

# Kaon mixing: chiral and continuum extrapolations

R Mukherjee

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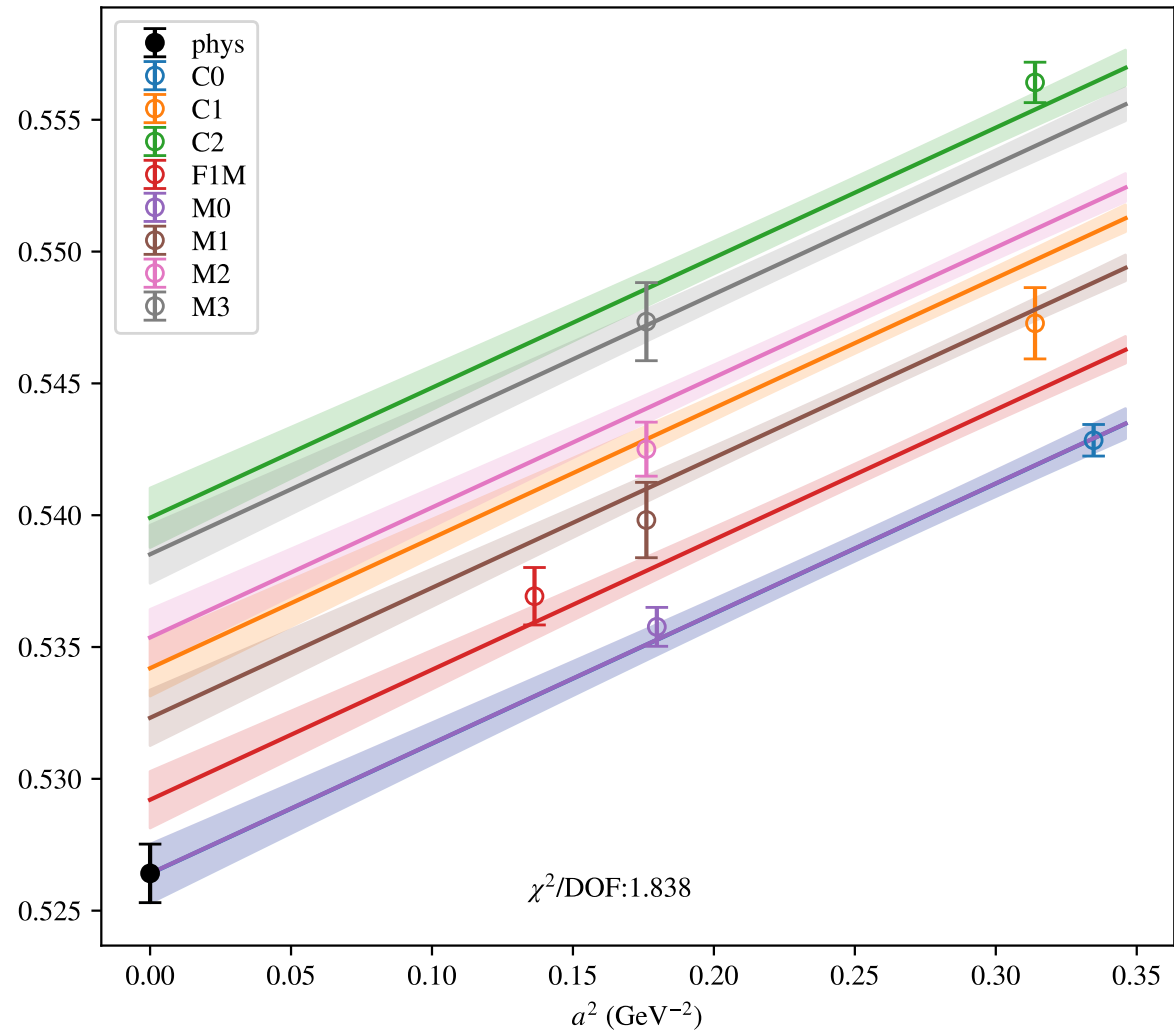
# 1 VVpAA

$\mu$ (GeV)	$a^2, m_\pi^2$	$a^2, m_\pi^2$ no C	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$
2.0	<b>0.5264(11)</b> : 1.838 (0.102)	<b>0.5310(48)</b> : 0.864 (0.421)	<b>0.5326(82)</b> : 2.159 (0.071)	<b>0.5282(12)</b> : 0.639 (0.635)
1.8	<b>0.5293(14)</b> : 1.315 (0.254)	<b>0.5328(53)</b> : 0.478 (0.62)	<b>0.5335(91)</b> : 1.6 (0.171)	<b>0.5310(14)</b> : 0.307 (0.873)
1.5	<b>0.5338(19)</b> : 0.734 (0.598)	<b>0.5356(64)</b> : 0.159 (0.853)	<b>0.536(11)</b> : 0.909 (0.457)	<b>0.5352(18)</b> : 0.099 (0.983)

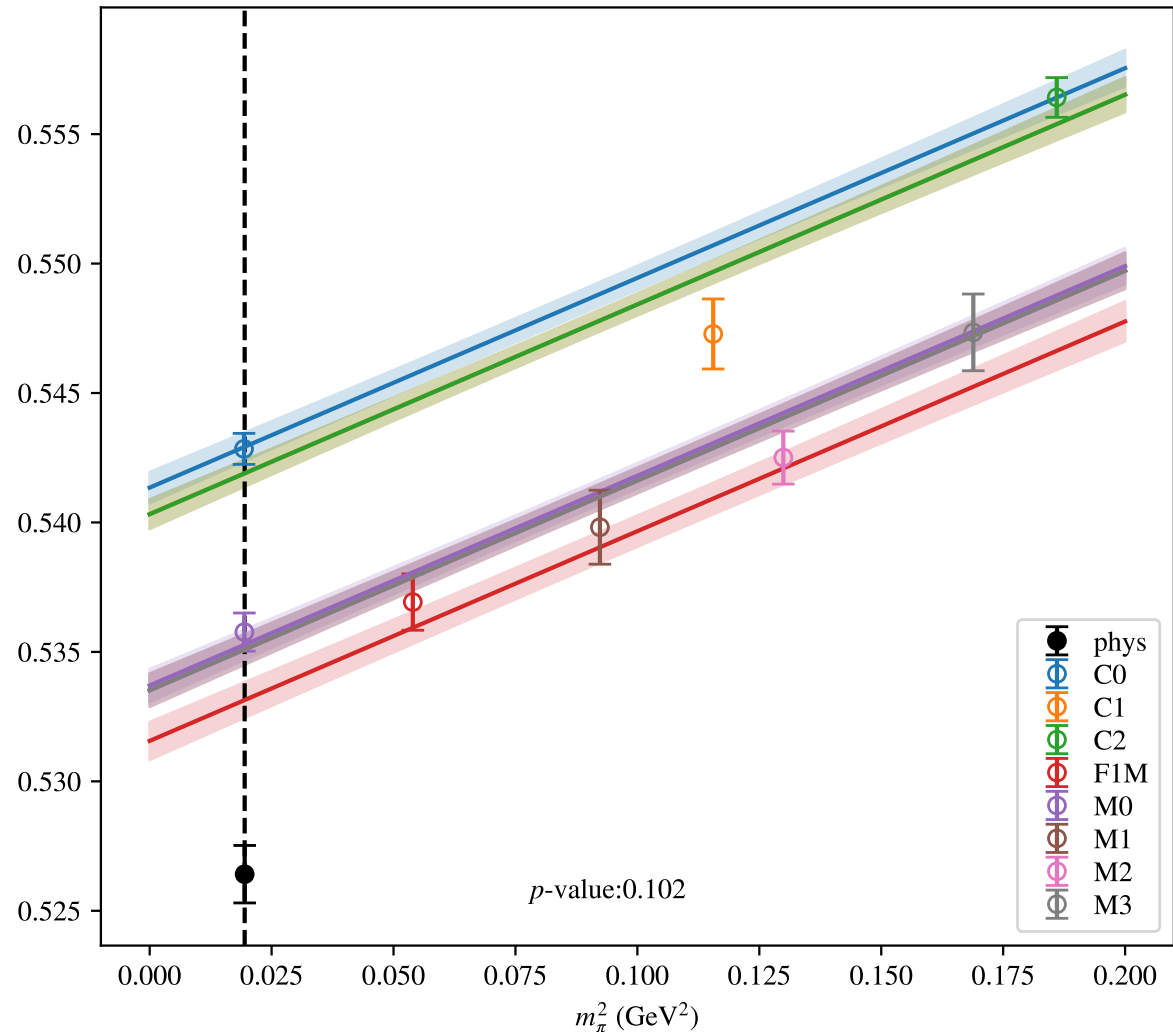
Table 1: Physical point value from chiral and continuum extrapolation at renormalisation scale  $\mu$ . Entries are **value(error)**:  $\chi^2/\text{DOF}$  ( $p$ -value).

$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

VVpAA

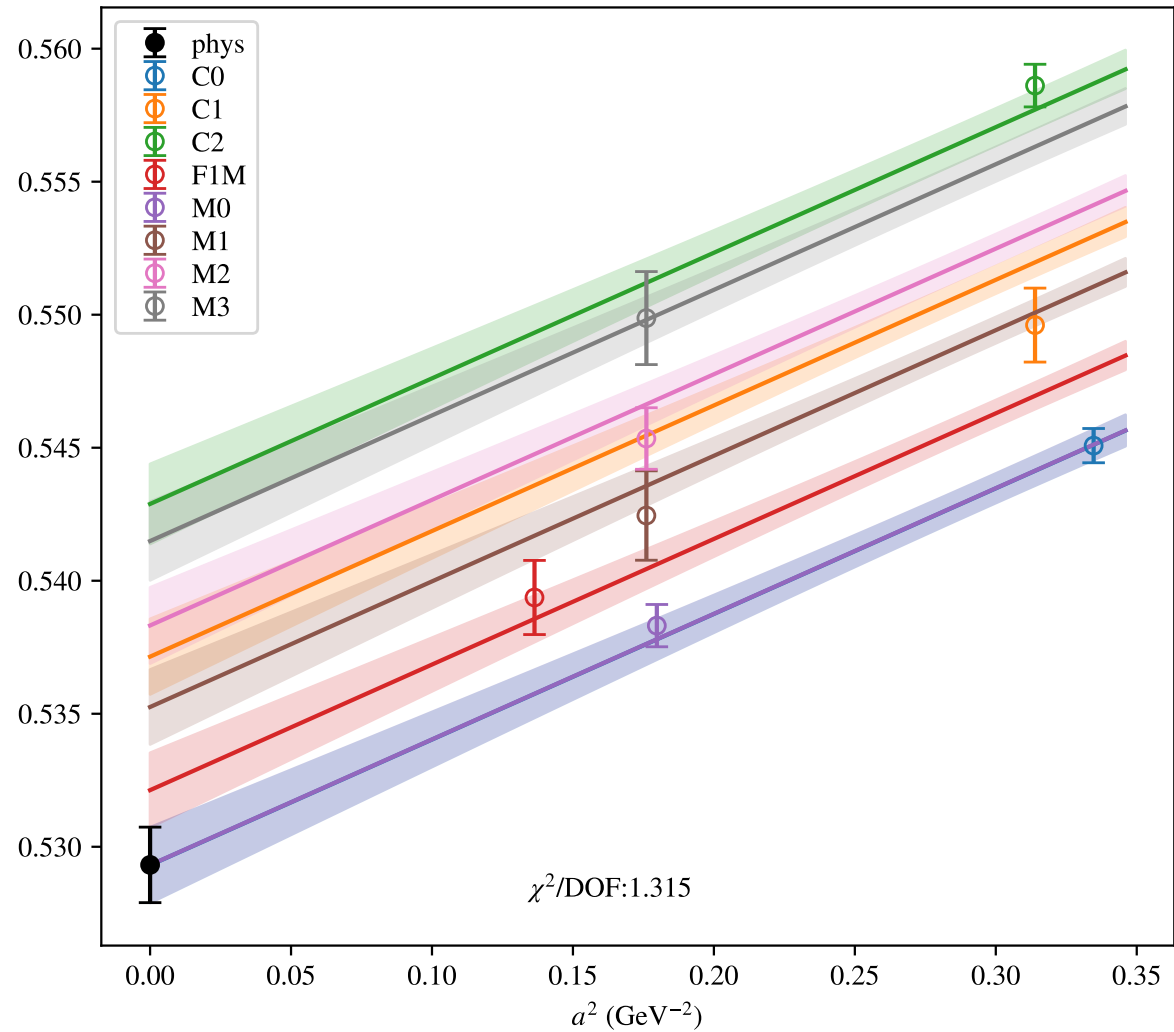


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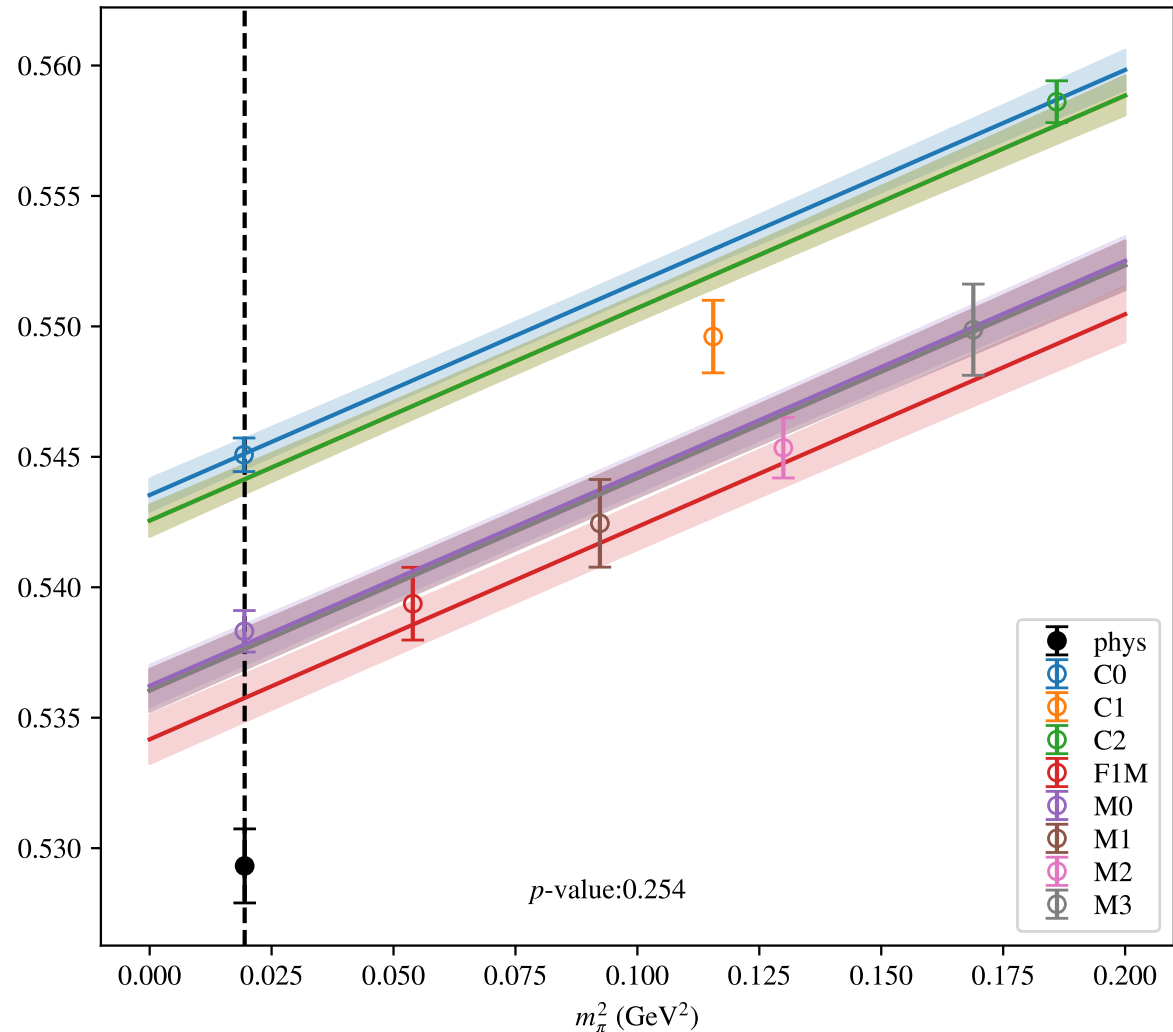


$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$

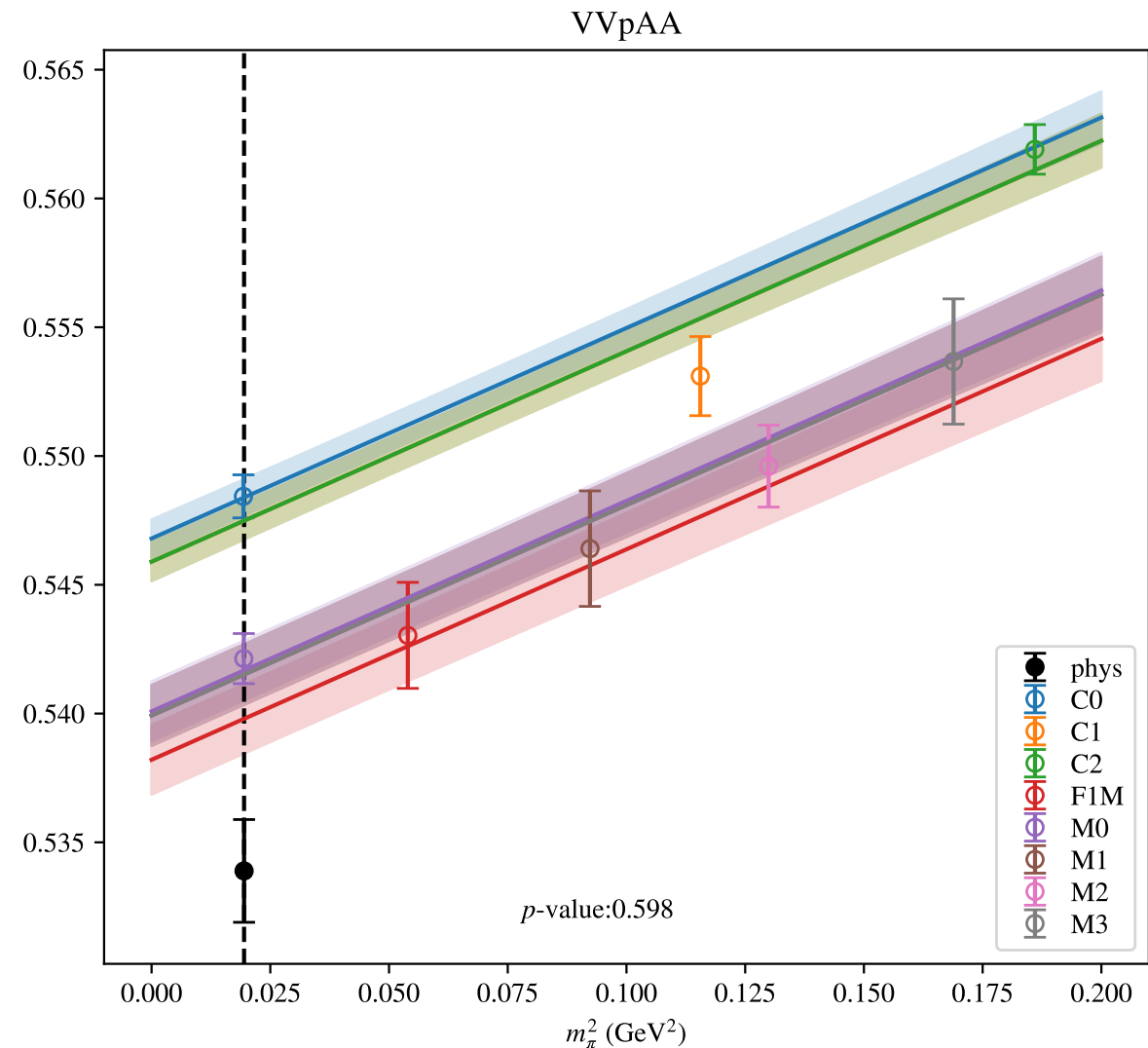
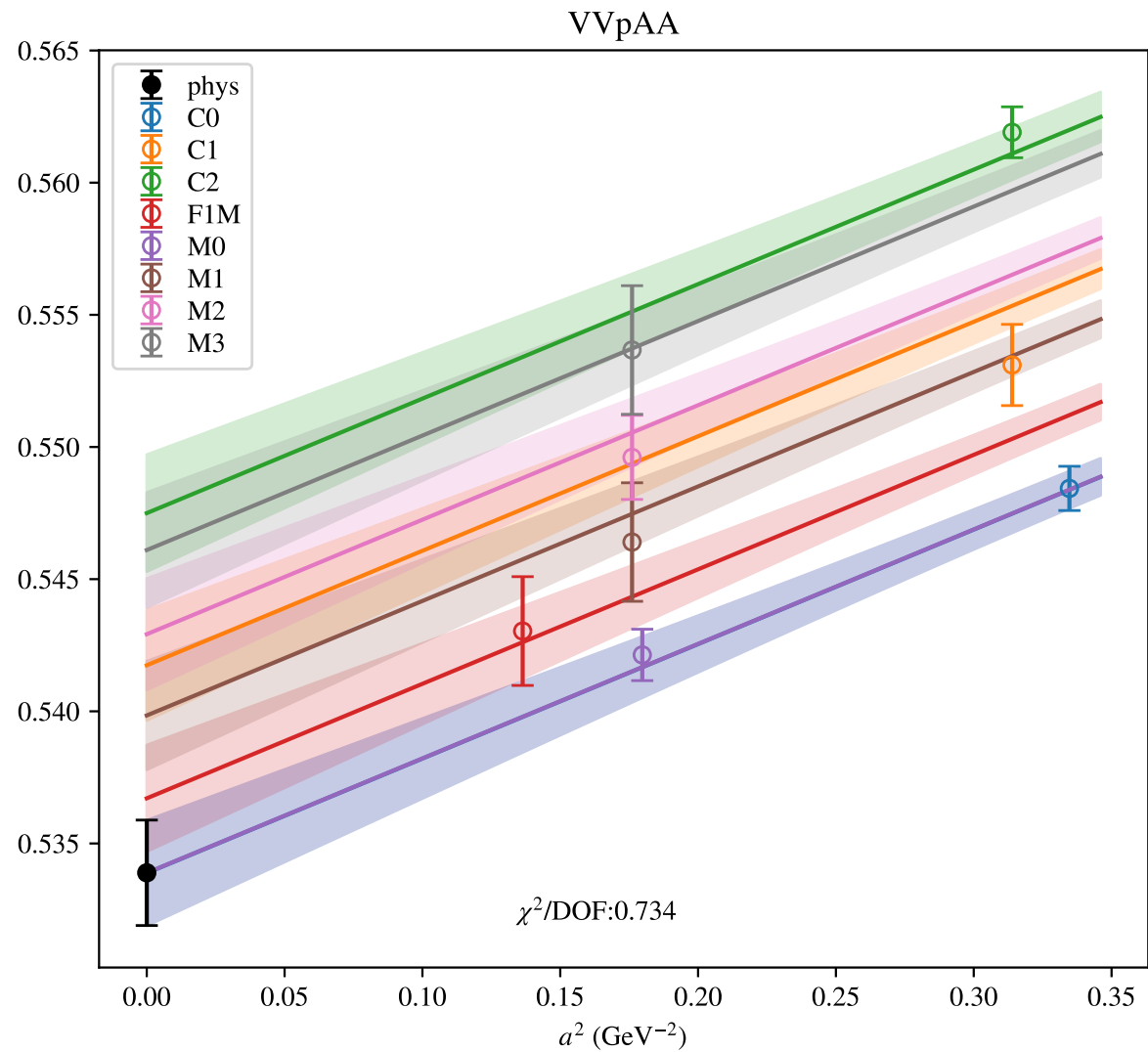
VVpAA



VVpAA

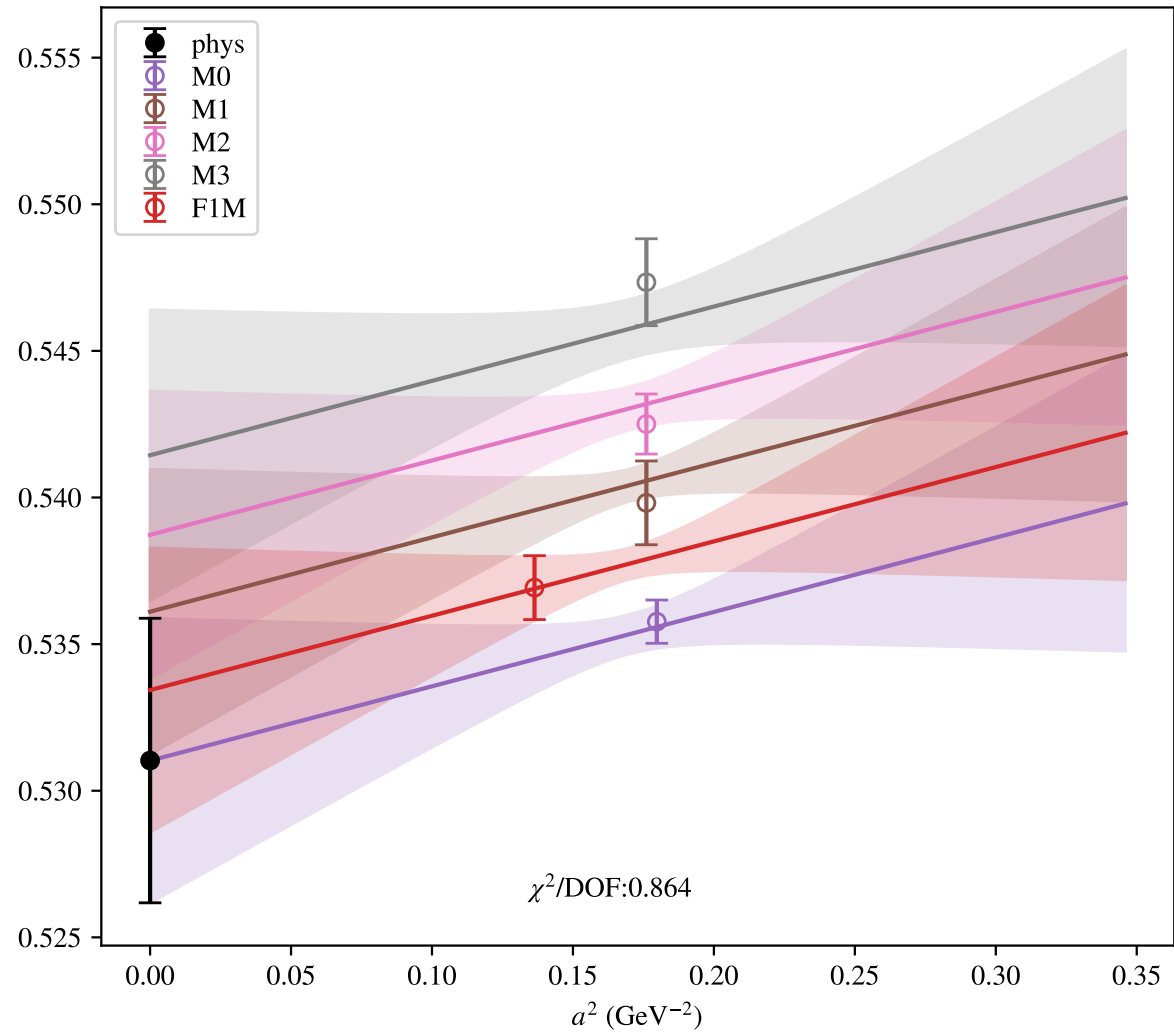


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

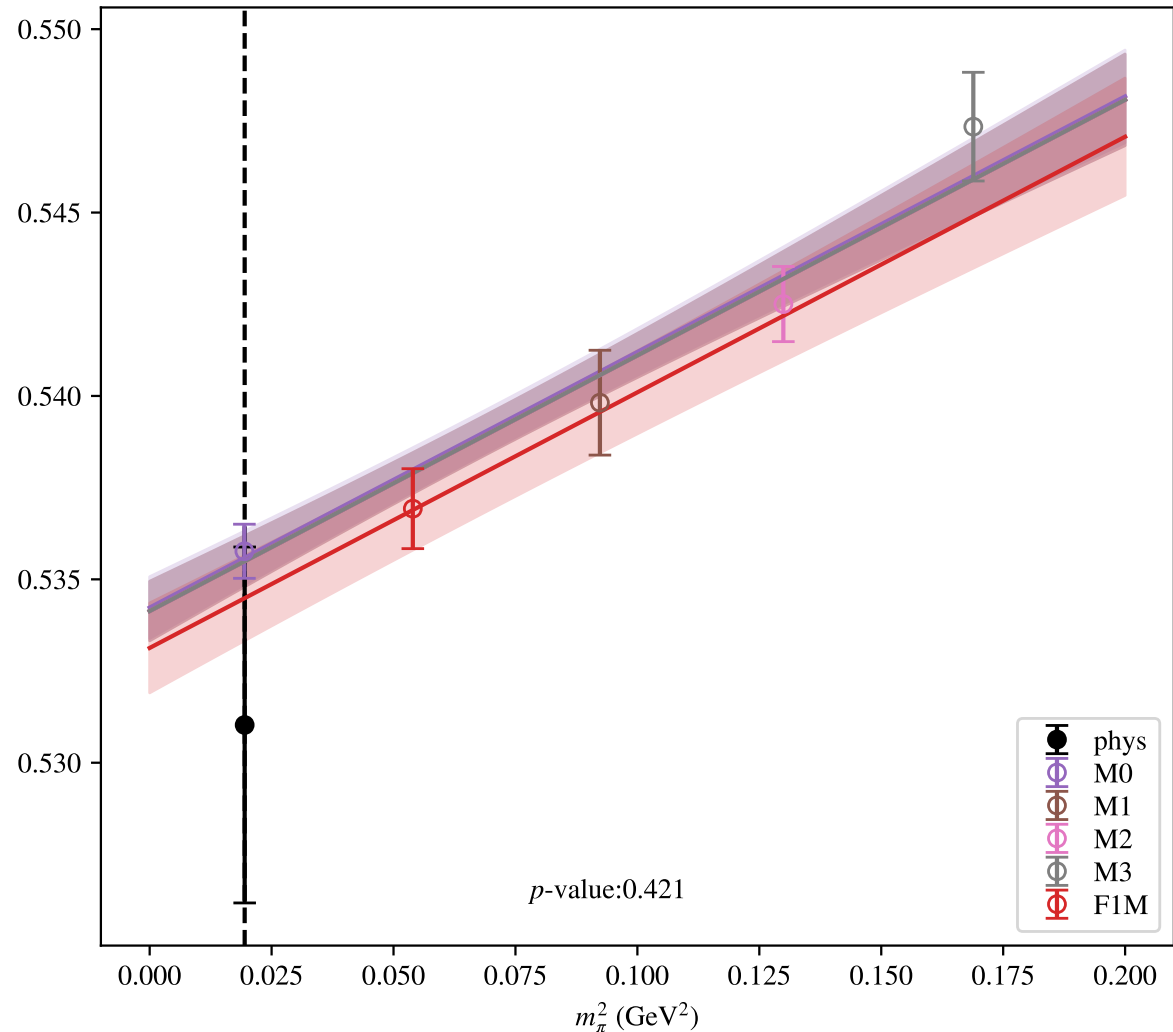


$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

VVpAA

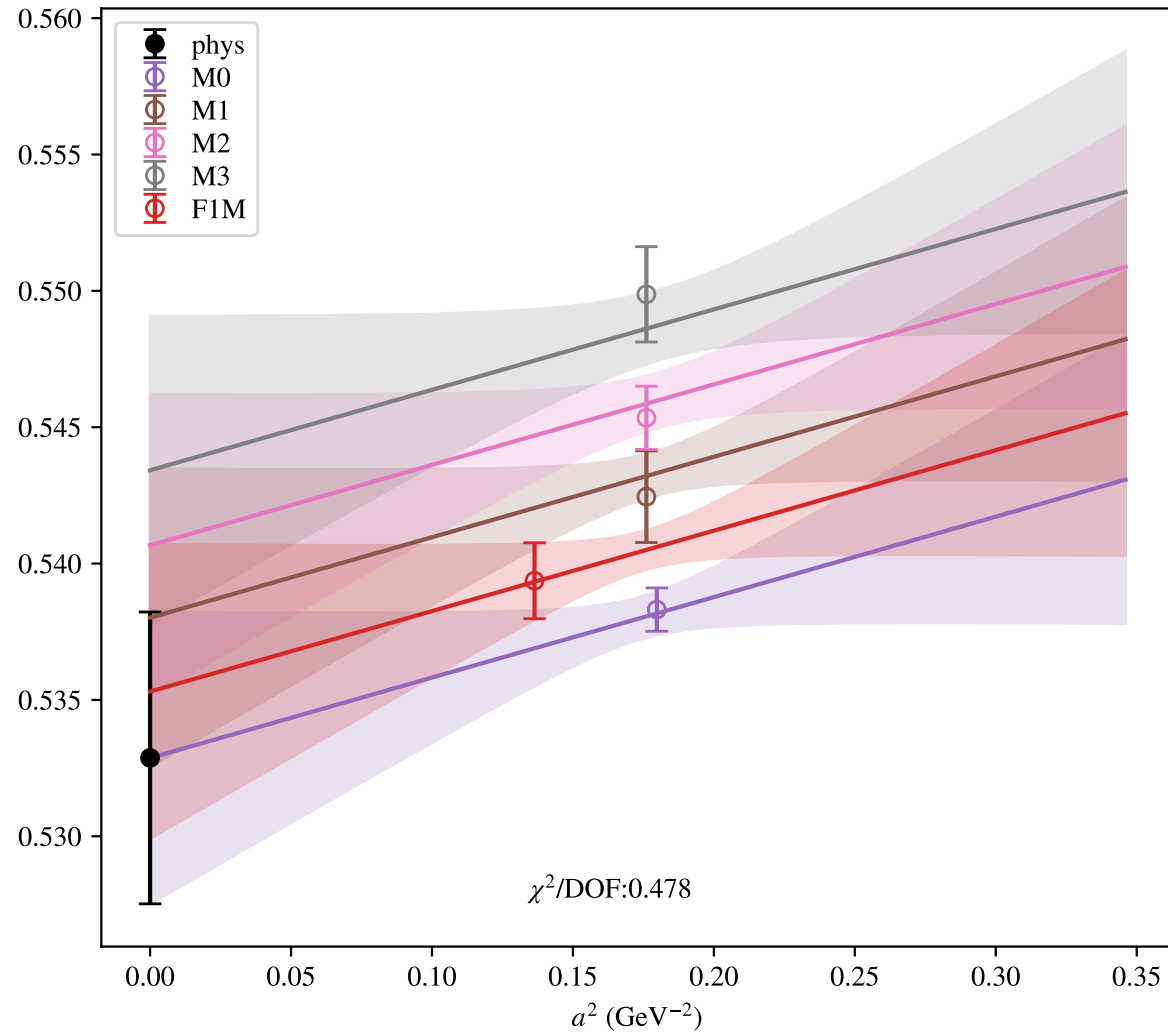


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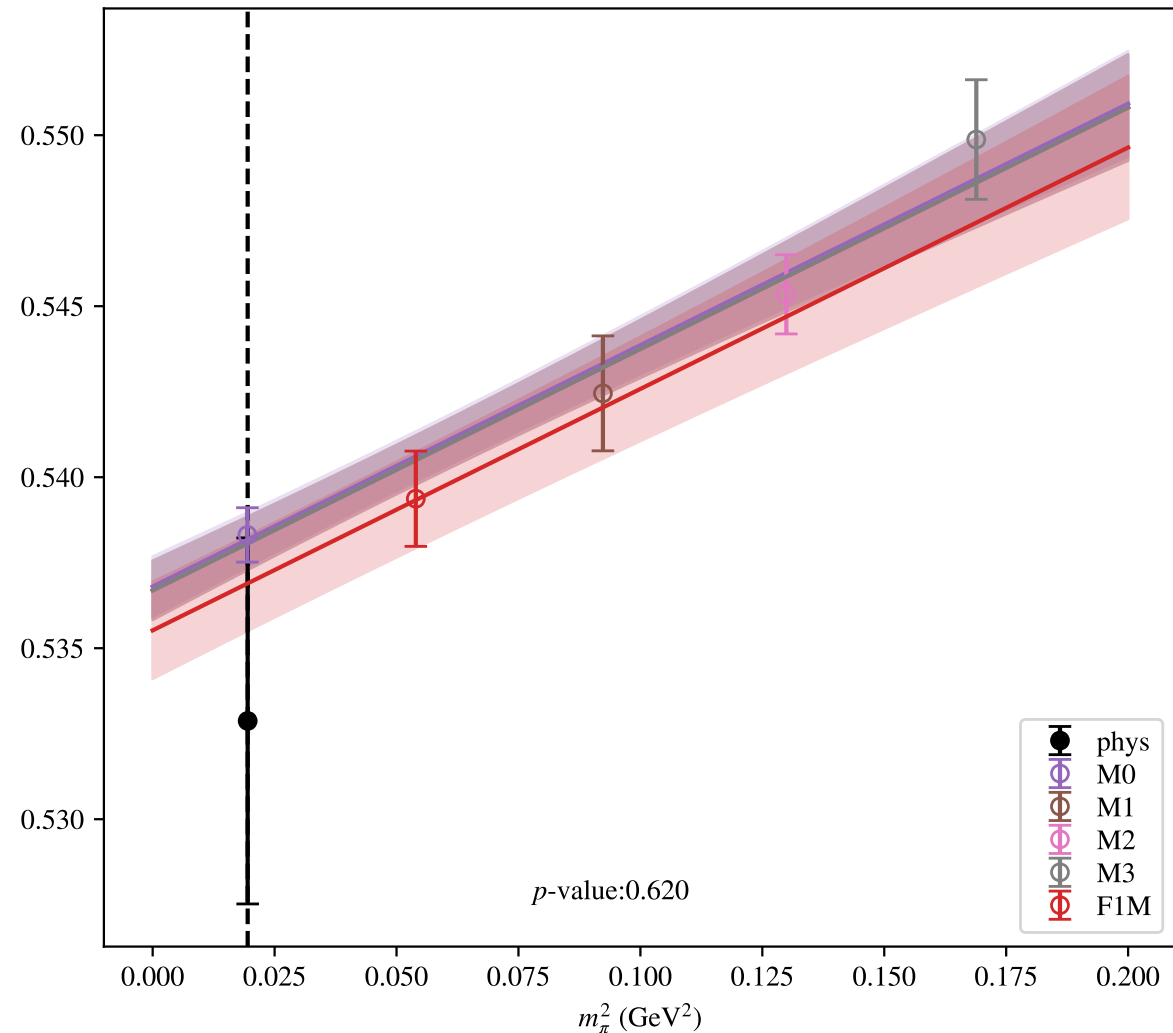


$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$

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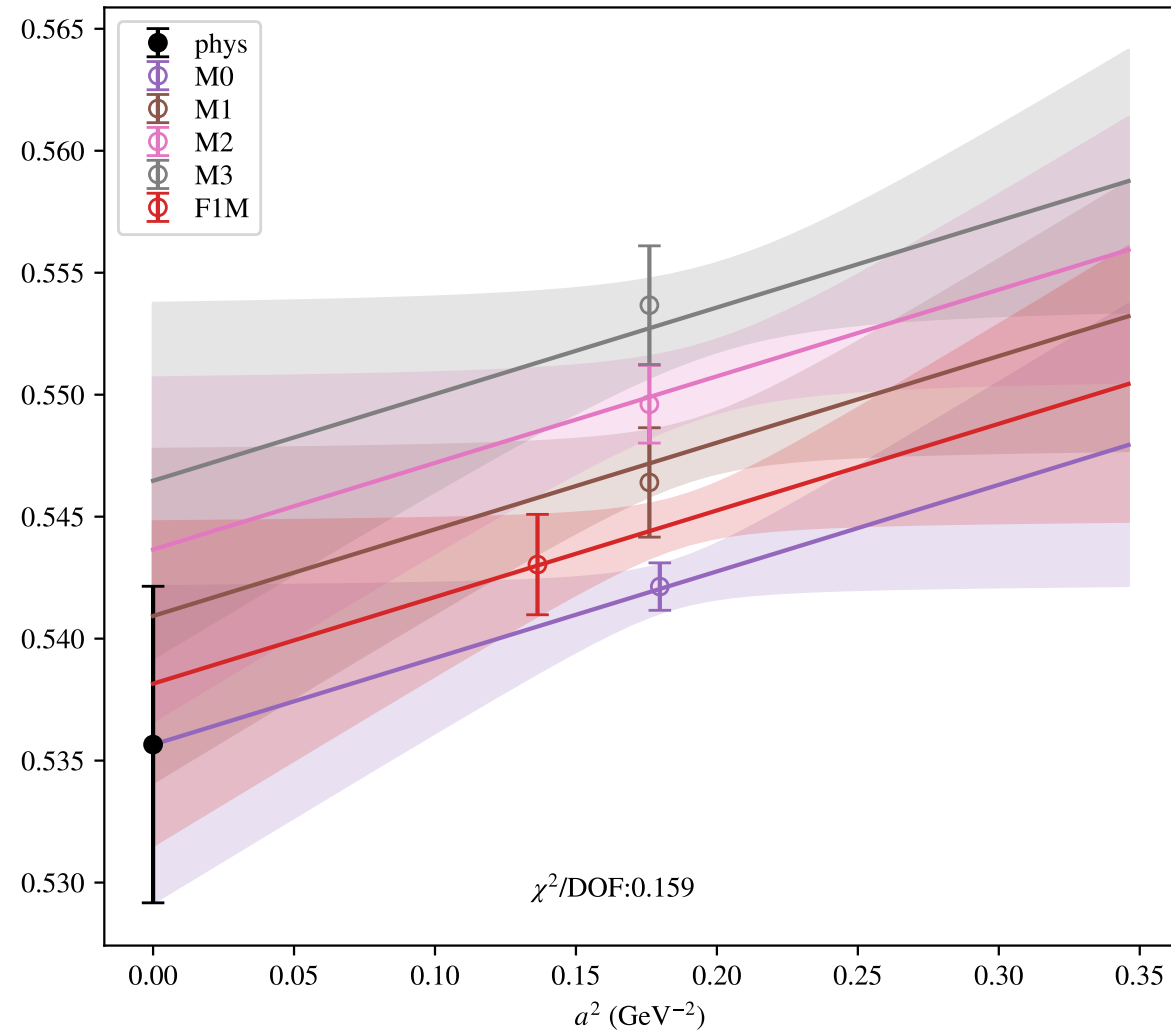


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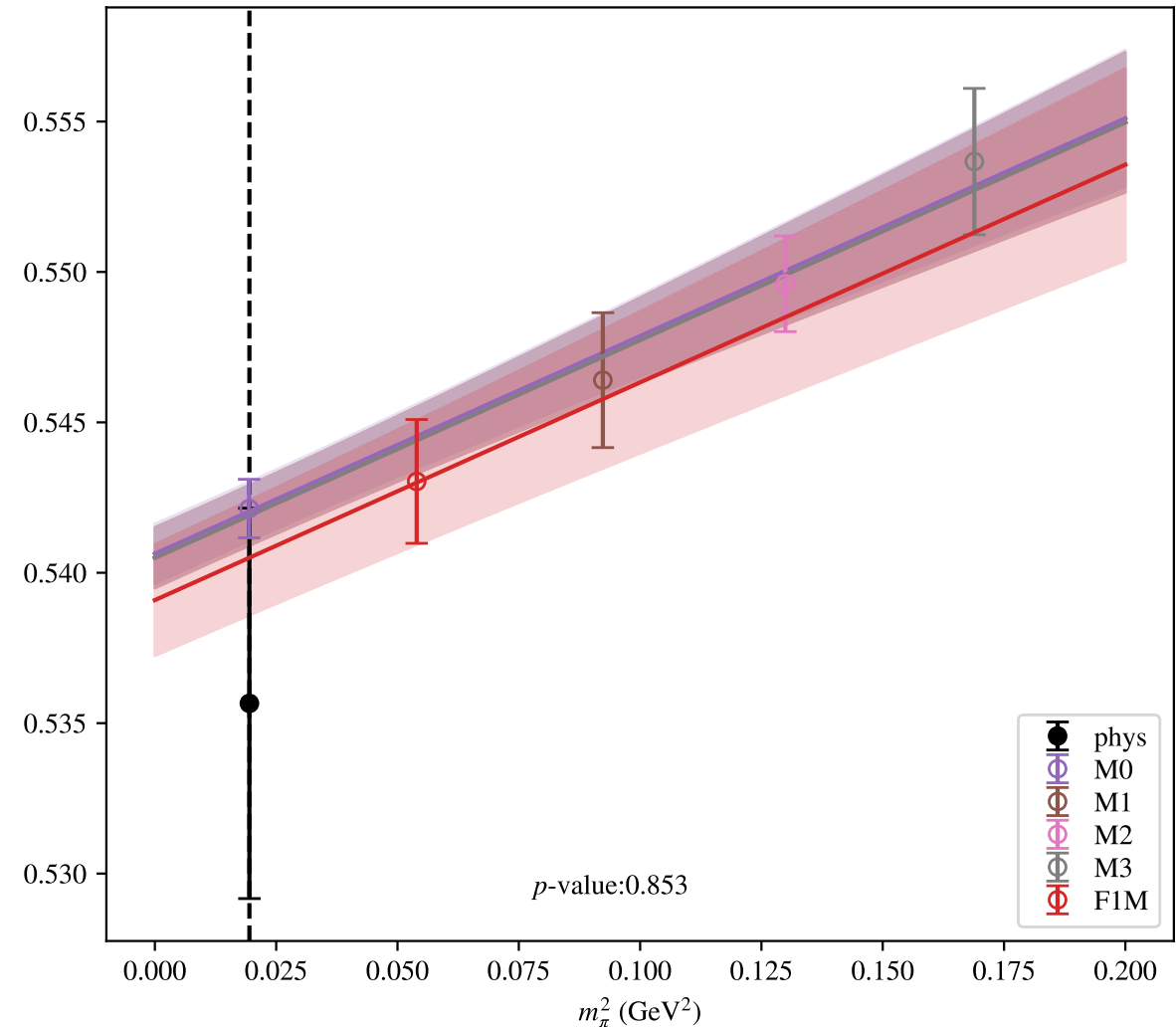


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

VVpAA



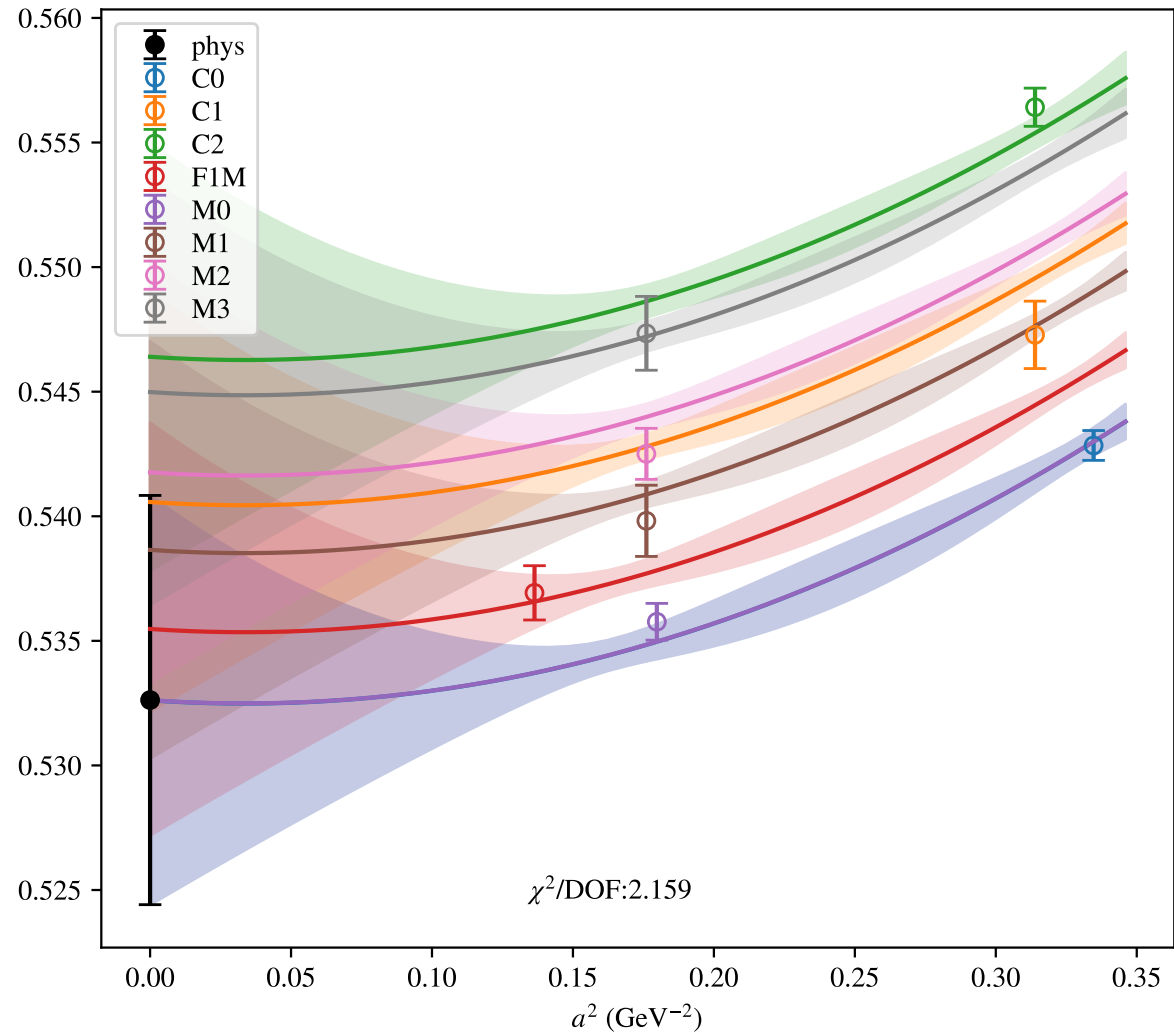
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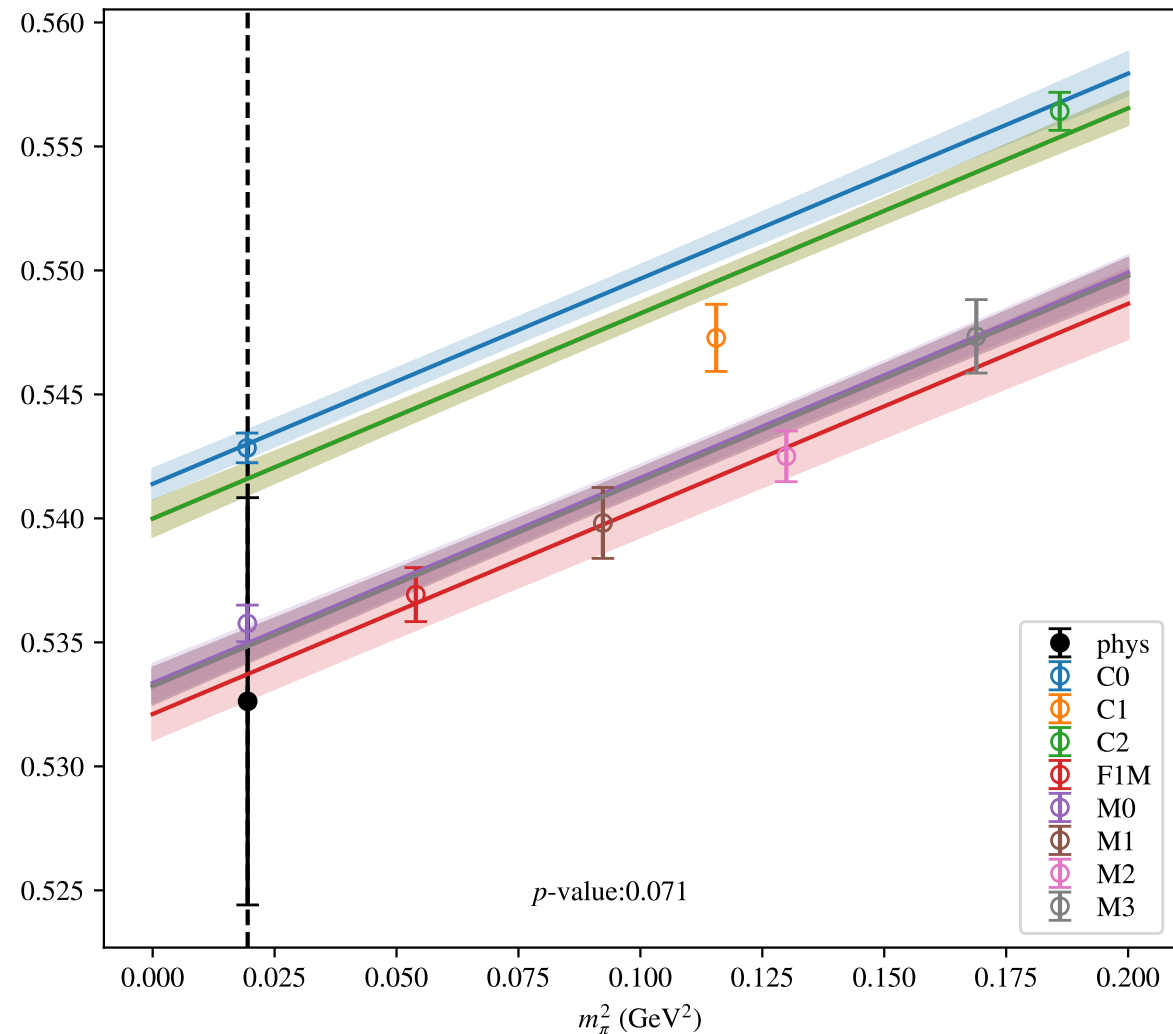


