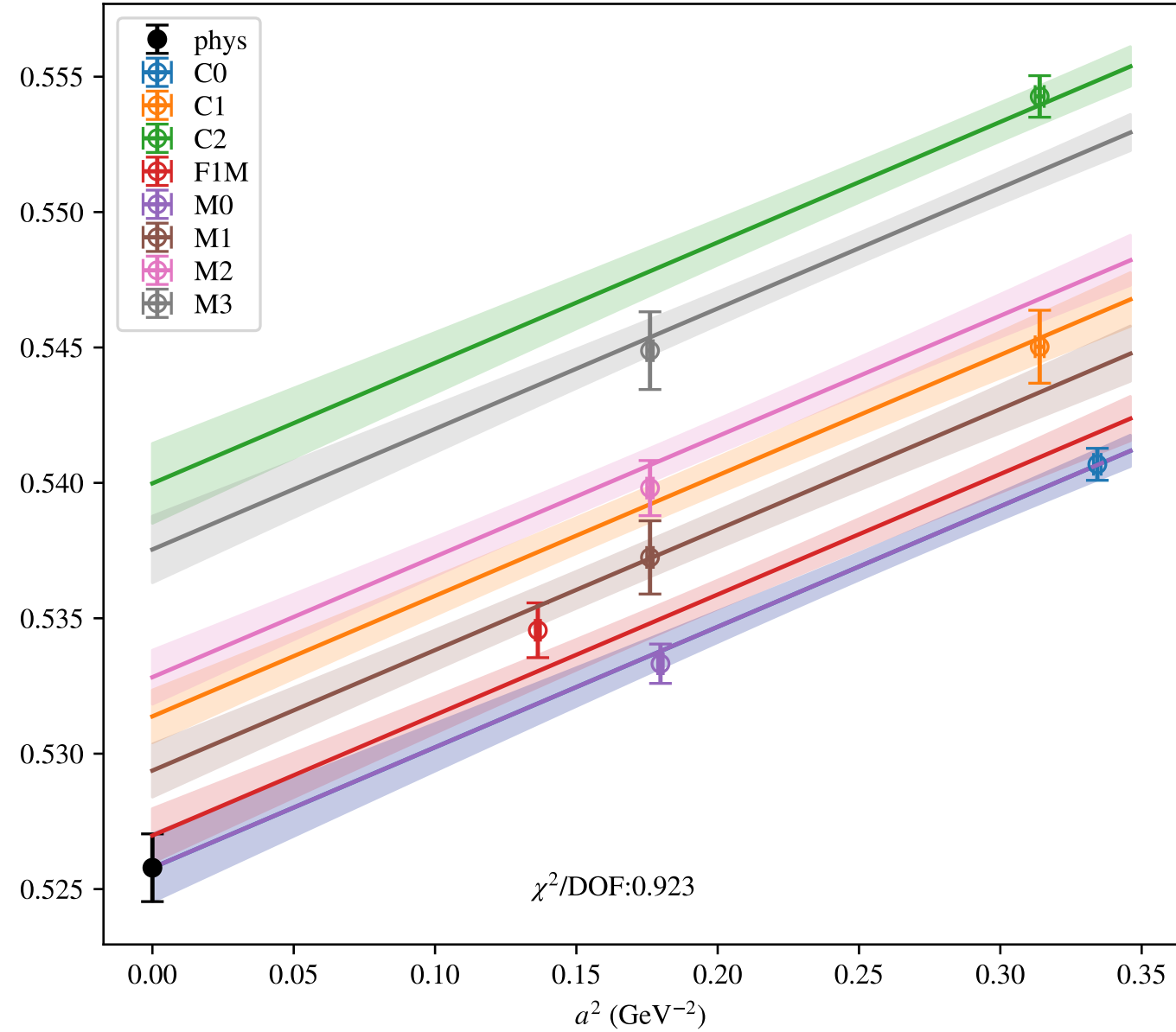


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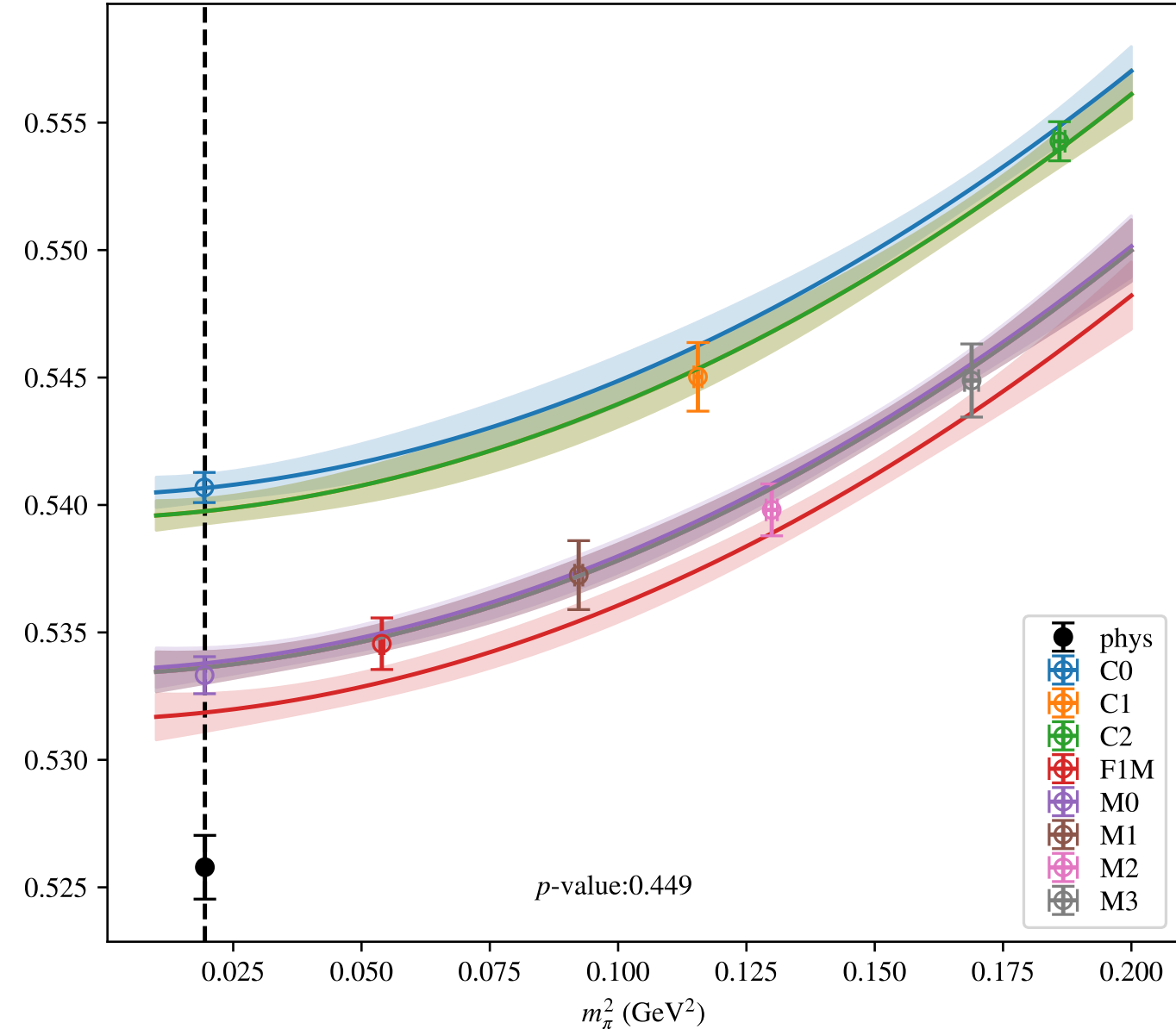


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.2 \text{ GeV}$$

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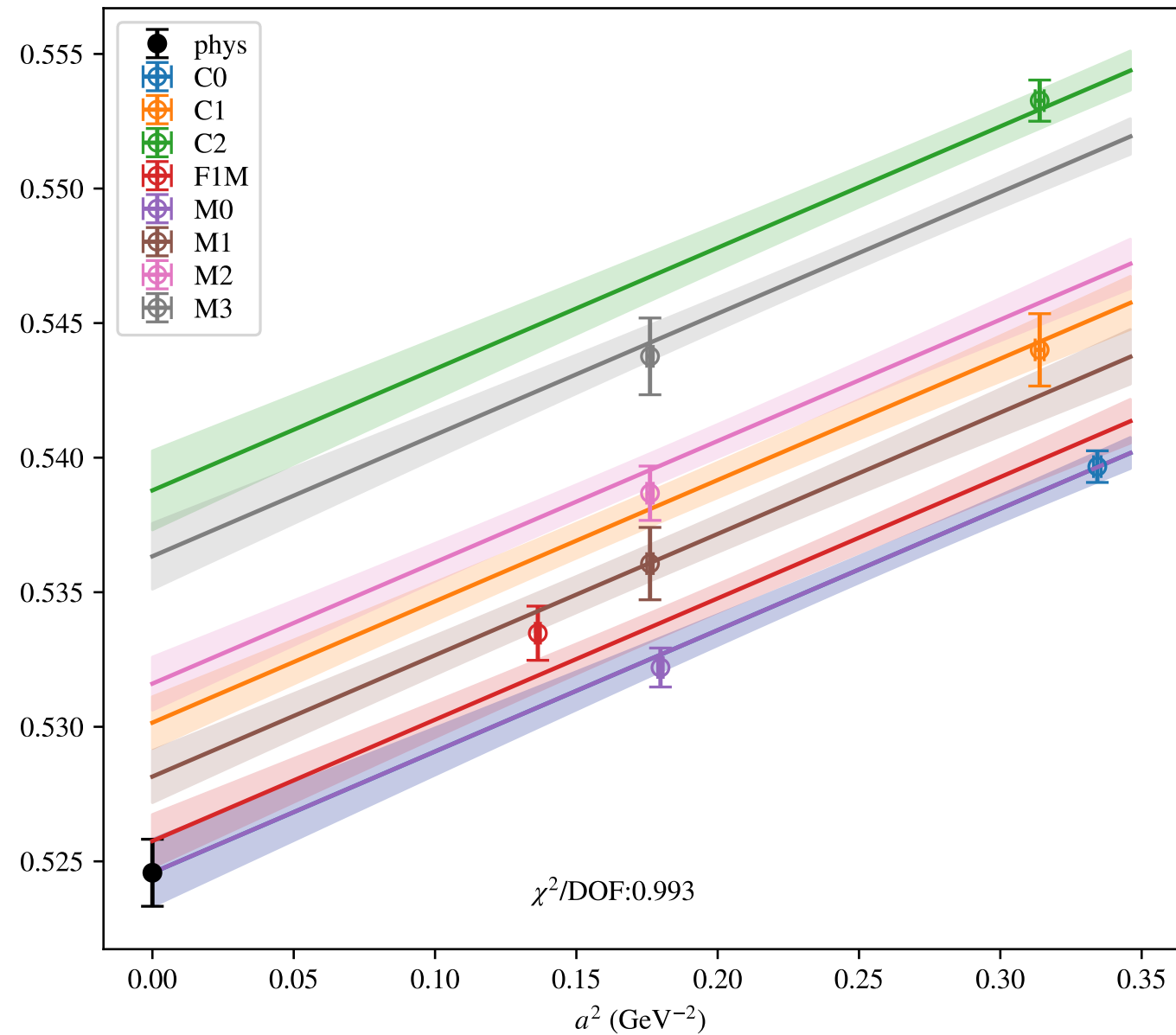


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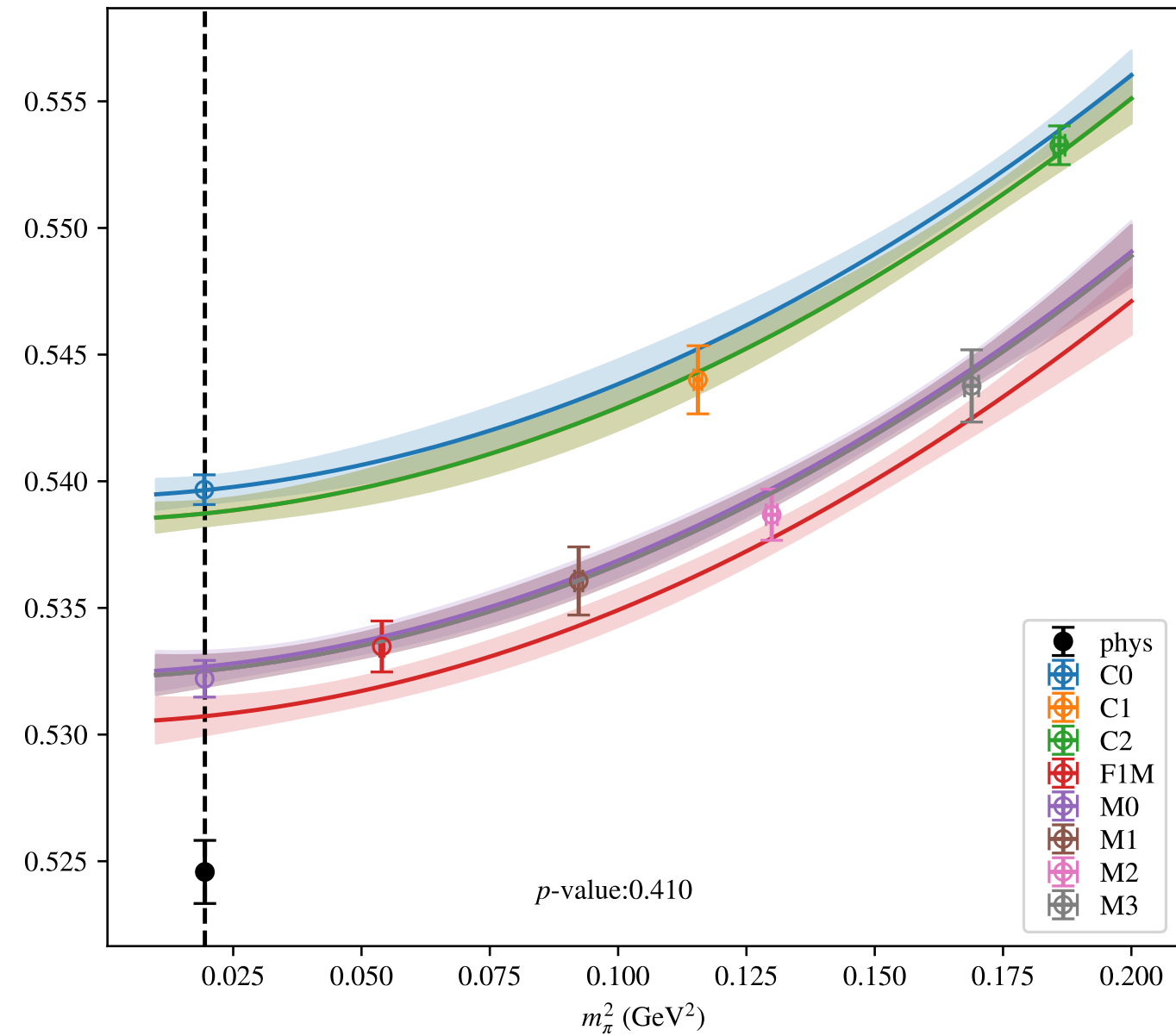


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.3 \text{ GeV}$$

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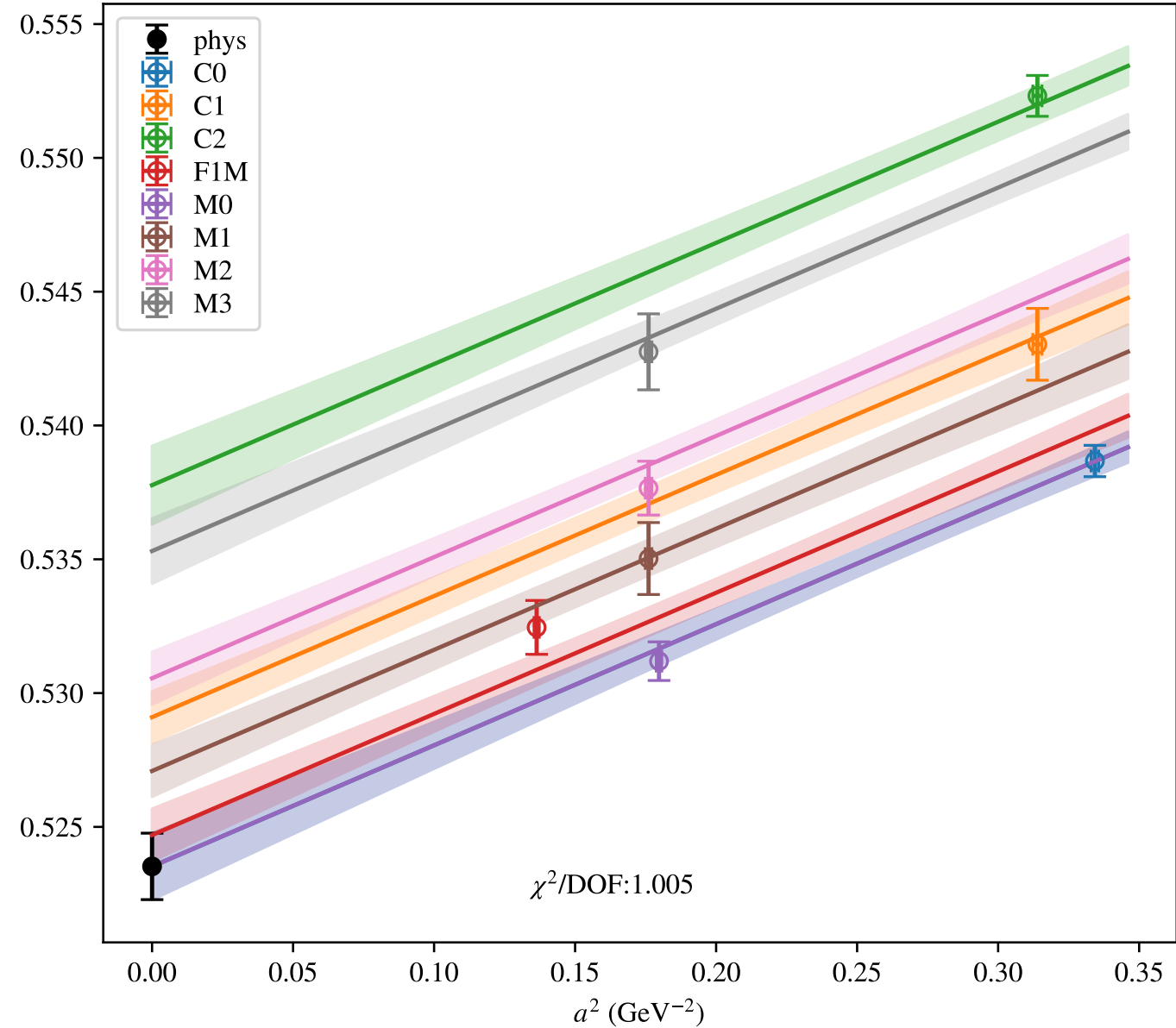


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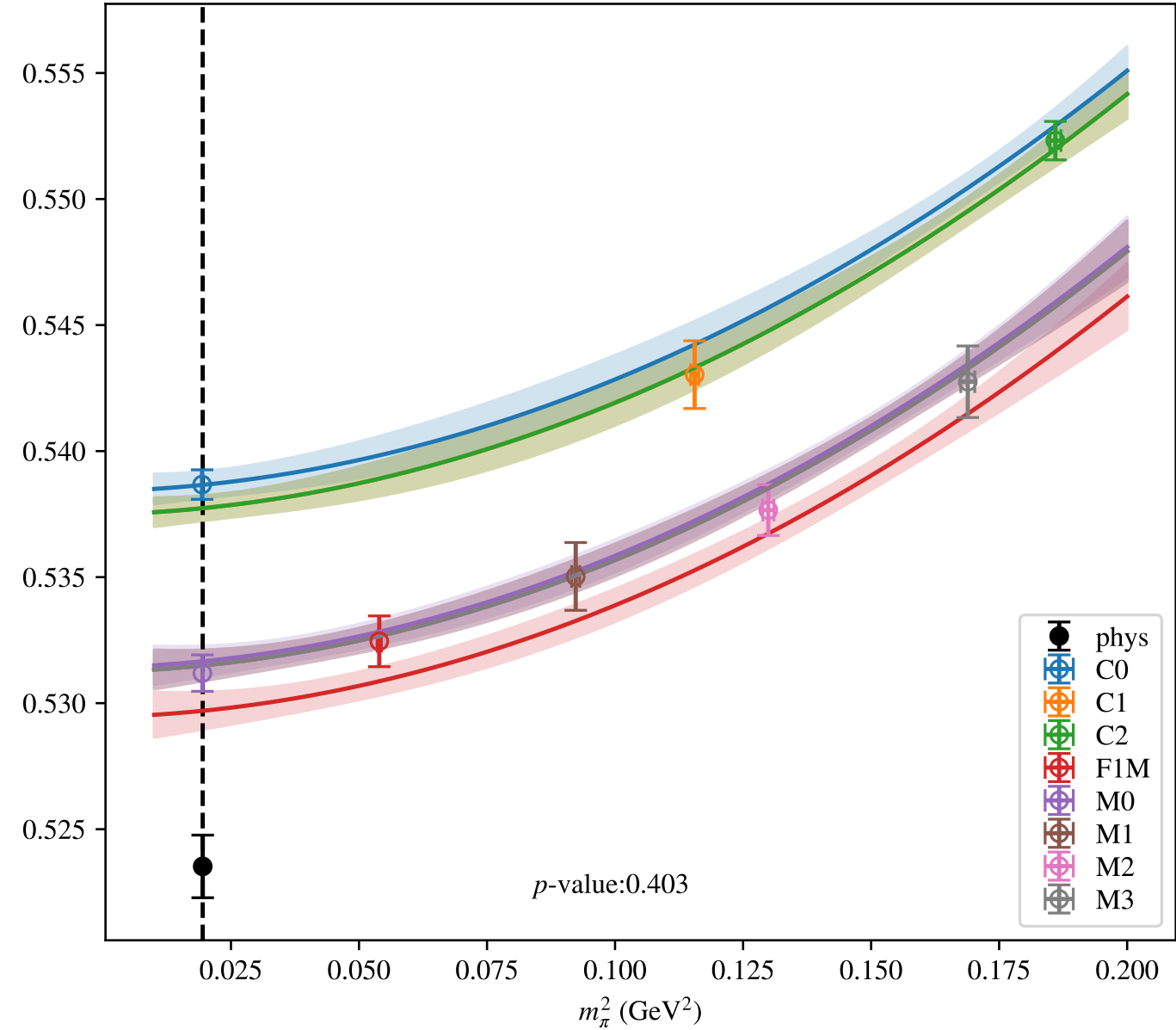


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.4 \text{ GeV}$$

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2 B_2

| μ (GeV) | a^2, m_π^2 | a^2, m_π^2 (no C) | a^2, a^4, m_π^2 | a^2, m_π^2, m_π^4 |
|-------------|----------------------------------|-----------------------------------|-----------------------------------|------------------------------------|
| 2.0 | 0.56711(94) : 5.155 (0.0) | 0.5919(54) : 0.934 (0.393) | 0.5997(87) : 3.111 (0.014) | 0.5658(10) : 4.514 (0.001) |
| 2.2 | 0.55003(96) : 5.515 (0.0) | 0.5735(52) : 1.315 (0.268) | 0.5766(86) : 4.635 (0.001) | 0.54879(99) : 4.735 (0.001) |
| 2.3 | 0.54271(92) : 5.479 (0.0) | 0.5658(52) : 1.442 (0.236) | 0.5690(85) : 4.496 (0.001) | 0.54154(97) : 4.8 (0.001) |
| 2.4 | 0.53684(91) : 5.295 (0.0) | 0.5576(51) : 1.384 (0.251) | 0.5584(83) : 4.966 (0.001) | 0.53575(97) : 4.991 (0.001) |

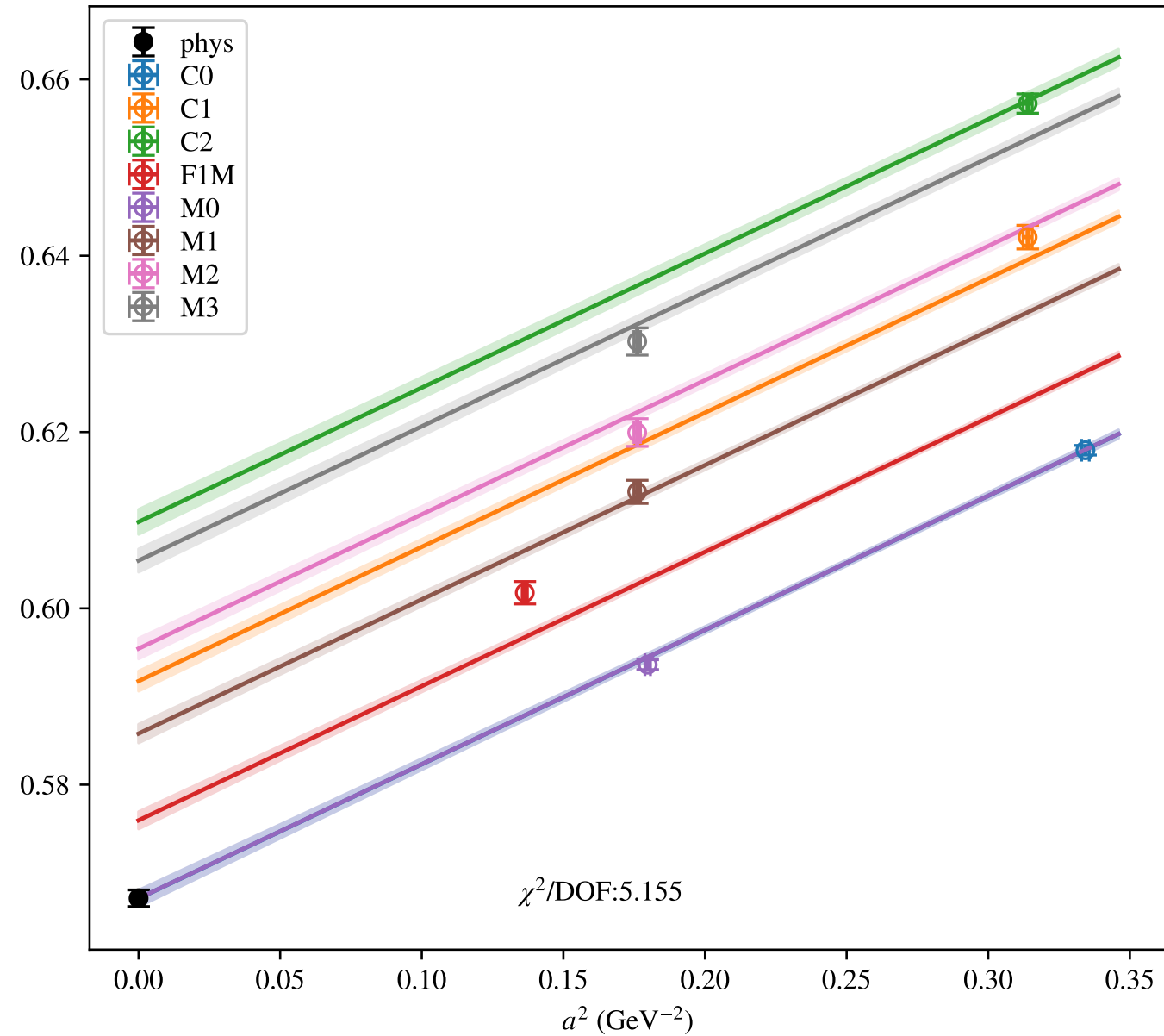
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| μ (GeV) | | a^2, m_π^2 | a^2, m_π^2 (no C) | a^2, a^4, m_π^2 | a^2, m_π^2, m_π^4 |
|-------------|----------|----------------|-----------------------|---------------------|-------------------------|
| 2.0 | α | 0.2684(65) | 0.016(53) | -0.2(12) | 0.2762(70) |
| | β | 0.00768(17) | 0.00709(26) | 0.00744(17) | 0.01003(71) |
| 2.2 | α | 0.3012(70) | 0.055(53) | -0.1(13) | 0.3090(73) |
| | β | 0.00738(15) | 0.00670(24) | 0.00719(15) | 0.00980(74) |
| 2.3 | α | 0.3187(71) | 0.074(53) | -0.1(13) | 0.3262(74) |
| | β | 0.00734(15) | 0.00667(24) | 0.00715(15) | 0.00964(73) |
| 2.4 | α | 0.3313(71) | 0.108(54) | -0.026 | 0.3384(75) |
| | β | 0.00734(14) | 0.00665(22) | 0.00718(14) | 0.00932(70) |

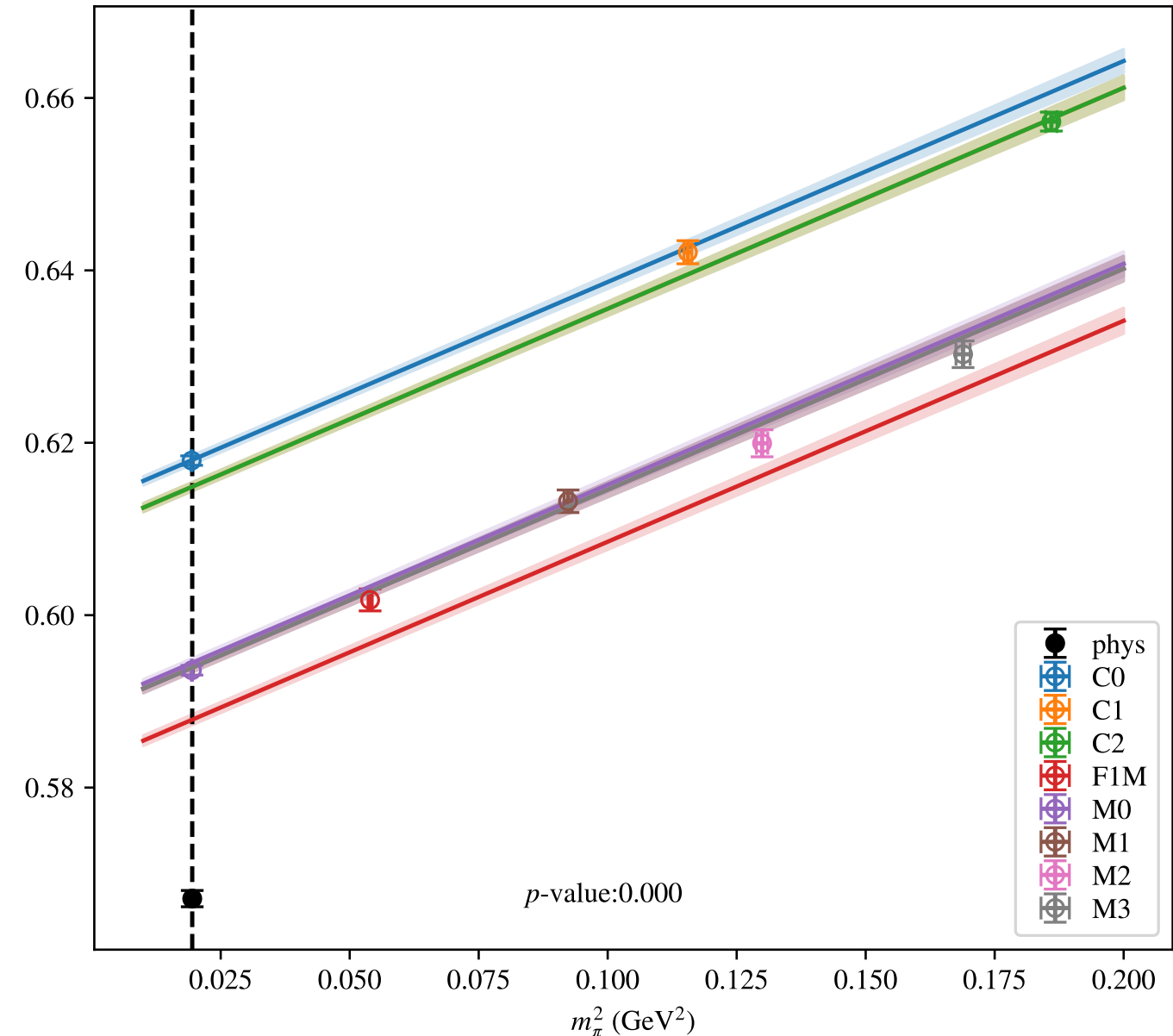
Table 4: Fit values of coefficients in $B = B_{phys} + \alpha a^2 + \beta \left(\frac{m_\pi^2}{f_\pi^2} - \frac{m_{\pi,PDG}^2}{f_\pi^2} \right) + \dots$

$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

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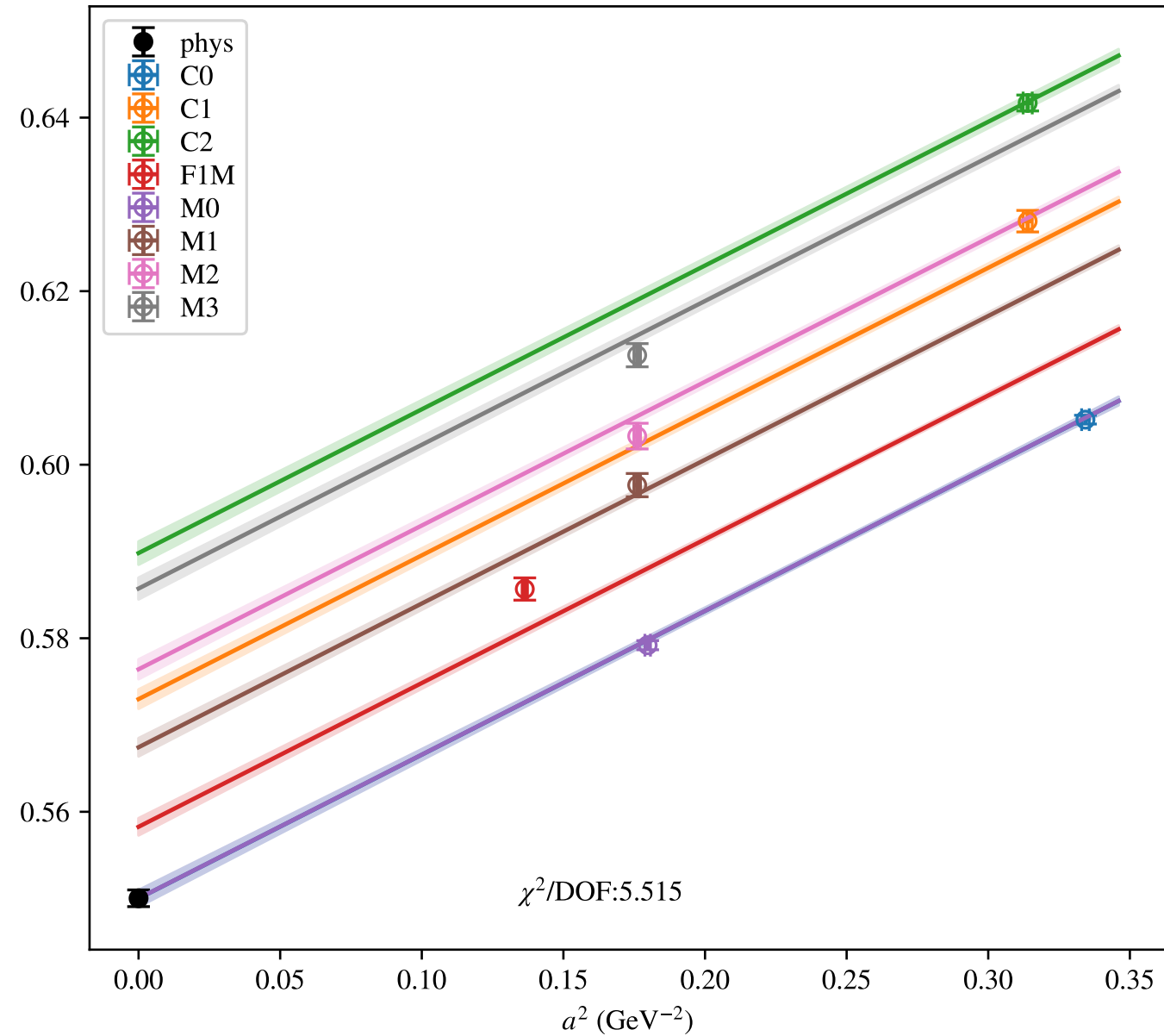


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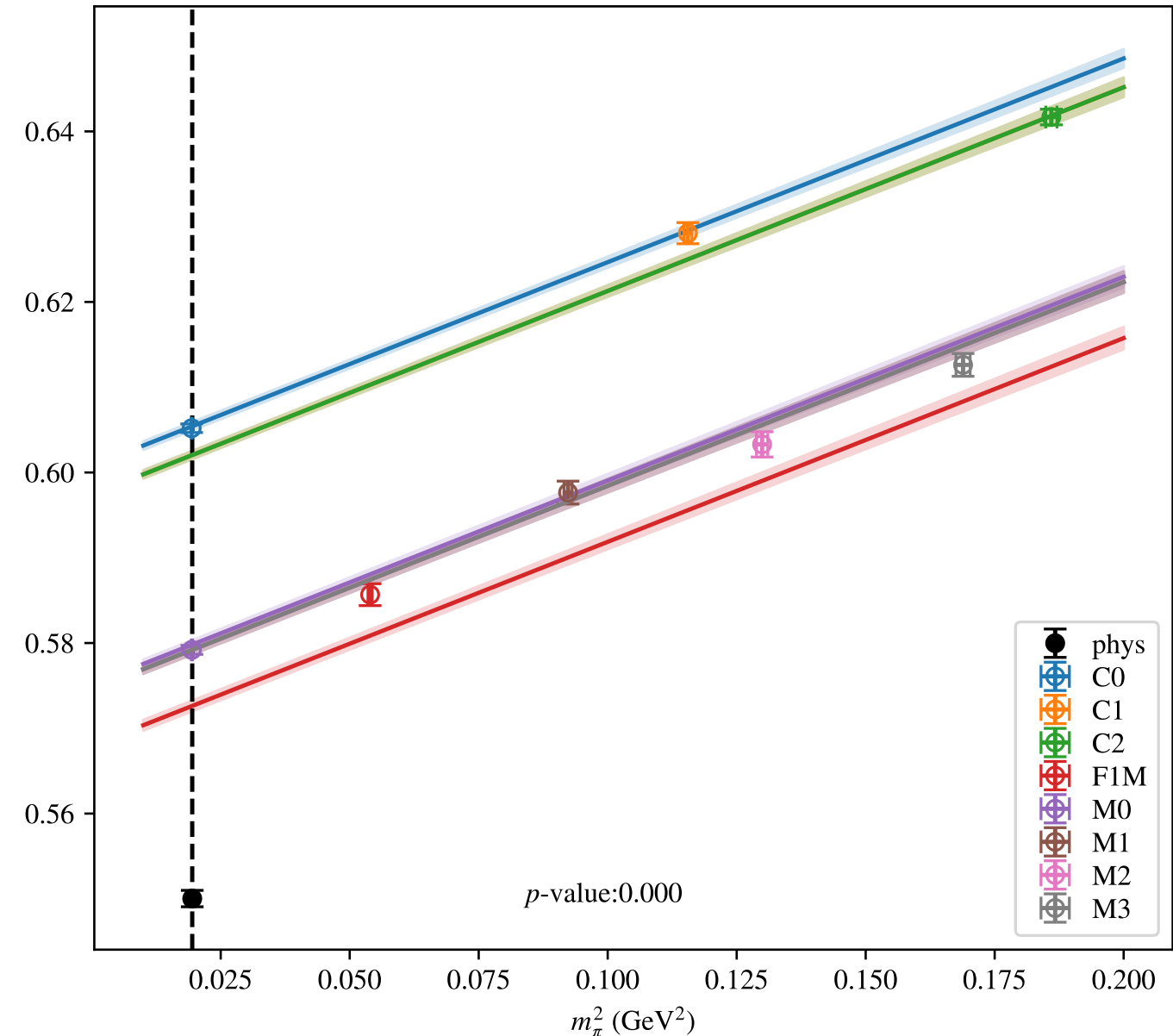


$$a^2, m_\pi^2, \mu = 2.2 \text{ GeV}$$

SSpPP

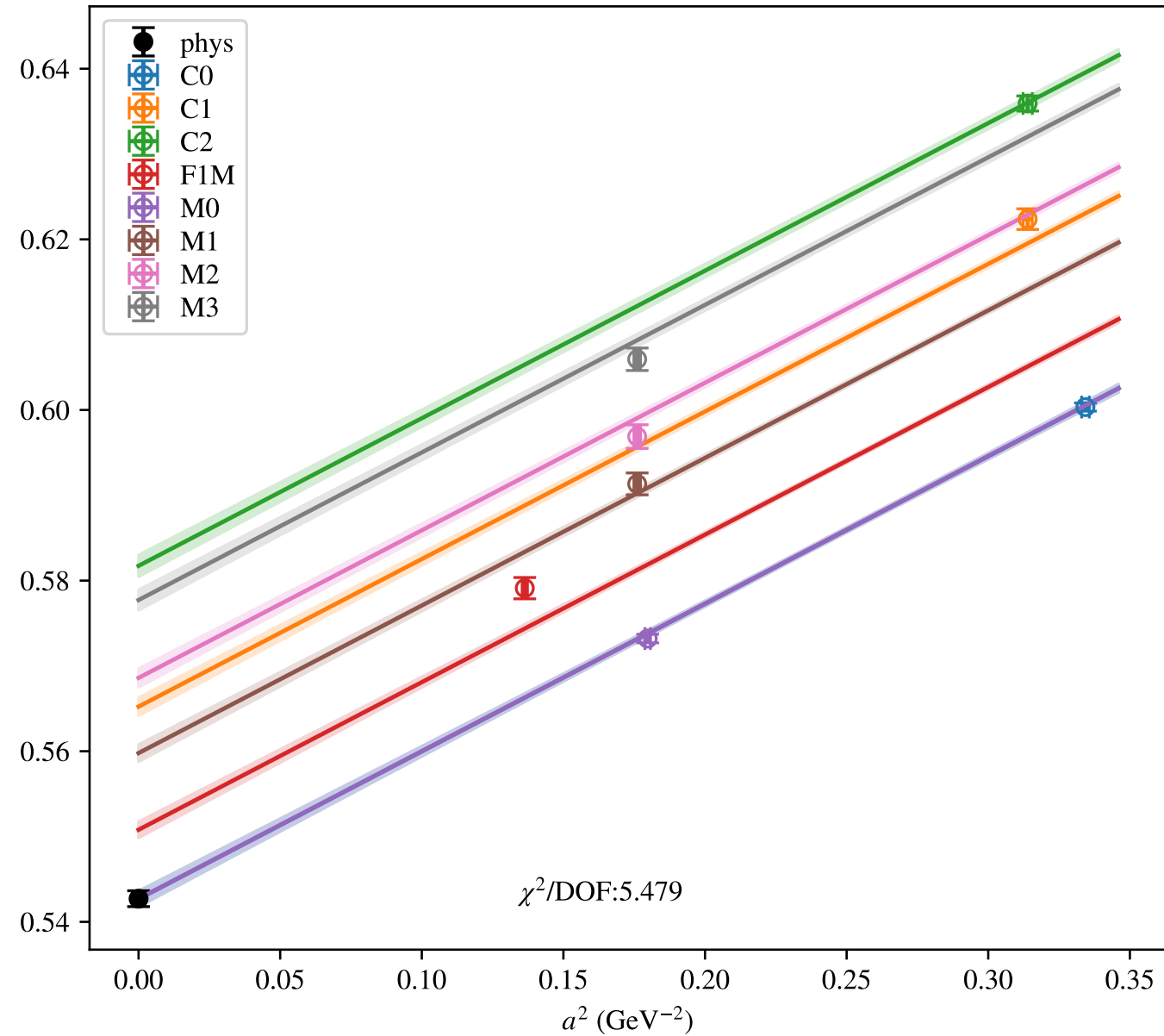


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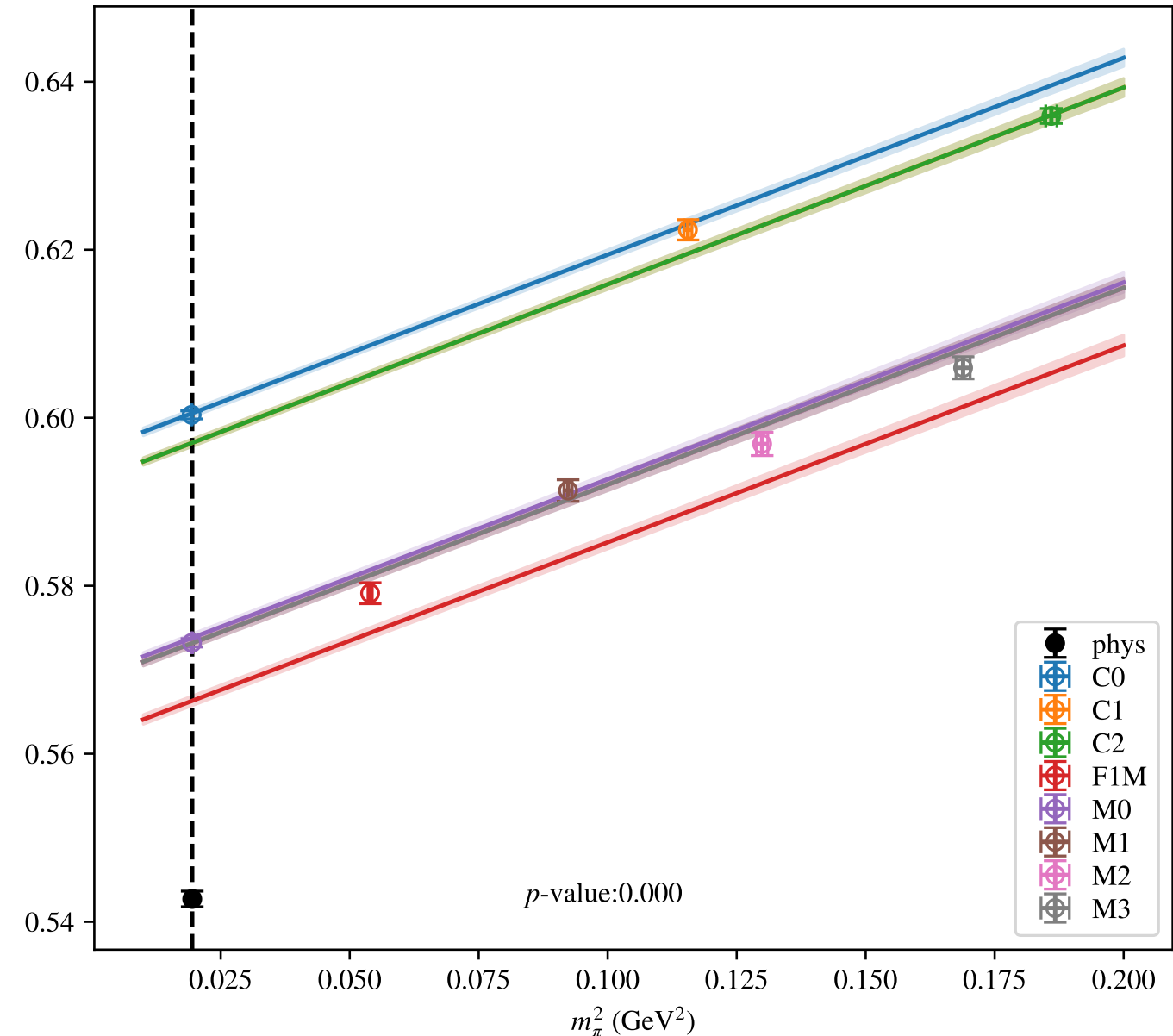


$$a^2, m_\pi^2, \mu = 2.3 \text{ GeV}$$

SSpPP

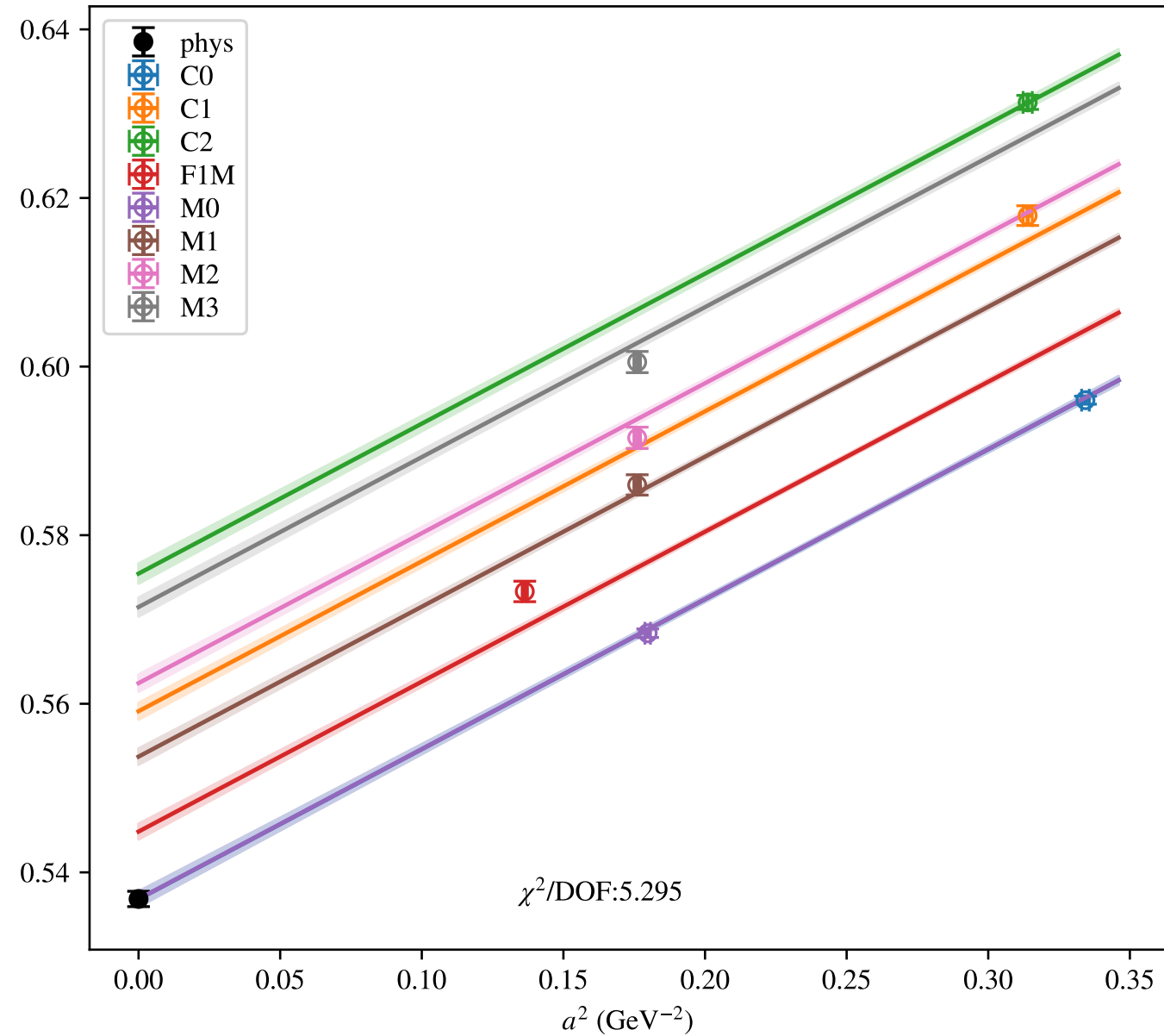


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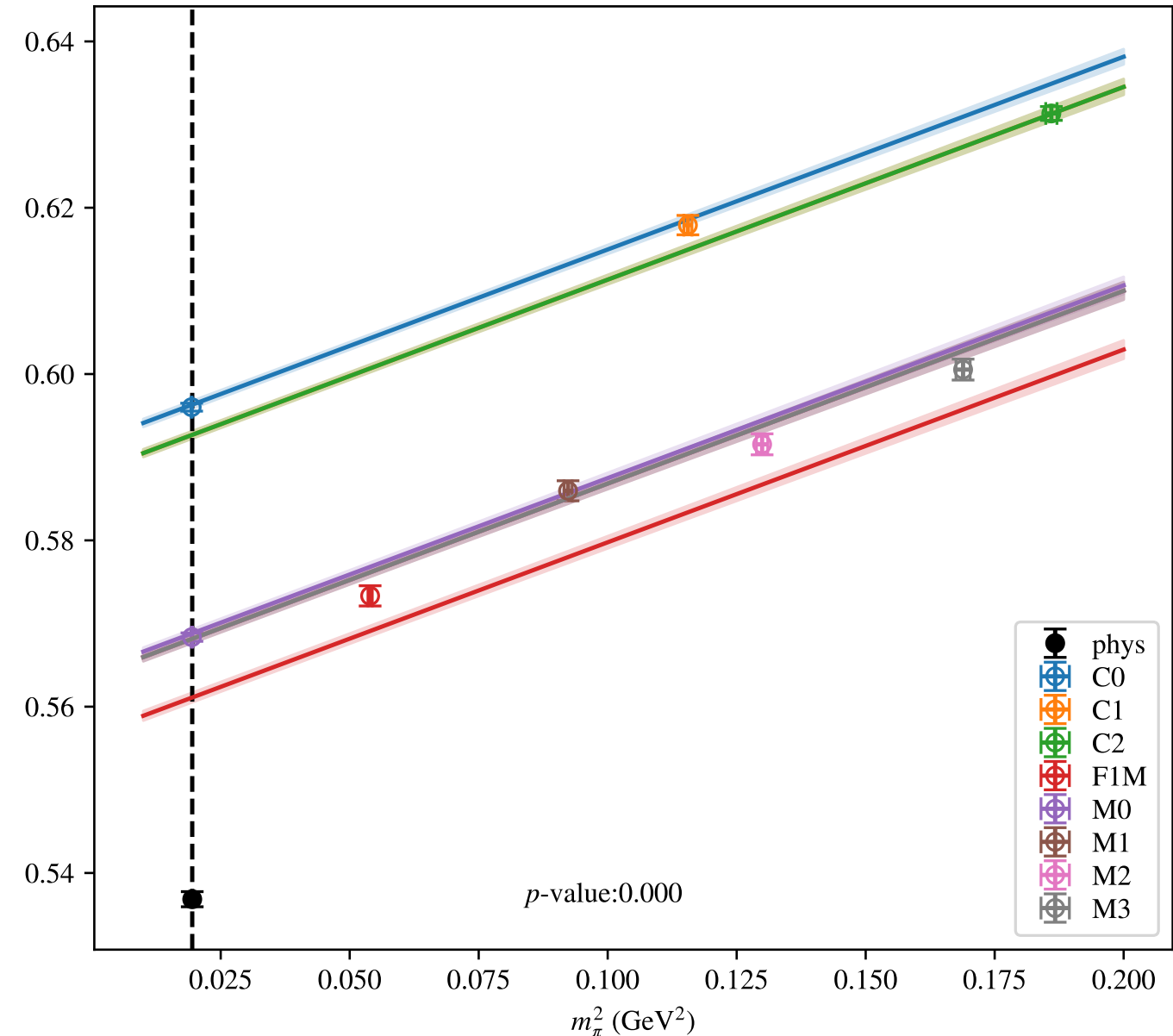


$$a^2, m_\pi^2, \mu = 2.4 \text{ GeV}$$

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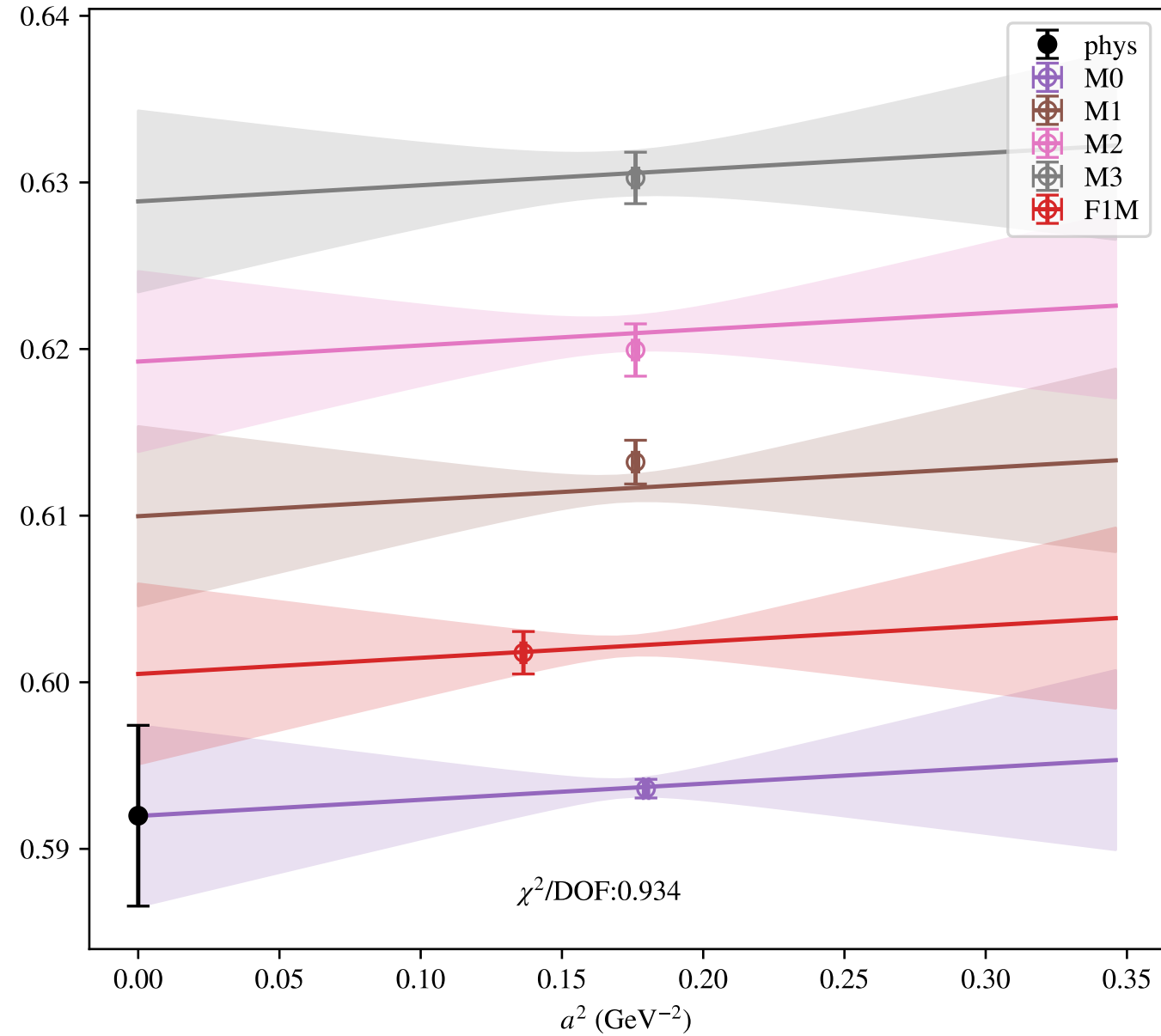


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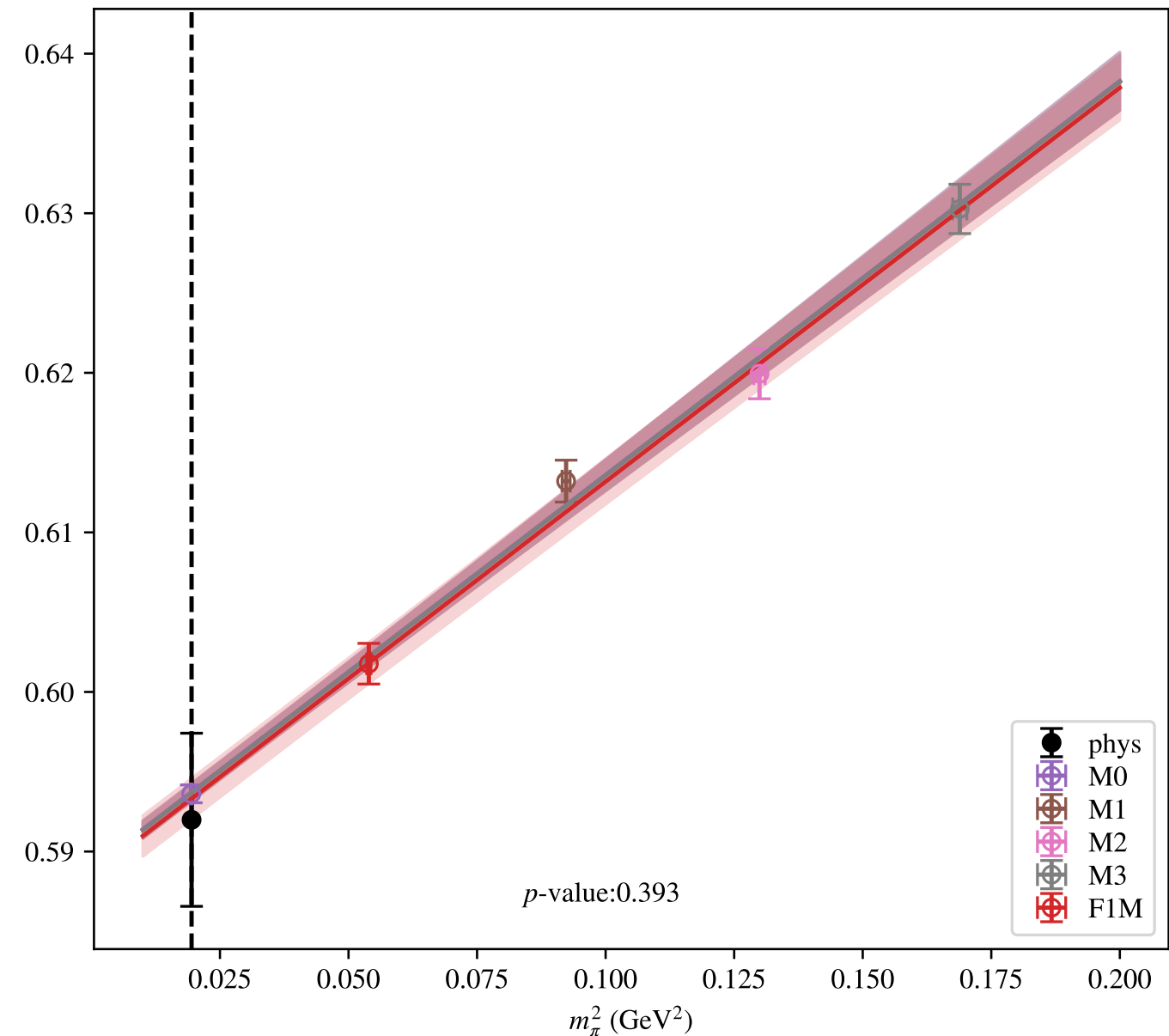


a^2, m_π^2 (no C), $\mu = 2.0$ GeV

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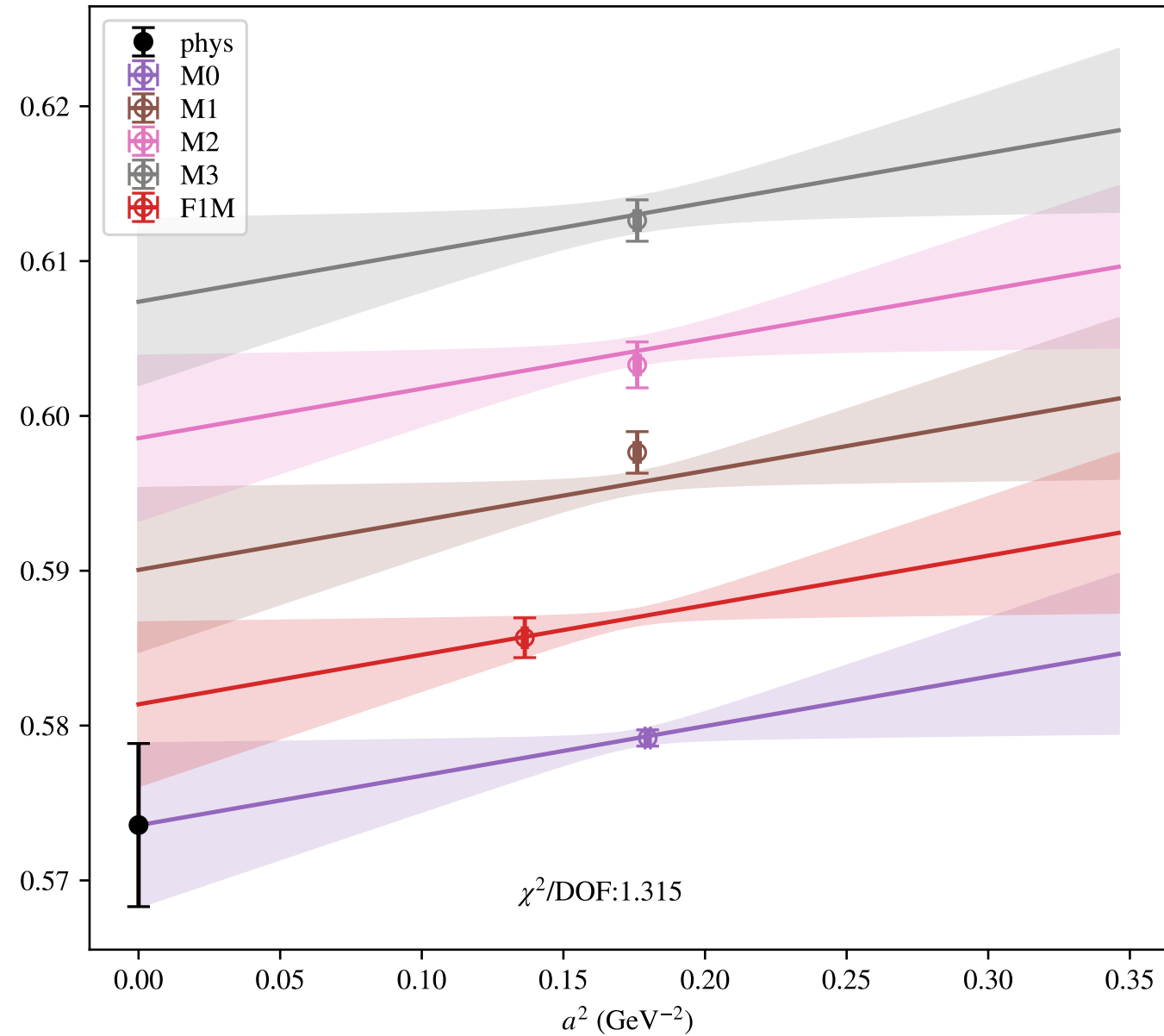


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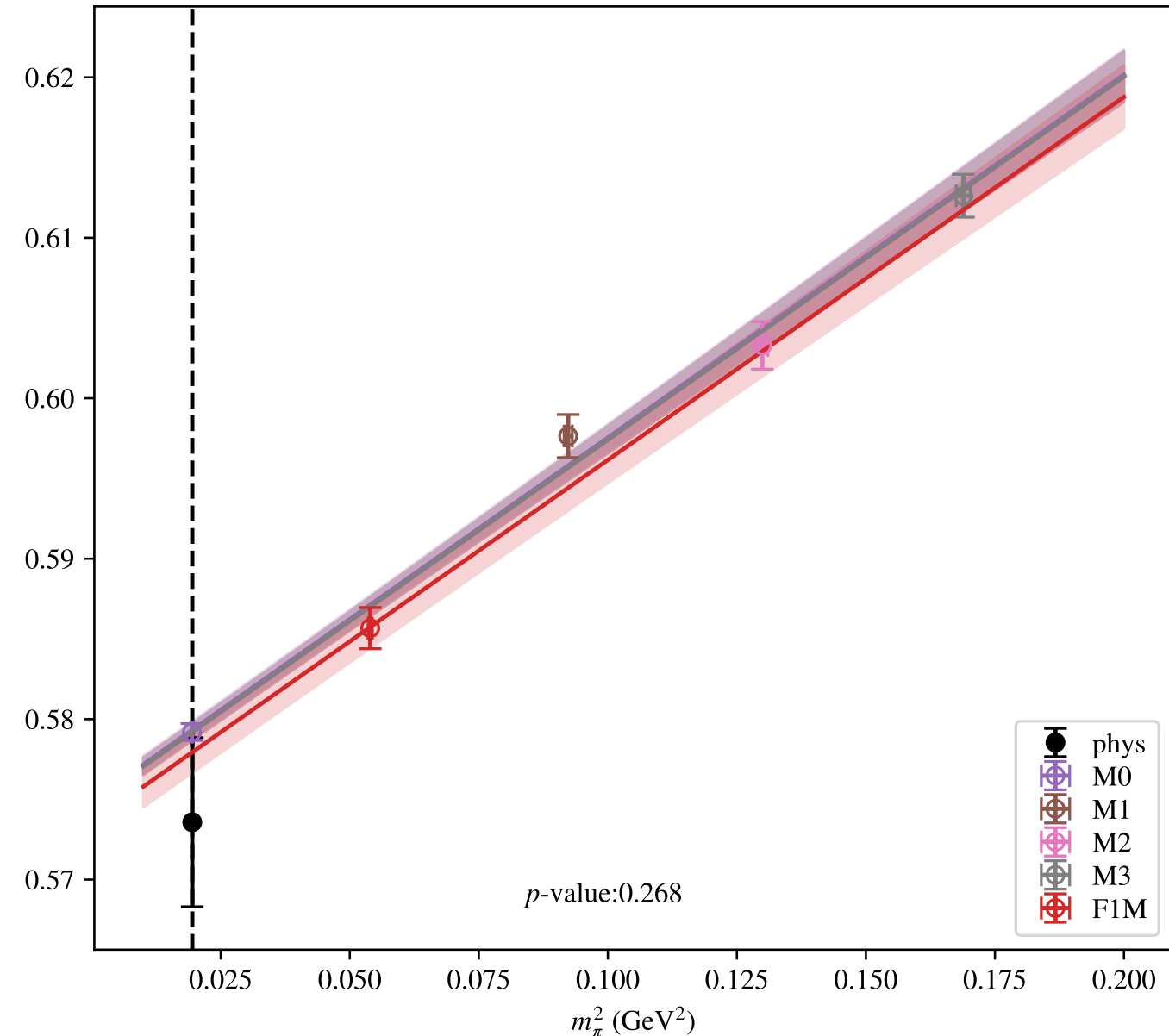


a^2, m_π^2 (no C), $\mu = 2.2$ GeV

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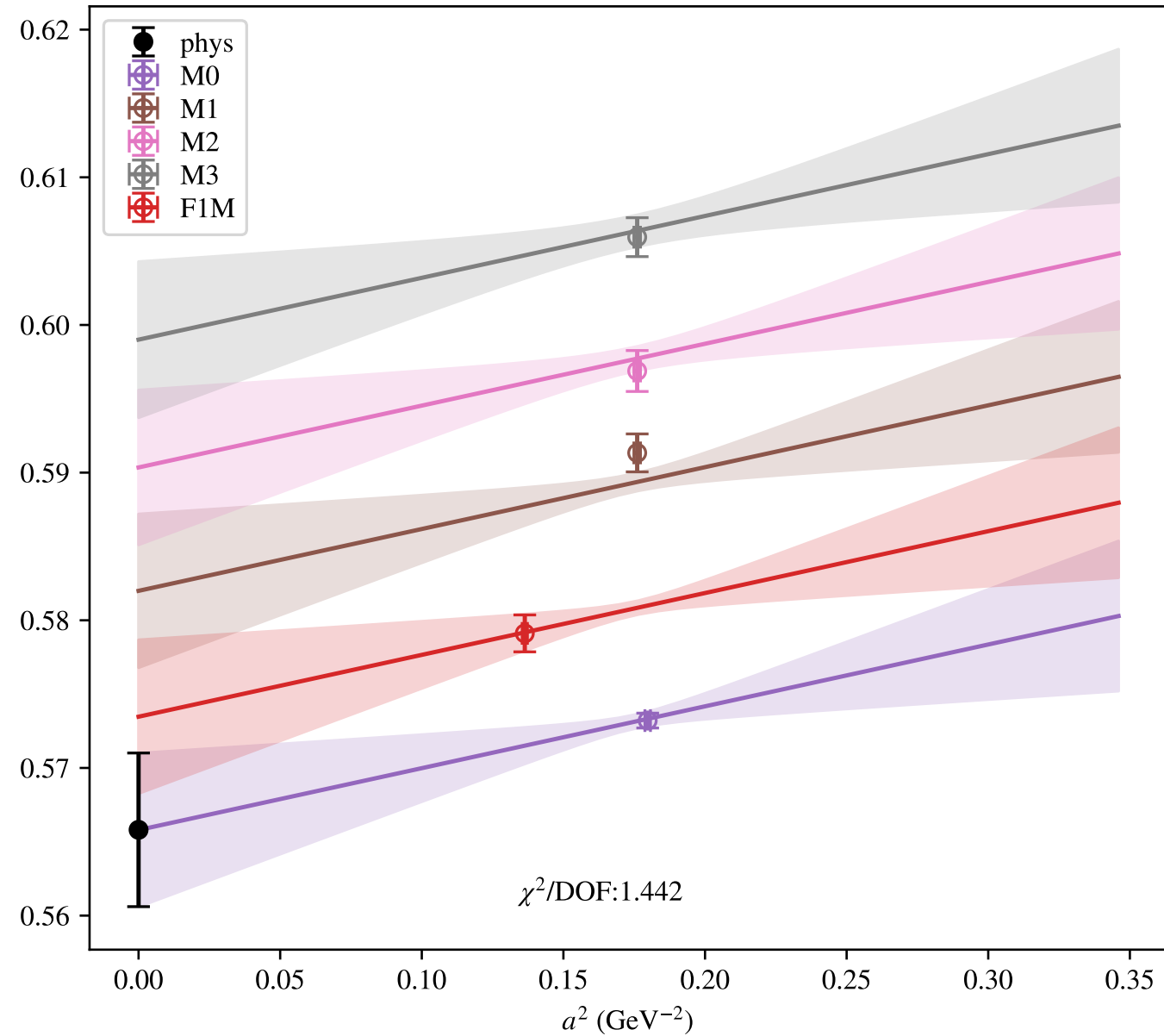


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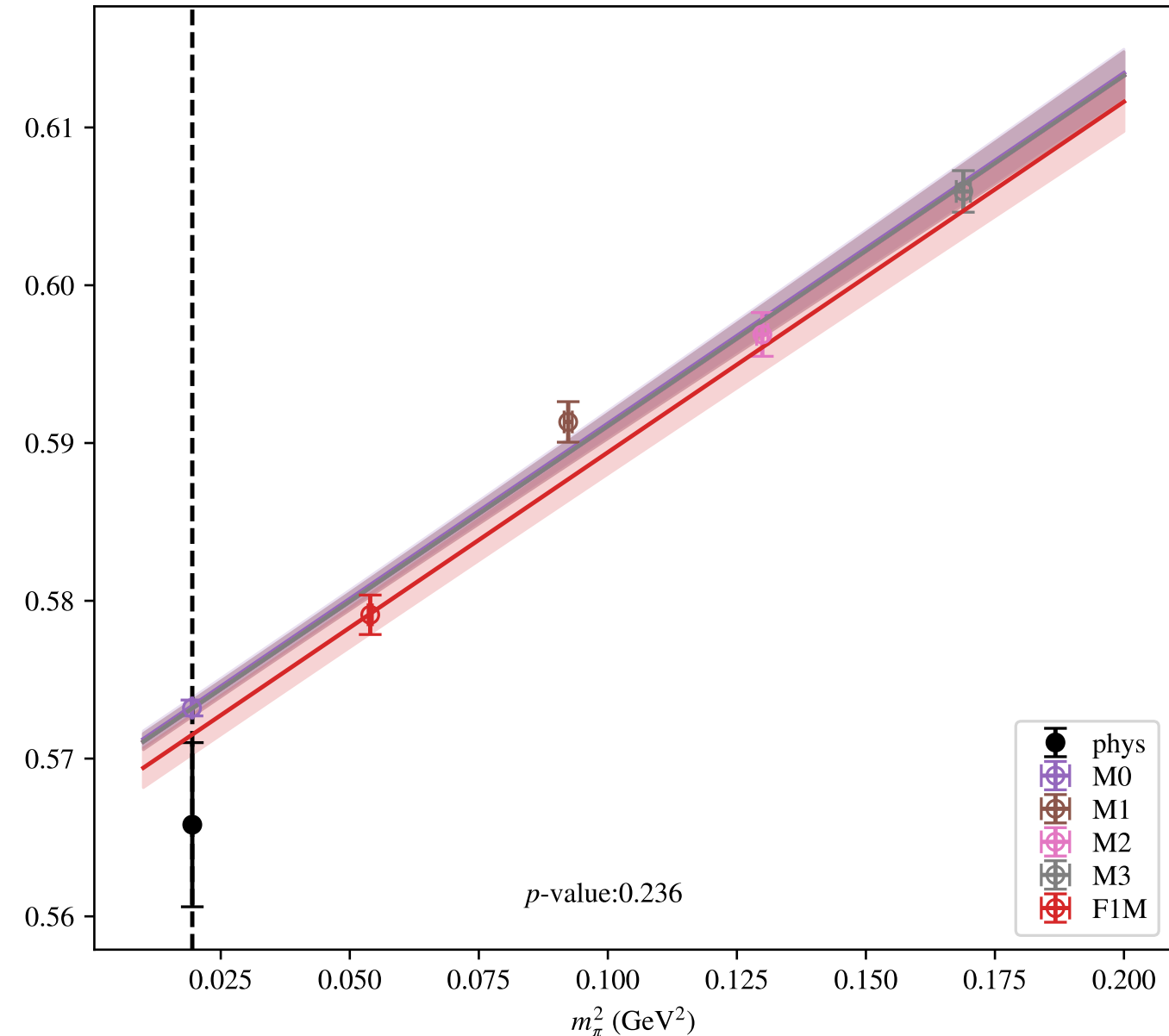


a^2, m_π^2 (no C), $\mu = 2.3$ GeV

SSpPP

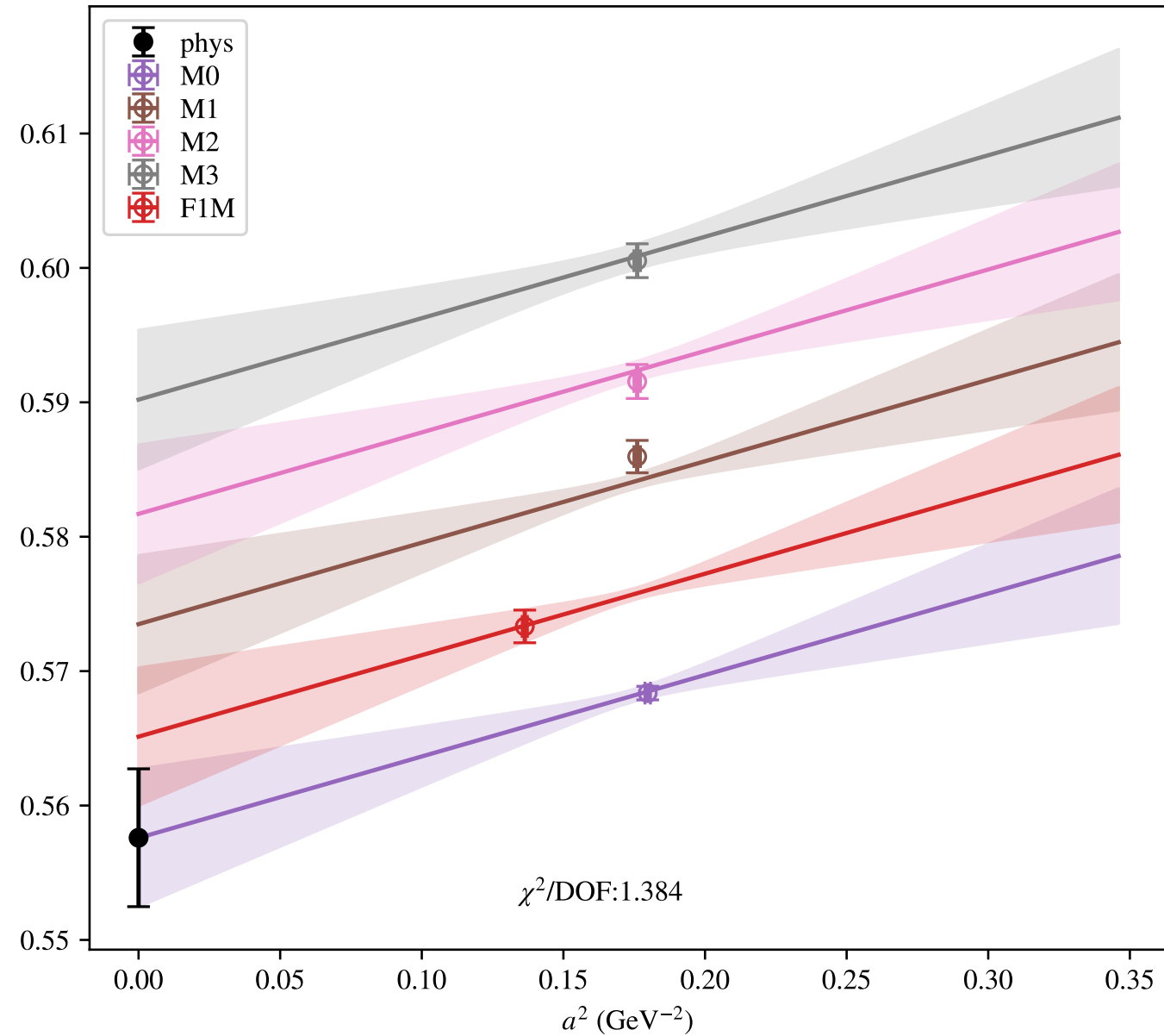


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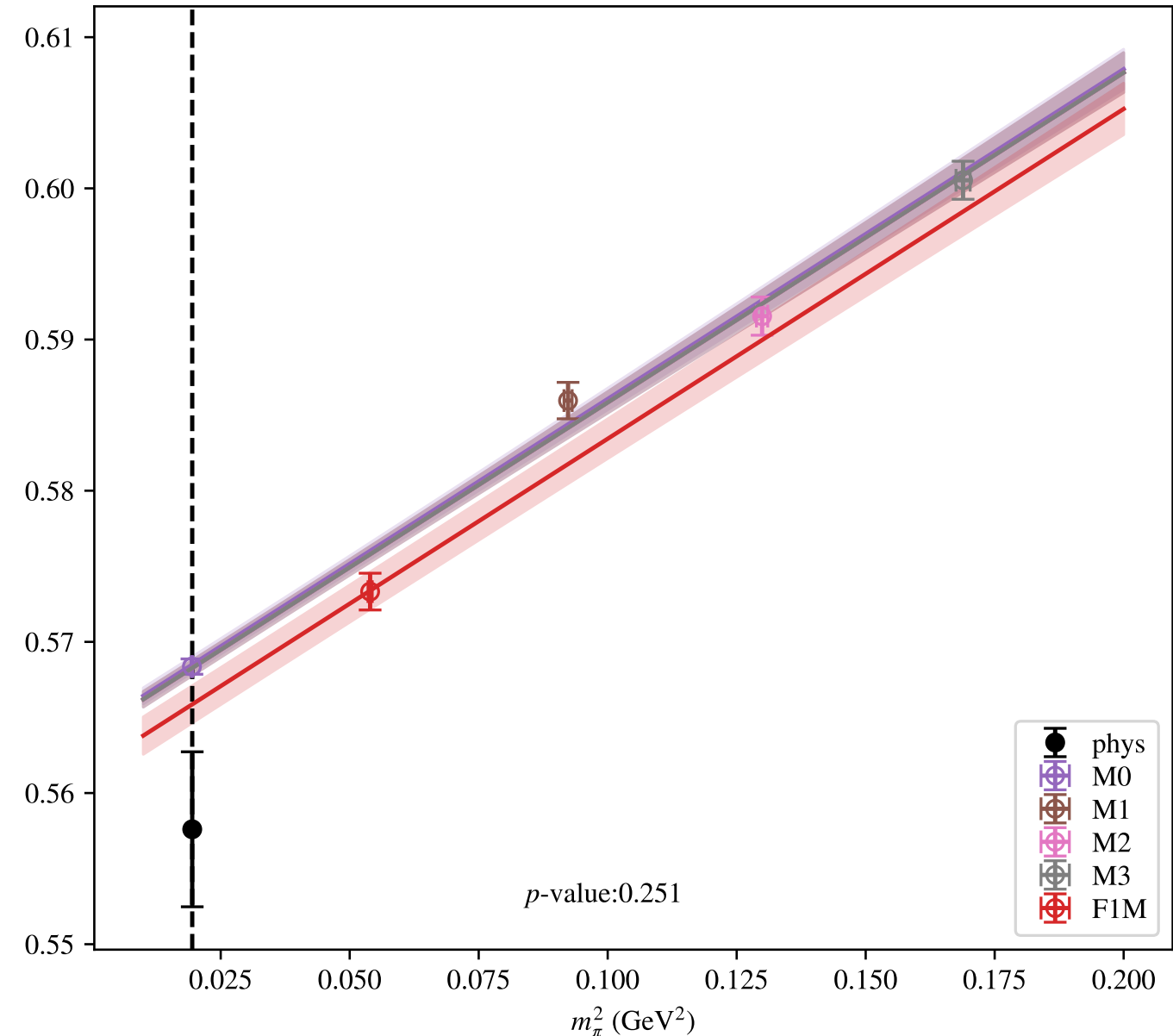


a^2, m_π^2 (no C), $\mu = 2.4$ GeV

SSpPP

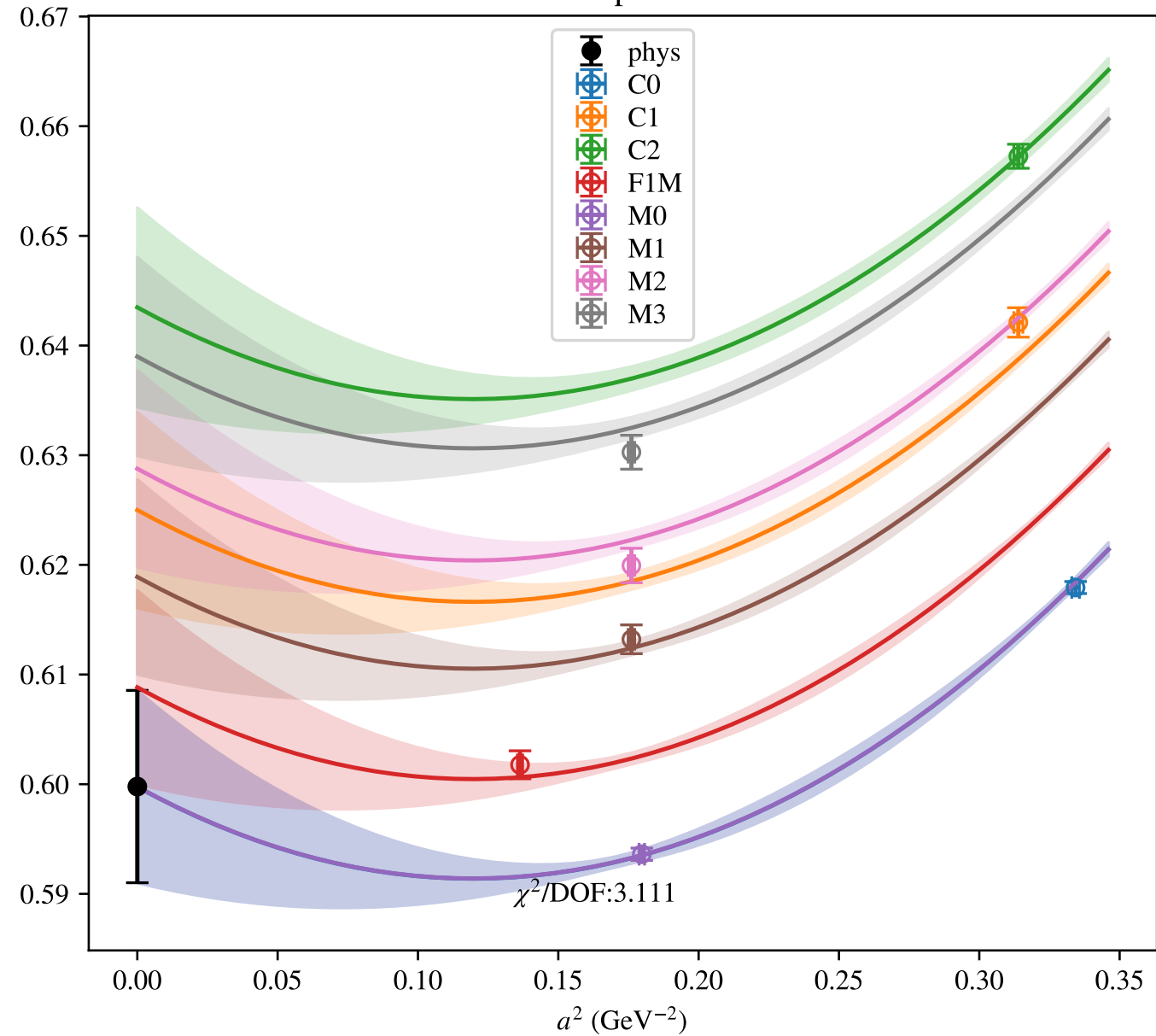


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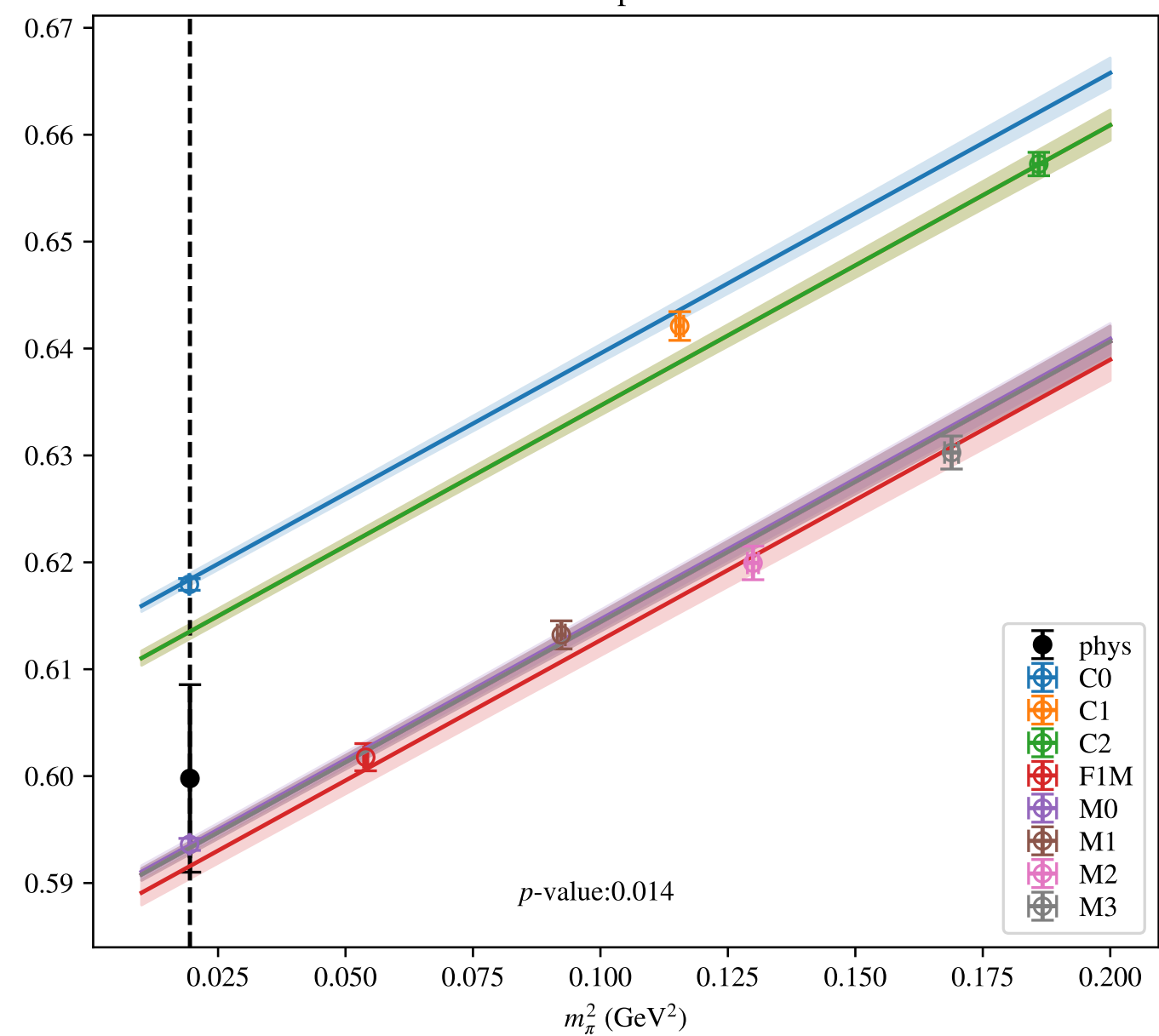


$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$

SSpPP

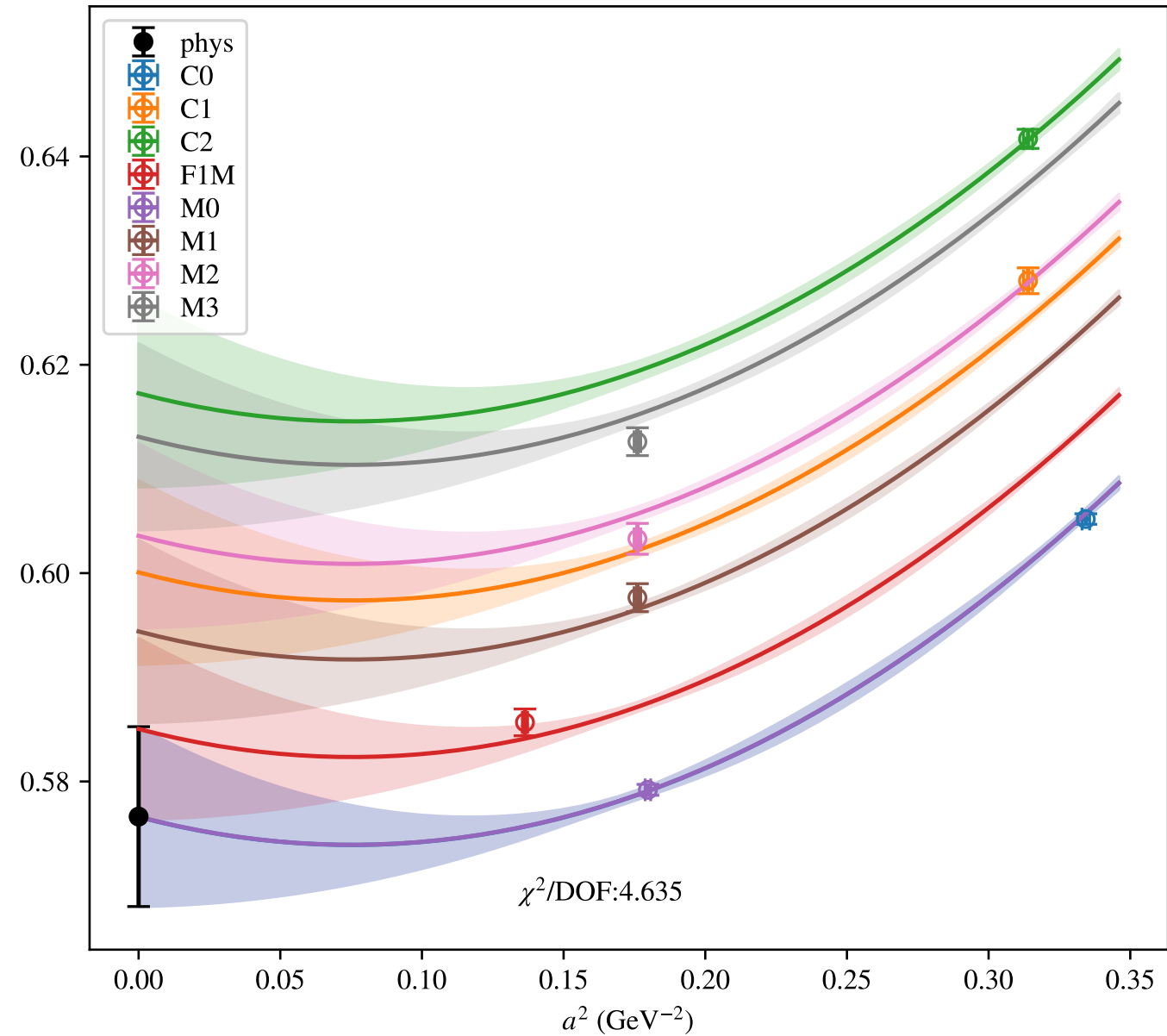


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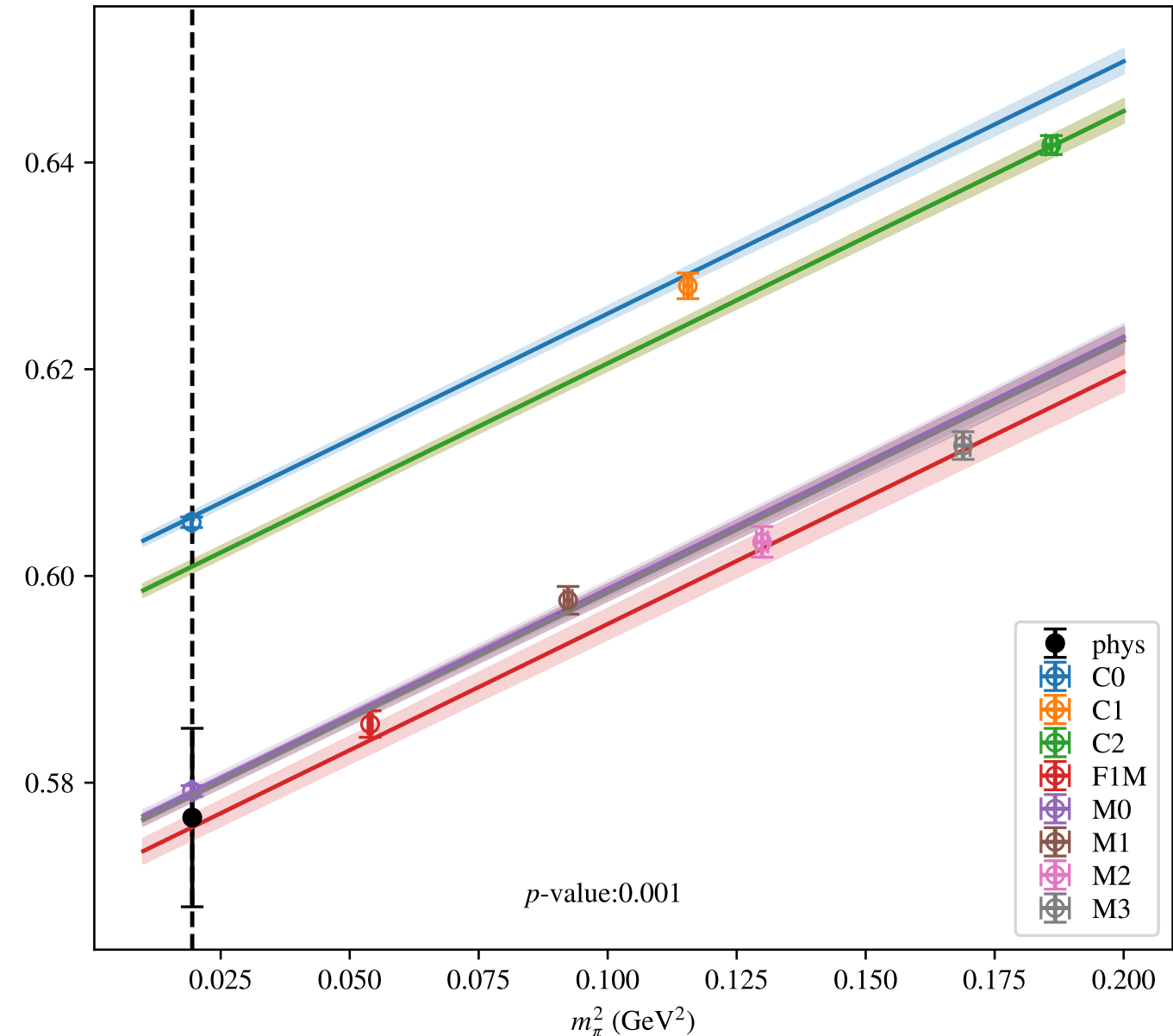