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

VVpAA

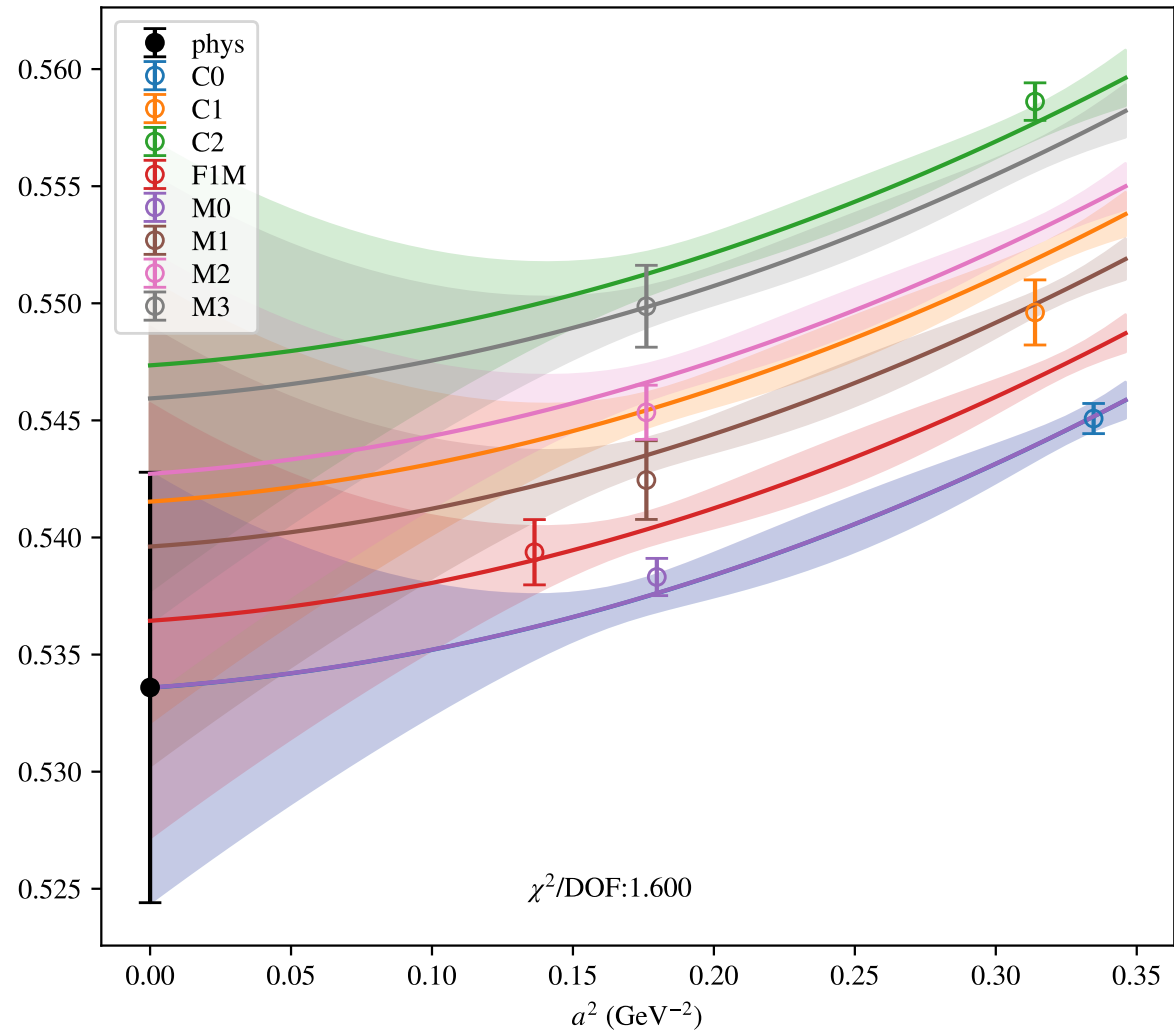


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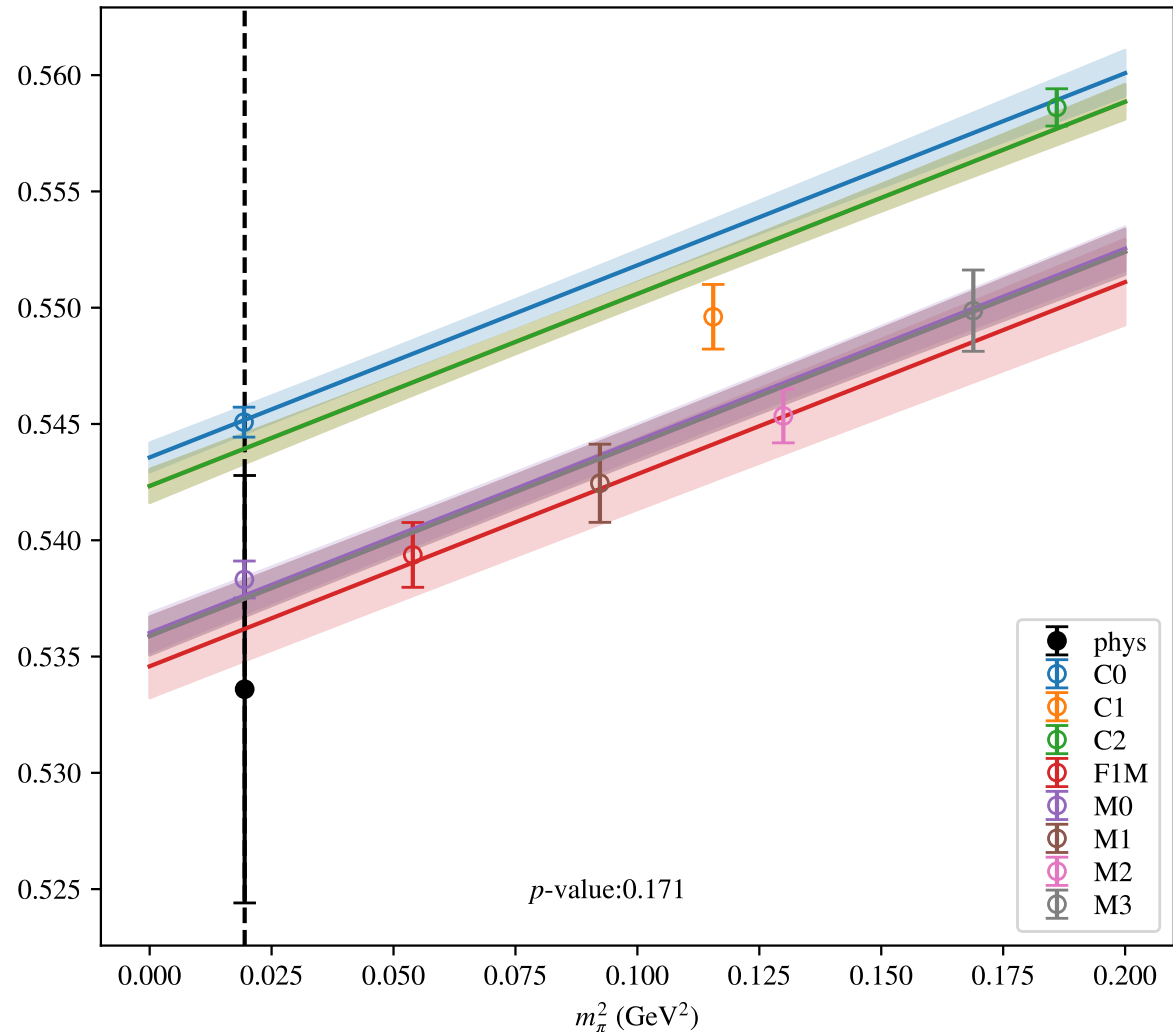


$$a^2, a^4, m_\pi^2, \mu = 1.8 \text{ GeV}$$

VVpAA

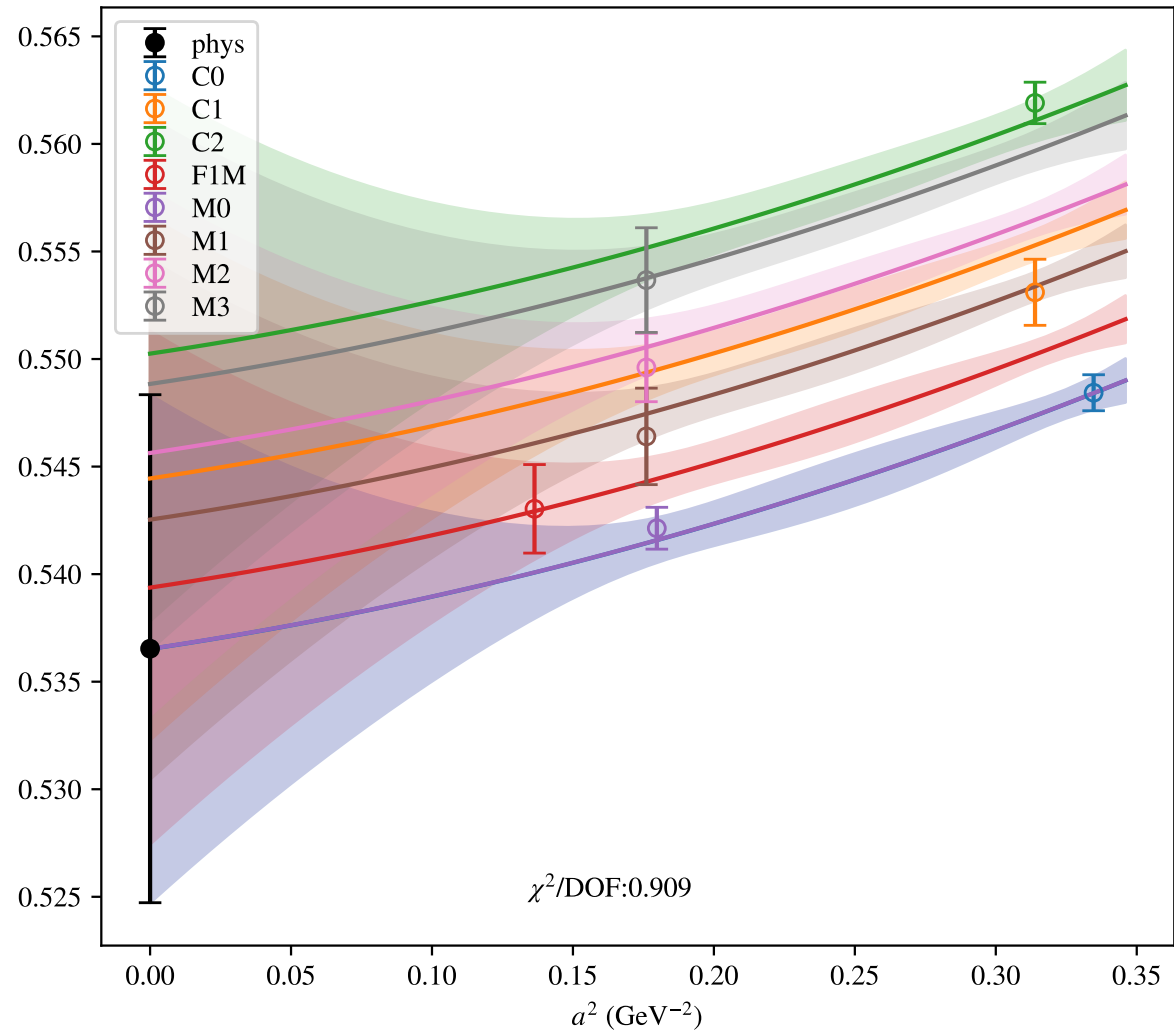


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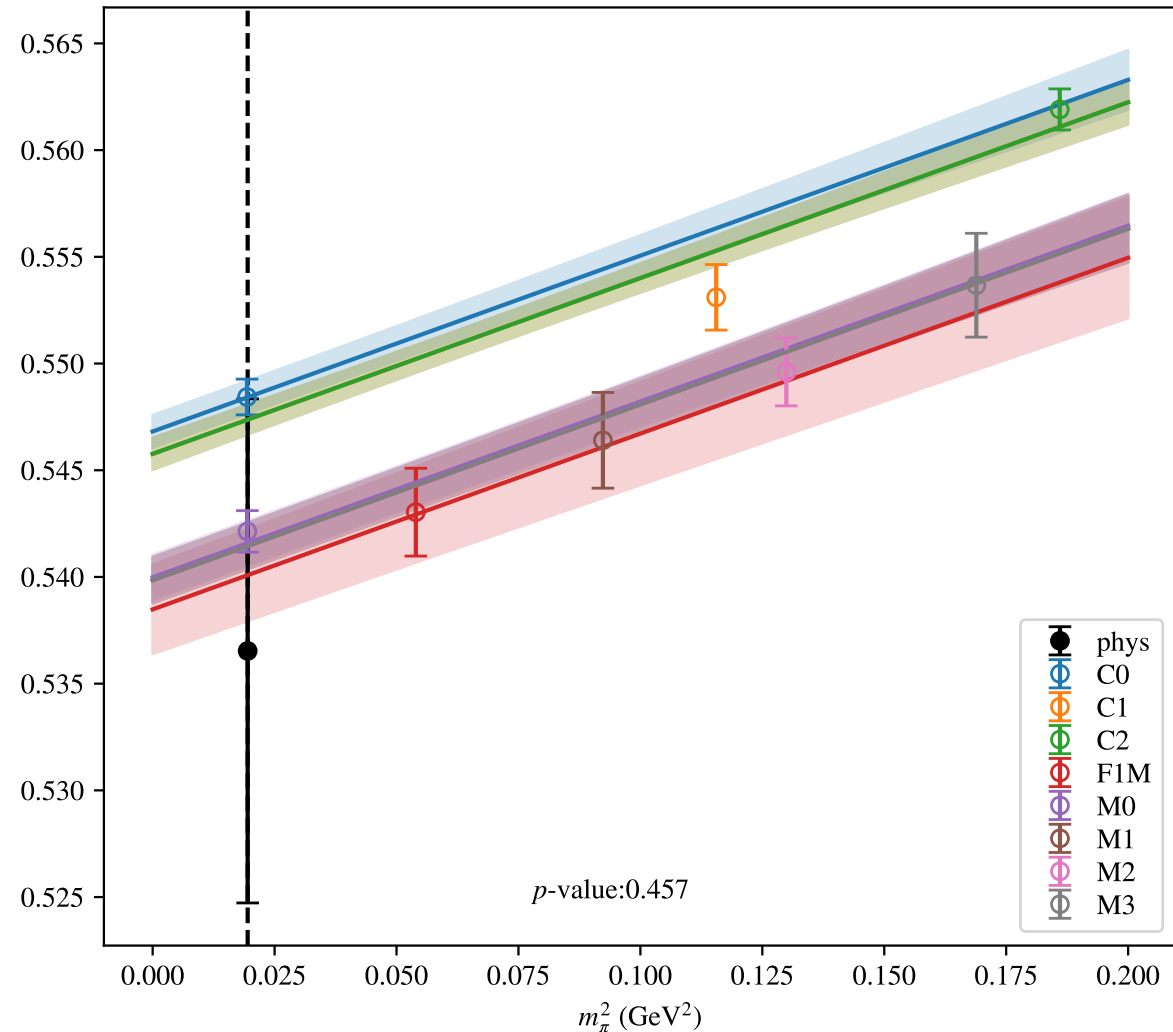


$$a^2, a^4, m_\pi^2, \mu = 1.5 \text{ GeV}$$

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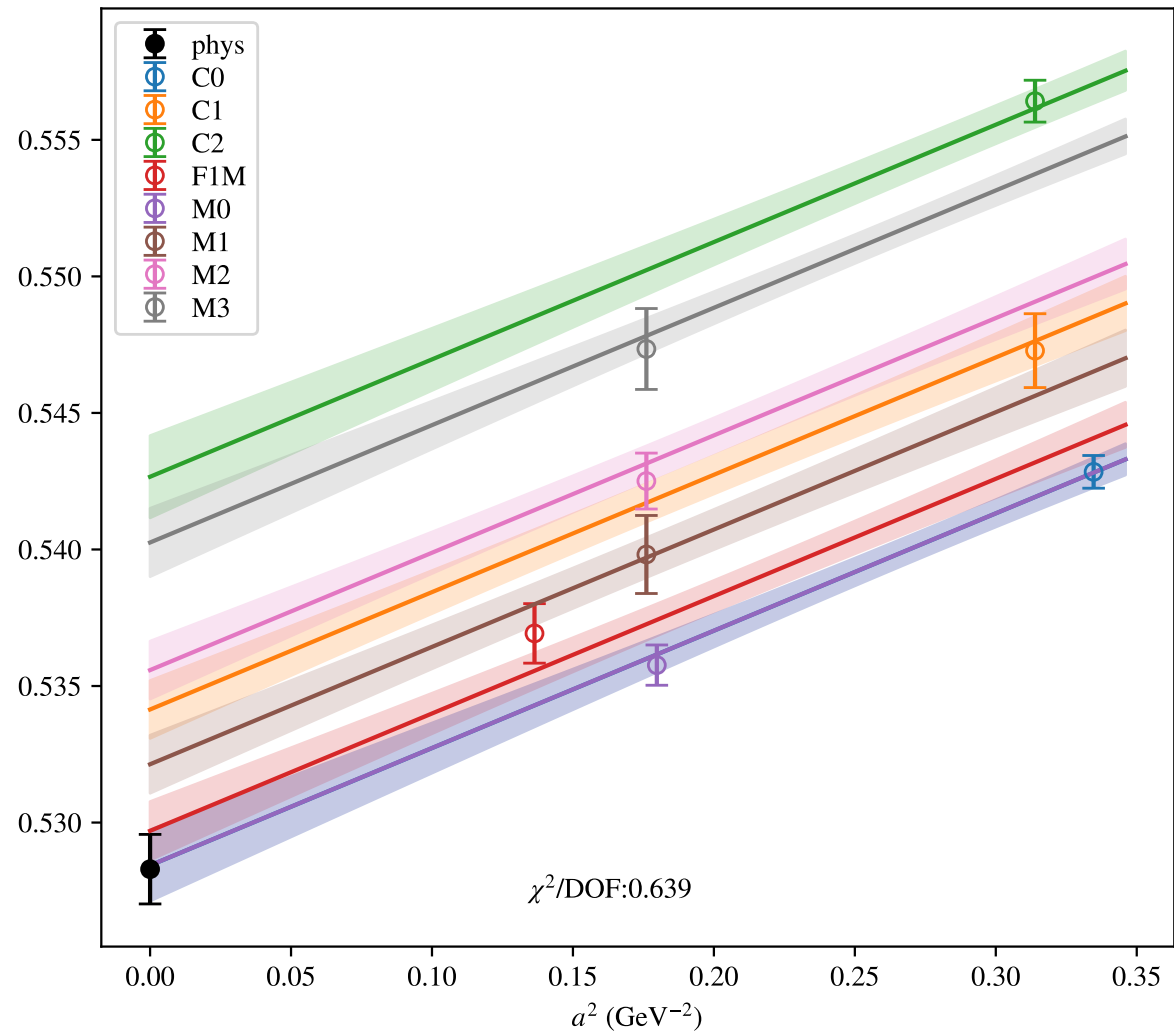


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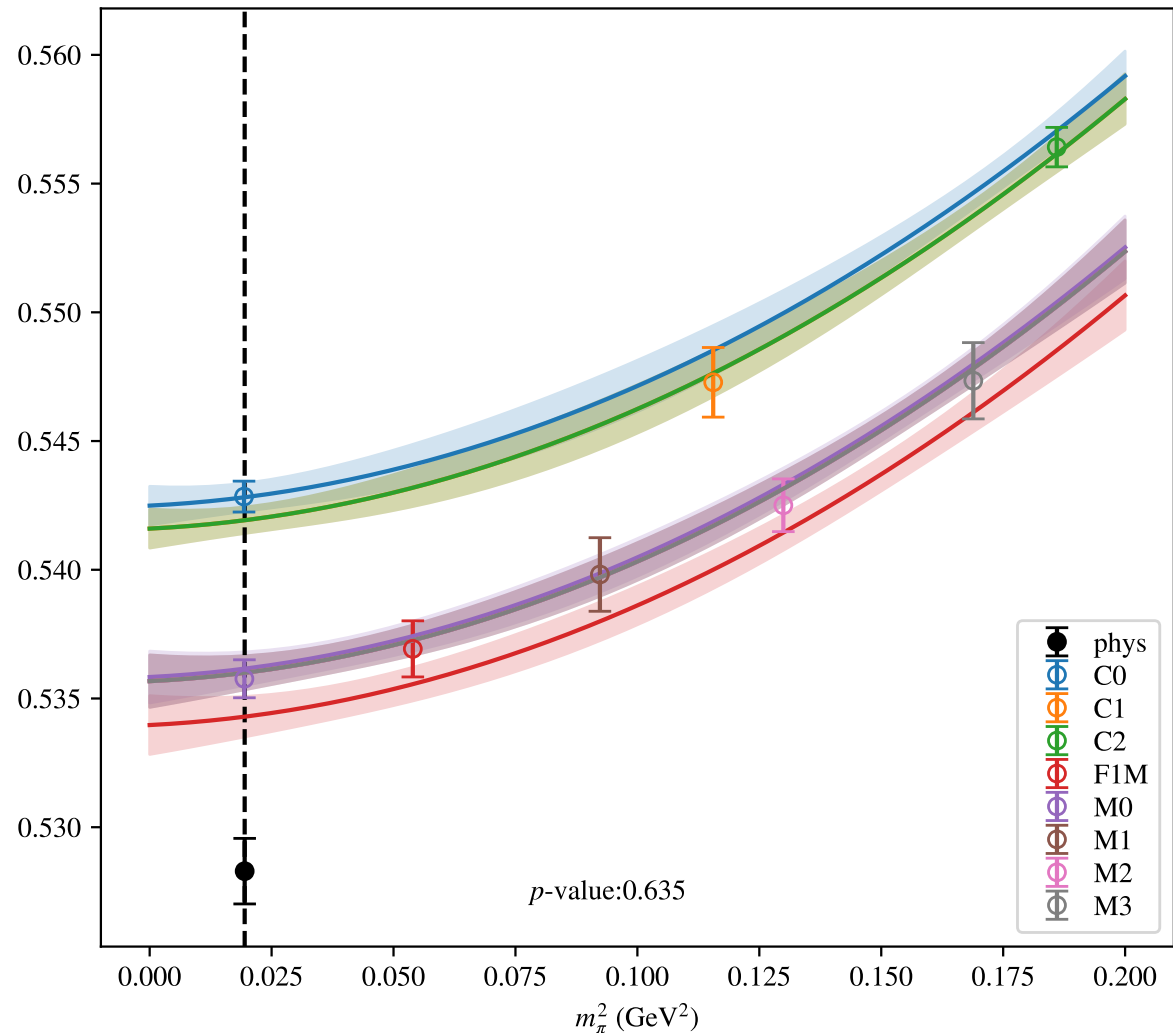


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

VVpAA

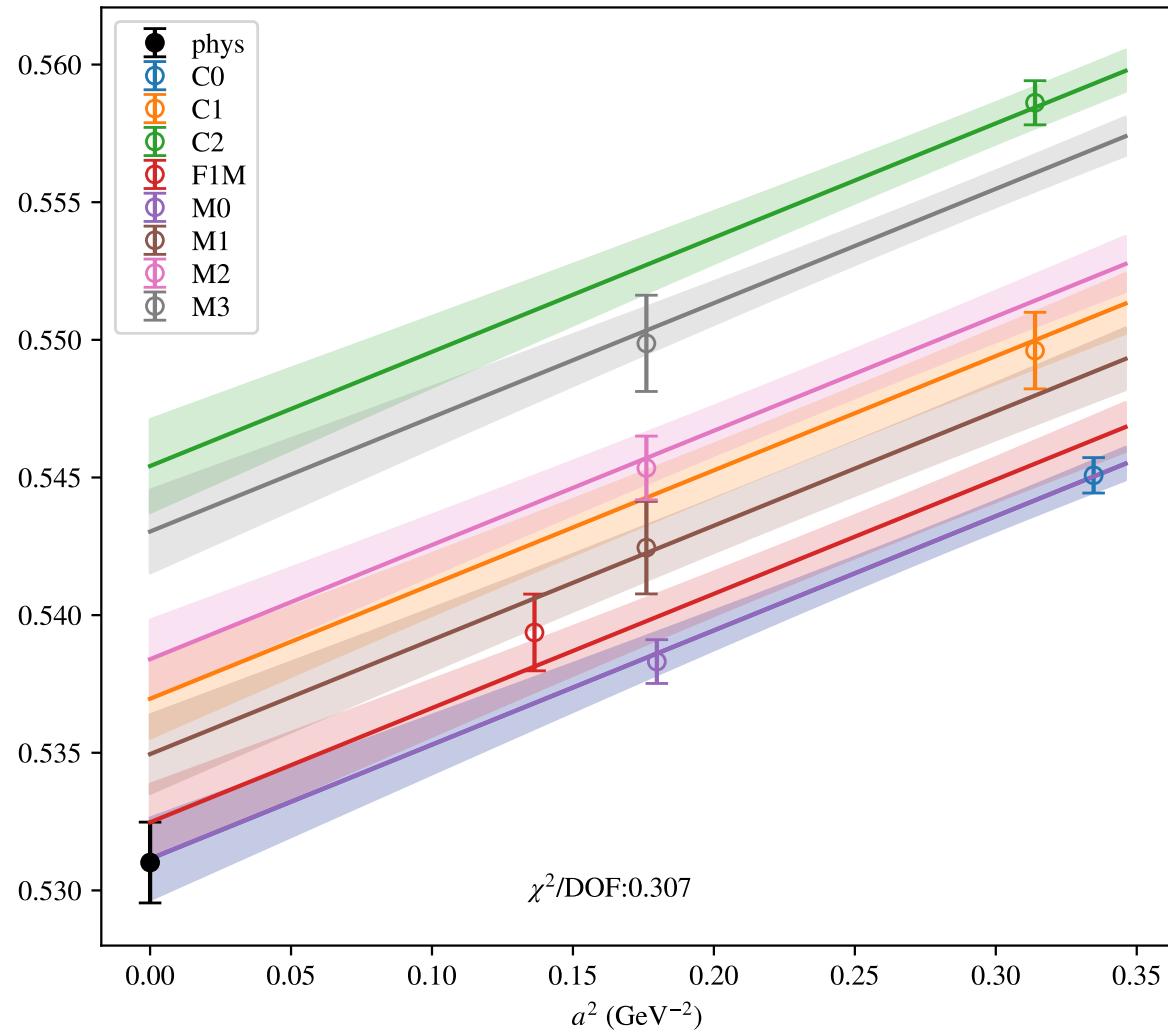


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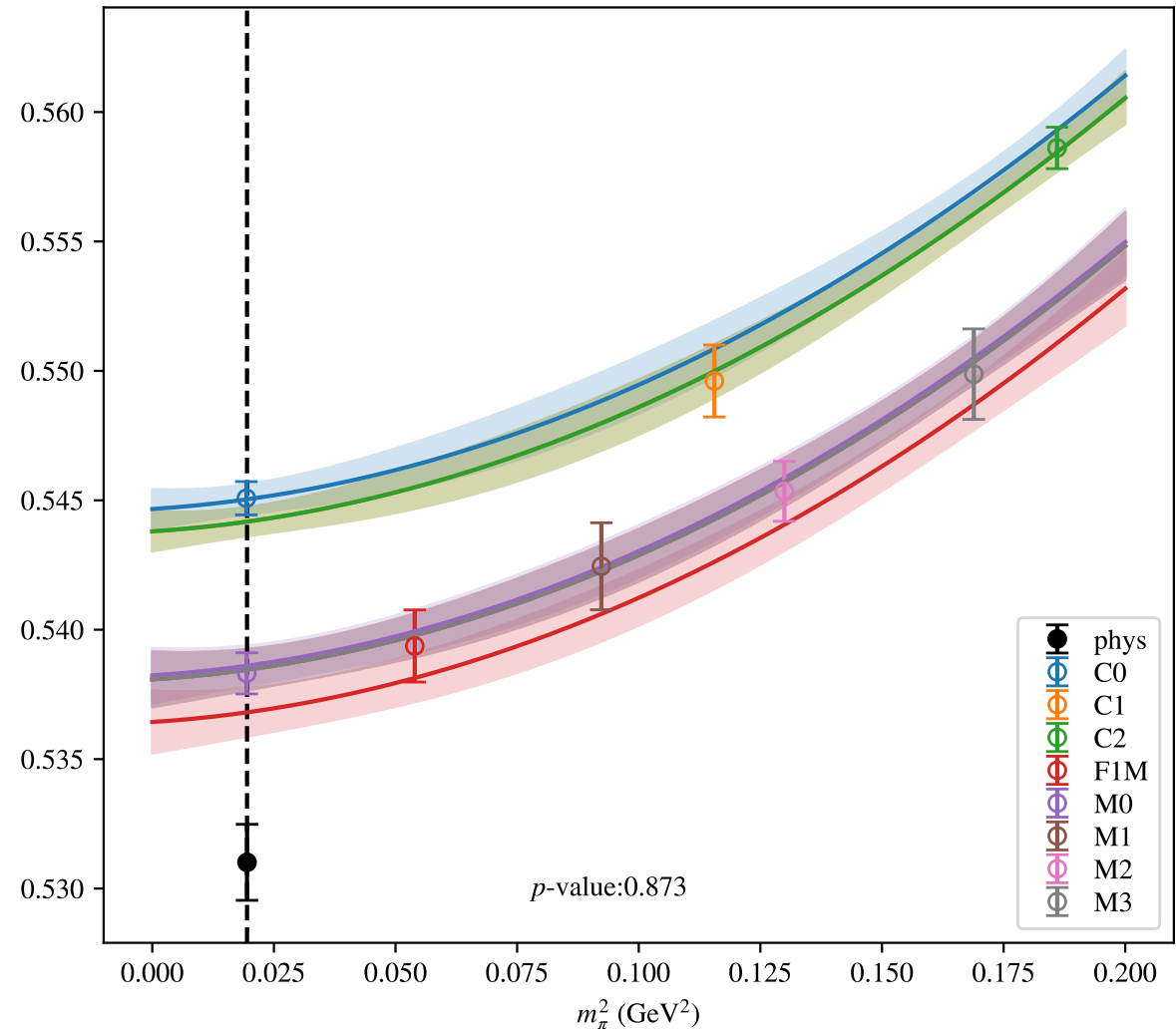


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.8 \text{ GeV}$$

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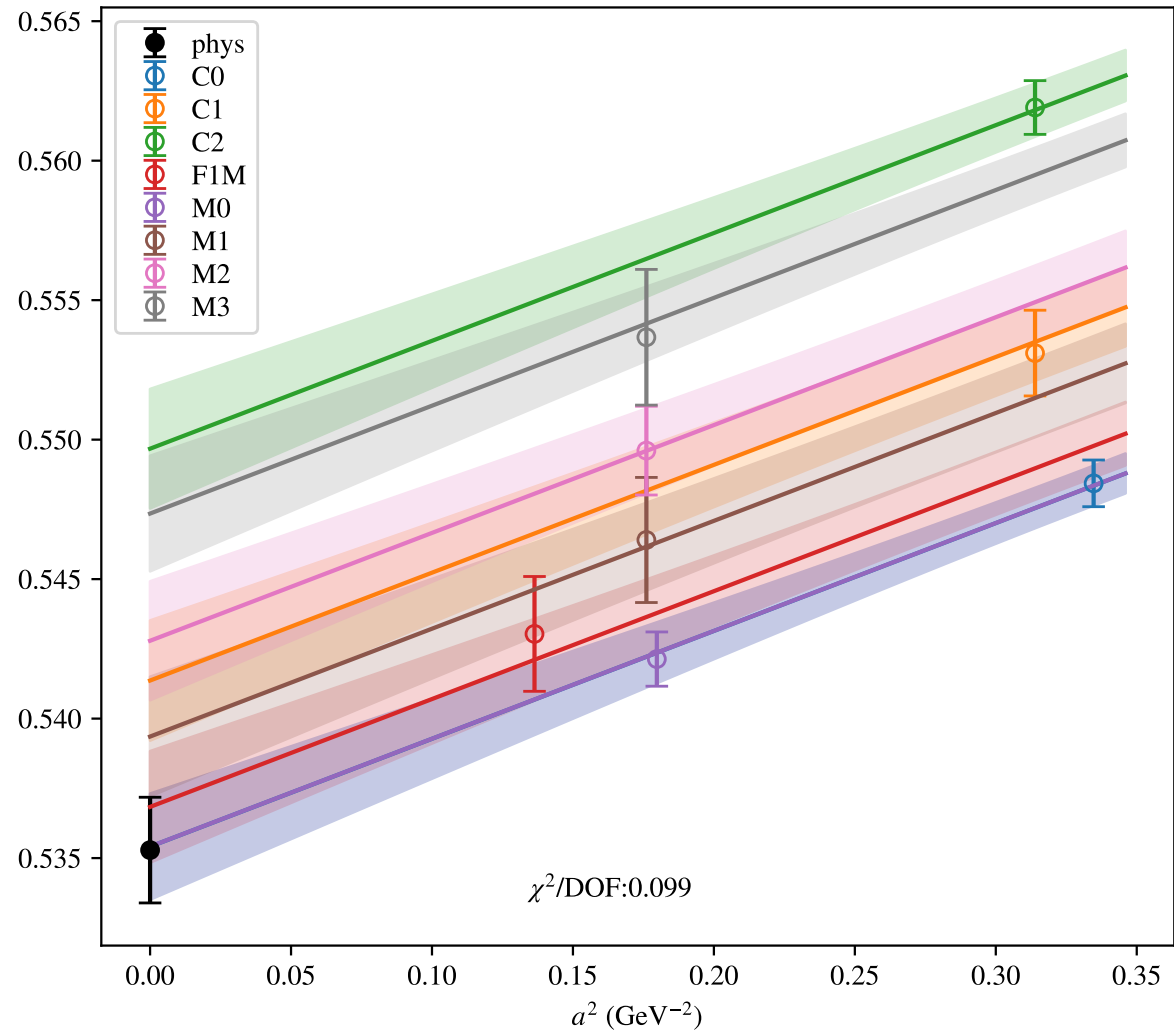


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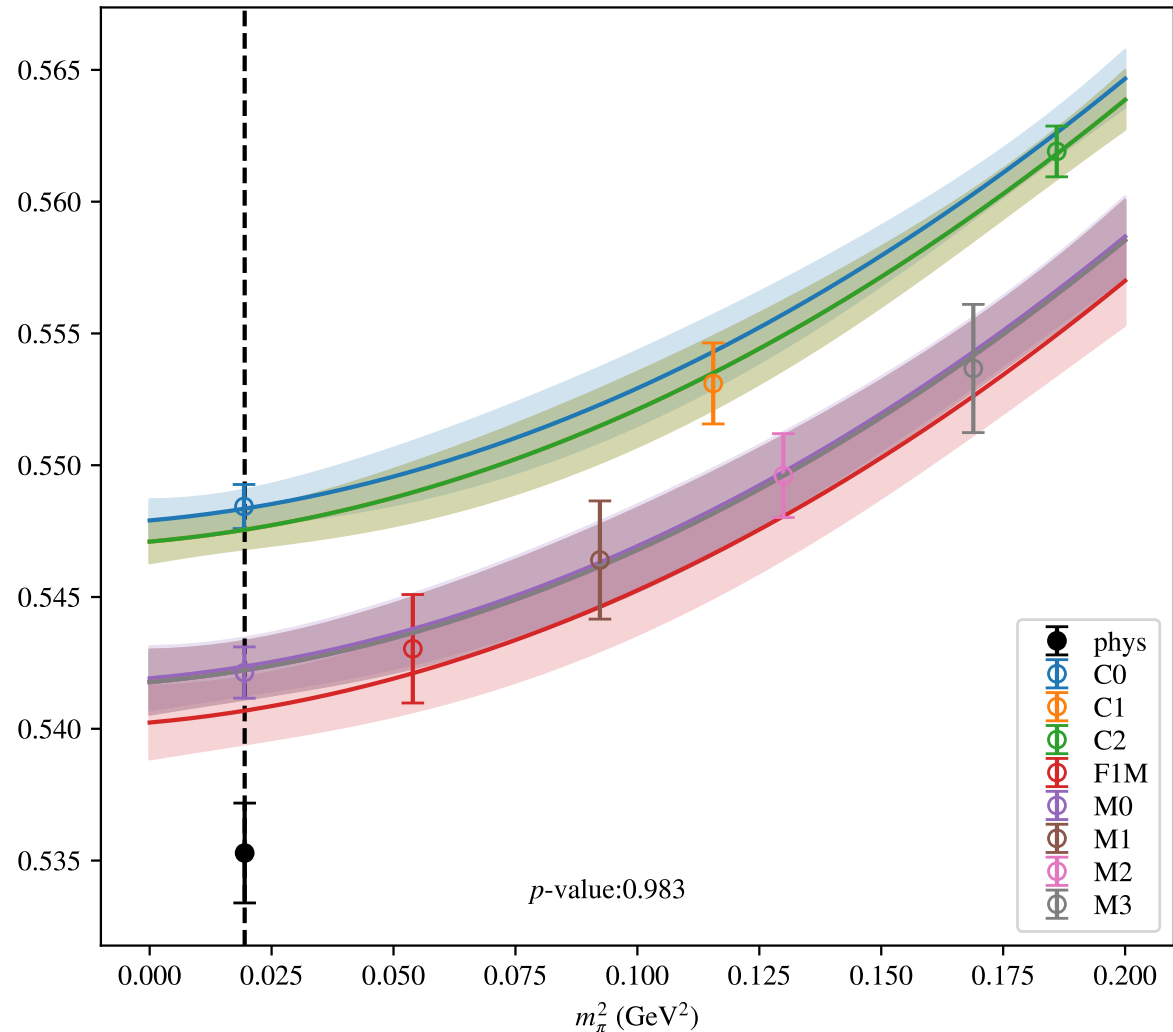


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.5 \text{ GeV}$$

VVpAA



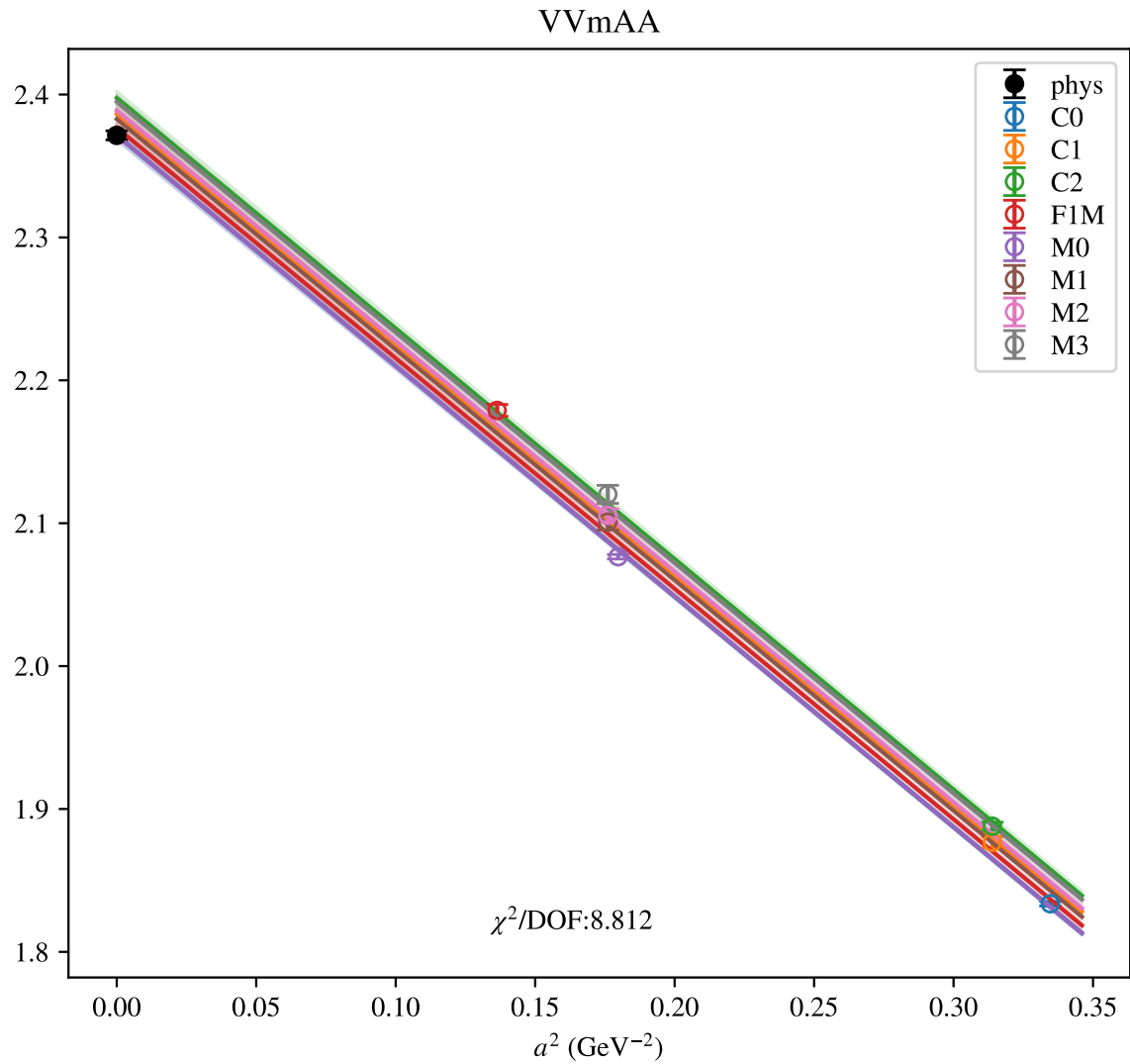
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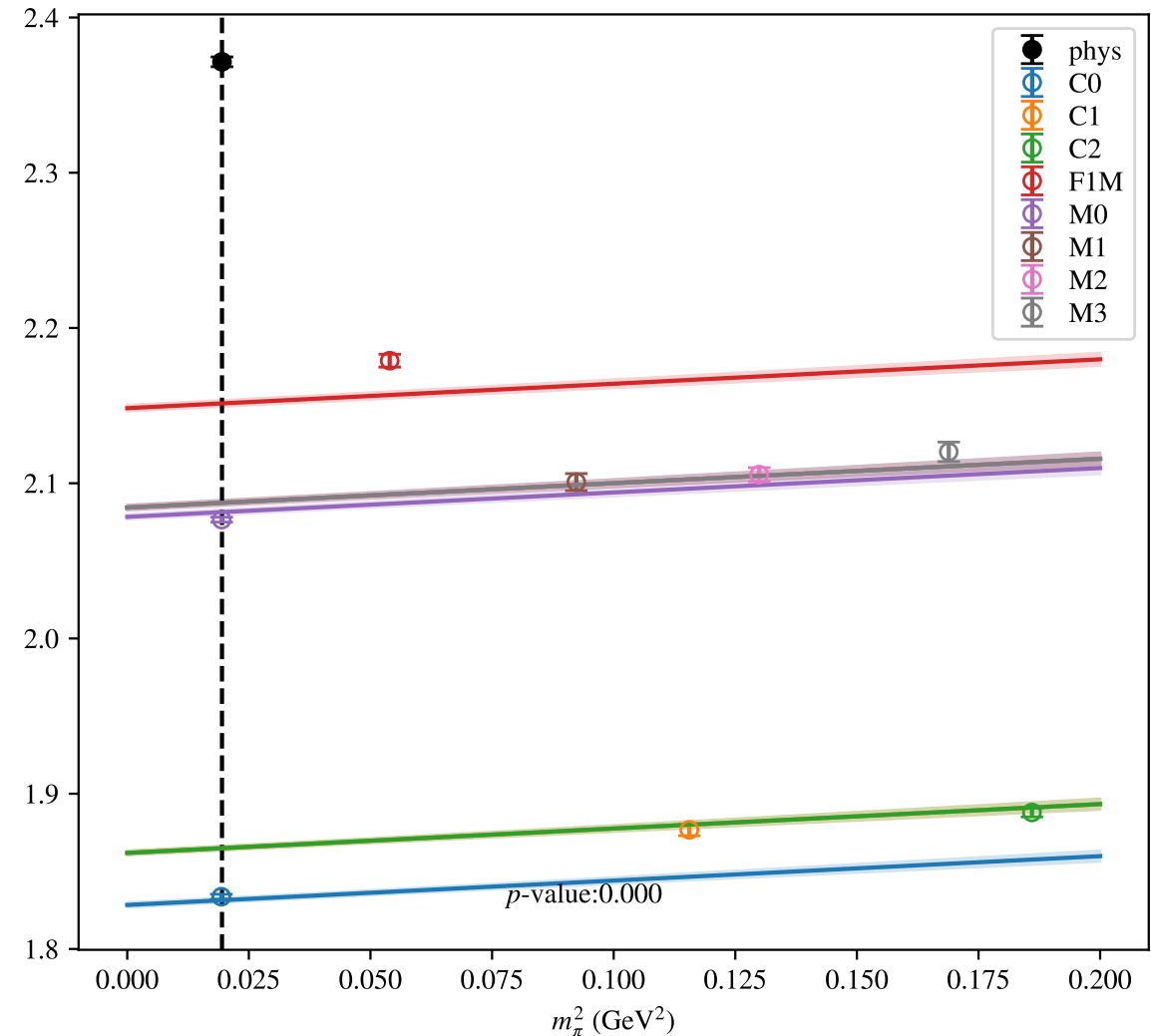
## 2 VVmA

$\mu$ (GeV)	$a^2, m_\pi^2$	$a^2, m_\pi^2$ no C	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$
2.0	<b>2.3714(31)</b> : 8.812 (0.0)	<b>2.470(16)</b> : 0.354 (0.702)	<b>2.555(26)</b> : 0.418 (0.796)	<b>2.3688(31)</b> : 9.437 (0.0)
1.8	<b>2.5968(42)</b> : 8.923 (0.0)	<b>2.692(19)</b> : 0.993 (0.371)	<b>2.821(31)</b> : 2.489 (0.041)	<b>2.5945(40)</b> : 10.51 (0.0)
1.5	<b>2.9376(65)</b> : 9.386 (0.0)	<b>3.026(26)</b> : 1.469 (0.23)	<b>3.232(43)</b> : 5.497 (0.0)	<b>2.9360(58)</b> : 11.616 (0.0)

Table 2: Physical point value from chiral and continuum extrapolation at renormalisation scale  $\mu$ . Entries are **value(error)**:  $\chi^2/\text{DOF}$  ( $p$ -value).