$$a^2, a^4, m_\pi^2, \mu = 2.2 \text{ GeV}$$

SSpPP

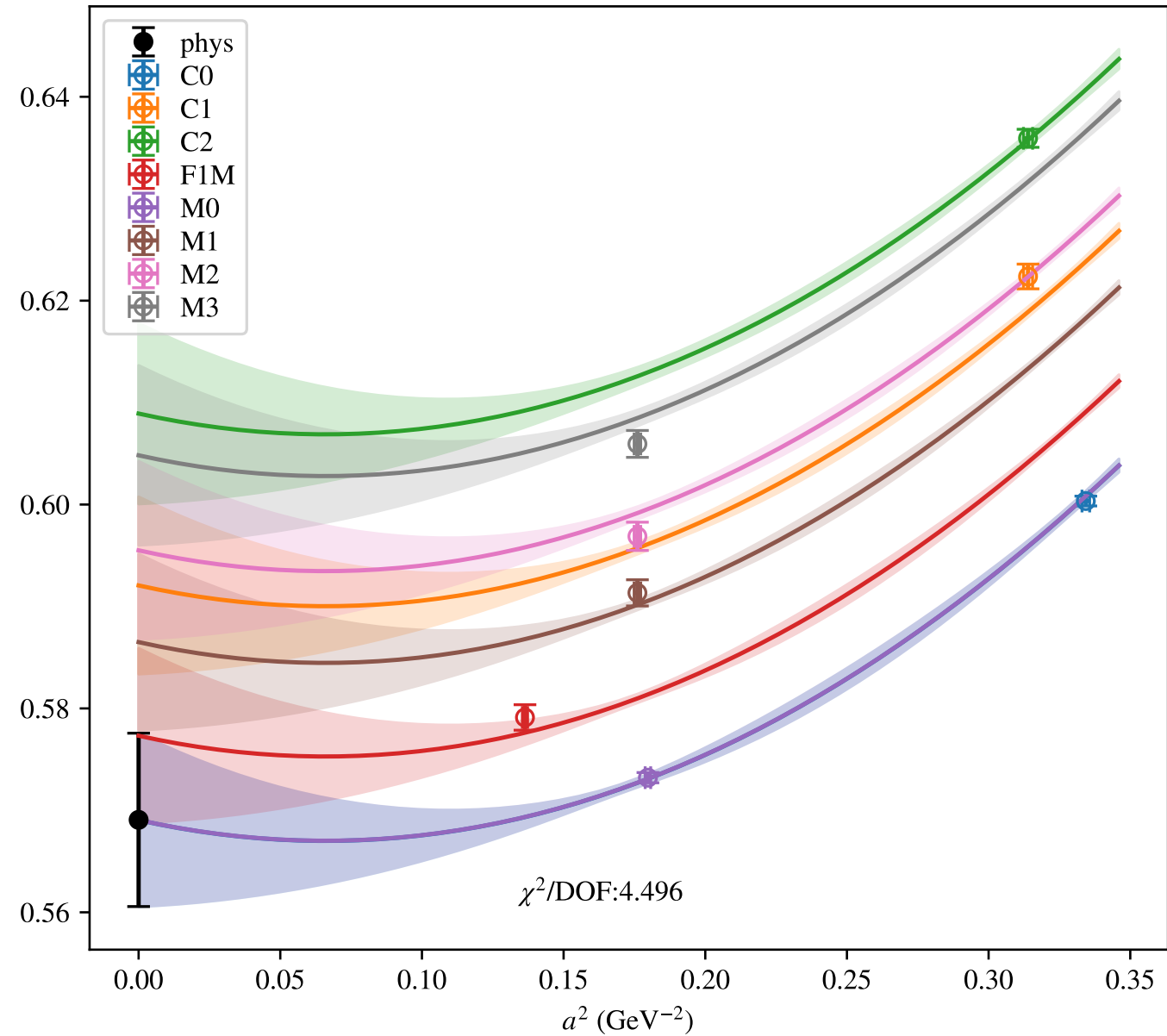


SSpPP

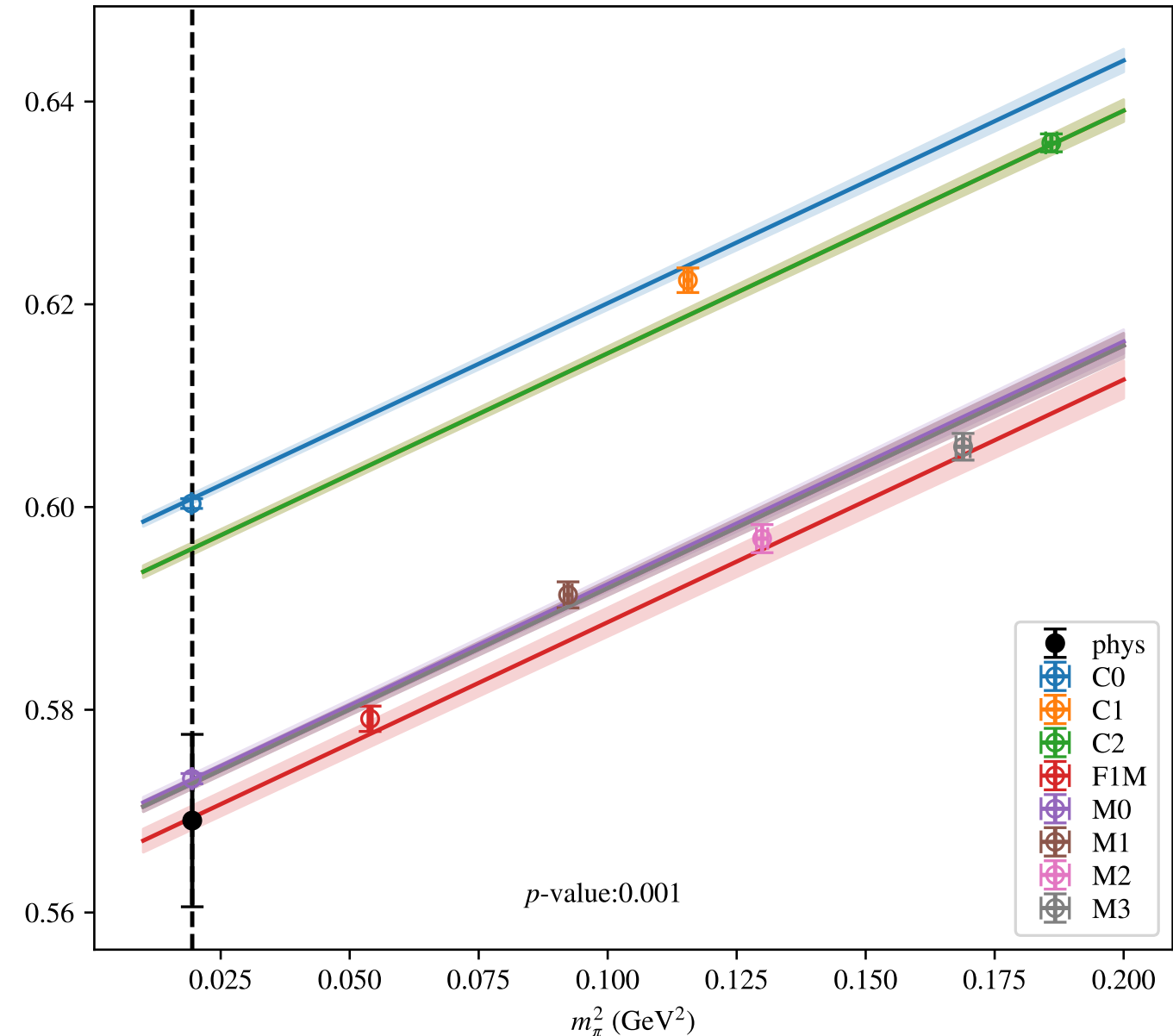


$$a^2, a^4, m_\pi^2, \mu = 2.3 \text{ GeV}$$

SSpPP

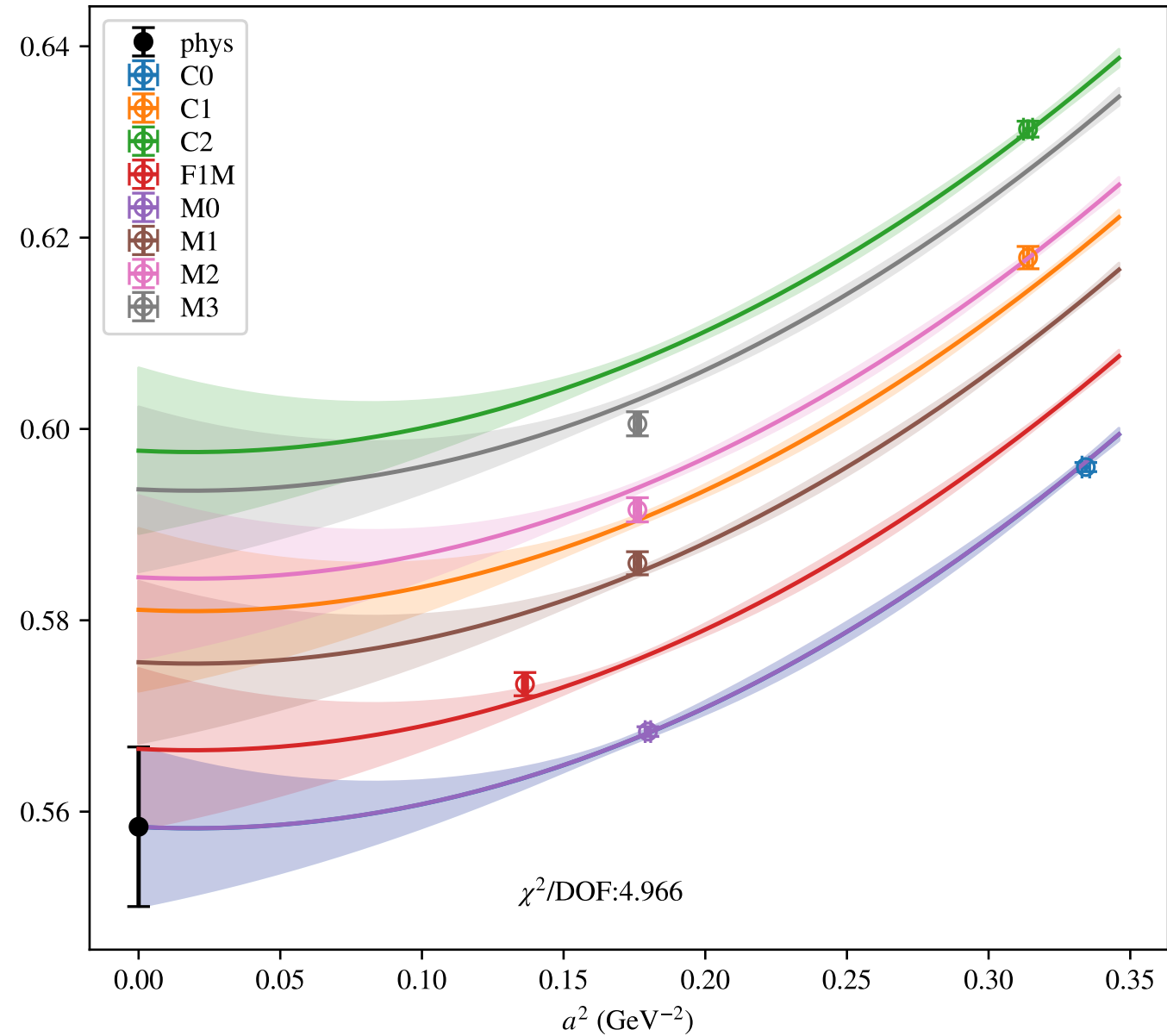


SSpPP

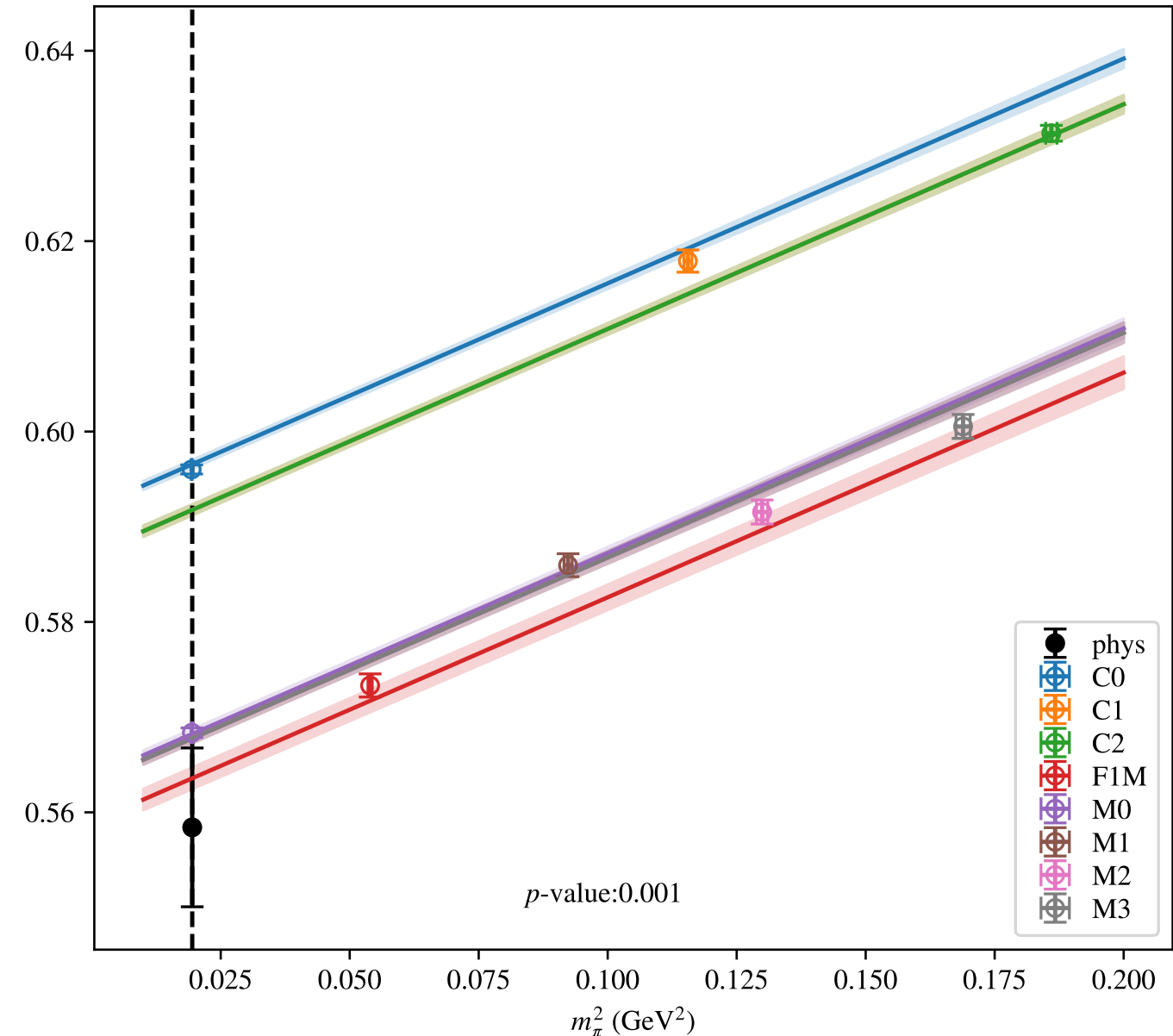


$$a^2, a^4, m_\pi^2, \mu = 2.4 \text{ GeV}$$

SSpPP

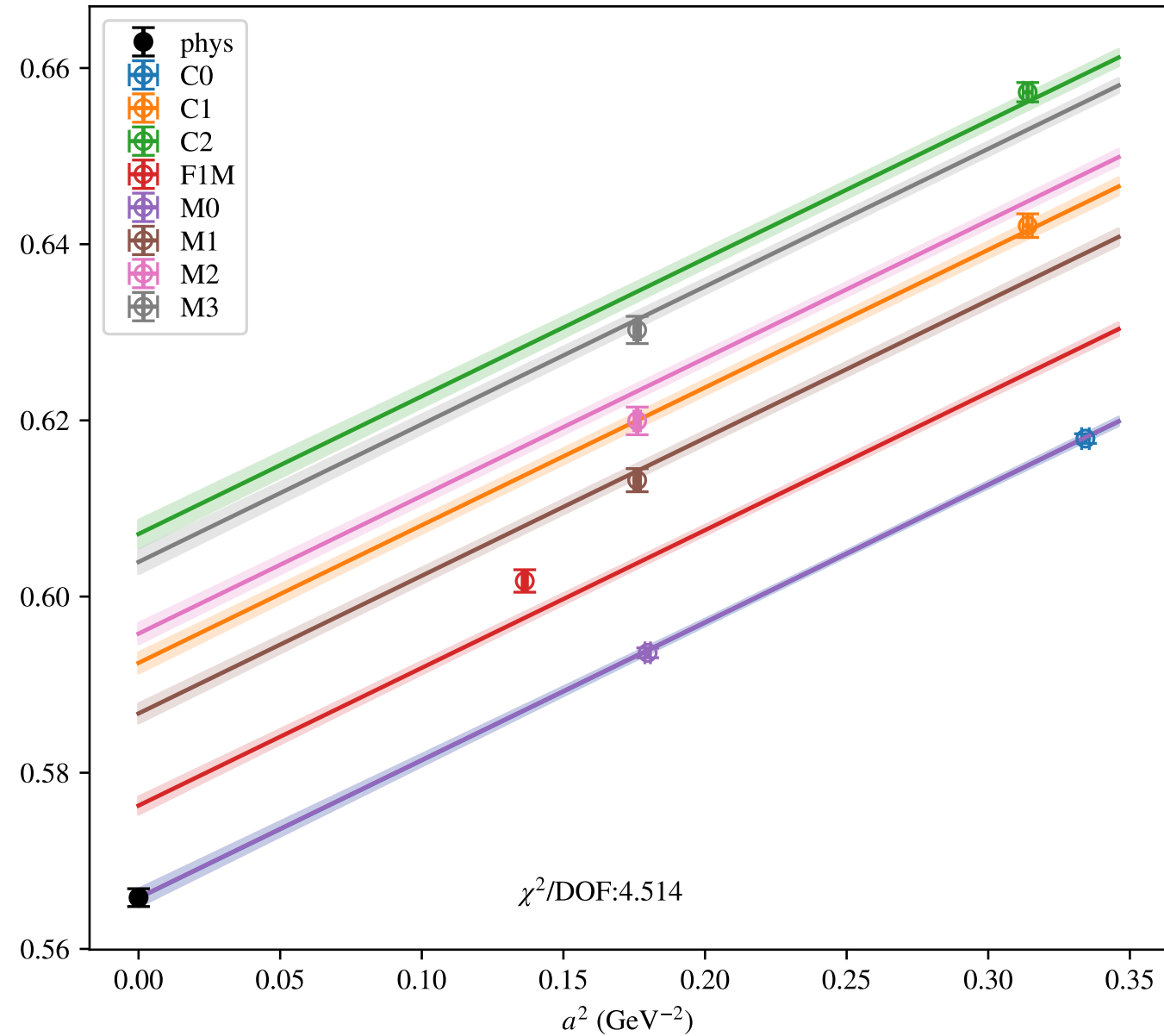


SSpPP

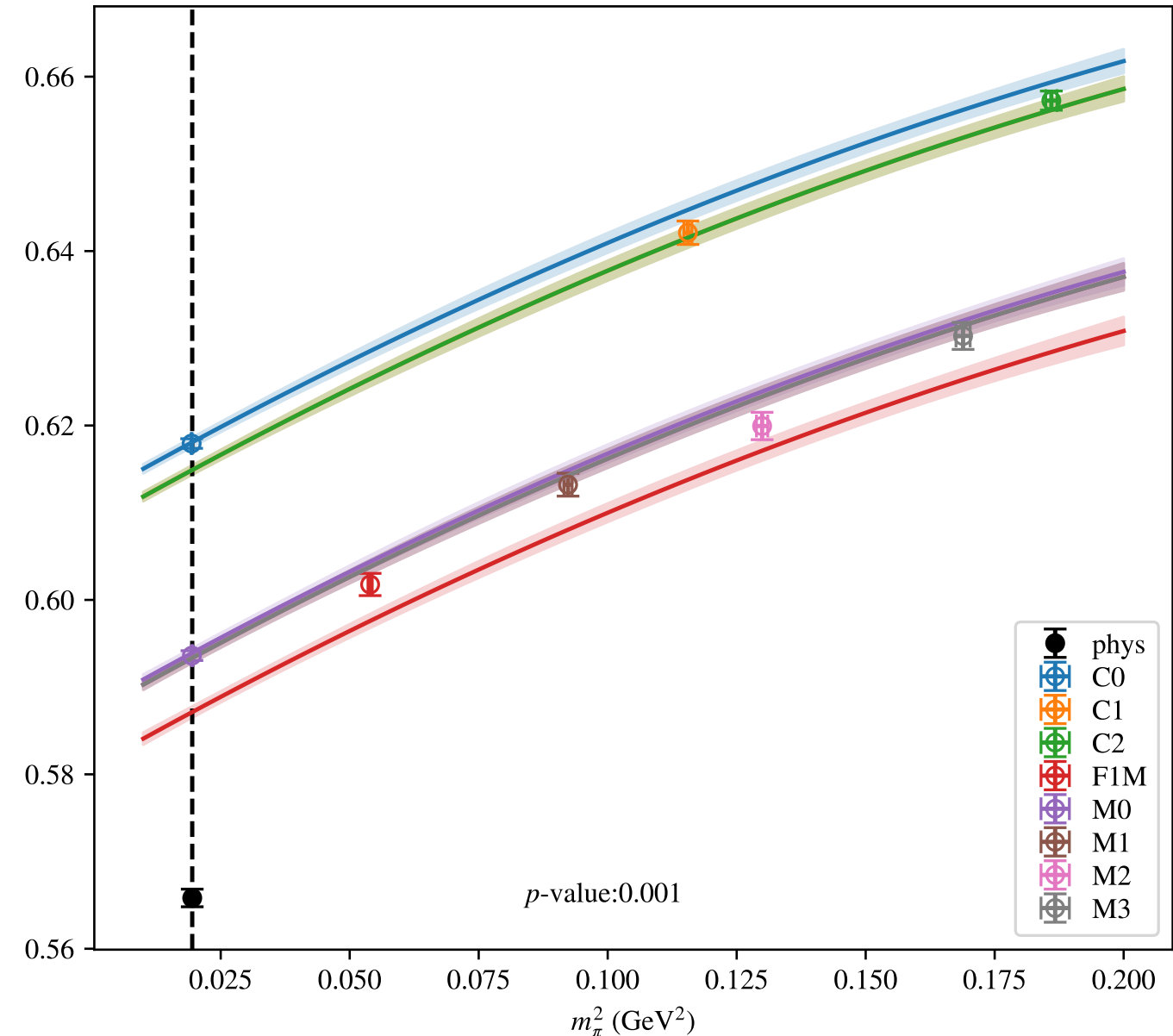


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

SSpPP

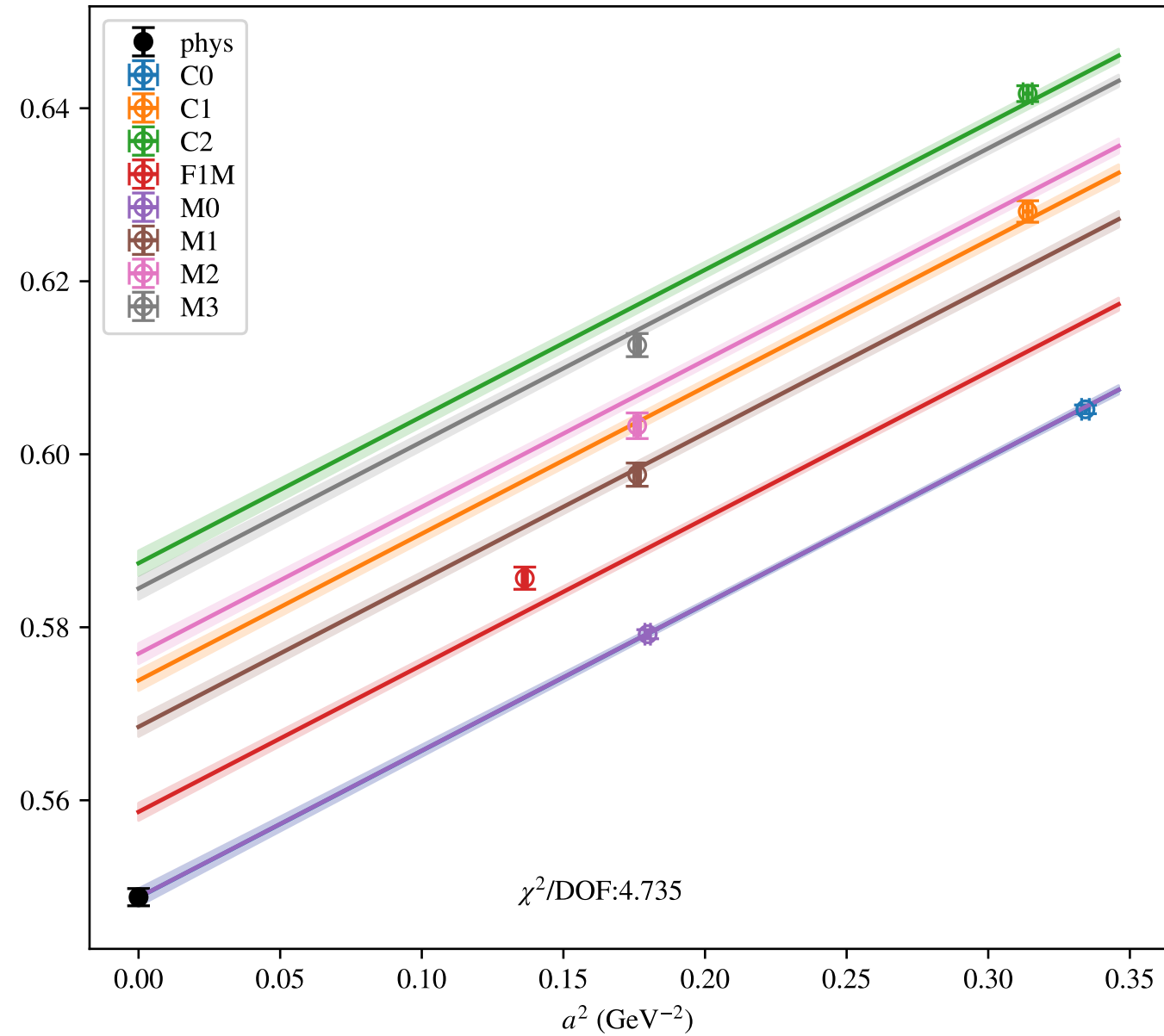


SSpPP

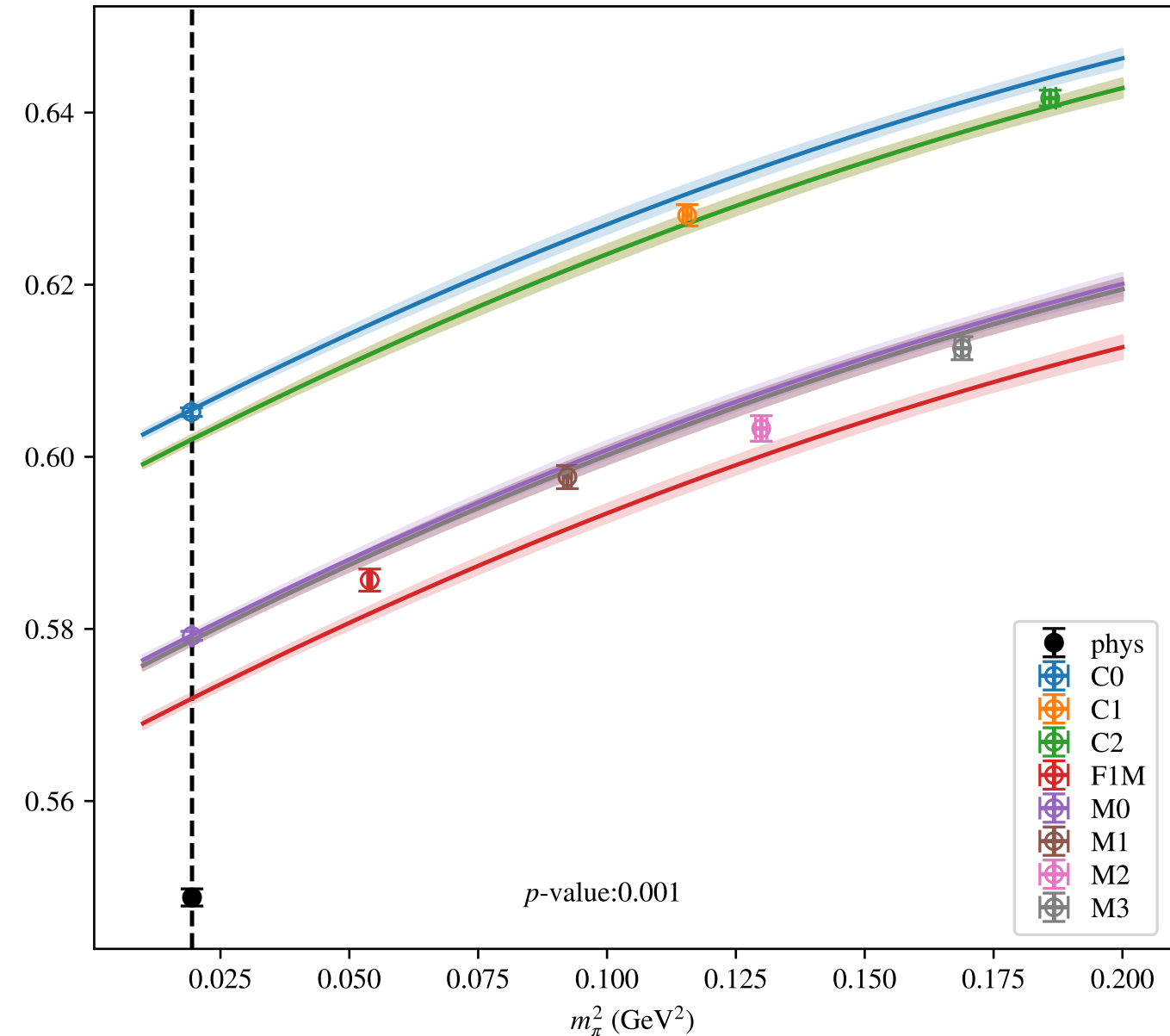


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.2 \text{ GeV}$$

SSpPP

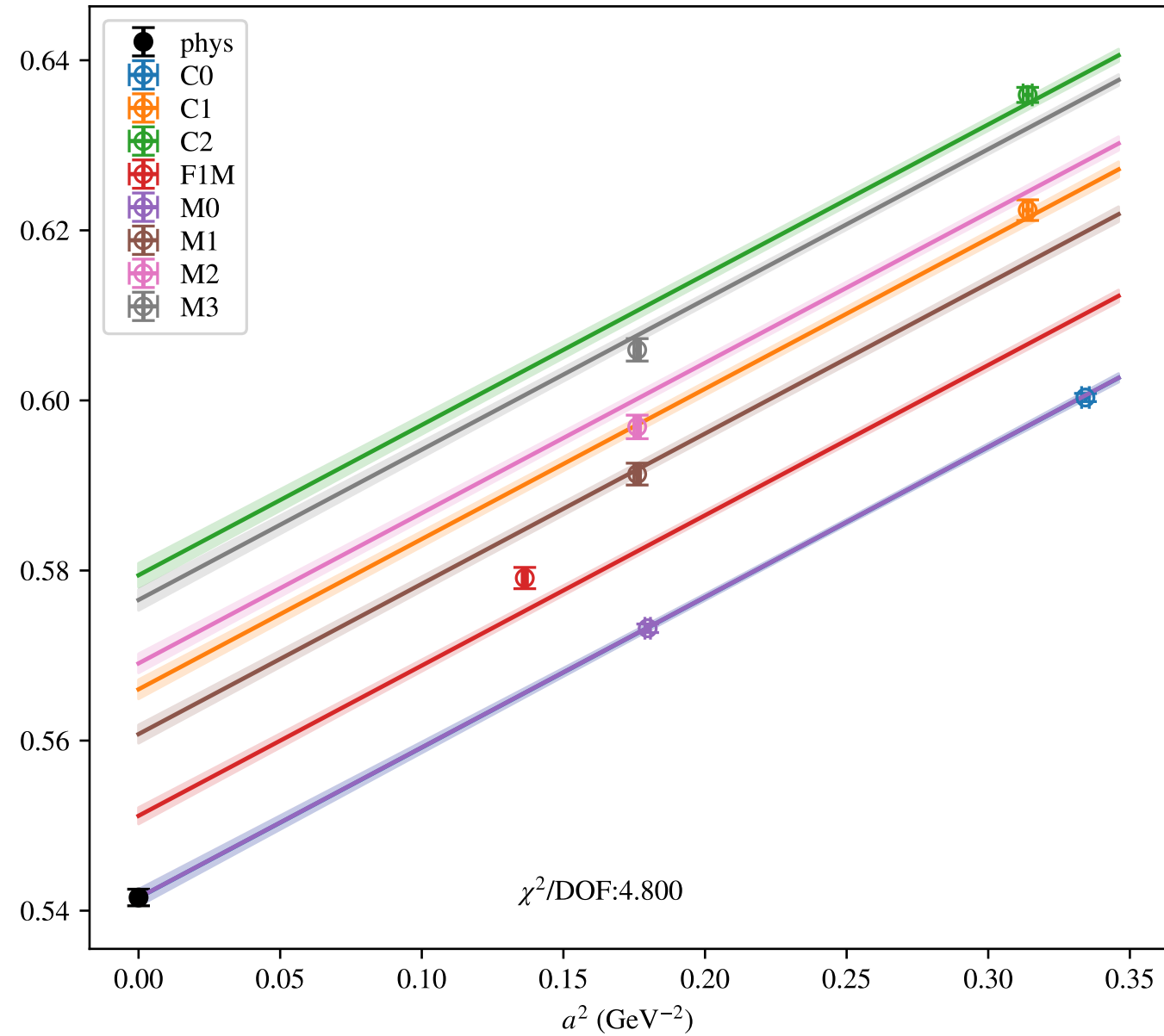


SSpPP

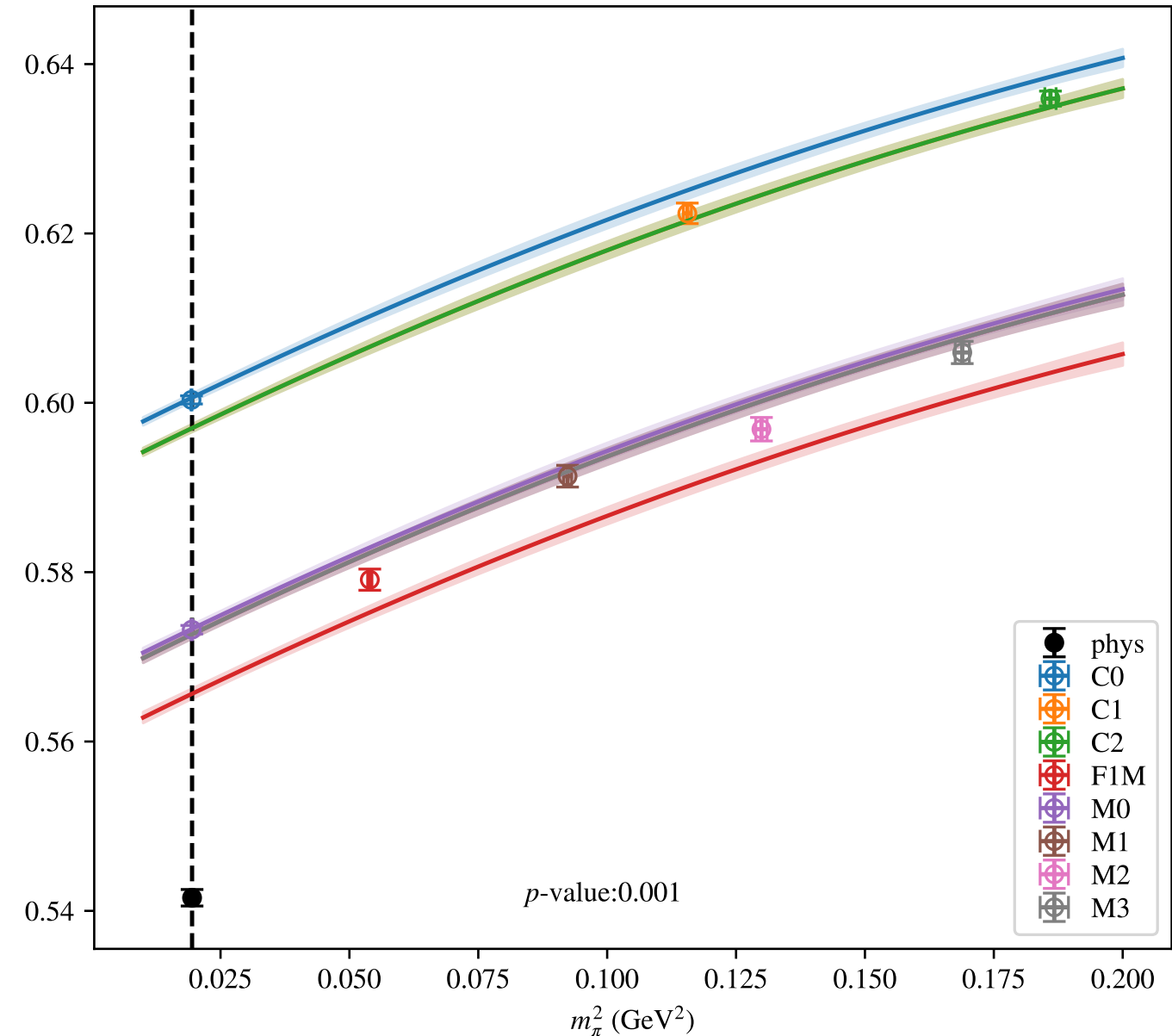


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.3 \text{ GeV}$$

SSpPP

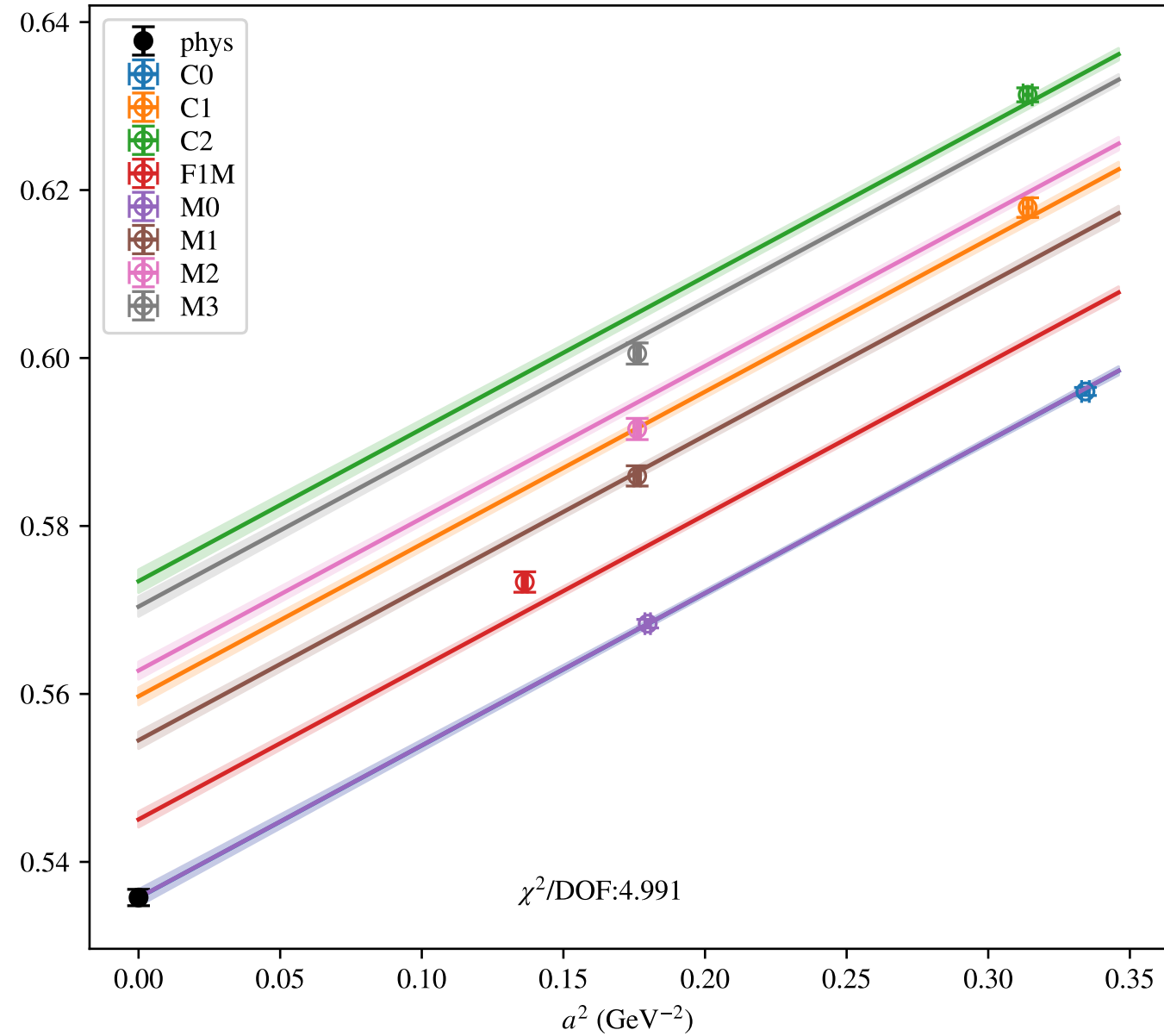


SSpPP

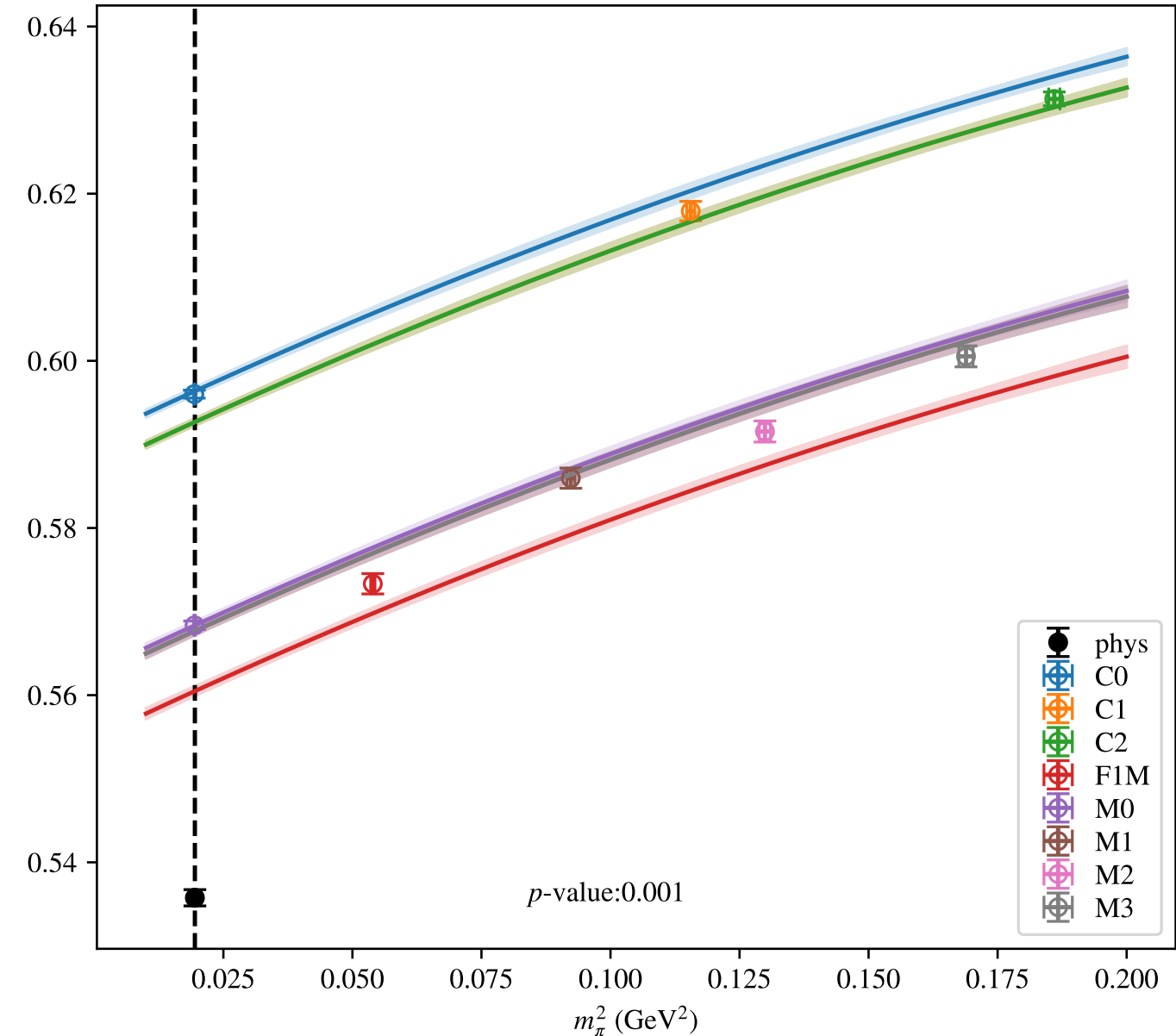


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.4 \text{ GeV}$$

SSpPP



SSpPP



3 B_3

| μ (GeV) | a^2, m_π^2 | a^2, m_π^2 (no C) | a^2, a^4, m_π^2 | a^2, m_π^2, m_π^4 |
|-------------|---------------------------------|-----------------------------------|-----------------------------------|---------------------------------|
| 2.0 | 0.4850(22) : 7.006 (0.0) | 0.5272(48) : 1.477 (0.228) | 0.5583(77) : 1.501 (0.199) | 0.4826(21) : 7.095 (0.0) |
| 2.2 | 0.4252(21) : 6.344 (0.0) | 0.4679(49) : 0.972 (0.378) | 0.4916(75) : 1.8 (0.126) | 0.4234(20) : 6.425 (0.0) |
| 2.3 | 0.4007(19) : 7.974 (0.0) | 0.4435(44) : 0.803 (0.448) | 0.4700(69) : 1.587 (0.175) | 0.3988(18) : 8.053 (0.0) |
| 2.4 | 0.3814(18) : 6.868 (0.0) | 0.4178(41) : 0.706 (0.494) | 0.4383(62) : 1.999 (0.092) | 0.3796(17) : 7.078 (0.0) |

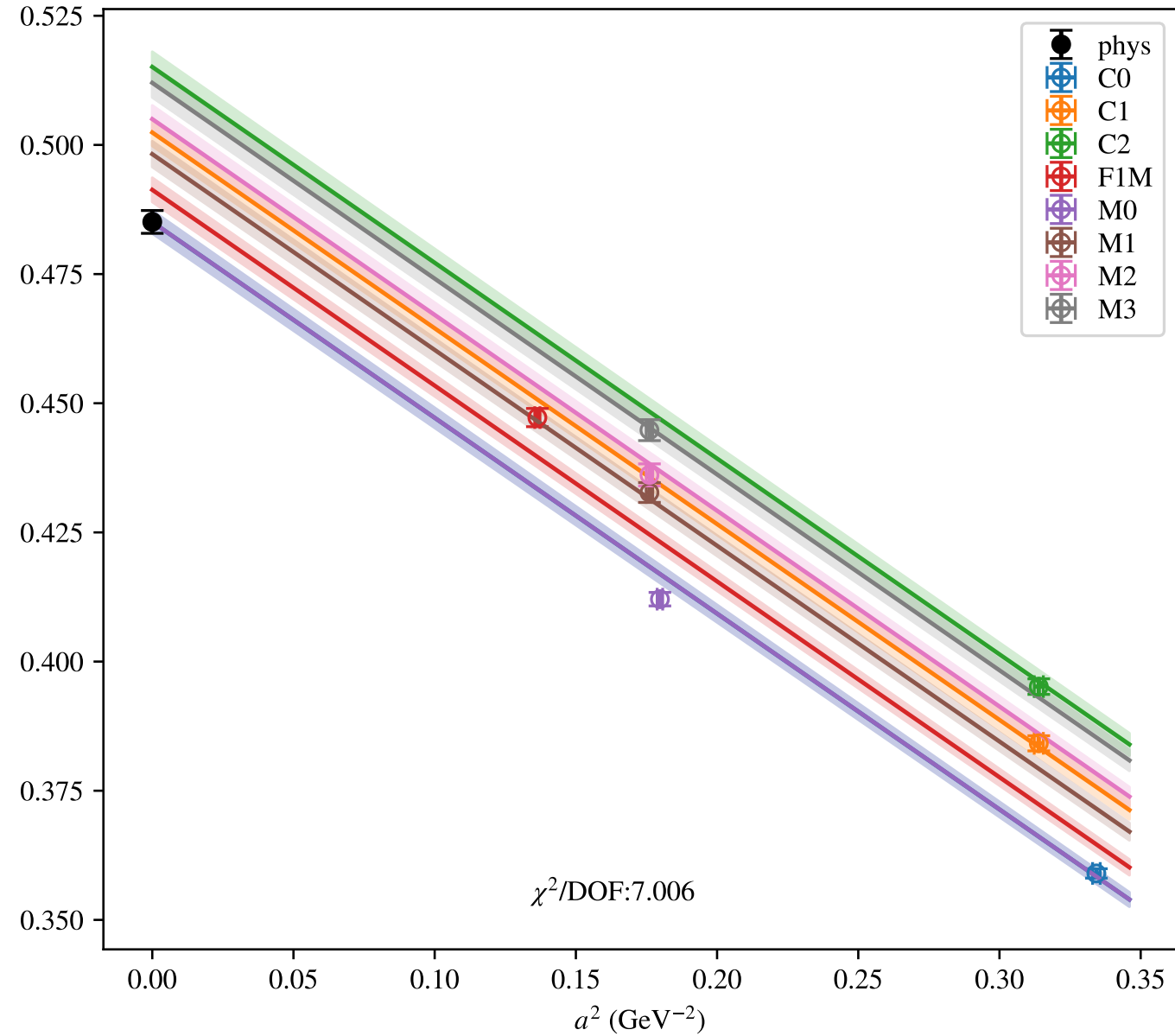
Table 5: Physical point value from chiral and continuum extrapolation at renormalisation scale μ . Entries are **value(error)**: χ^2/DOF (p -value).

| μ (GeV) | | a^2, m_π^2 | a^2, m_π^2 (no C) | a^2, a^4, m_π^2 | a^2, m_π^2, m_π^4 |
|-------------|----------|----------------|-----------------------|---------------------|-------------------------|
| 2.0 | α | -0.781(45) | -1.20(41) | -1.89(98) | -0.771(47) |
| | β | 0.00631(17) | 0.00657(25) | 0.00619(15) | 0.00989(61) |
| 2.2 | α | -0.760(50) | -1.23(40) | -1.89(95) | -0.751(50) |
| | β | 0.00616(14) | 0.00601(21) | 0.00601(12) | 0.00946(64) |
| 2.3 | α | -0.747(51) | -1.25(39) | -1.98(92) | -0.737(49) |
| | β | 0.00608(19) | 0.00600(26) | 0.00591(15) | 0.00990(72) |
| 2.4 | α | -0.749(51) | -1.20(40) | -1.84(97) | -0.740(51) |
| | β | 0.00609(14) | 0.00595(20) | 0.00600(12) | 0.00925(62) |

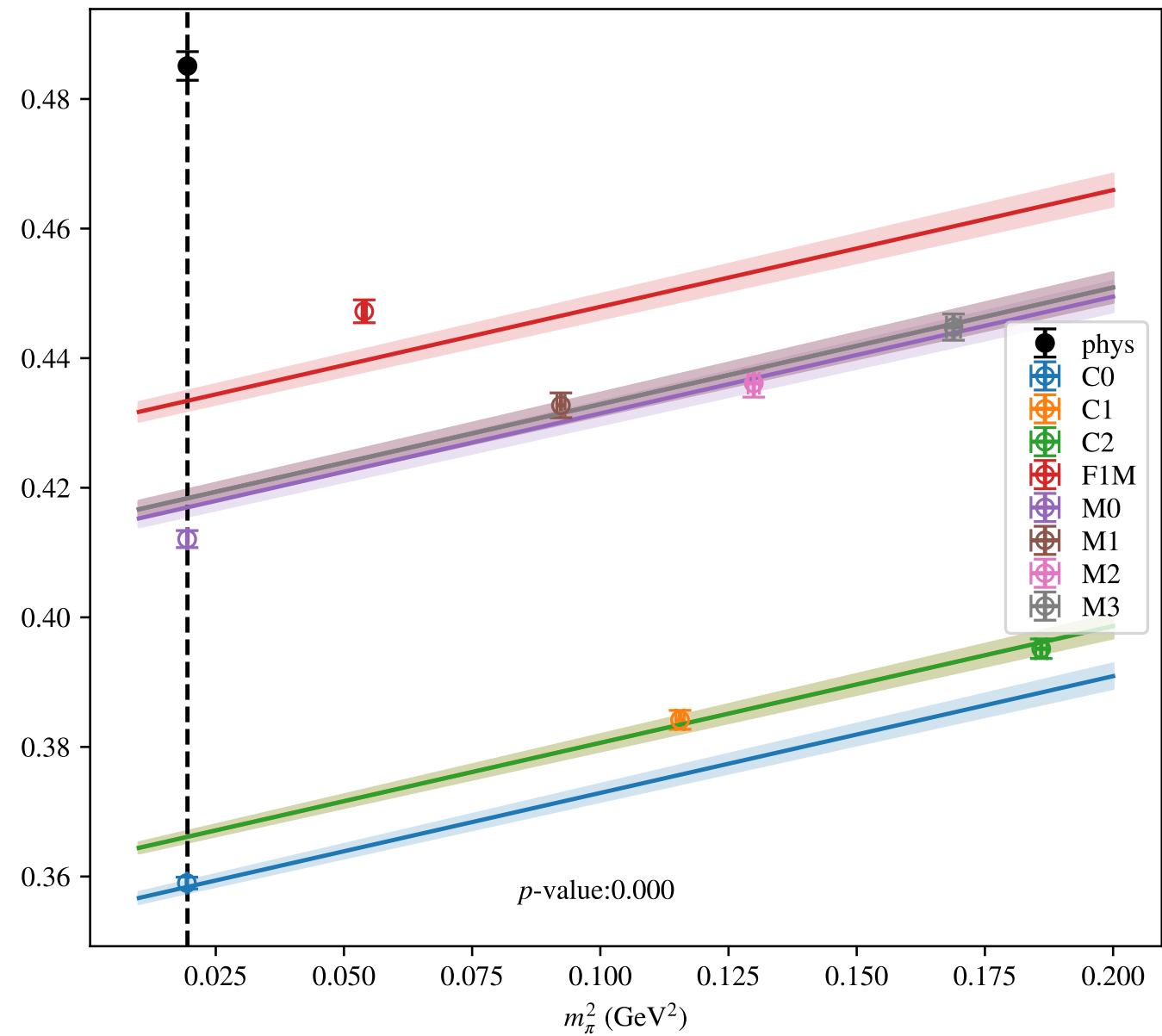
Table 6: Fit values of coefficients in $B = B_{phys} + \alpha a^2 + \beta \left(\frac{m_\pi^2}{f_\pi^2} - \frac{m_{\pi,PDG}^2}{f_\pi^2} \right) + \dots$

$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

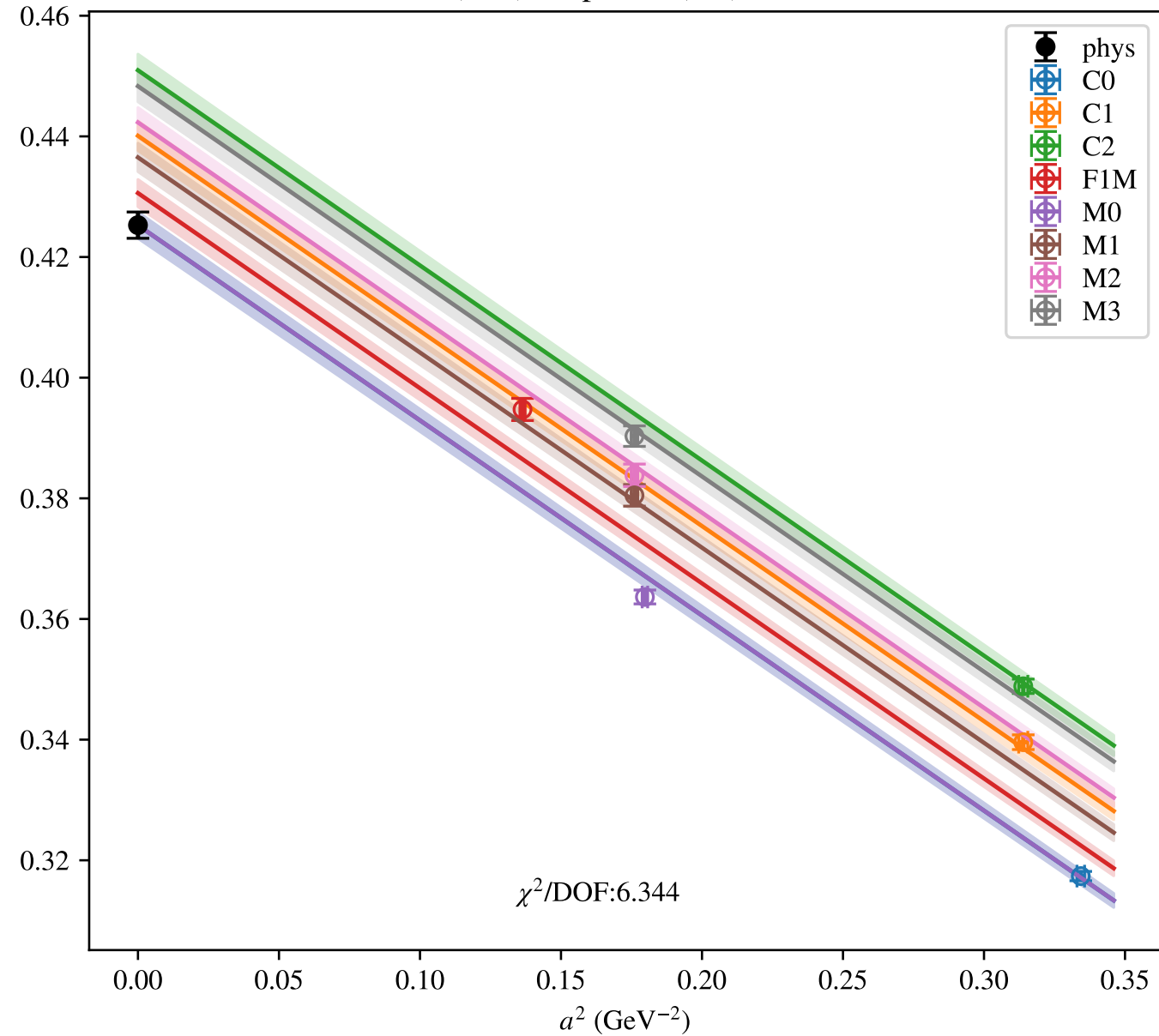


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

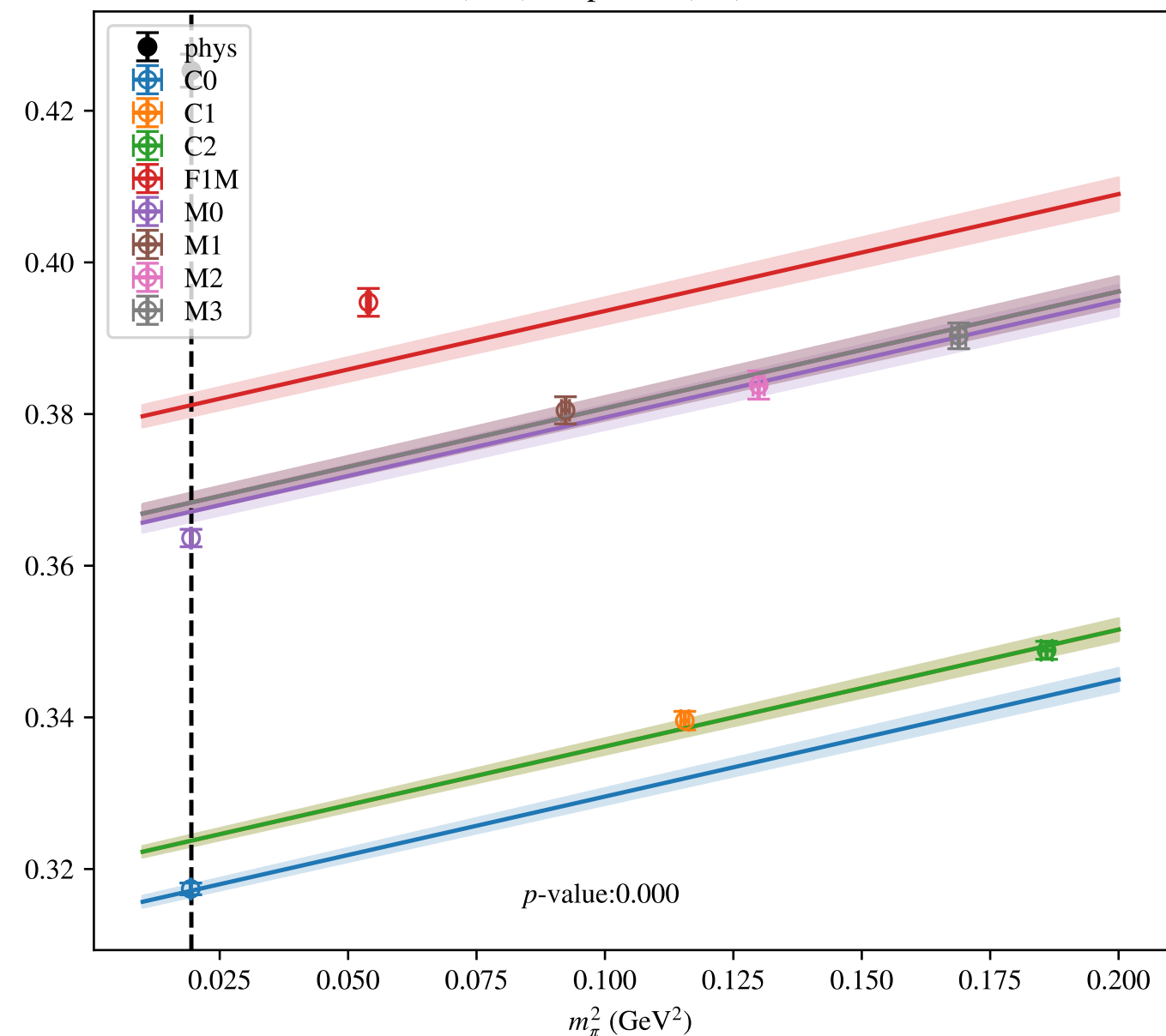


$$a^2, m_\pi^2, \mu = 2.2 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

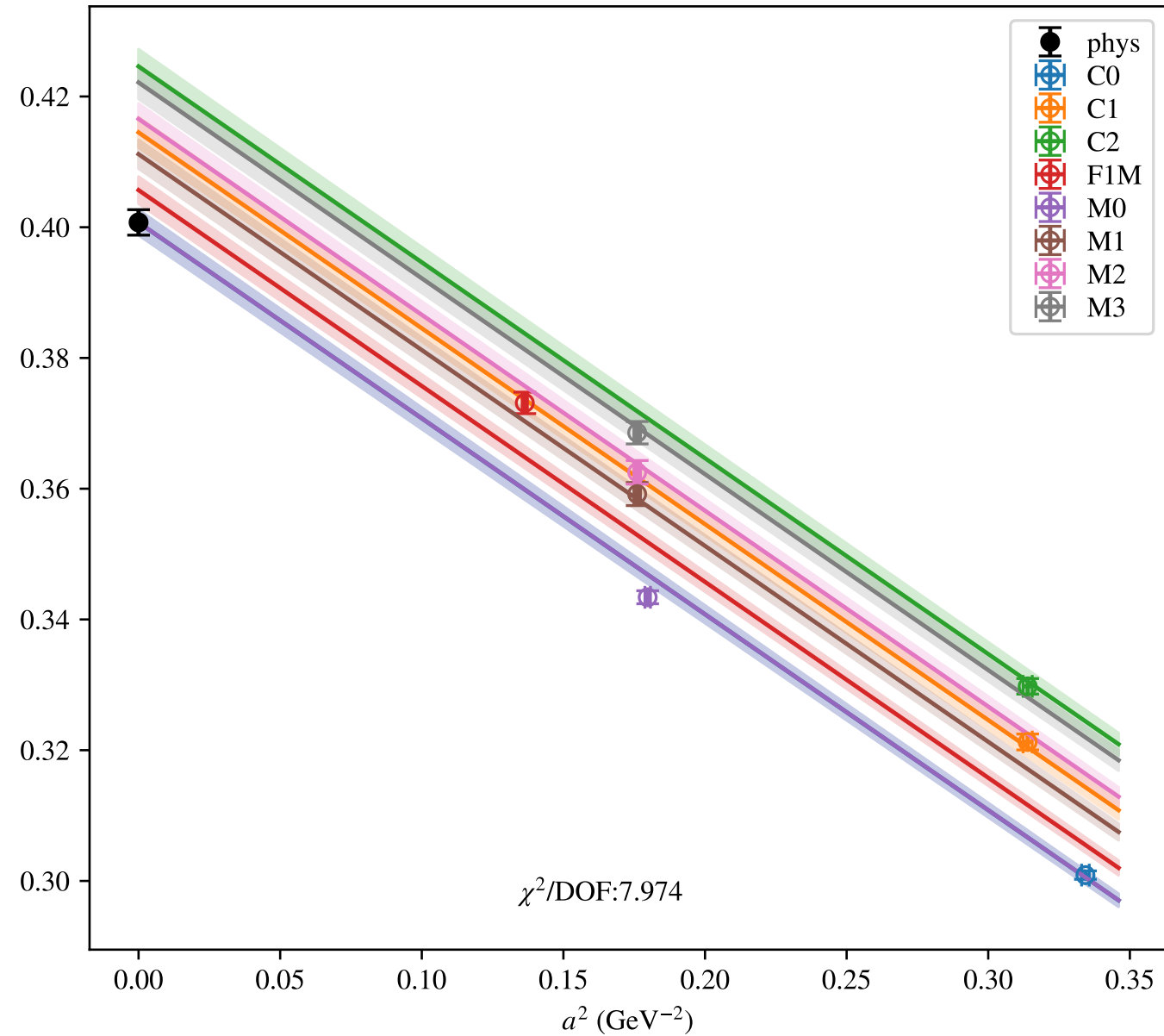


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

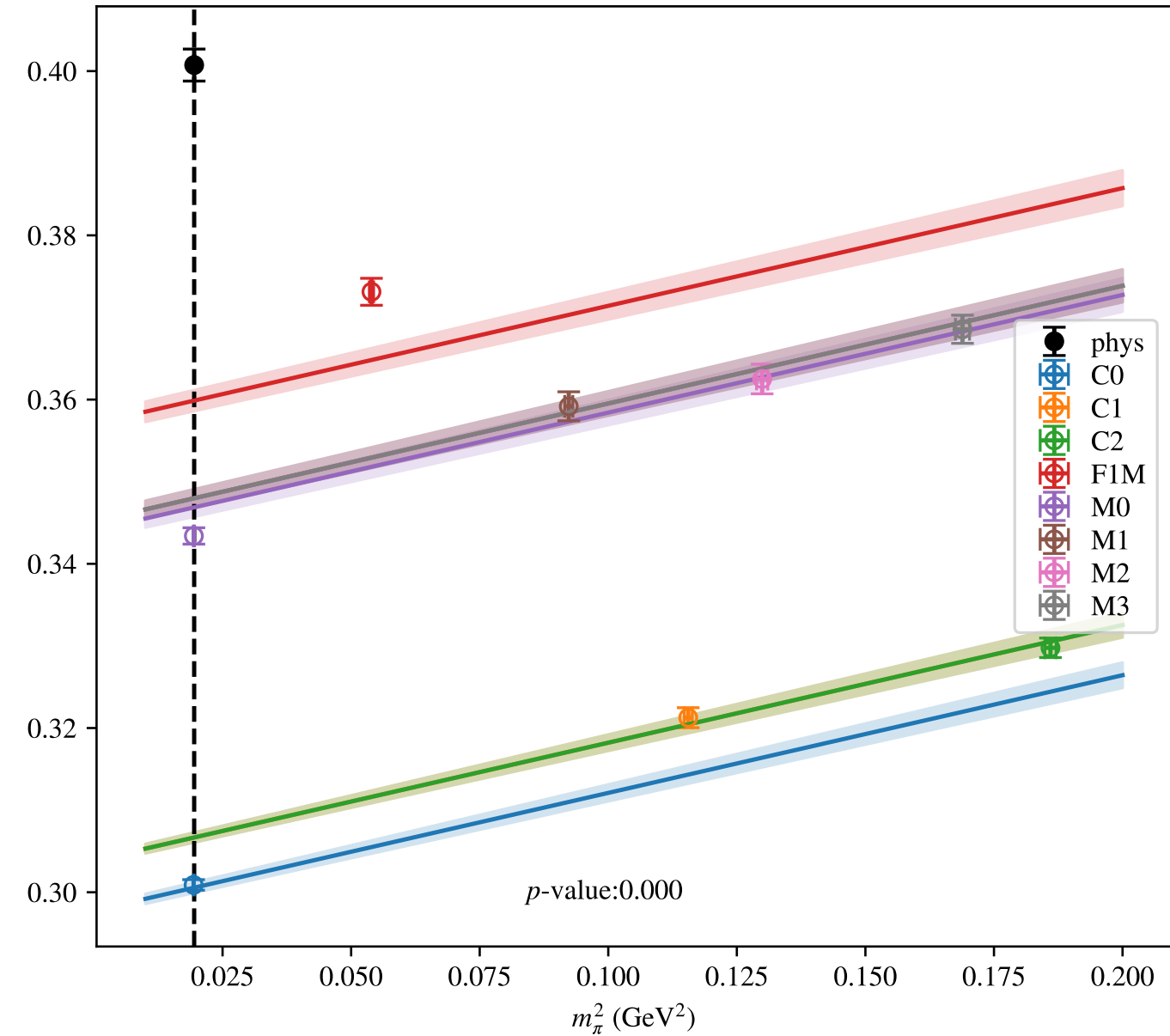


$$a^2, m_\pi^2, \mu = 2.3 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

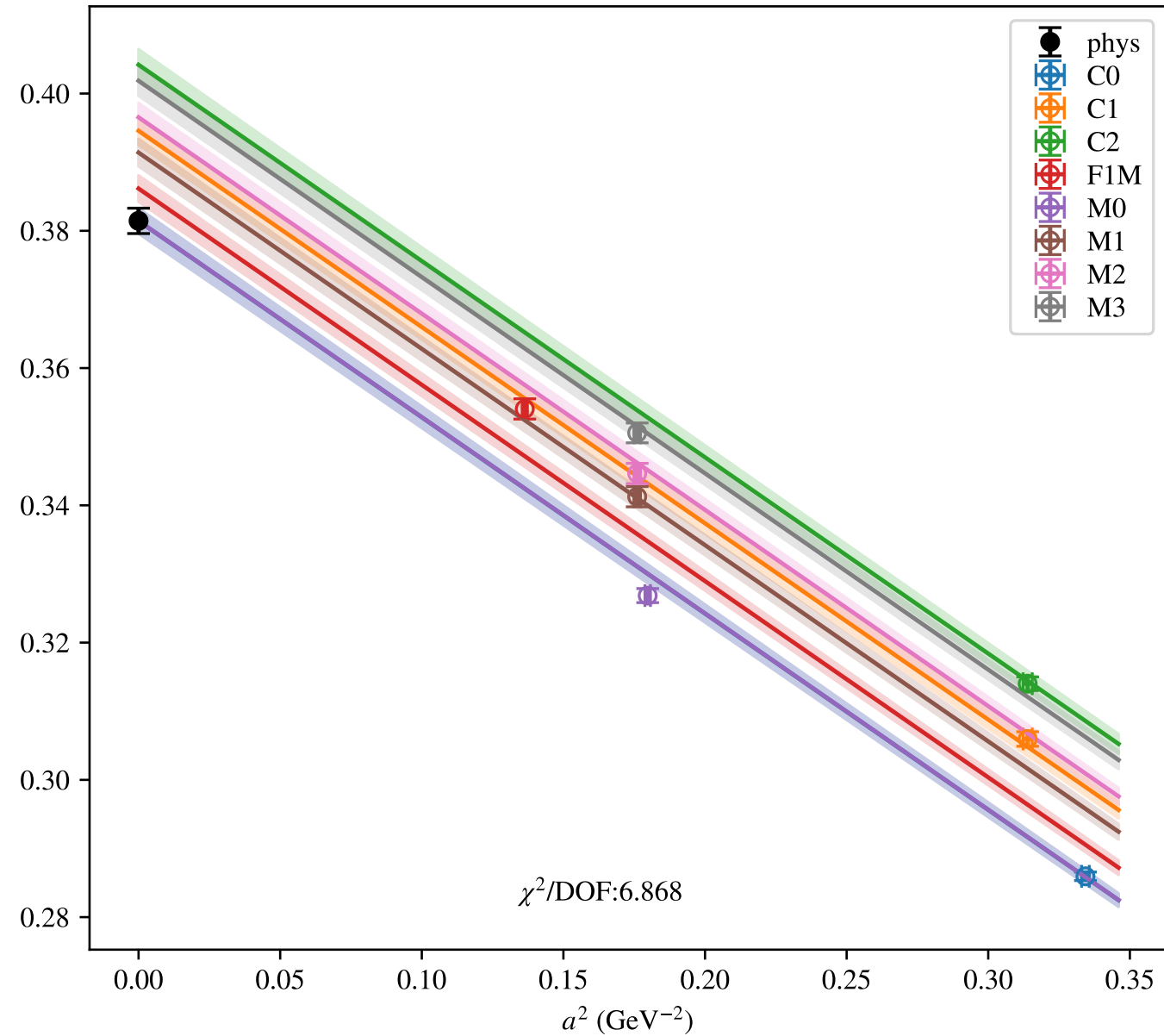


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

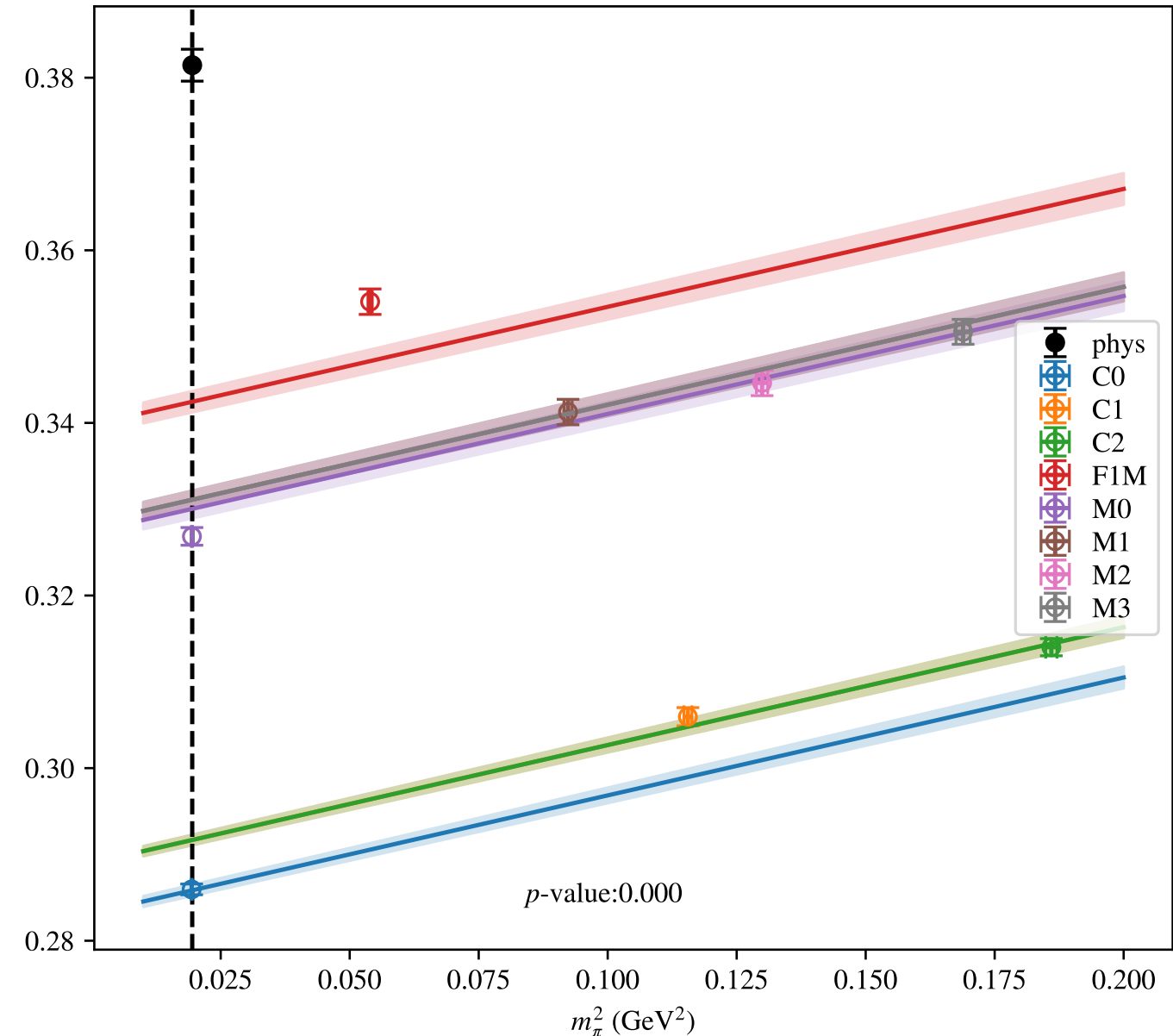


$$a^2, m_\pi^2, \mu = 2.4 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

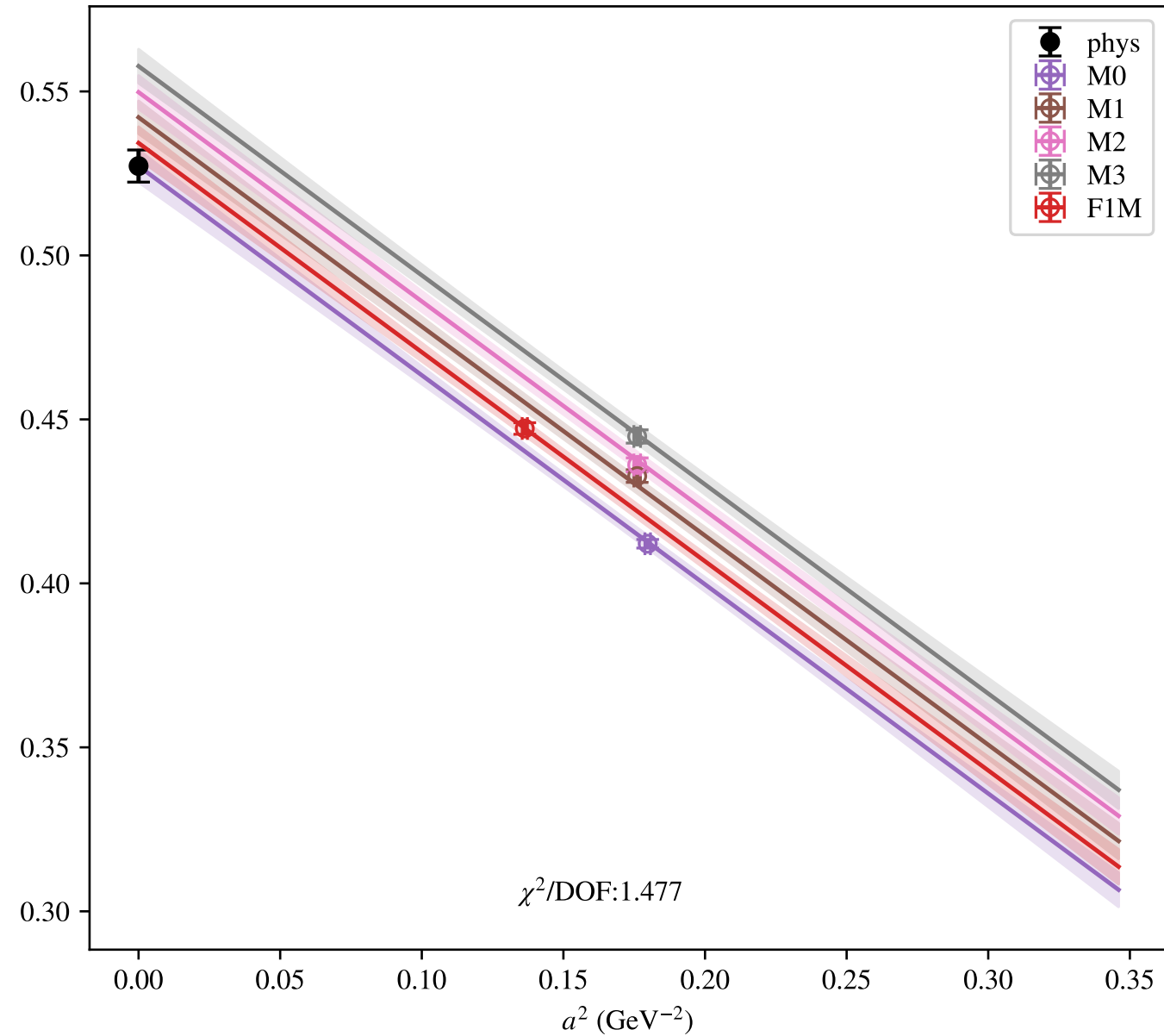


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

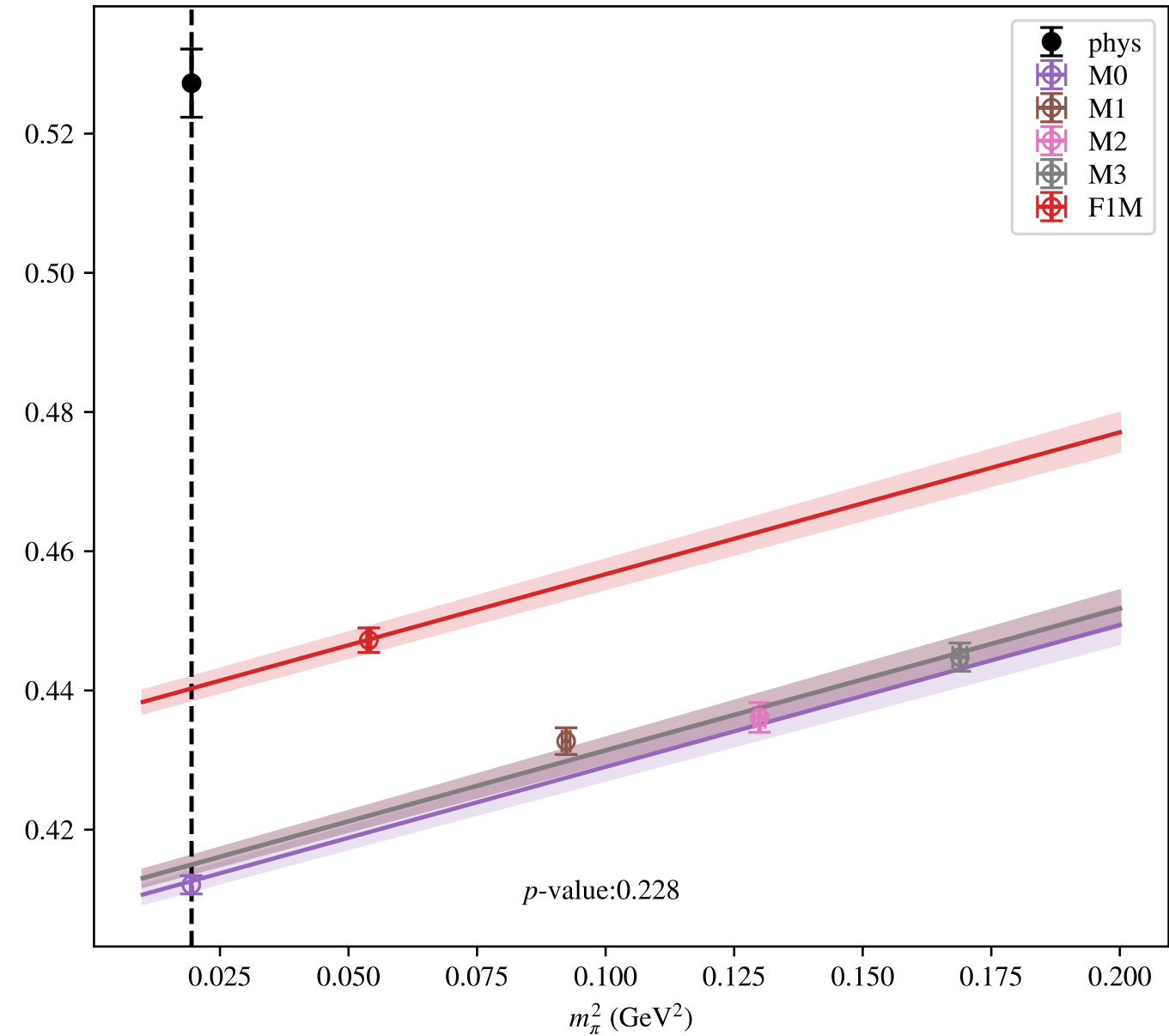


a^2, m_π^2 (no C), $\mu = 2.0$ GeV

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

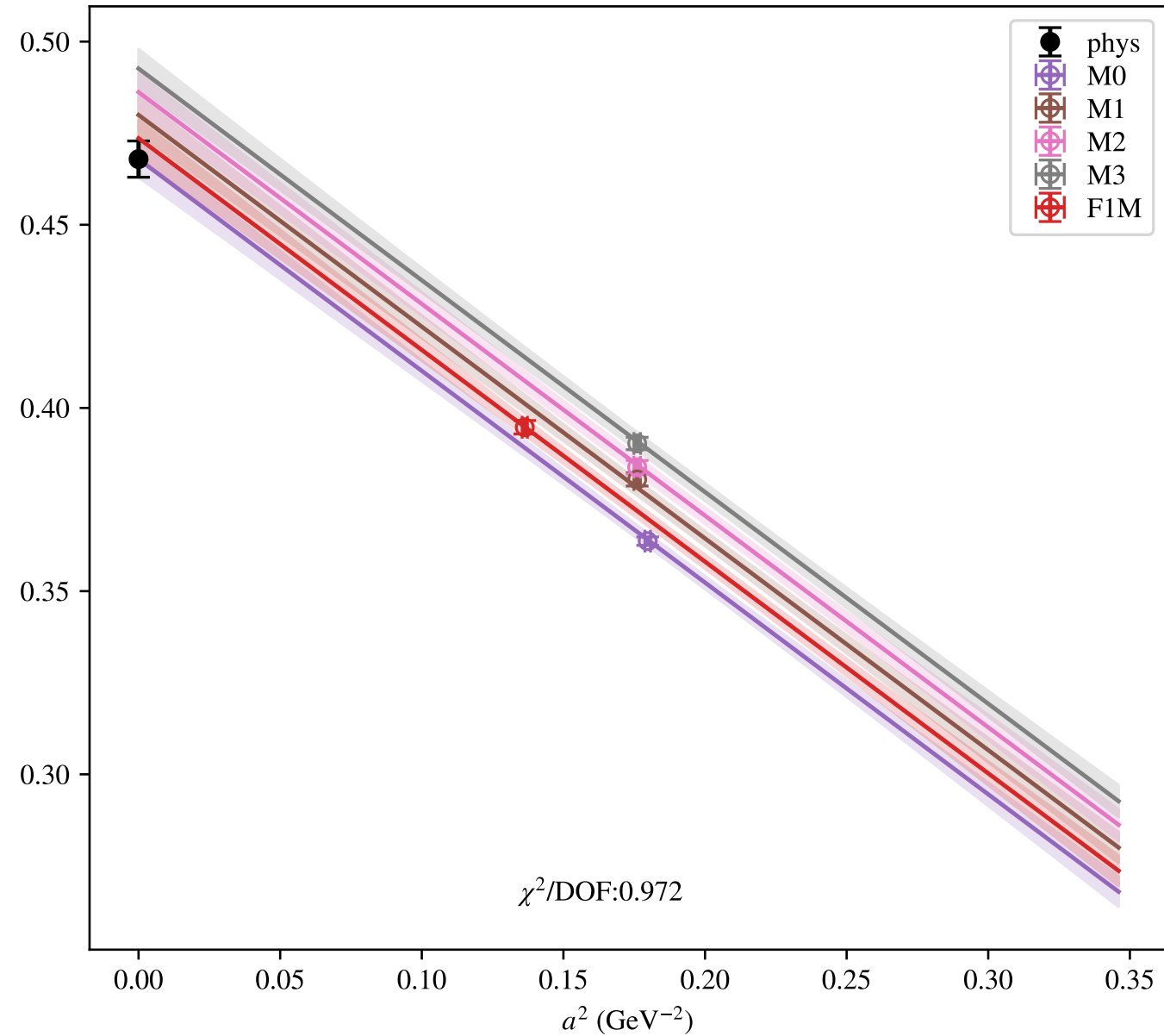


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

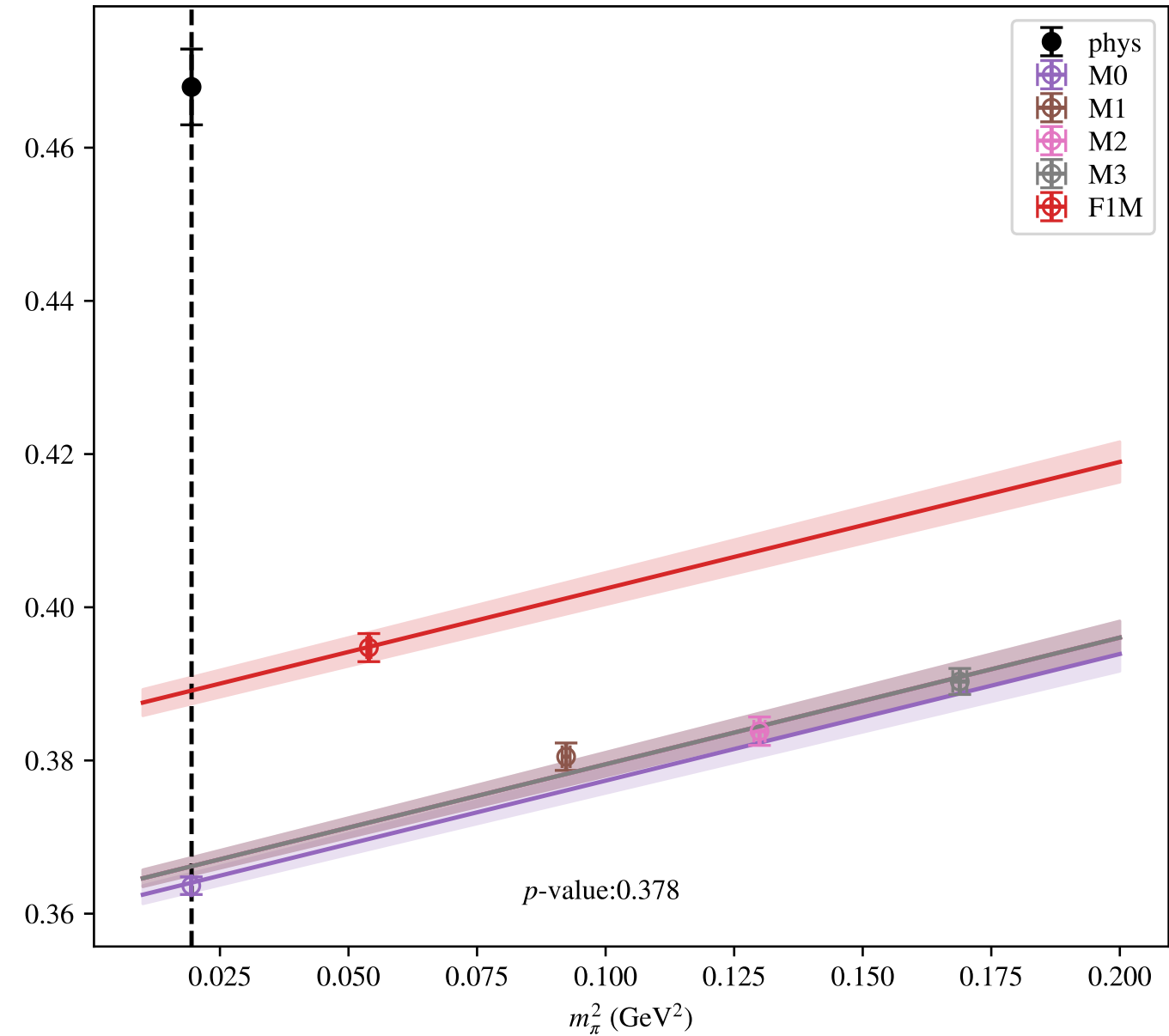


a^2, m_π^2 (no C), $\mu = 2.2$ GeV

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

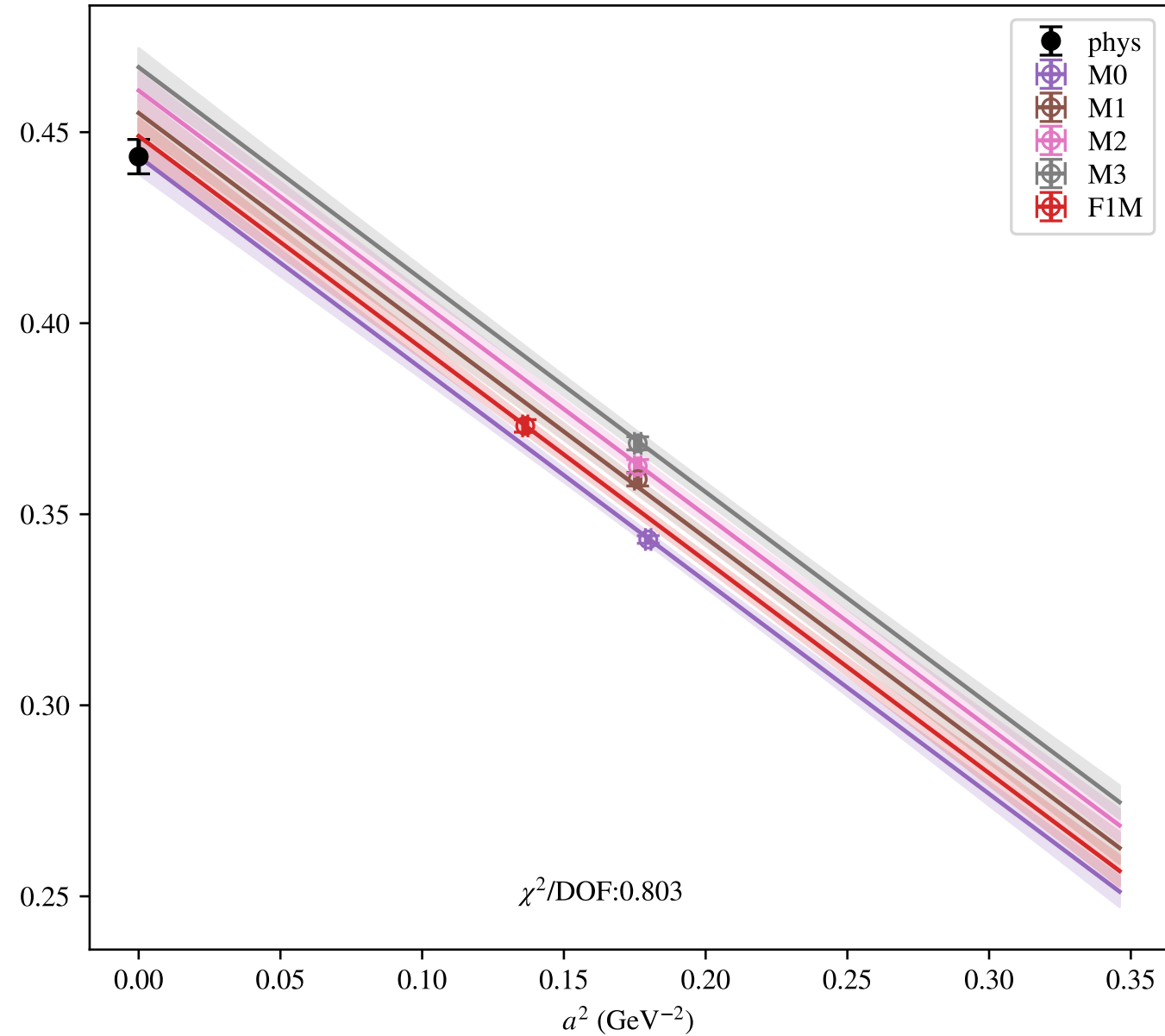


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

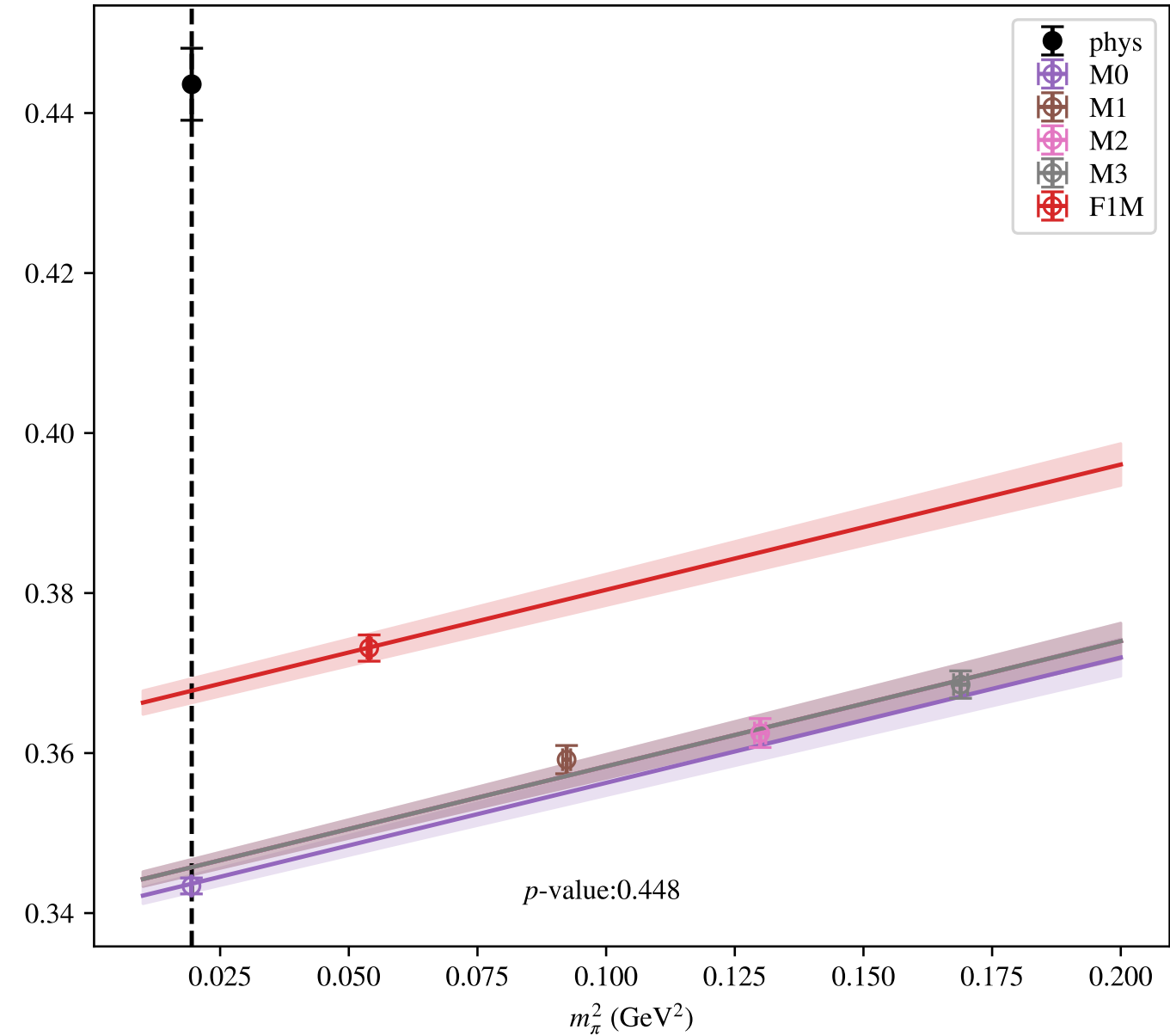


a^2, m_π^2 (no C), $\mu = 2.3$ GeV

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

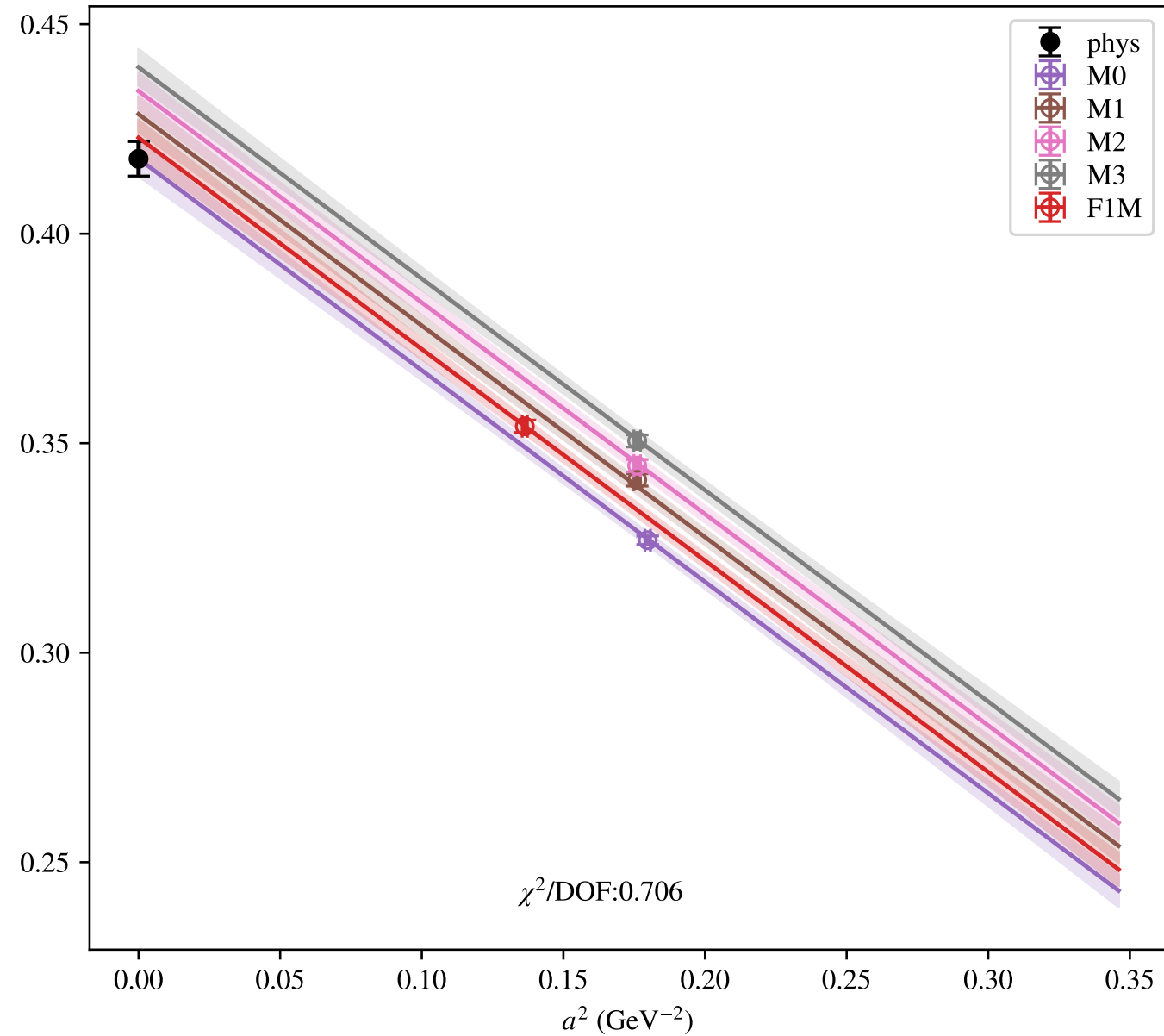


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

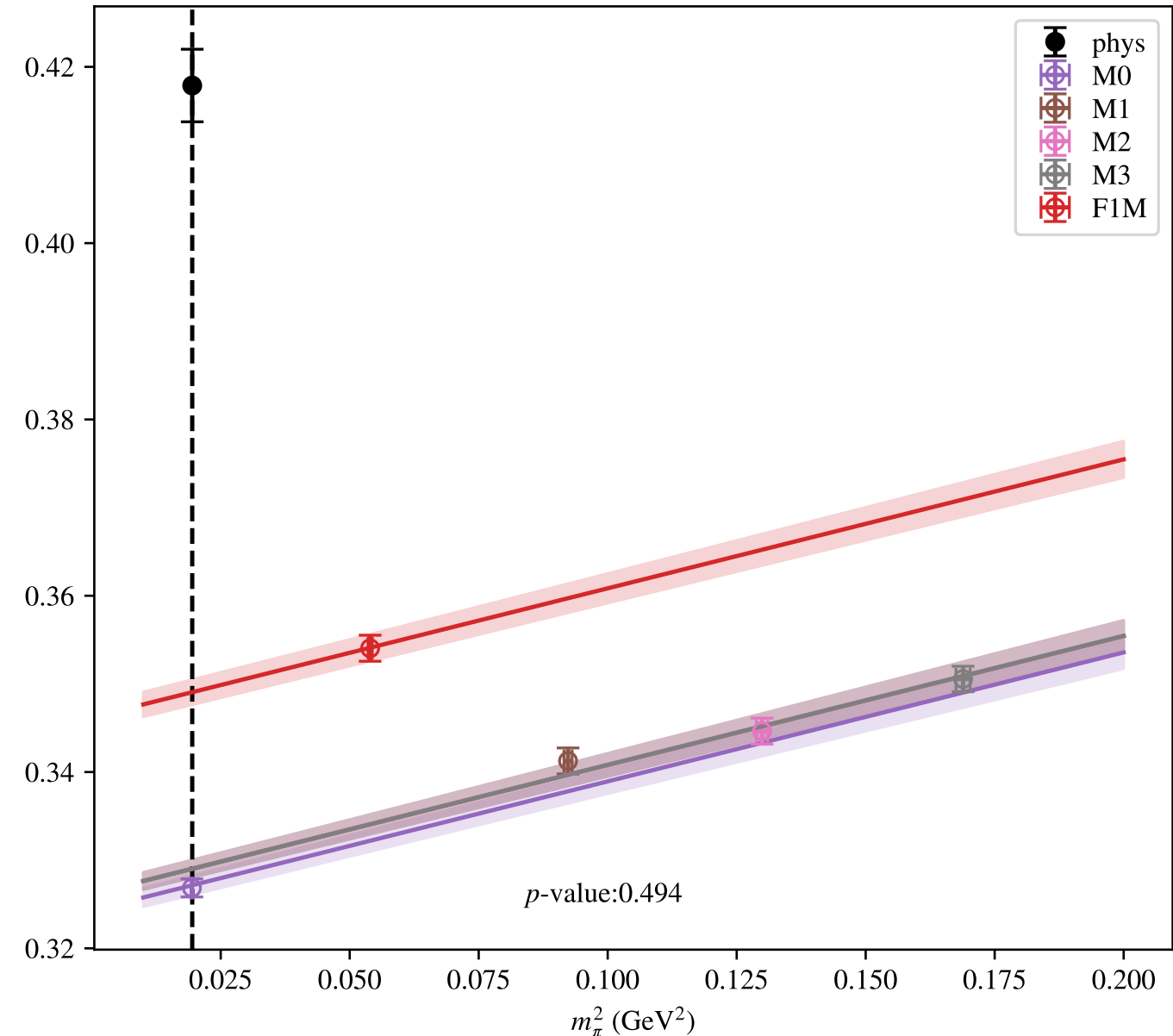


a^2, m_π^2 (no C), $\mu = 2.4$ GeV

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

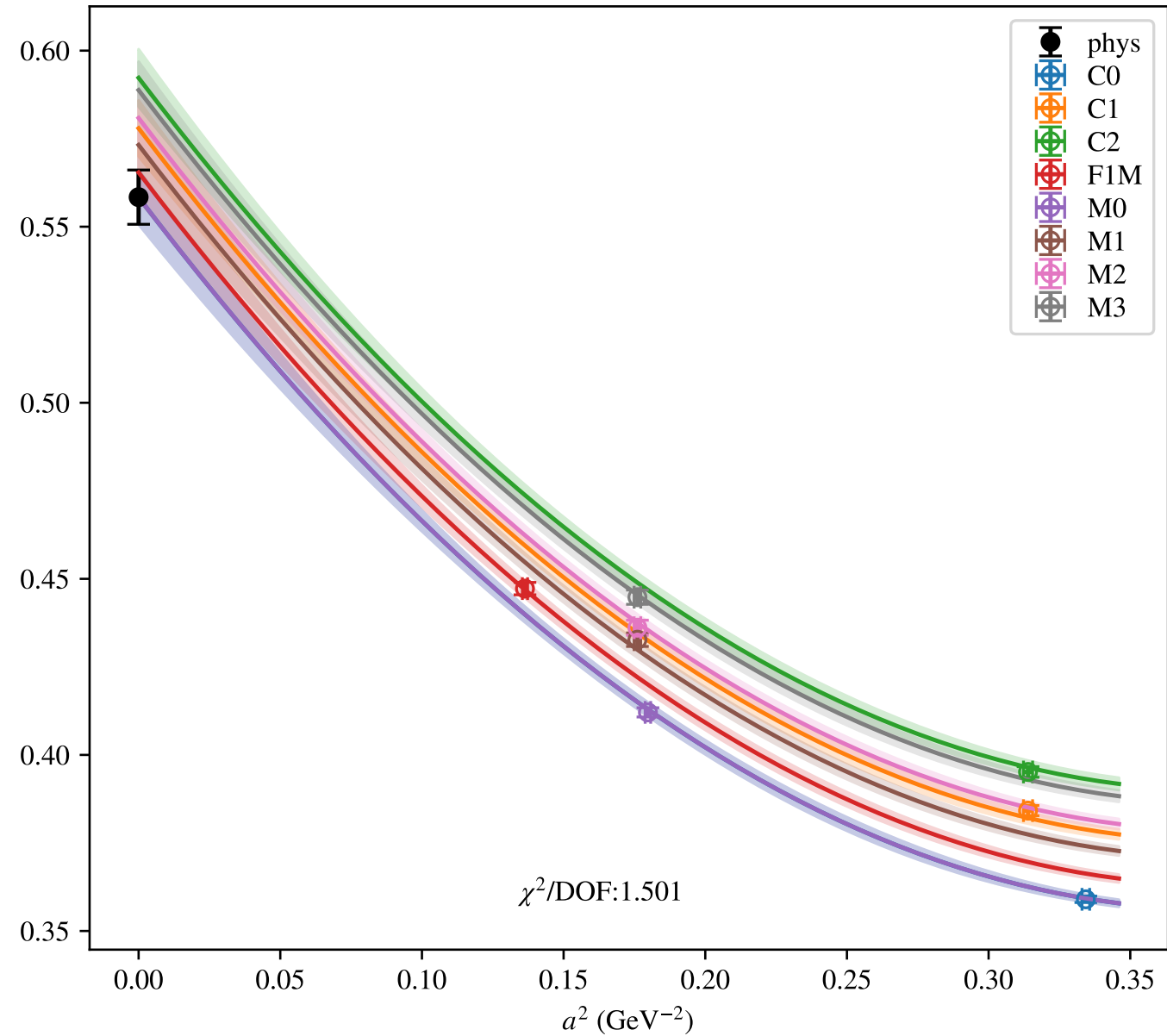


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

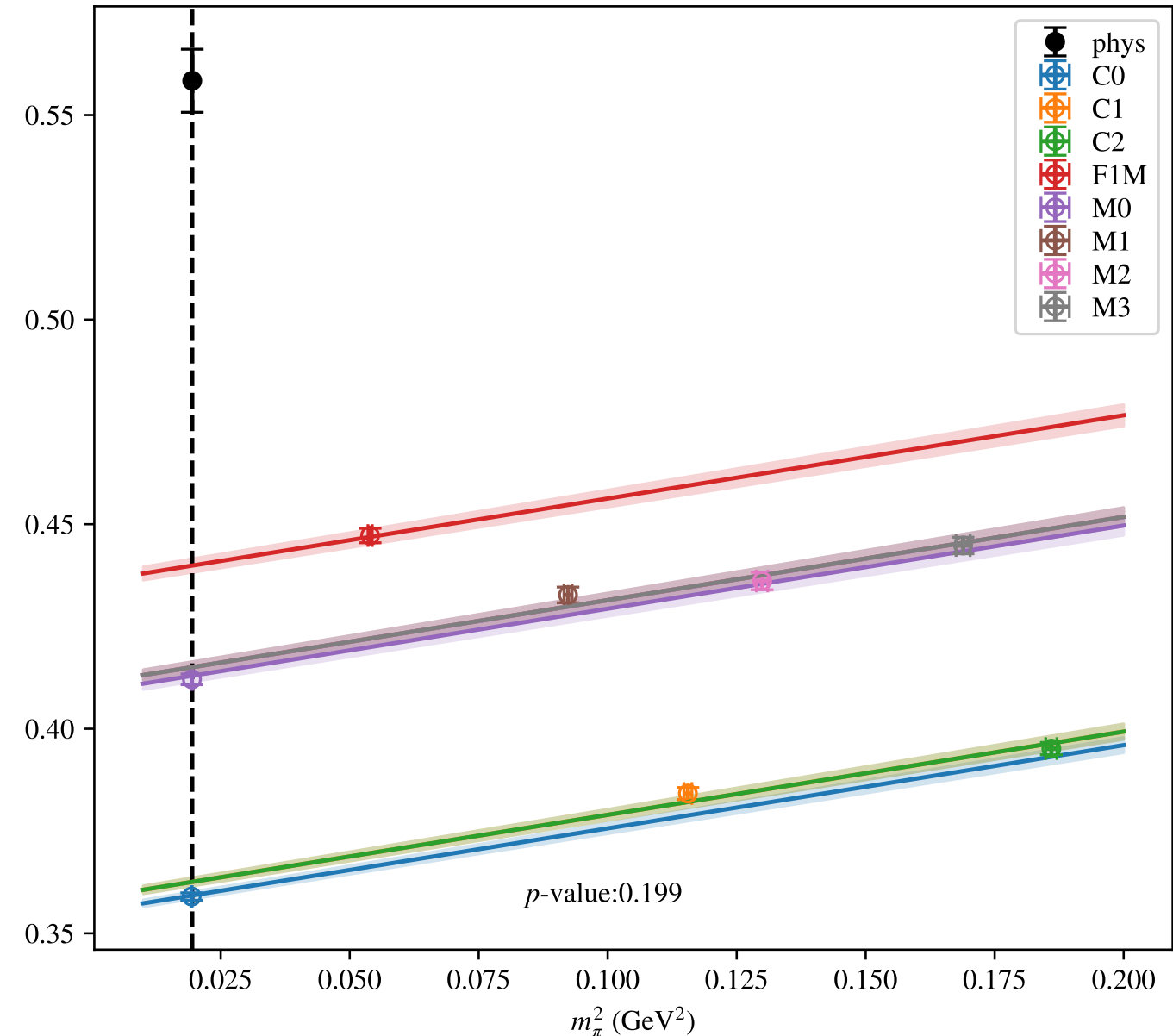


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

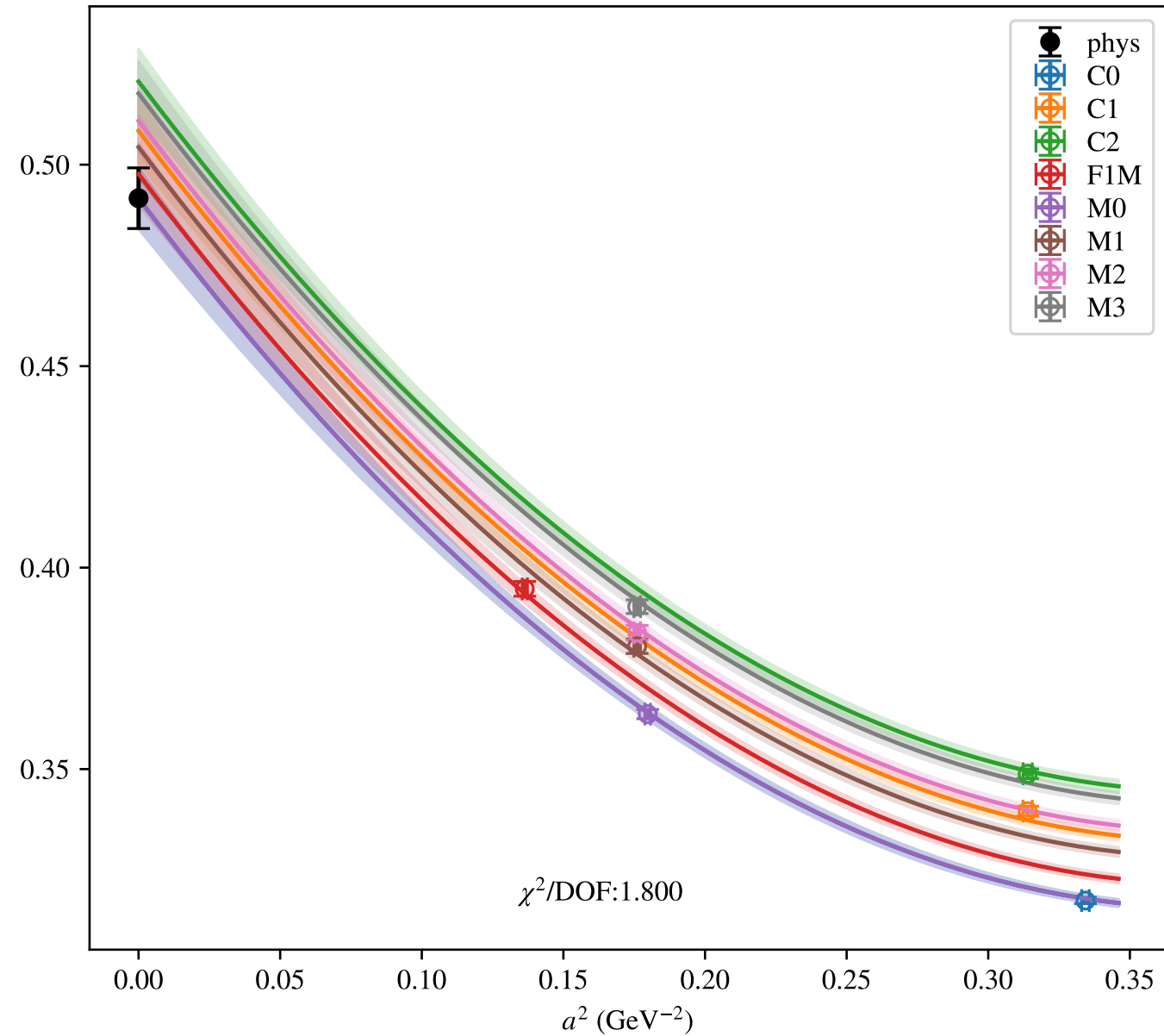


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

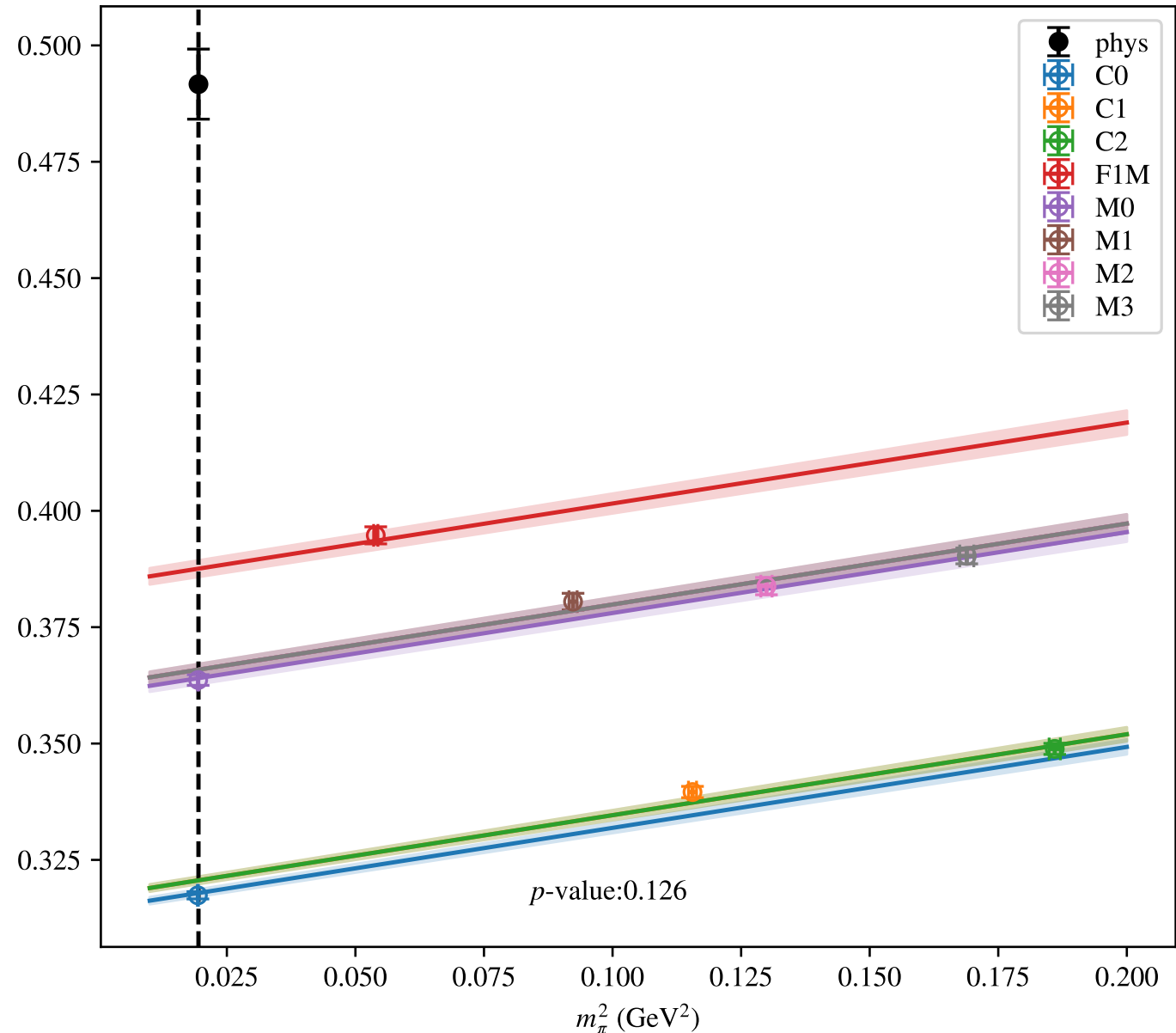


$$a^2, a^4, m_\pi^2, \mu = 2.2 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

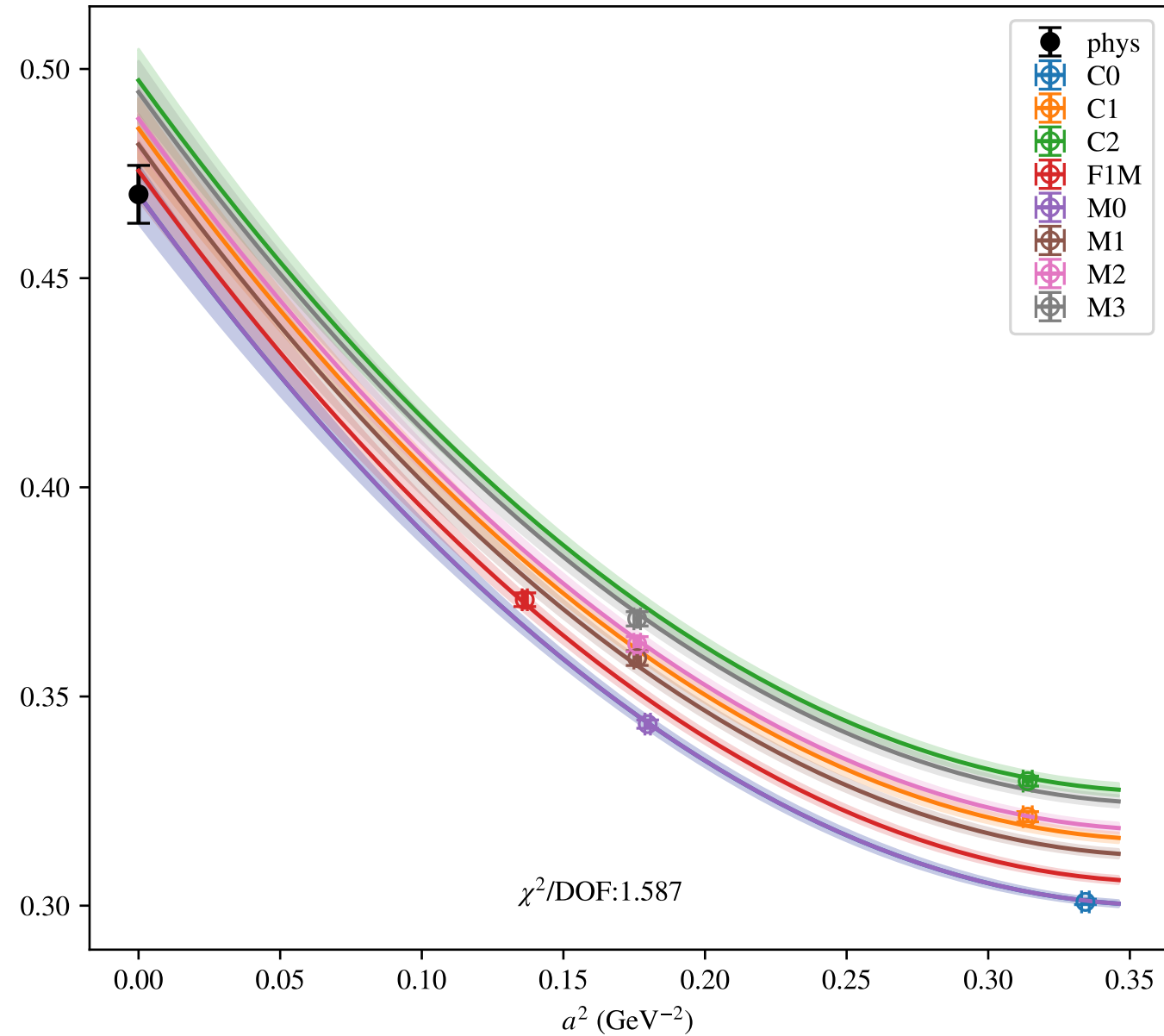


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

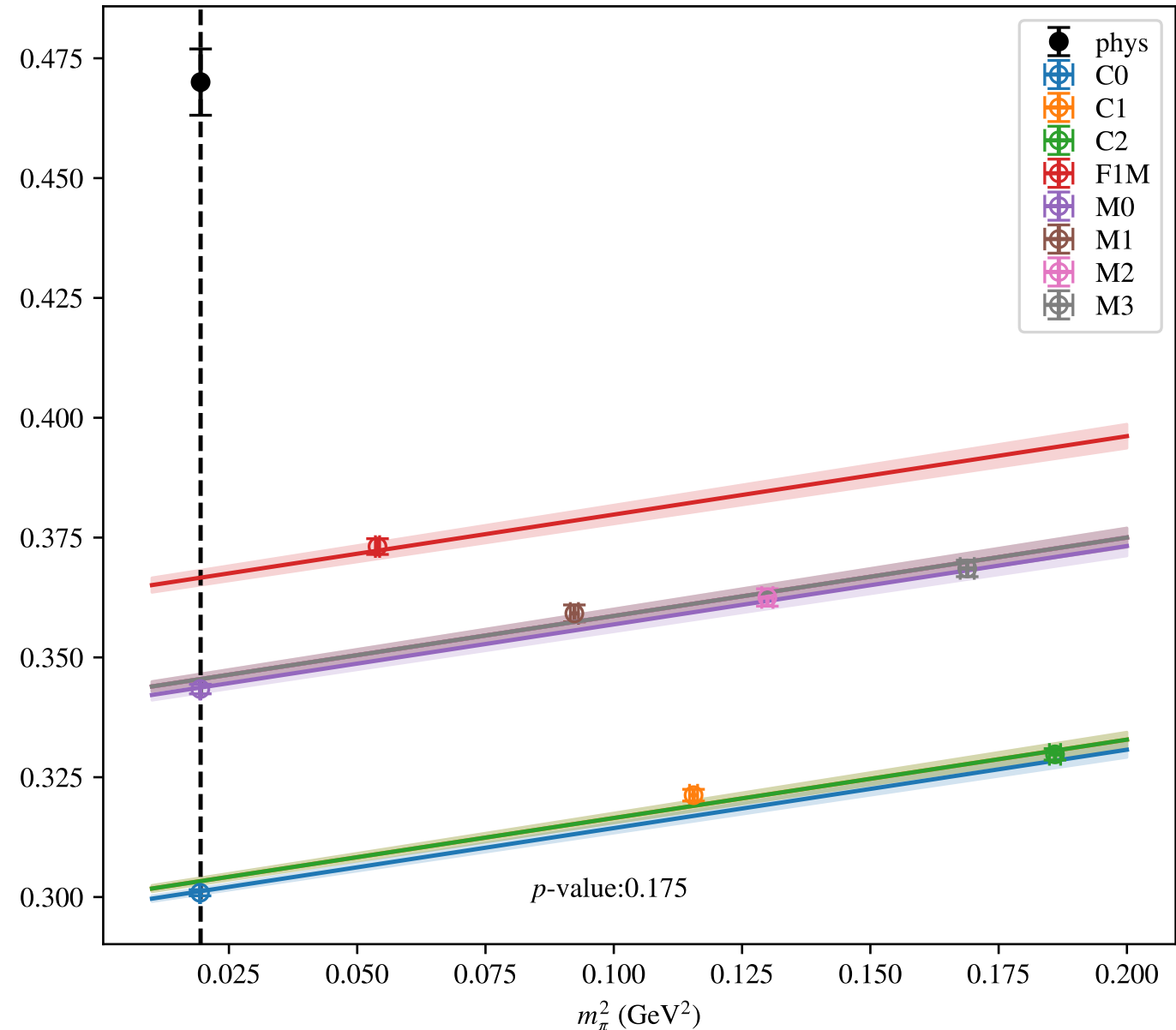


$$a^2, a^4, m_\pi^2, \mu = 2.3 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

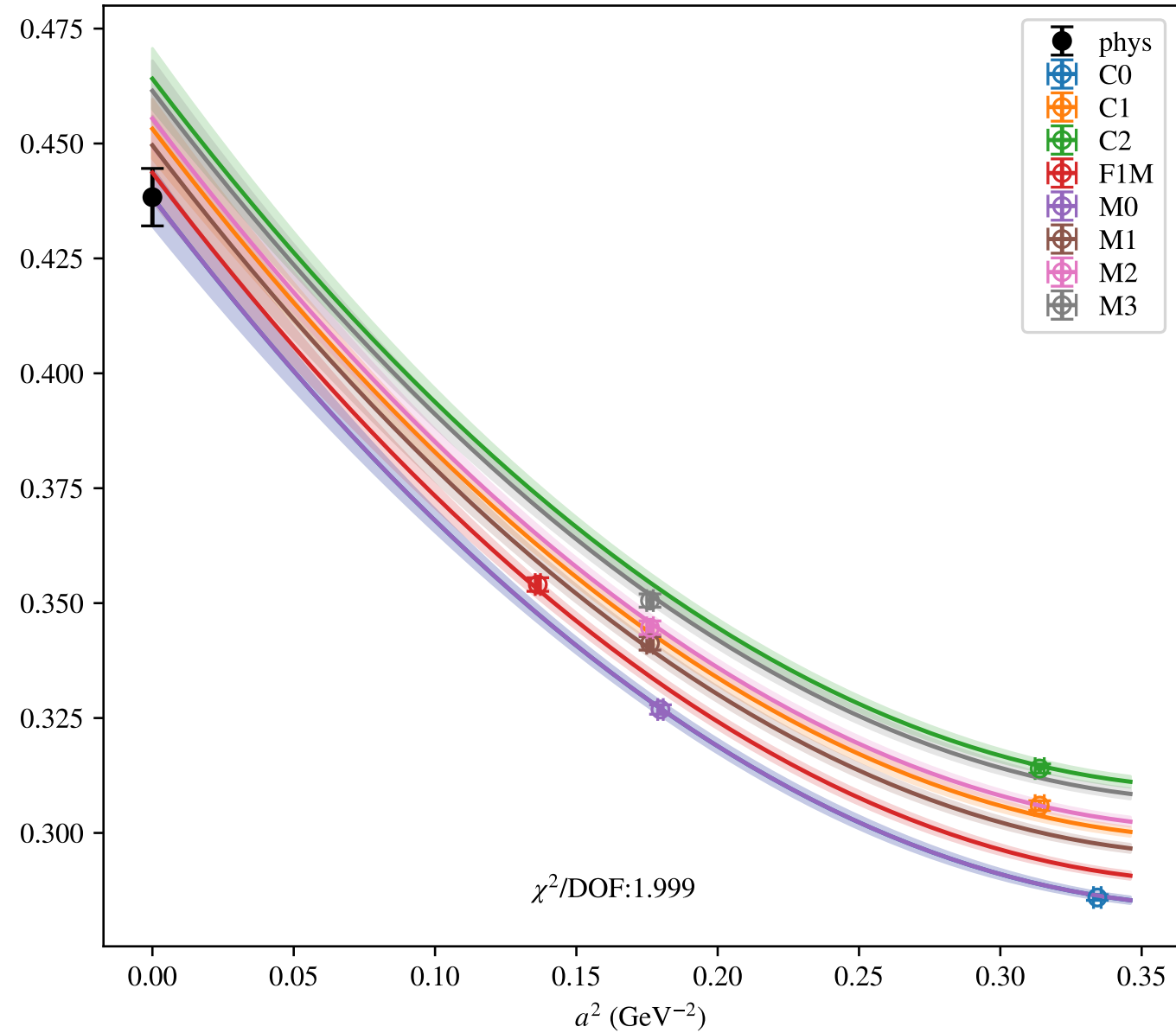


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

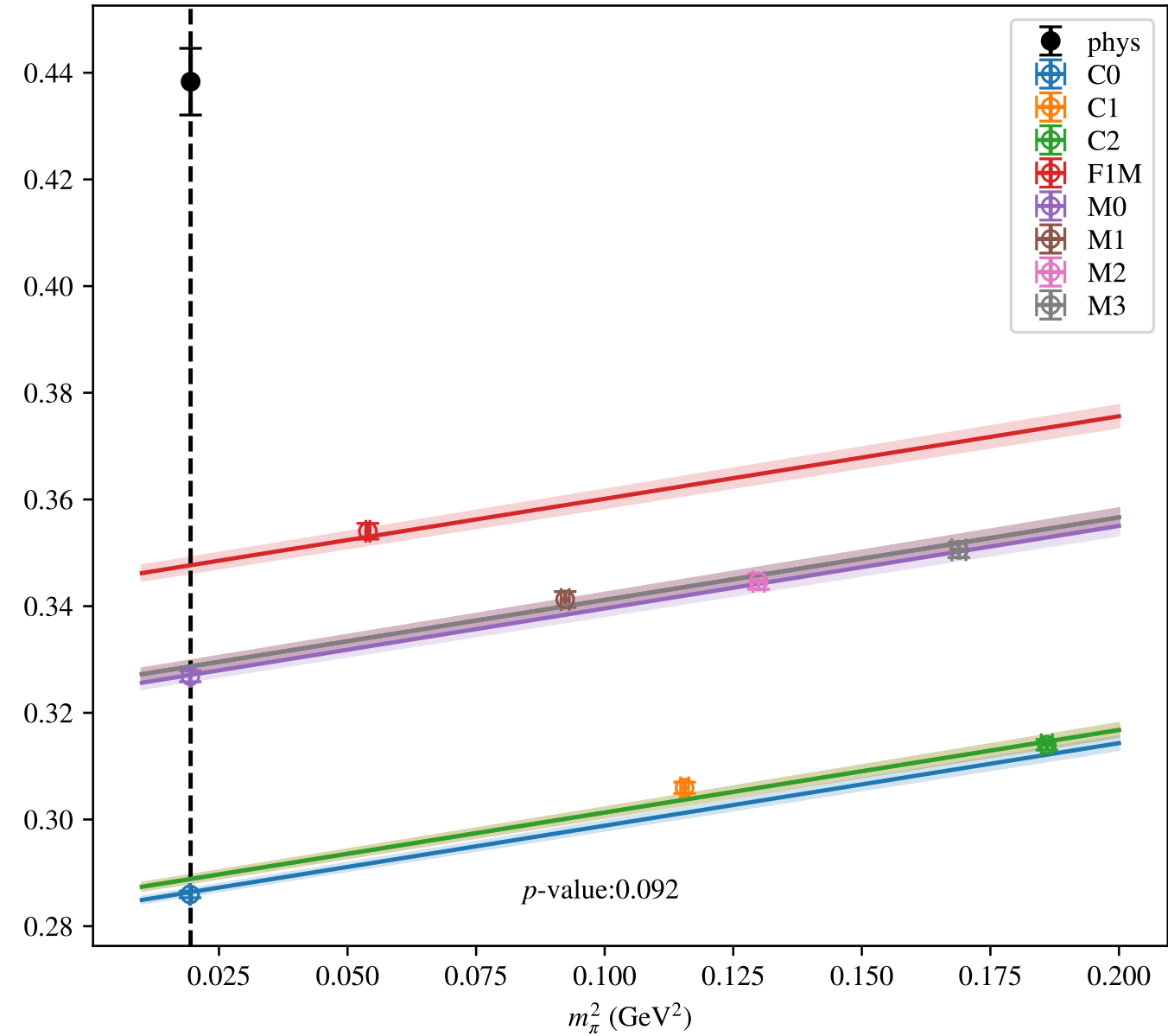


$$a^2, a^4, m_\pi^2, \mu = 2.4 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

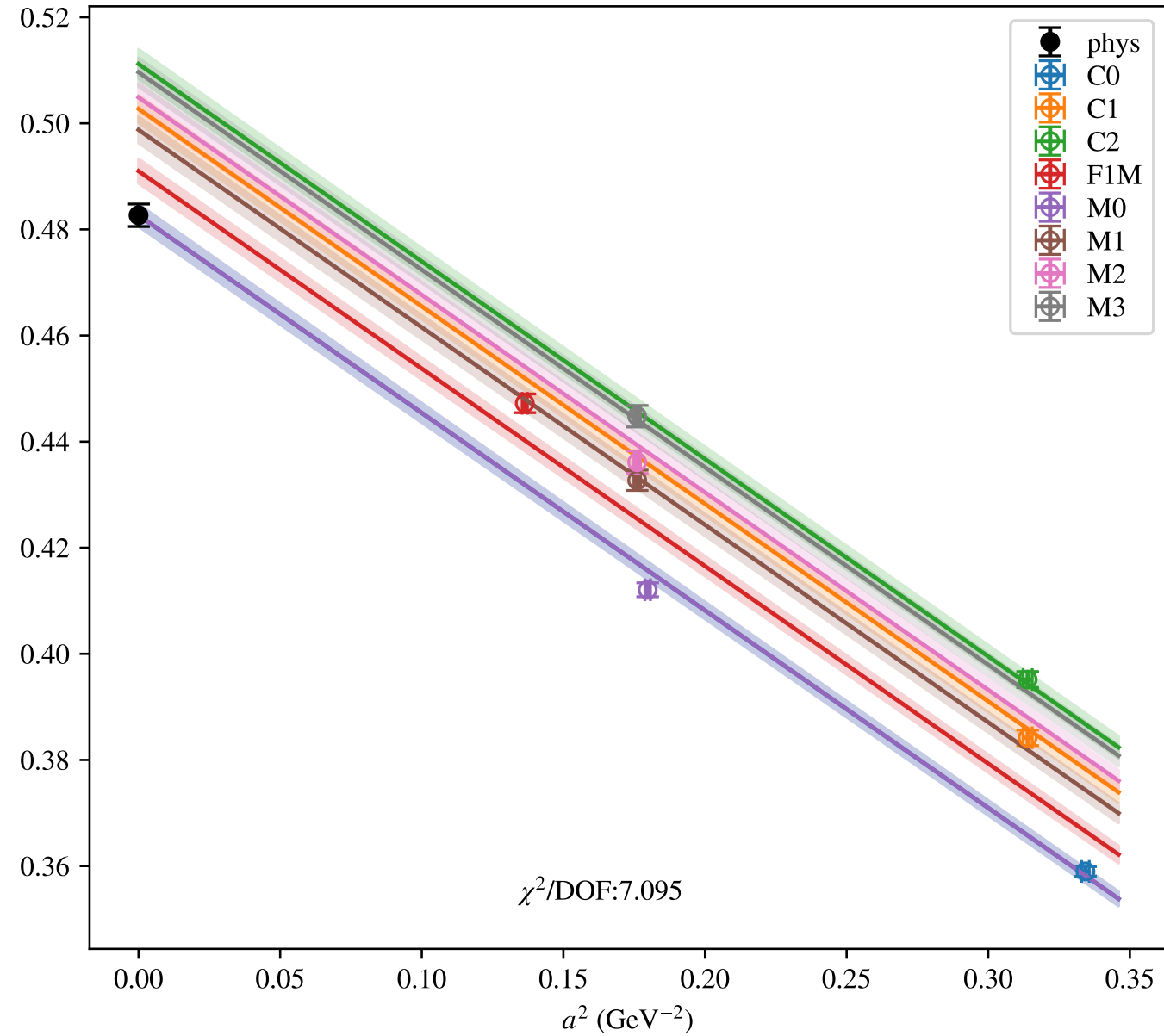


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

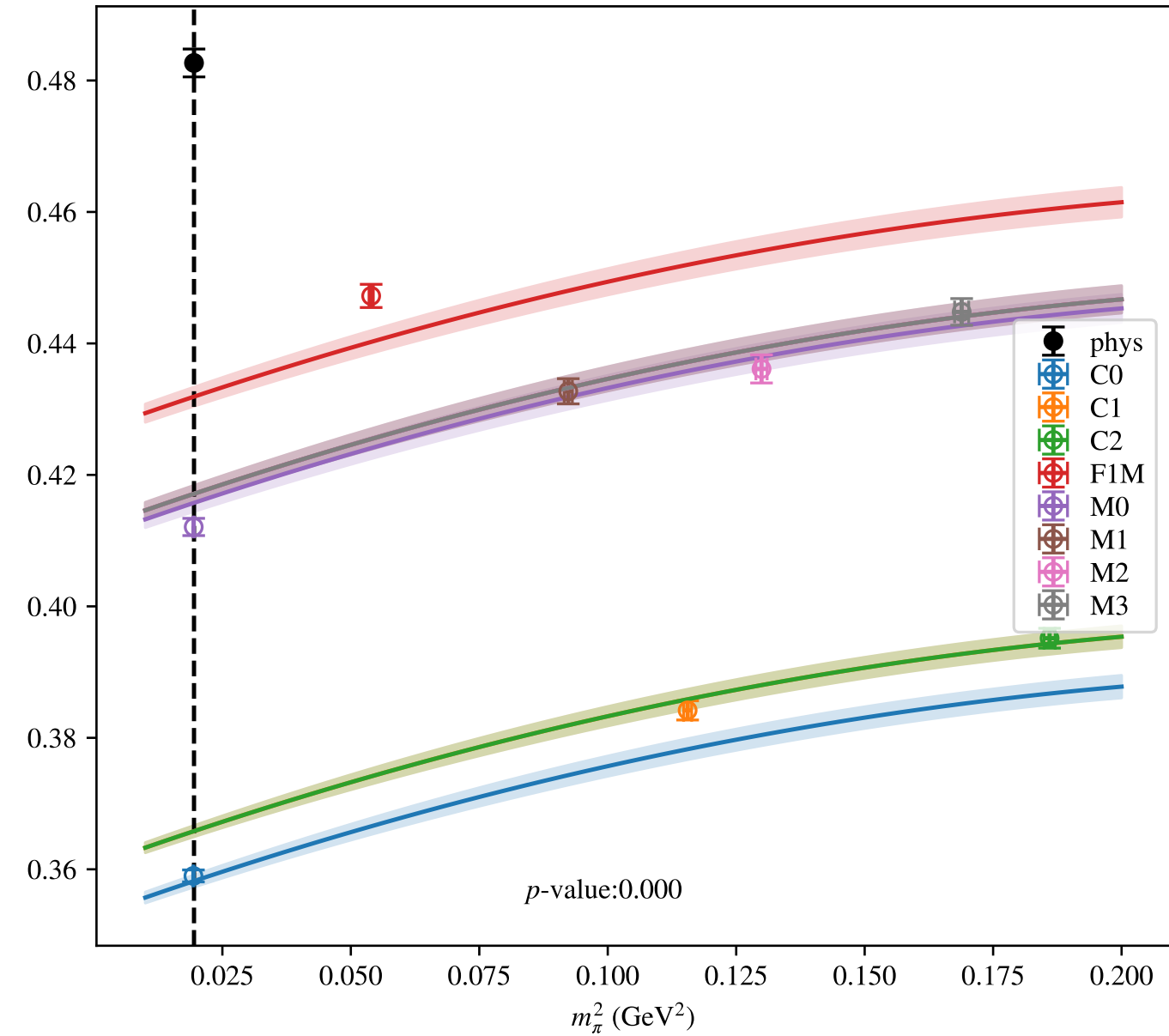


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

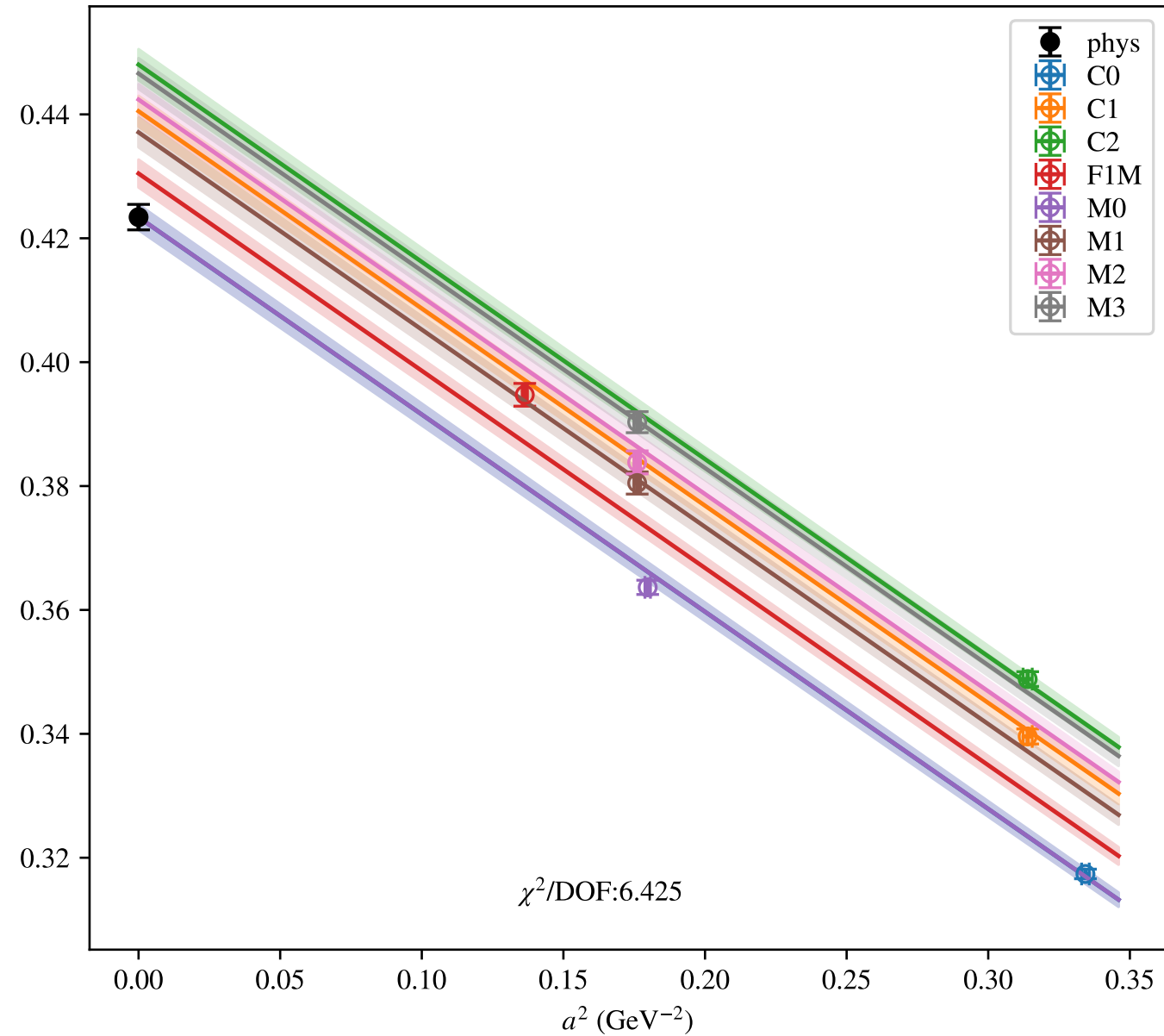


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

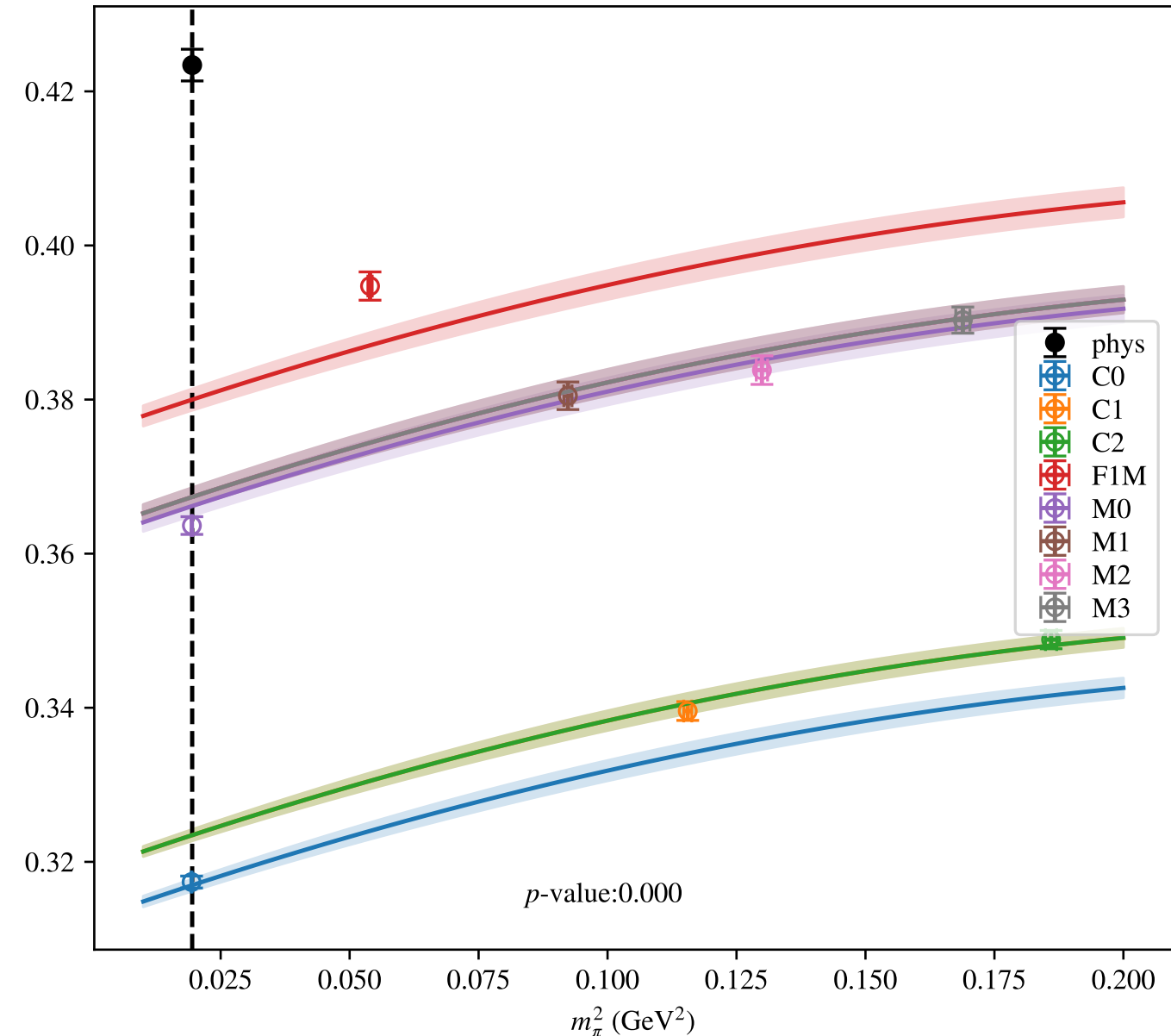


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.2 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