$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$ 

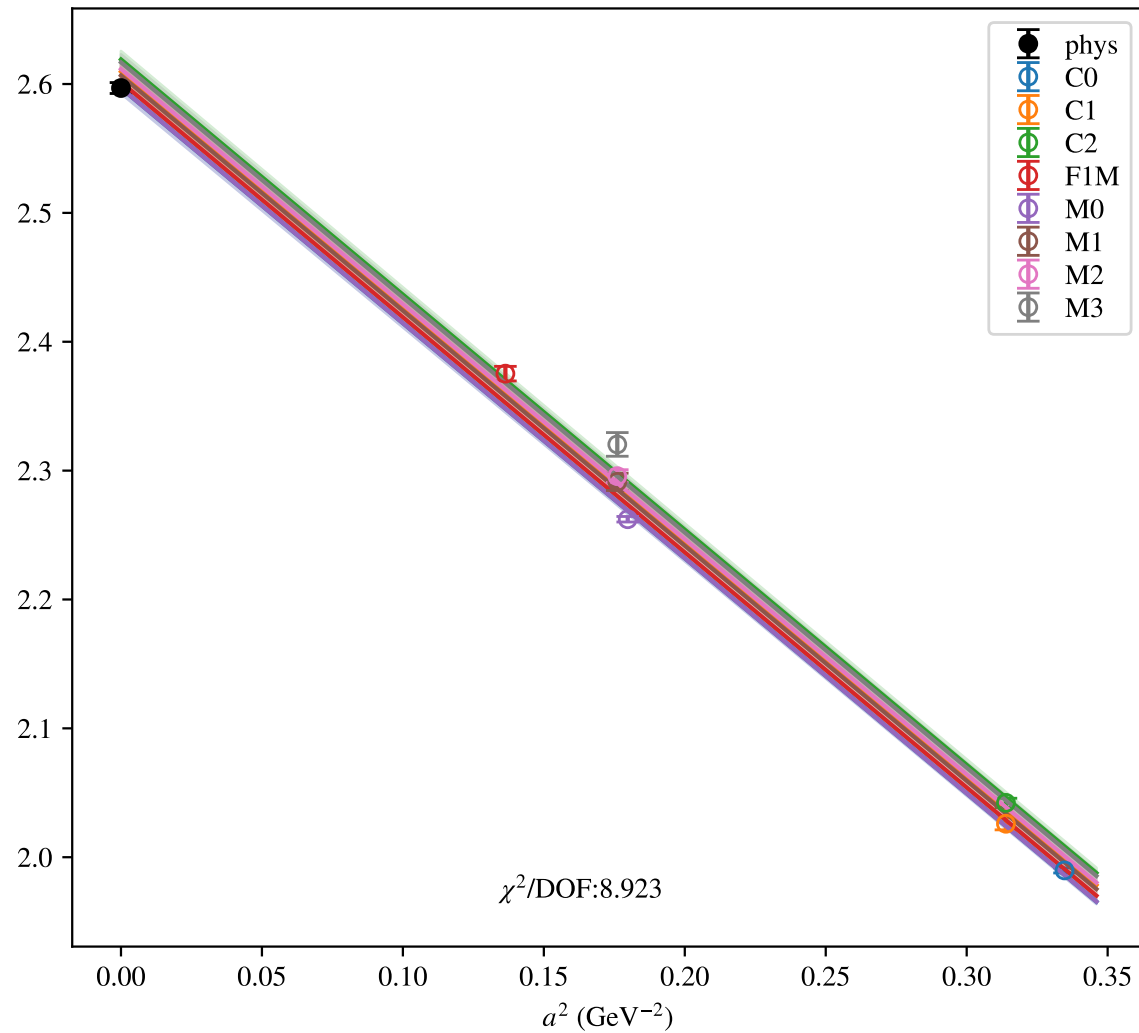
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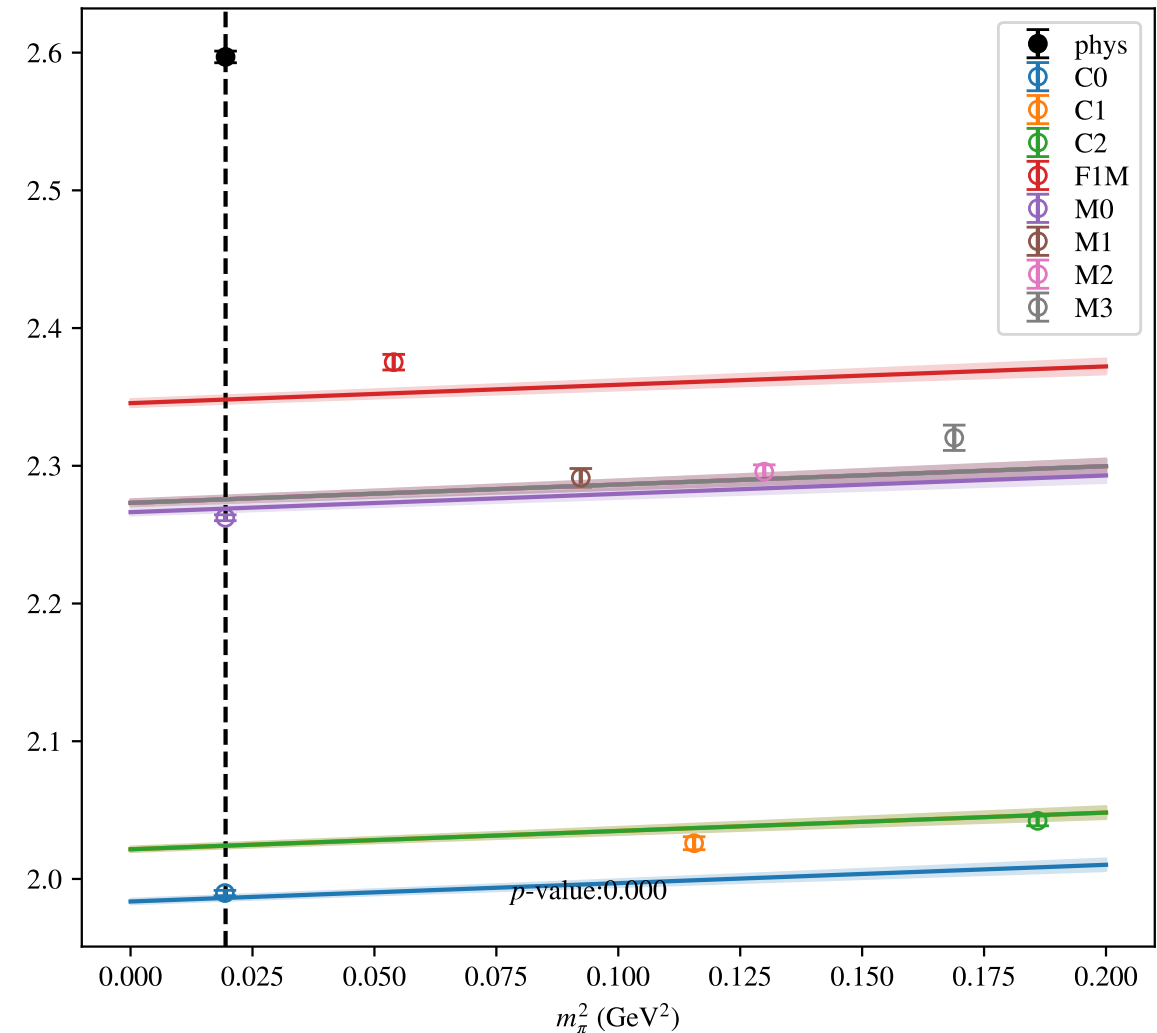


$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$

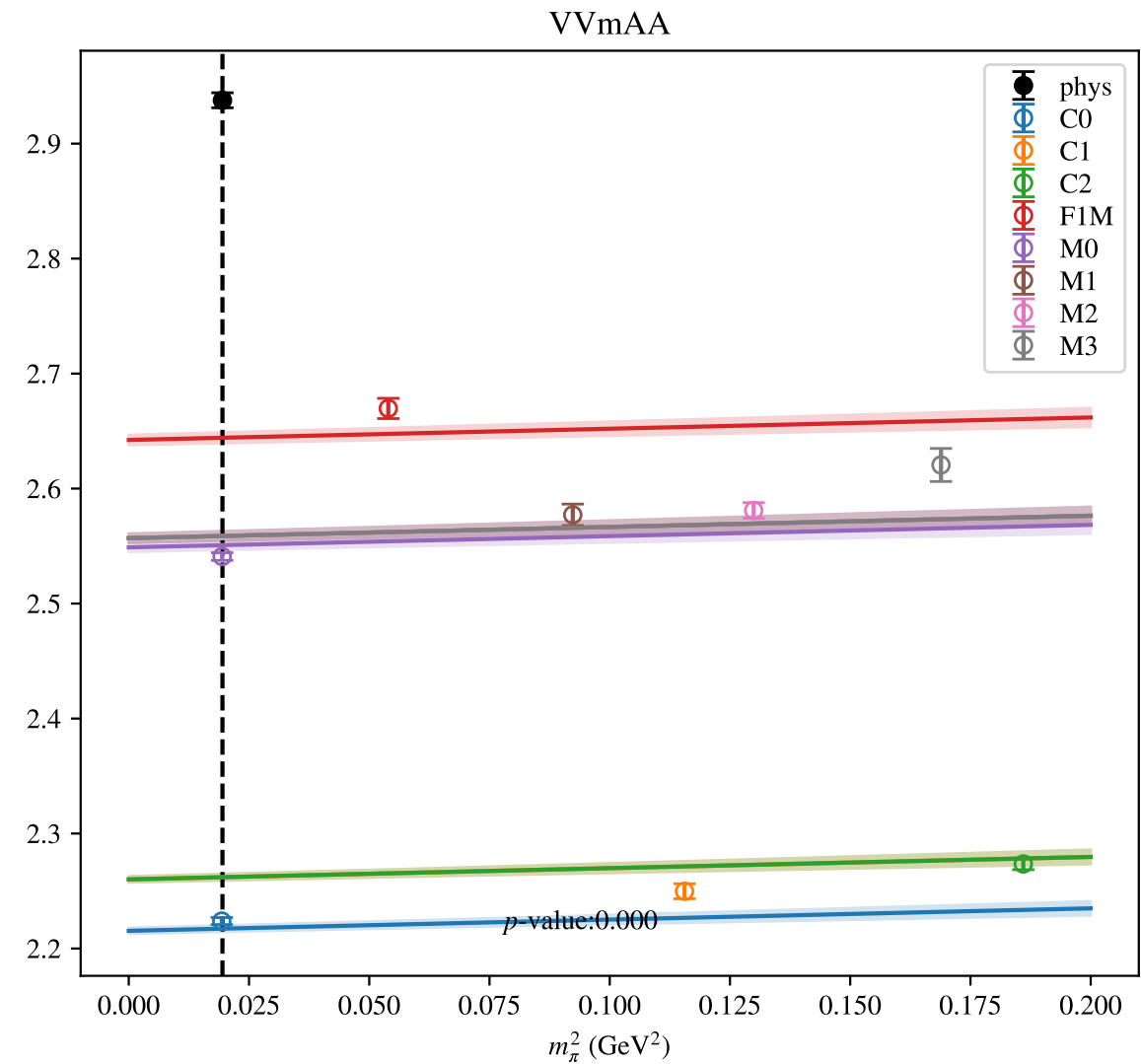
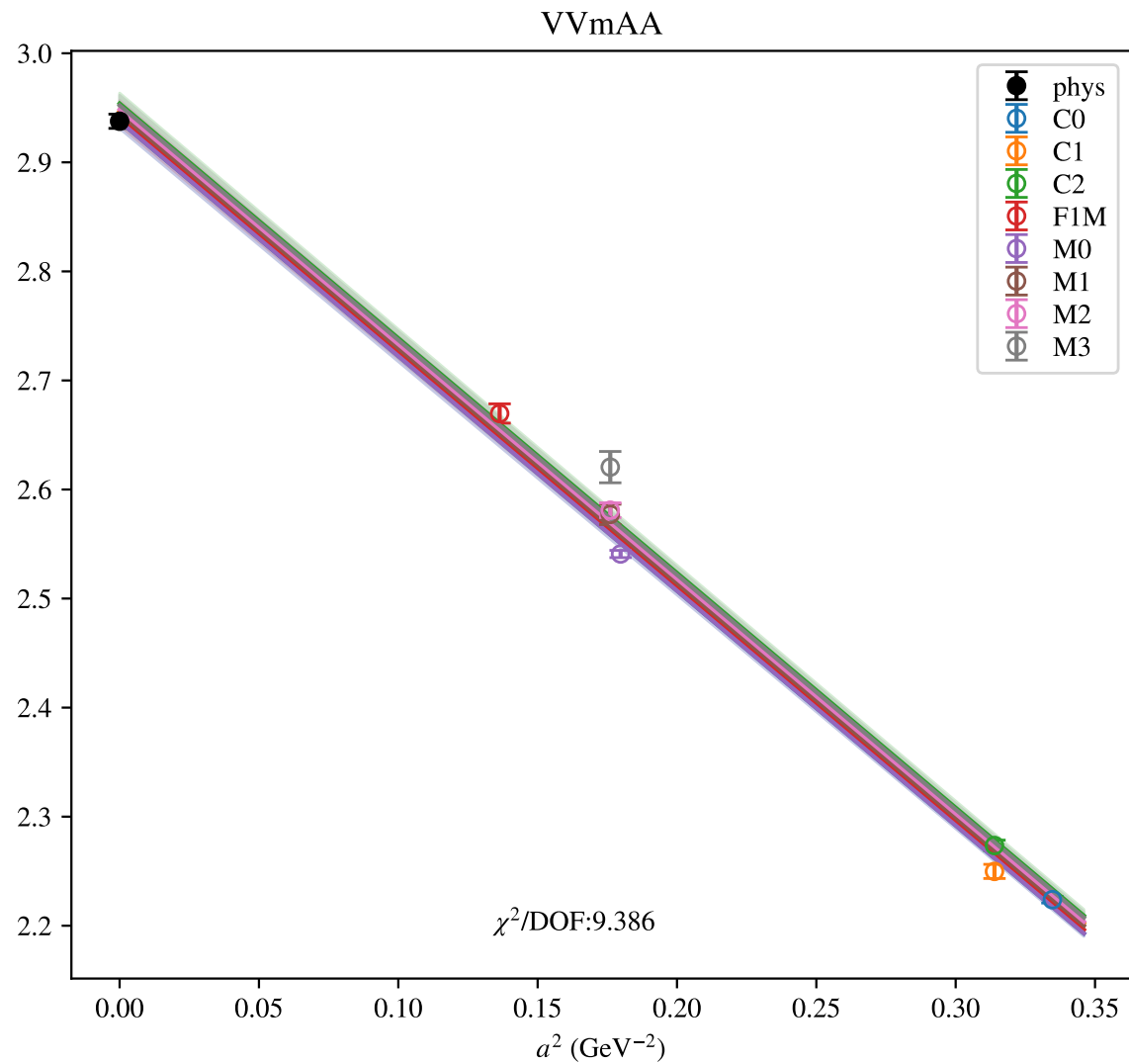
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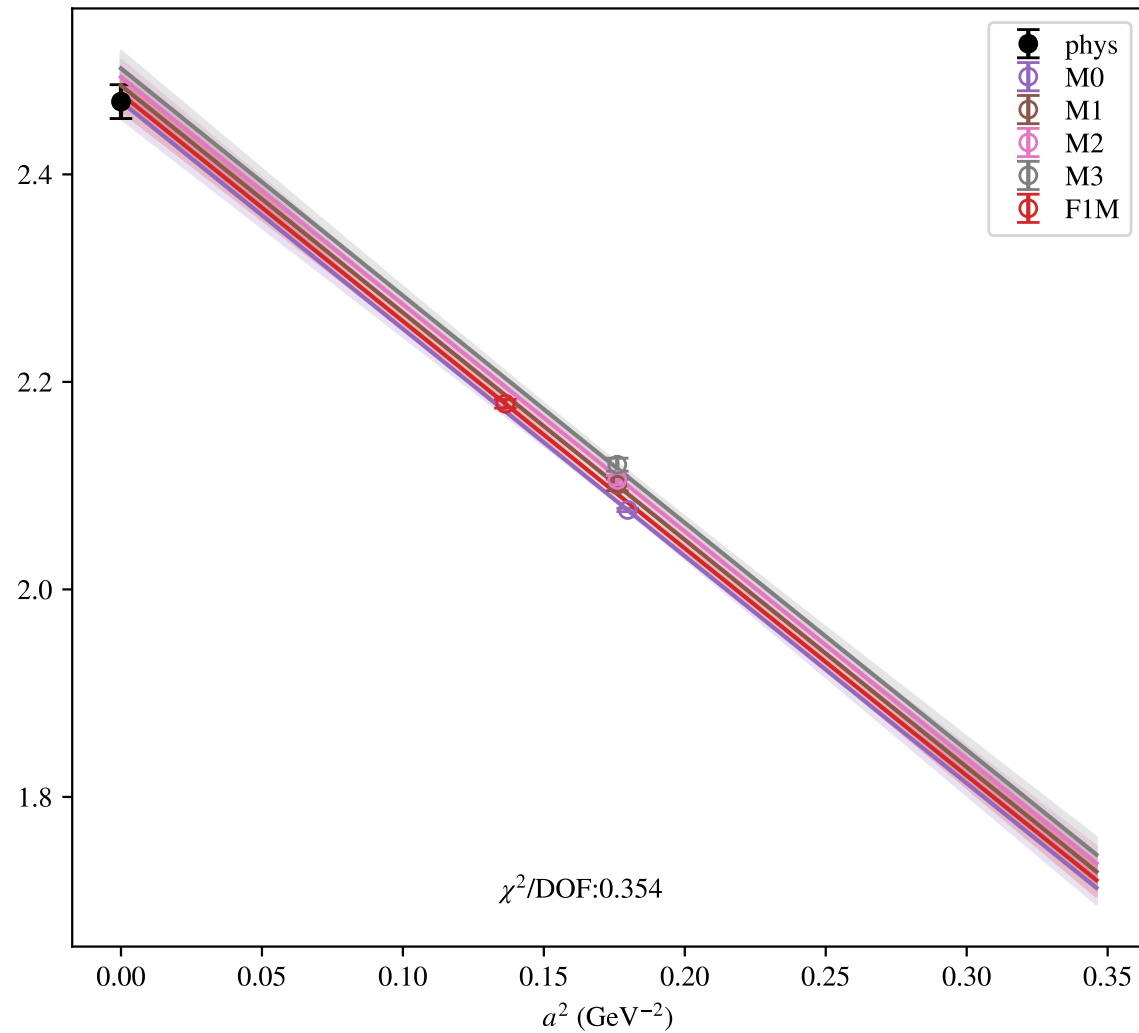


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

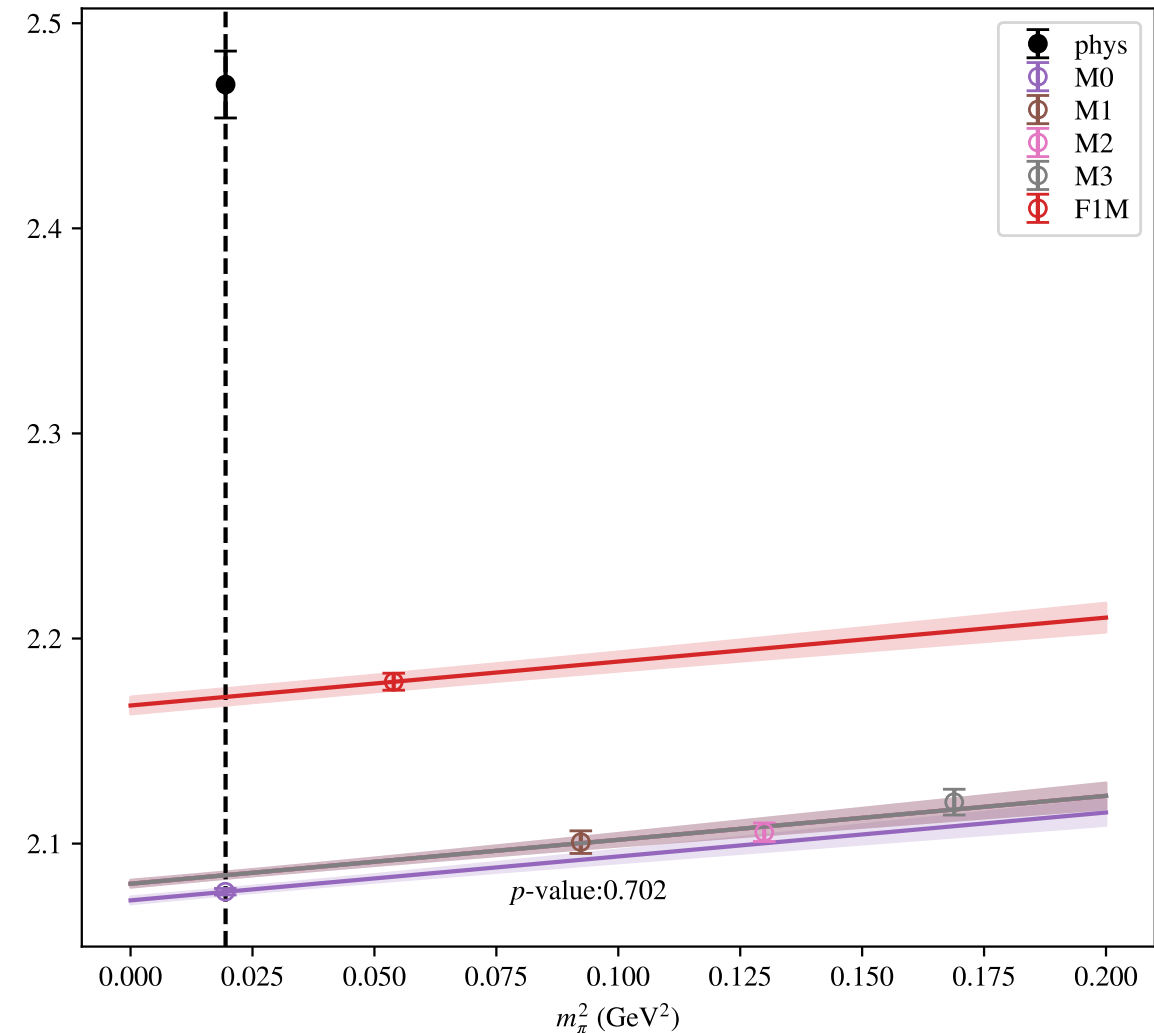


$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

VVmAA

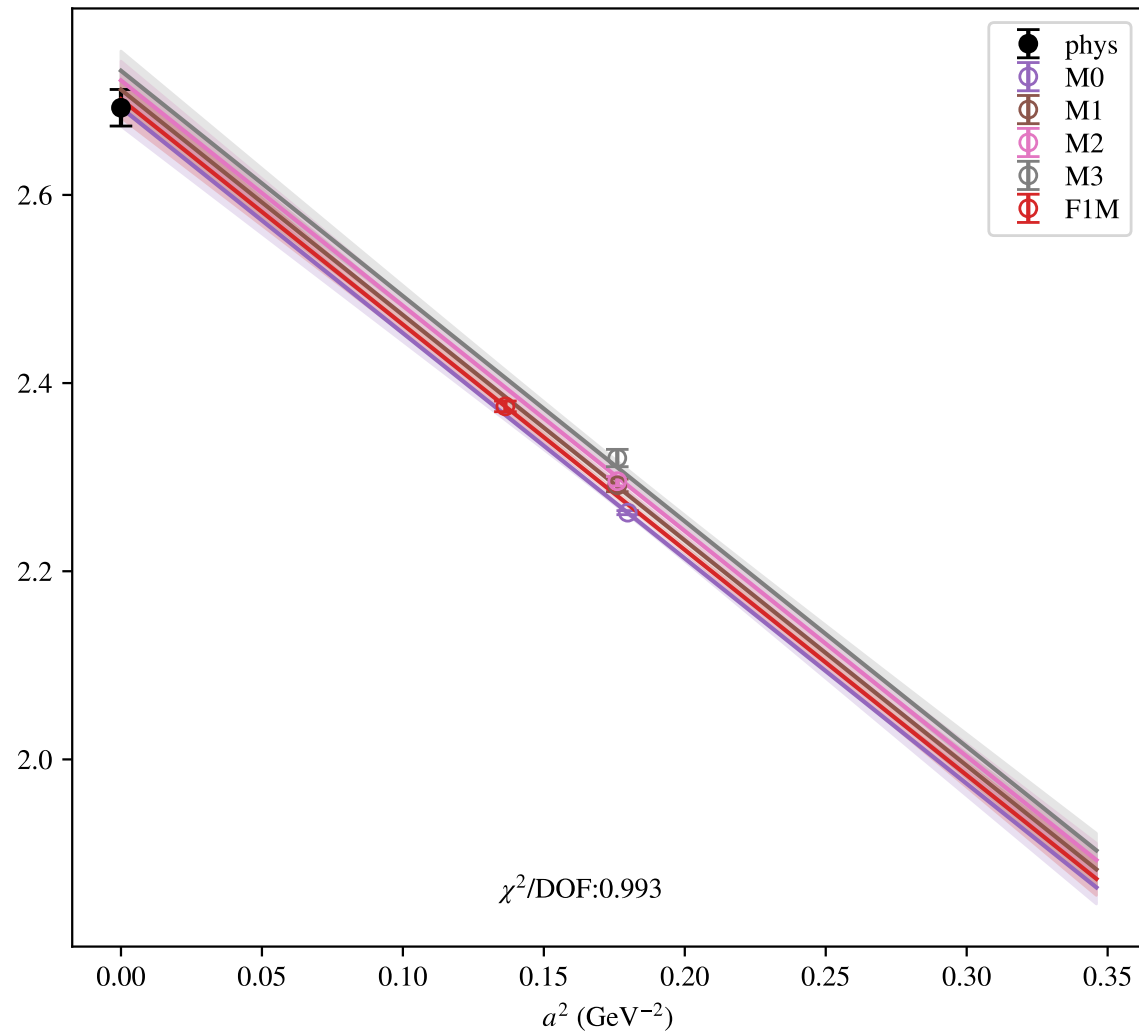


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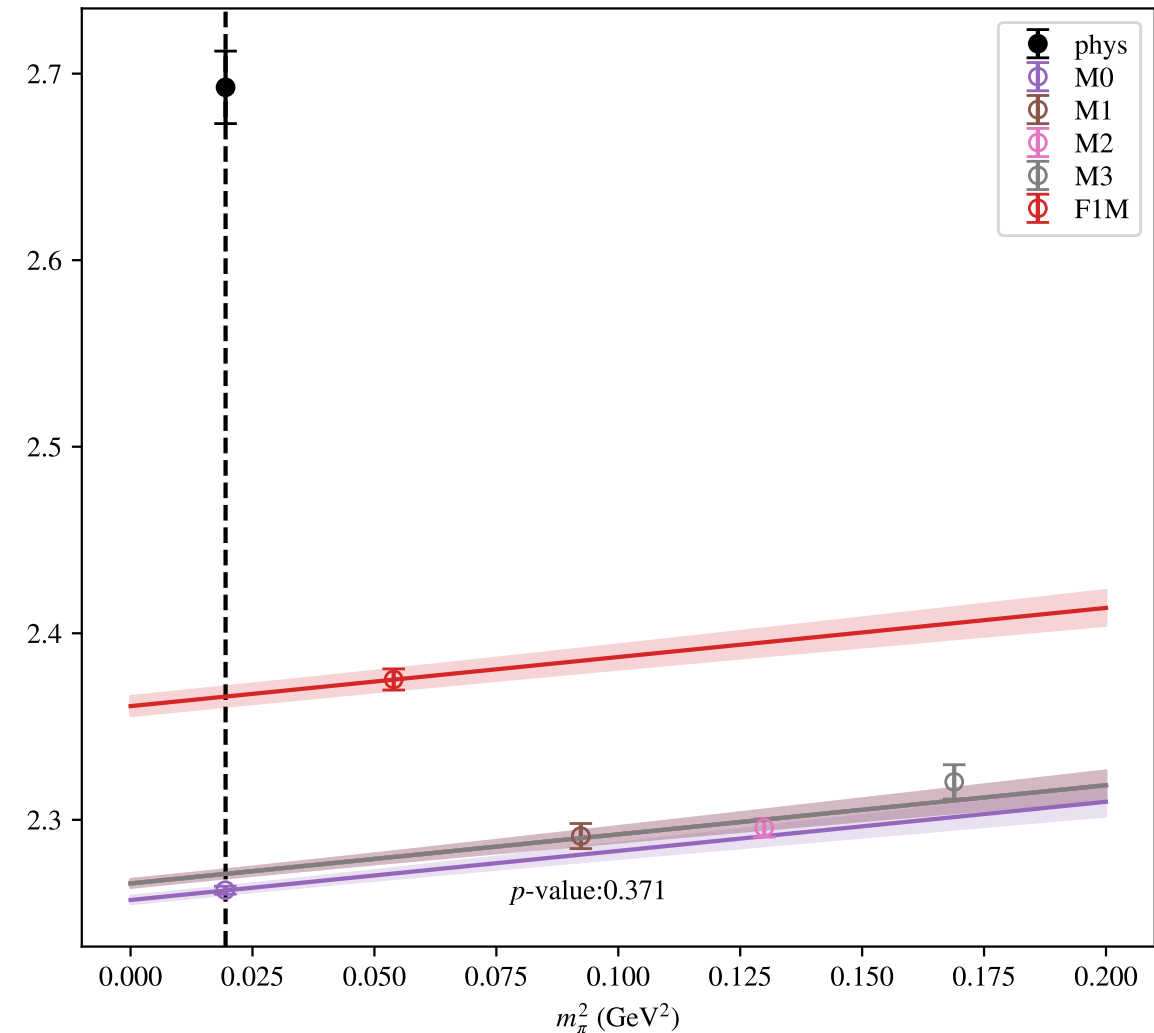


$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$

VVmAA

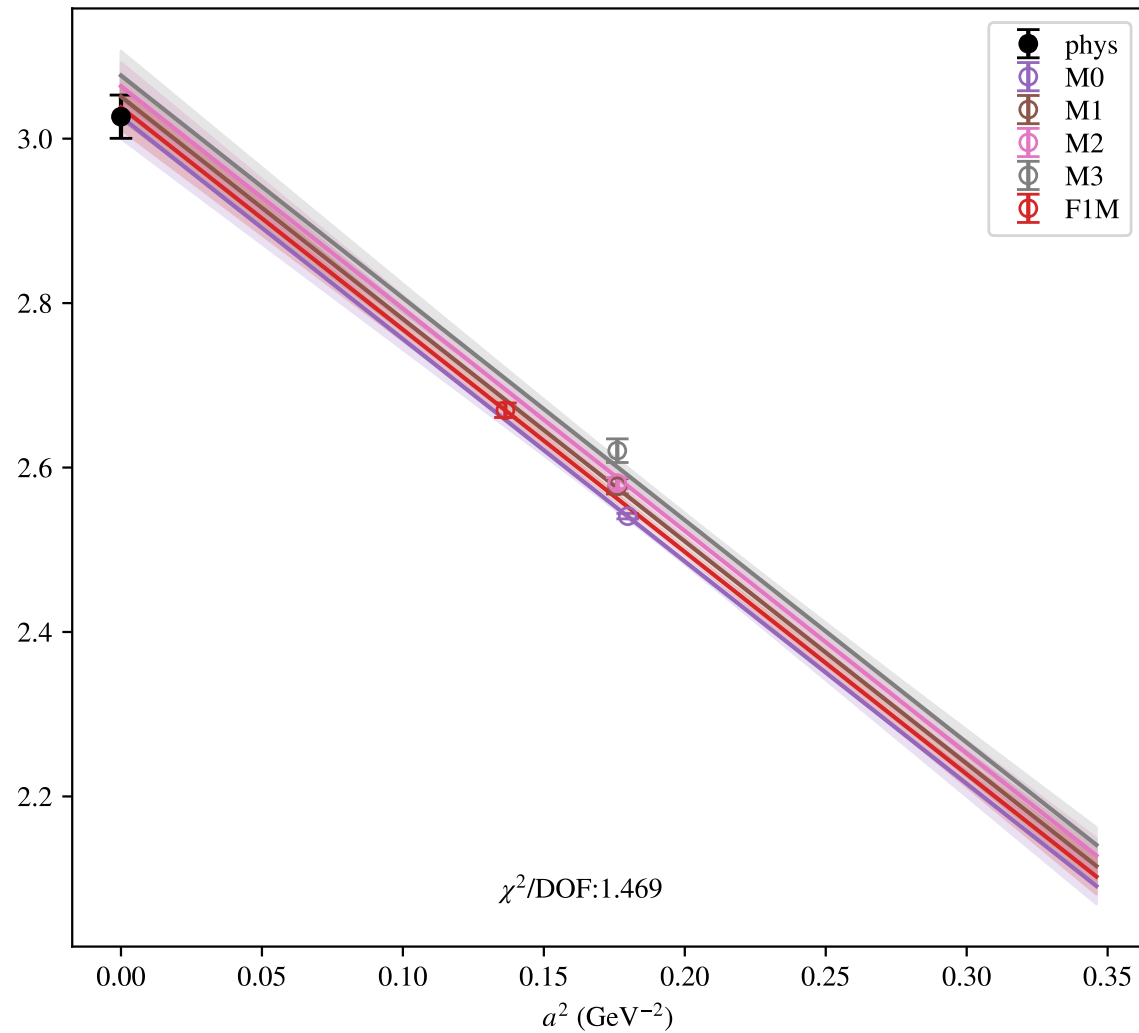


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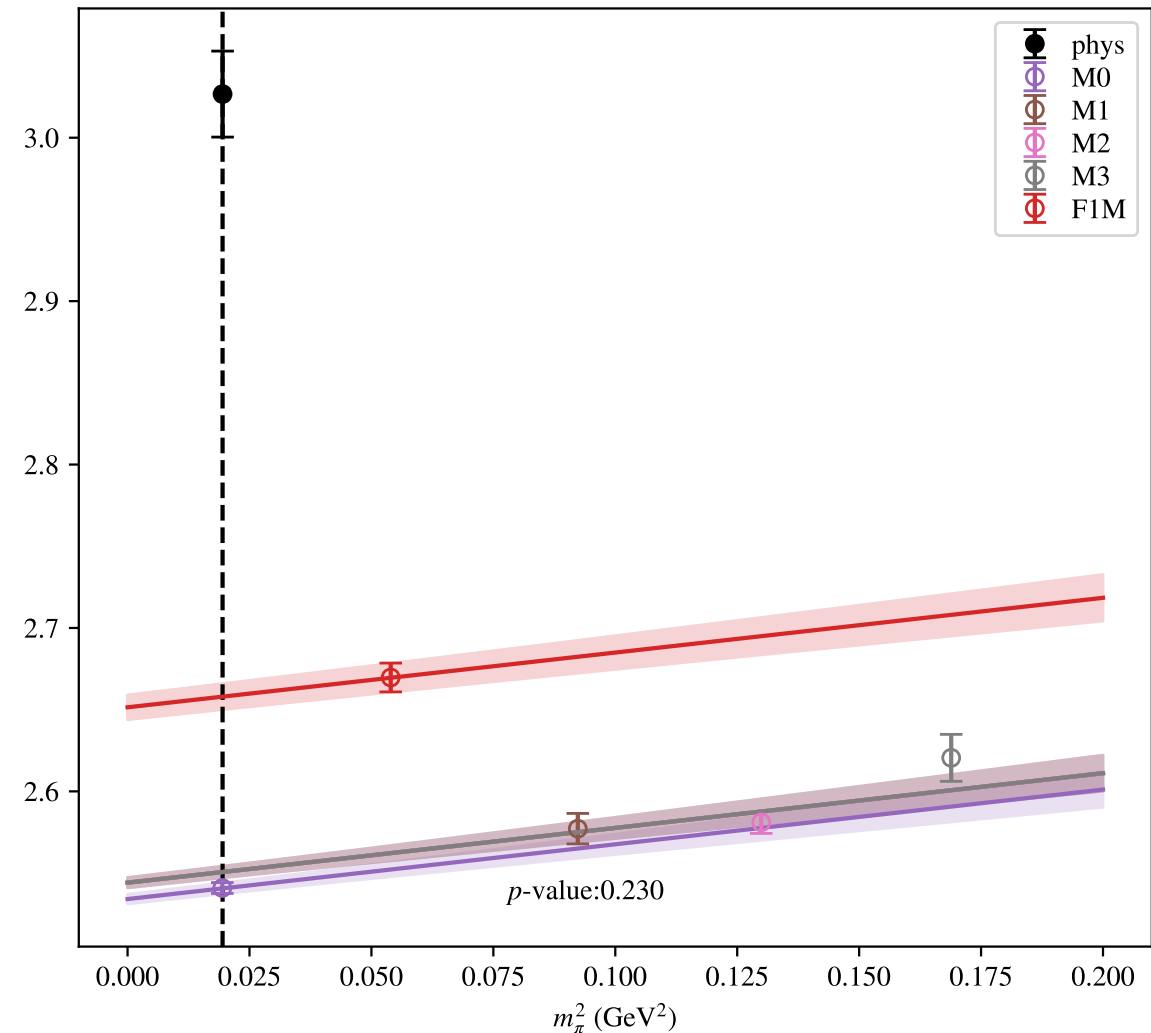


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

VVmAA

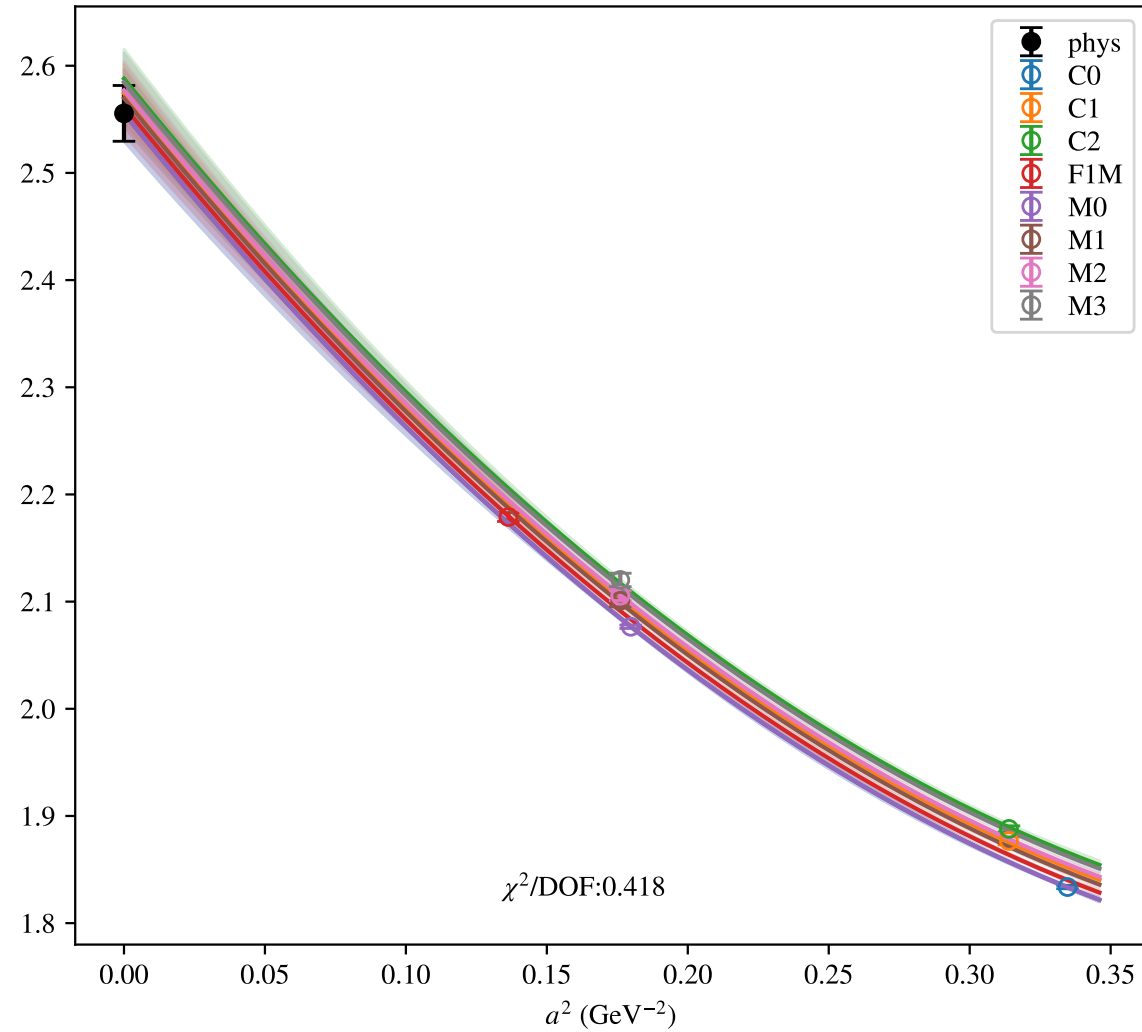


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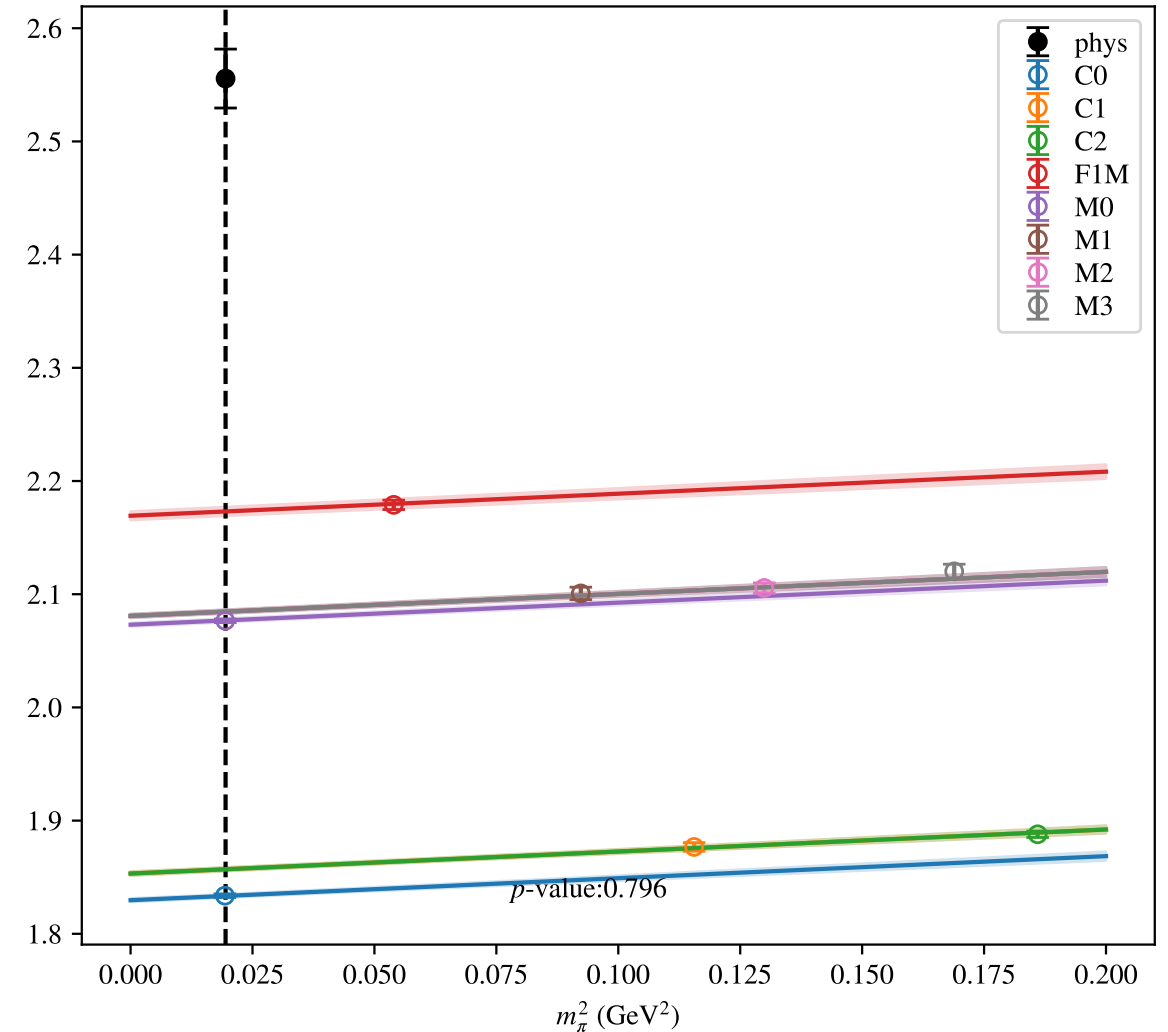


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

VVmAA

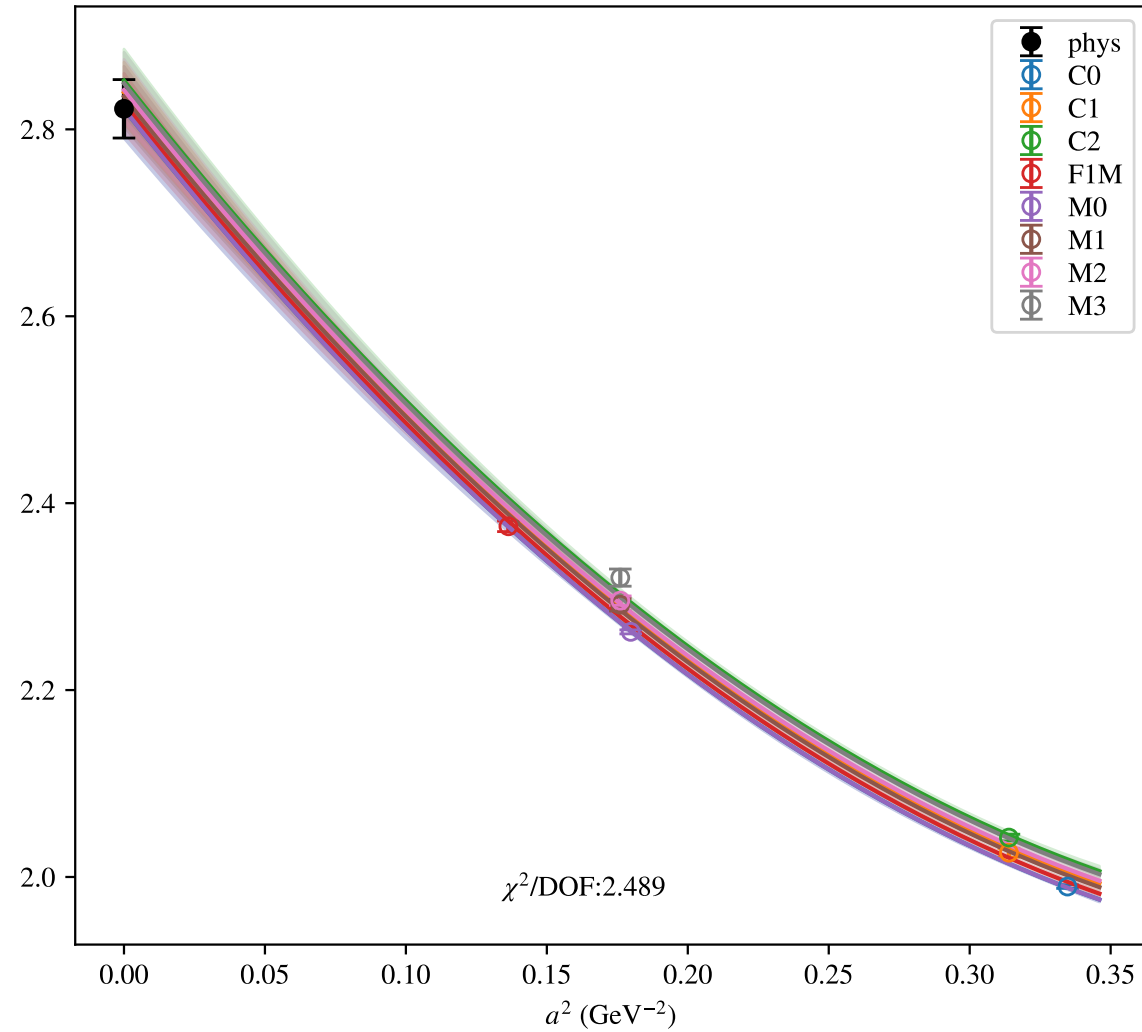


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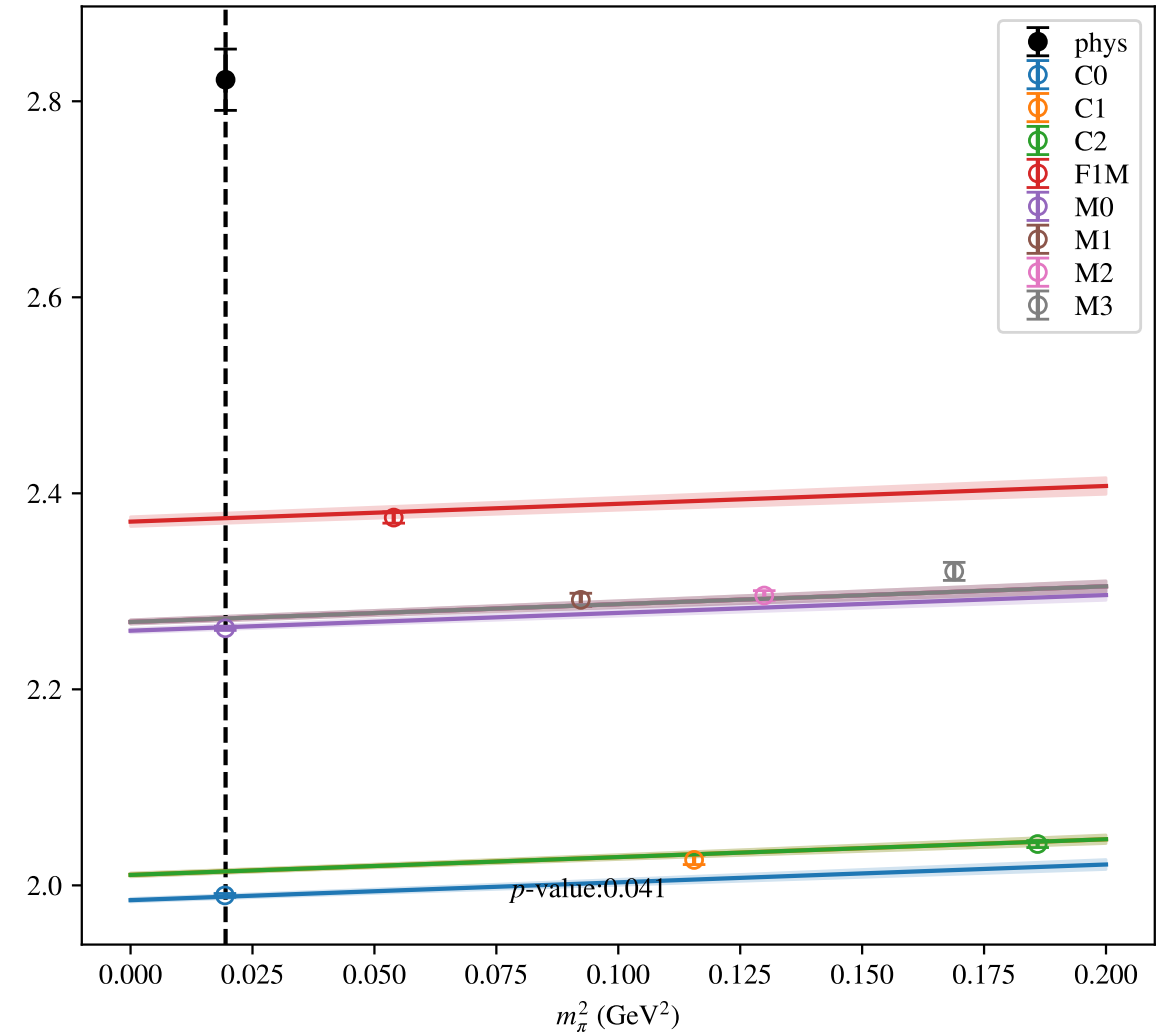