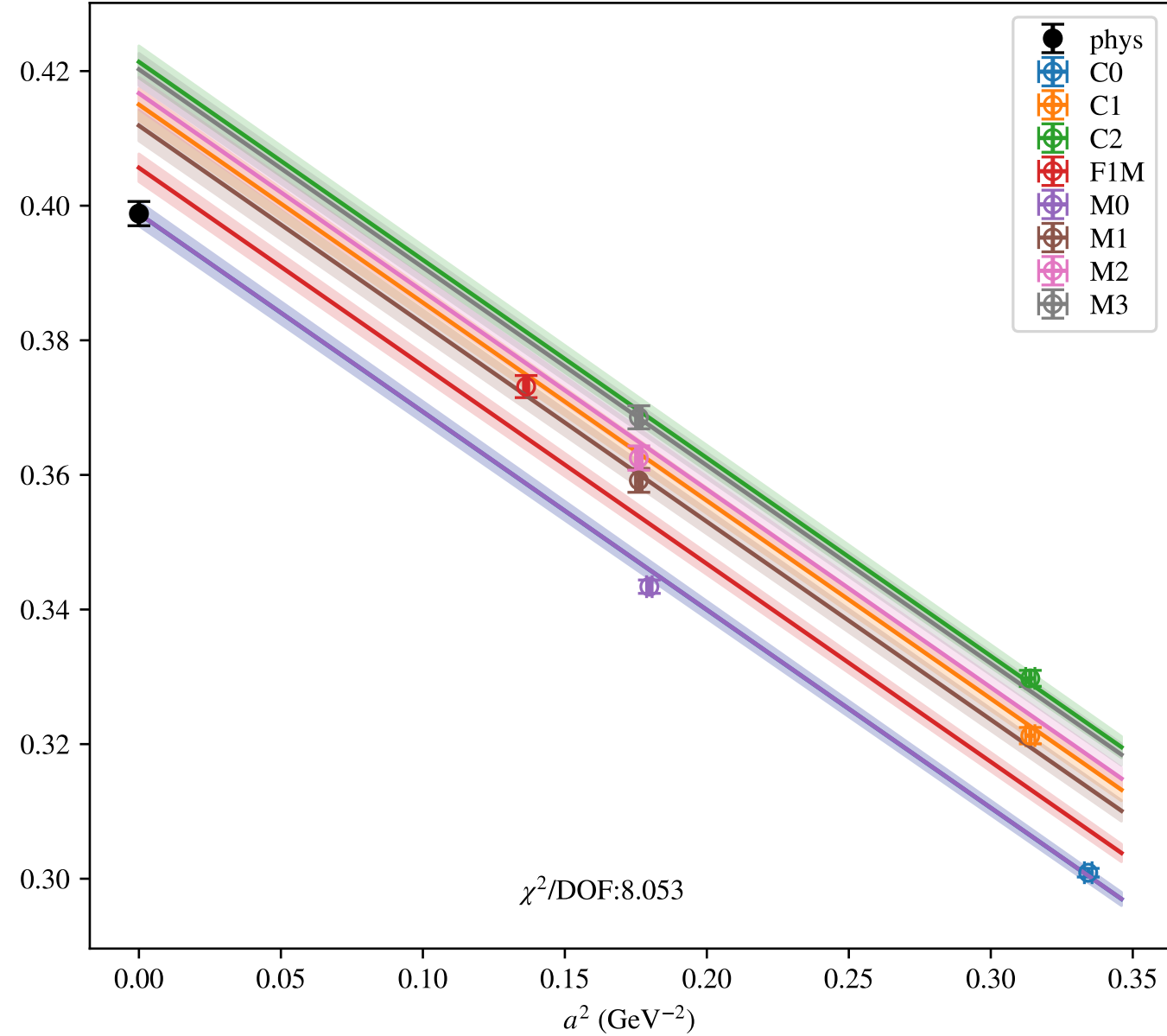


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

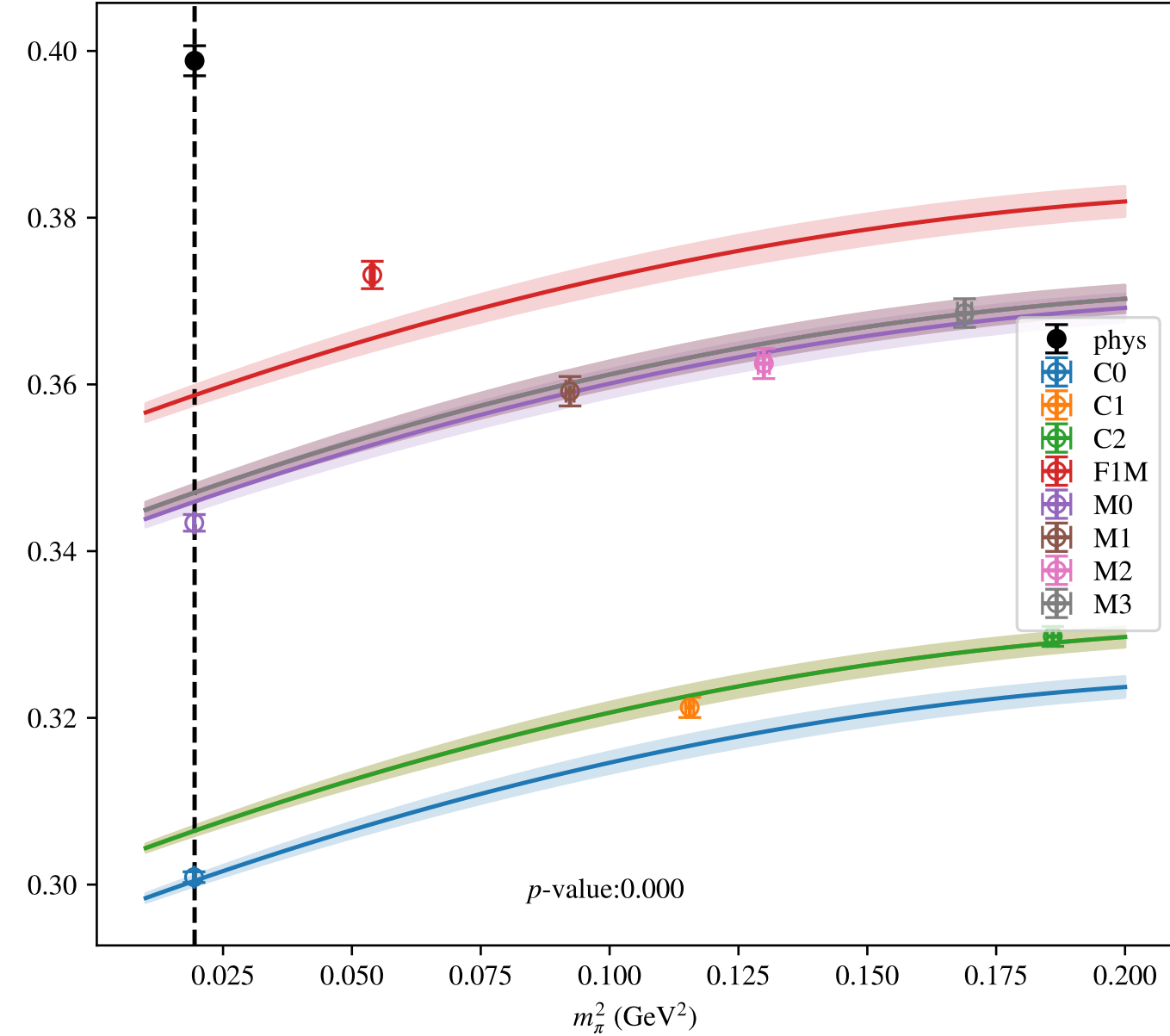


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.3 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

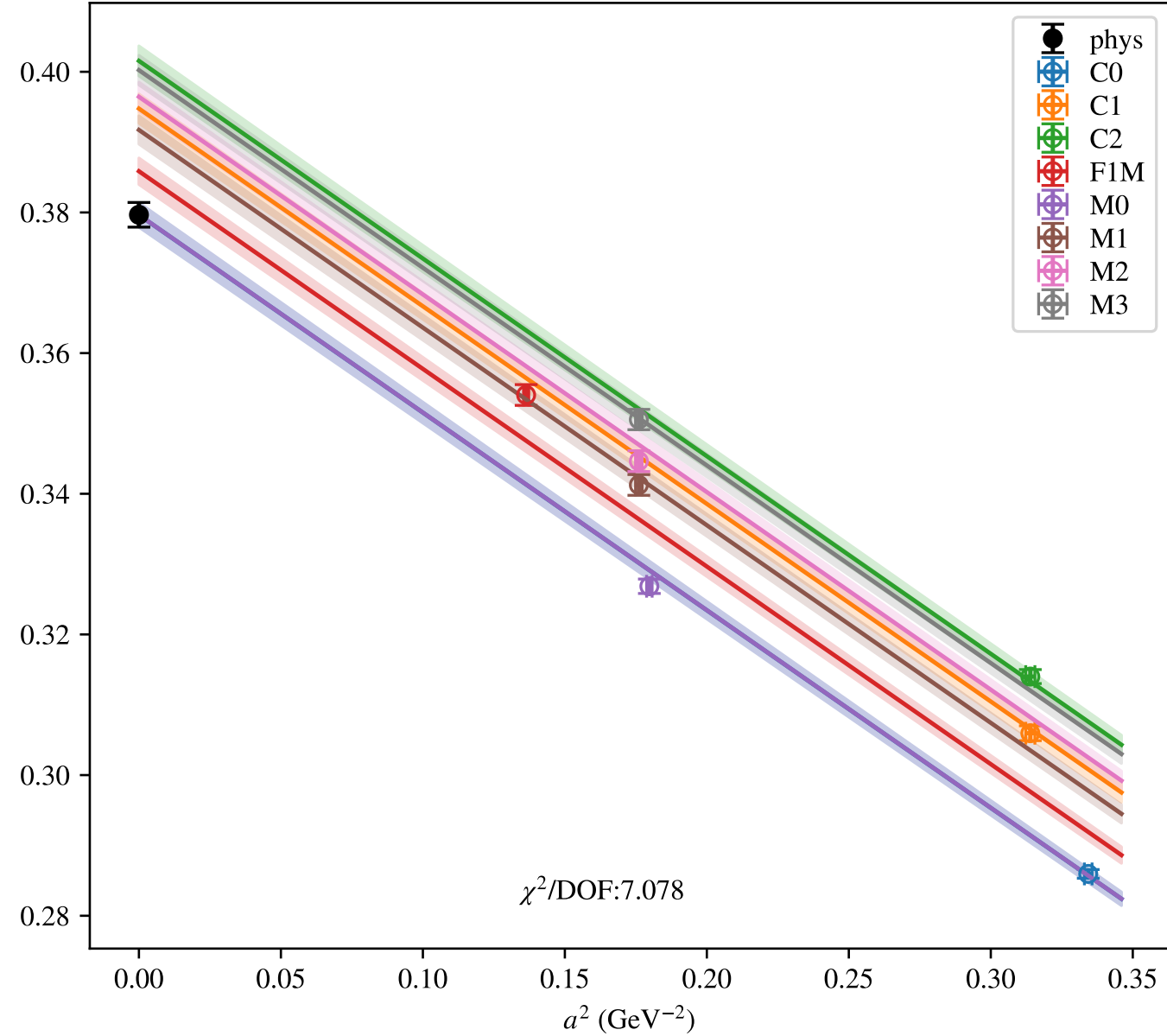


$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$

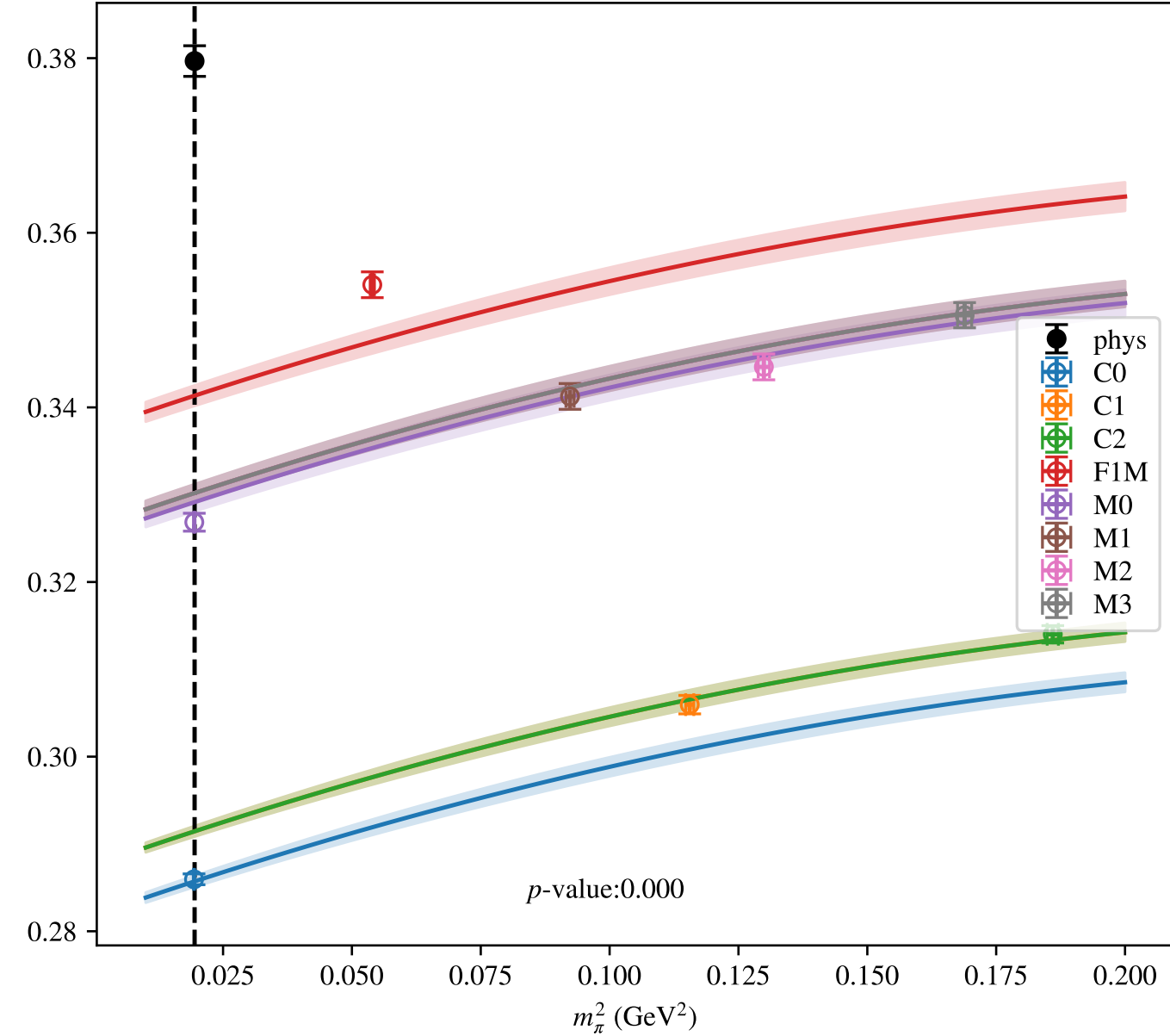


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.4 \text{ GeV}$$

$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$



$(-0.5)*\text{SSpPP} + (0.5)*\text{TT}$



4 B_4

| μ (GeV) | a^2, m_π^2 | a^2, m_π^2 (no C) | a^2, a^4, m_π^2 | a^2, m_π^2, m_π^4 |
|-------------|---------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| 2.0 | 0.9140(12) : 5.64 (0.0) | 0.8853(65) : 3.392 (0.034) | 0.868(10) : 2.094 (0.079) | 0.9159(14) : 4.752 (0.001) |
| 2.2 | 0.9104(12) : 5.627 (0.0) | 0.8803(63) : 2.059 (0.128) | 0.861(10) : 1.37 (0.242) | 0.9121(14) : 5.199 (0.0) |
| 2.3 | 0.9090(12) : 5.971 (0.0) | 0.8781(63) : 2.161 (0.115) | 0.859(10) : 1.532 (0.19) | 0.9108(13) : 5.437 (0.0) |
| 2.4 | 0.9080(12) : 6.697 (0.0) | 0.8757(62) : 2.496 (0.082) | 0.8559(99) : 1.713 (0.144) | 0.9100(13) : 6.06 (0.0) |

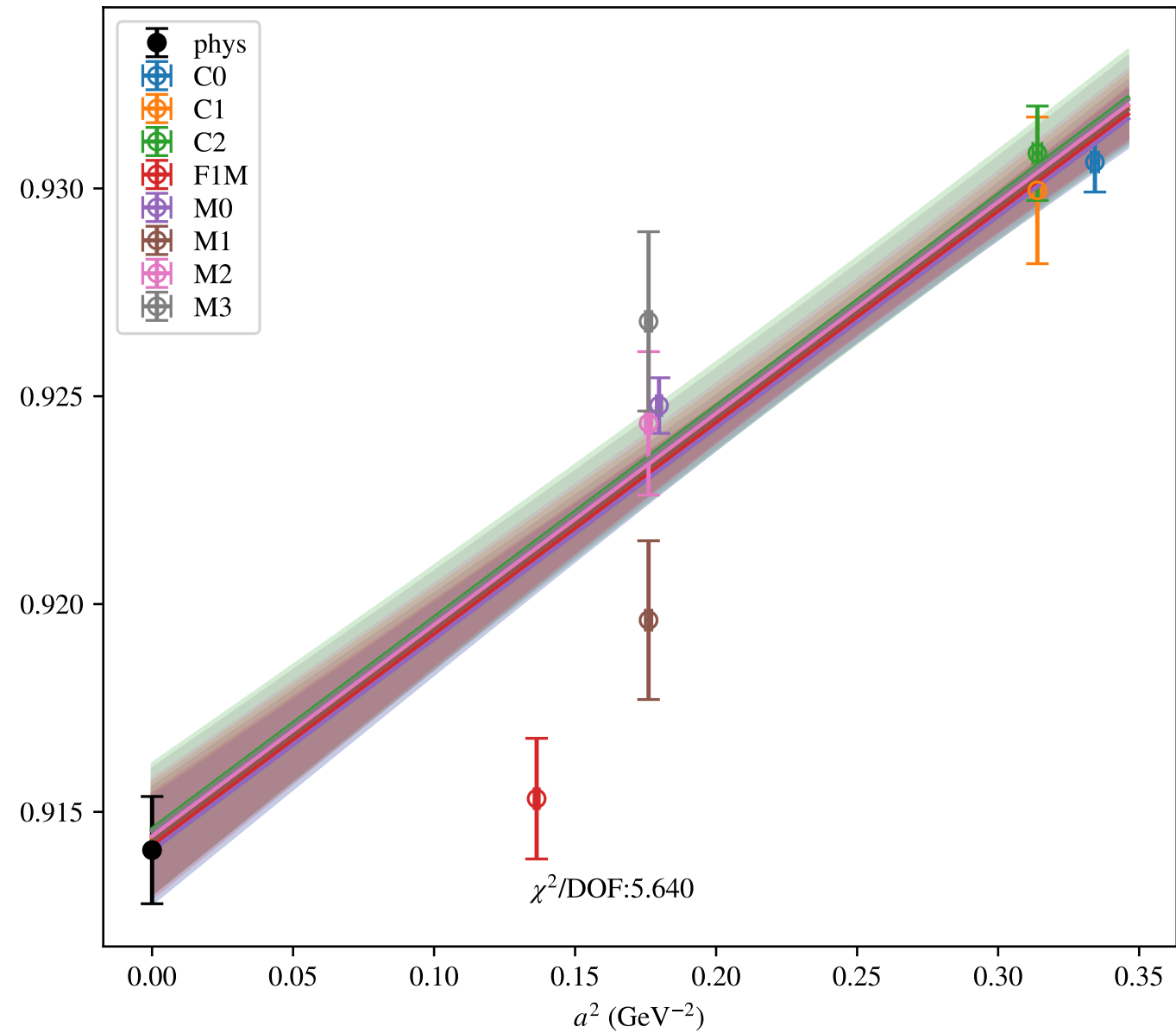
Table 7: Physical point value from chiral and continuum extrapolation at renormalisation scale μ . Entries are **value(error)**: χ^2/DOF (p -value).

| μ (GeV) | | a^2, m_π^2 | a^2, m_π^2 (no C) | a^2, a^4, m_π^2 | a^2, m_π^2, m_π^4 |
|-------------|----------|----------------|-----------------------|---------------------|-------------------------|
| 2.0 | α | 0.0556(54) | 0.246(43) | 0.53(11) | 0.0486(59) |
| | β | 0.0 | 0.0 | -0.0001(14) | -0.0019(63) |
| 2.2 | α | 0.0542(53) | 0.254(42) | 0.56(11) | 0.0480(58) |
| | β | -0.0002(12) | -0.0002(22) | -0.0004(13) | -0.0019(66) |
| 2.3 | α | 0.0544(52) | 0.261(42) | 0.57(11) | 0.0477(57) |
| | β | -0.0003(11) | -0.0003(21) | -0.0005(13) | -0.0020(64) |
| 2.4 | α | 0.0540(52) | 0.271(42) | 0.60(11) | 0.0468(57) |
| | β | -0.0003(11) | -0.0003(20) | -0.0005(12) | -0.0022(62) |

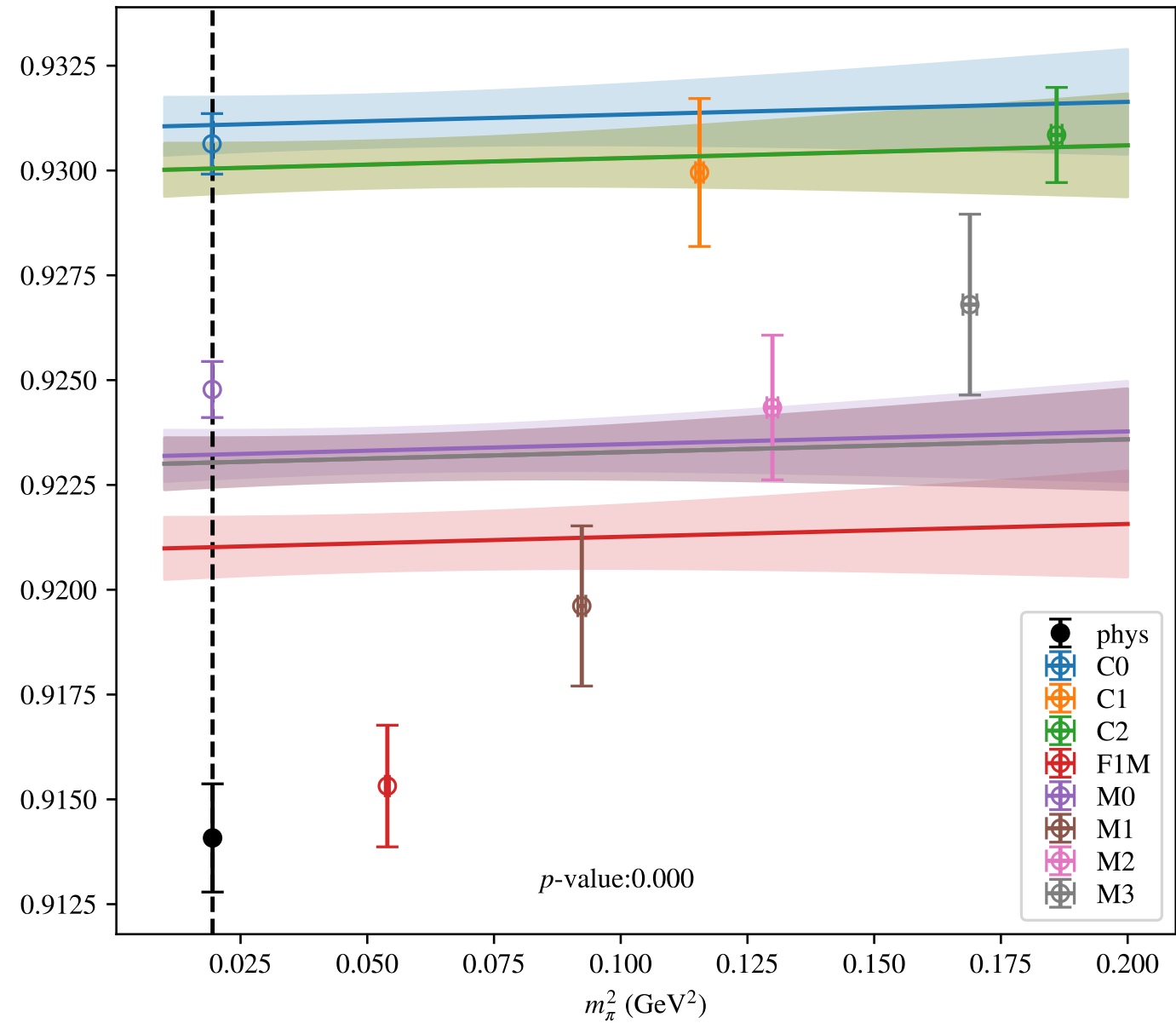
Table 8: Fit values of coefficients in $B = B_{phys} + \alpha a^2 + \beta \left(\frac{m_\pi^2}{f_\pi^2} - \frac{m_{\pi,PDG}^2}{f_\pi^2} \right) + \dots$

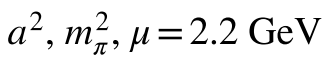
$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$

SSmPP



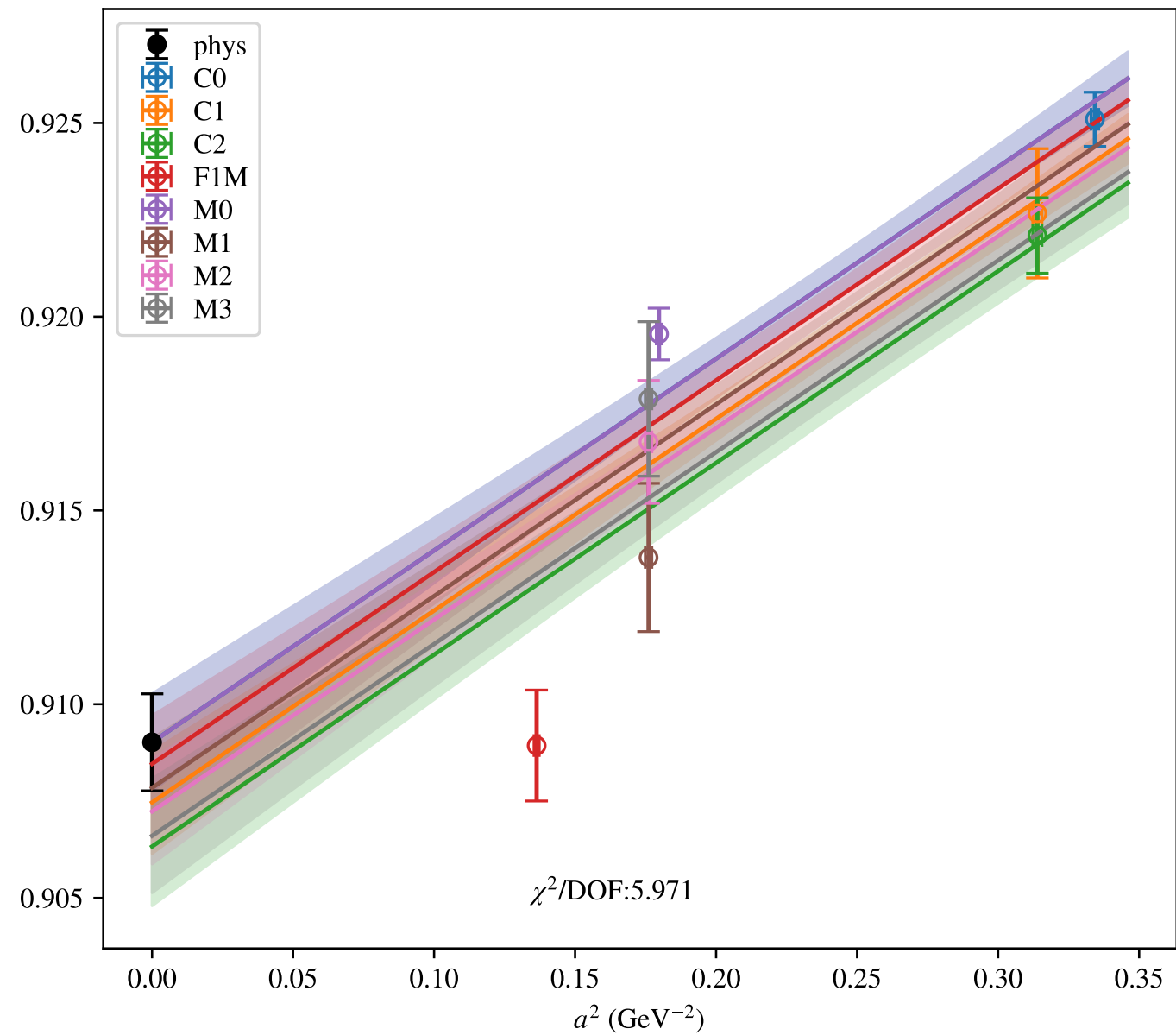
SSmPP



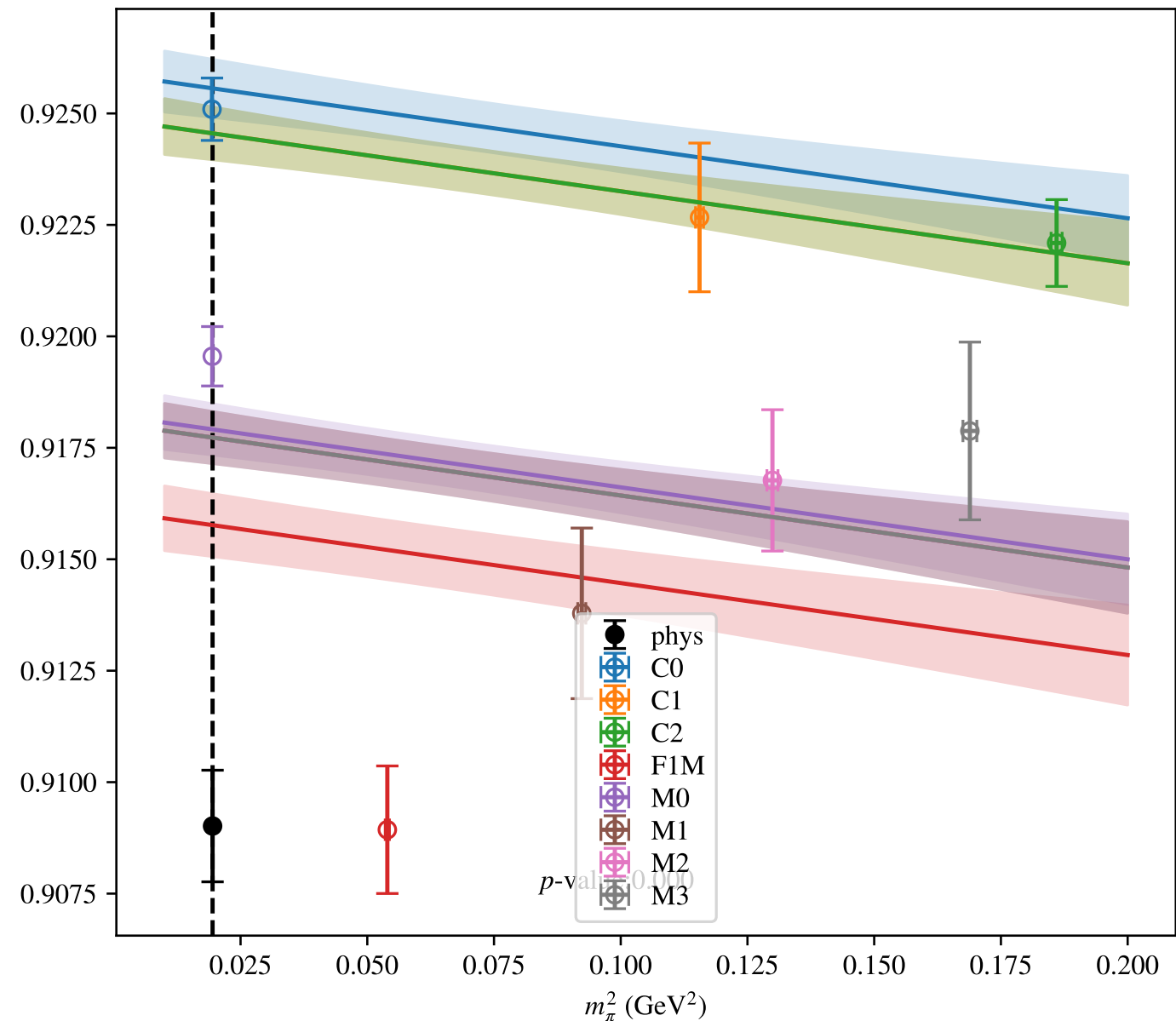
$a^2, m_\pi^2, \mu = 2.2 \text{ GeV}$ 

$$a^2, m_\pi^2, \mu = 2.3 \text{ GeV}$$

SSmPP

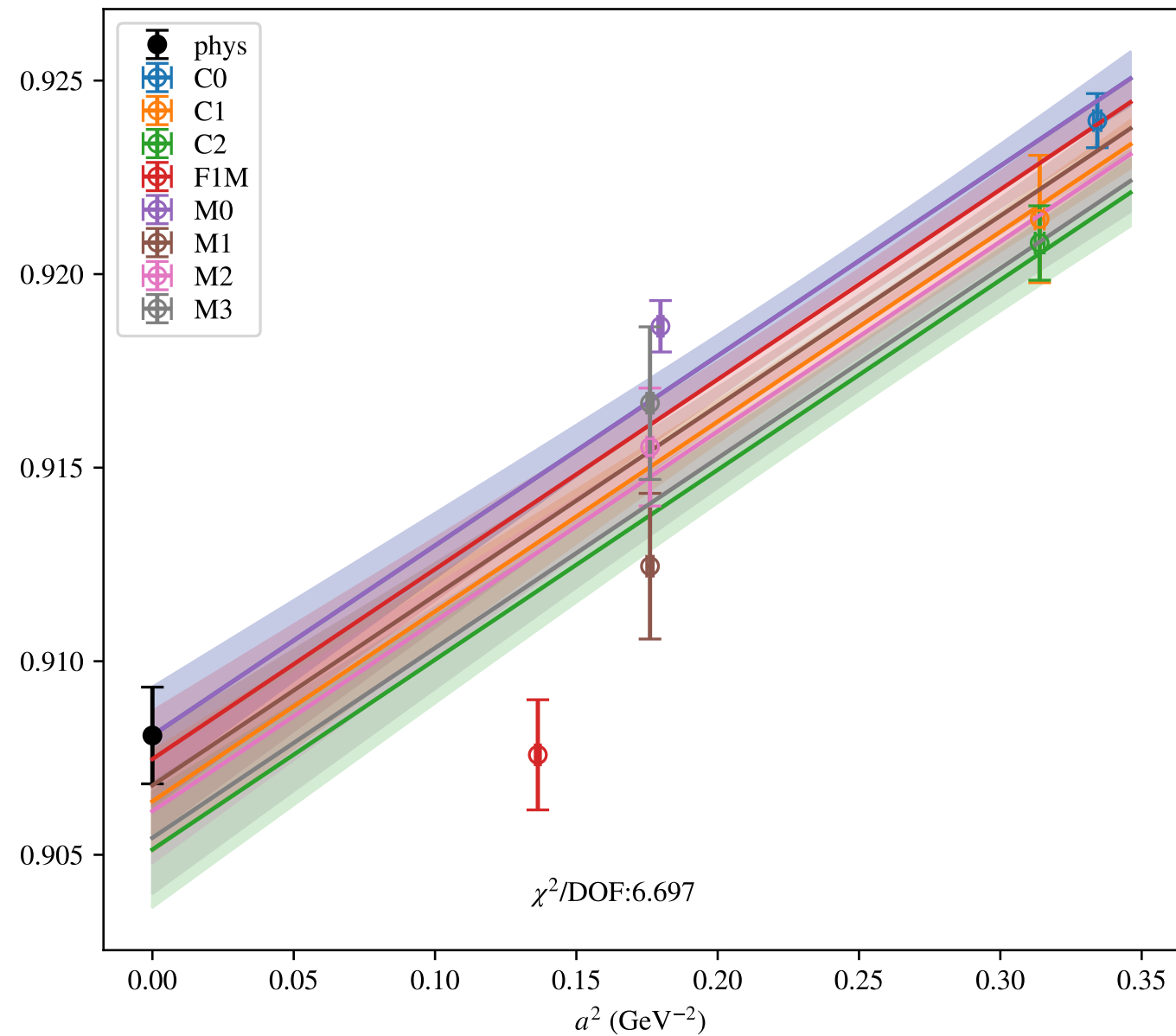


SSmPP

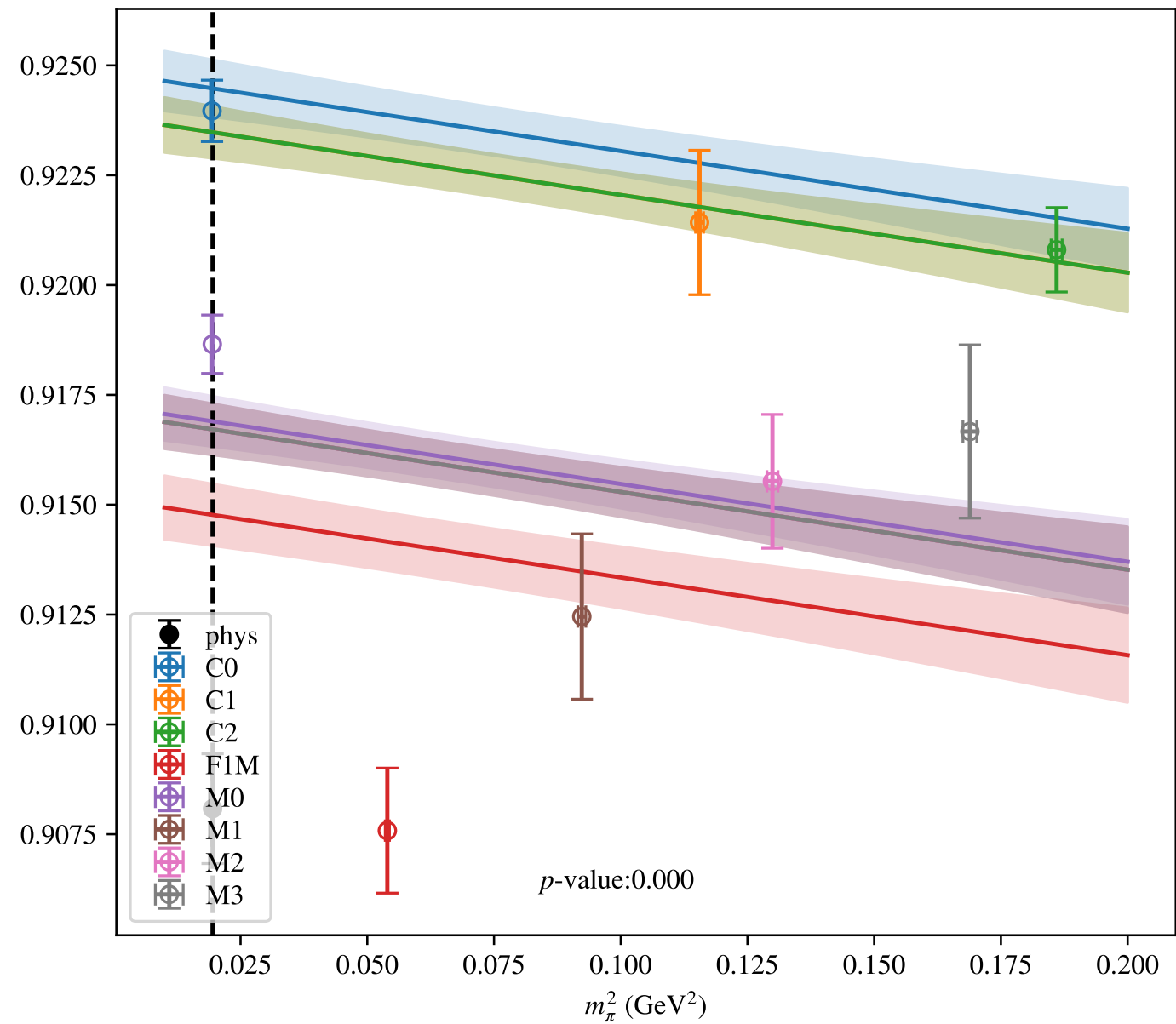


$$a^2, m_\pi^2, \mu = 2.4 \text{ GeV}$$

SSmPP

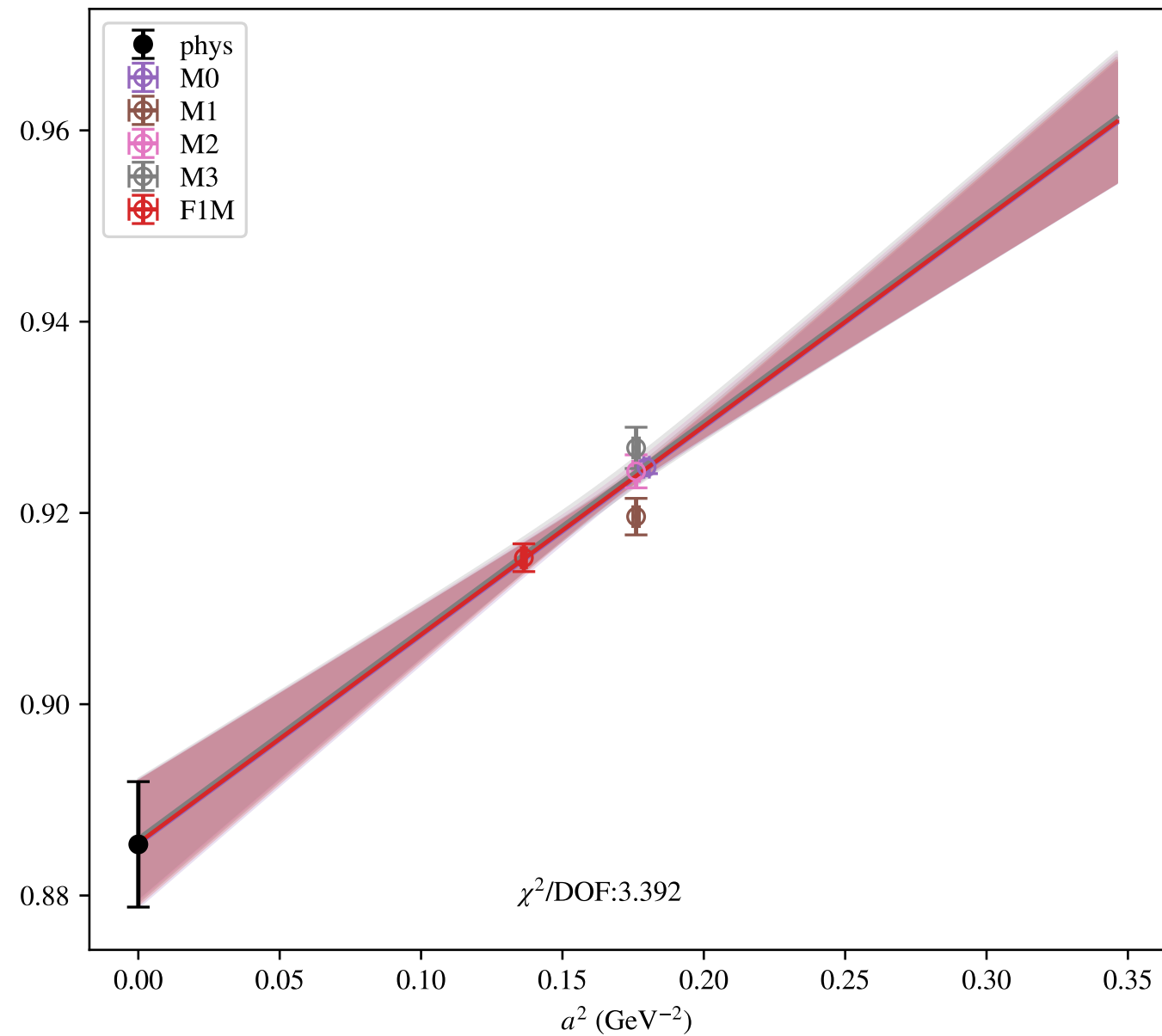


SSmPP

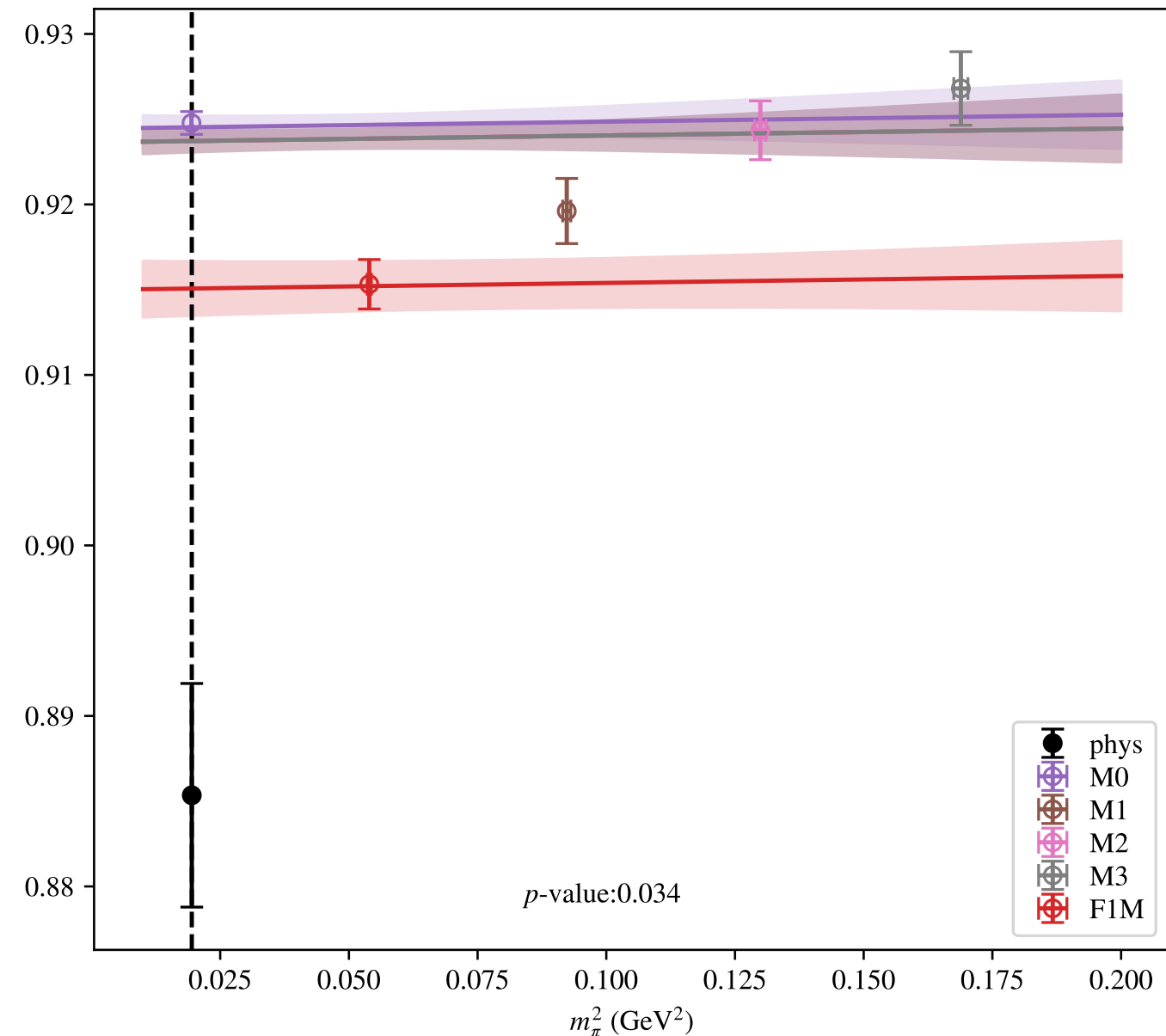


a^2, m_π^2 (no C), $\mu = 2.0$ GeV

SSmPP

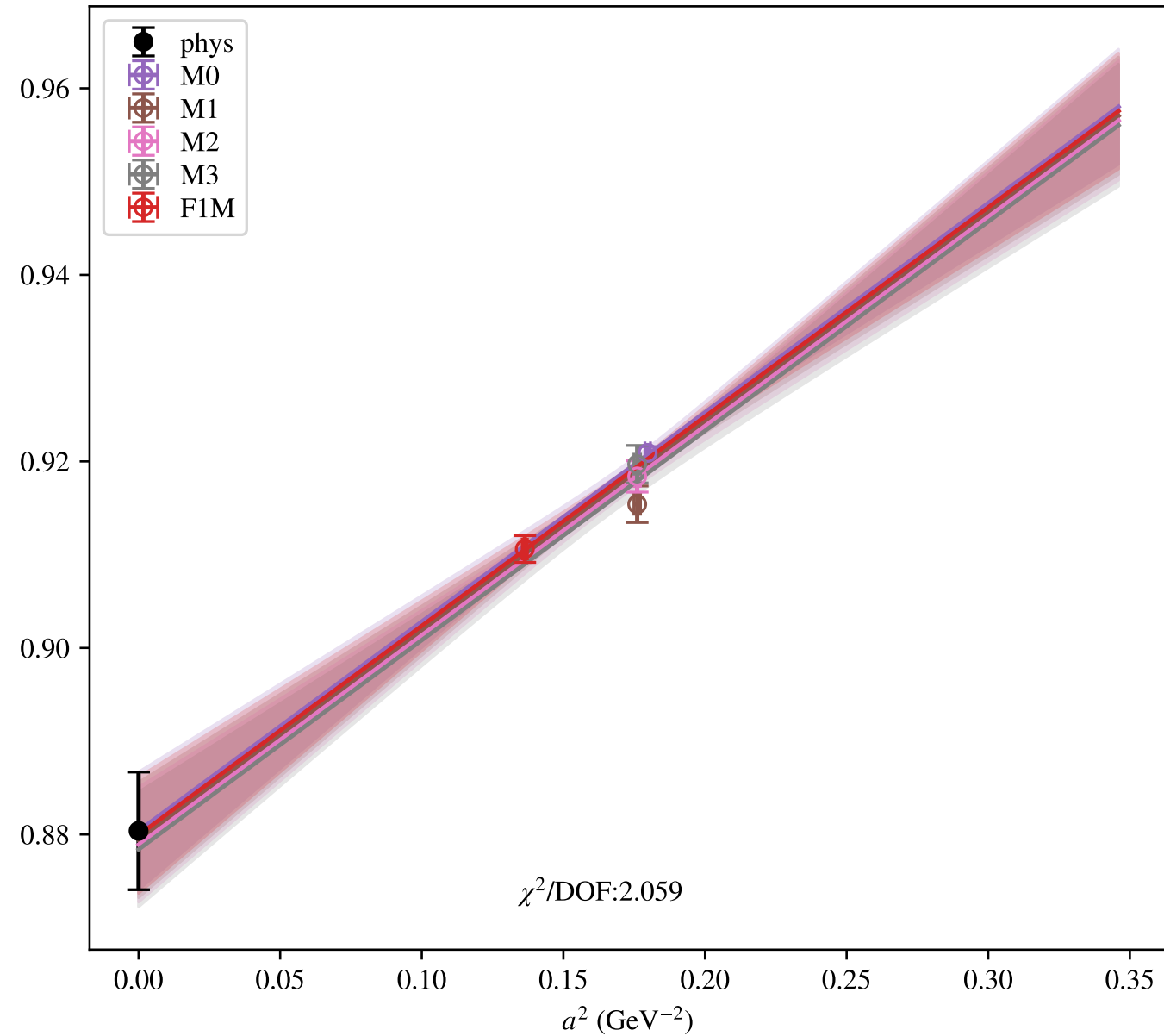


SSmPP

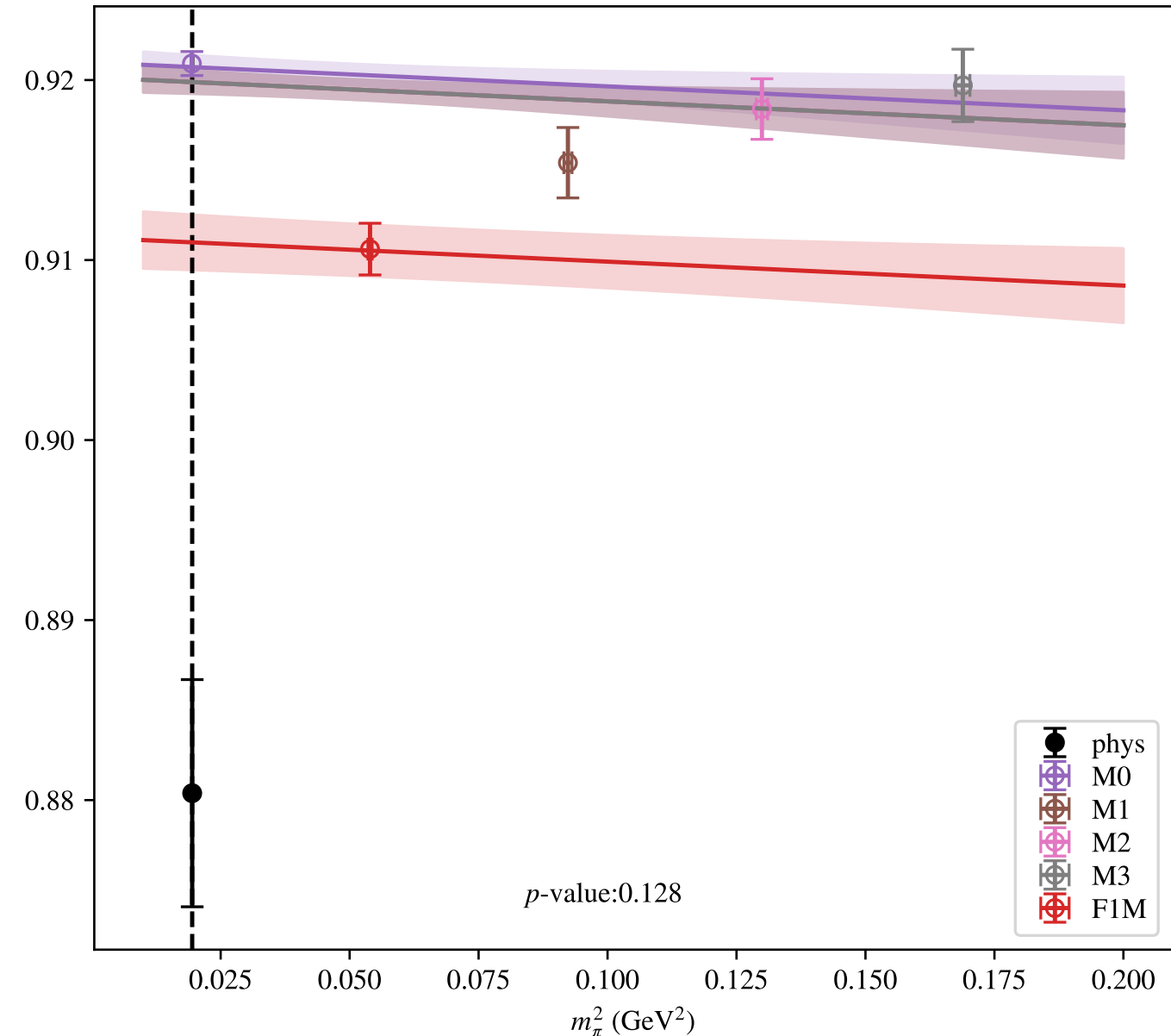


a^2, m_π^2 (no C), $\mu = 2.2$ GeV

SSmPP

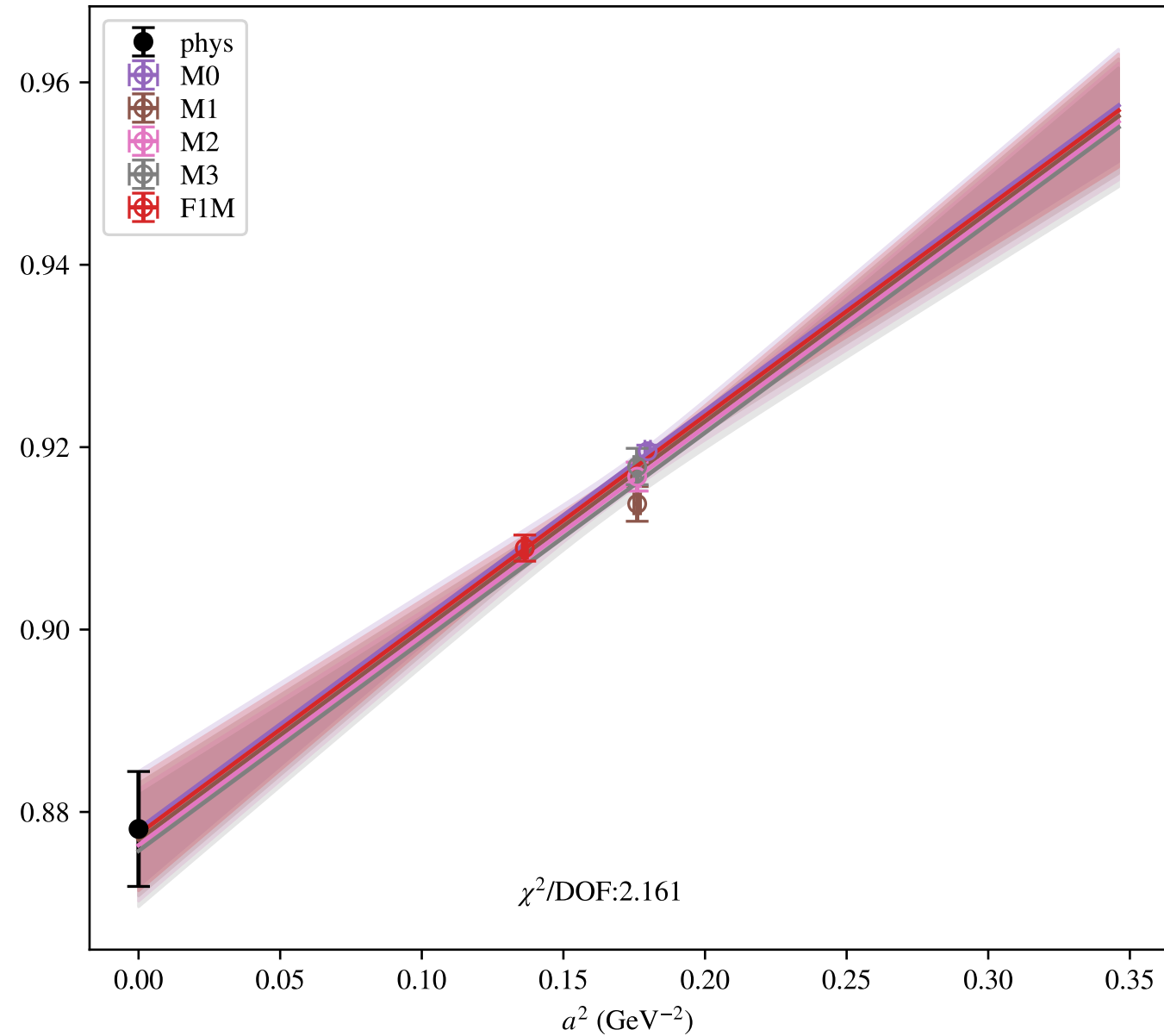


SSmPP

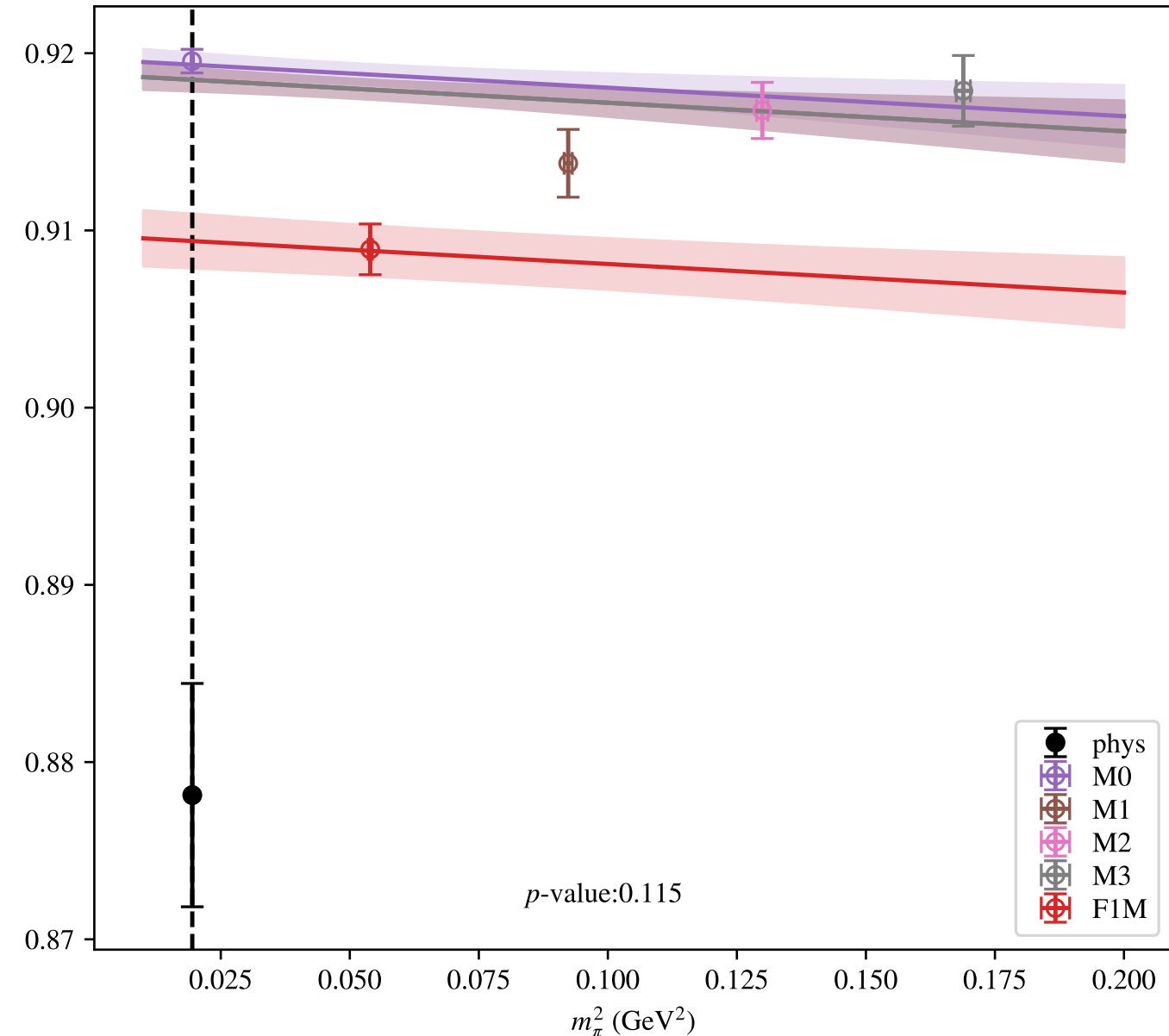


a^2, m_π^2 (no C), $\mu = 2.3$ GeV

SSmPP

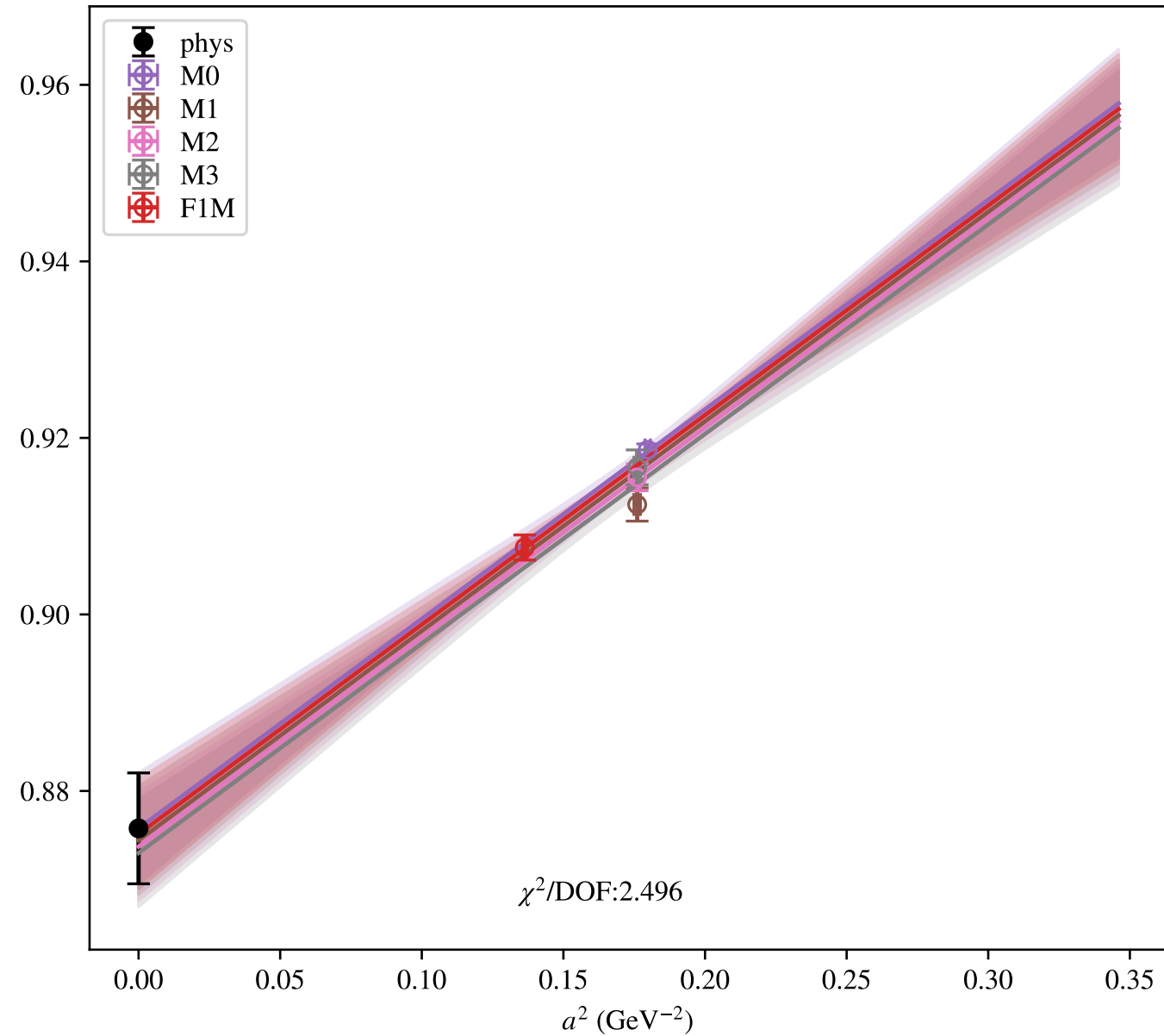


SSmPP

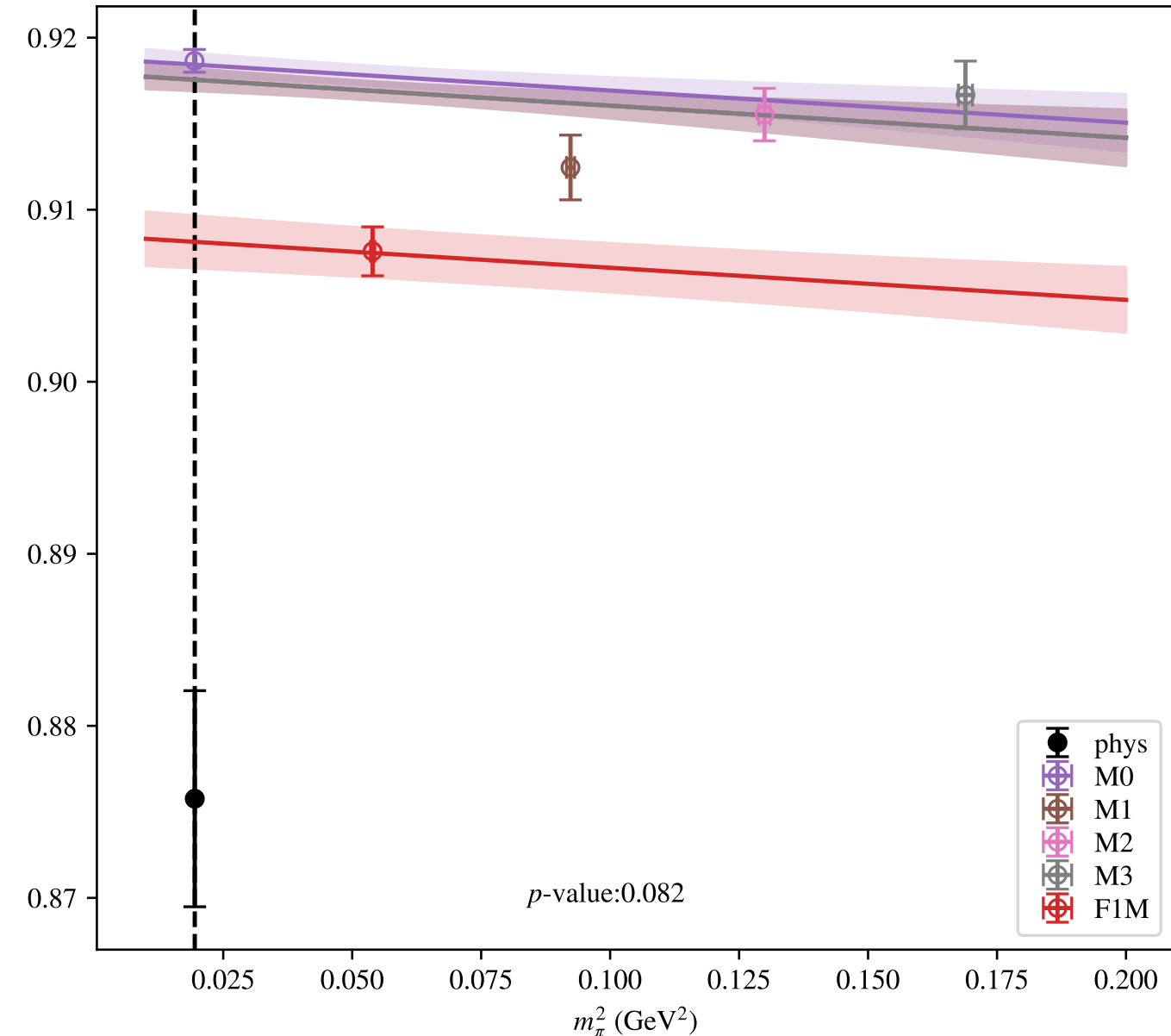


a^2, m_π^2 (no C), $\mu = 2.4$ GeV

SSmPP

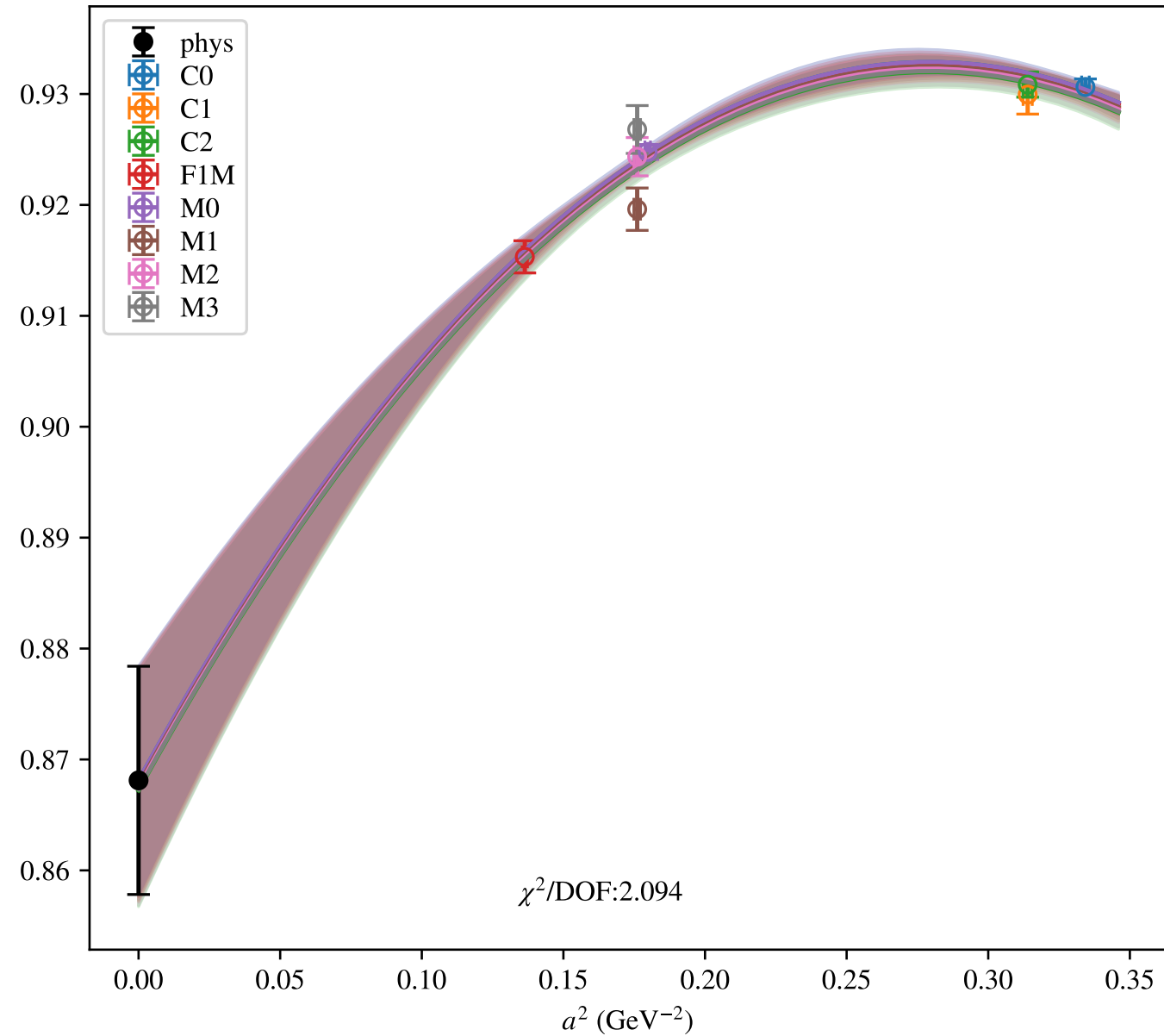


SSmPP

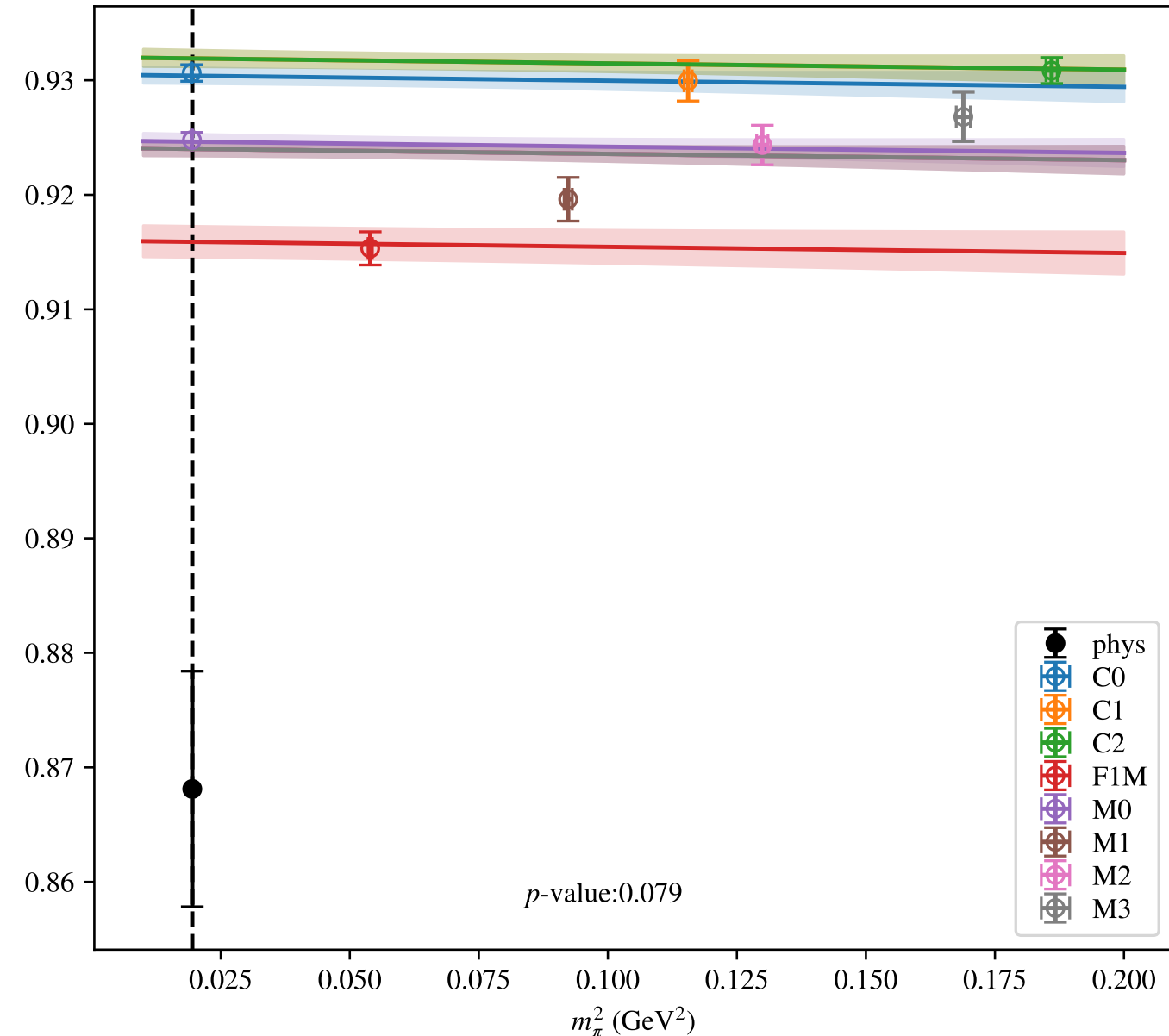


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

SSmPP

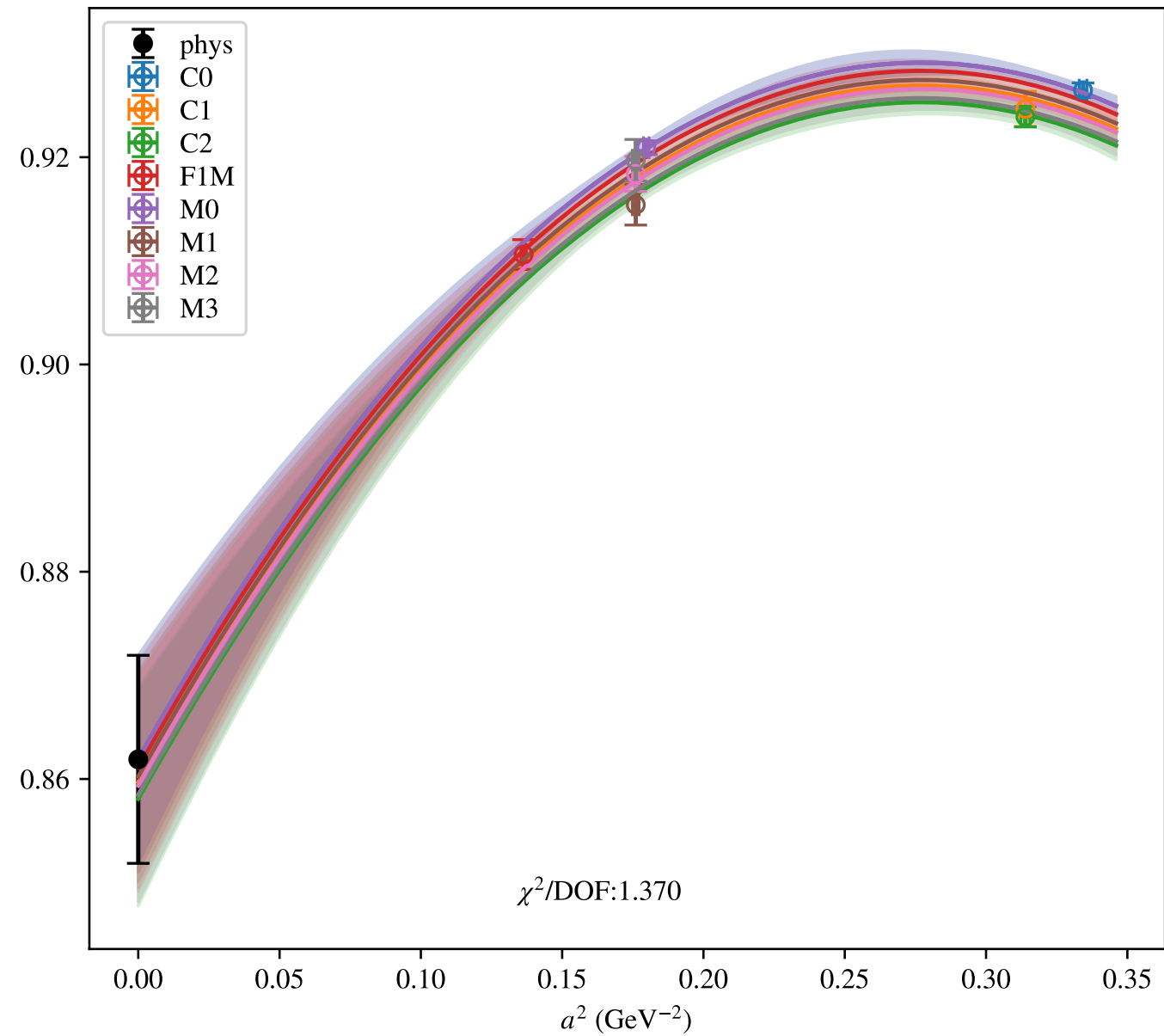


SSmPP

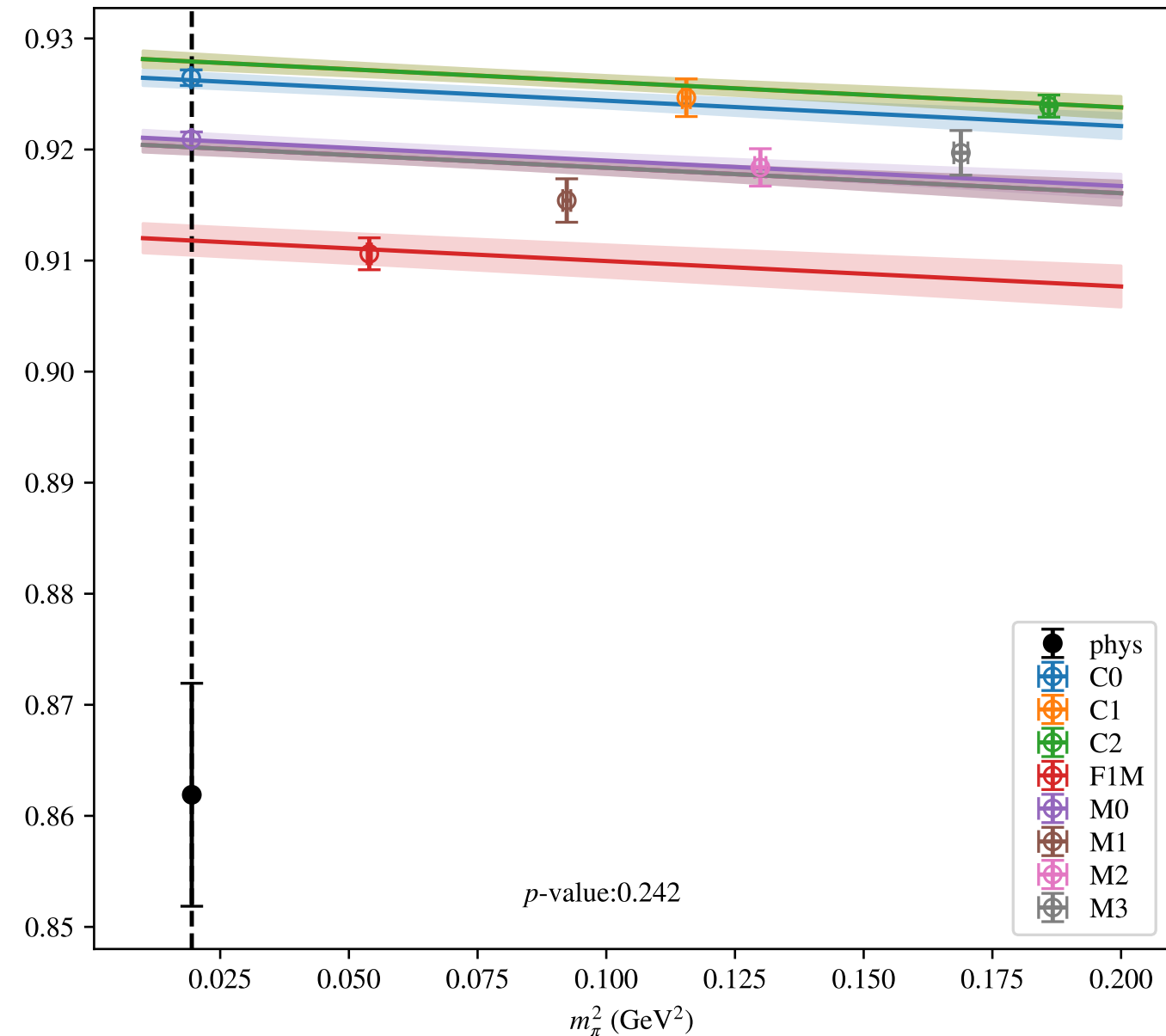


$$a^2, a^4, m_\pi^2, \mu = 2.2 \text{ GeV}$$

SSmPP

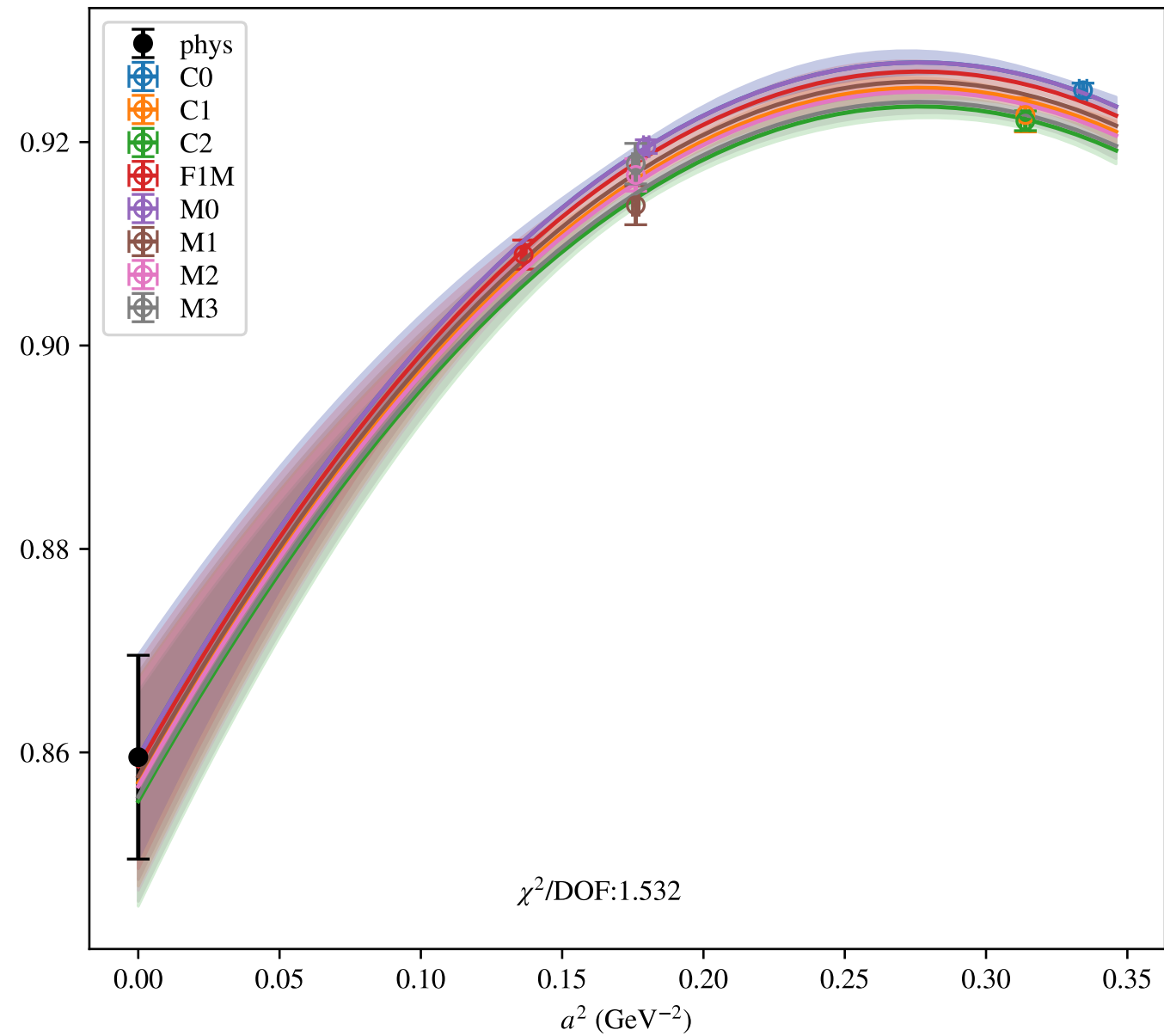


SSmPP

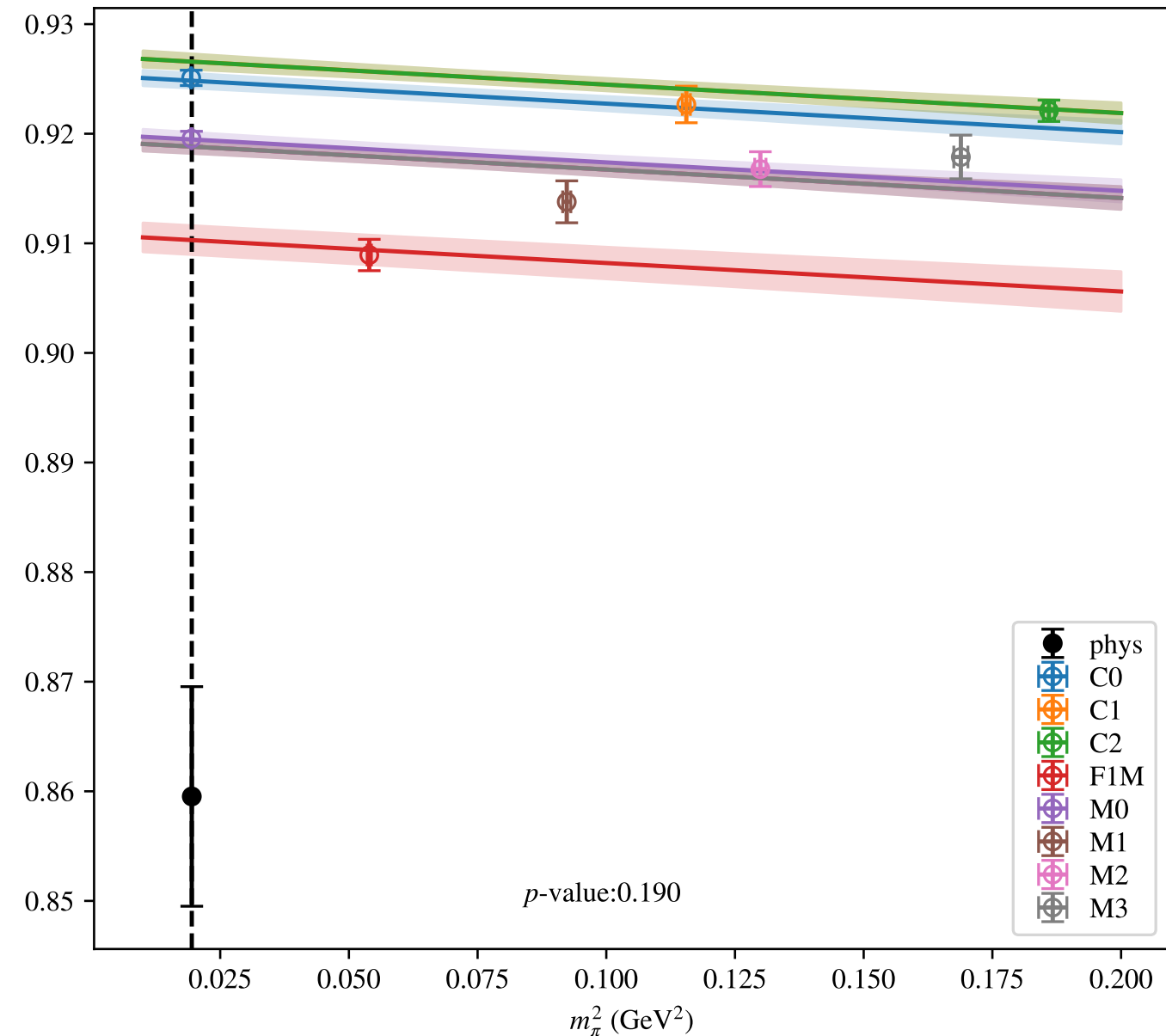


$$a^2, a^4, m_\pi^2, \mu = 2.3 \text{ GeV}$$

SSmPP

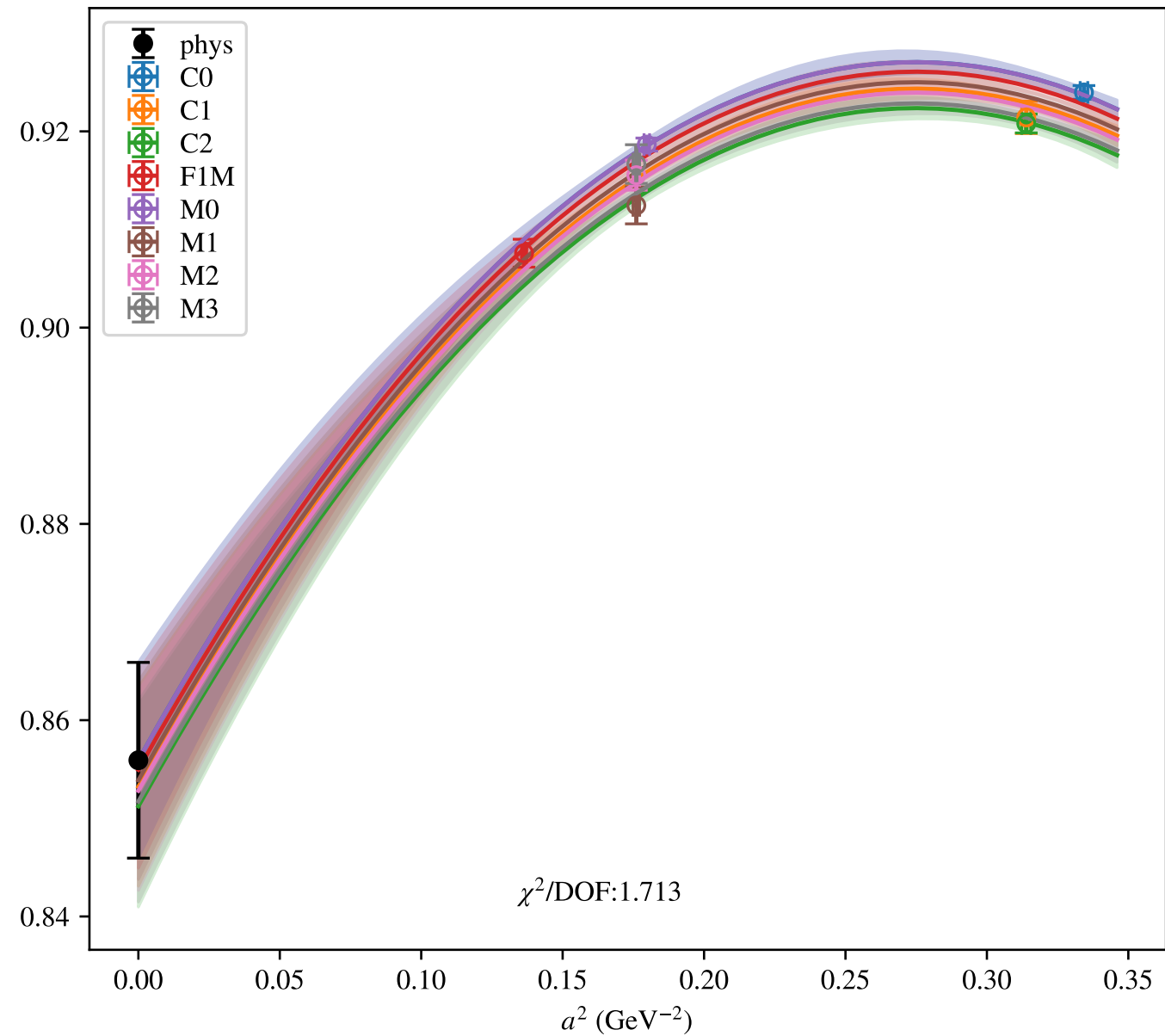


SSmPP

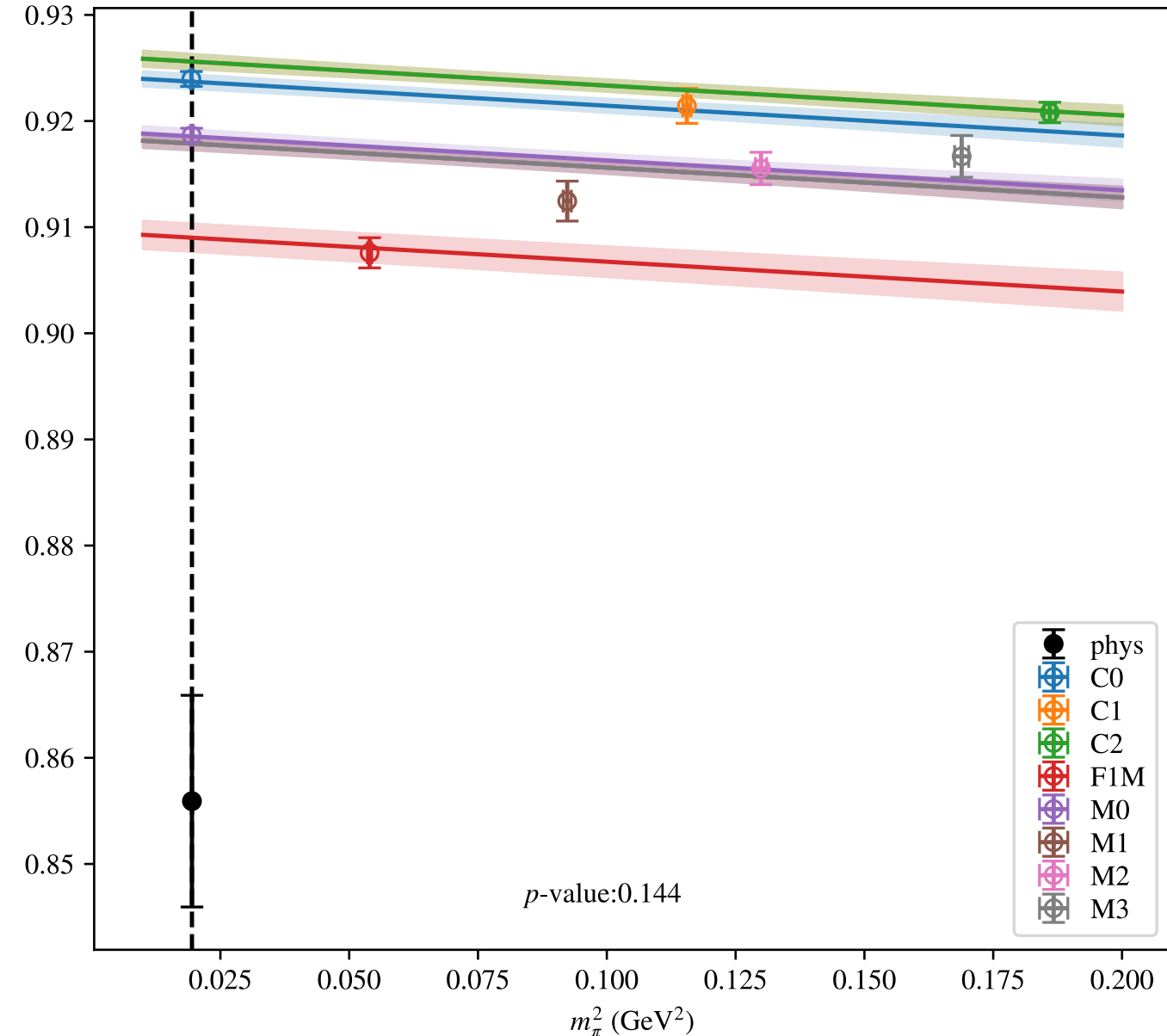


$$a^2, a^4, m_\pi^2, \mu = 2.4 \text{ GeV}$$

SSmPP

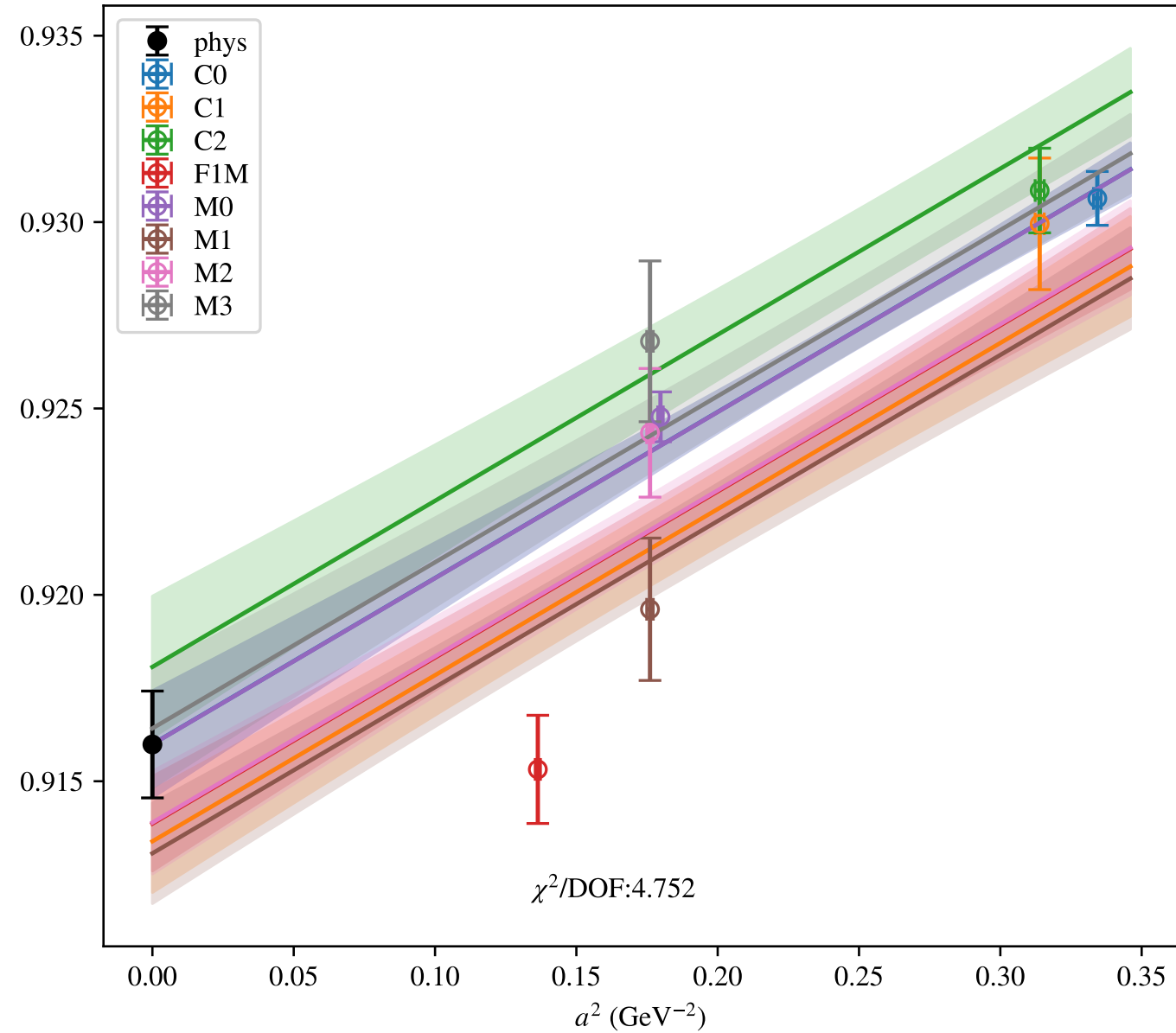


SSmPP

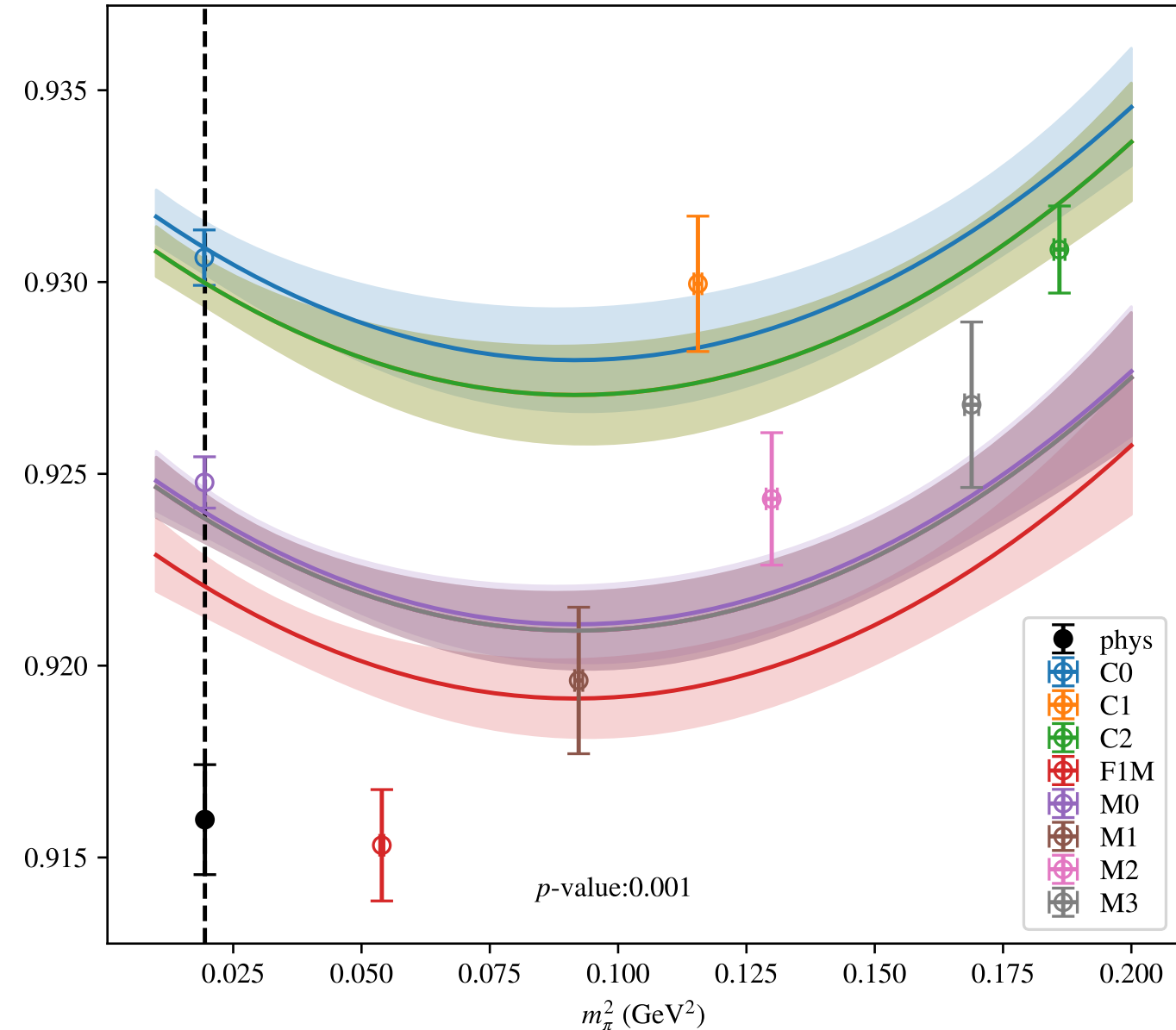


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

SSmPP

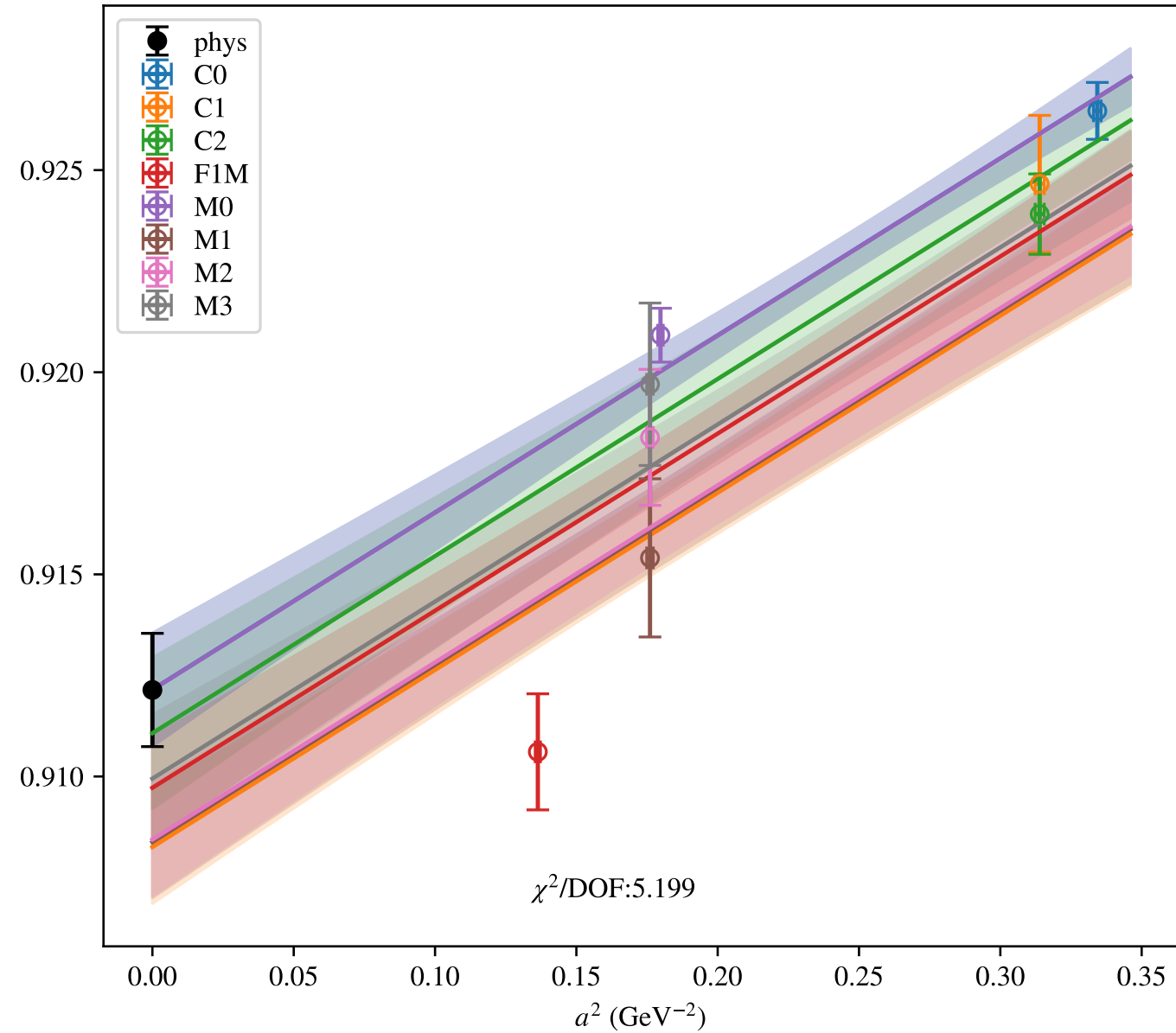


SSmPP

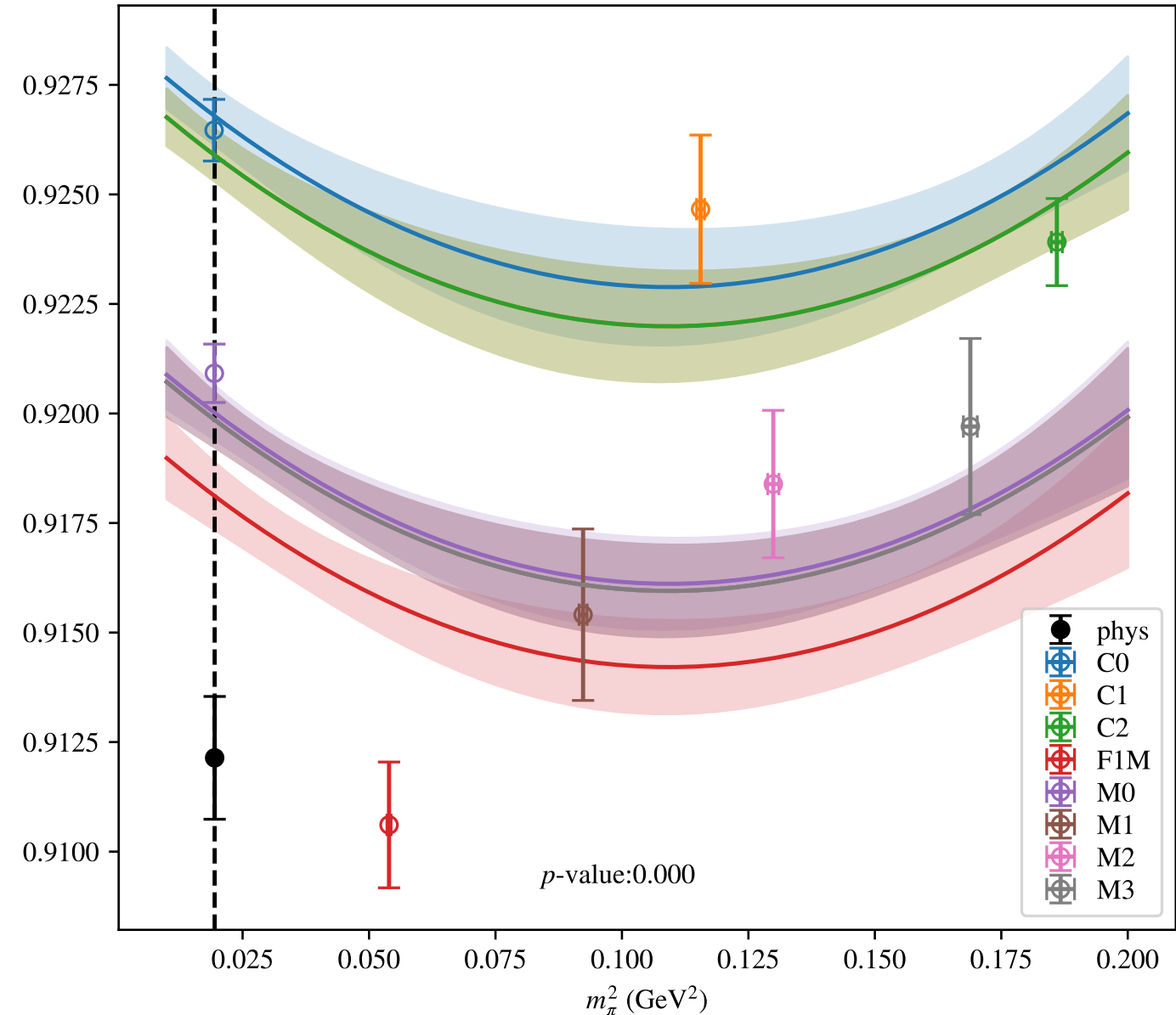


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.2 \text{ GeV}$$

SSmPP

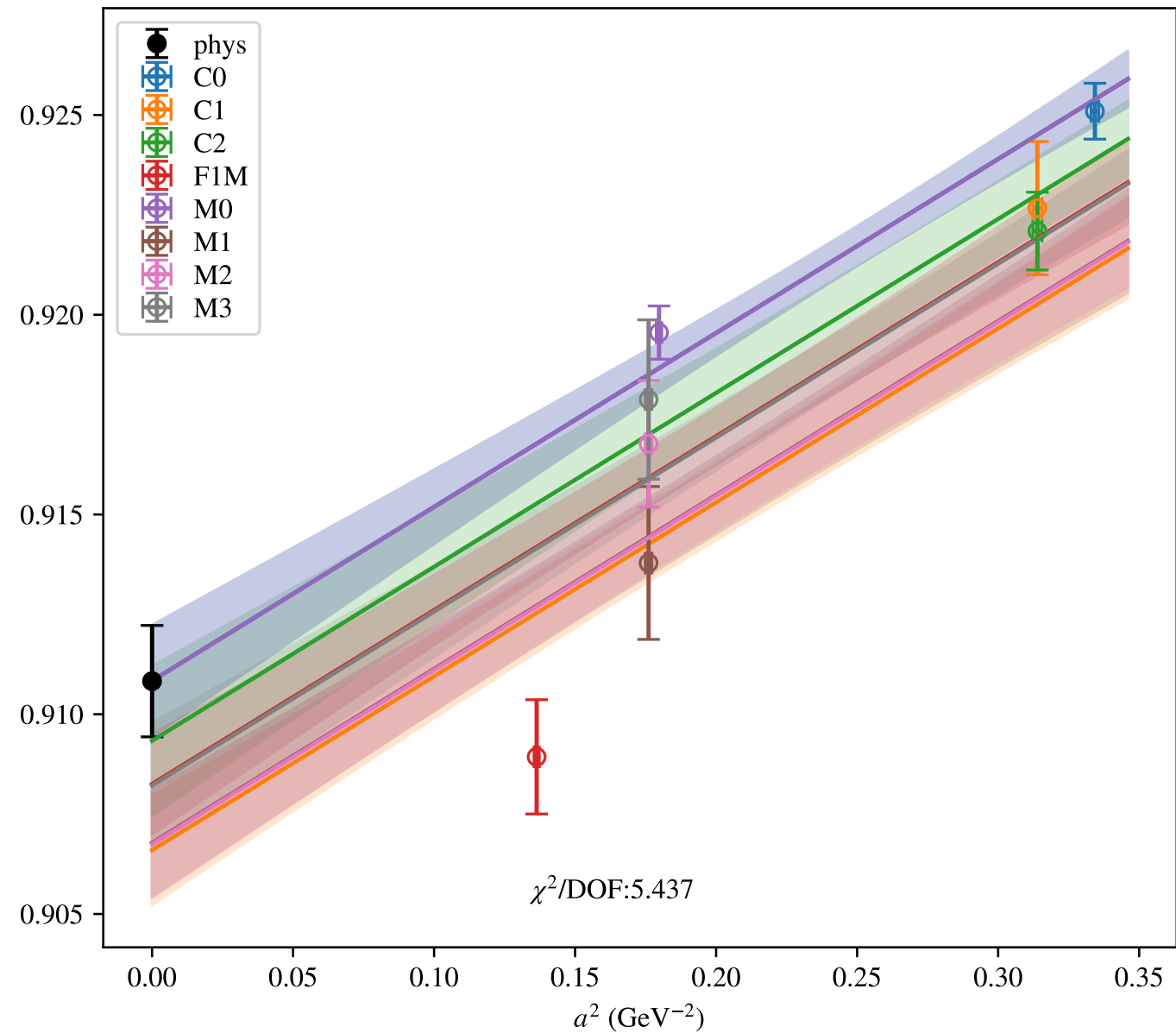


SSmPP

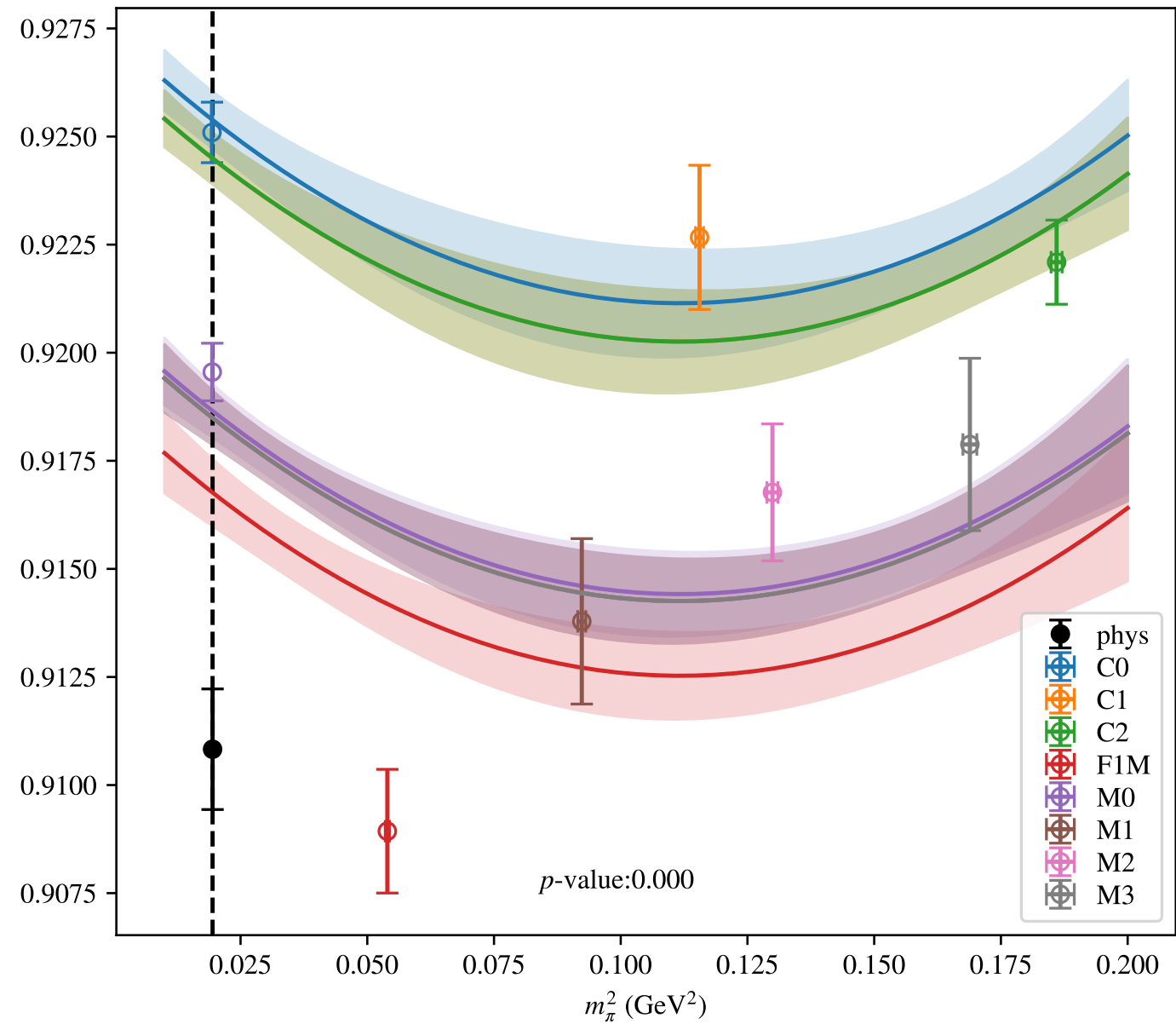


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.3 \text{ GeV}$$

SSmPP

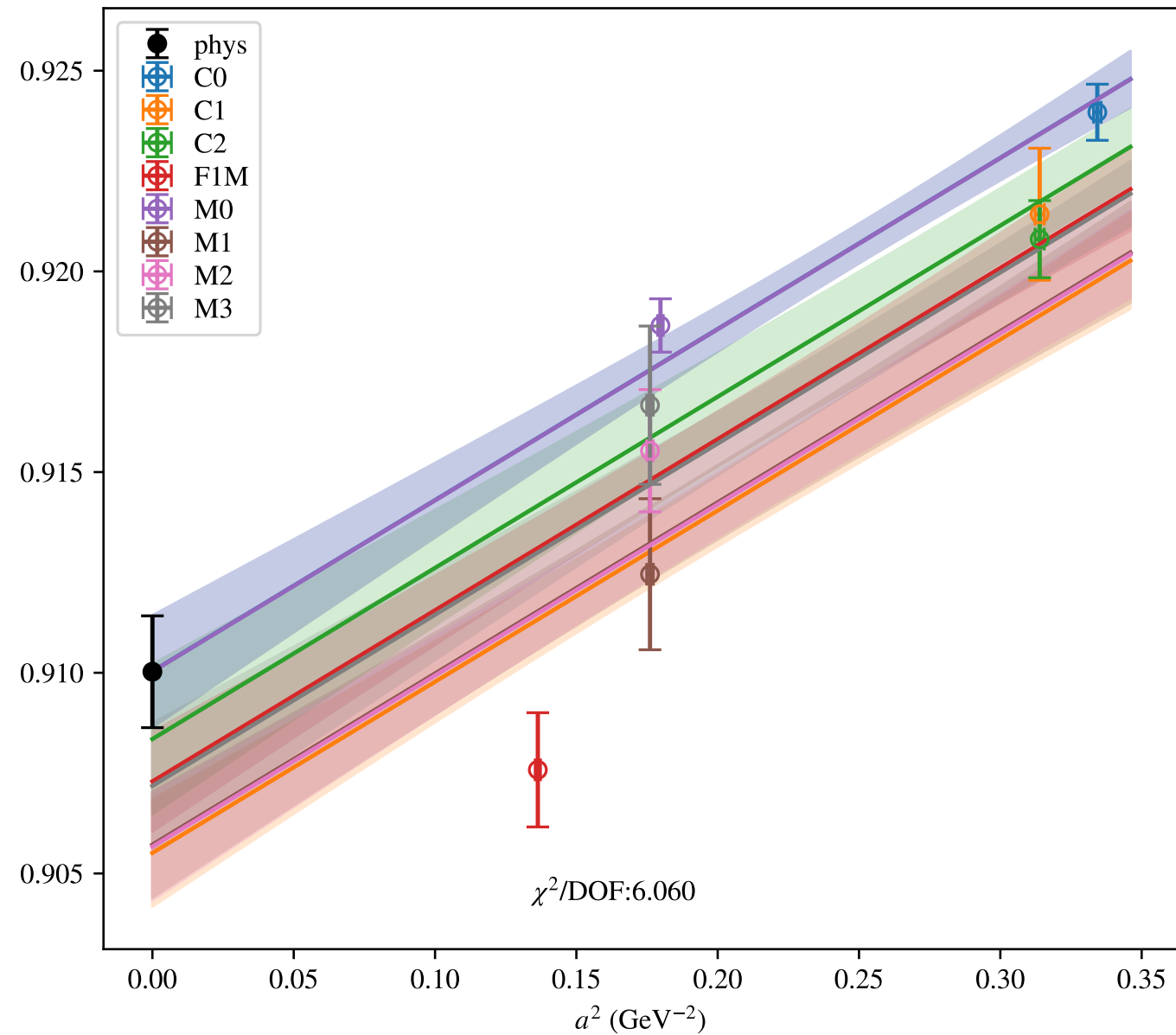


SSmPP

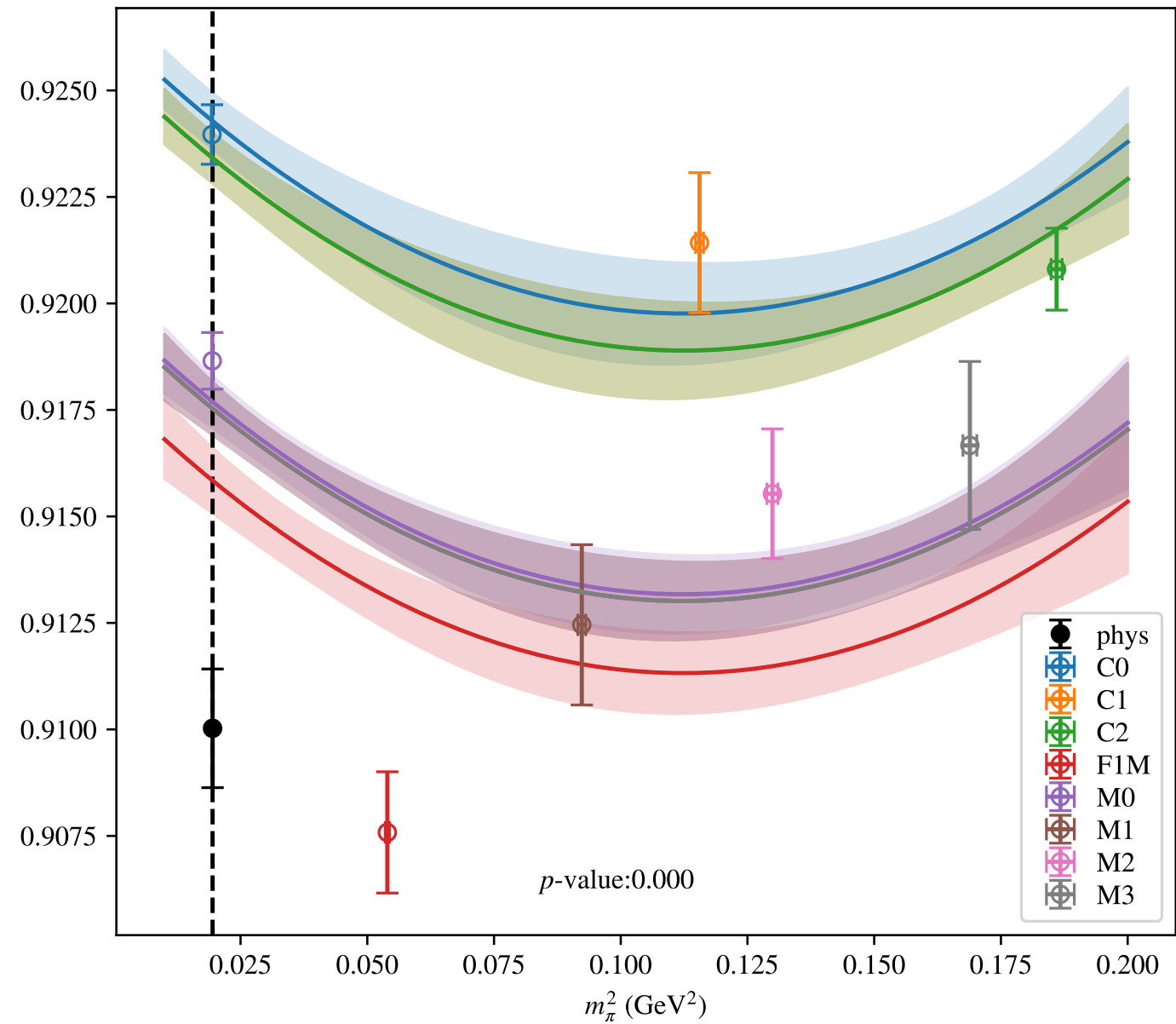


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.4 \text{ GeV}$$

SSmPP



SSmPP



5 B_5

| μ (GeV) | a^2, m_π^2 | a^2, m_π^2 (no C) | a^2, a^4, m_π^2 | a^2, m_π^2, m_π^4 |
|-------------|------------------------------------|-----------------------------------|-----------------------------------|------------------------------------|
| 2.0 | 0.5412(10) : 0.775 (0.568) | 0.5415(42) : 1.461 (0.232) | 0.5447(66) : 0.917 (0.453) | 0.54157(96) : 0.839 (0.5) |
| 2.2 | 0.5213(10) : 0.499 (0.777) | 0.5215(41) : 1.153 (0.316) | 0.5231(64) : 0.611 (0.654) | 0.52163(95) : 0.524 (0.718) |
| 2.3 | 0.51291(97) : 0.588 (0.709) | 0.5130(40) : 1.295 (0.274) | 0.5151(63) : 0.713 (0.583) | 0.51317(90) : 0.623 (0.646) |
| 2.4 | 0.50605(95) : 0.829 (0.529) | 0.5043(39) : 1.935 (0.144) | 0.5046(60) : 1.026 (0.392) | 0.50648(90) : 0.765 (0.548) |