$$a^2, a^4, m_\pi^2, \mu = 1.8 \text{ GeV}$$

VVmAA

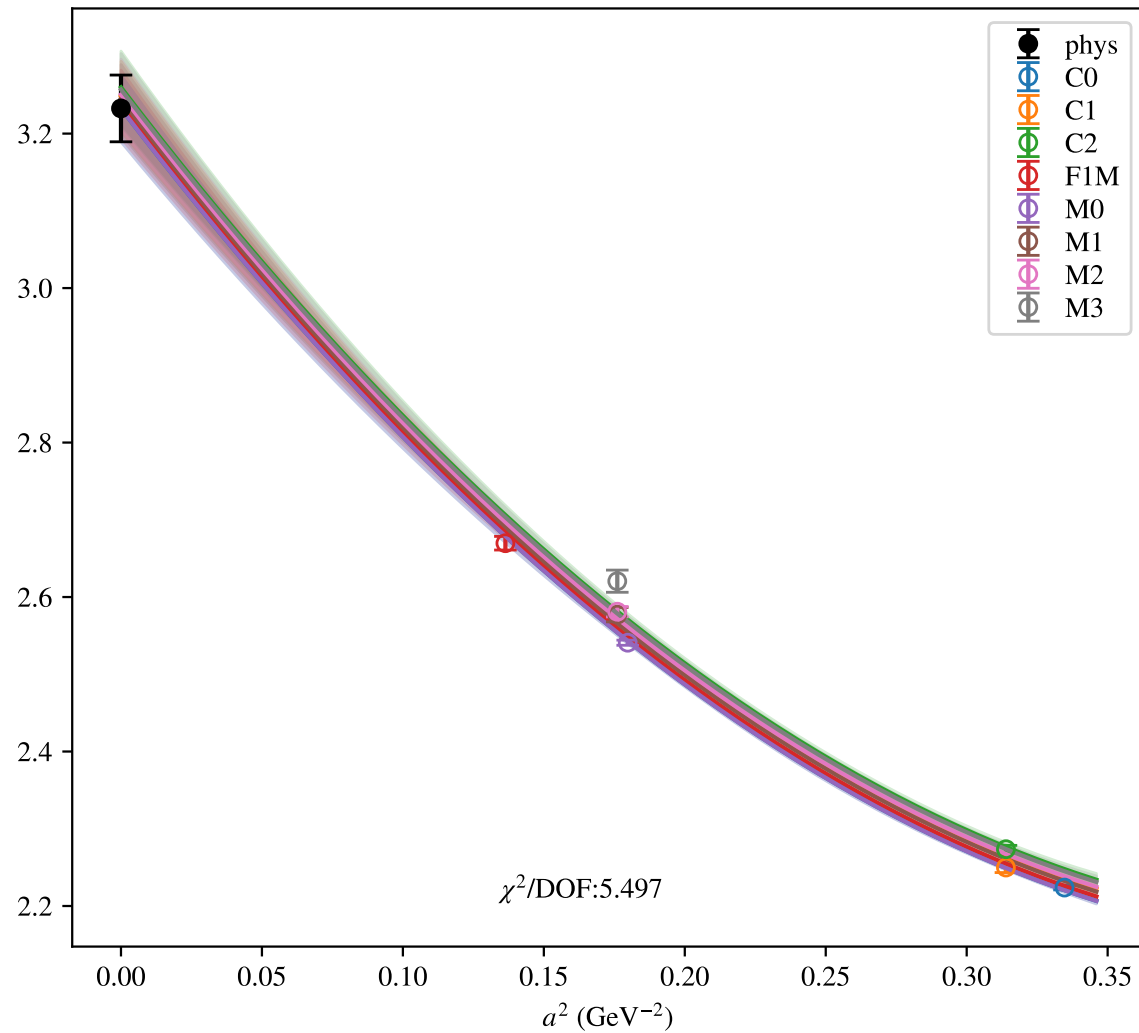


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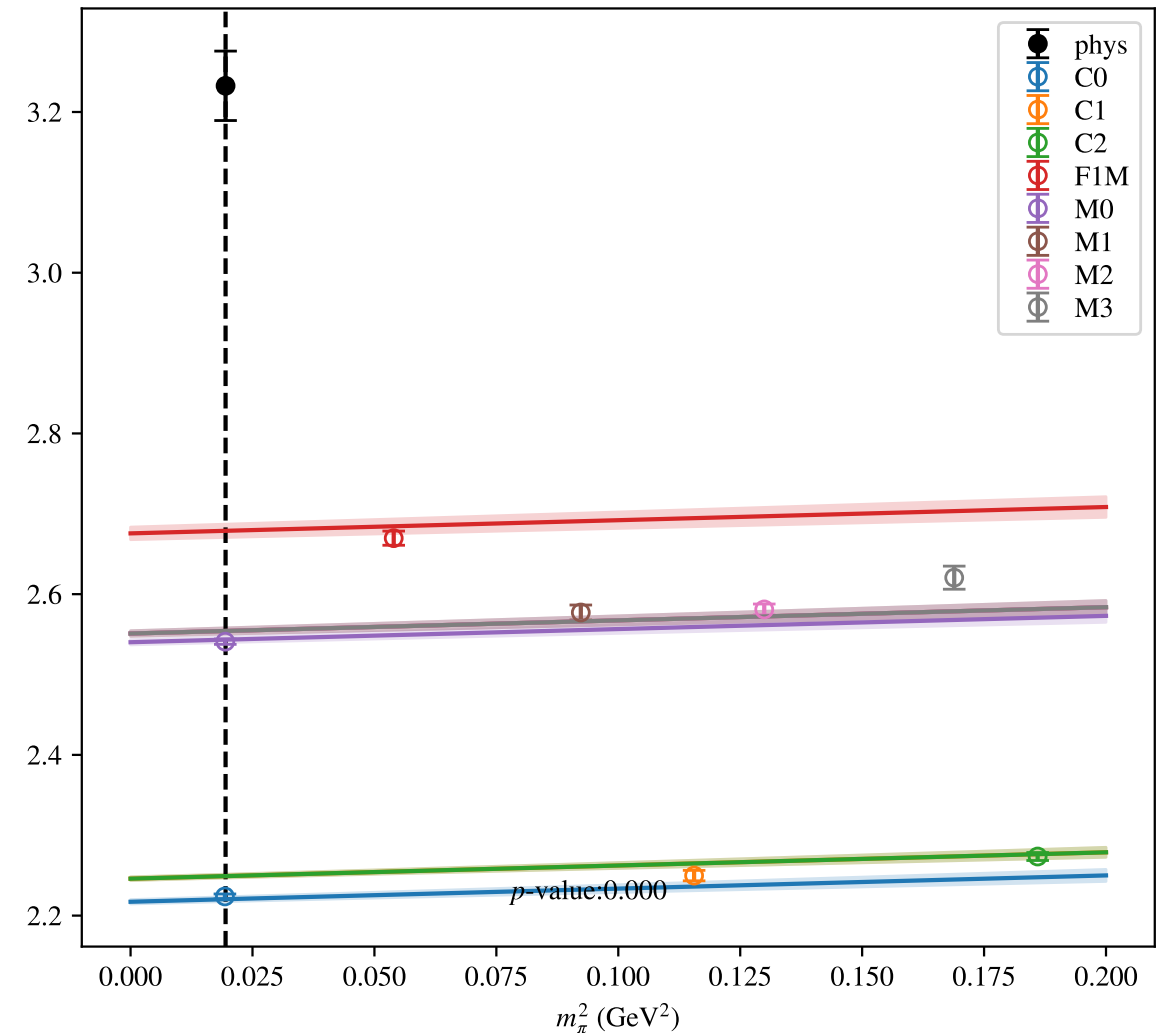


$$a^2, a^4, m_\pi^2, \mu = 1.5 \text{ GeV}$$

VVmA



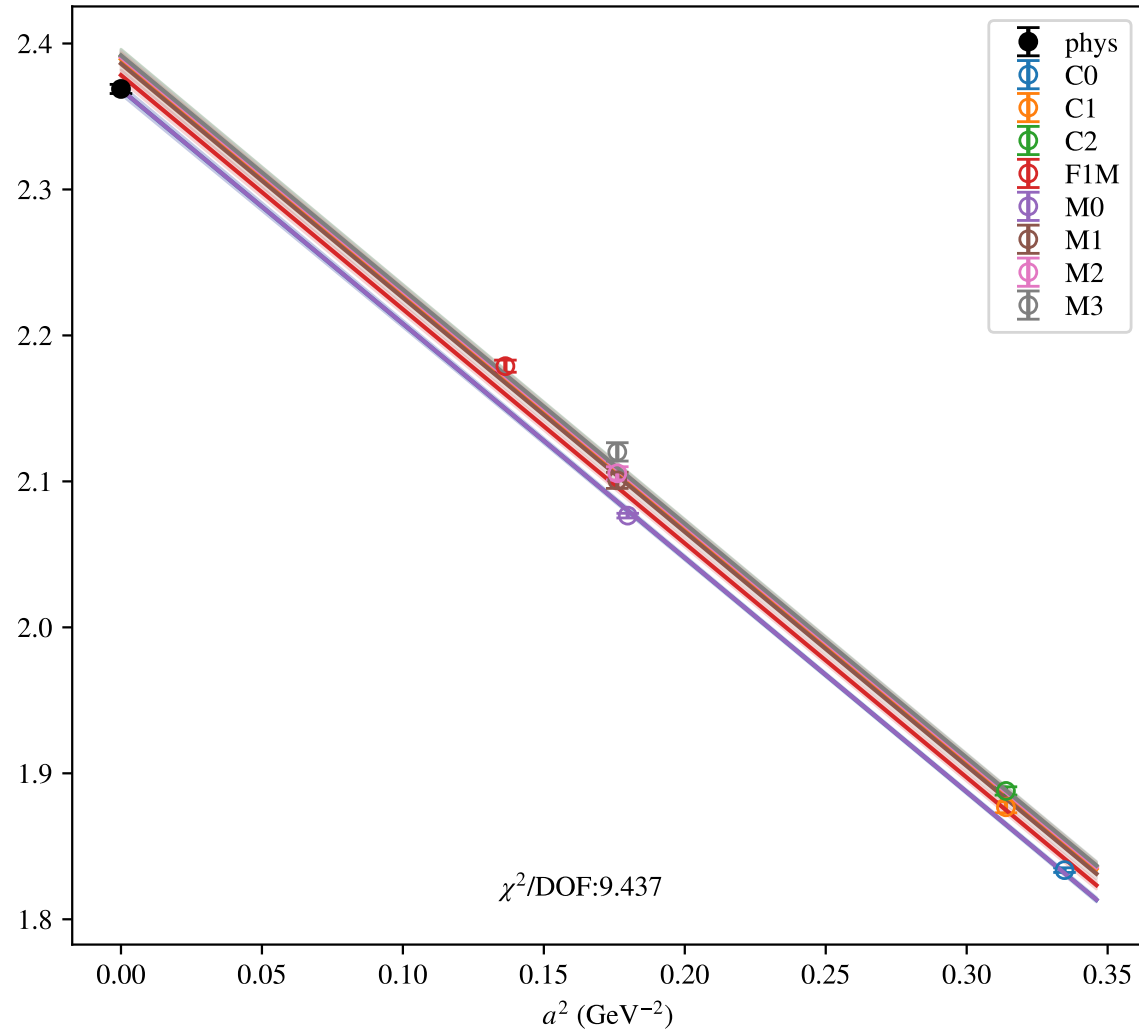
VVmA



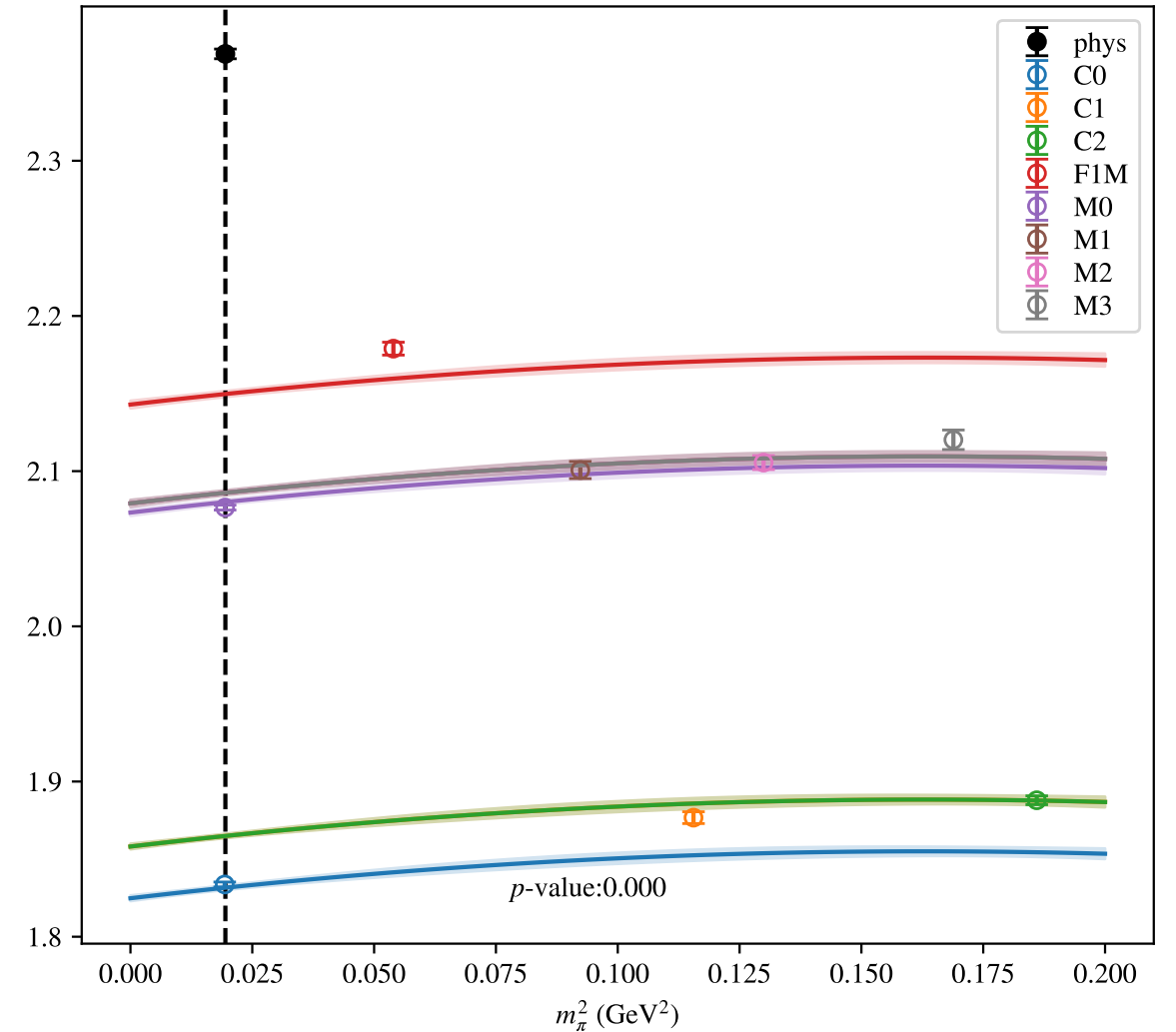


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

VVmA

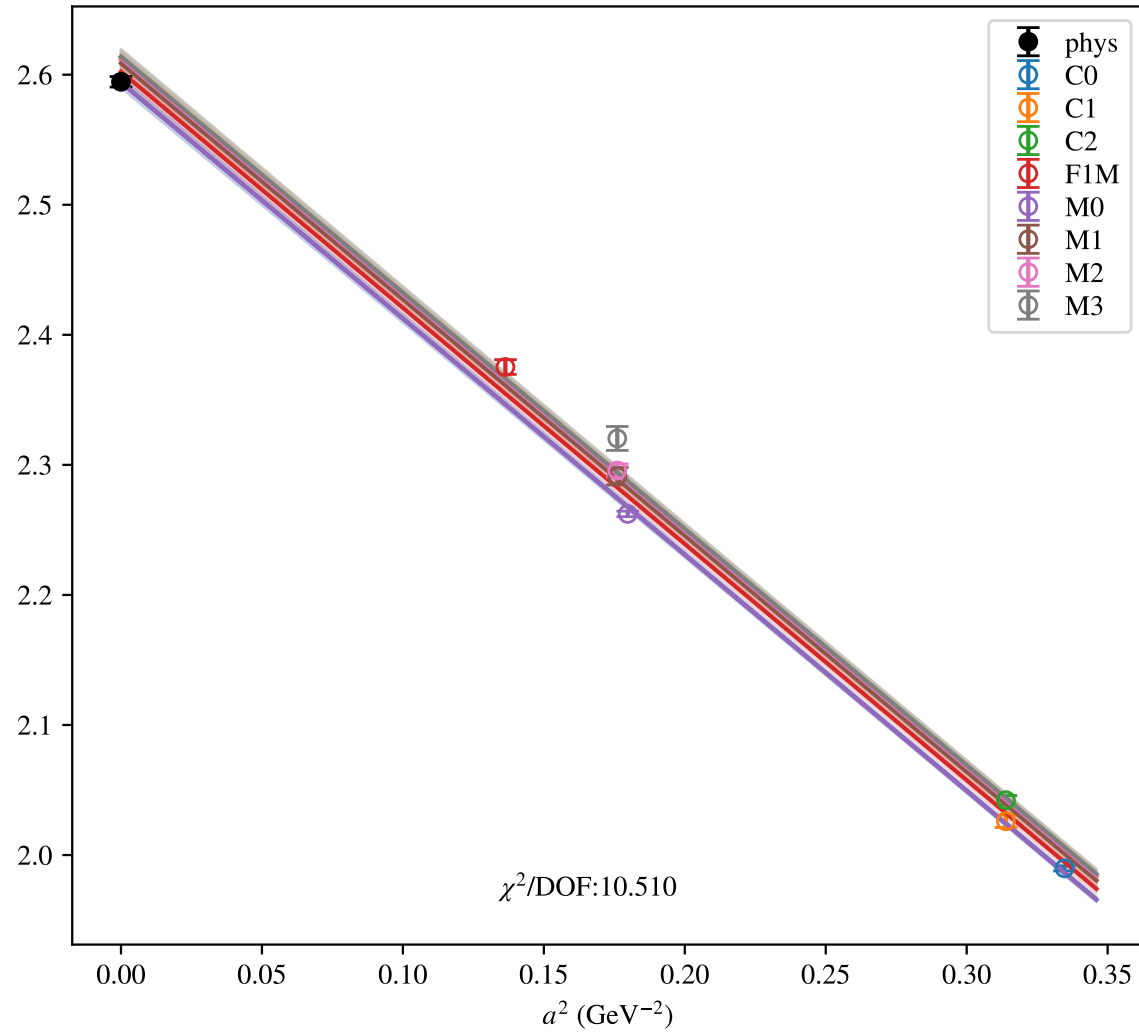


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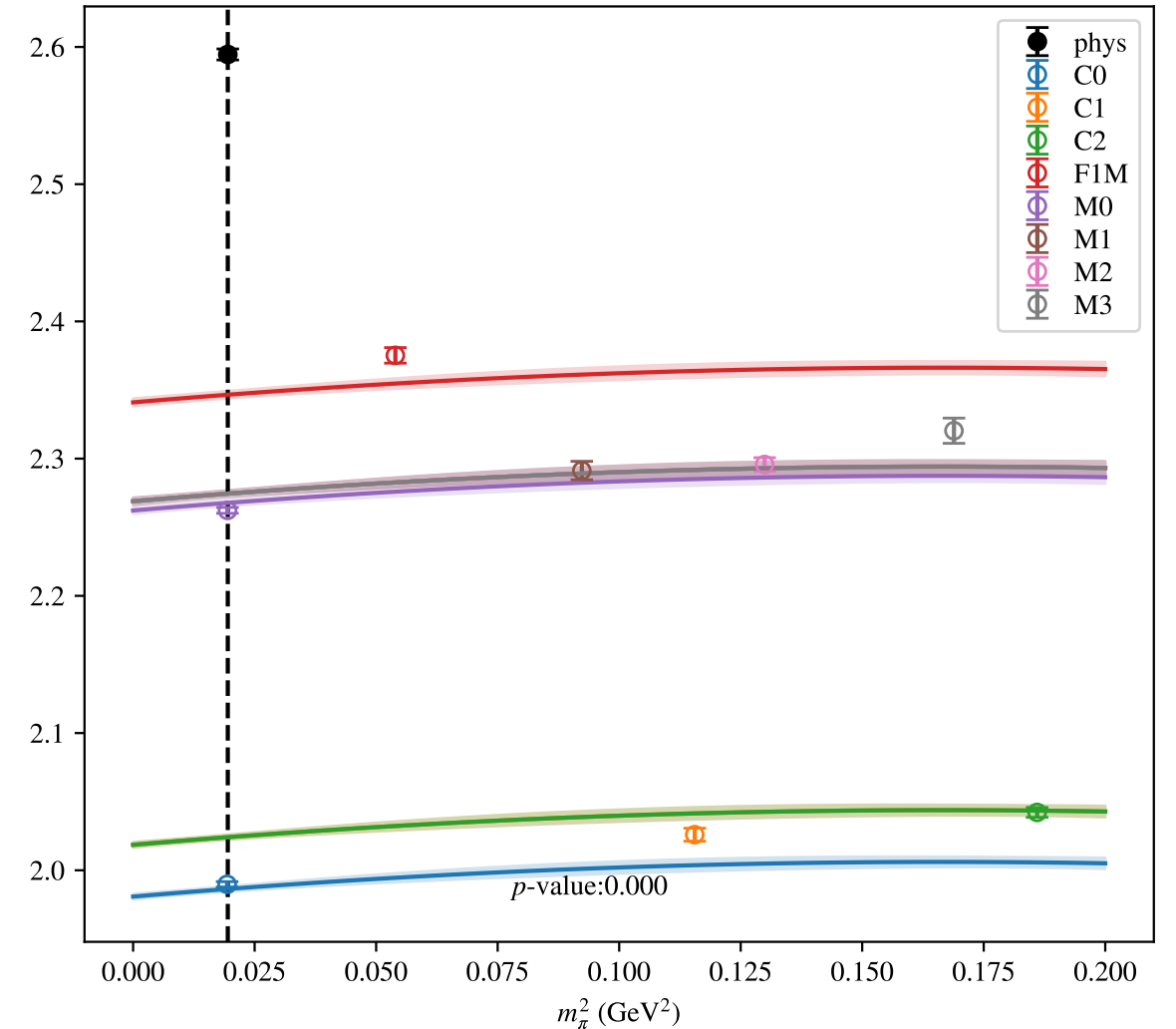


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.8 \text{ GeV}$$

VVmAA

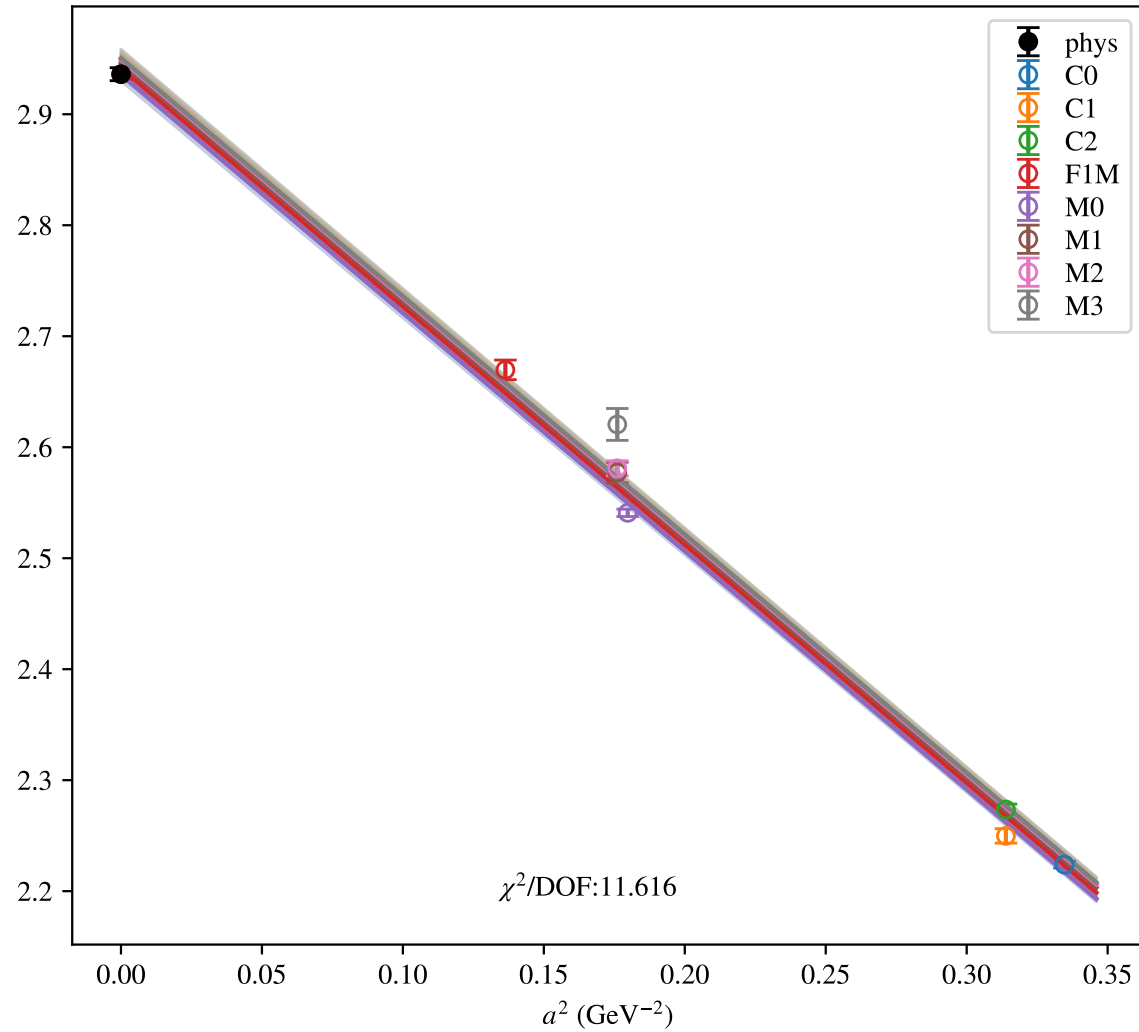


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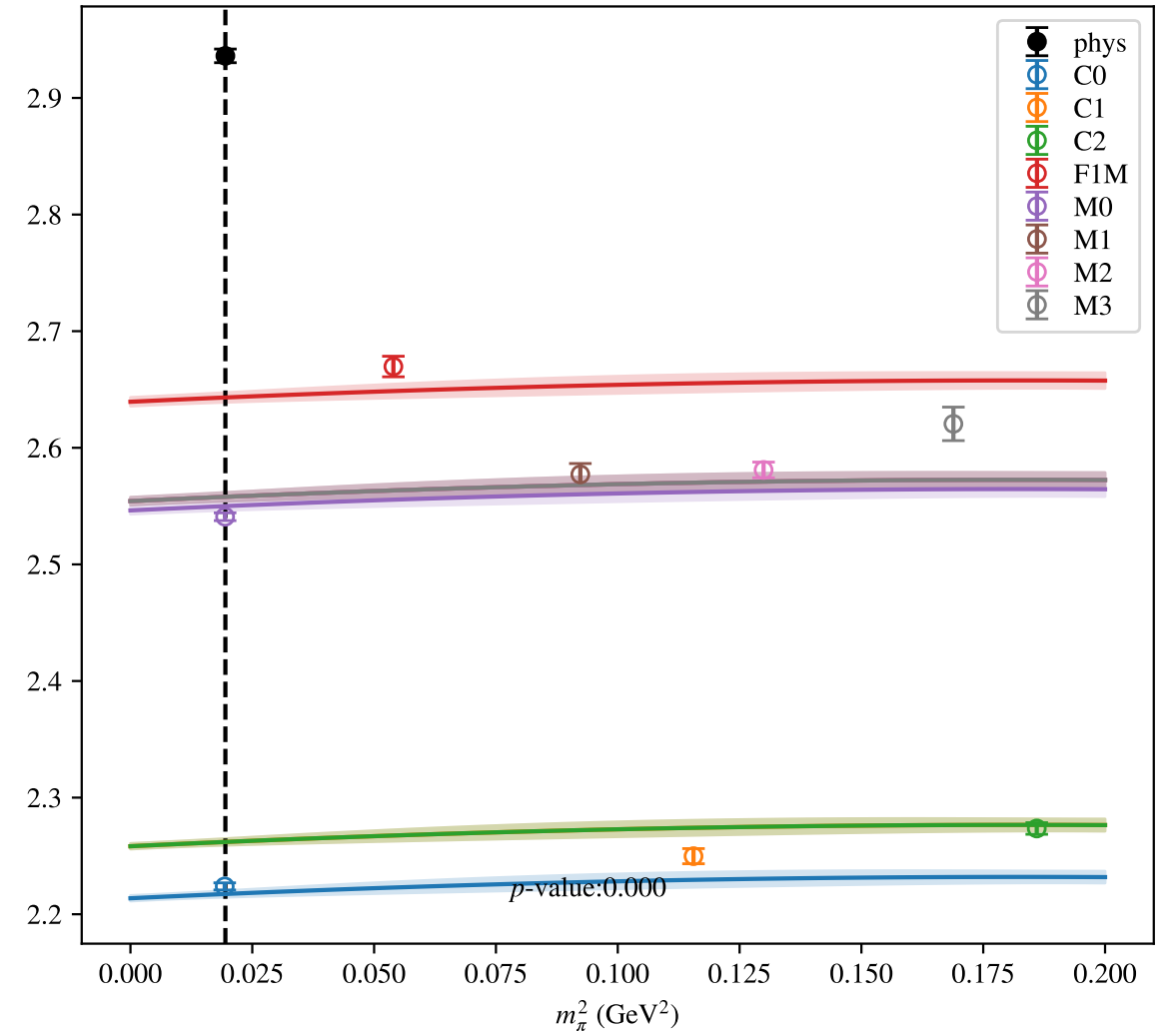


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.5 \text{ GeV}$$

VVmAA



VVmAA



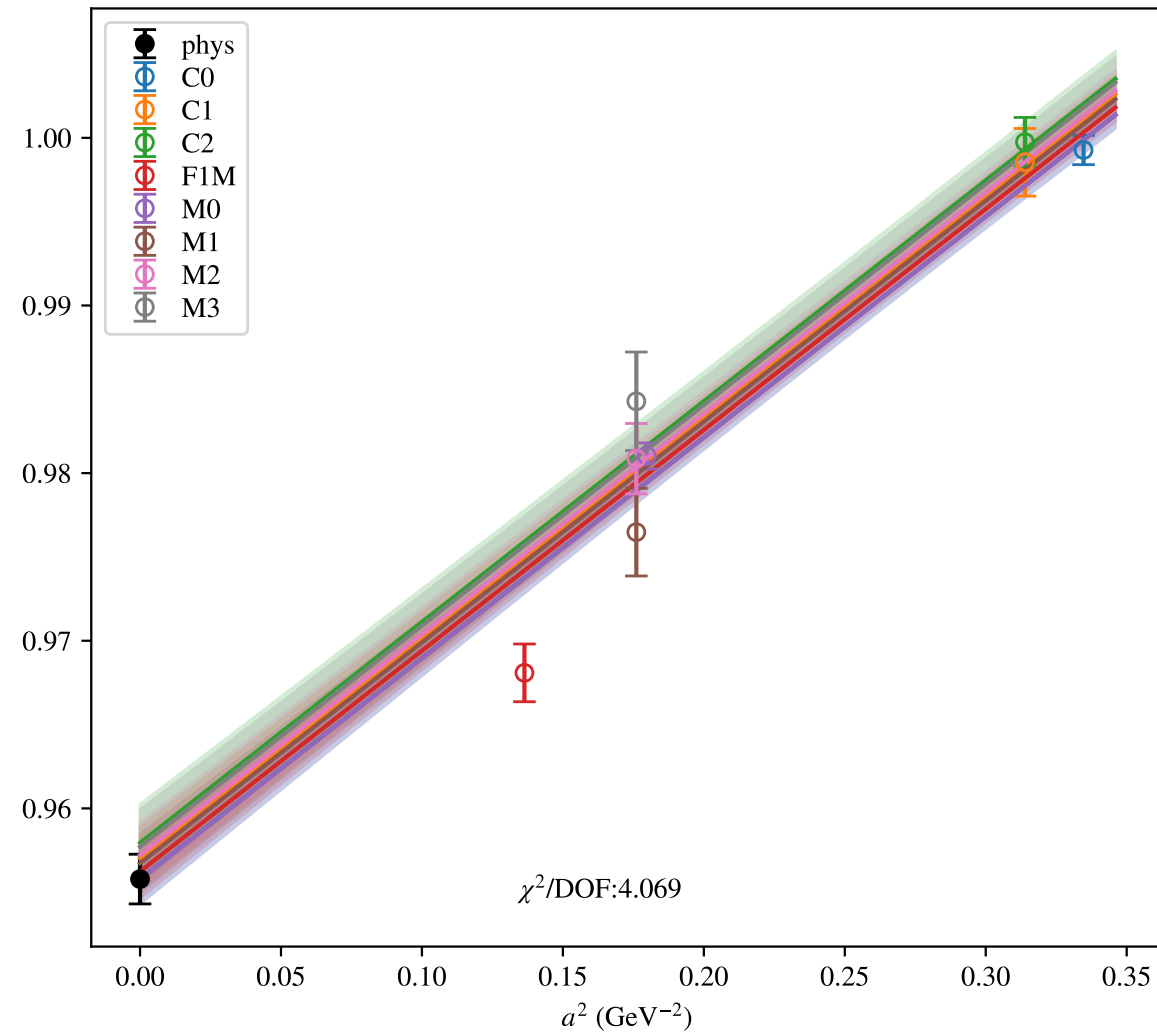
### 3 SSmPP

$\mu$ (GeV)	$a^2, m_\pi^2$	$a^2, m_\pi^2$ no C	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$
2.0	<b>0.9557(14)</b> : 4.069 (0.001)	<b>0.9259(67)</b> : 1.711 (0.181)	<b>0.906(11)</b> : 1.012 (0.4)	<b>0.9572(15)</b> : 3.563 (0.007)
1.8	<b>0.9631(18)</b> : 2.42 (0.033)	<b>0.9365(74)</b> : 2.117 (0.12)	<b>0.922(12)</b> : 1.336 (0.254)	<b>0.9647(17)</b> : 1.96 (0.098)
1.5	<b>0.9741(27)</b> : 1.372 (0.231)	<b>0.9523(94)</b> : 2.007 (0.134)	<b>0.947(15)</b> : 1.41 (0.228)	<b>0.9759(24)</b> : 1.054 (0.378)

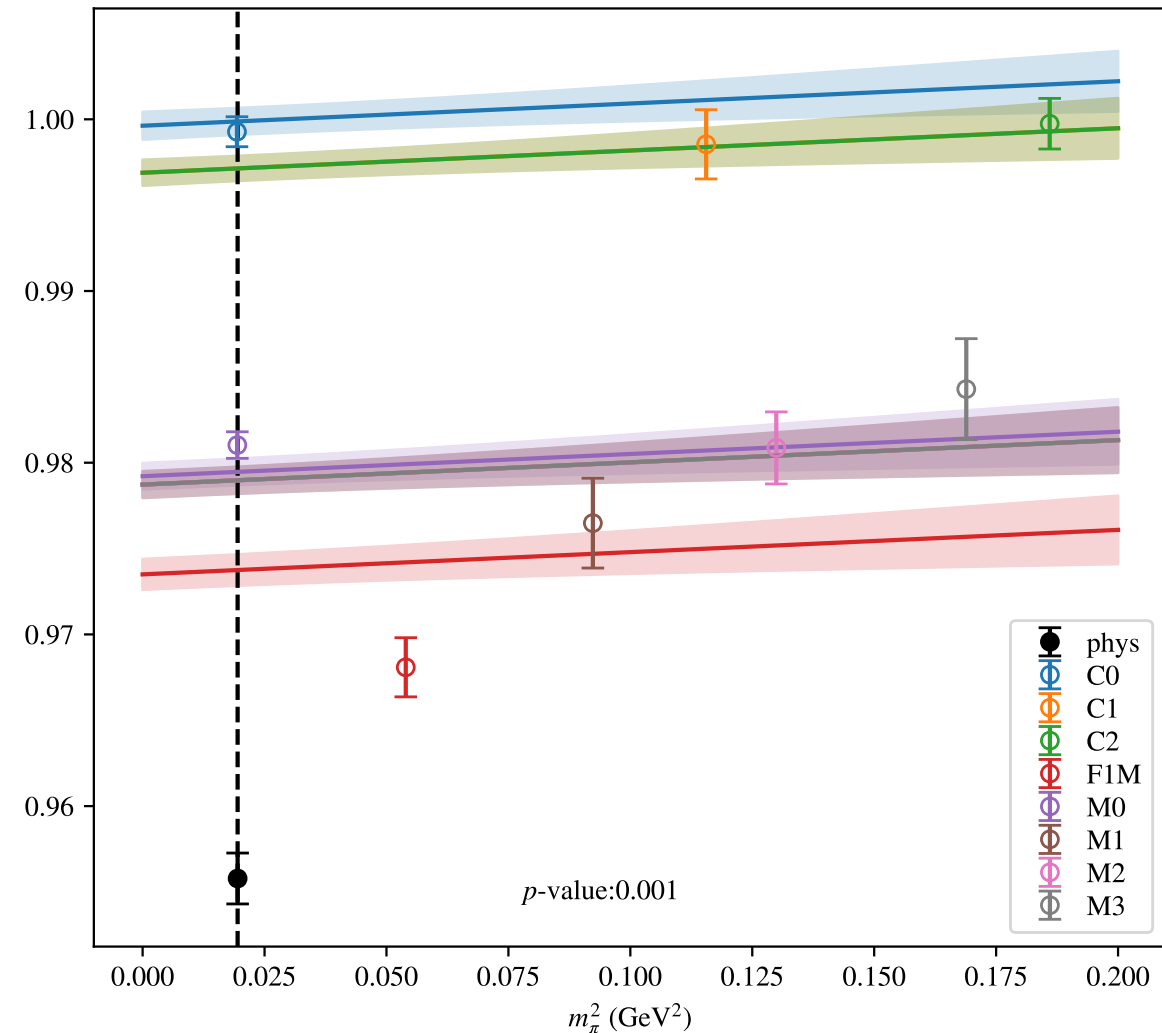
Table 3: Physical point value from chiral and continuum extrapolation at renormalisation scale  $\mu$ . Entries are **value(error)**:  $\chi^2/\text{DOF}$  ( $p$ -value).

$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

SSmPP

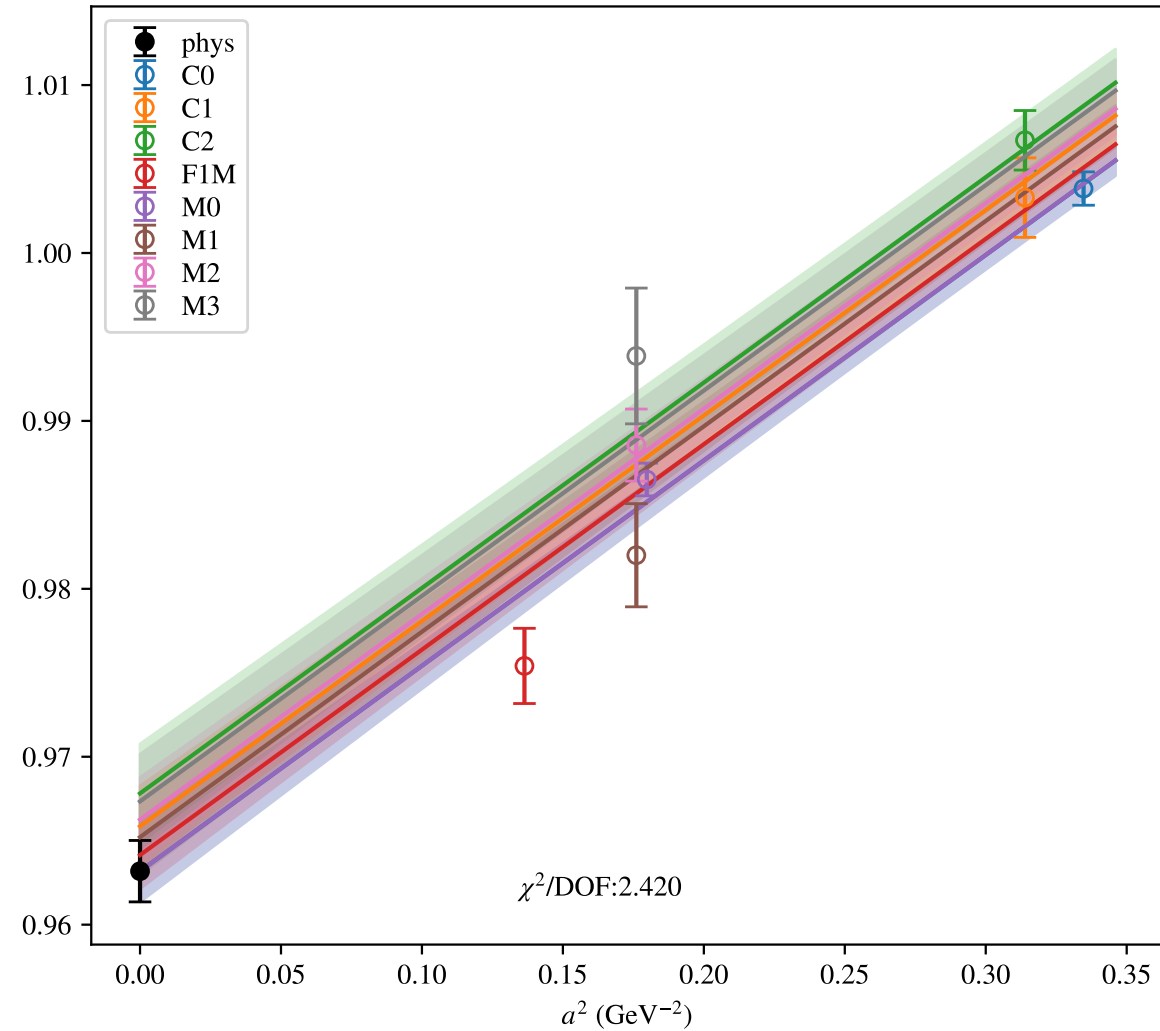


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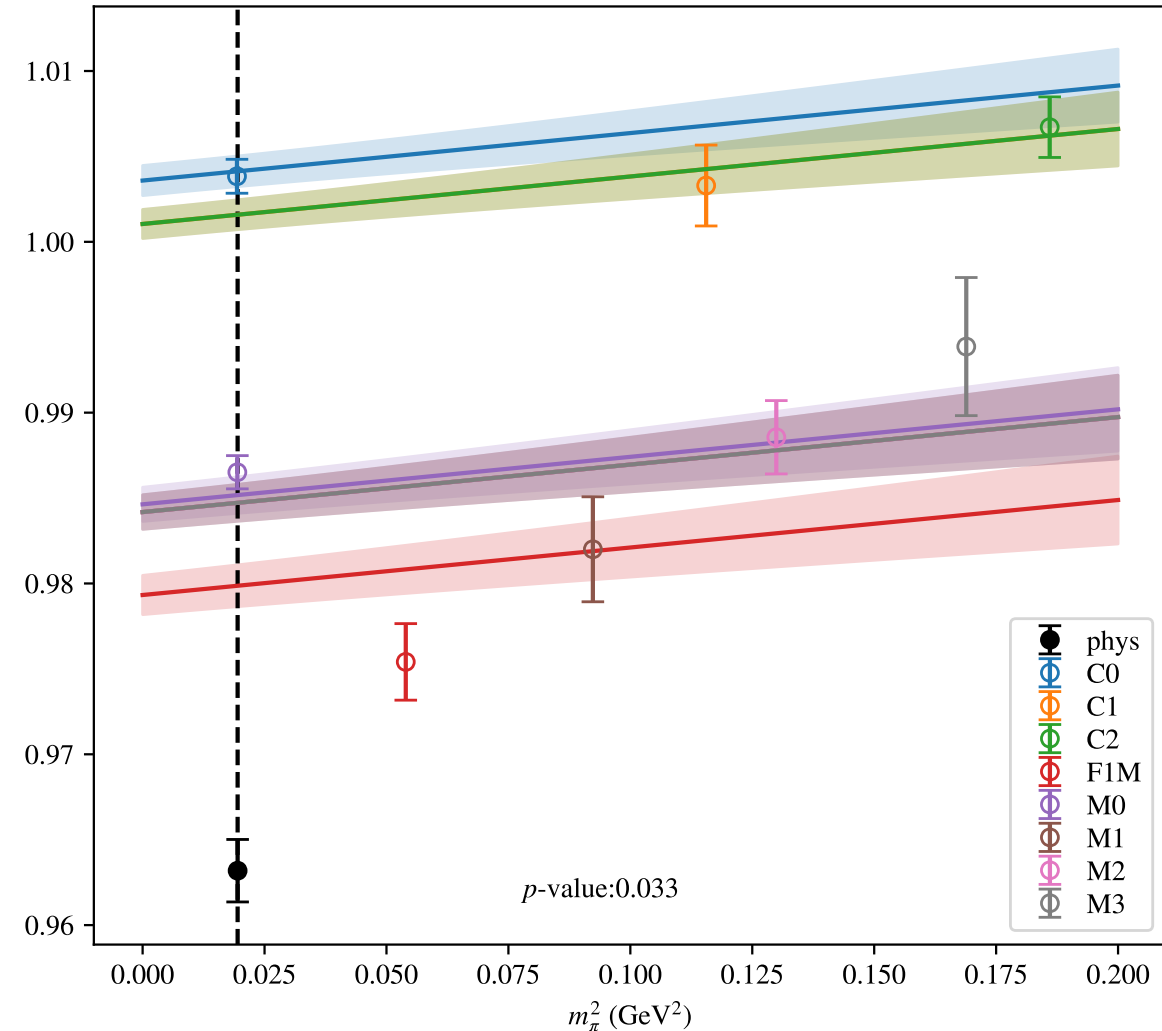


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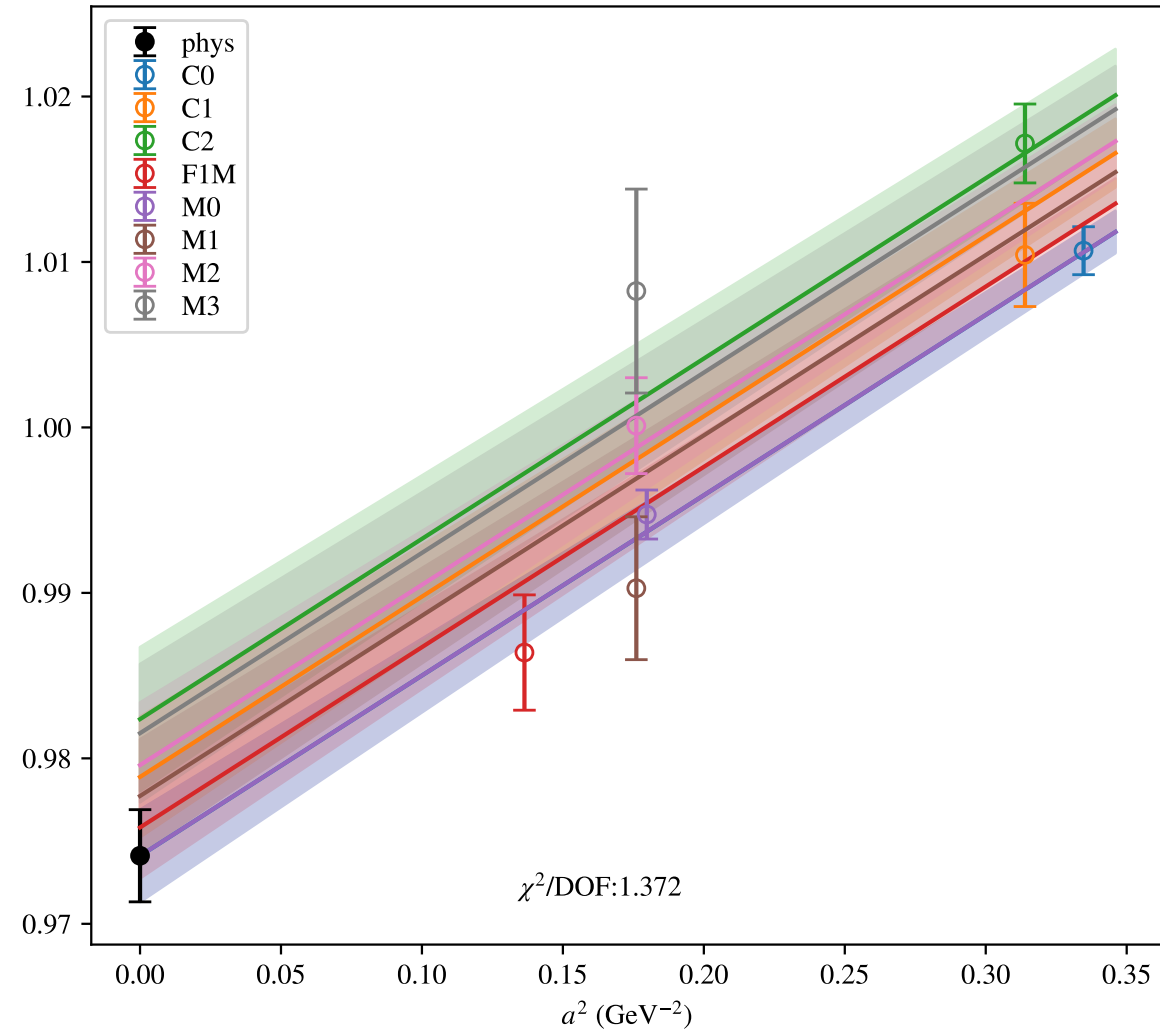