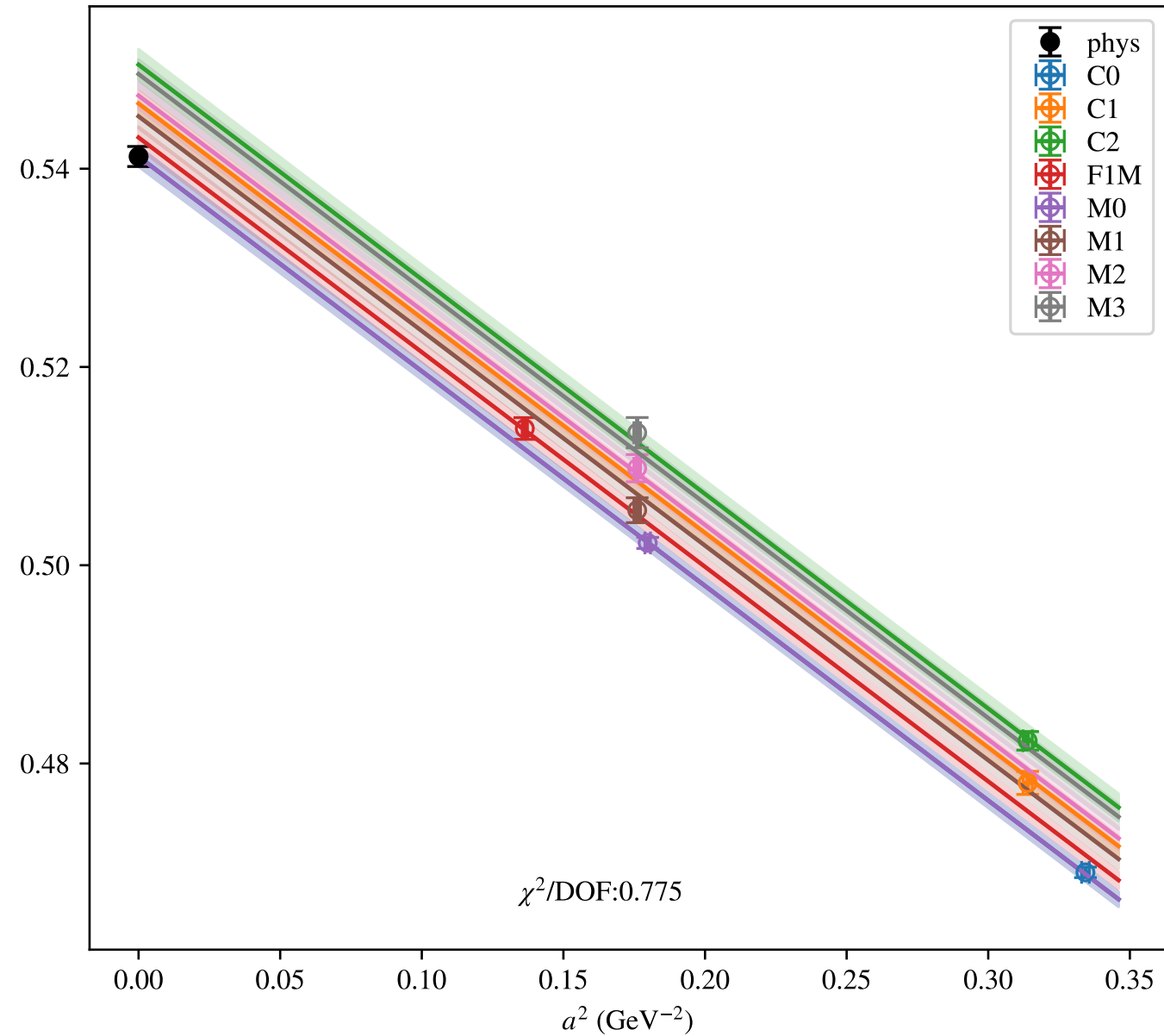
Table 9: Physical point value from chiral and continuum extrapolation at renormalisation scale μ . Entries are **value(error)**: χ^2/DOF (p -value).

| μ (GeV) | | a^2, m_π^2 | a^2, m_π^2 (no C) | a^2, a^4, m_π^2 | a^2, m_π^2, m_π^4 |
|-------------|----------|----------------|-----------------------|---------------------|-------------------------|
| 2.0 | α | -0.400(43) | -0.40(42) | -0.4(10) | -0.401(46) |
| | β | 0.00174(15) | 0.00194(25) | 0.00176(15) | 0.00117(65) |
| 2.2 | α | -0.393(43) | -0.39(41) | -0.4(10) | -0.394(45) |
| | β | 0.00139(13) | 0.00147(23) | 0.00139(13) | 0.00091(68) |
| 2.3 | α | -0.388(43) | -0.39(41) | -0.4(10) | -0.390(44) |
| | β | 0.00128(13) | 0.00139(22) | 0.00129(13) | 0.00080(67) |
| 2.4 | α | -0.389(43) | -0.37(41) | -0.3(10) | -0.391(45) |
| | β | 0.00125(11) | 0.00132(20) | 0.00124(12) | 0.00053(62) |

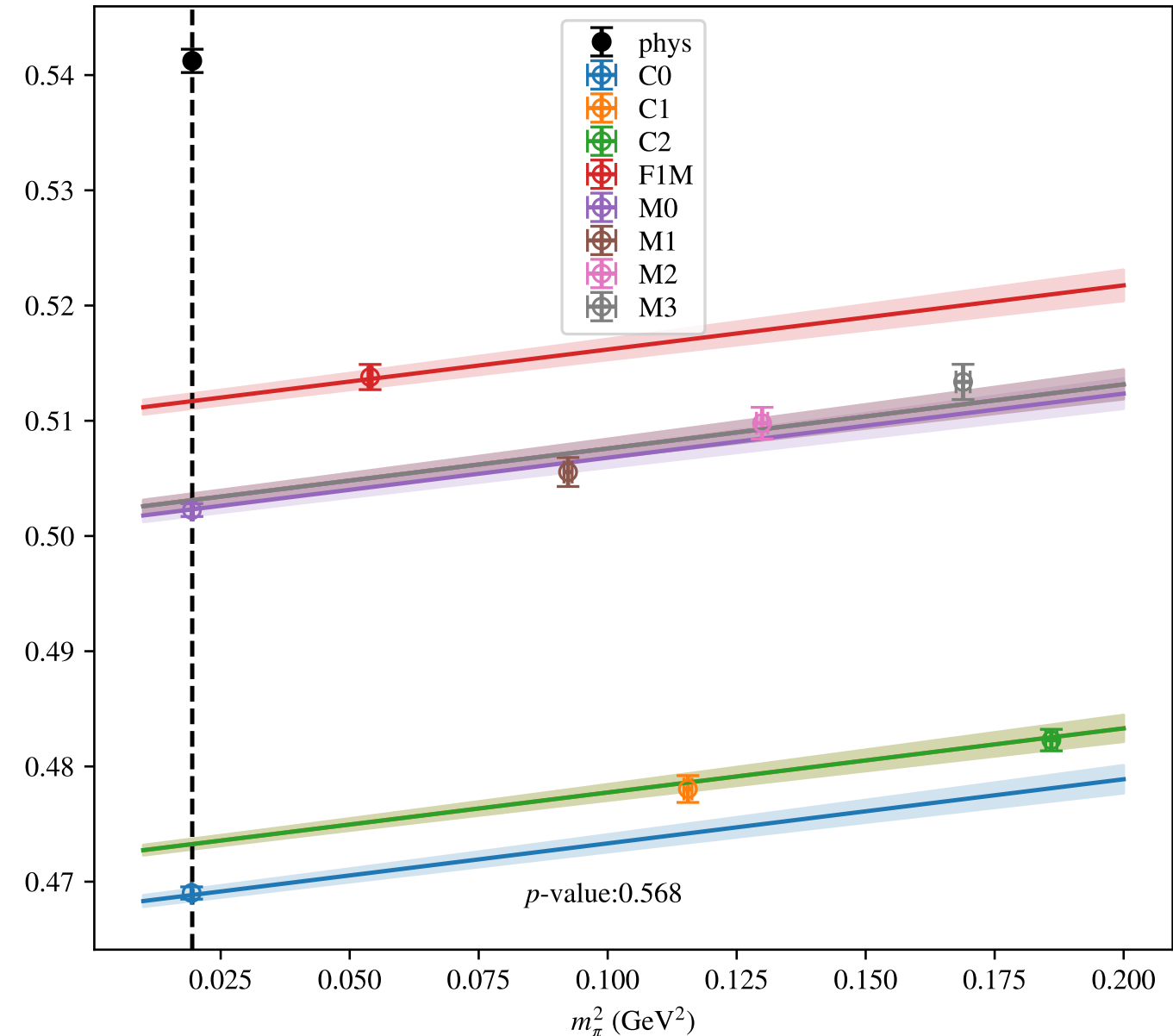
Table 10: Fit values of coefficients in $B = B_{phys} + \alpha a^2 + \beta \left(\frac{m_\pi^2}{f_\pi^2} - \frac{m_{\pi,PDG}^2}{f_\pi^2} \right) + \dots$