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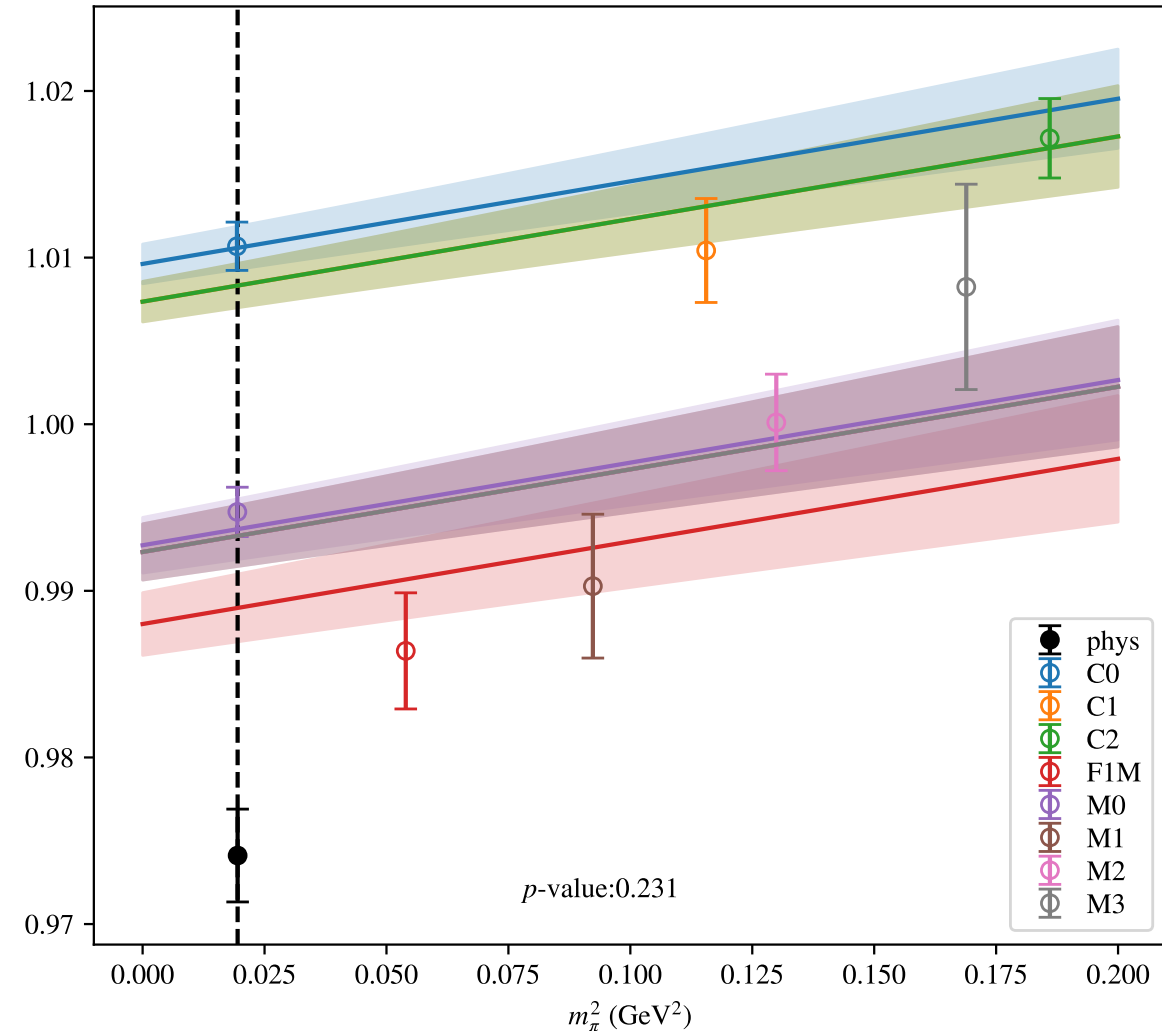


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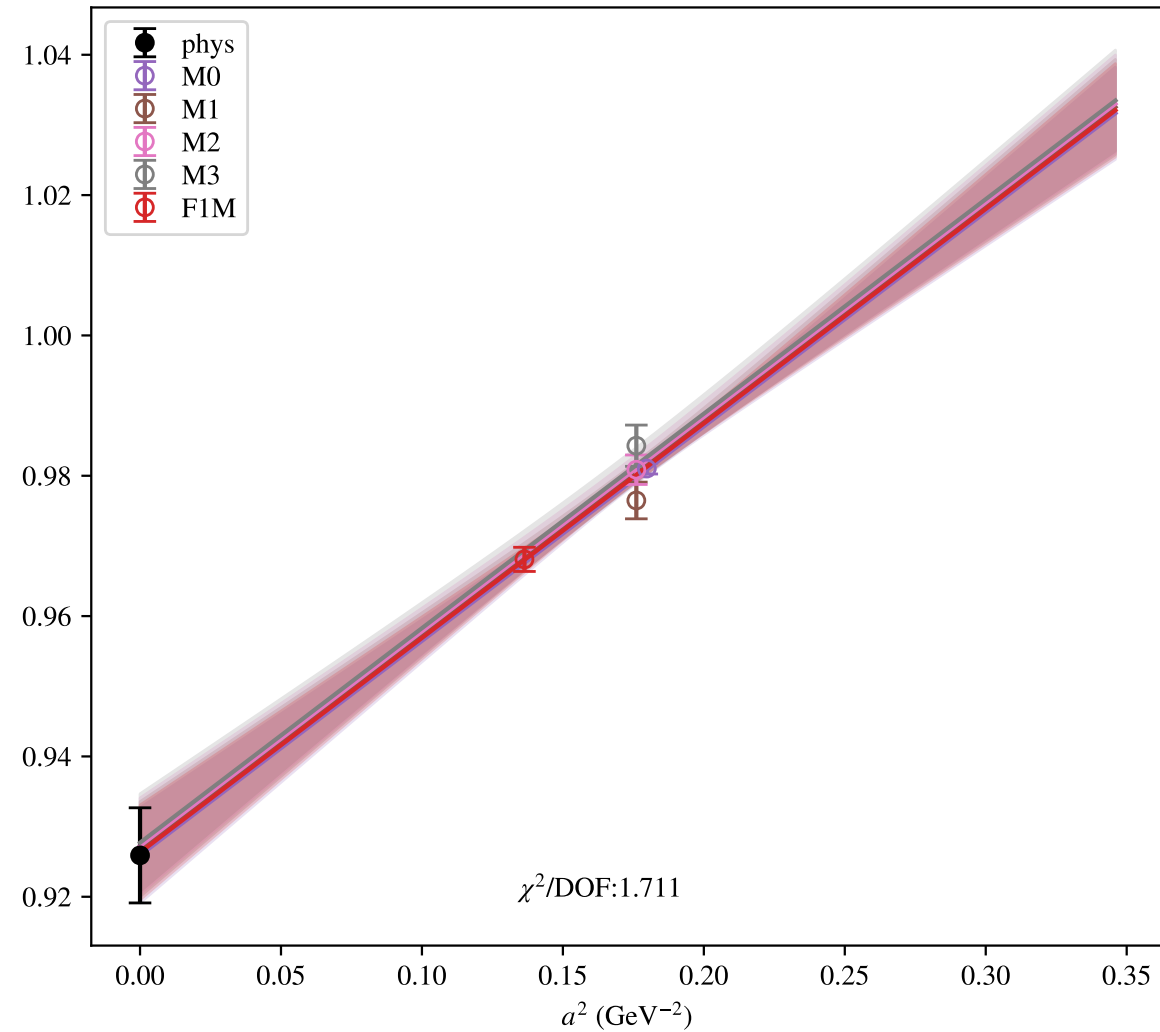


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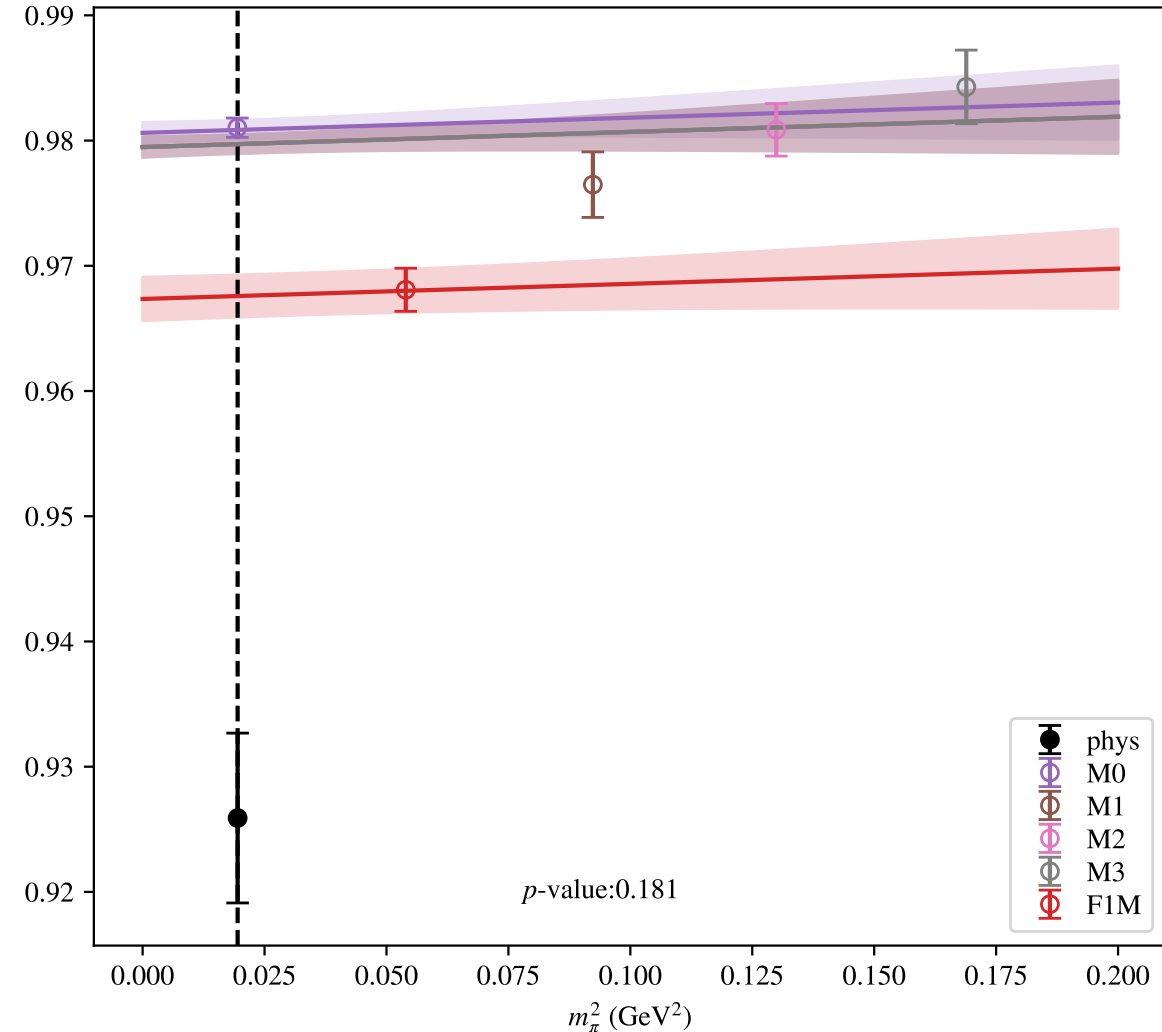


$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

SSmPP



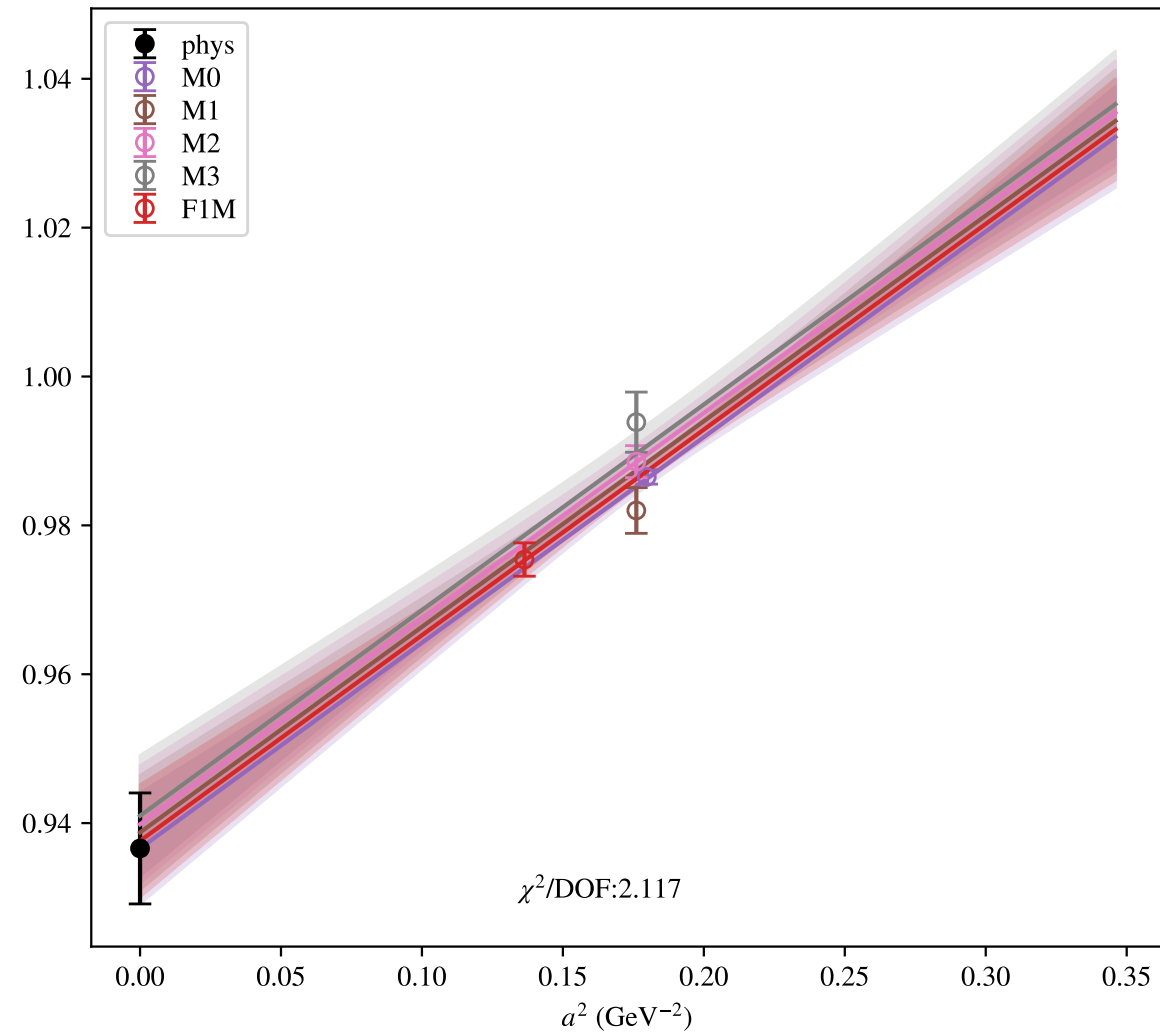
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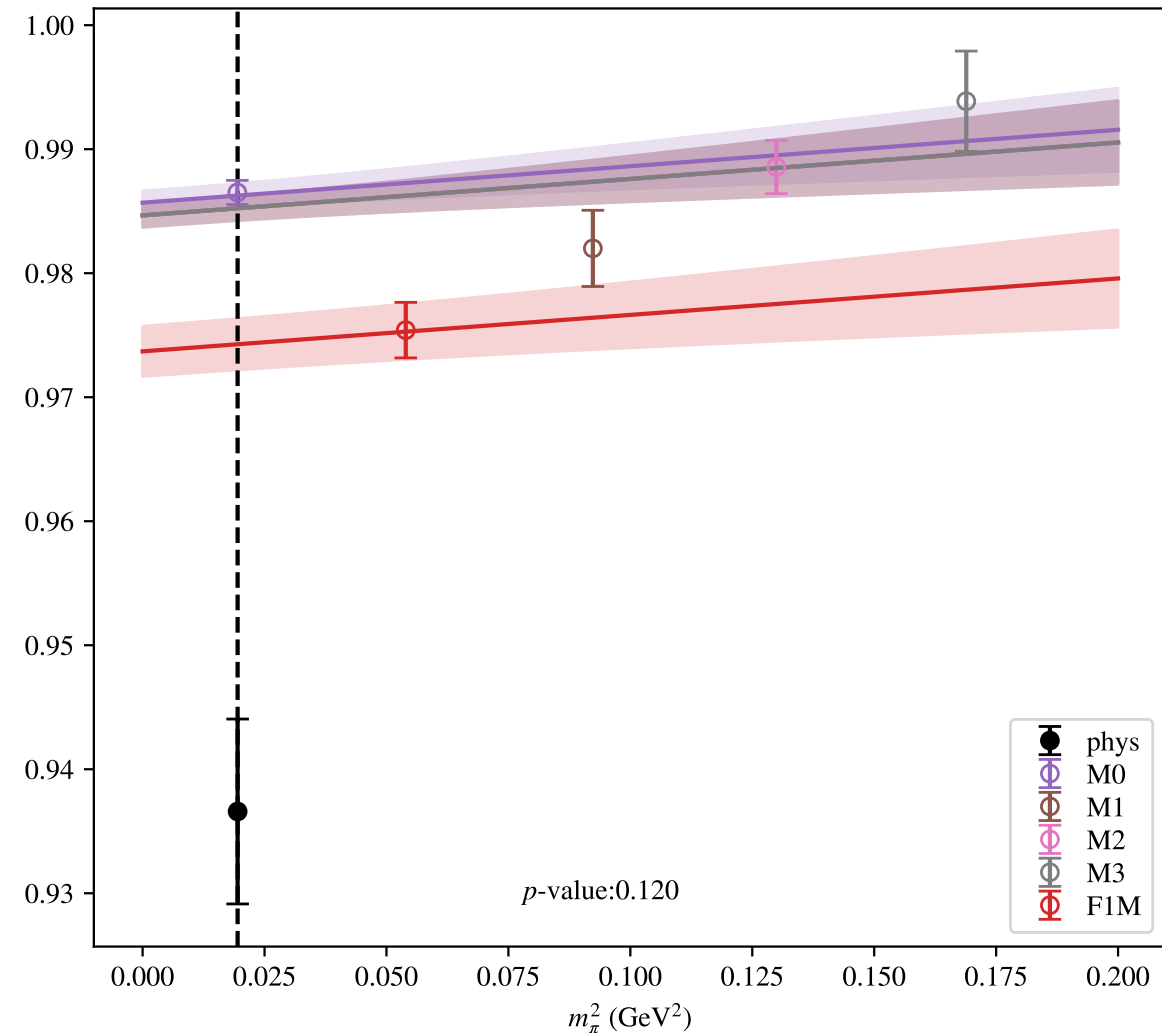


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SSmPP

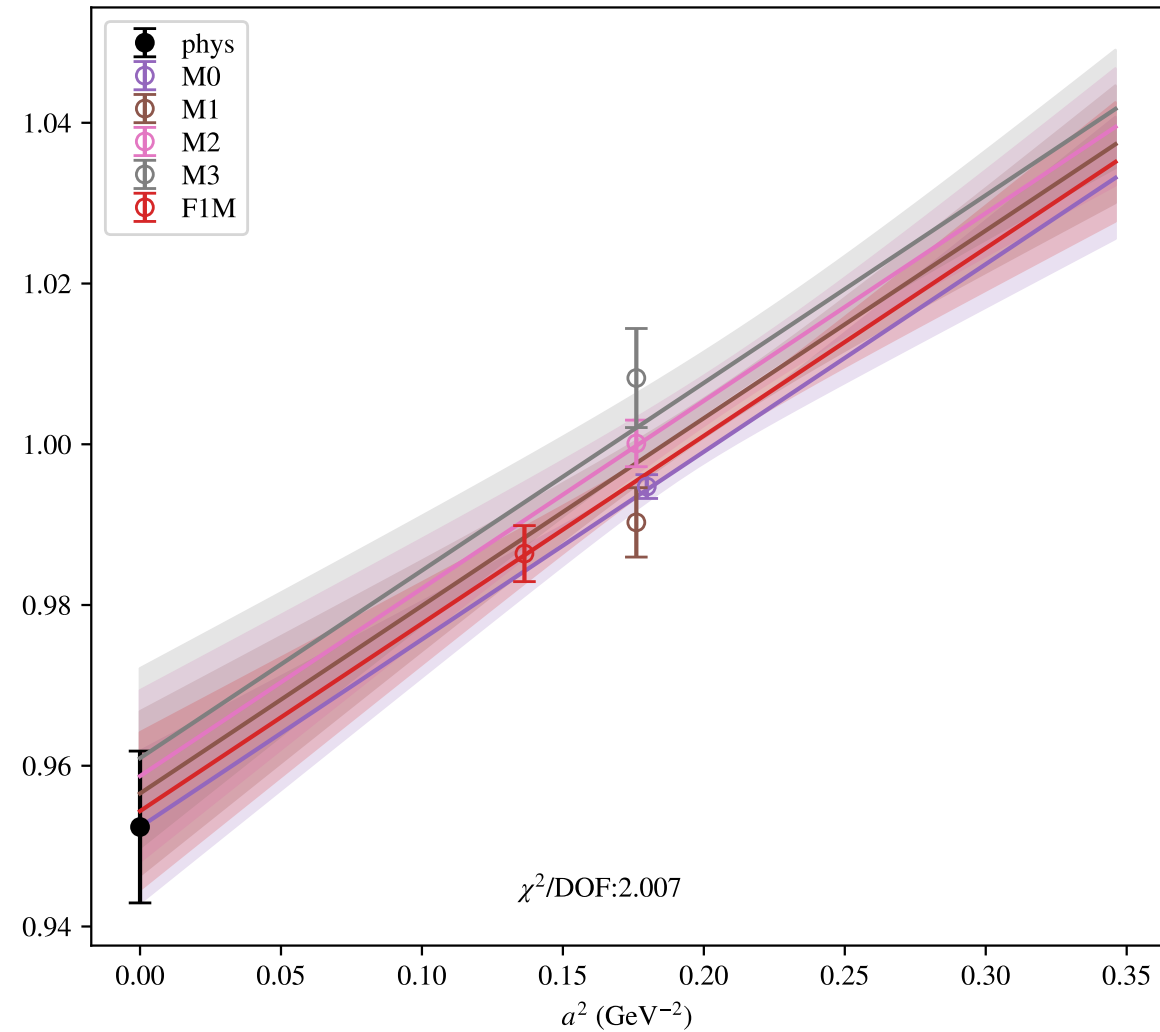


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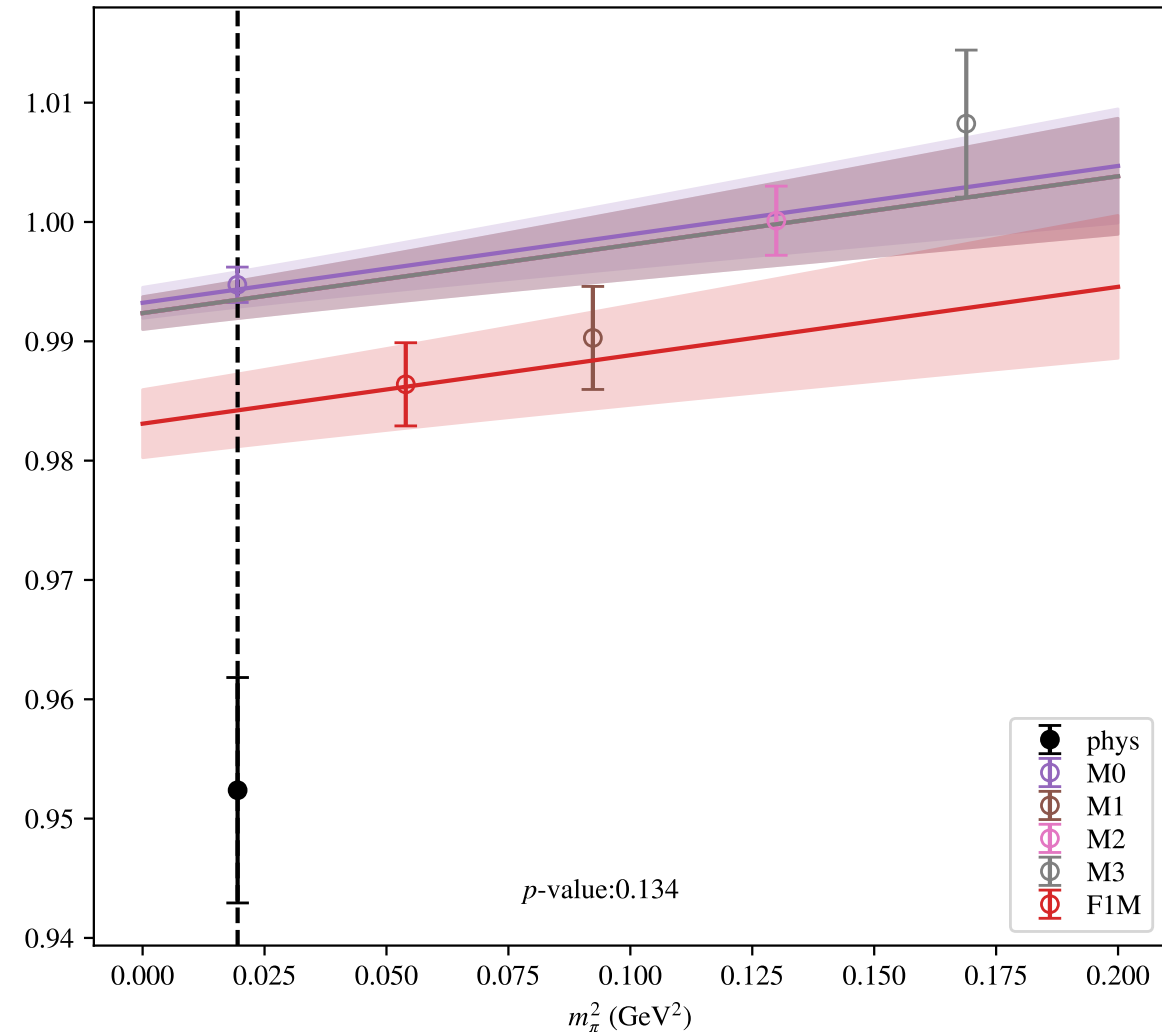


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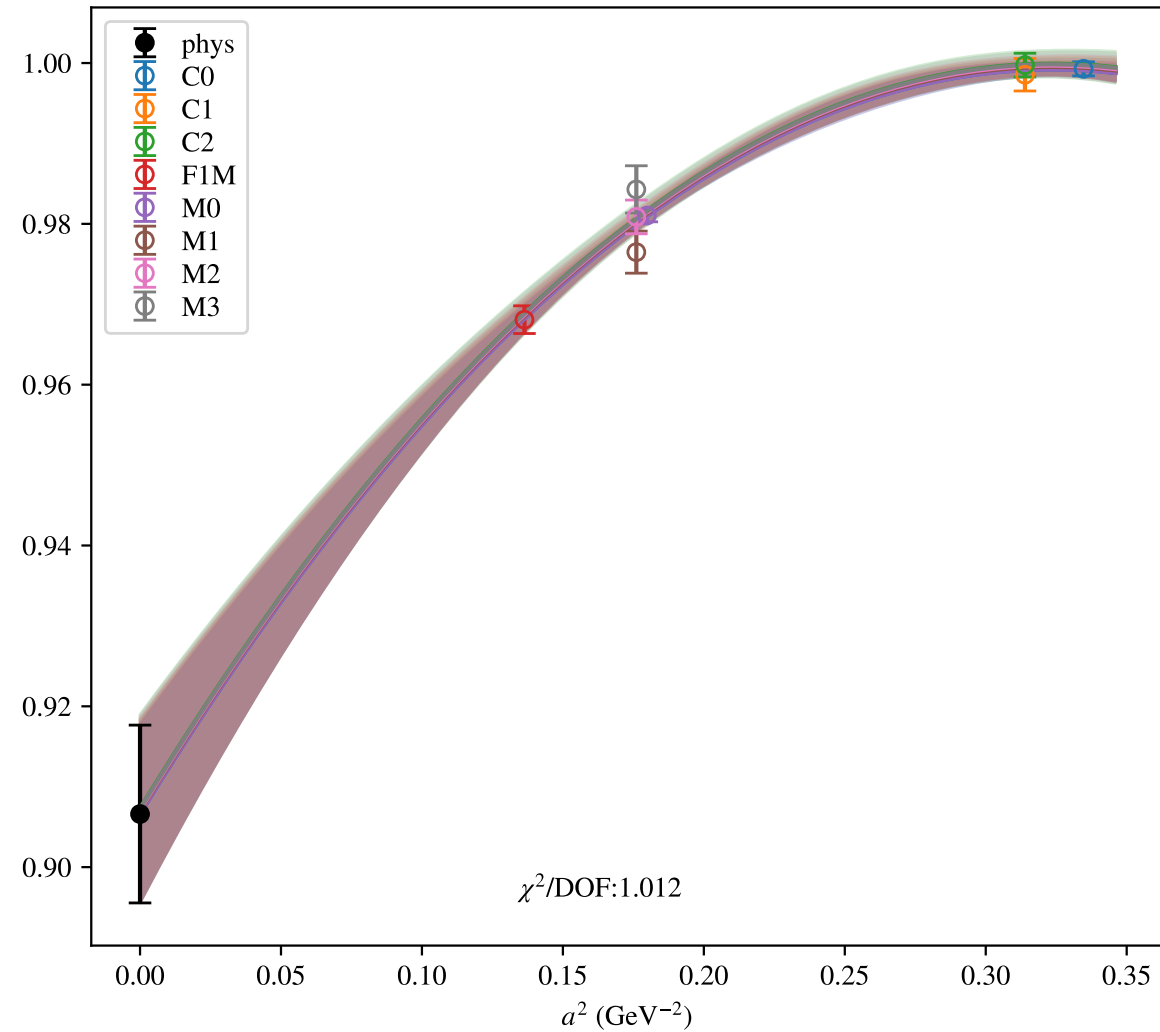


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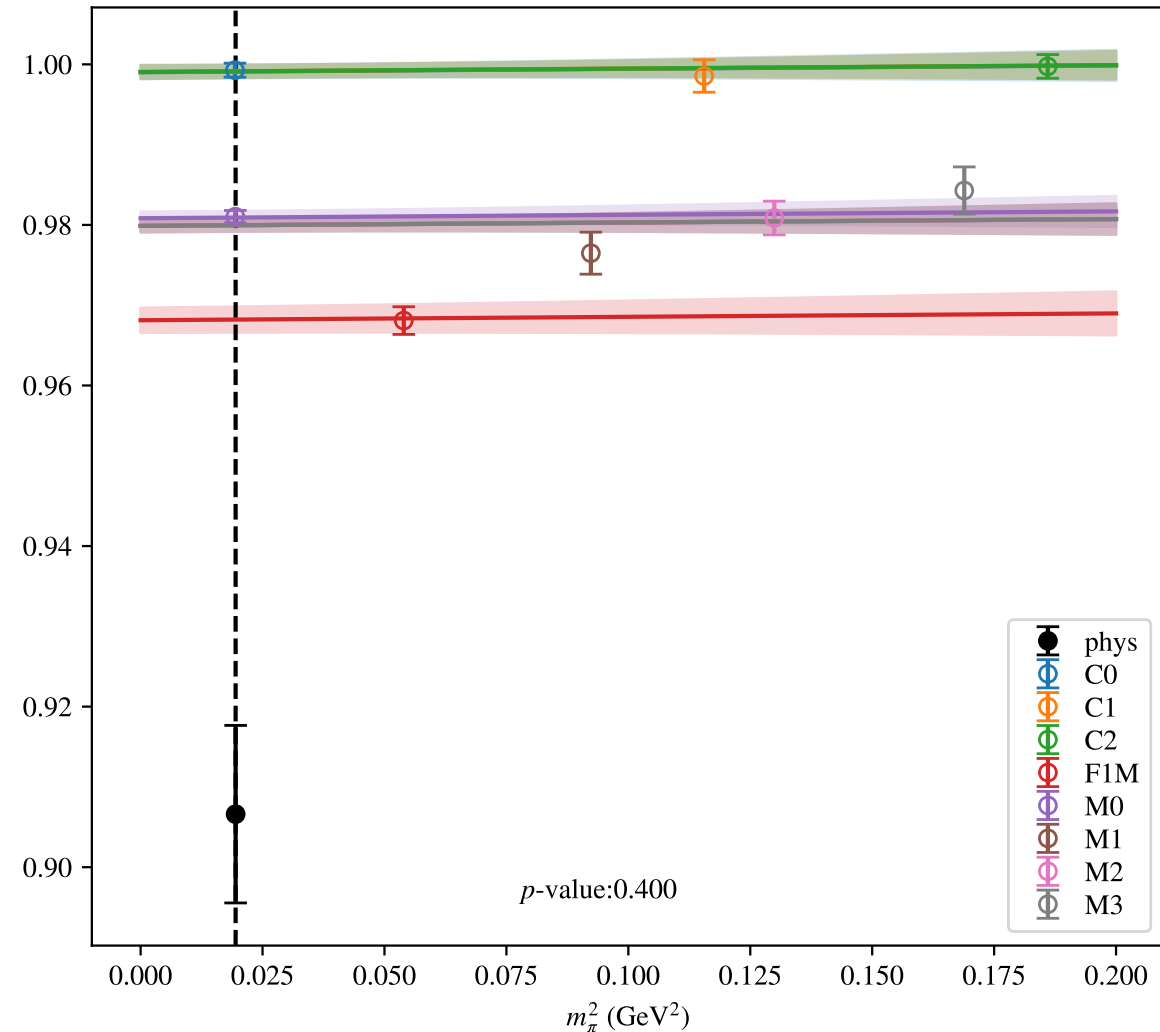


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

SSmPP

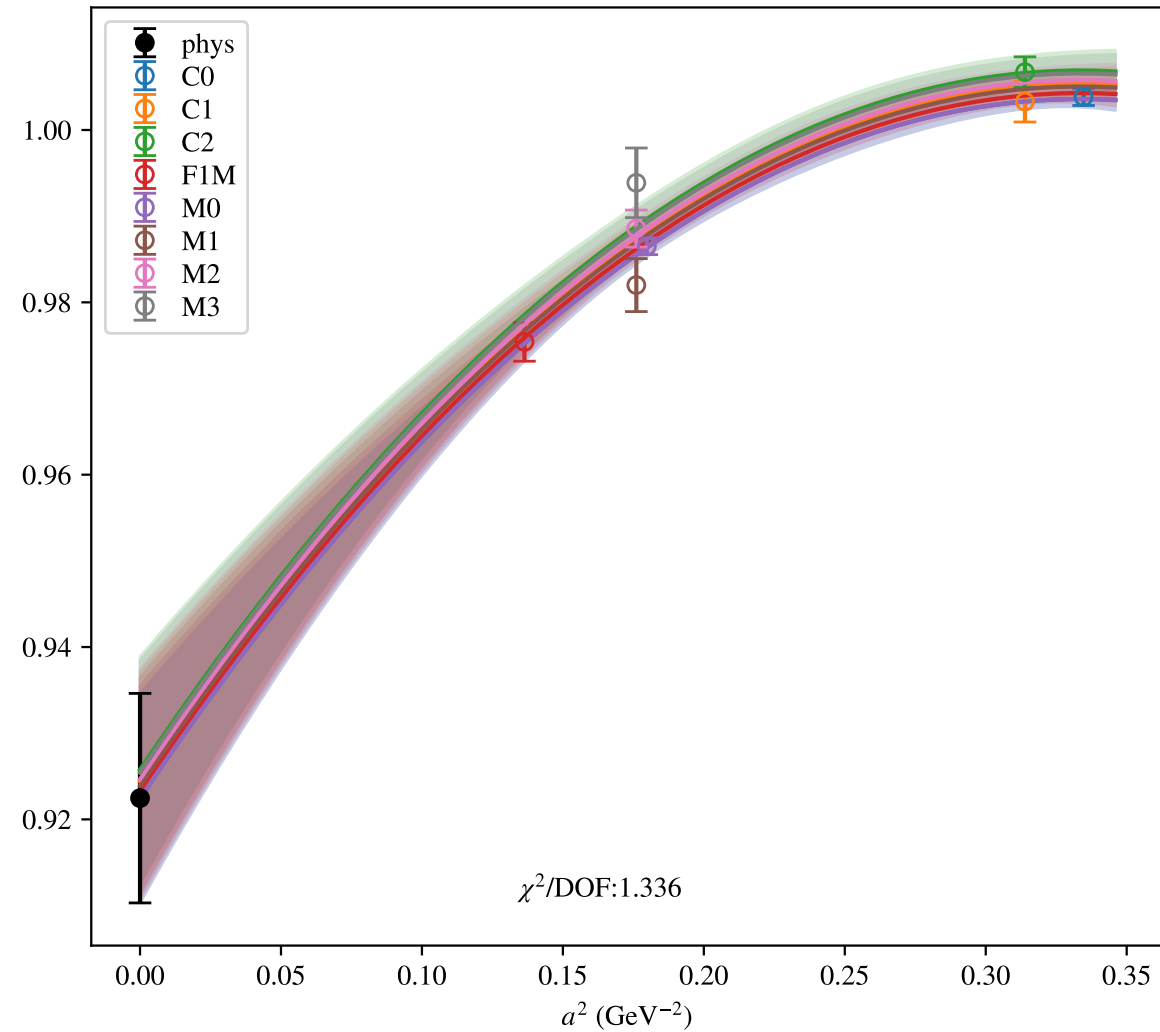


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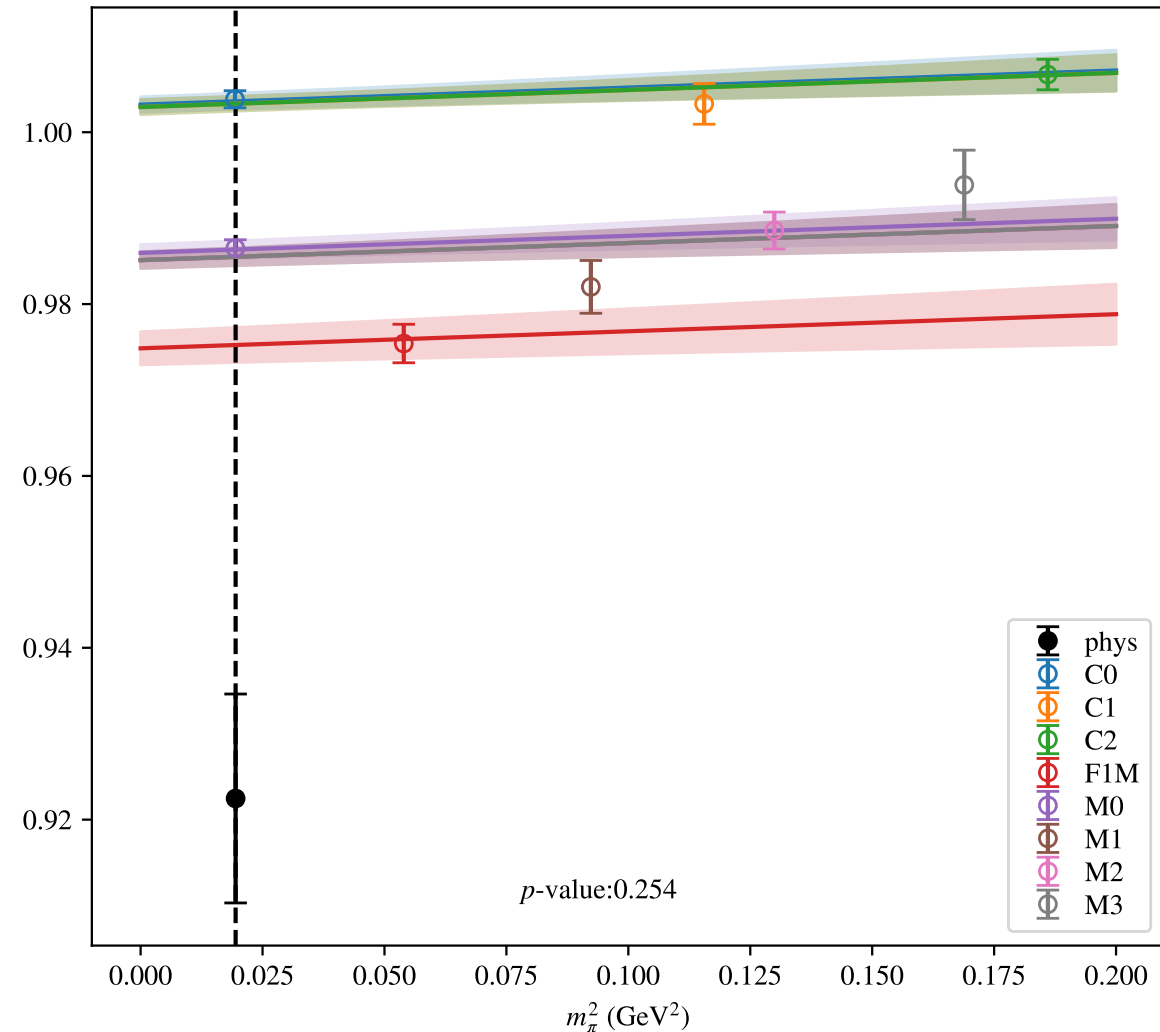


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SSmPP

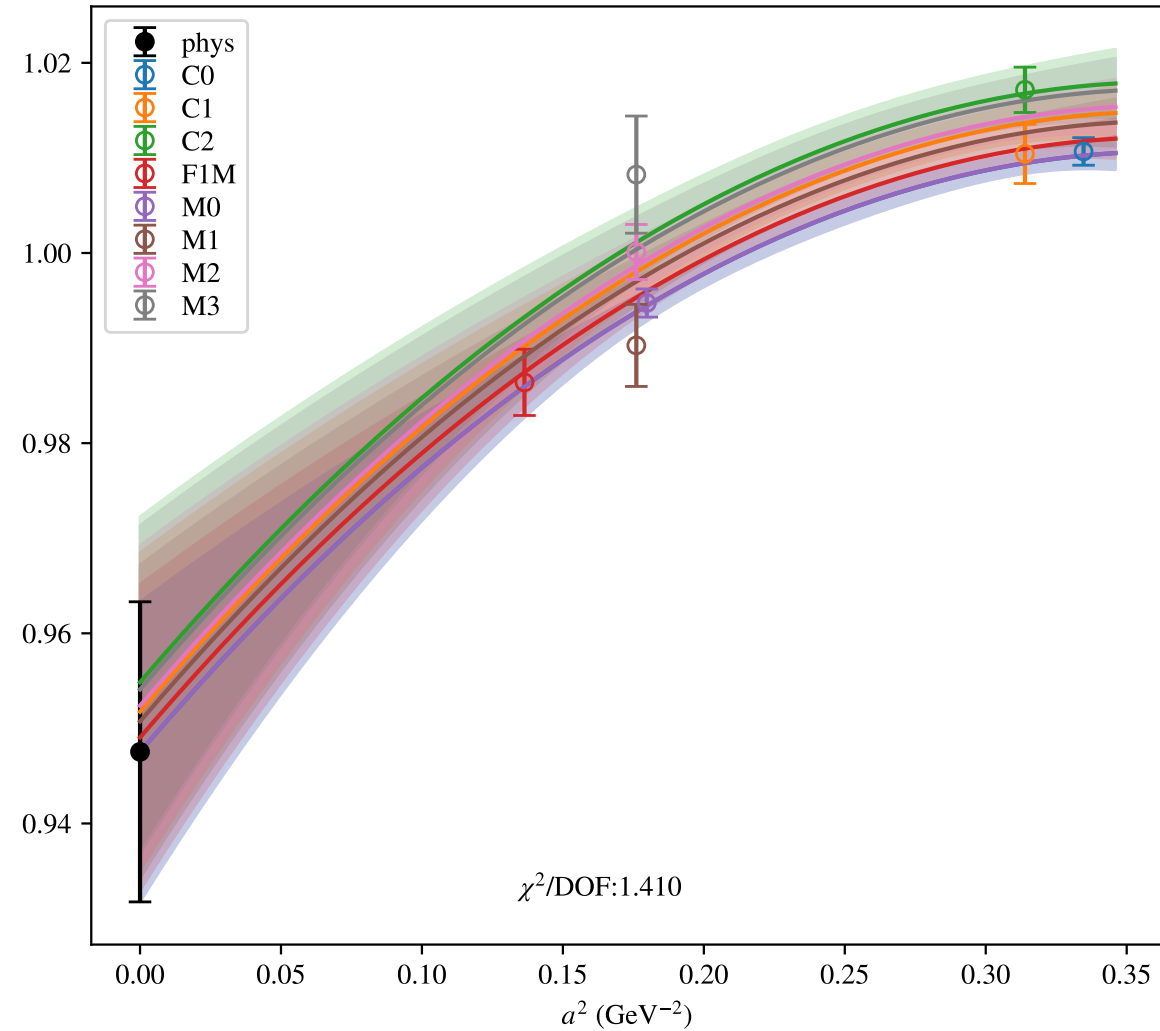


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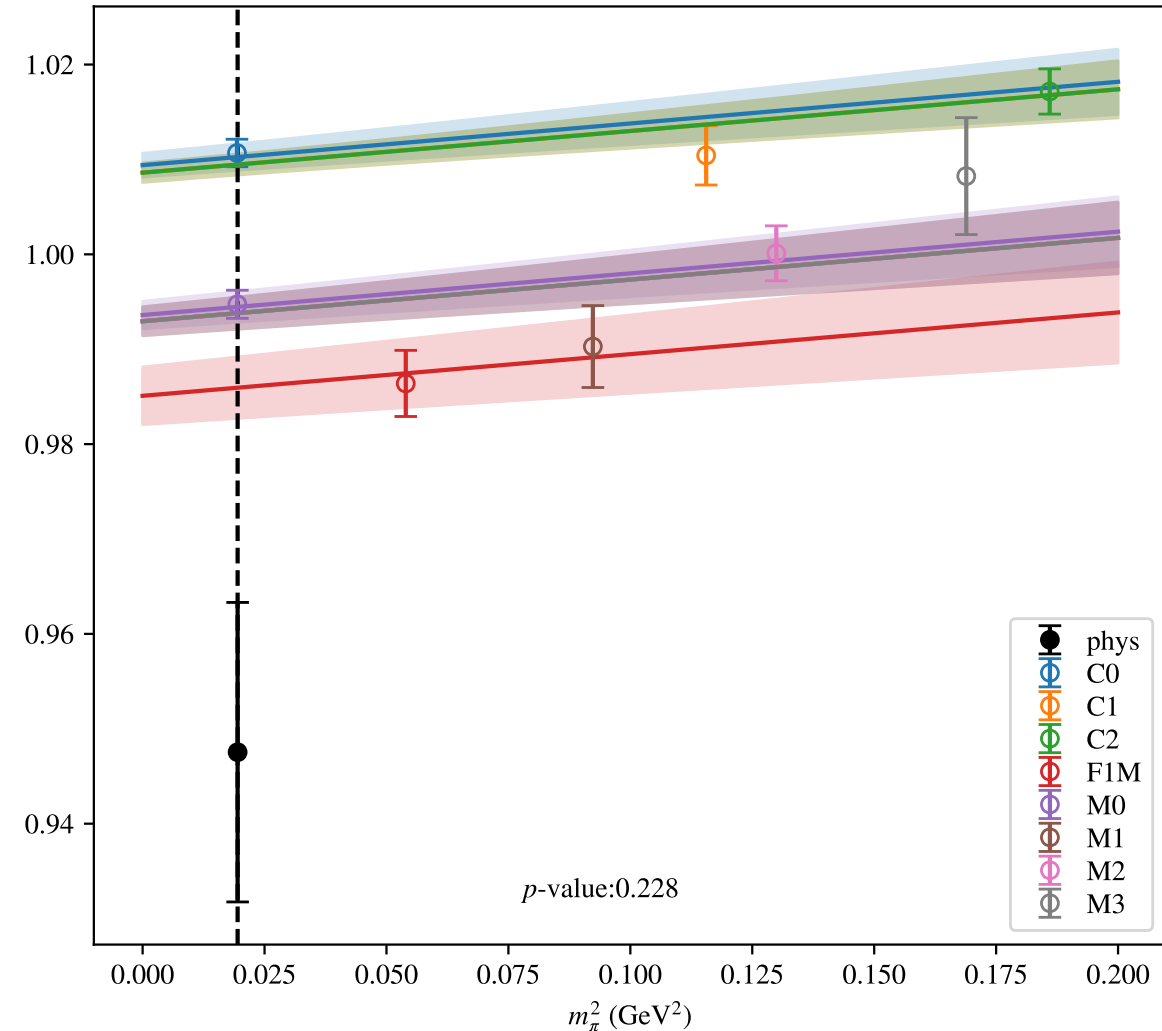


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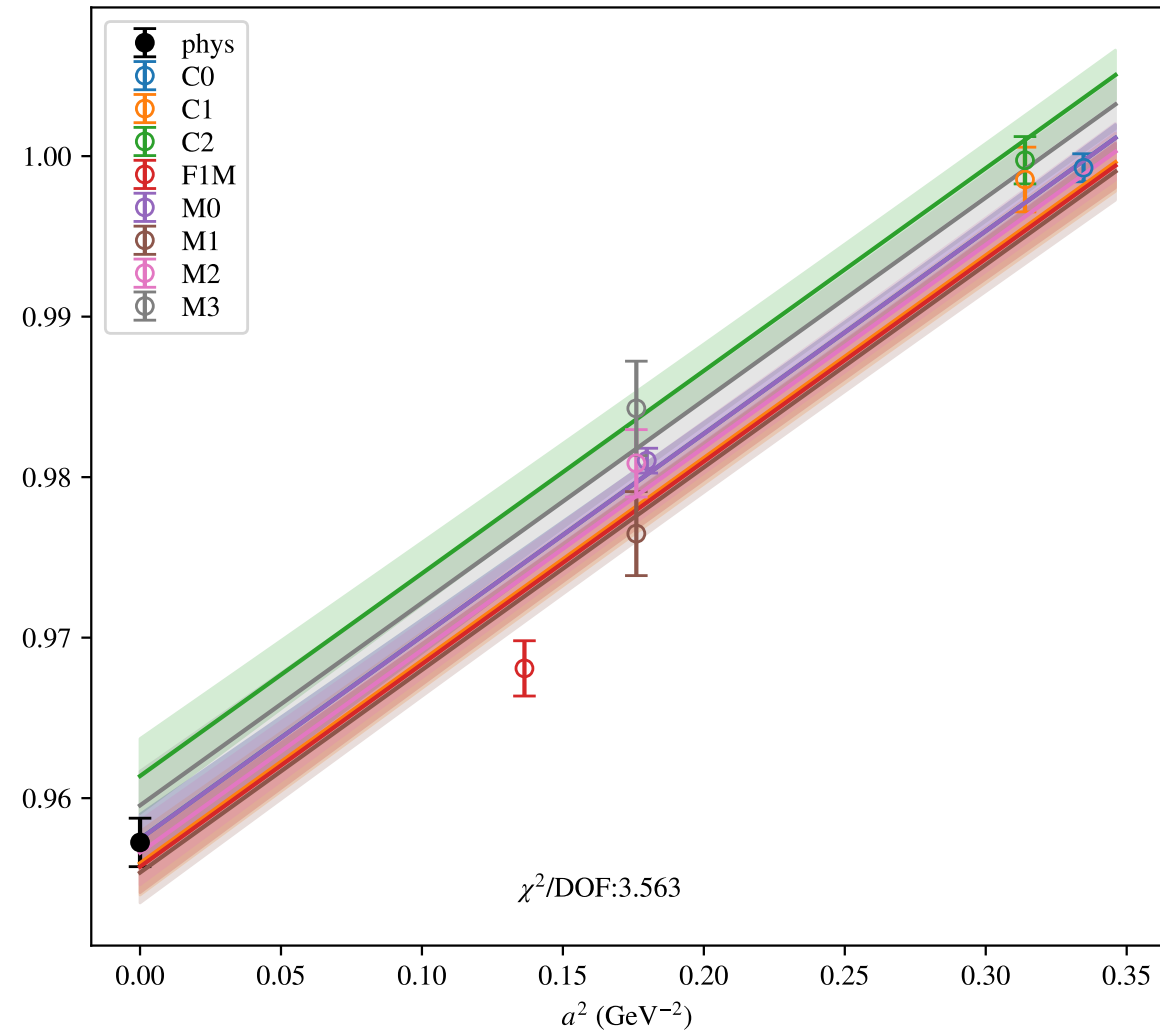


SSmPP

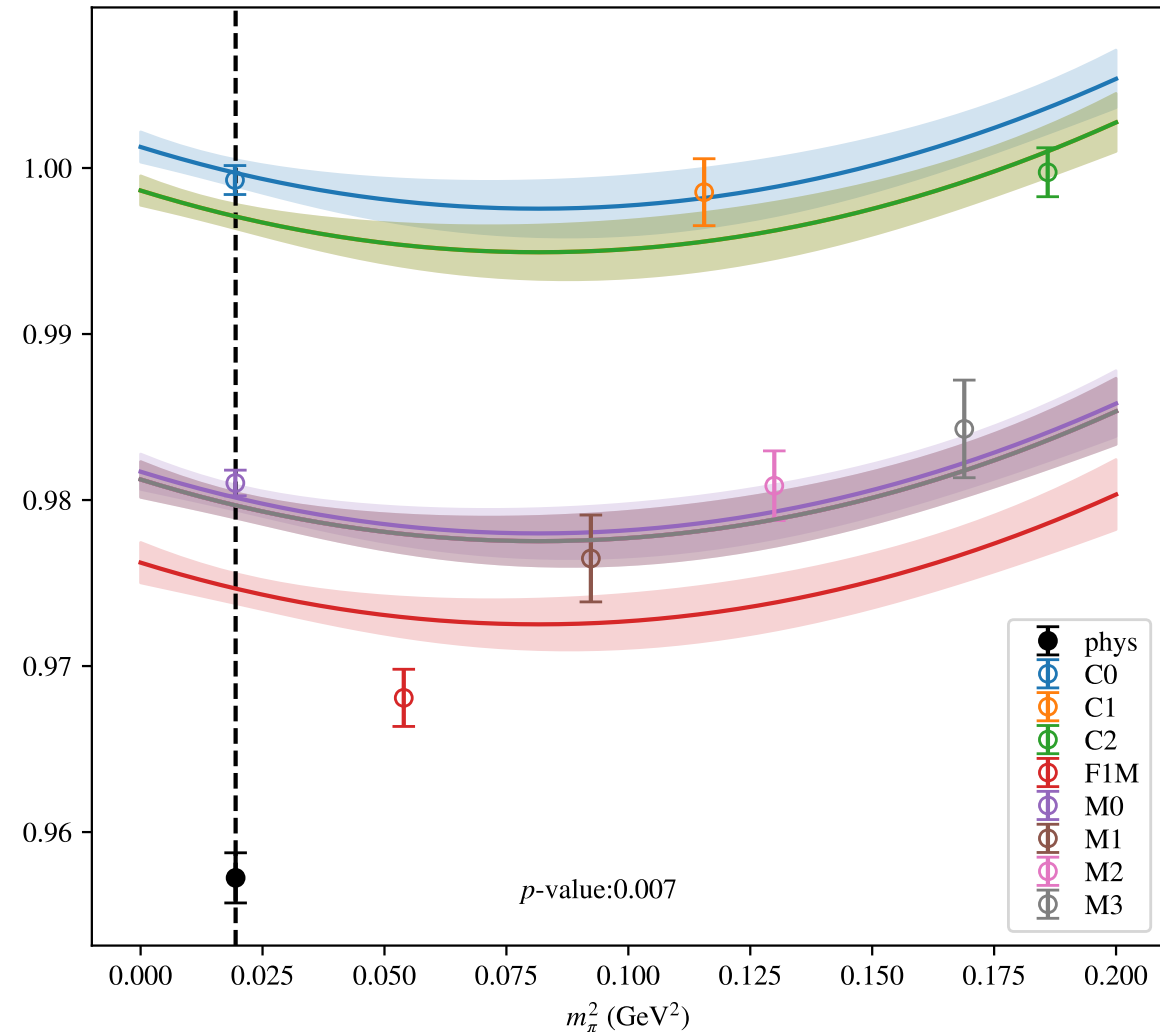


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

SSmPP

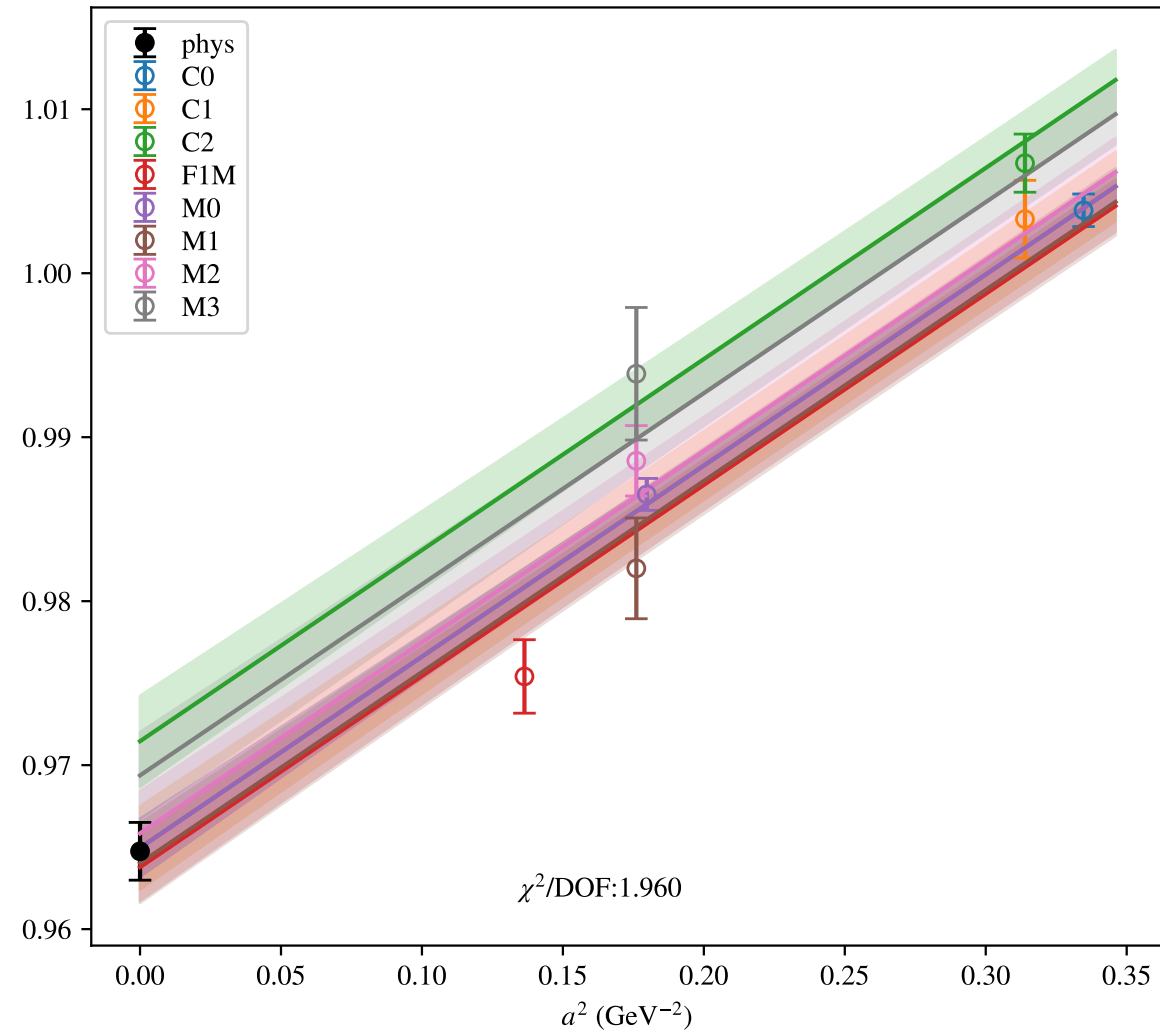


SSmPP

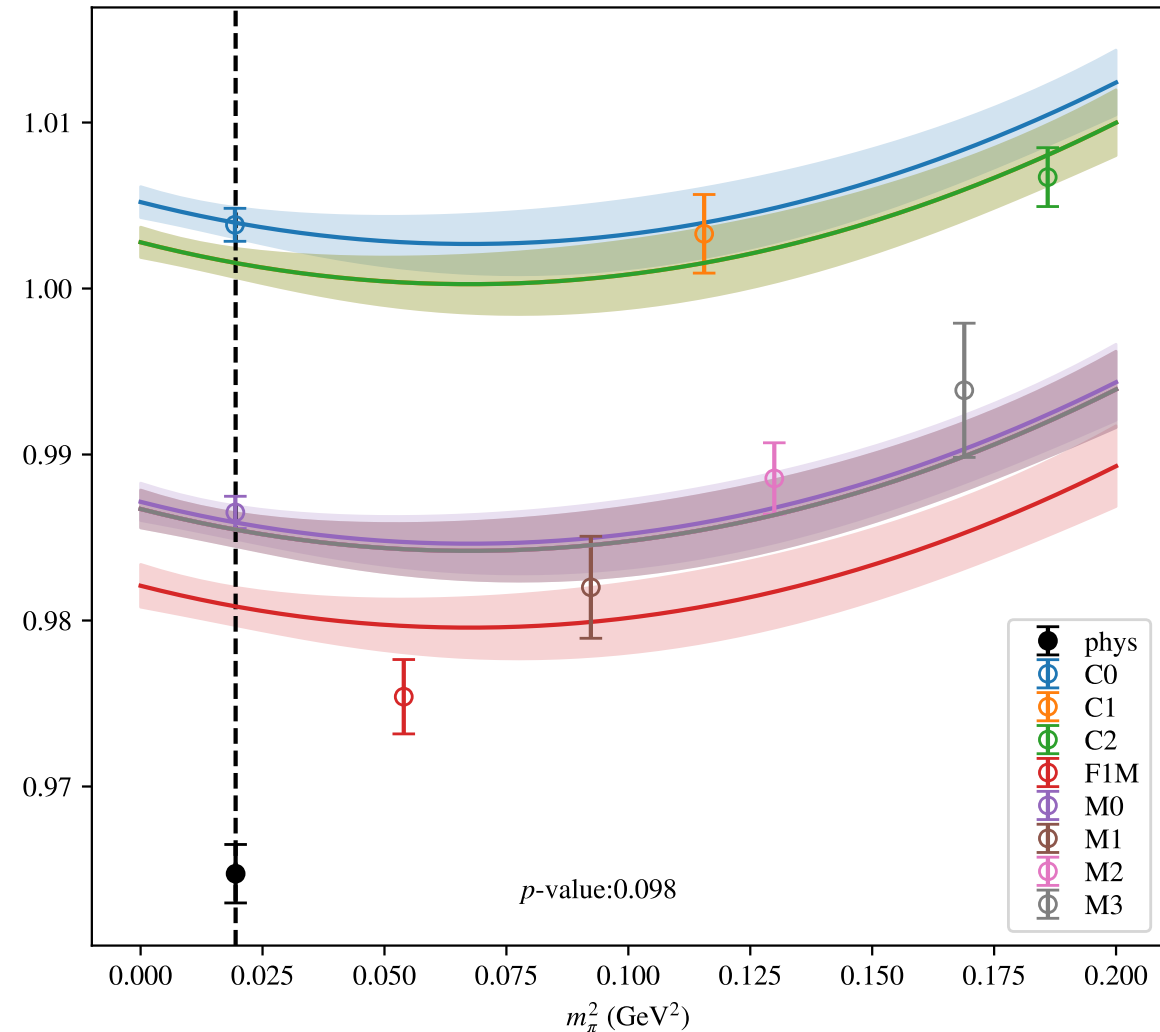


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.8 \text{ GeV}$$

SSmPP

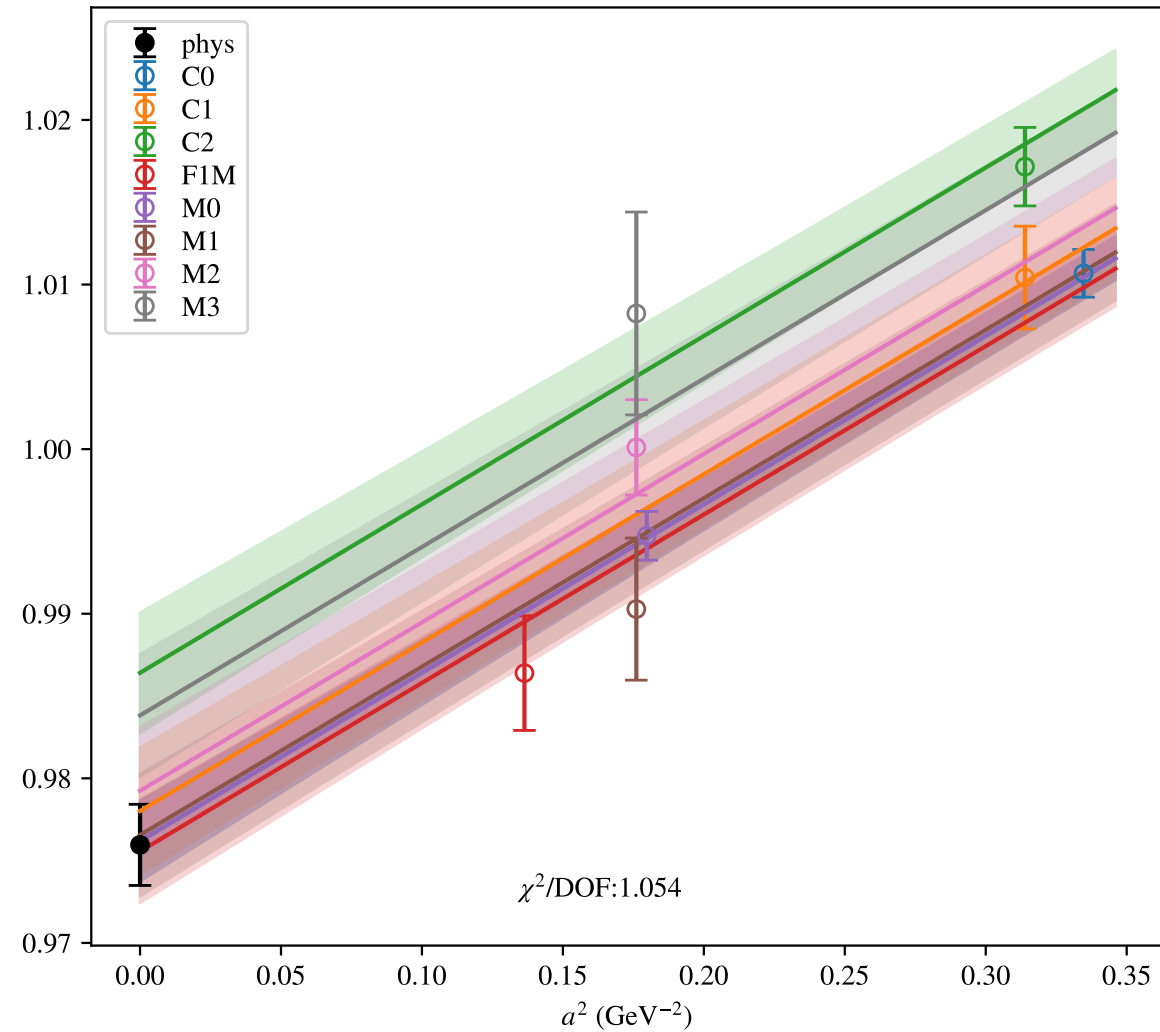


SSmPP

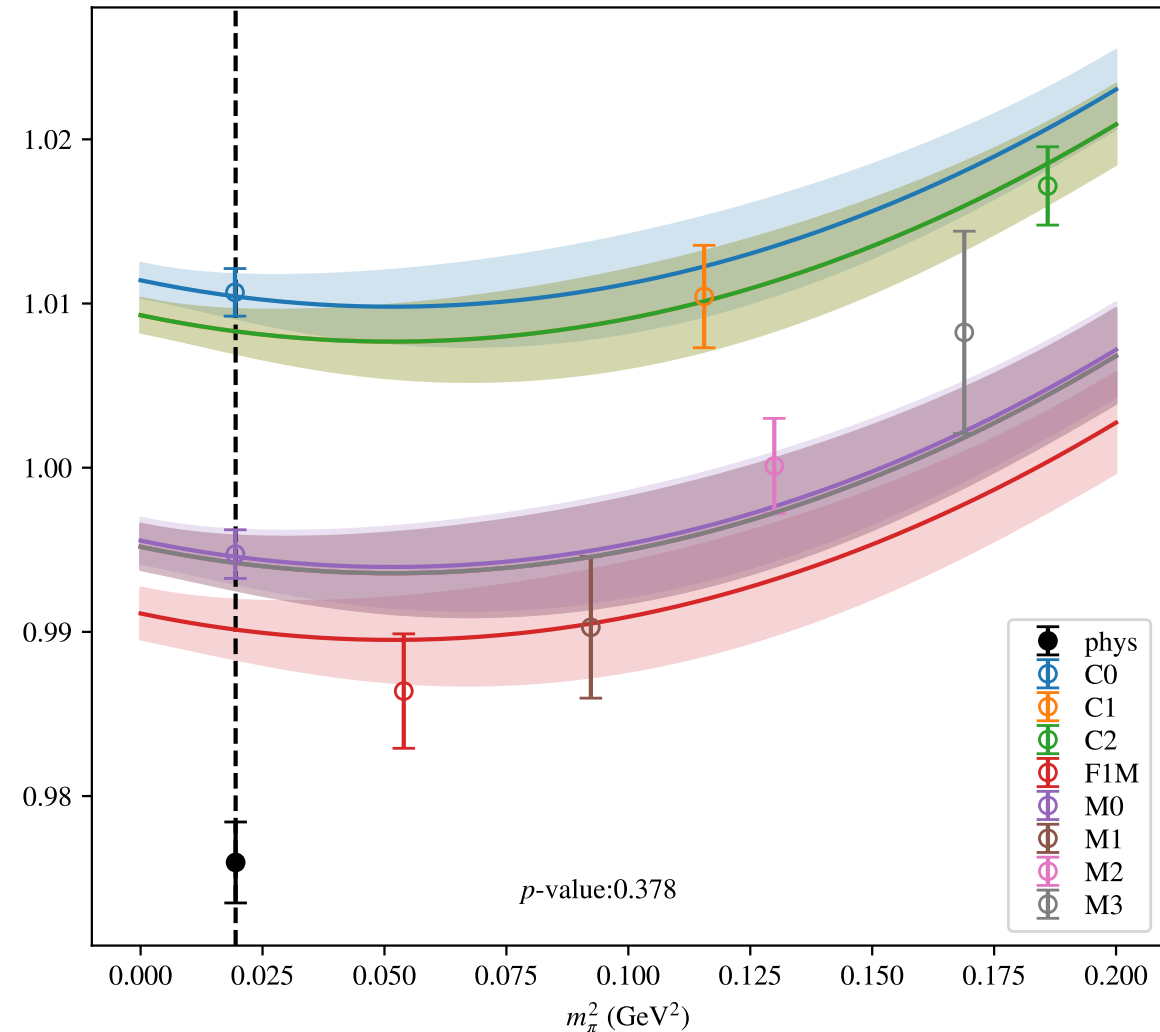


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.5 \text{ GeV}$$

SSmPP



SSmPP





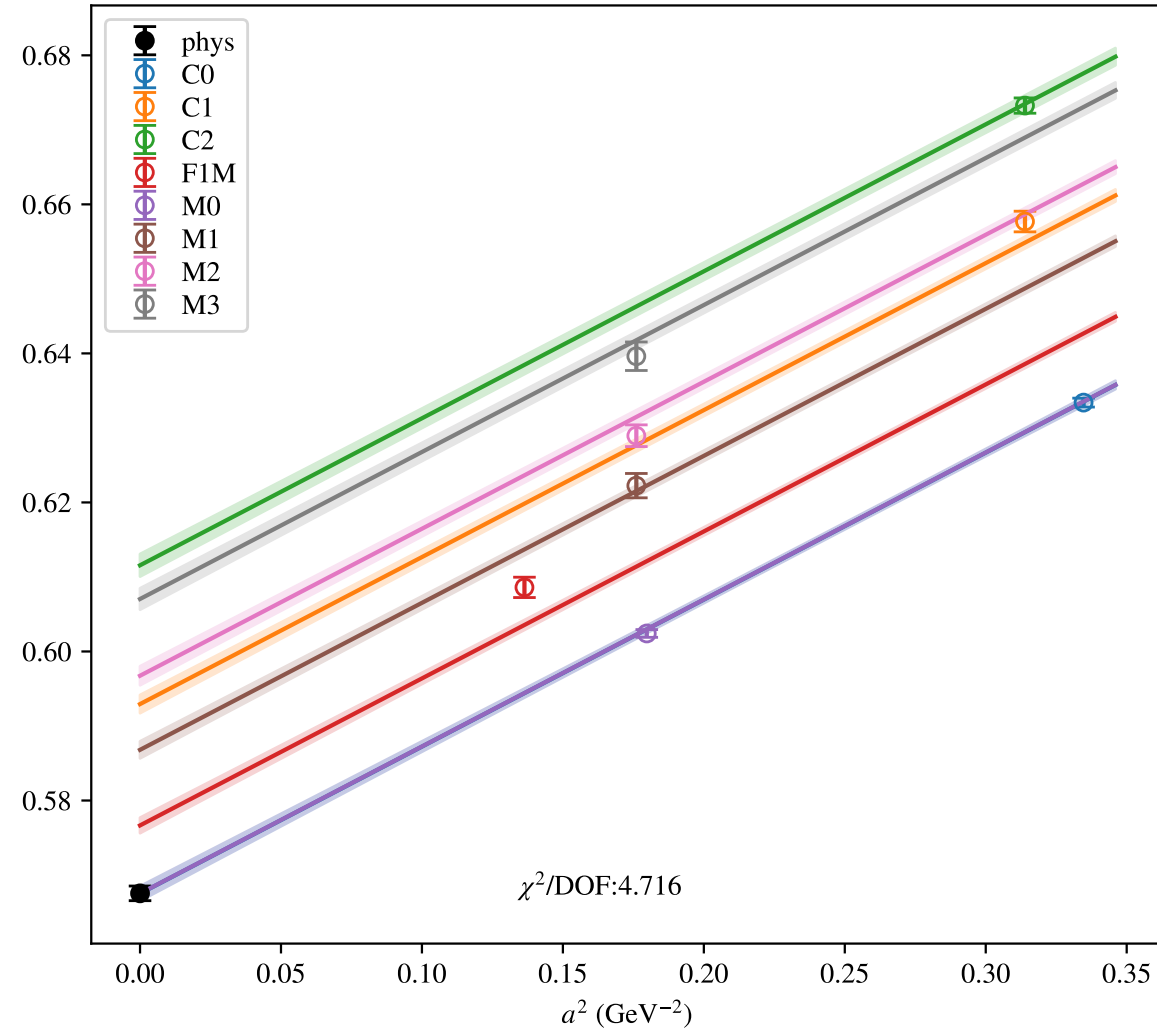
## 4 SSpPP

$\mu$ (GeV)	$a^2, m_\pi^2$	$a^2, m_\pi^2$ no C	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$
2.0	<b>0.56754(98)</b> : 4.716 (0.0)	<b>0.5922(55)</b> : 0.797 (0.451)	<b>0.5969(91)</b> : 3.427 (0.008)	<b>0.5666(10)</b> : 4.48 (0.001)
1.8	<b>0.5840(12)</b> : 3.149 (0.008)	<b>0.6114(59)</b> : 0.41 (0.664)	<b>0.6210(99)</b> : 1.37 (0.242)	<b>0.5832(11)</b> : 3.371 (0.009)
1.5	<b>0.6089(17)</b> : 1.887 (0.093)	<b>0.6400(71)</b> : 0.233 (0.792)	<b>0.657(12)</b> : 0.272 (0.896)	<b>0.6084(15)</b> : 2.247 (0.061)

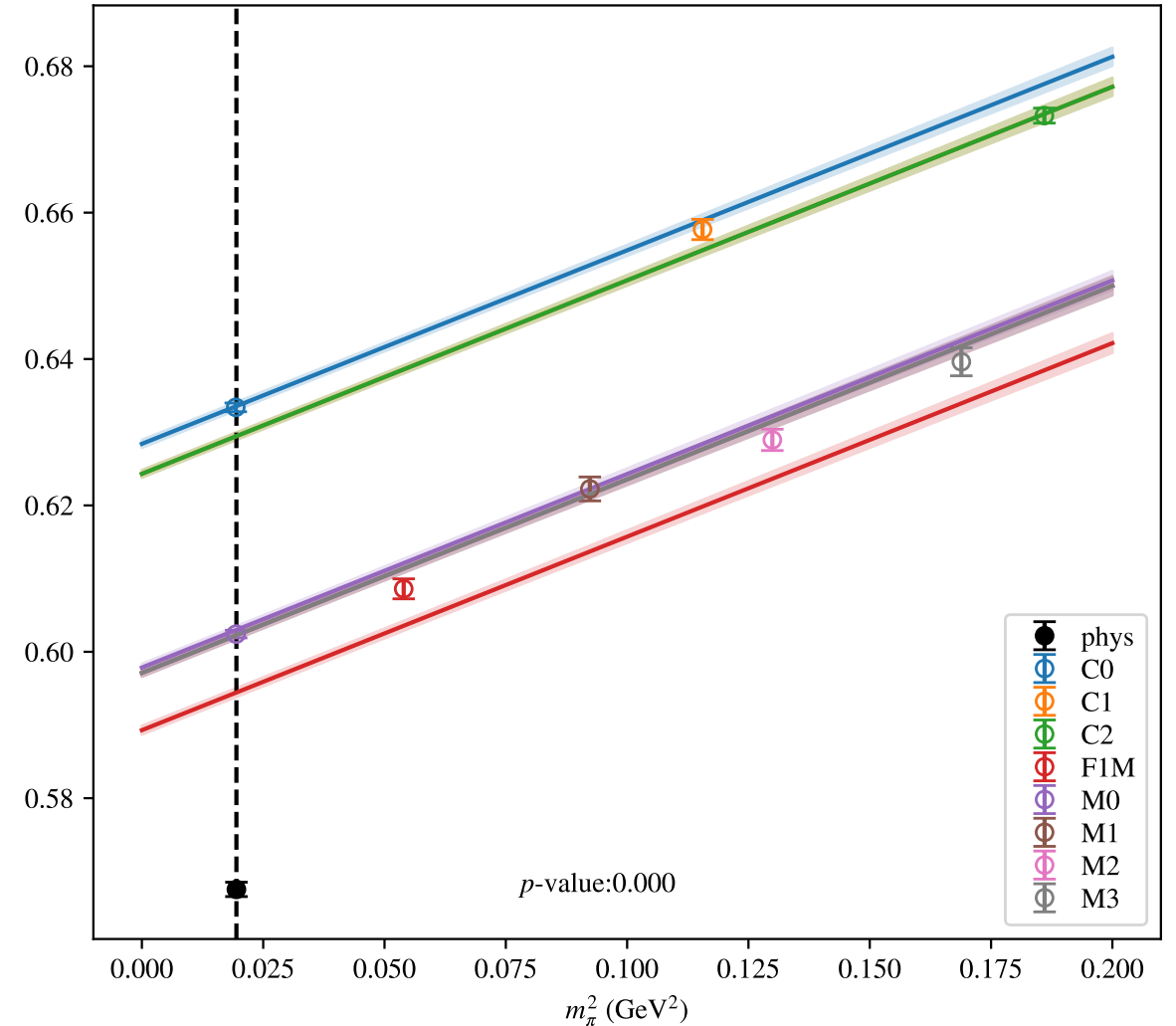
Table 4: Physical point value from chiral and continuum extrapolation at renormalisation scale  $\mu$ . Entries are **value(error)**:  $\chi^2/\text{DOF}$  ( $p$ -value).

$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

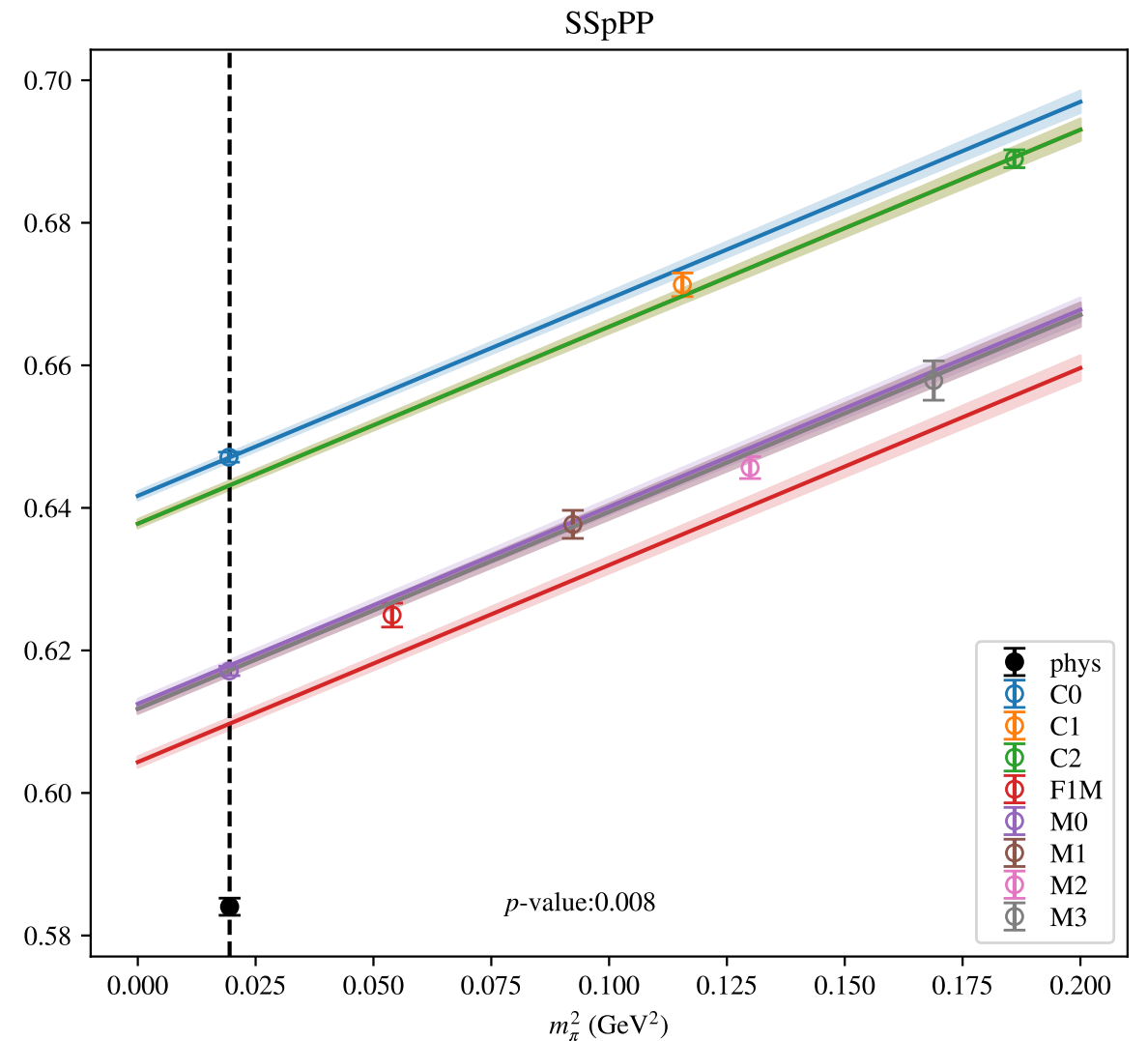
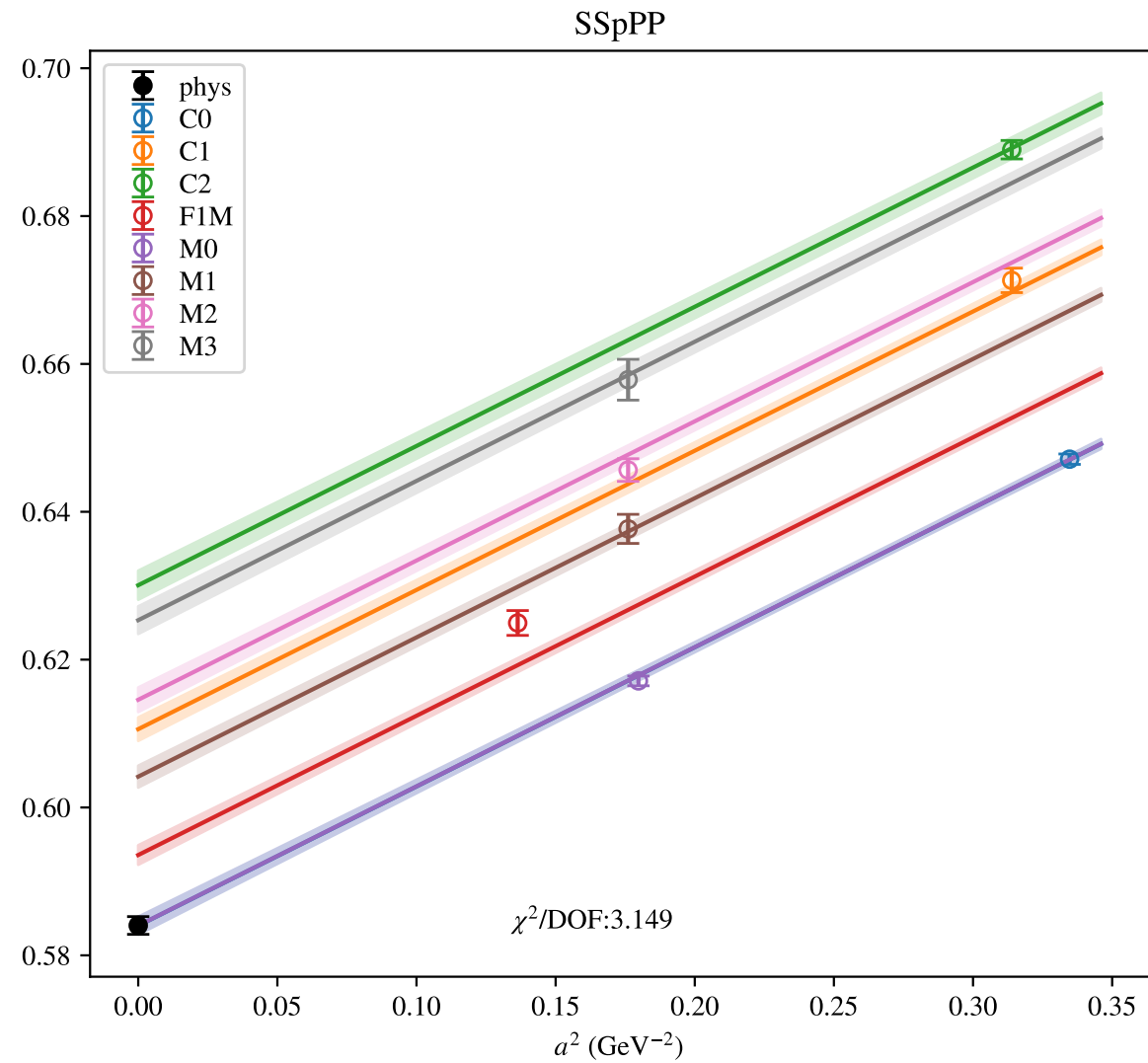
SSpPP



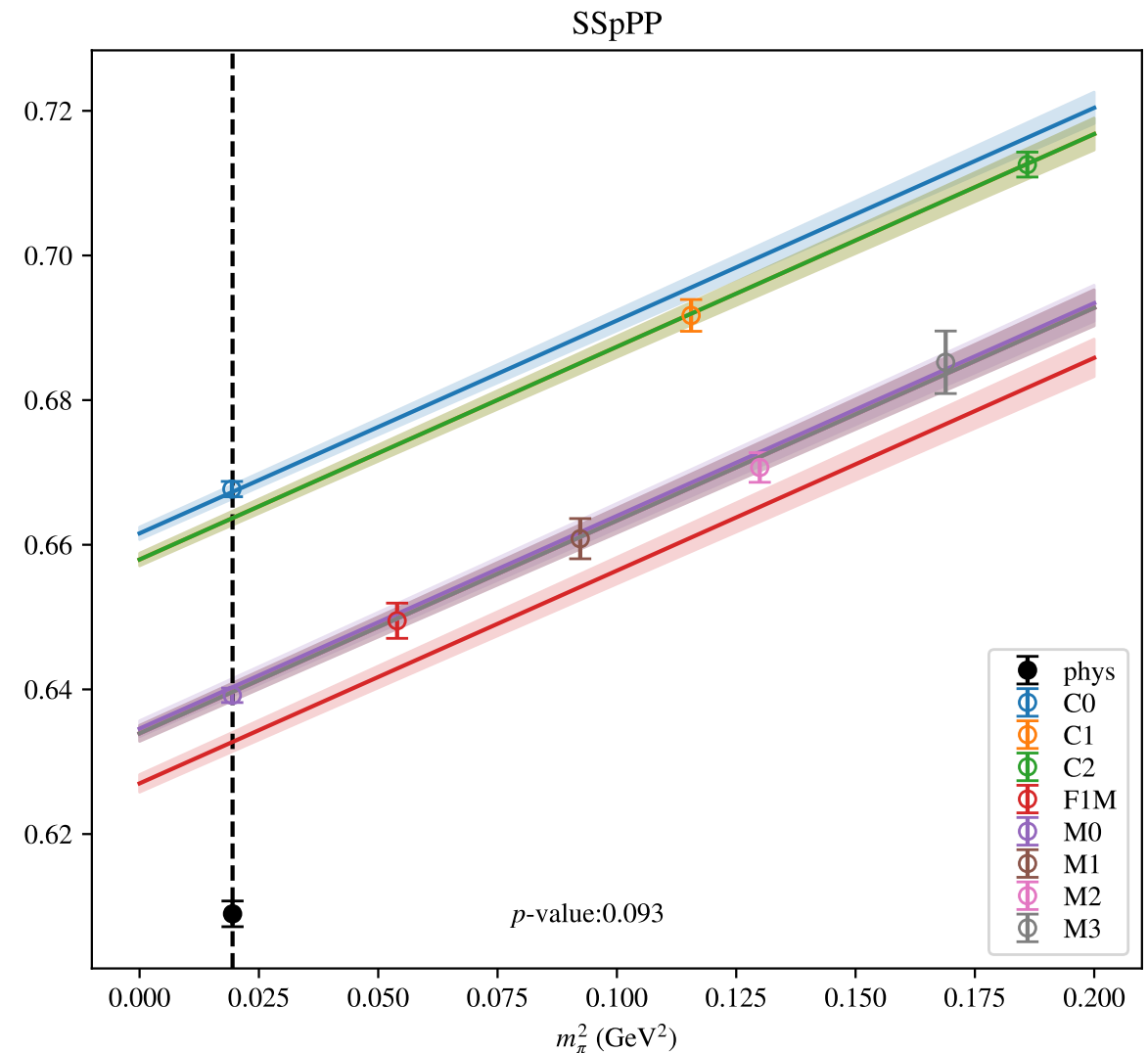
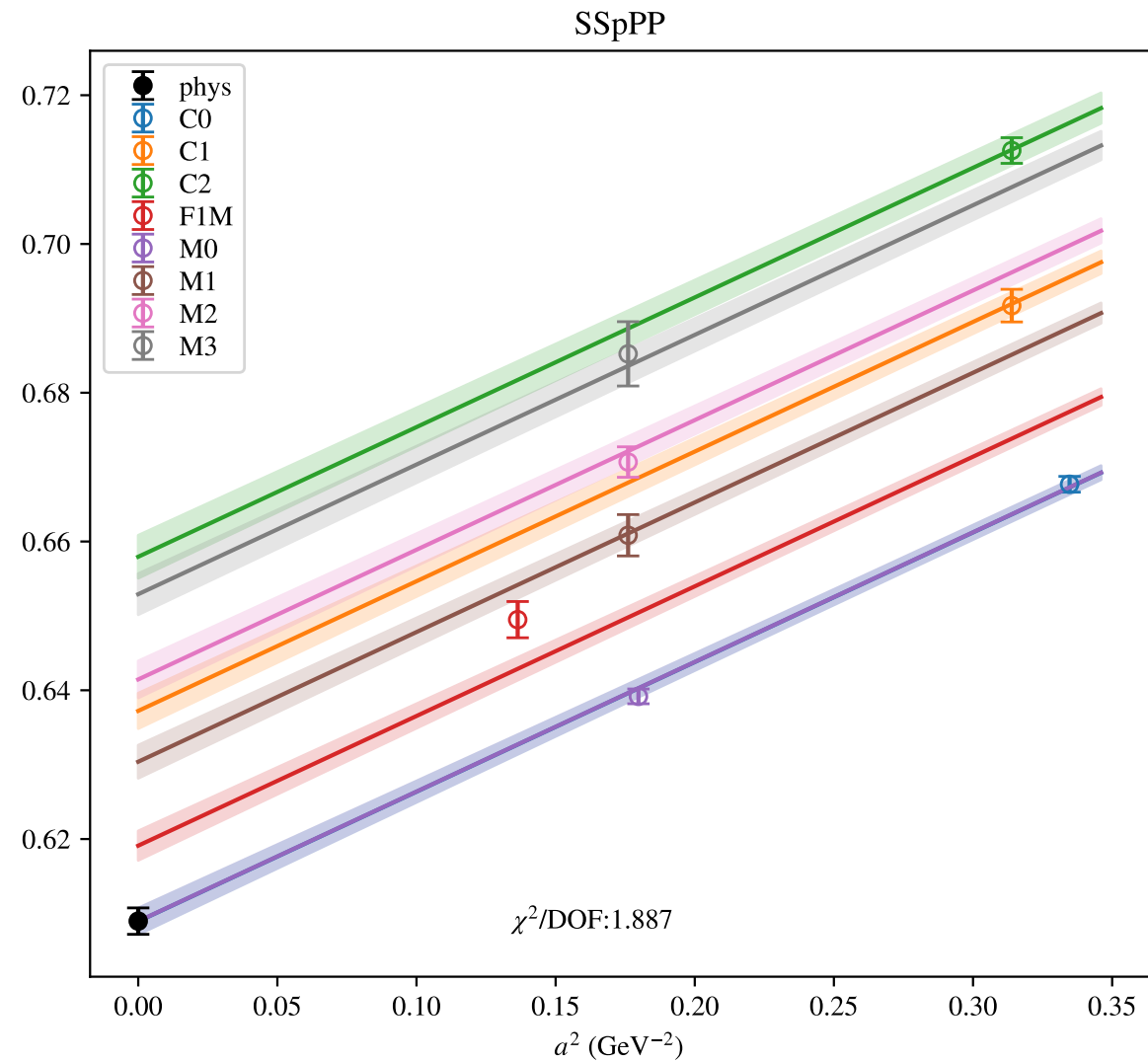
SSpPP



$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$

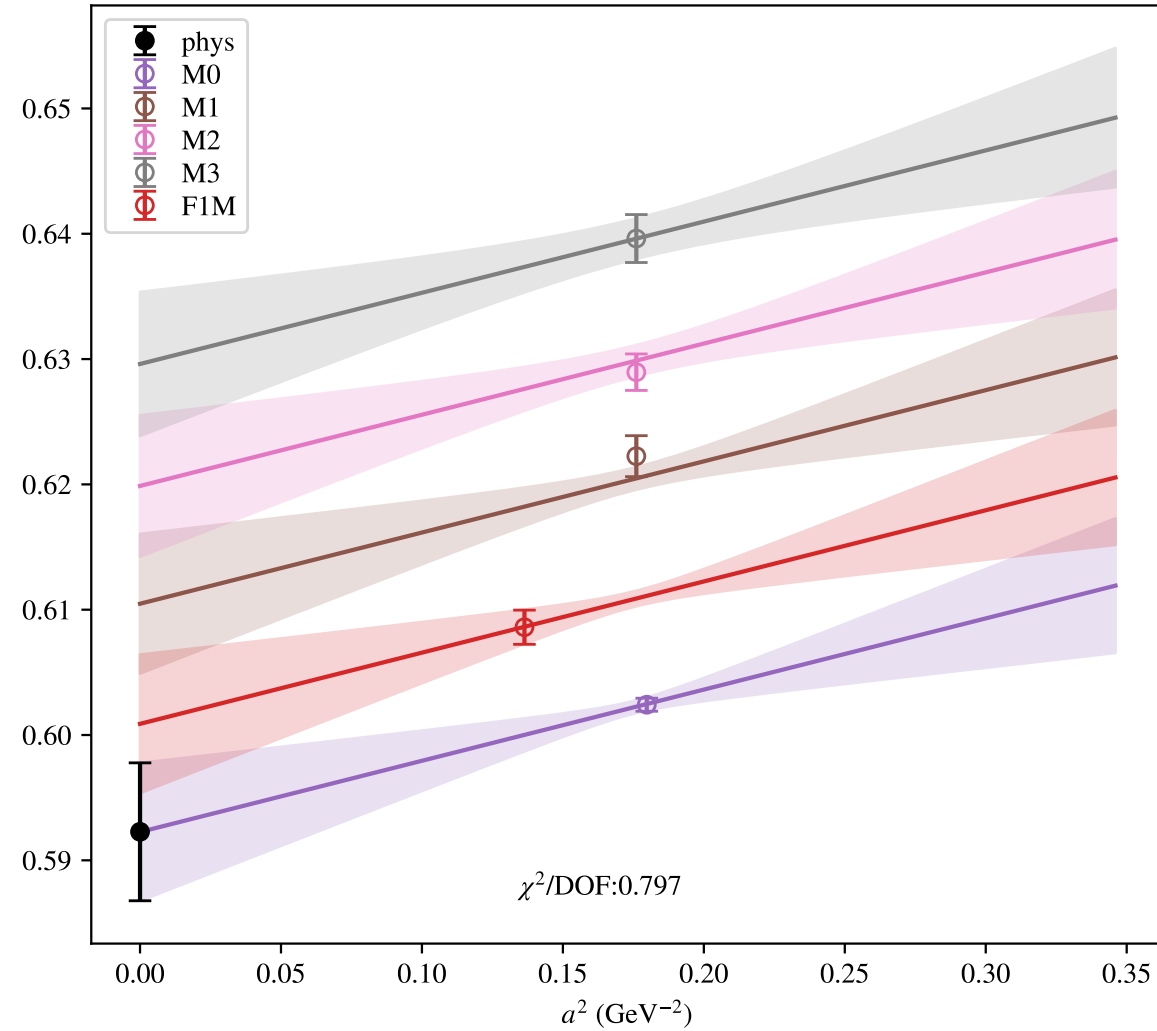


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

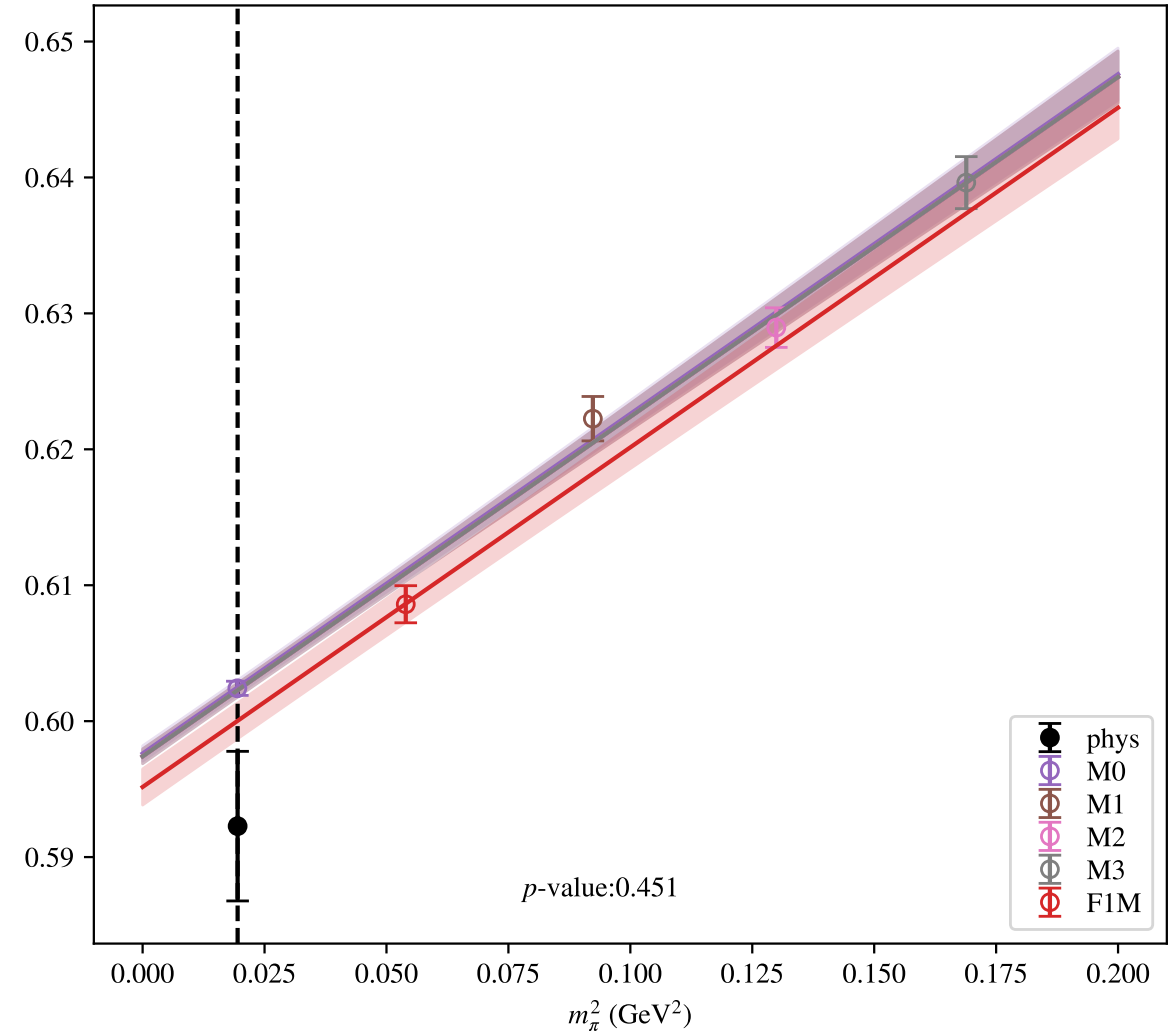


$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

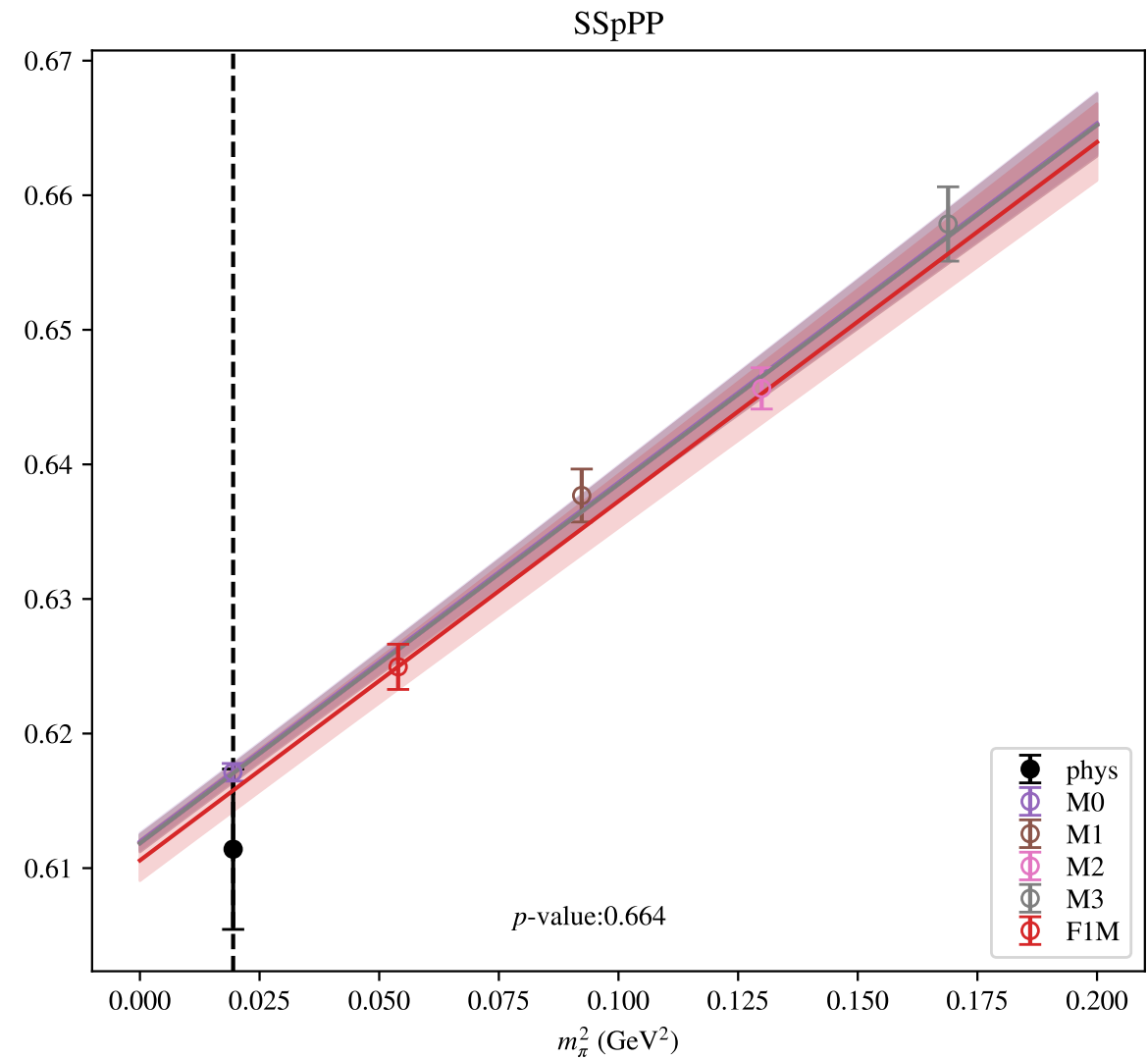
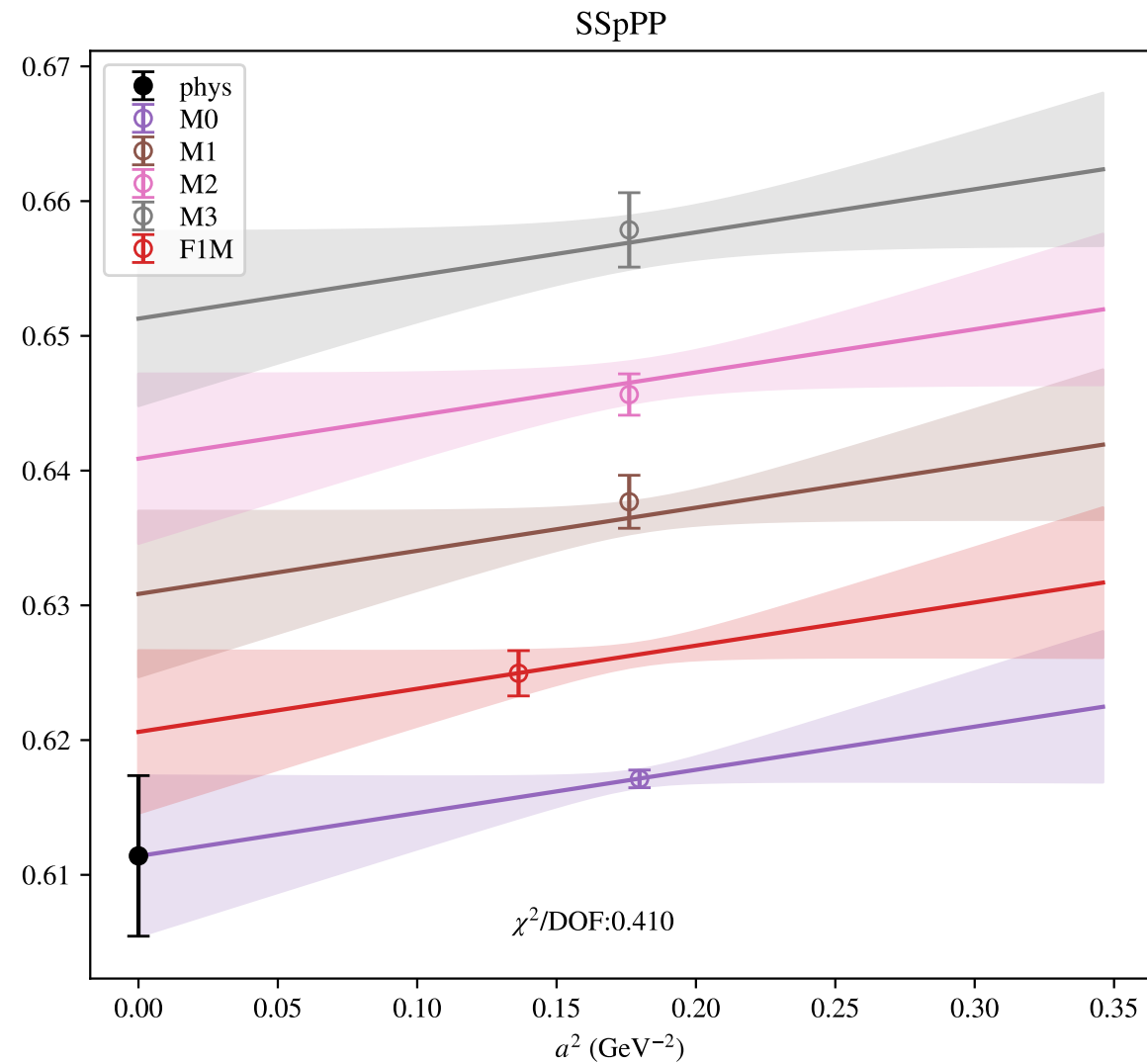
SSpPP