$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

$(-0.5)*VVmAA$

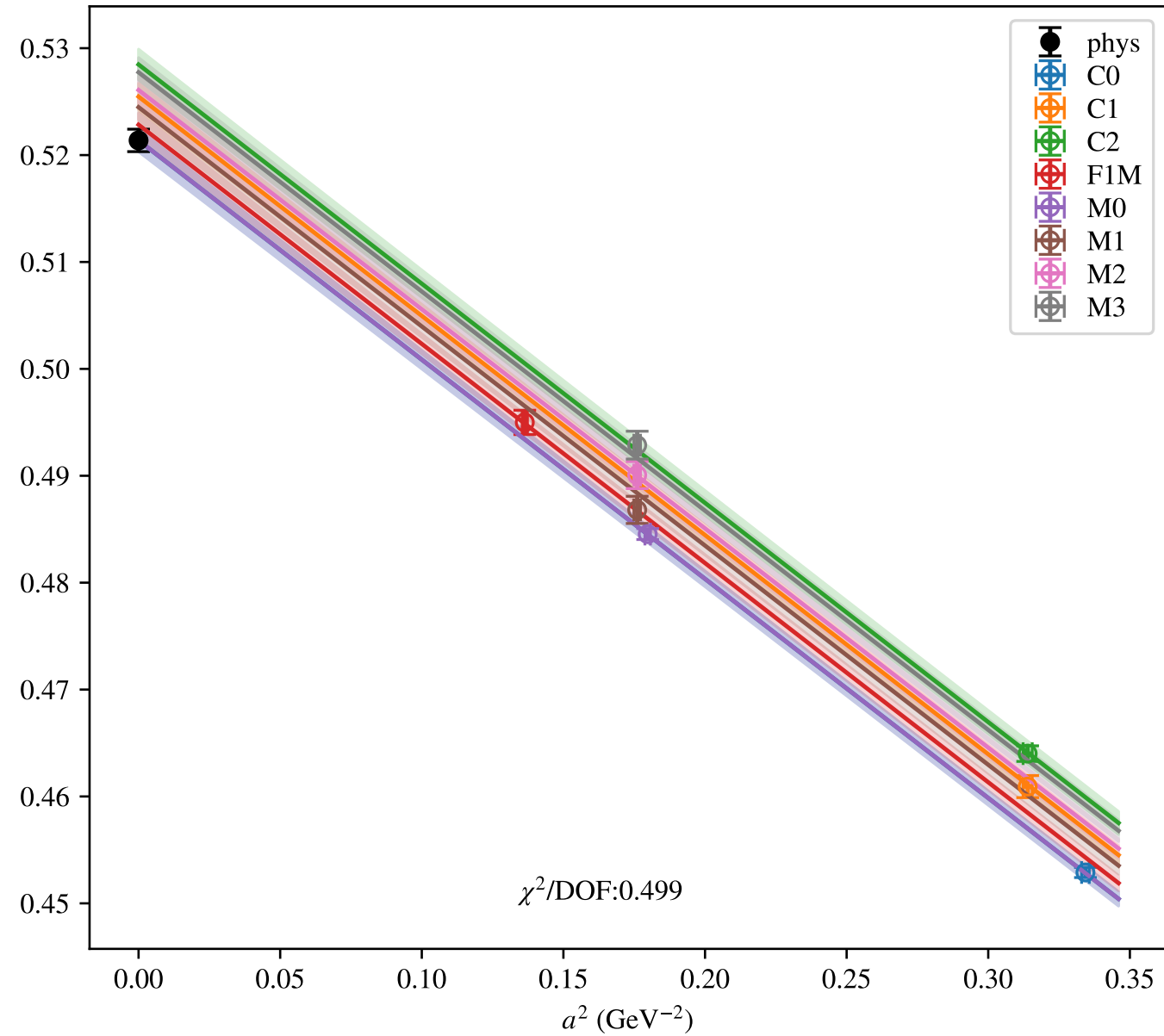


$(-0.5)*VVmAA$

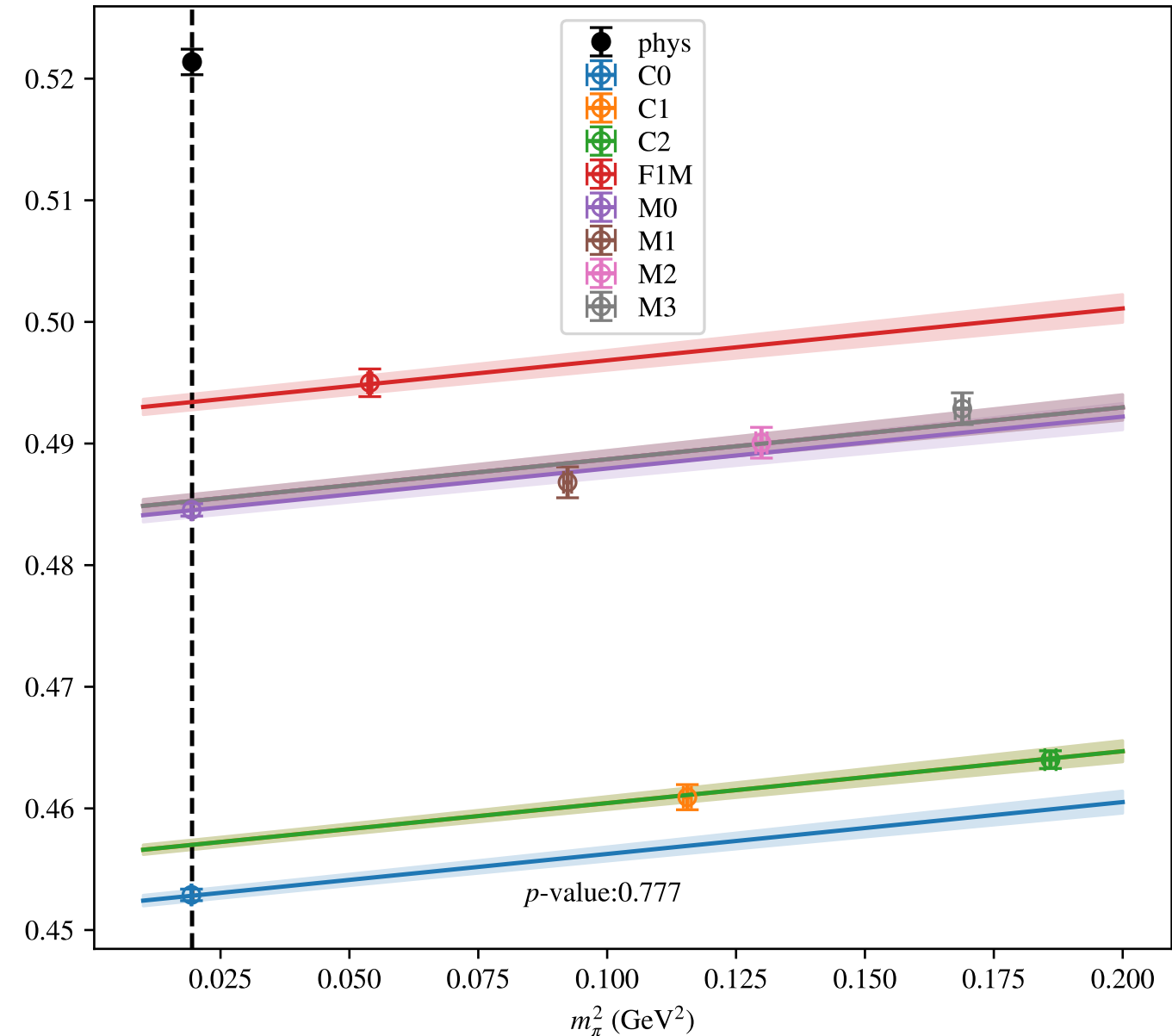


$$a^2, m_\pi^2, \mu = 2.2 \text{ GeV}$$

$(-0.5)*VVmAA$

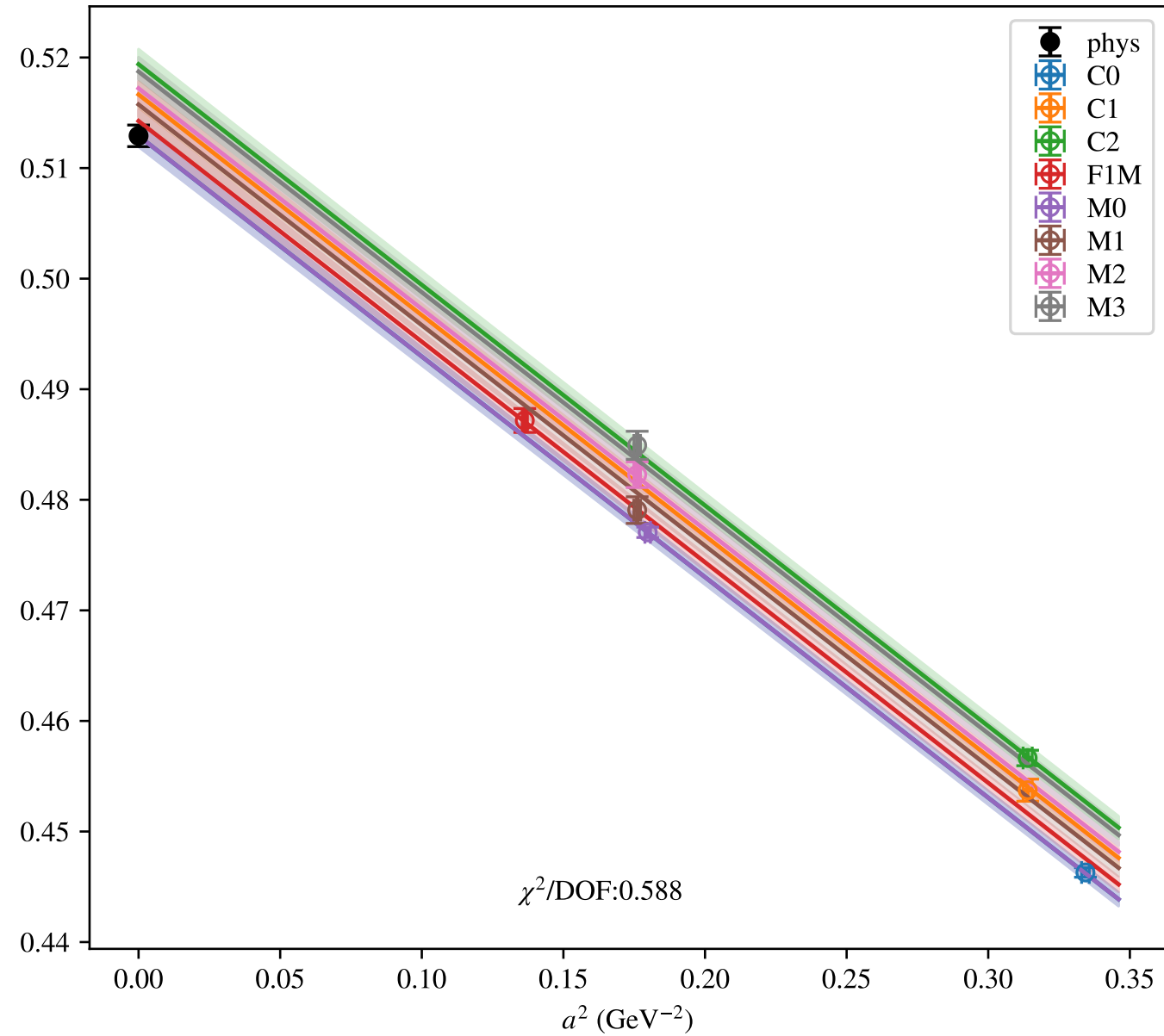


$(-0.5)*VVmAA$

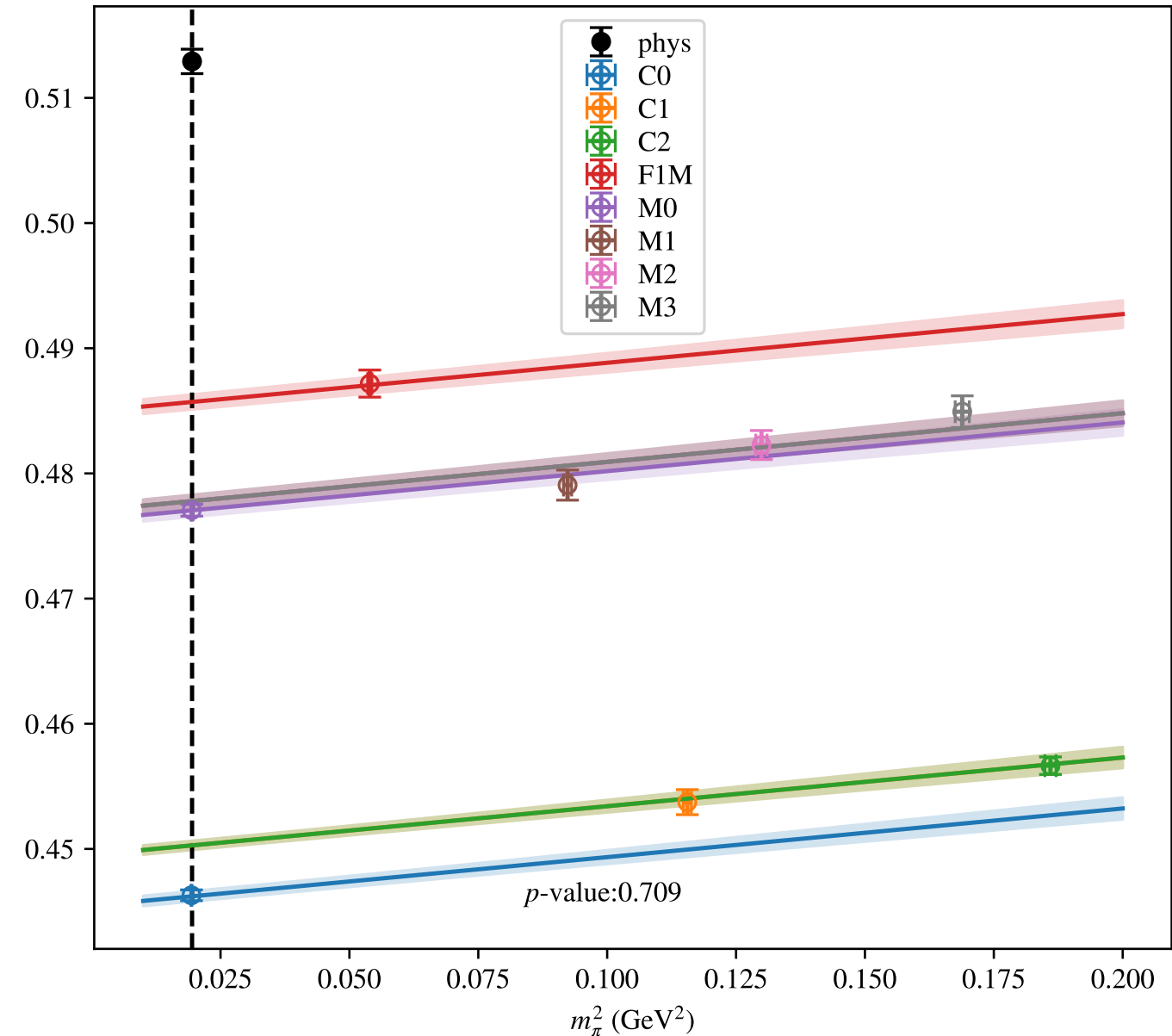


$$a^2, m_\pi^2, \mu = 2.3 \text{ GeV}$$

$(-0.5)*VVmAA$

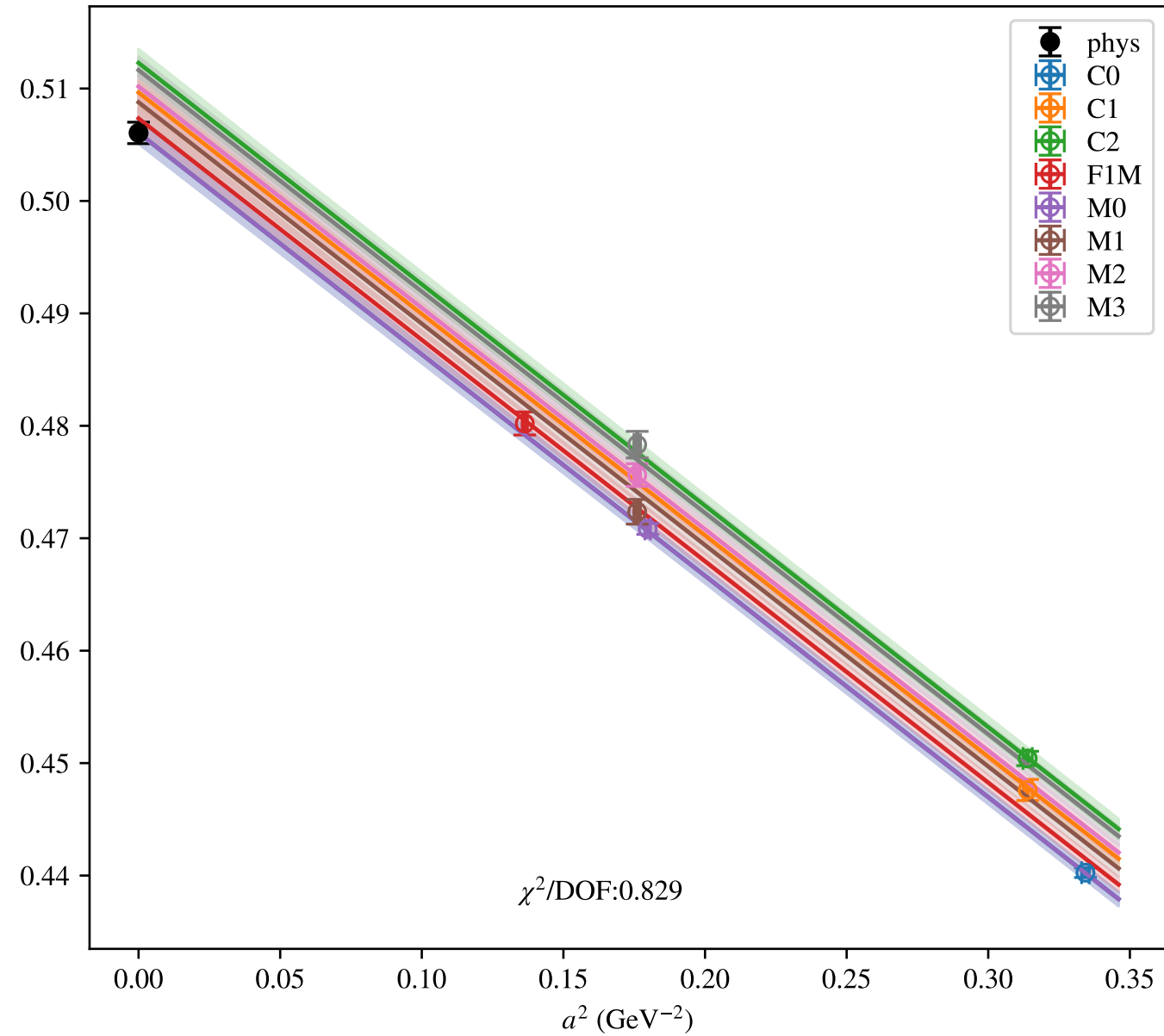


$(-0.5)*VVmAA$

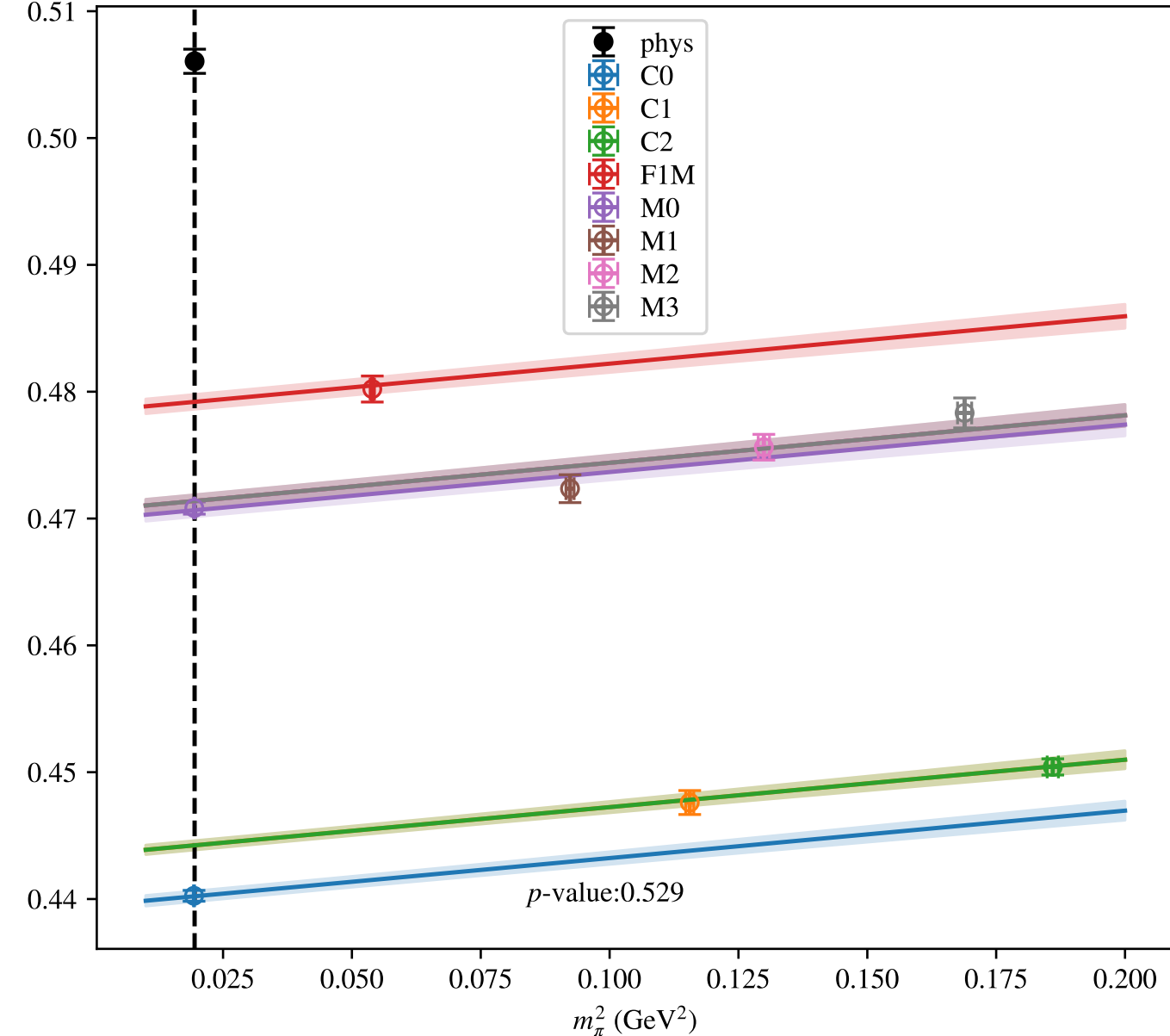


$$a^2, m_\pi^2, \mu = 2.4 \text{ GeV}$$

$(-0.5)*VVmAA$

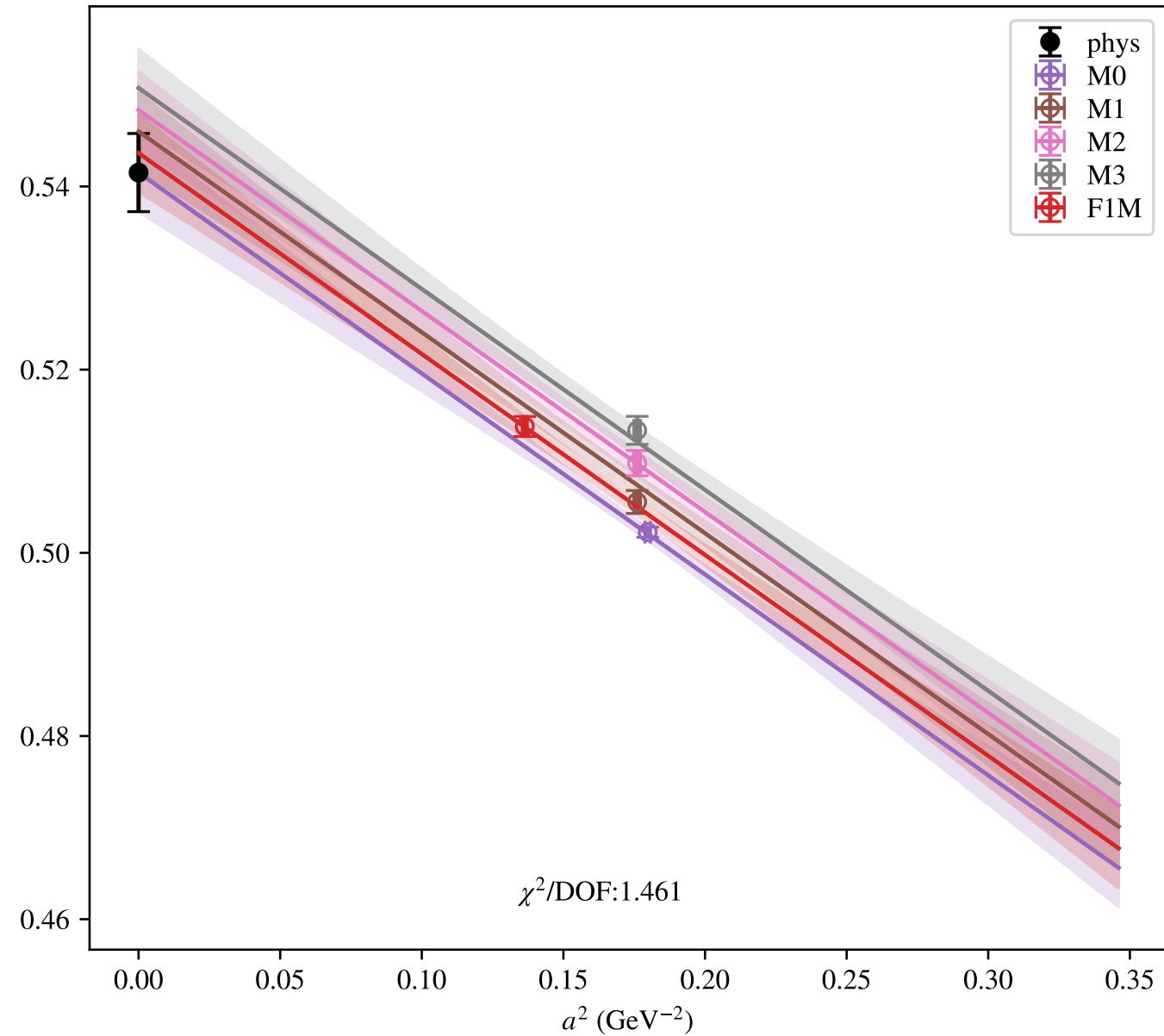


$(-0.5)*VVmAA$

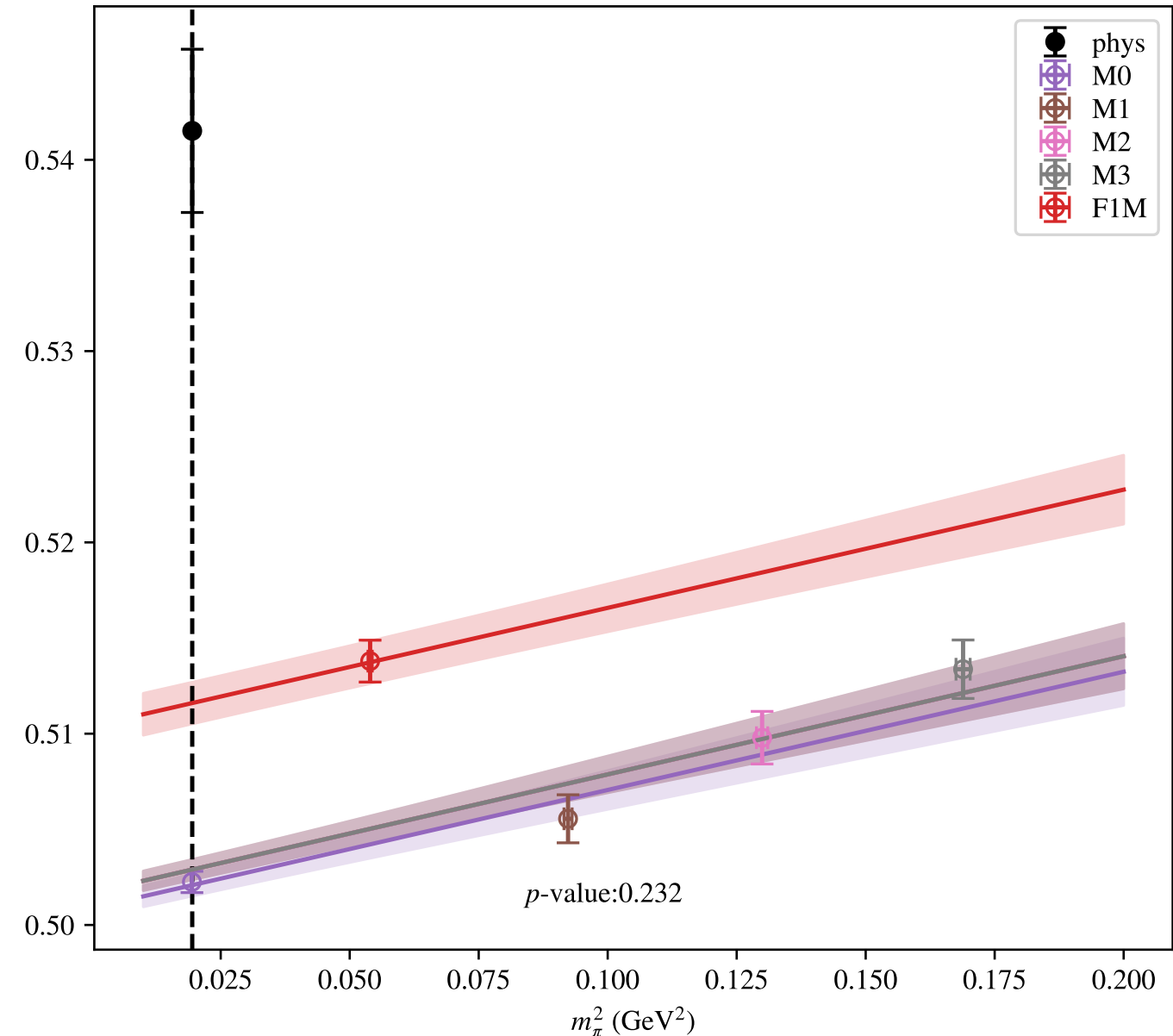


a^2, m_π^2 (no C), $\mu = 2.0$ GeV

$(-0.5)*VVmAA$

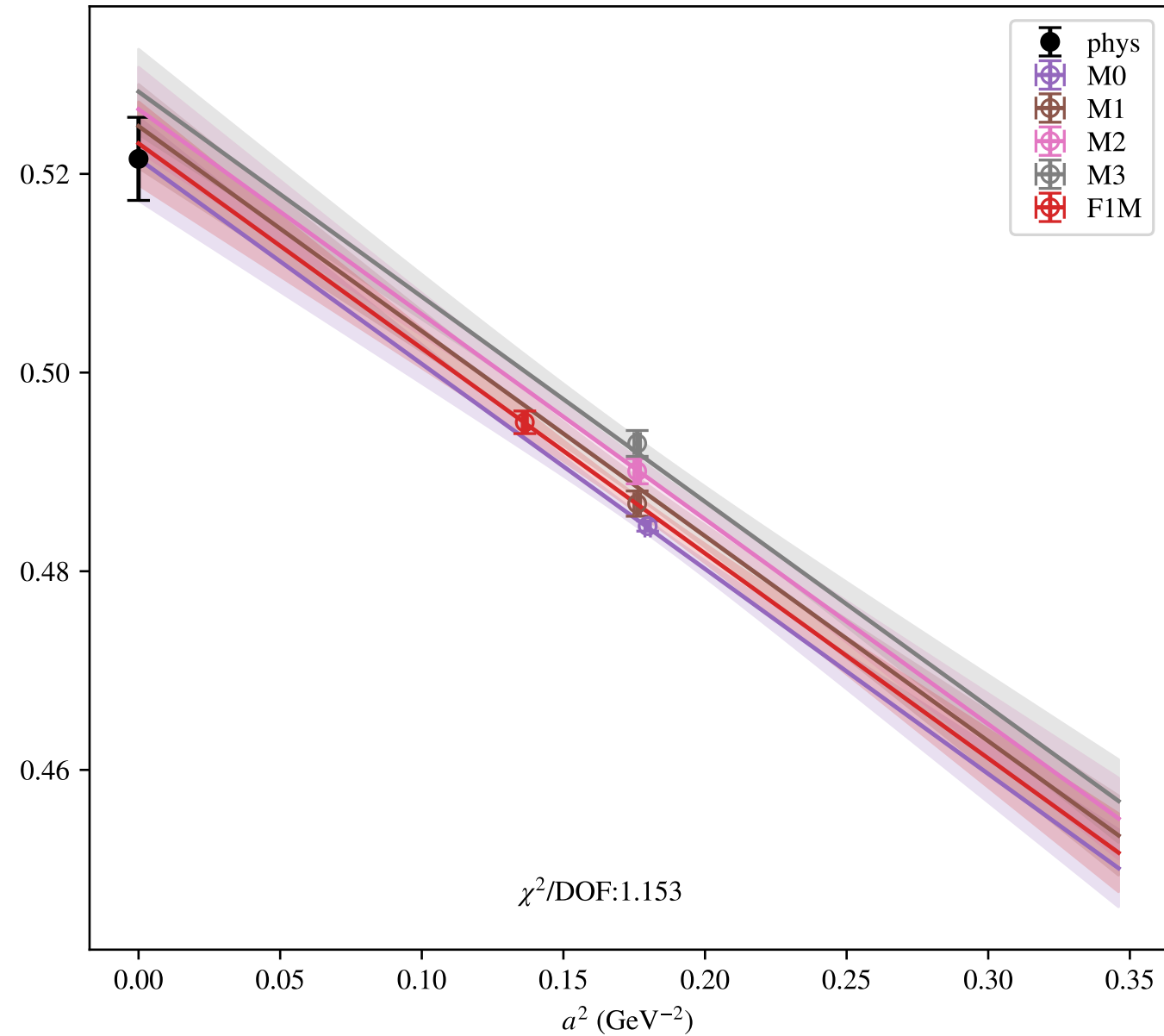


$(-0.5)*VVmAA$

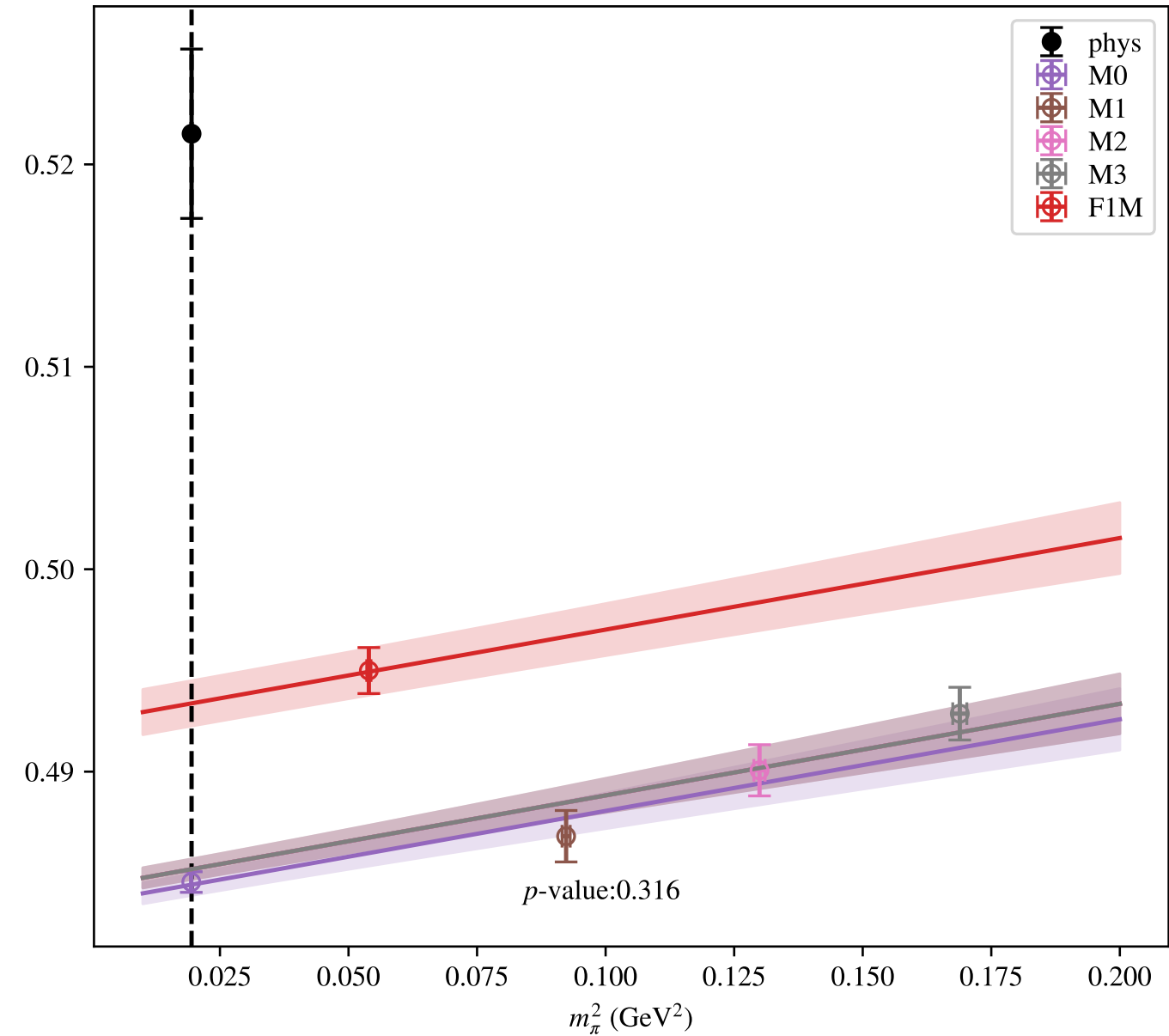


a^2, m_π^2 (no C), $\mu = 2.2$ GeV

$(-0.5)*VVmAA$

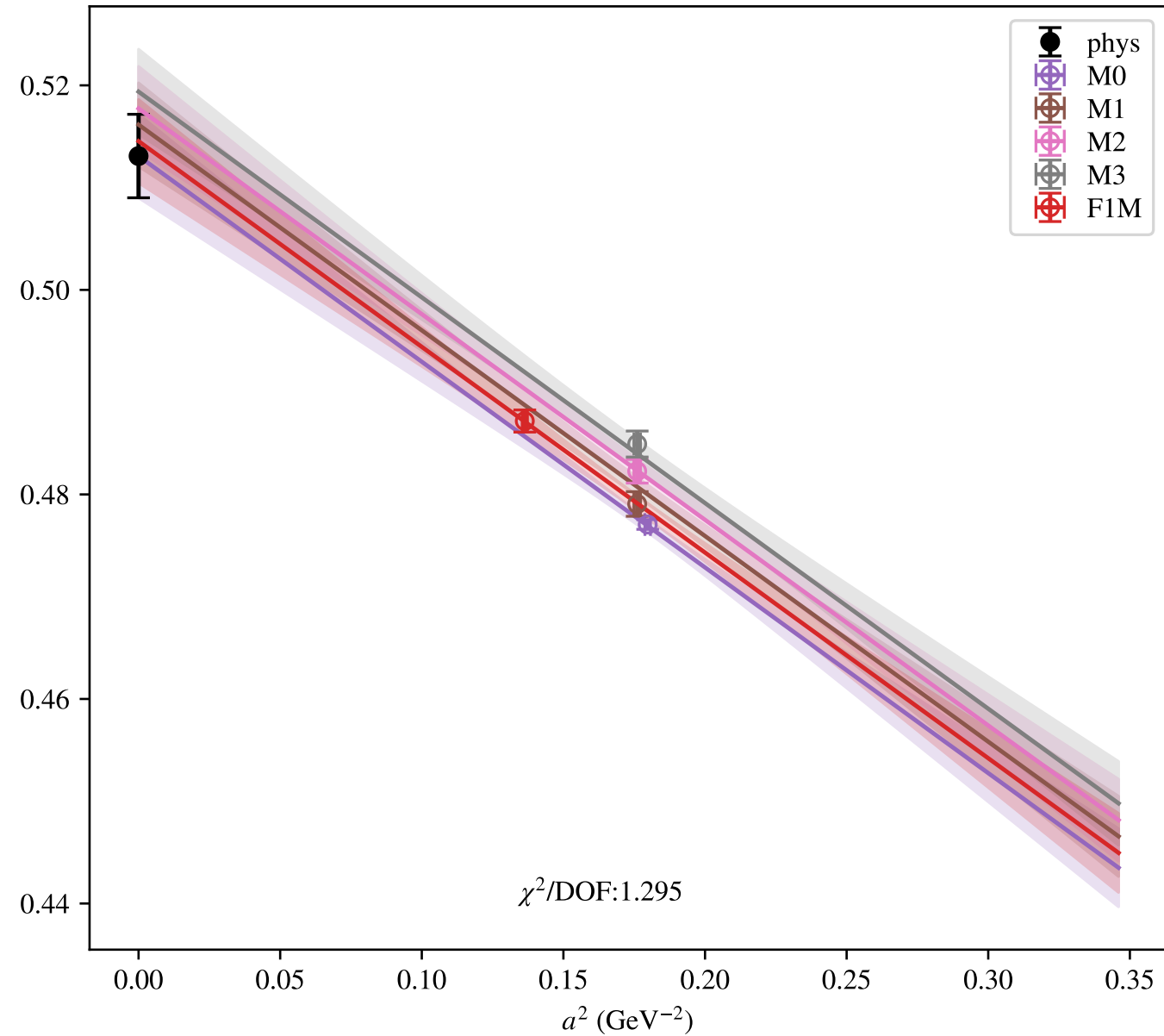


$(-0.5)*VVmAA$

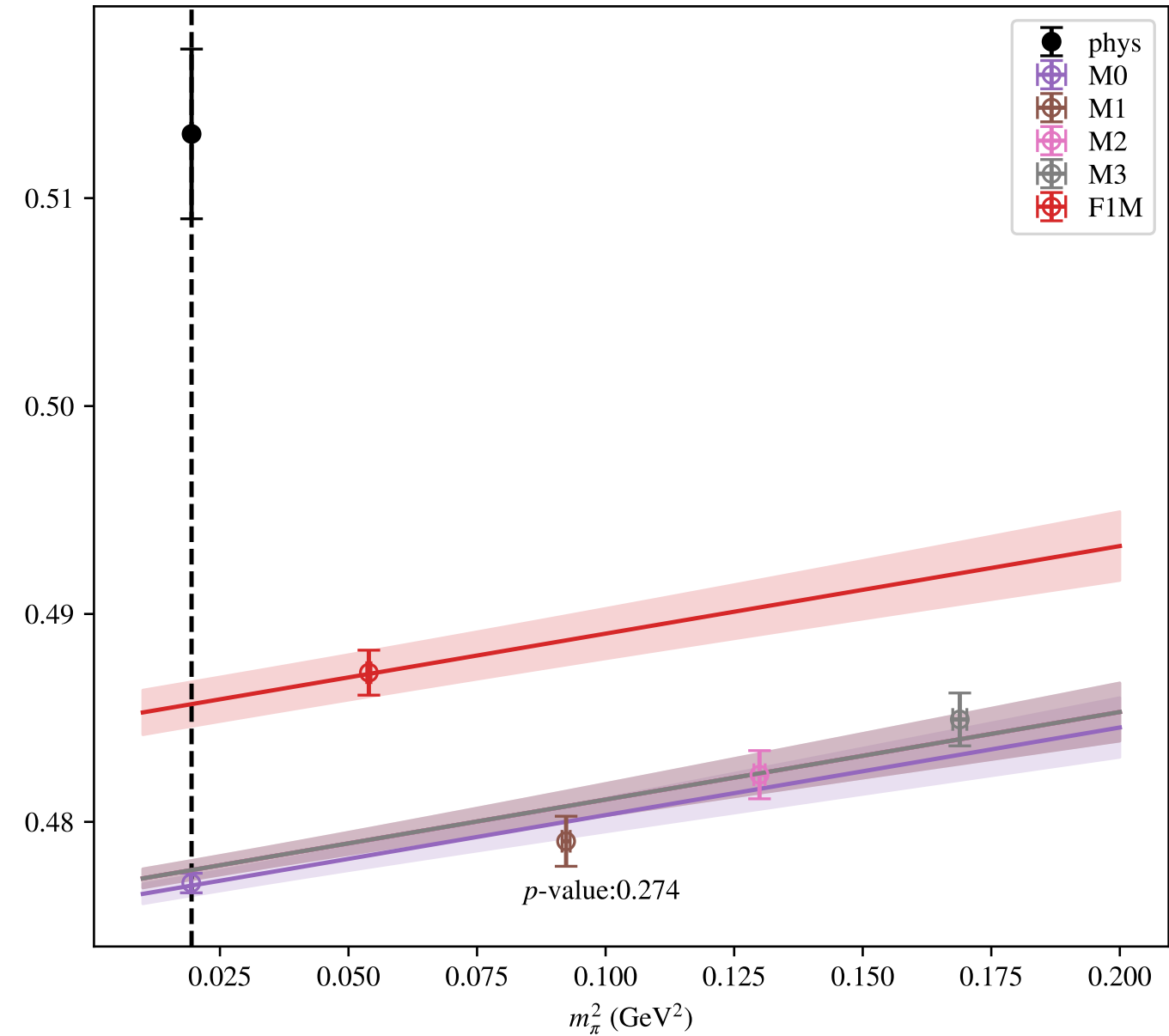


a^2, m_π^2 (no C), $\mu = 2.3$ GeV

$(-0.5)*VVmAA$

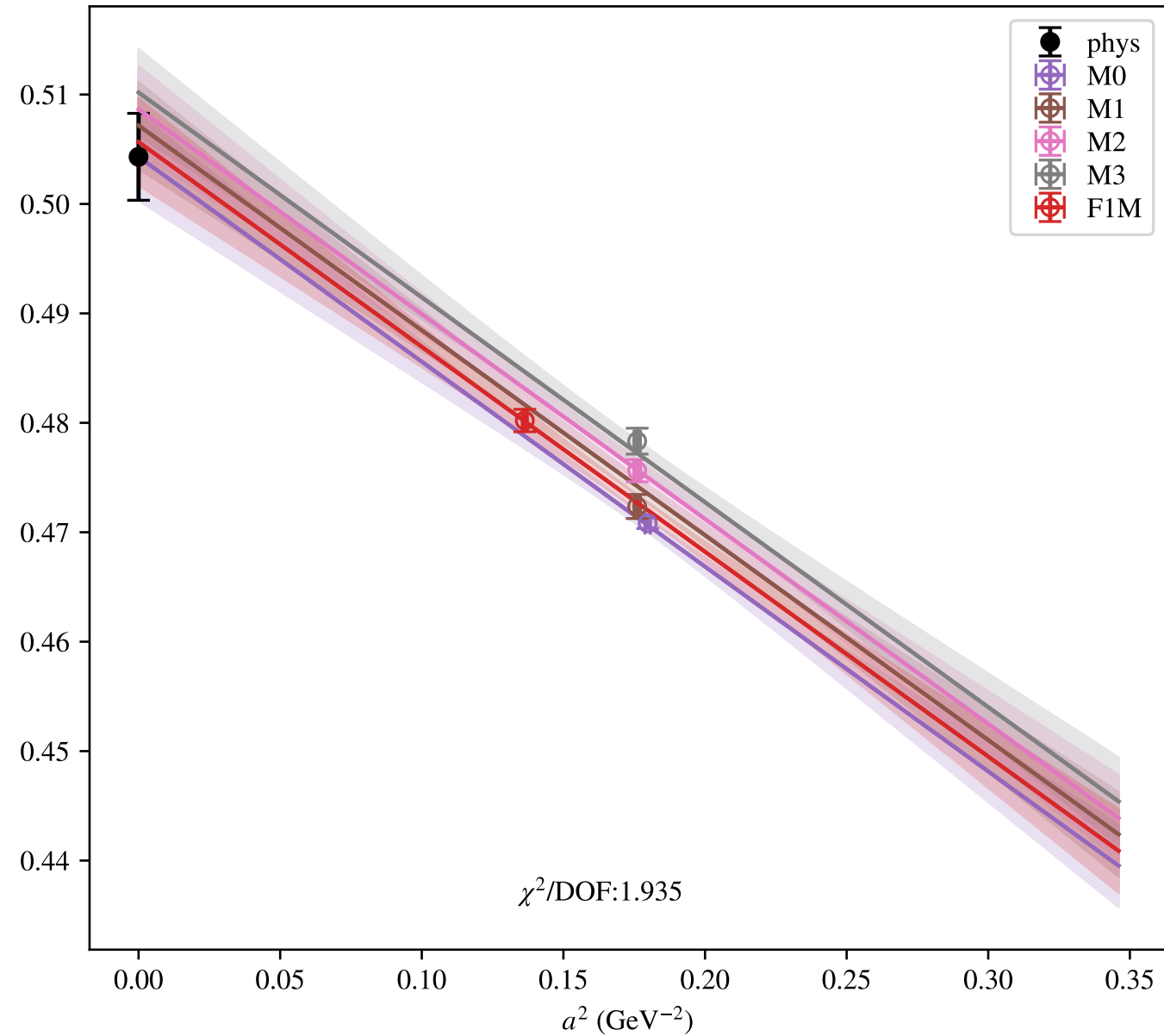


$(-0.5)*VVmAA$

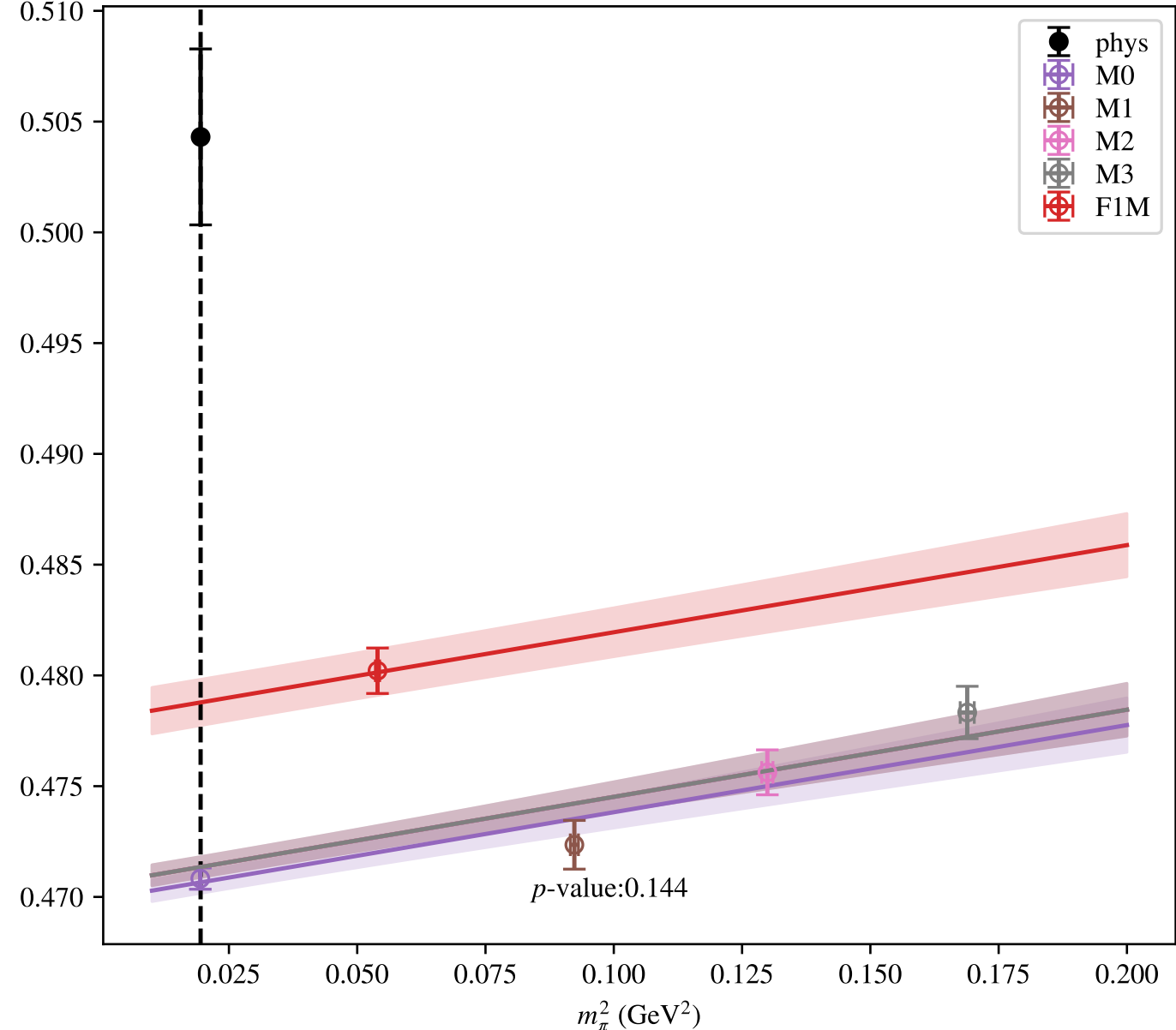


a^2, m_π^2 (no C), $\mu = 2.4$ GeV

$(-0.5)*VVmAA$

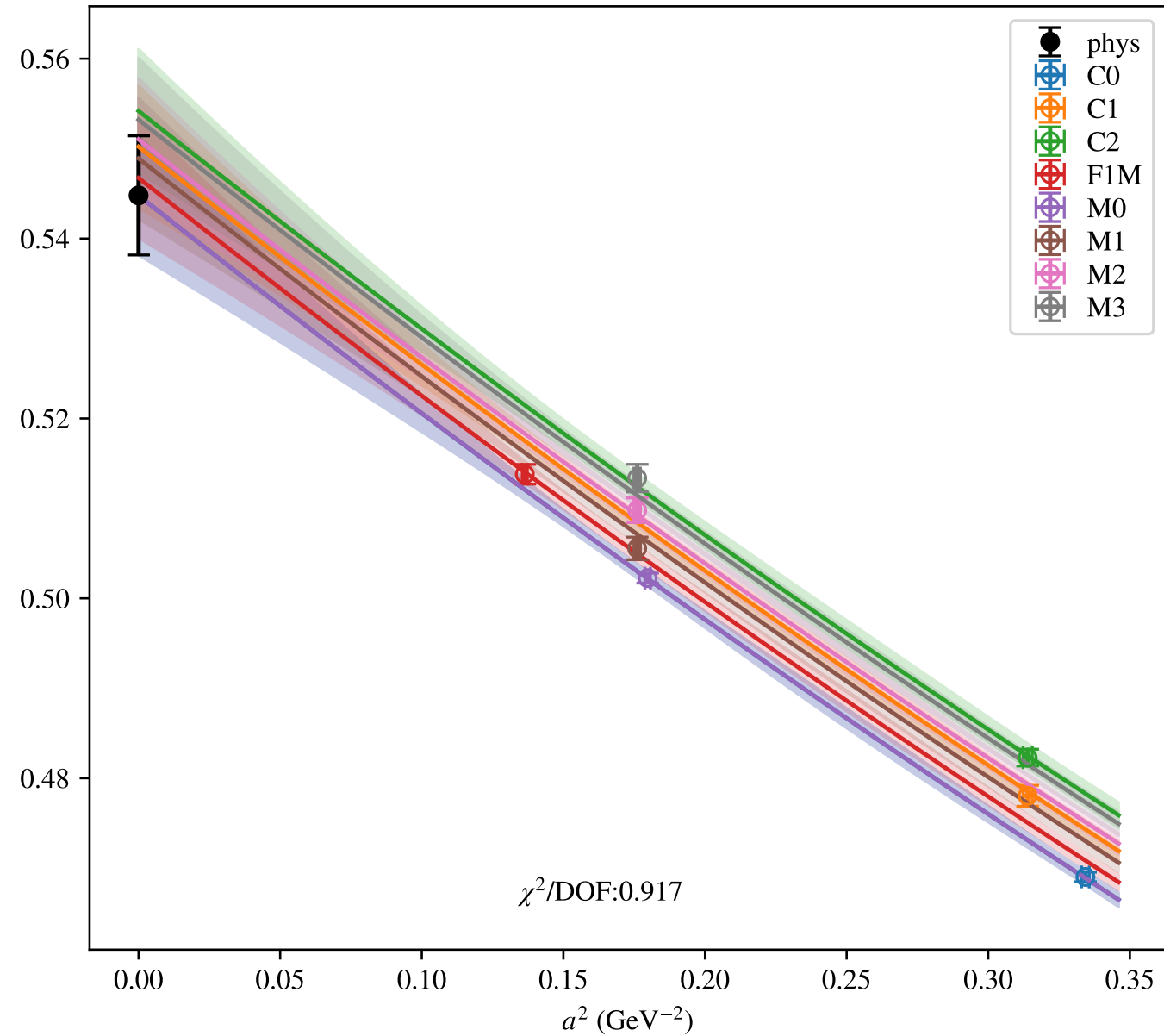


$(-0.5)*VVmAA$

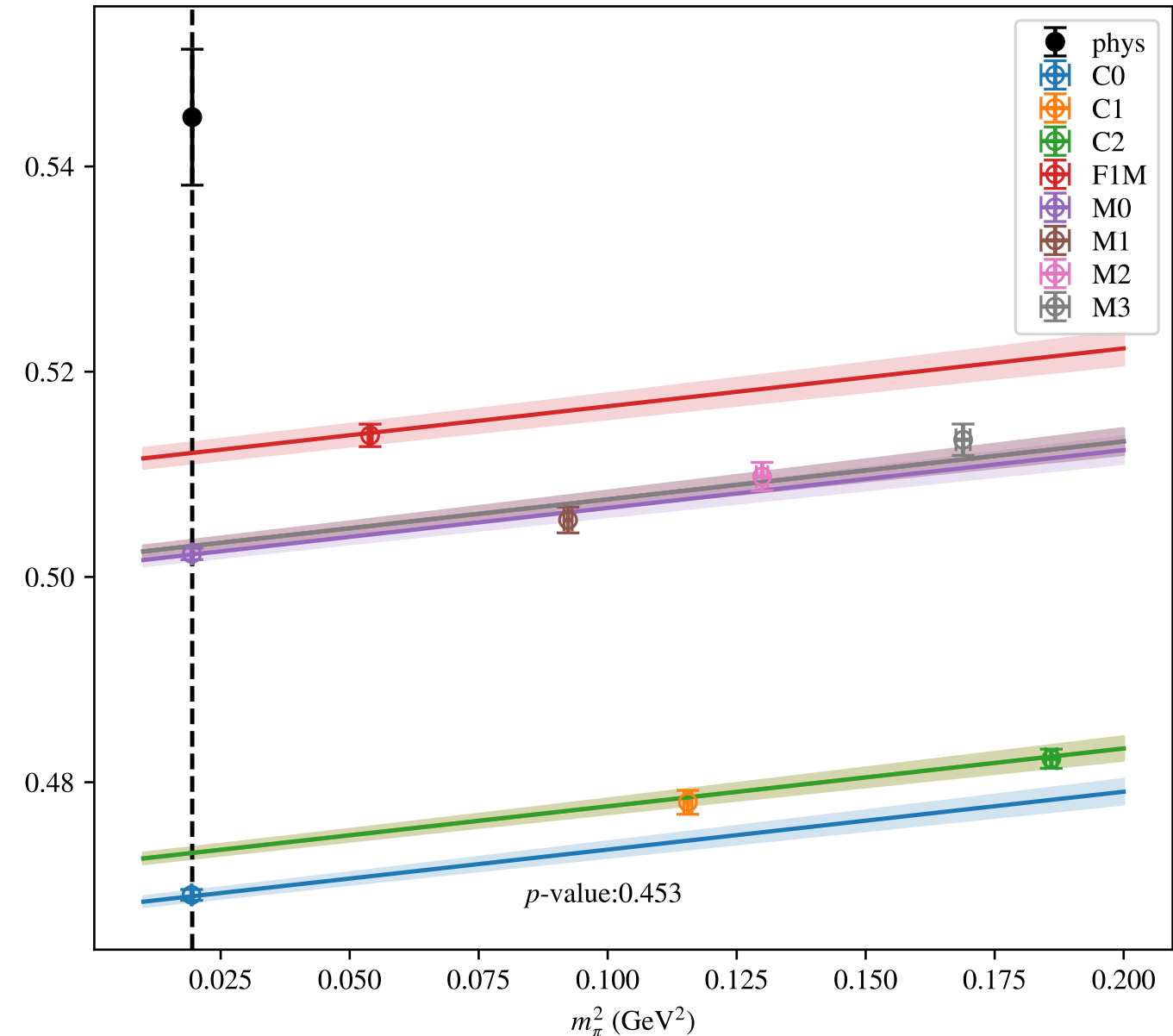


$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$

$(-0.5)*VVmAA$

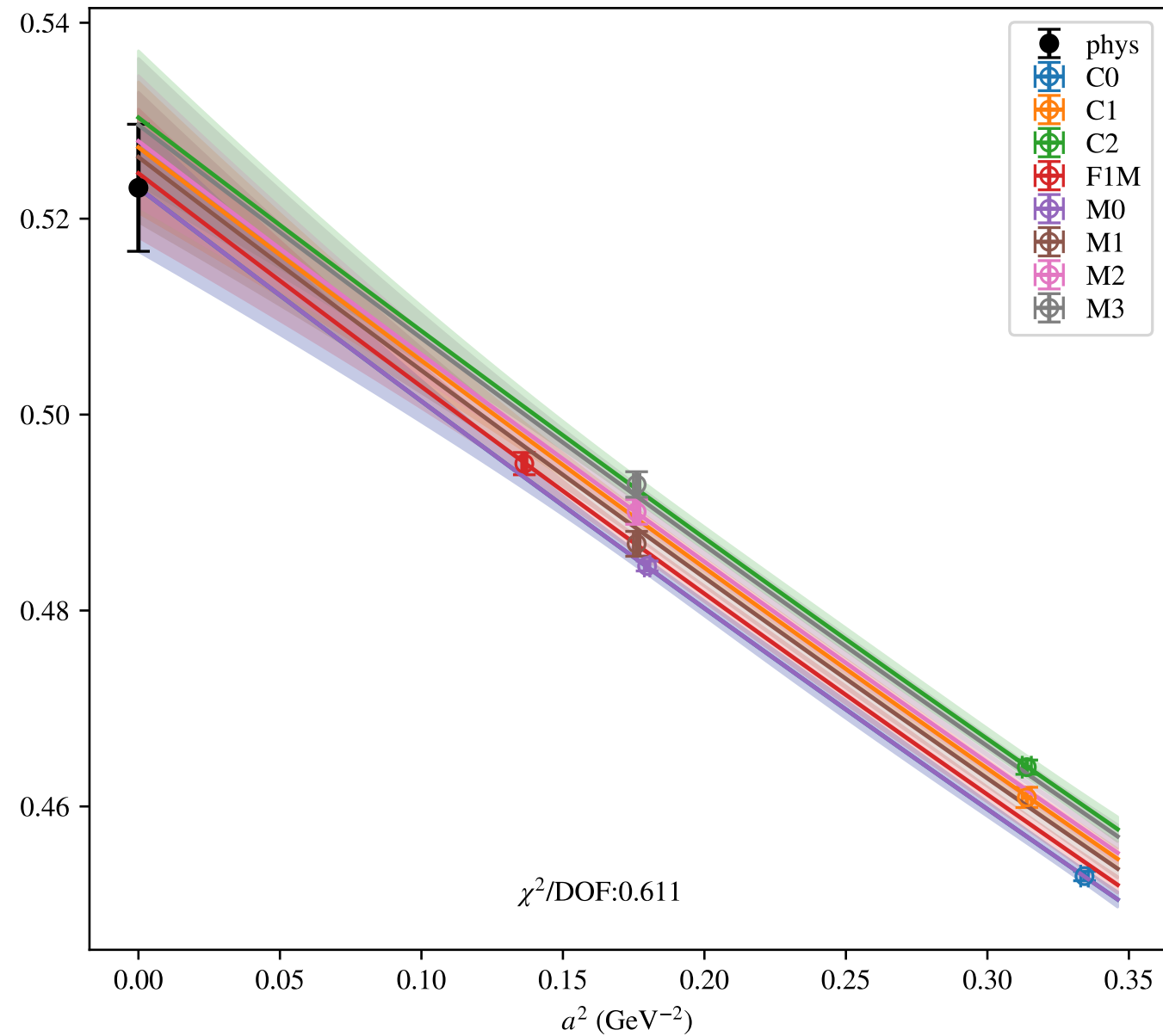


$(-0.5)*VVmAA$

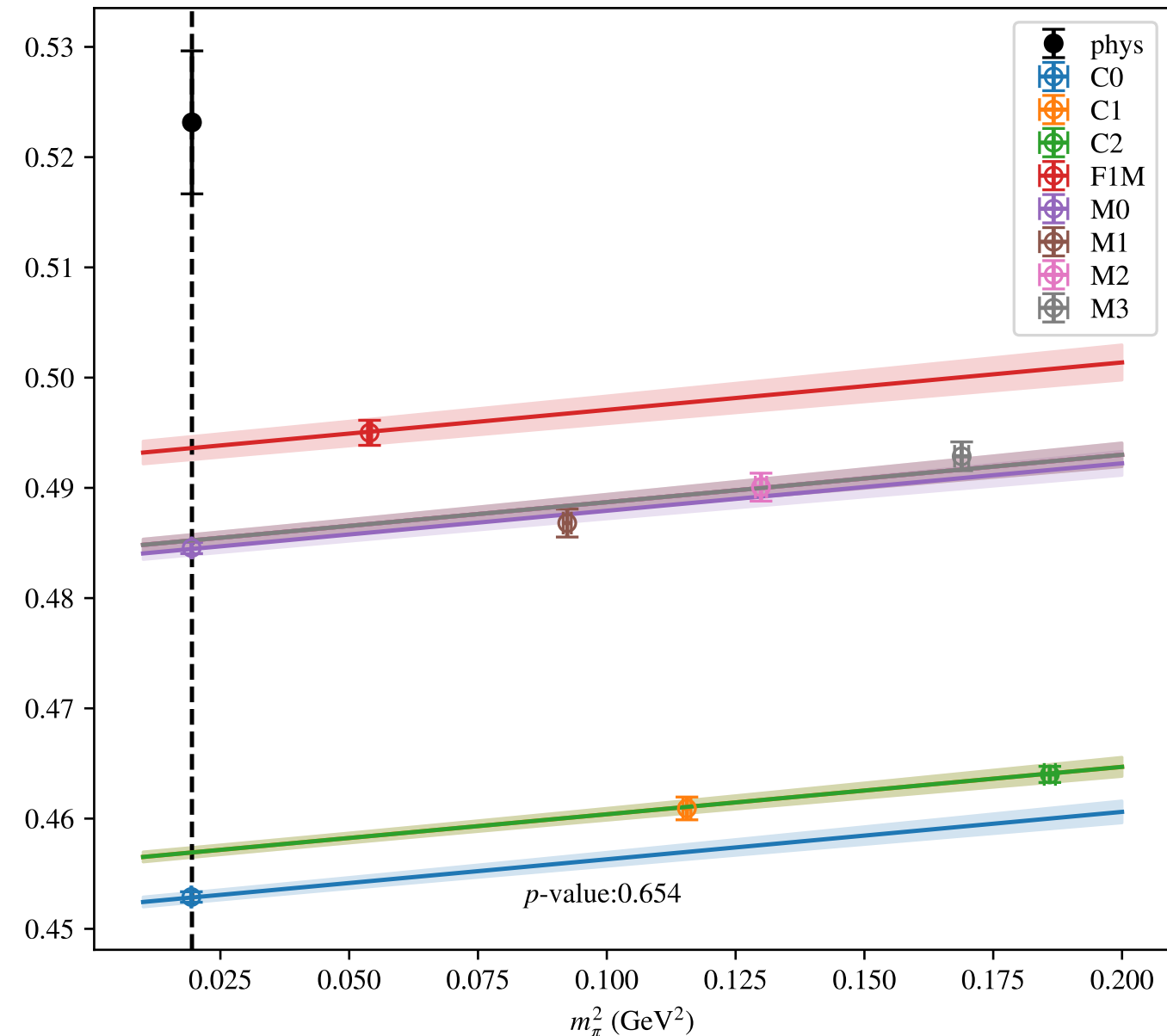


$$a^2, a^4, m_\pi^2, \mu = 2.2 \text{ GeV}$$

$(-0.5)*VVmAA$

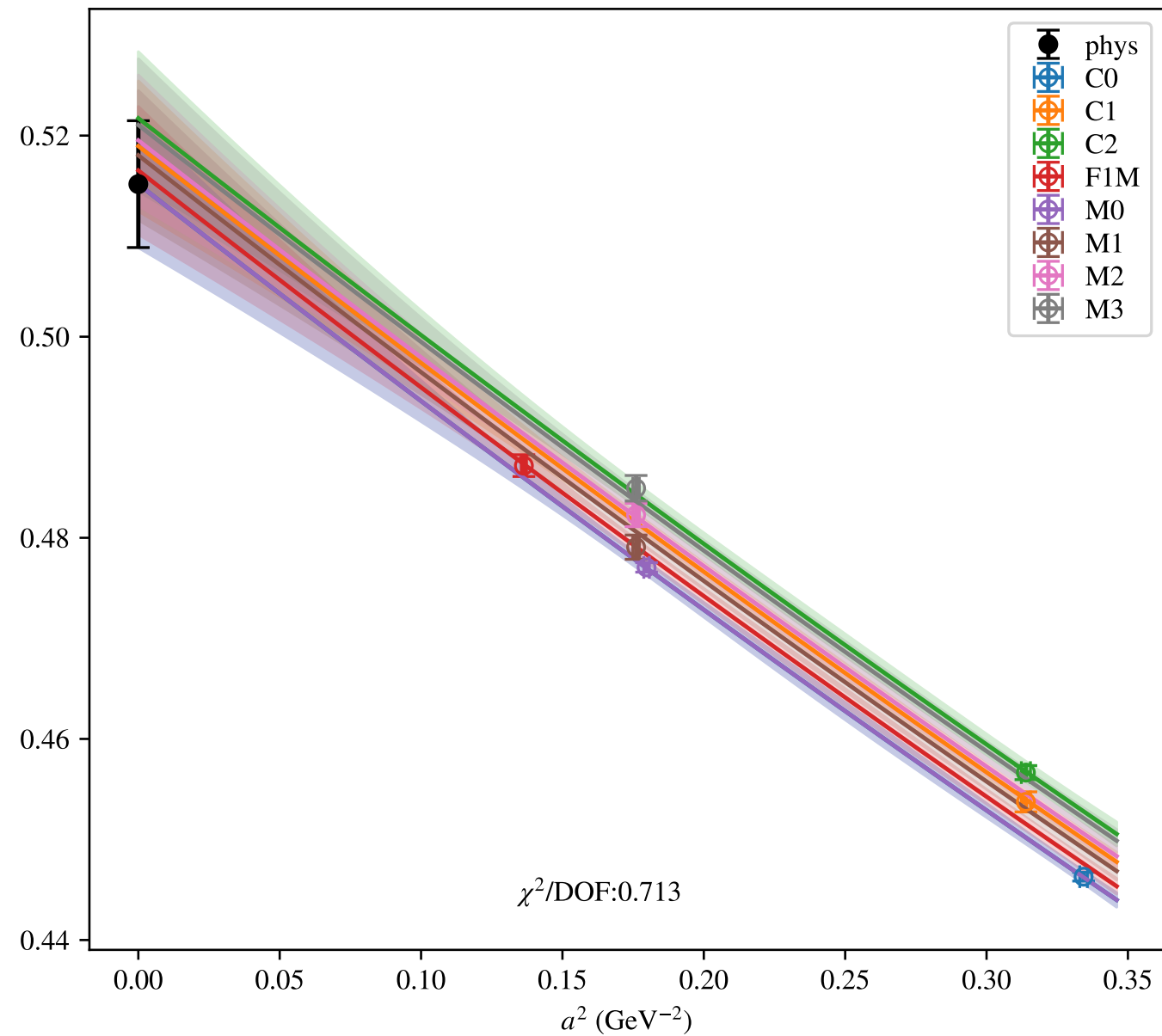


$(-0.5)*VVmAA$

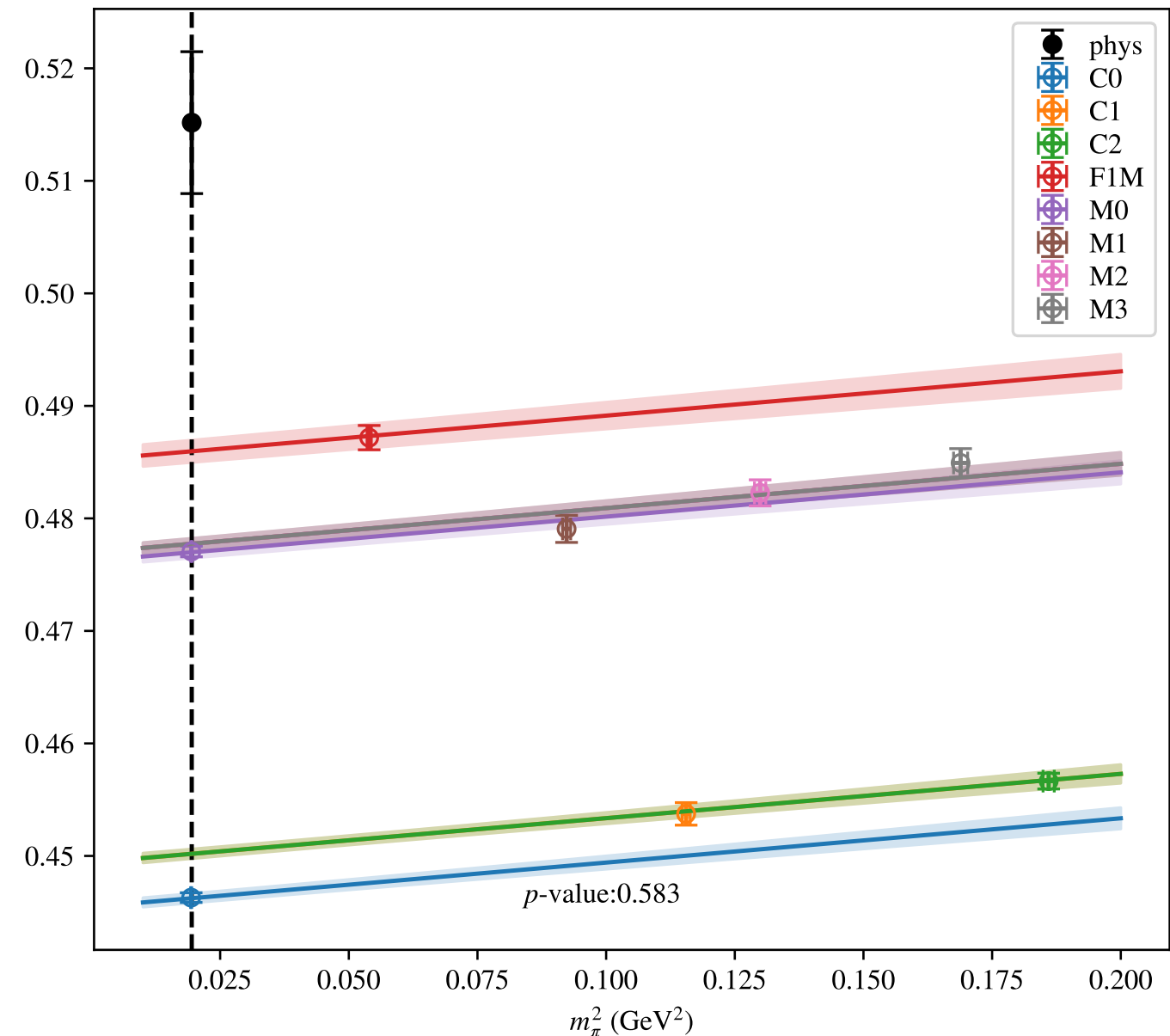


$$a^2, a^4, m_\pi^2, \mu = 2.3 \text{ GeV}$$

$(-0.5)*VVmAA$

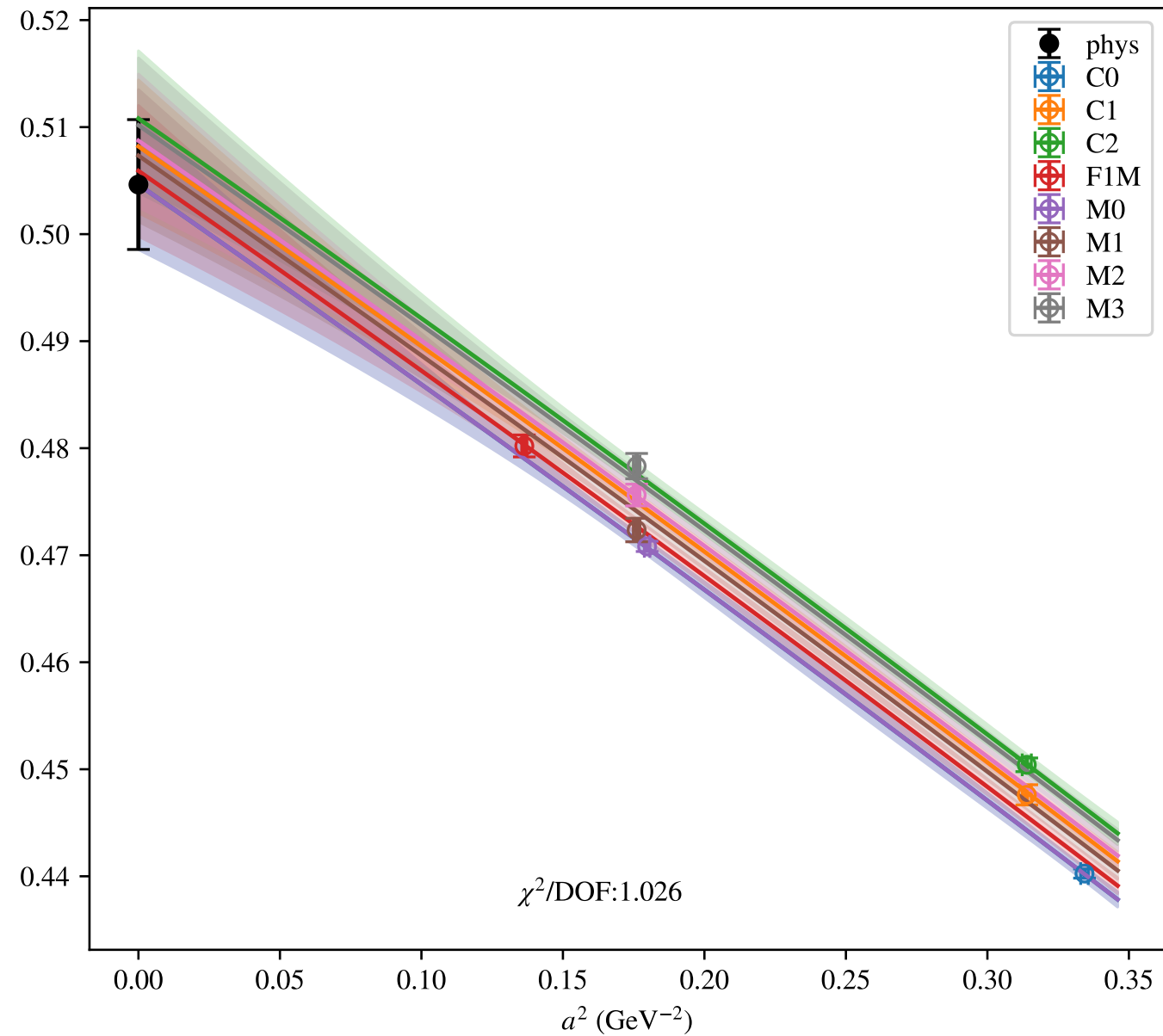


$(-0.5)*VVmAA$

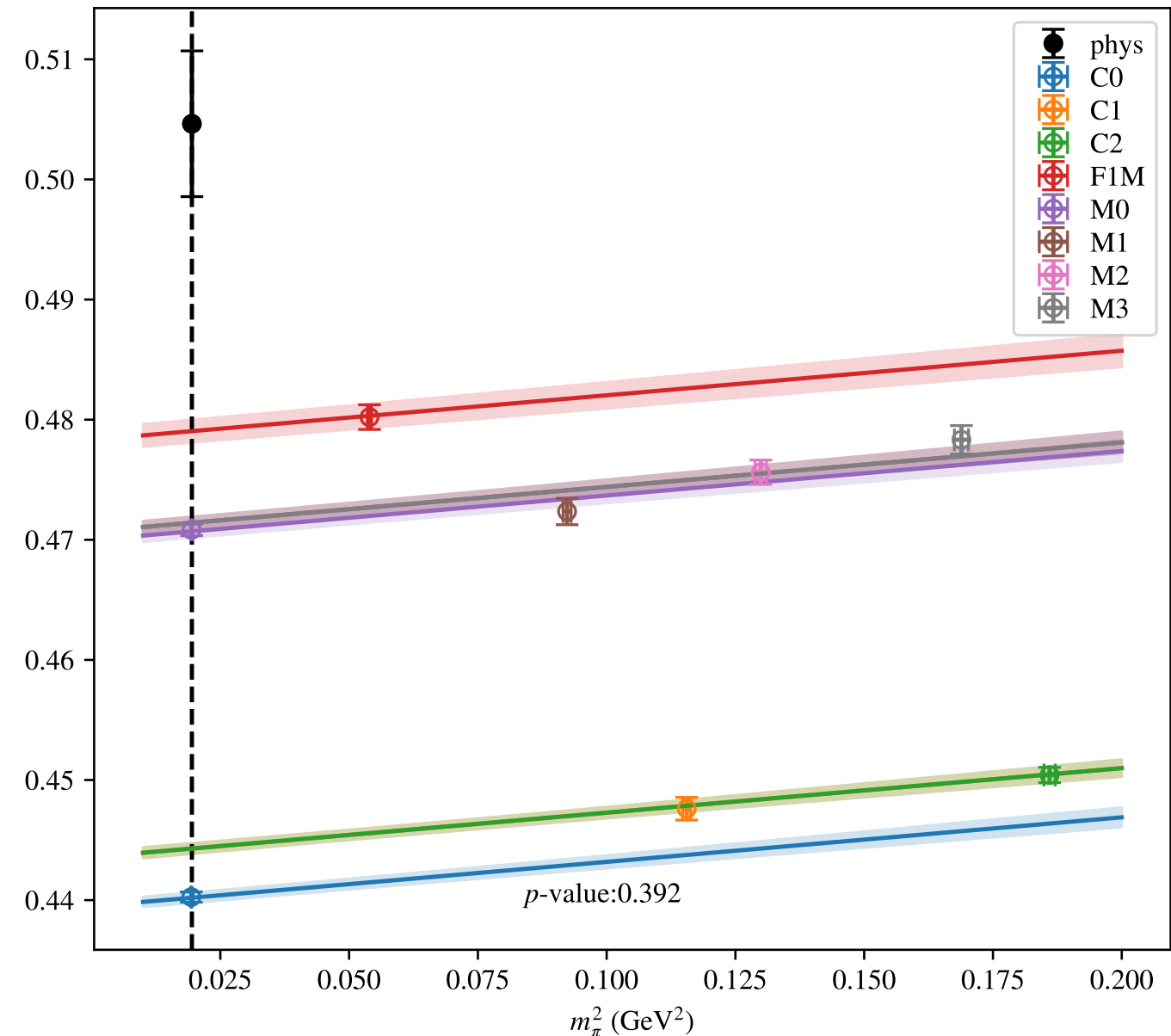


$$a^2, a^4, m_\pi^2, \mu = 2.4 \text{ GeV}$$

$(-0.5)*VVmAA$

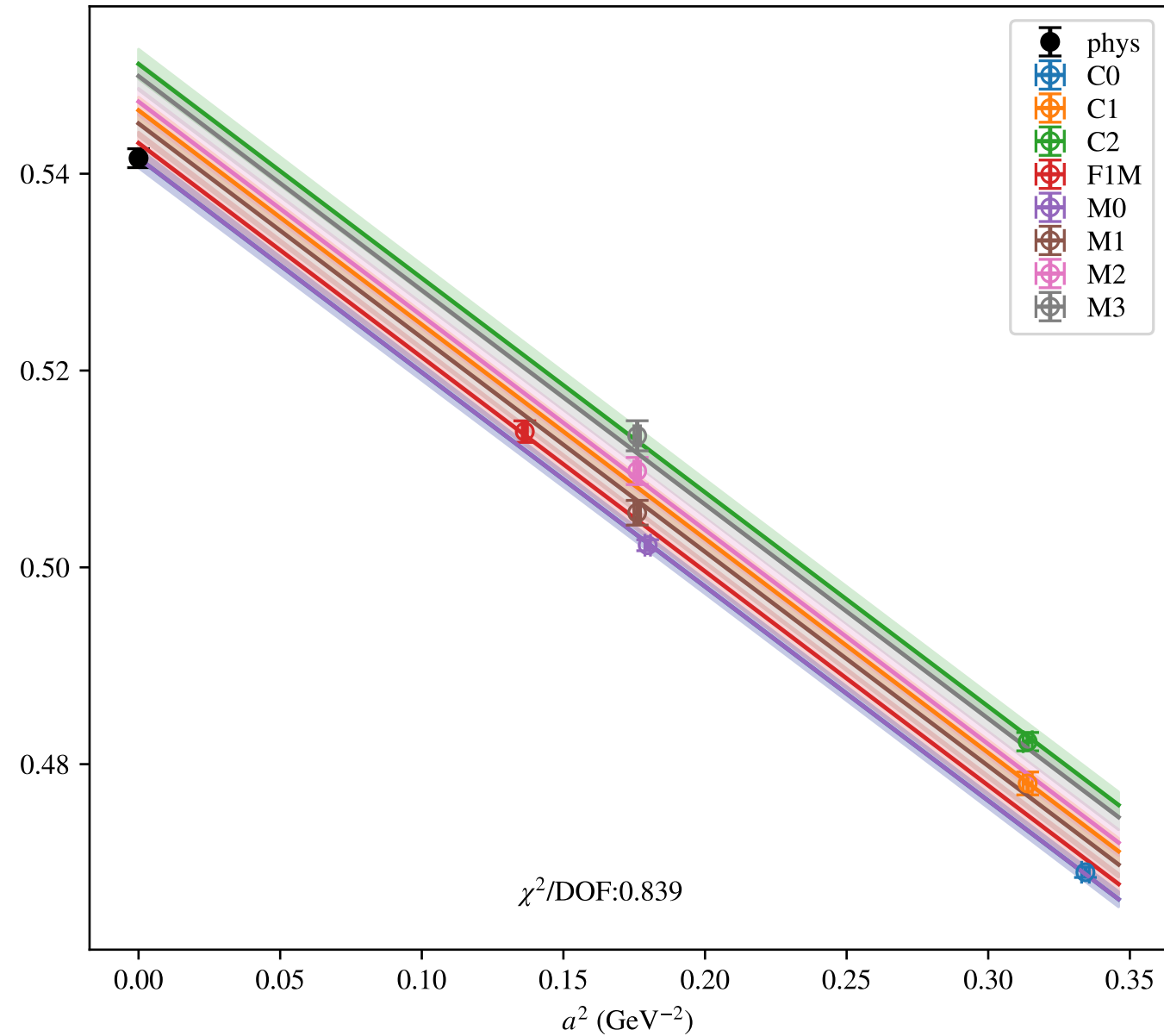


$(-0.5)*VVmAA$

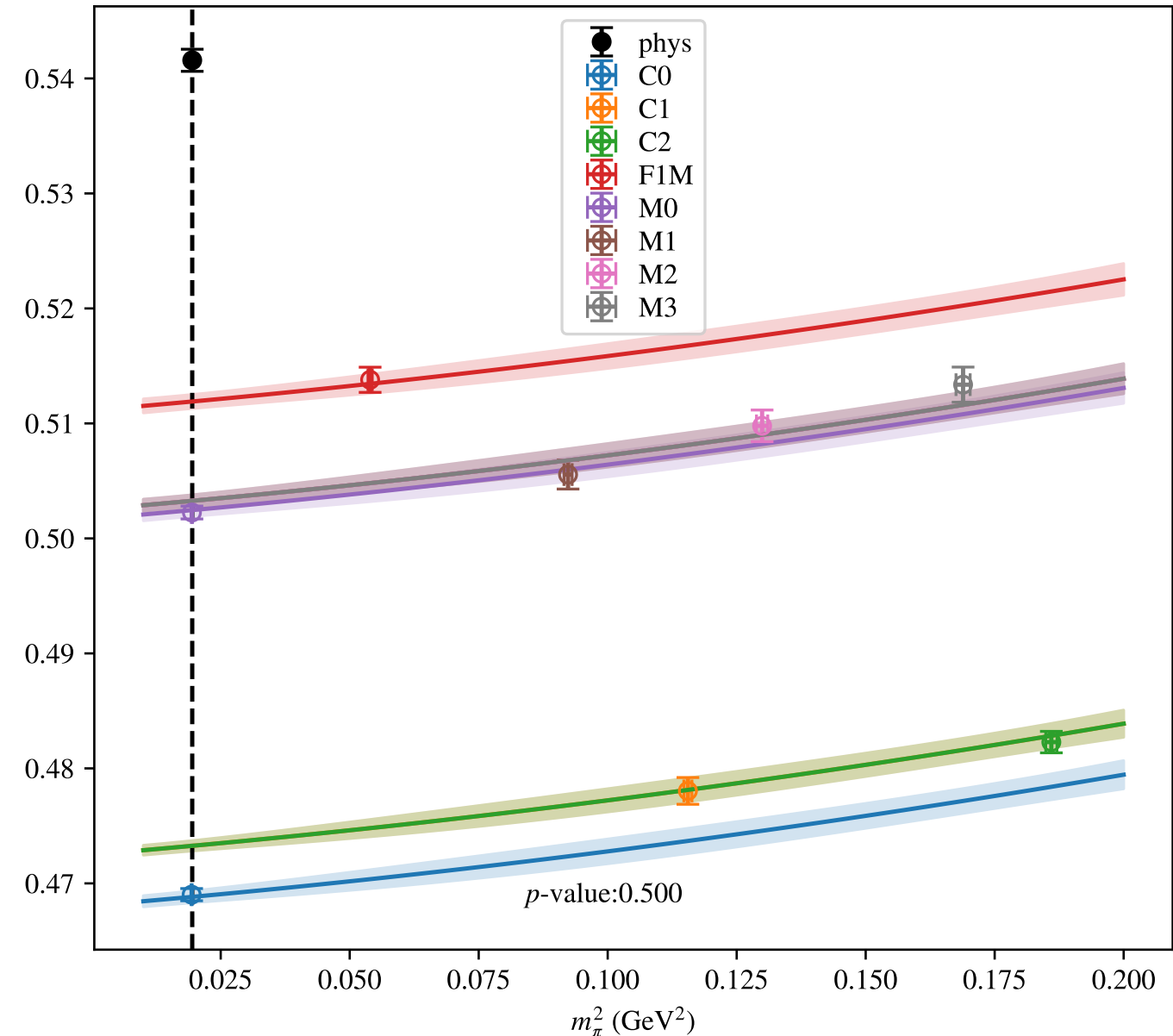


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

$(-0.5)*VVmAA$

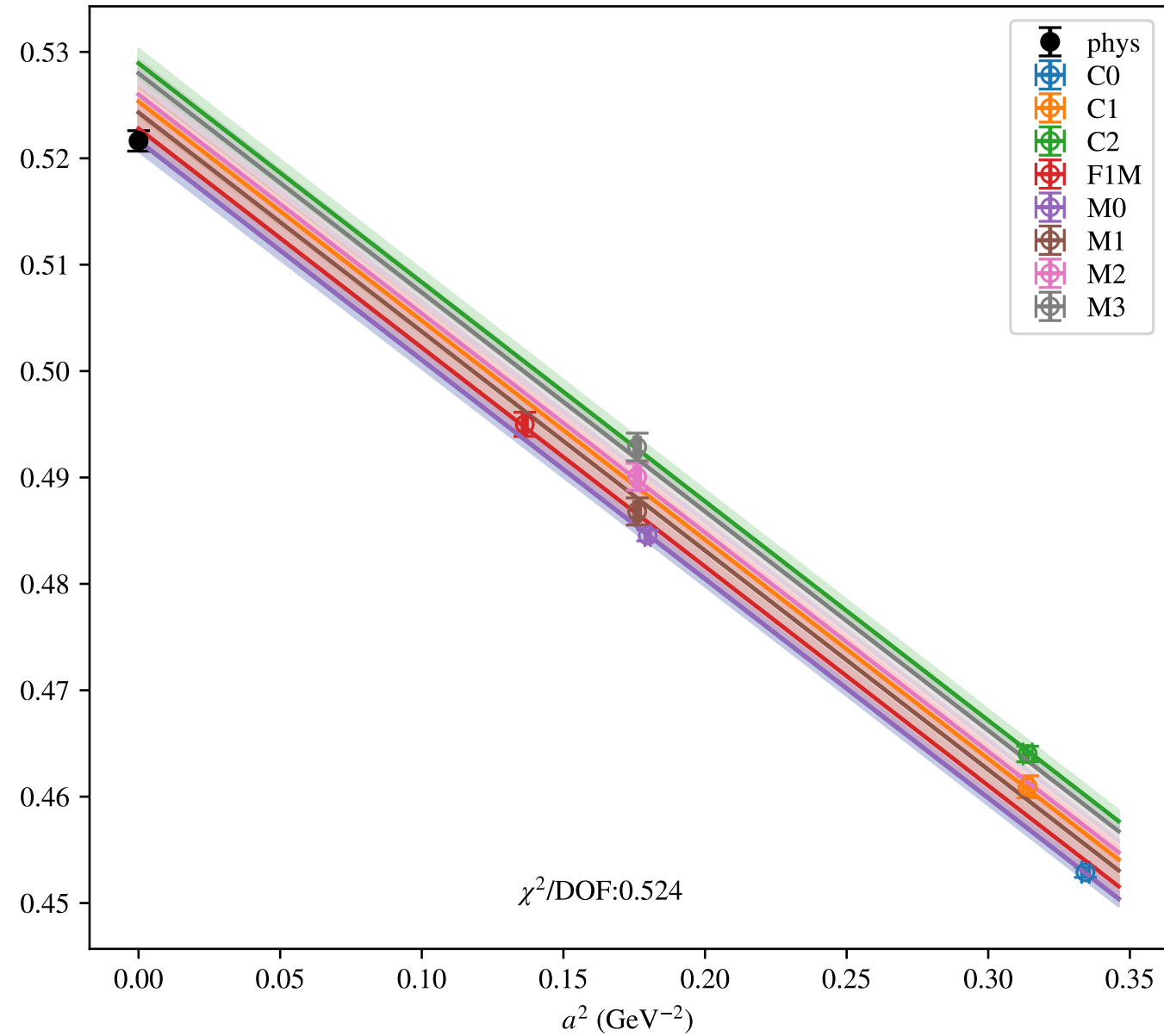


$(-0.5)*VVmAA$

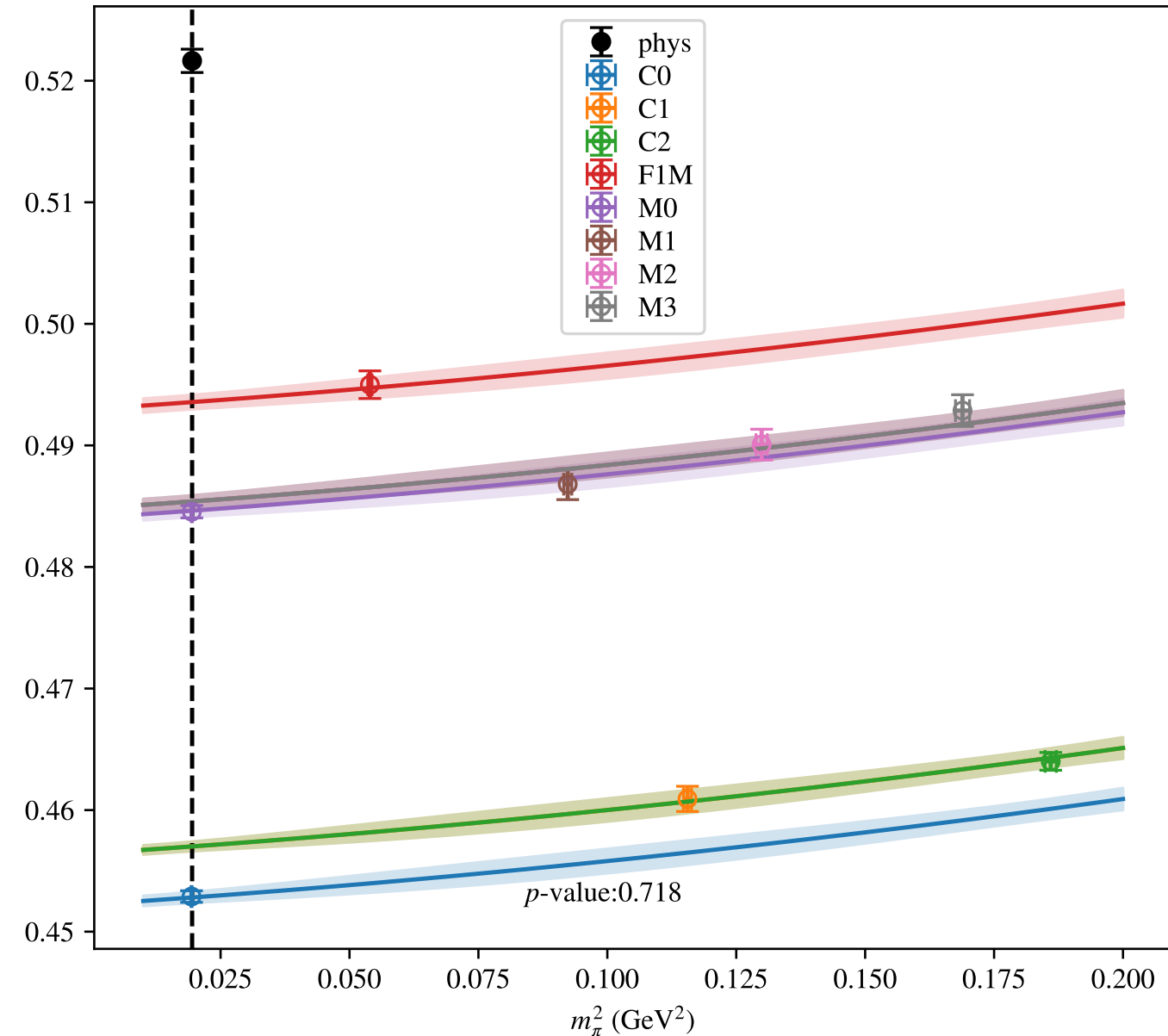


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.2 \text{ GeV}$$

$(-0.5)*VVmAA$

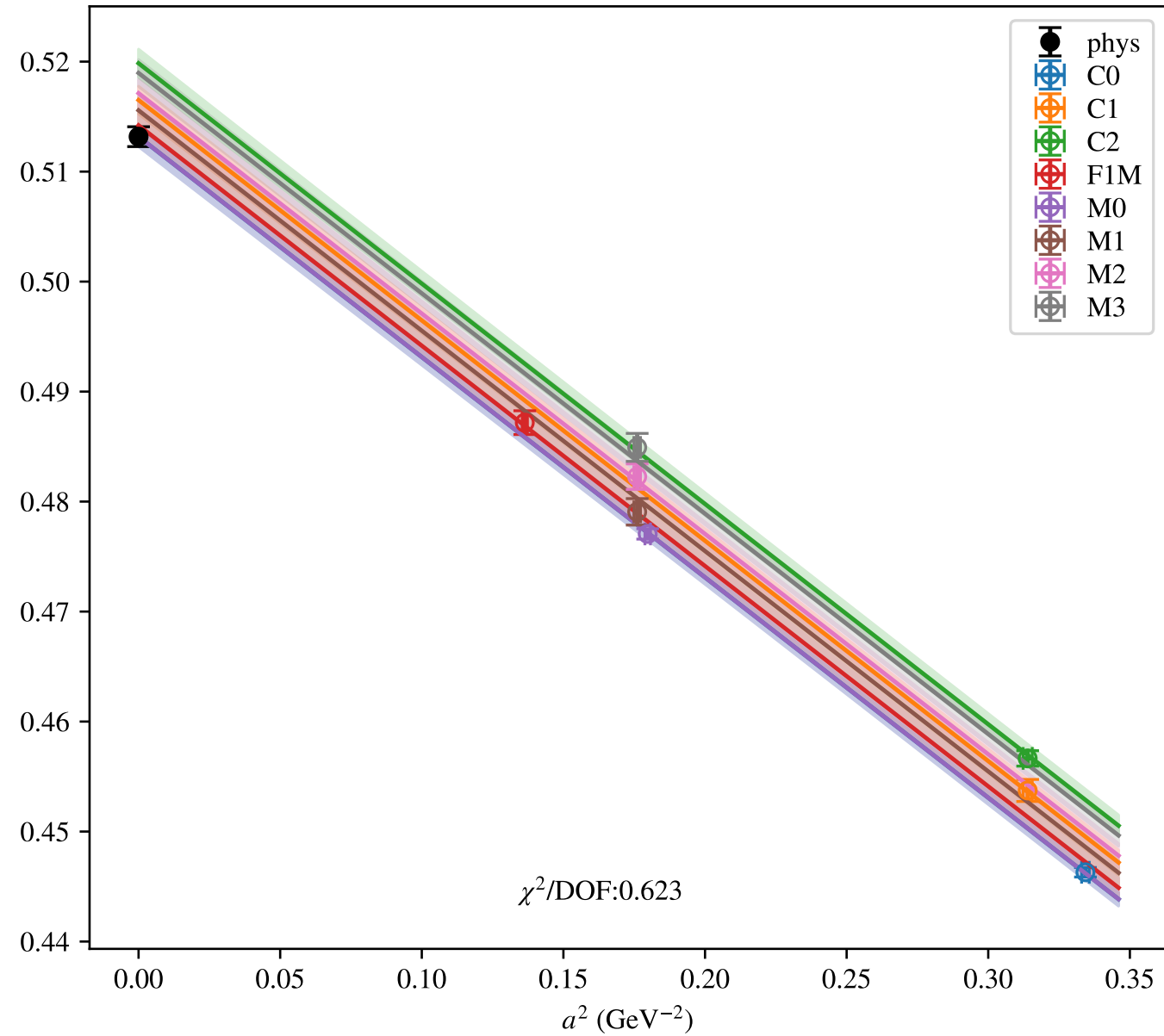


$(-0.5)*VVmAA$

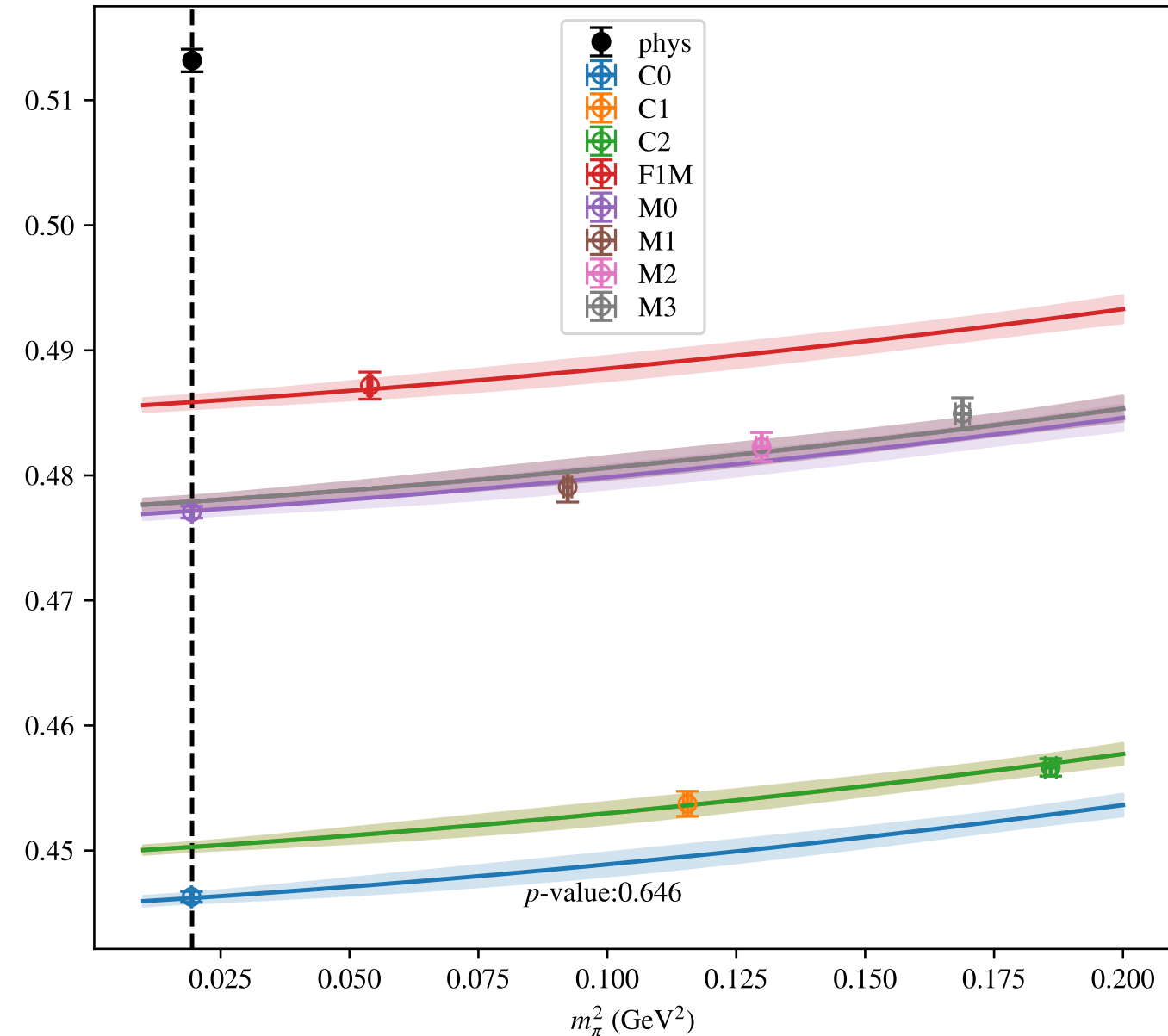


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.3 \text{ GeV}$$

$(-0.5)*VVmAA$

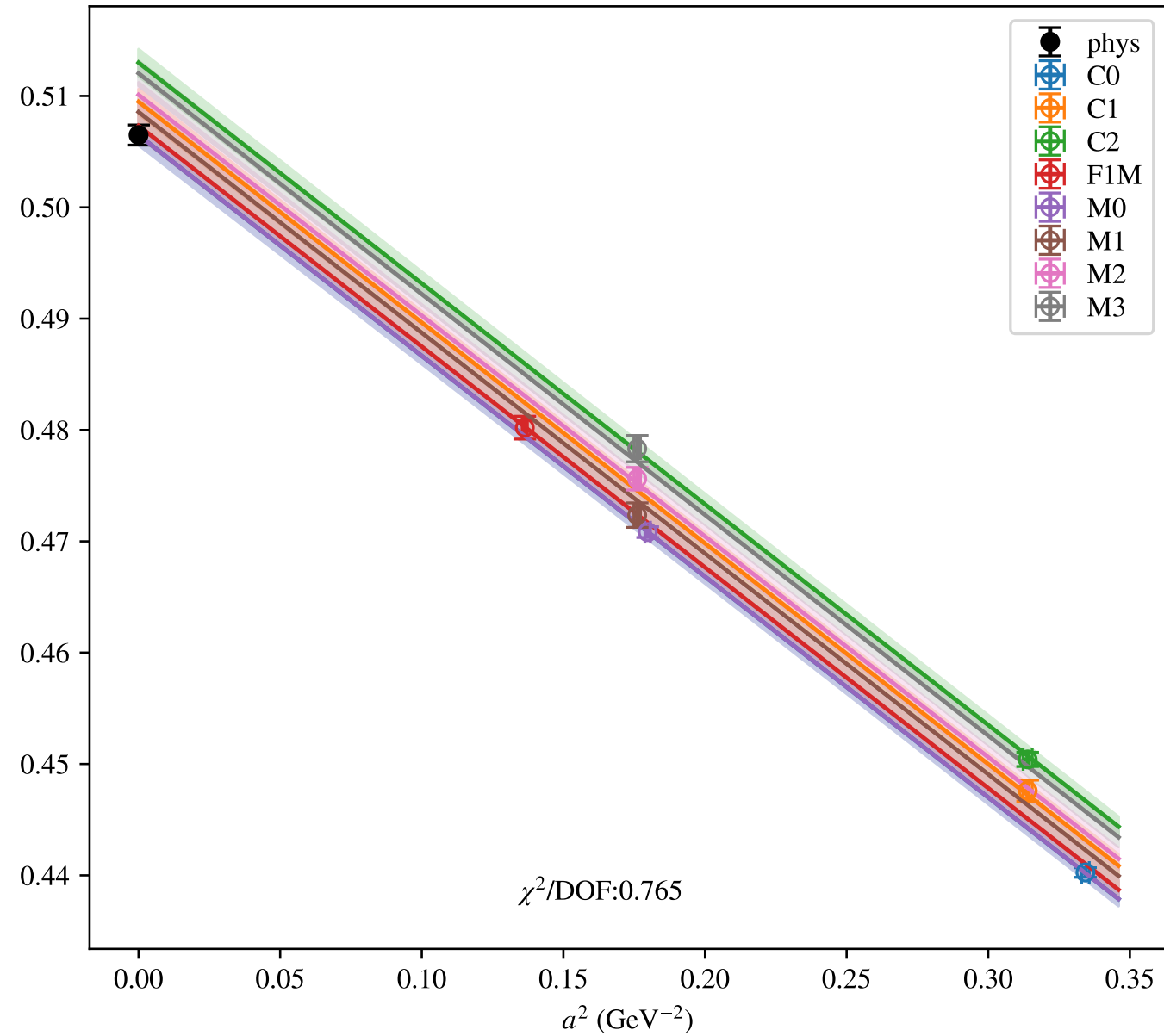


$(-0.5)*VVmAA$



$$a^2, m_\pi^2, m_\pi^4, \mu = 2.4 \text{ GeV}$$

$(-0.5)*VVmAA$



$(-0.5)*VVmAA$