SSpPP

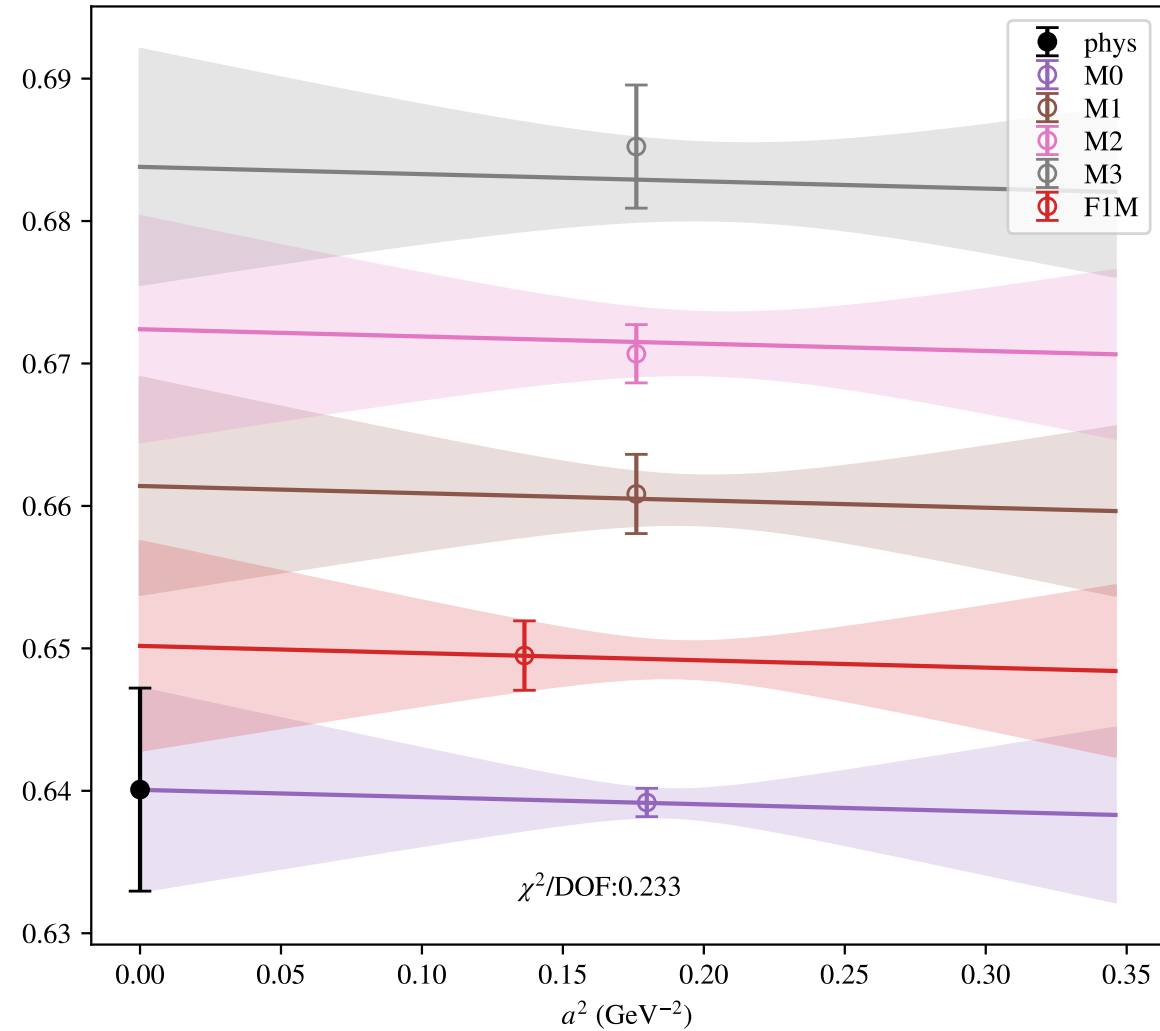


$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$

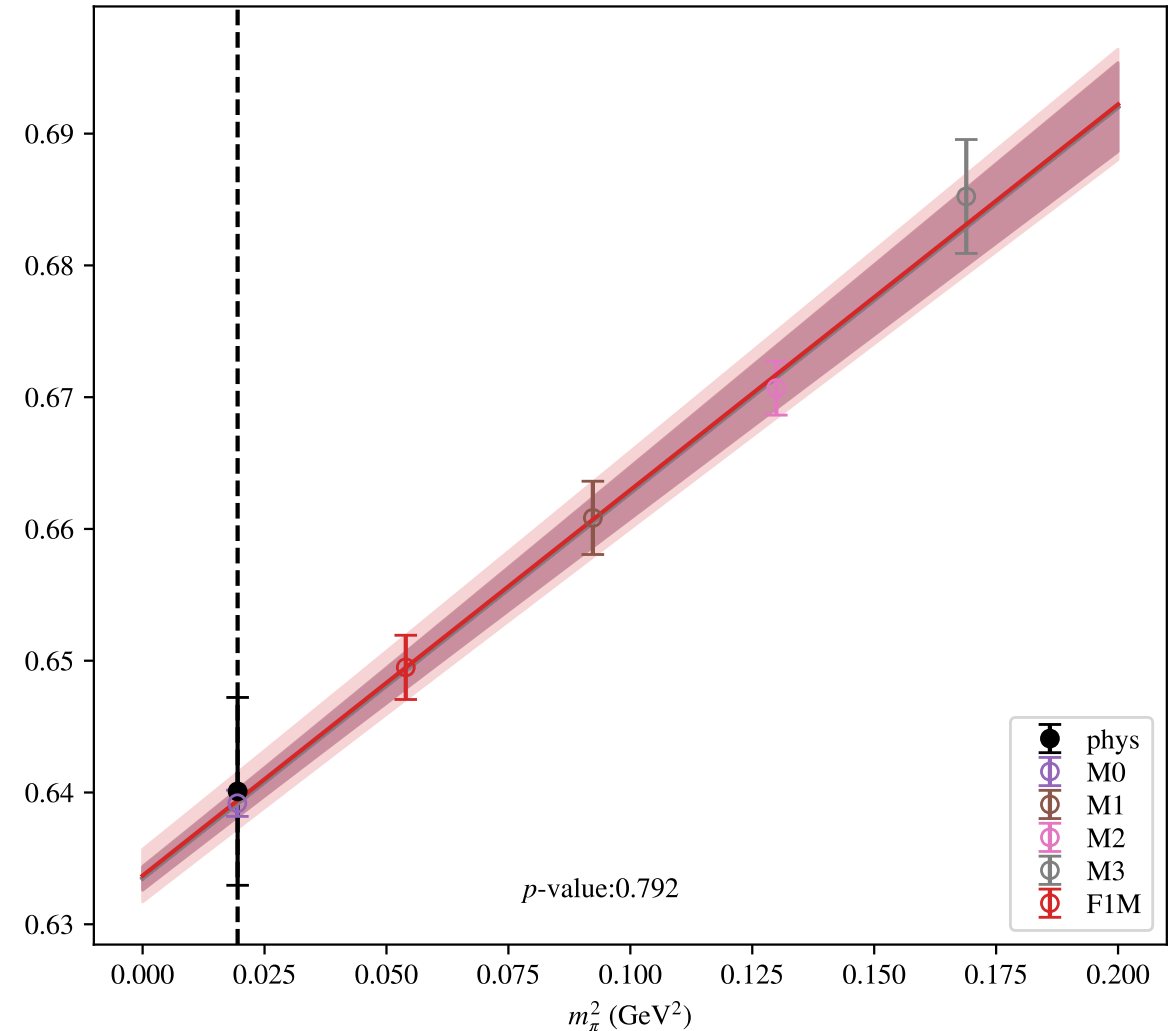


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

SSpPP

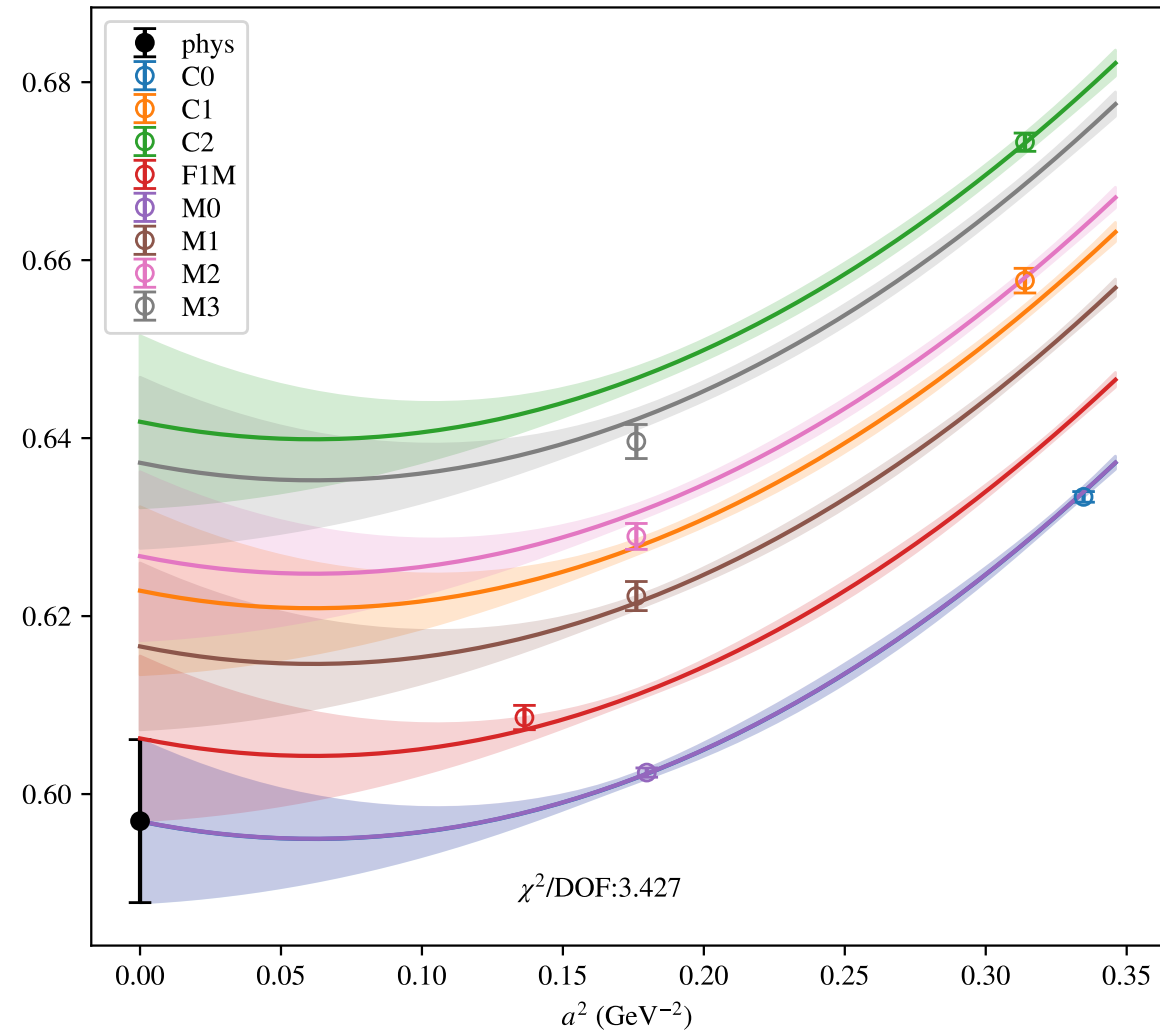


SSpPP

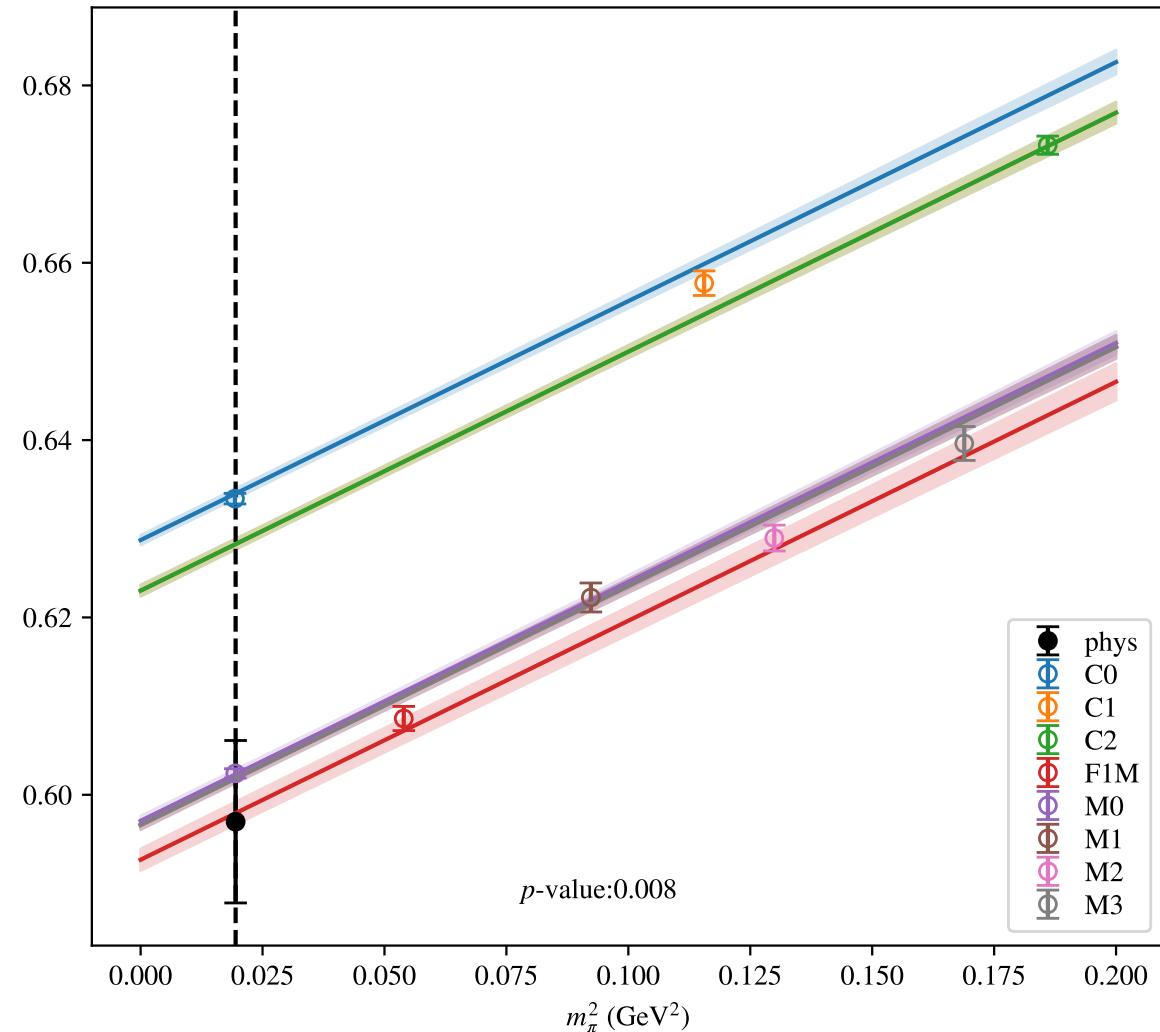


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

SSpPP



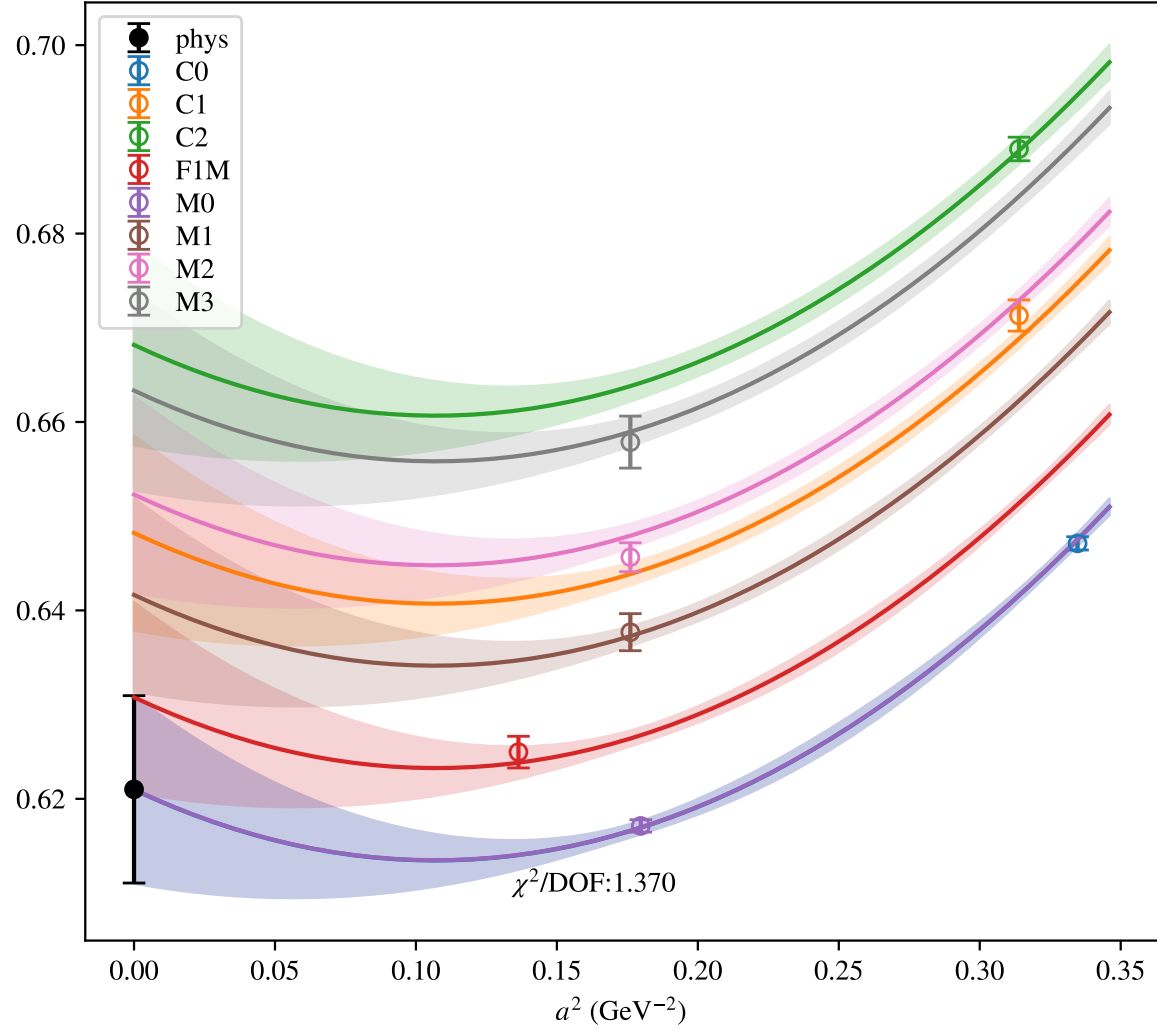
SSpPP



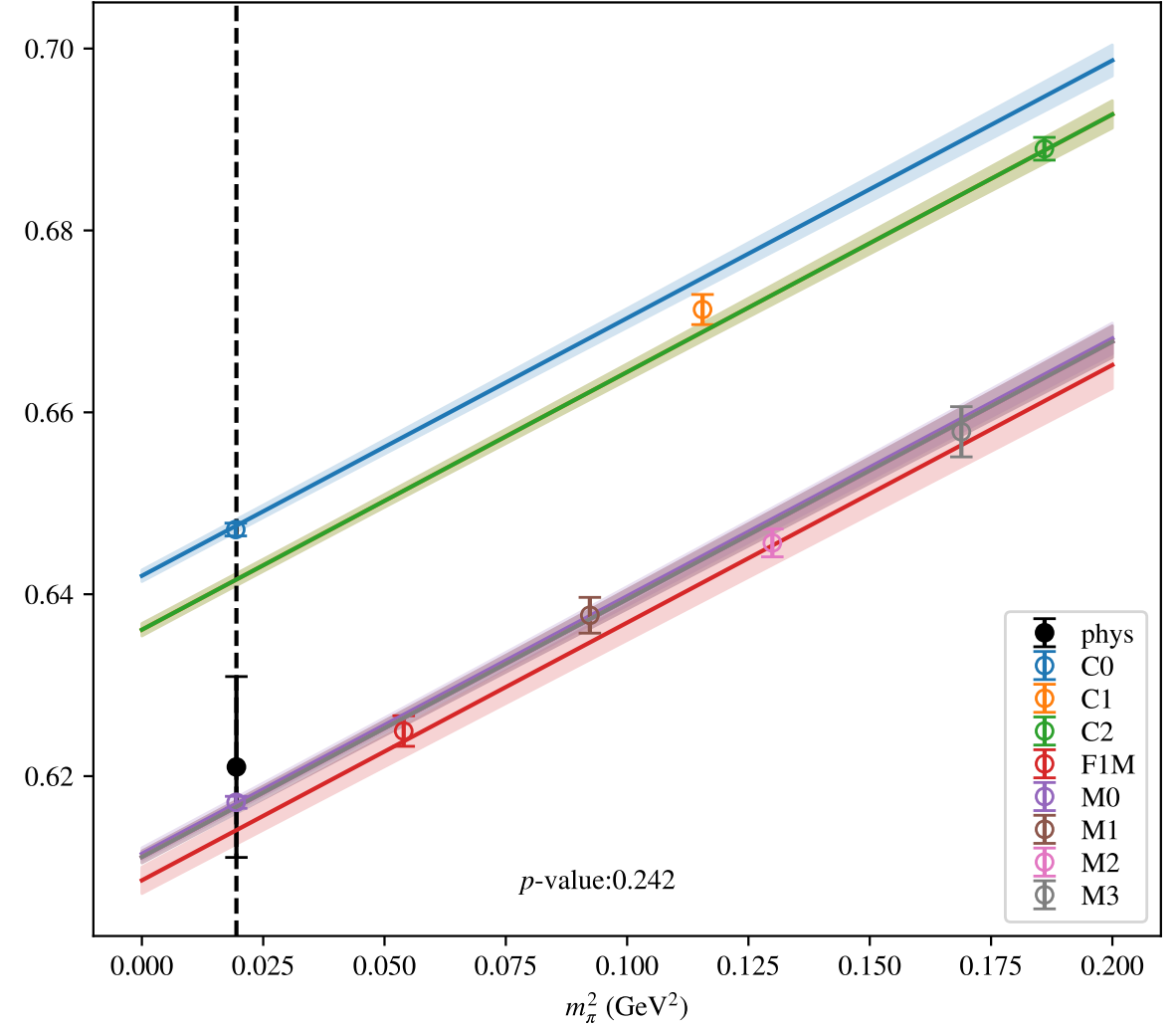


$$a^2, a^4, m_\pi^2, \mu = 1.8 \text{ GeV}$$

SSpPP

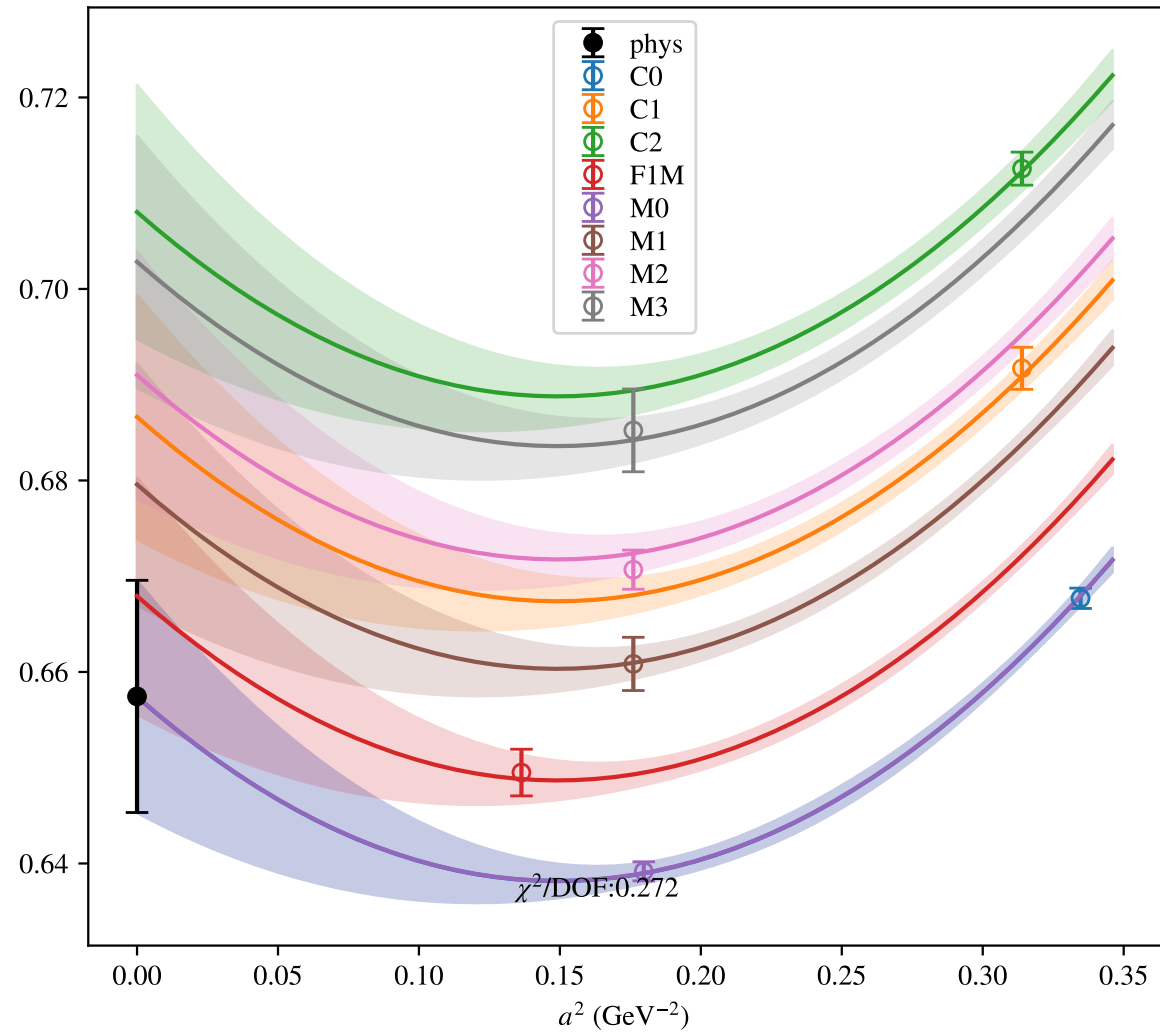


SSpPP

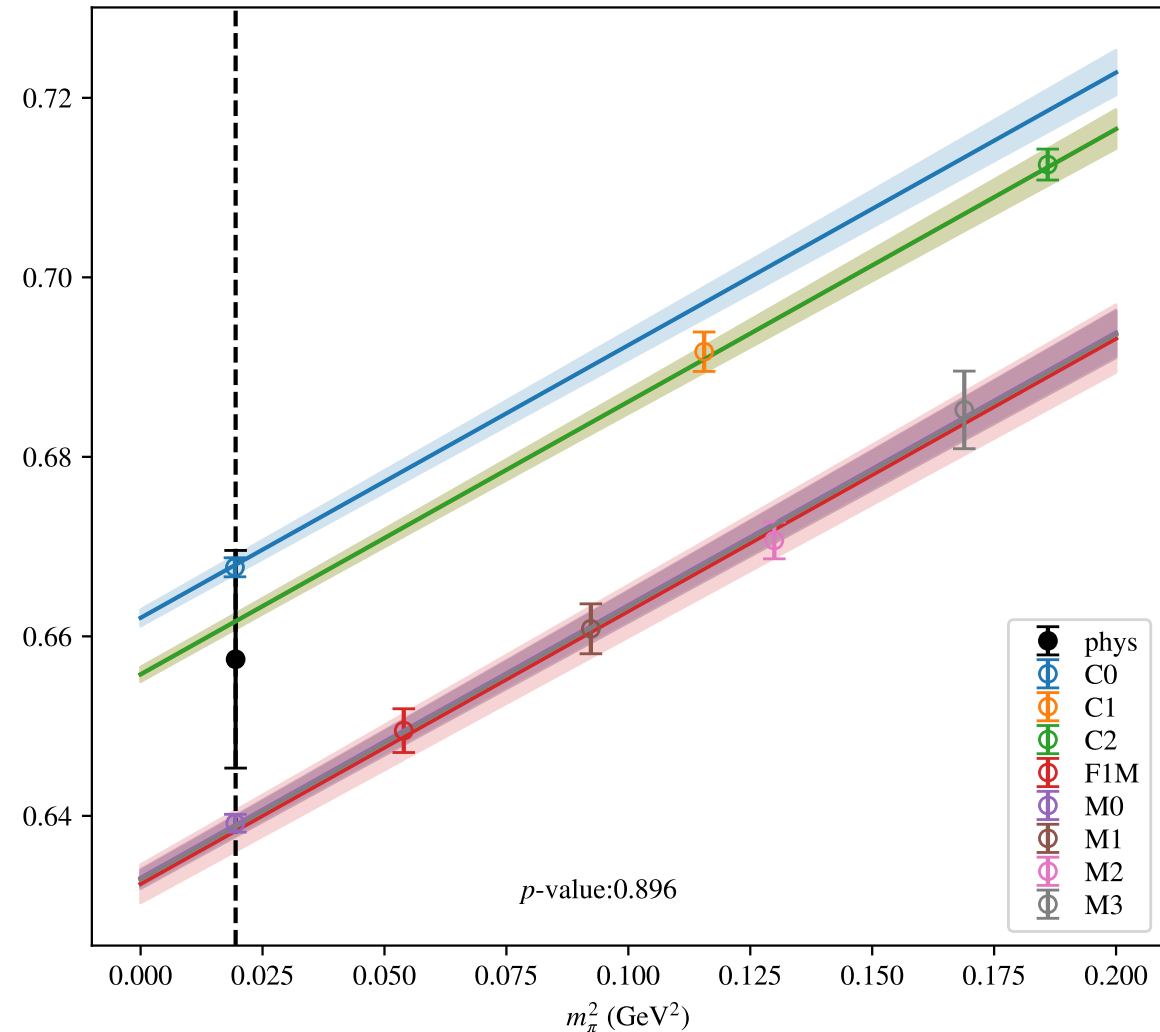


$$a^2, a^4, m_\pi^2, \mu = 1.5 \text{ GeV}$$

SSpPP

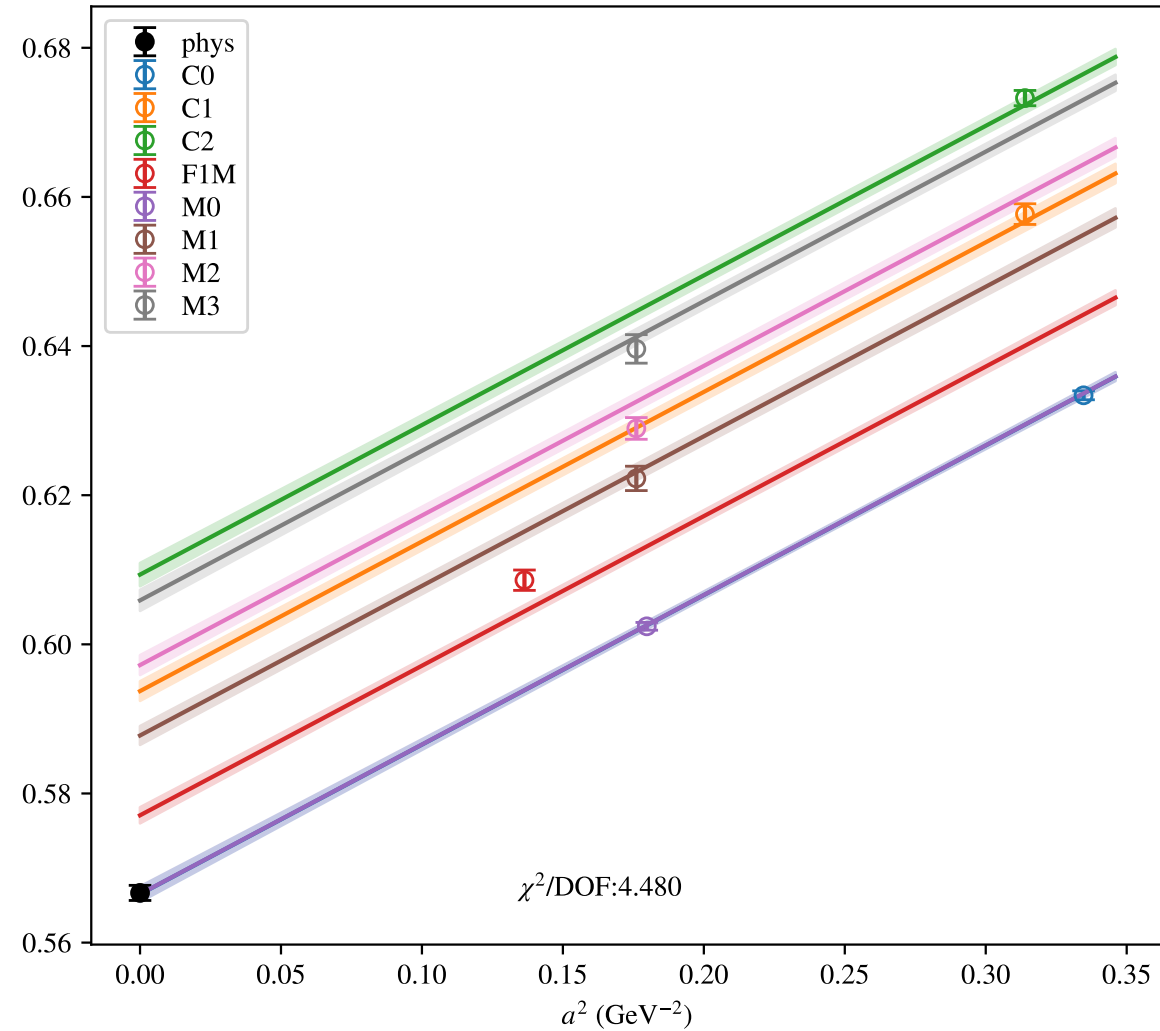


SSpPP

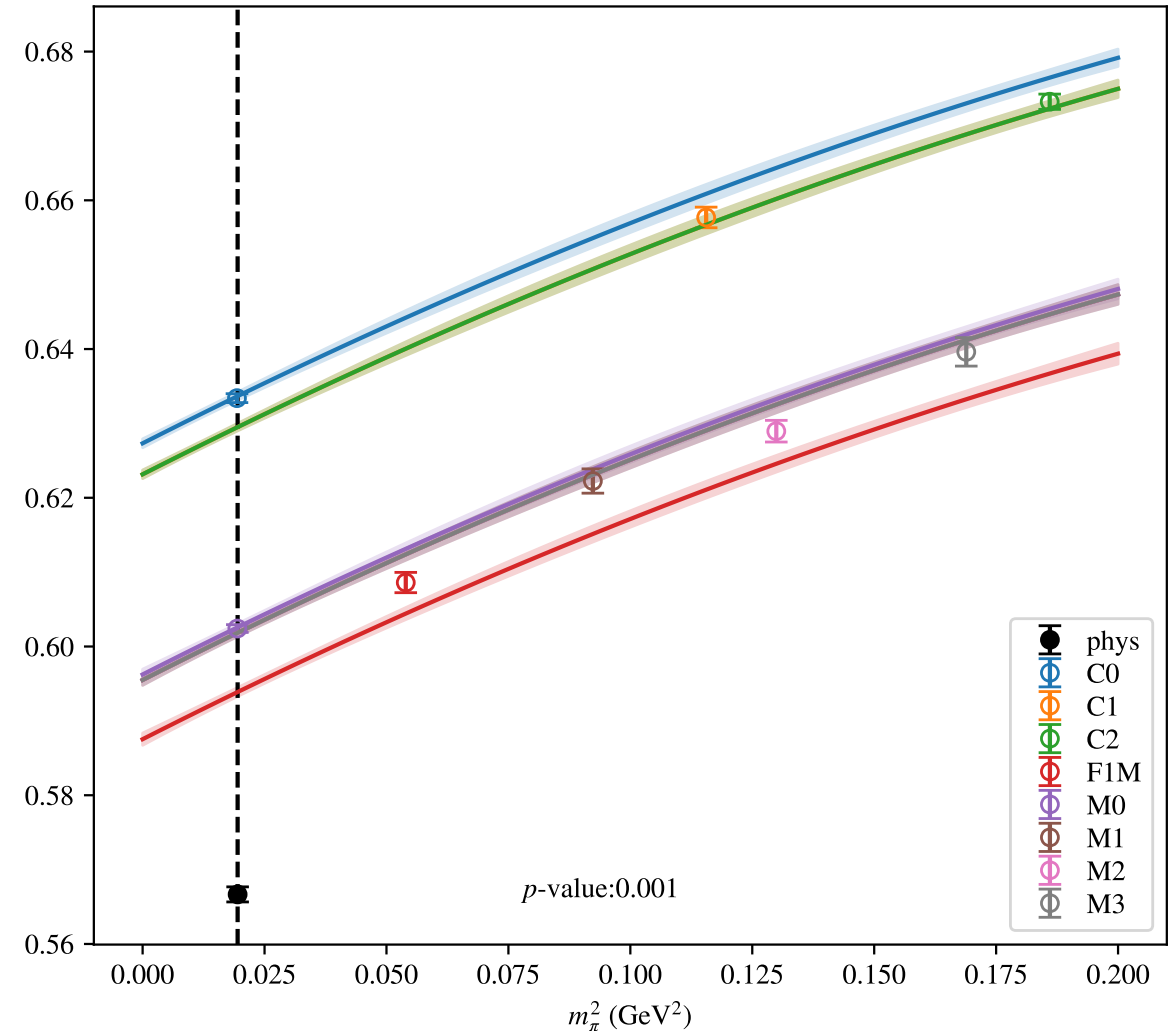


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

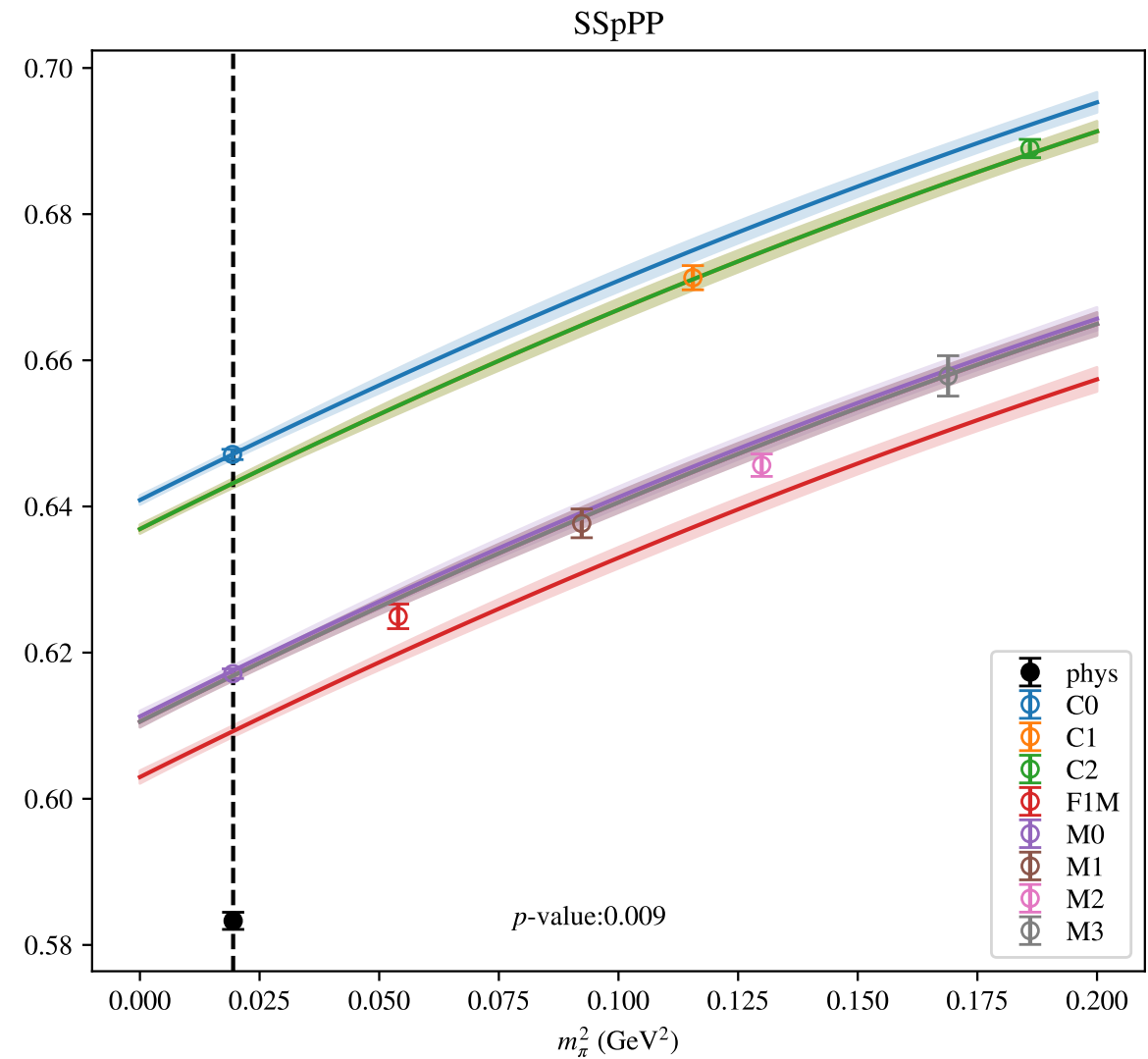
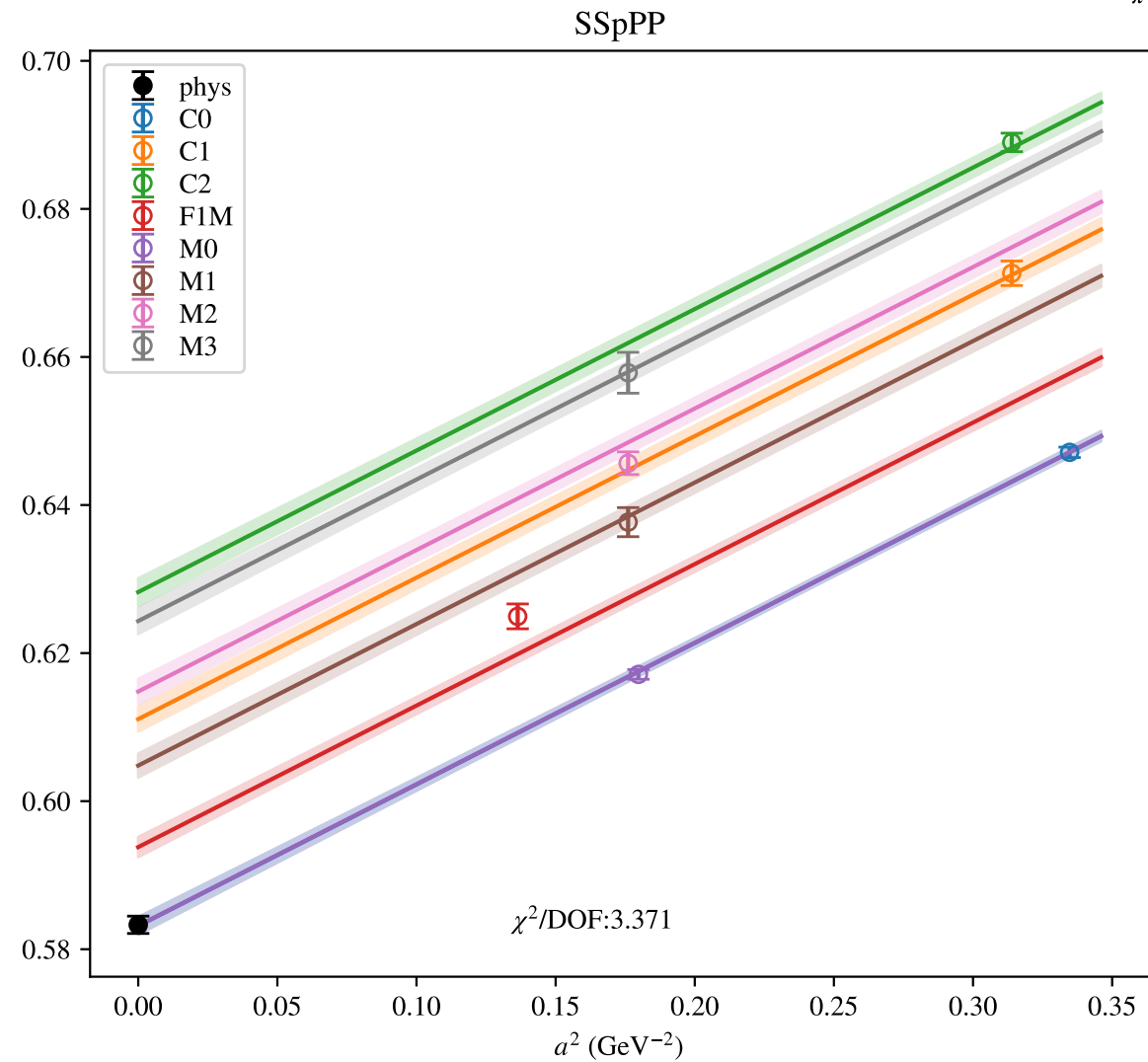
SSpPP



SSpPP



$$a^2, m_\pi^2, m_\pi^4, \mu = 1.8 \text{ GeV}$$



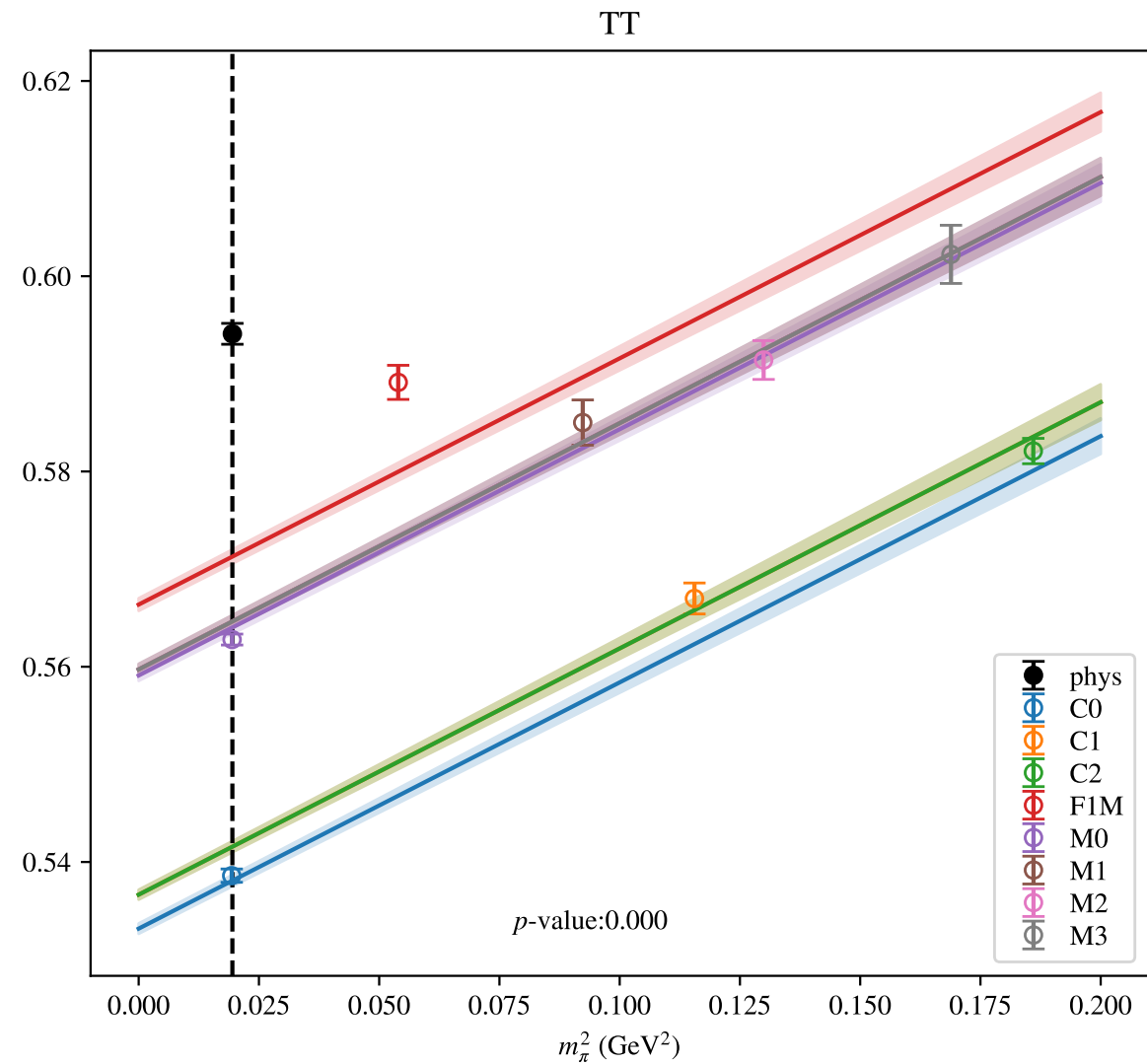
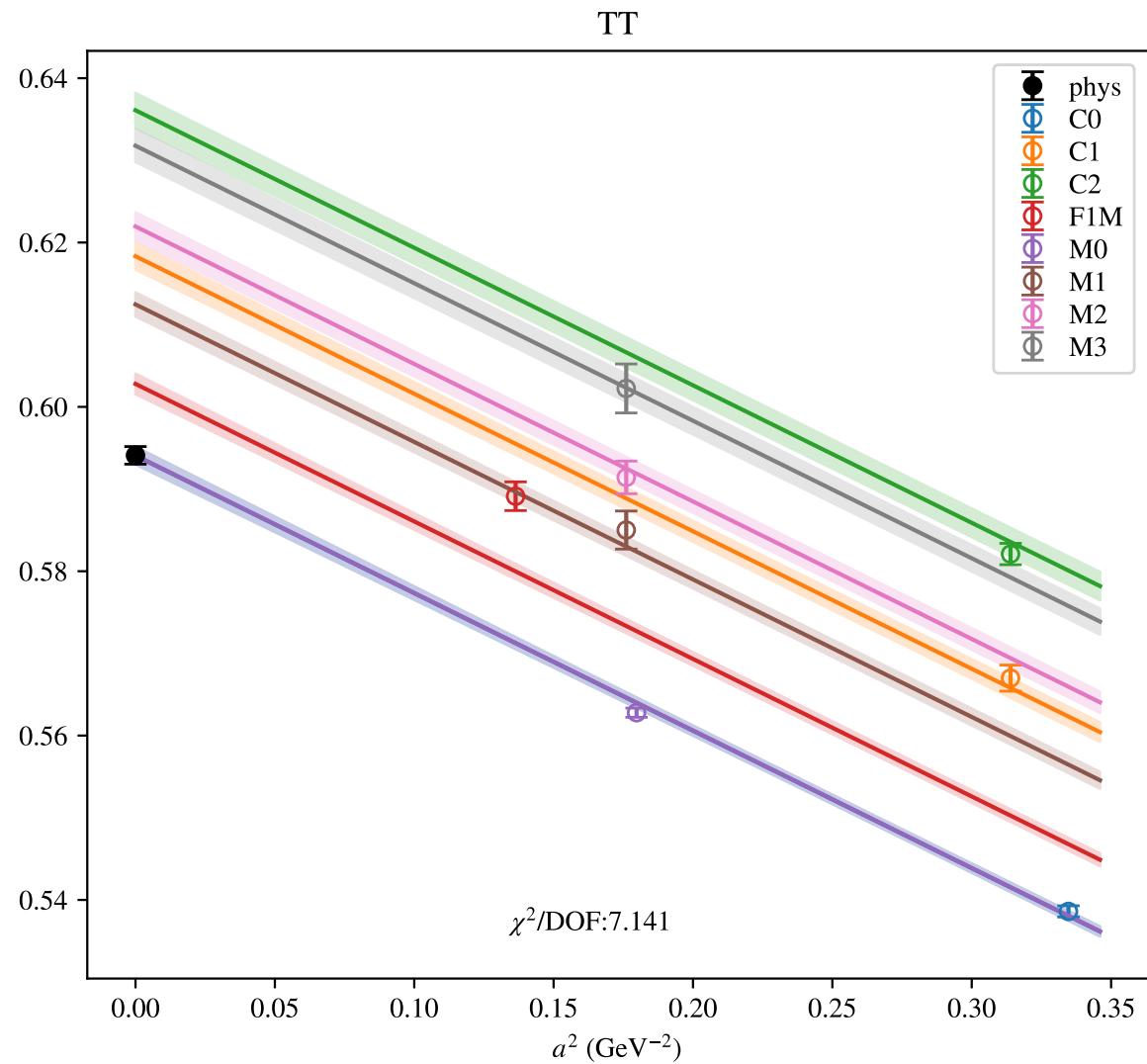


## 5 TT

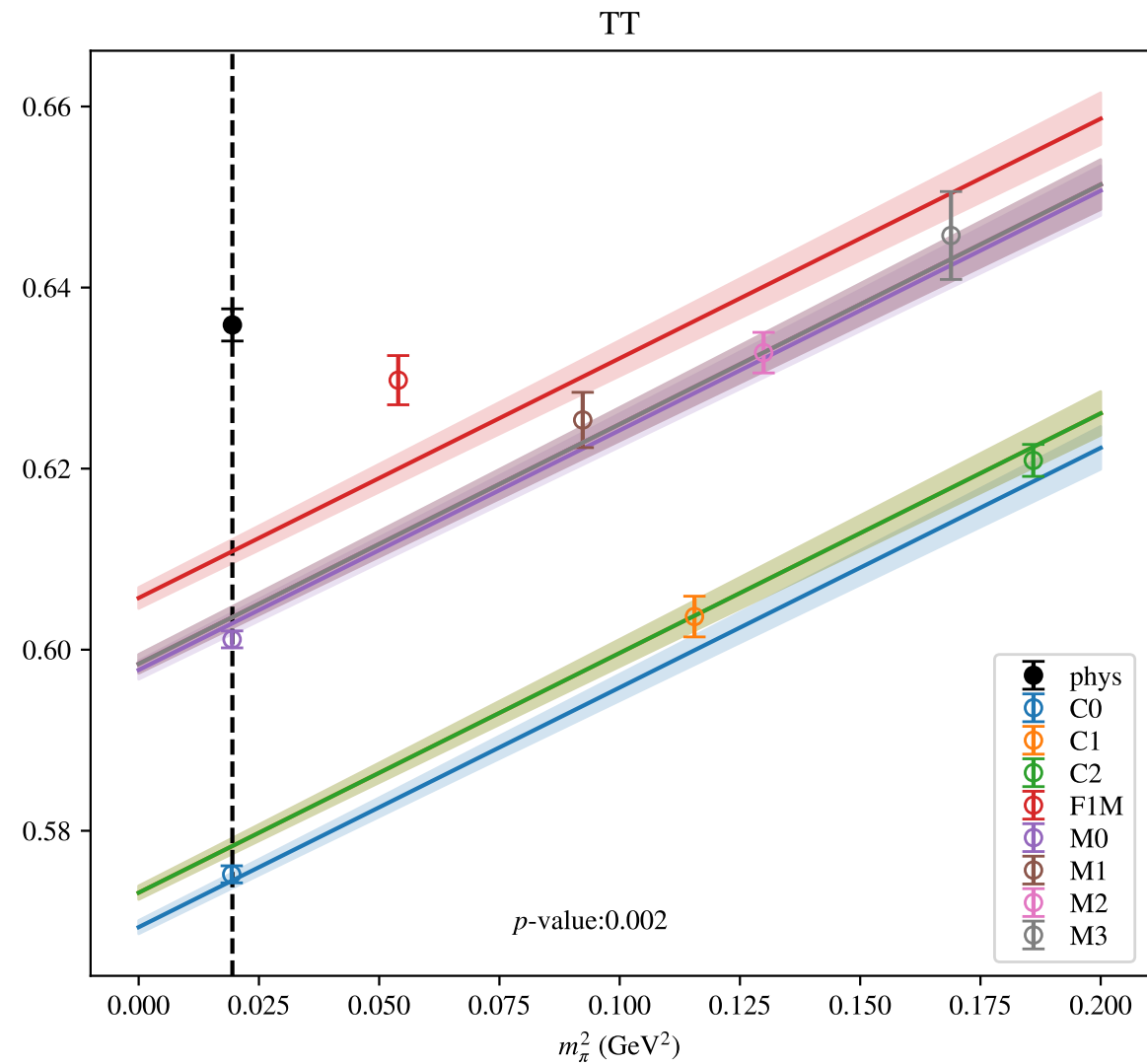
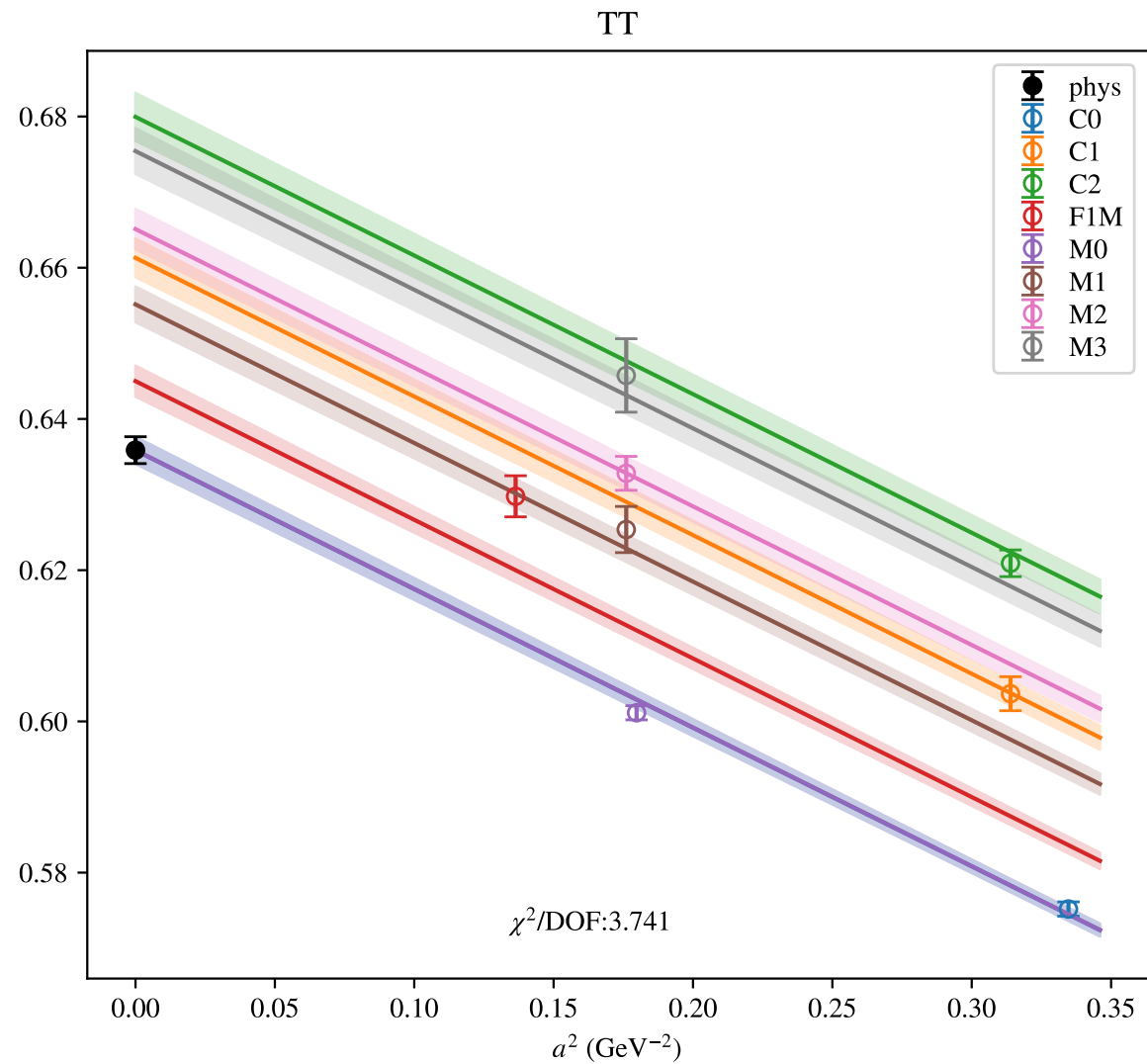
$\mu$ (GeV)	$a^2, m_\pi^2$	$a^2, m_\pi^2$ no C	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$
2.0	<b>0.5940(10)</b> : 7.141 (0.0)	<b>0.6355(60)</b> : 0.563 (0.569)	<b>0.658(10)</b> : 1.281 (0.275)	<b>0.5930(10)</b> : 6.036 (0.0)
1.8	<b>0.6358(17)</b> : 3.741 (0.002)	<b>0.6796(79)</b> : 0.363 (0.695)	<b>0.711(13)</b> : 0.386 (0.819)	<b>0.6345(15)</b> : 3.542 (0.007)
1.5	<b>0.6990(32)</b> : 2.017 (0.073)	<b>0.745(12)</b> : 0.194 (0.823)	<b>0.791(20)</b> : 0.255 (0.907)	<b>0.6975(26)</b> : 2.13 (0.074)

Table 5: Physical point value from chiral and continuum extrapolation at renormalisation scale  $\mu$ . Entries are **value(error)**:  $\chi^2/\text{DOF}$  ( $p$ -value).

$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$



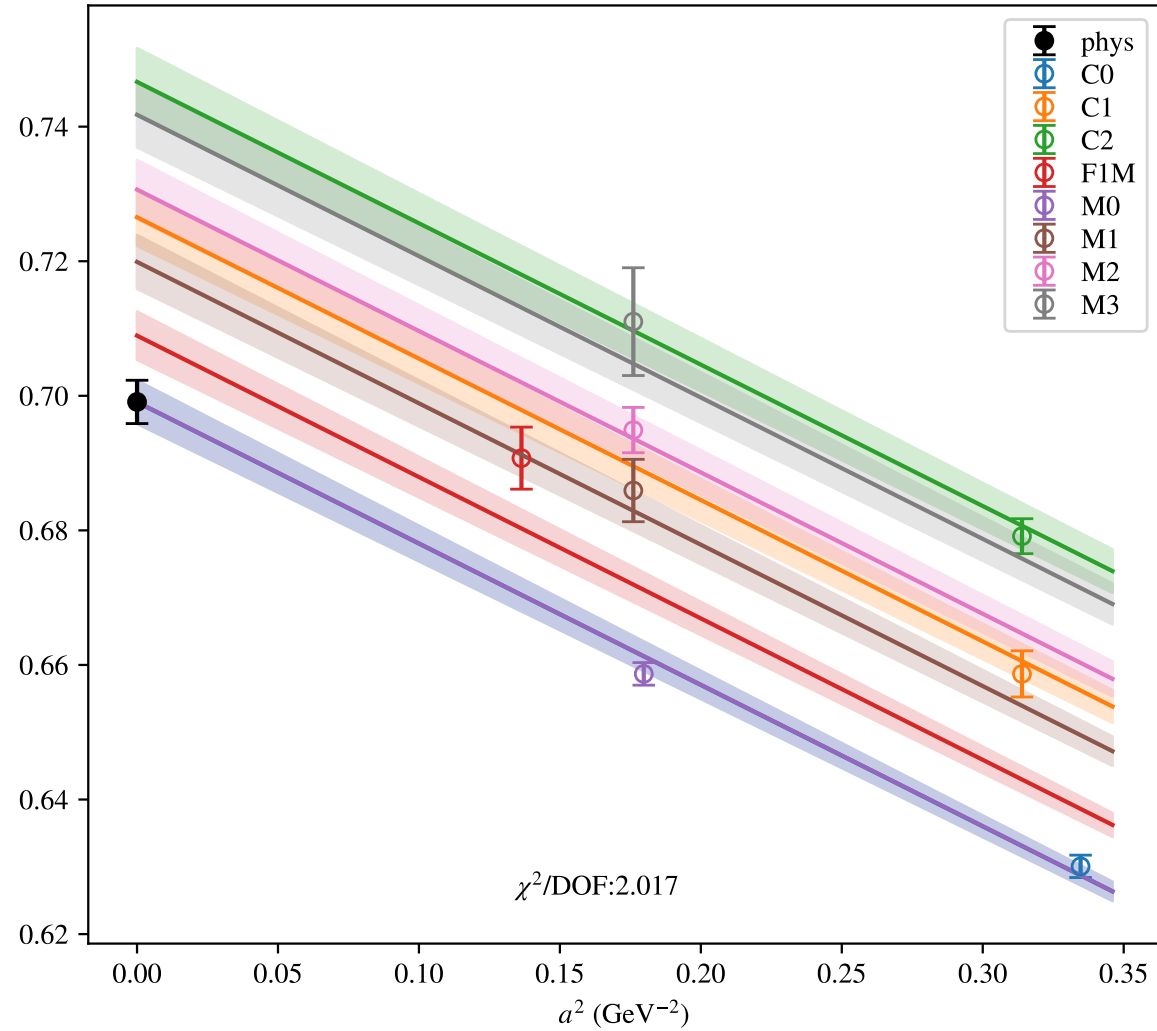
$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$



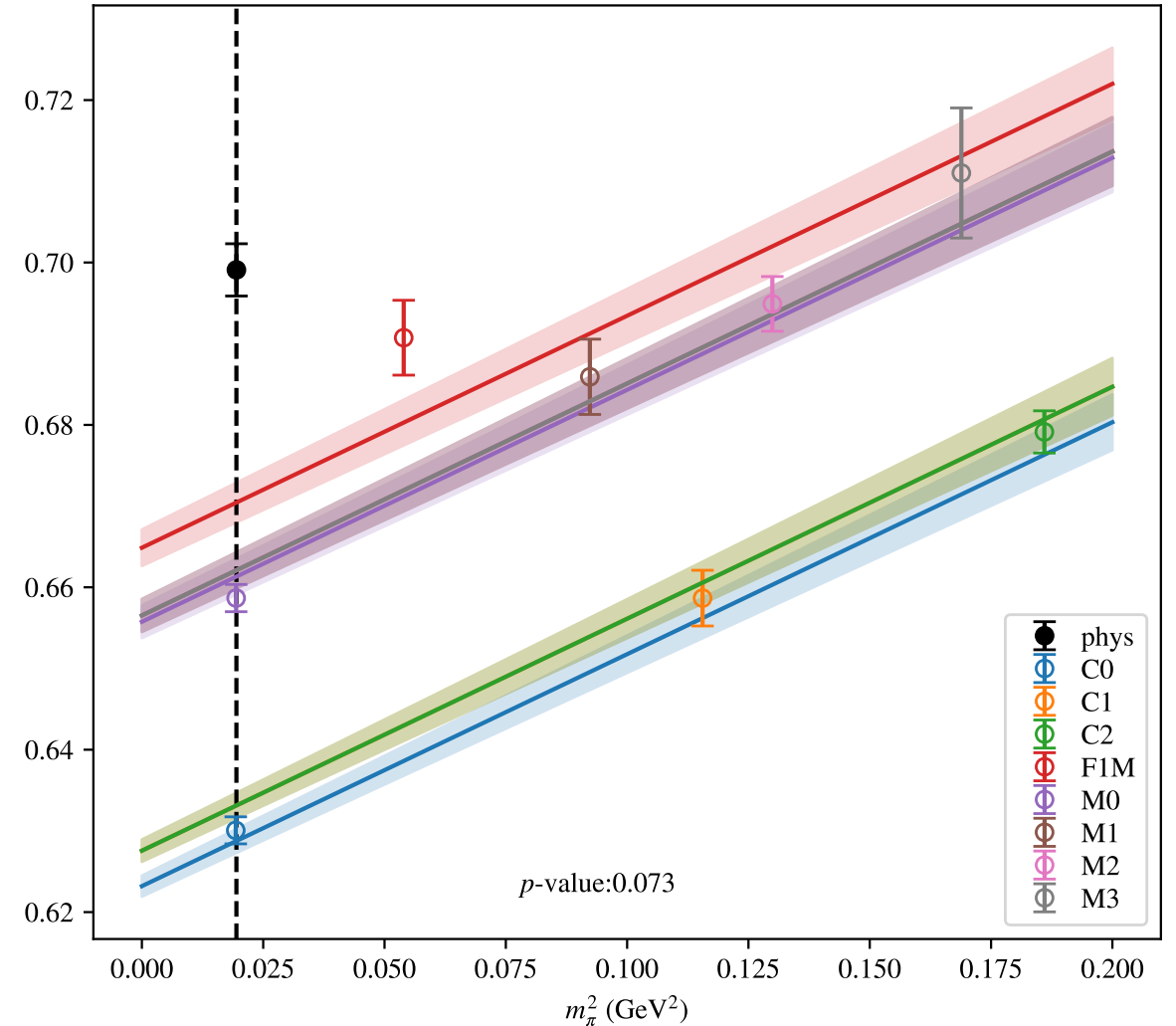


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

TT

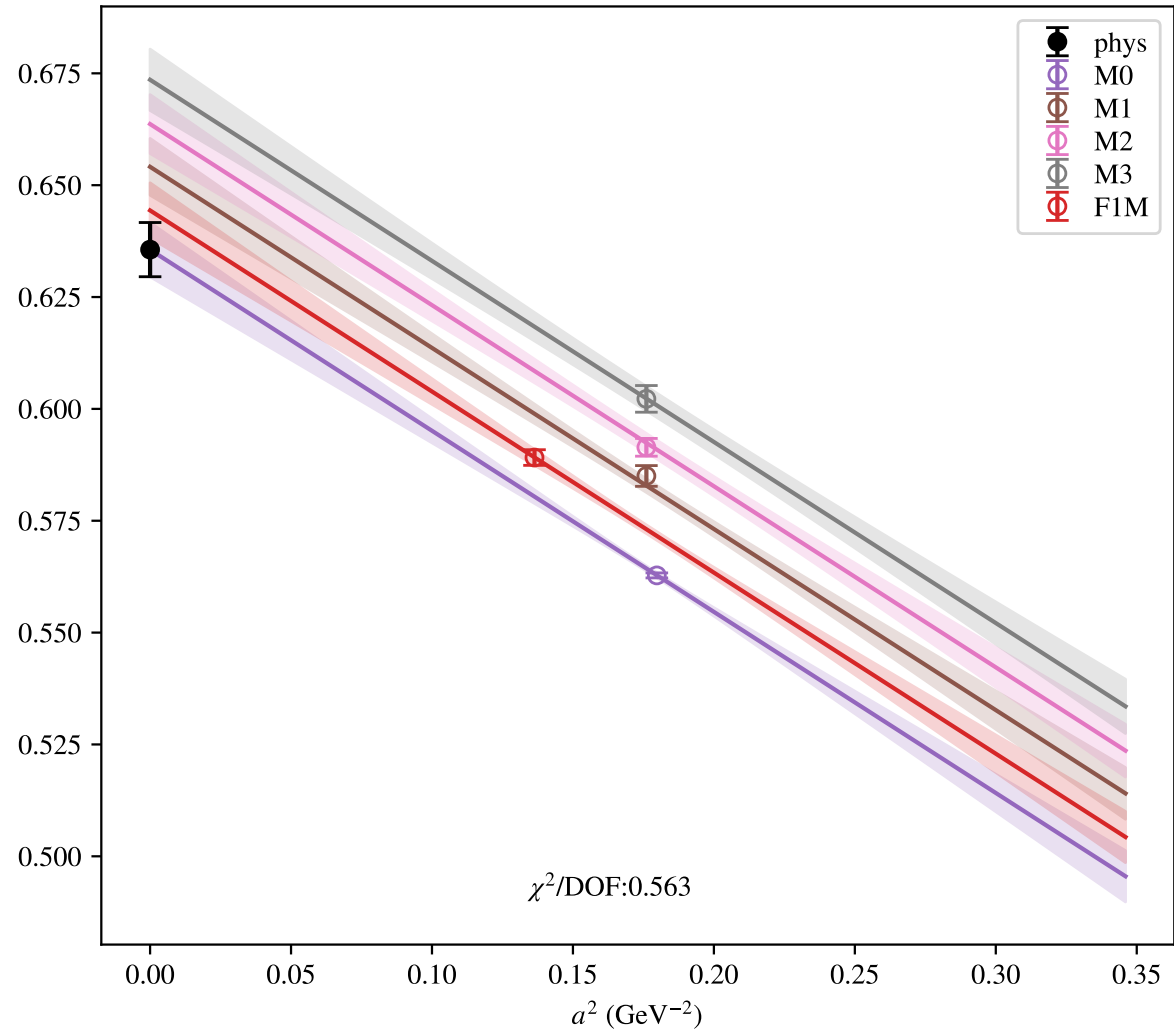


TT

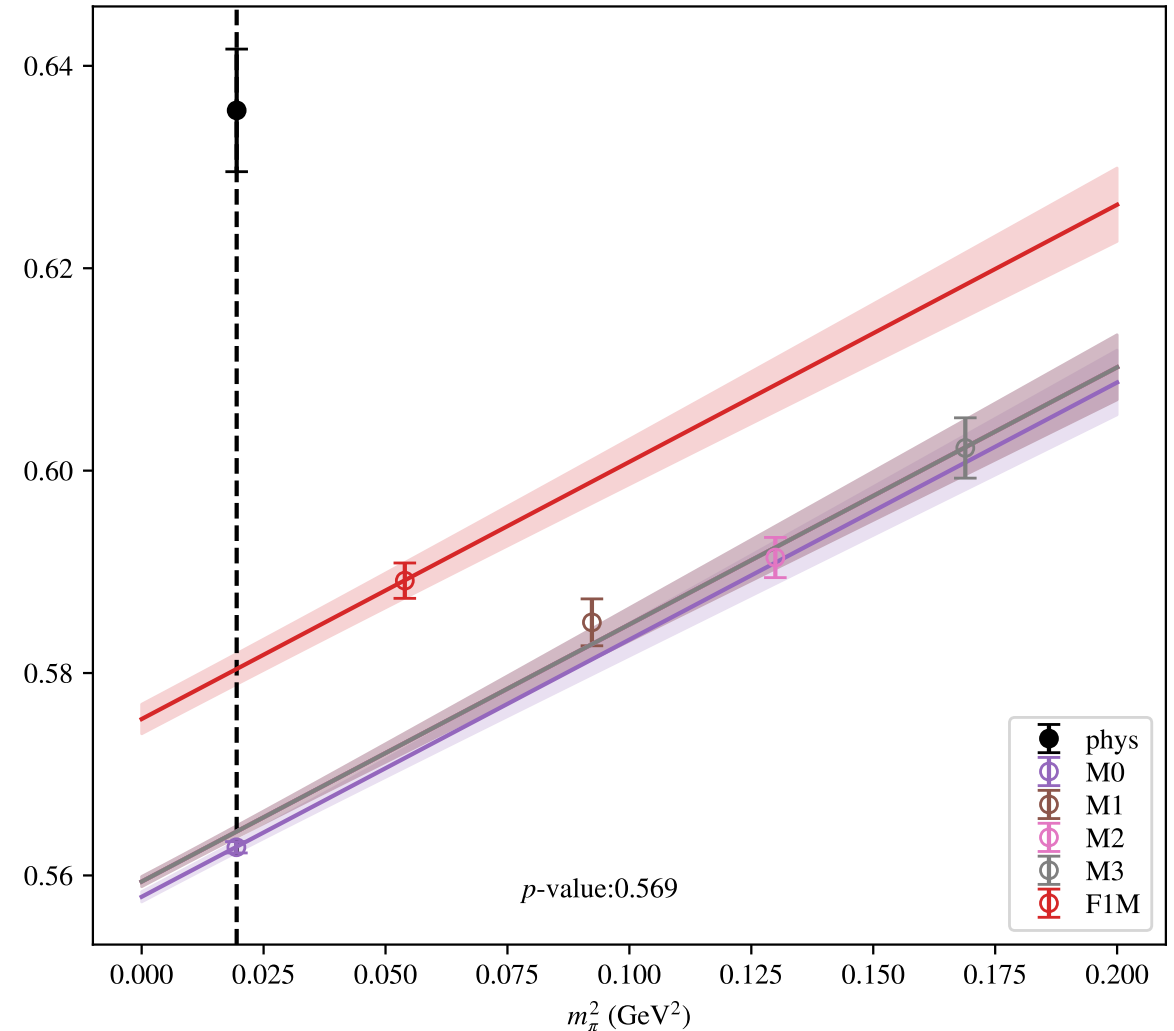


$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

TT

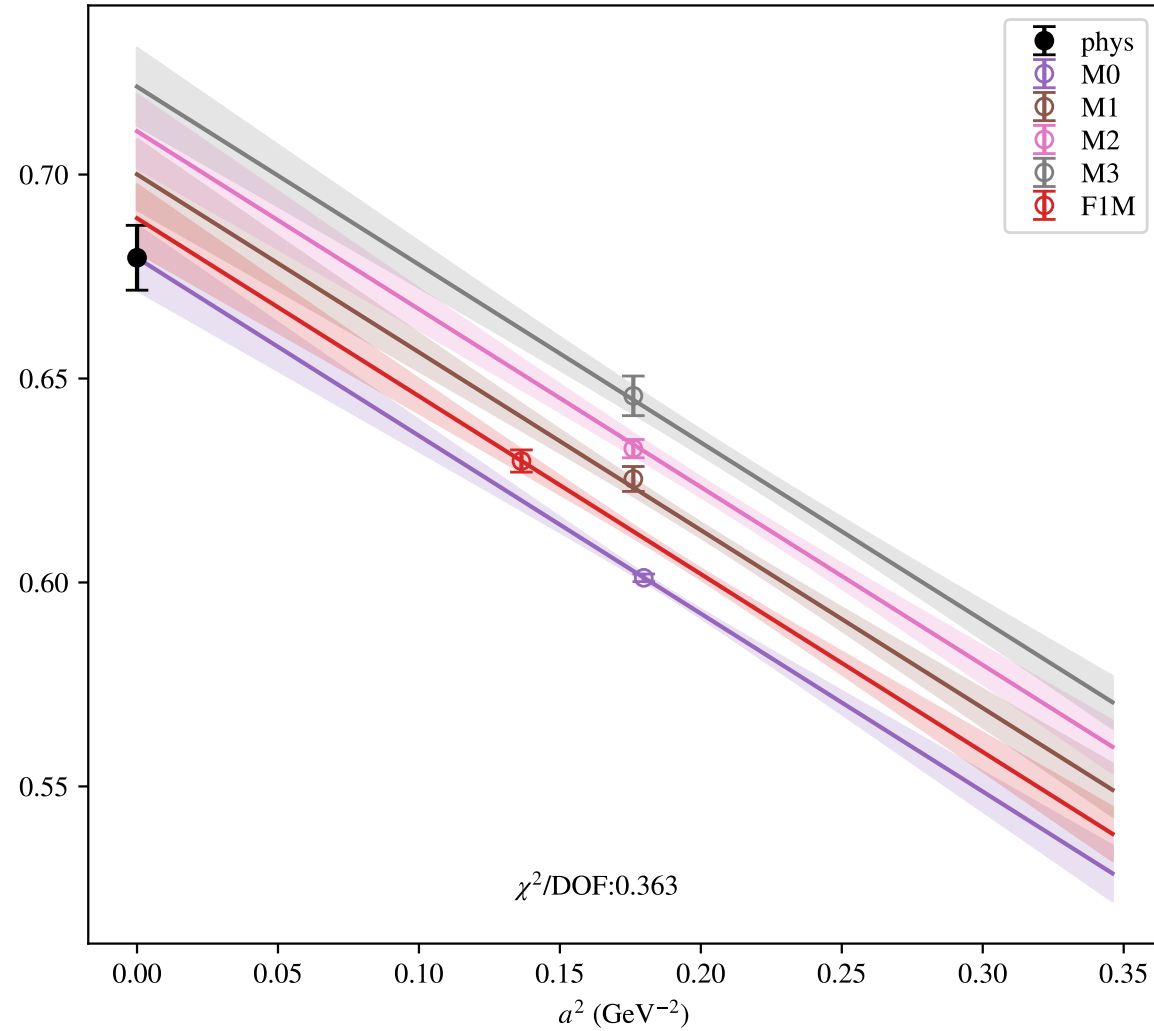


TT

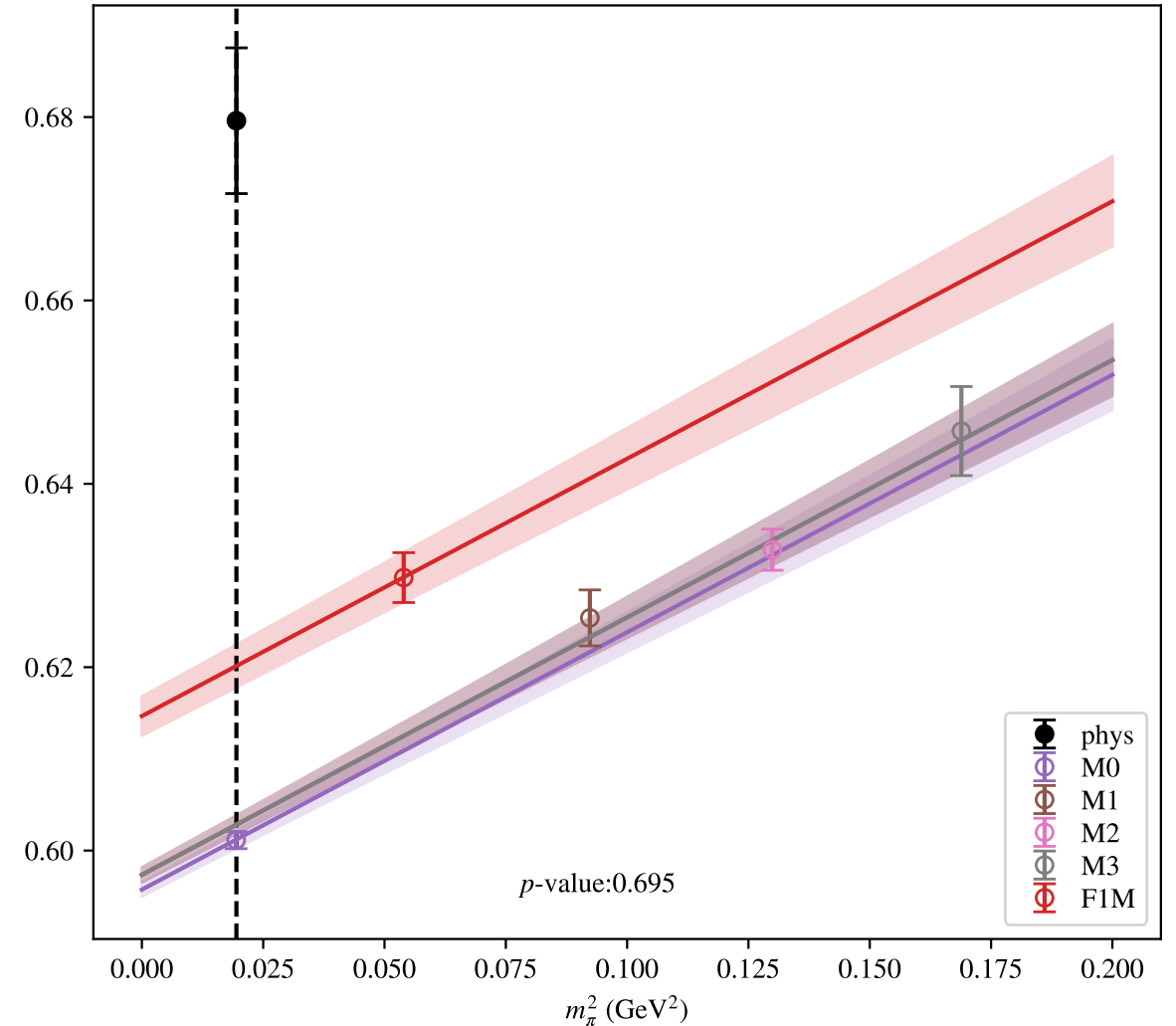


$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$

TT

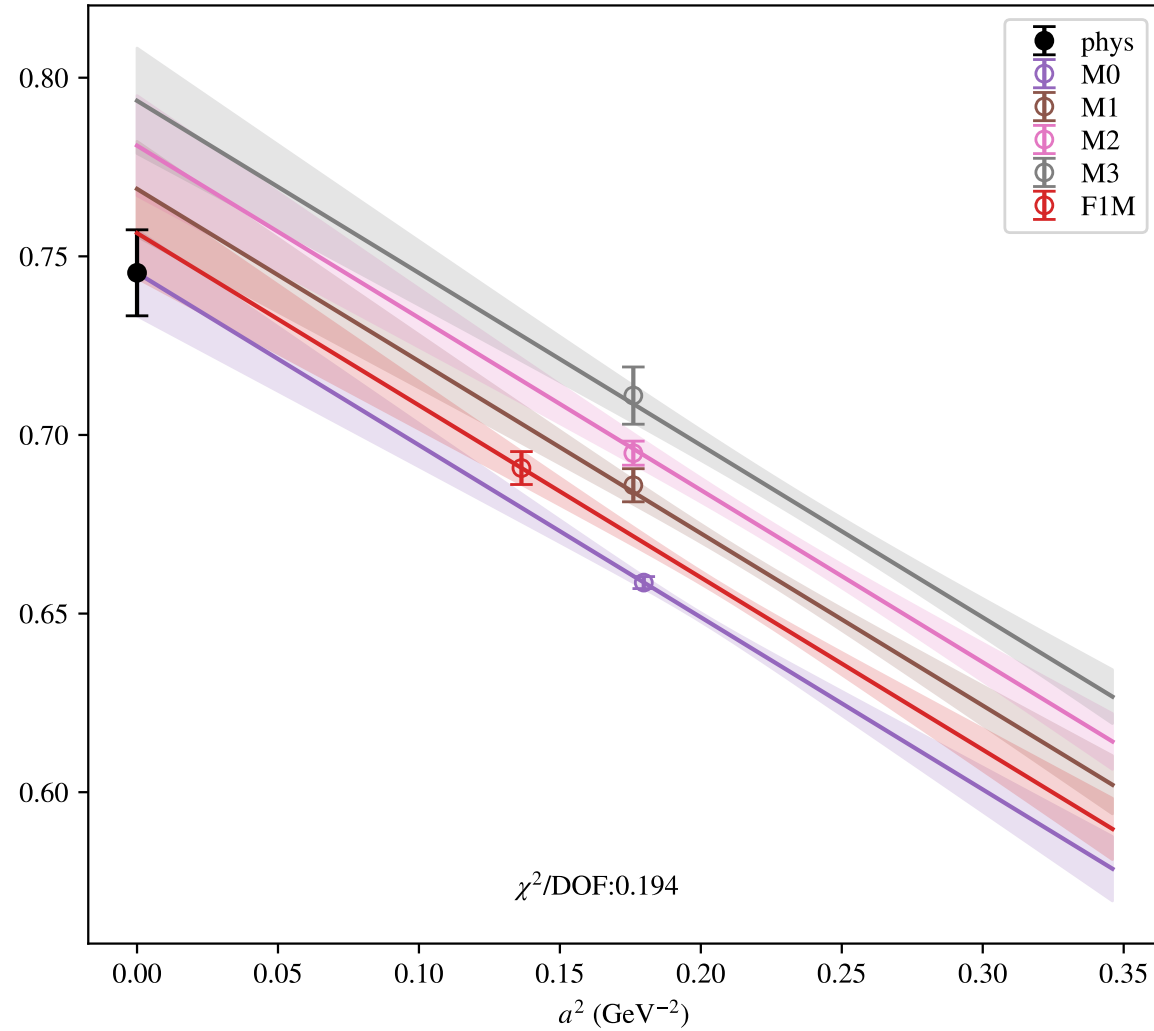


TT

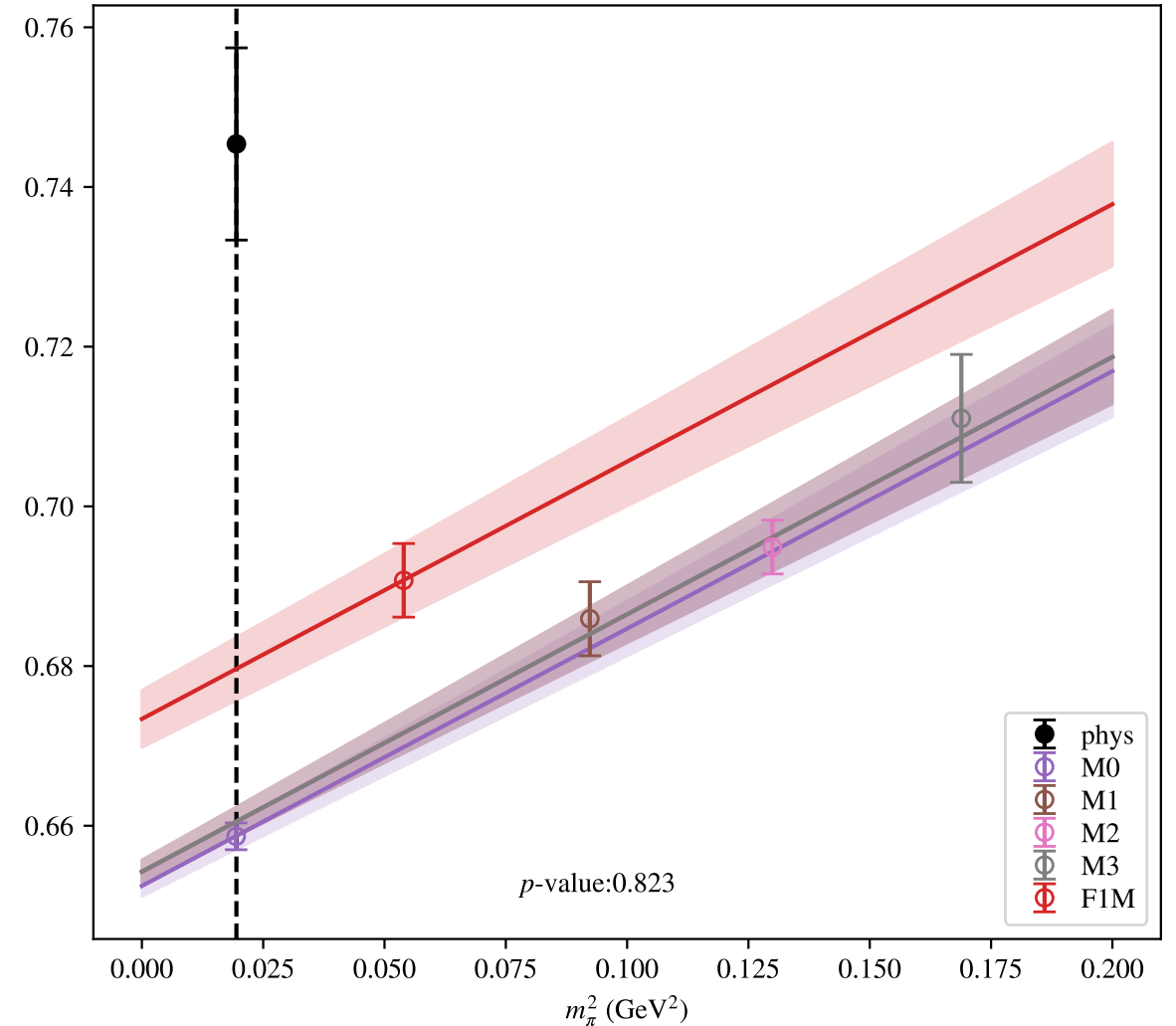


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

TT

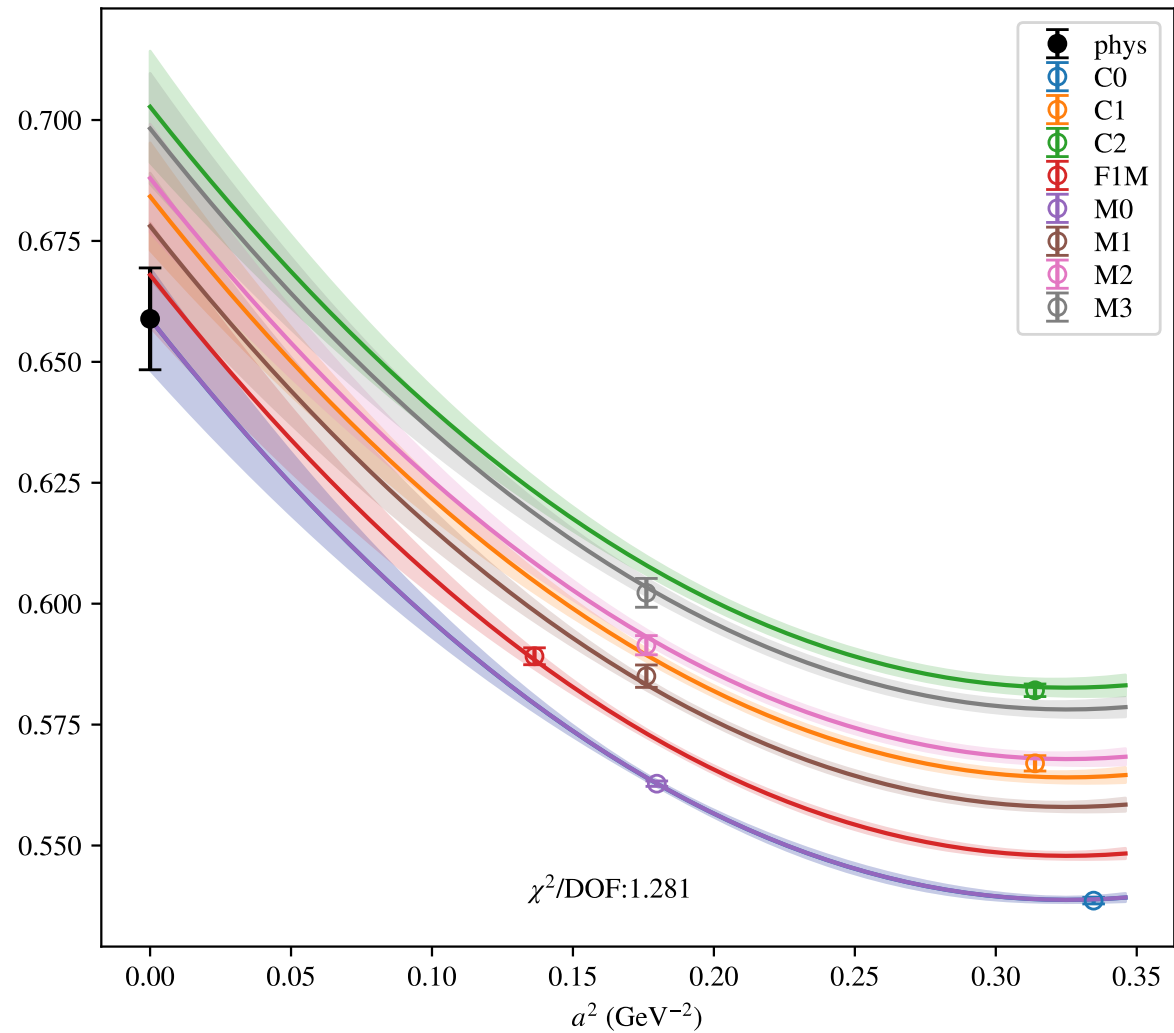


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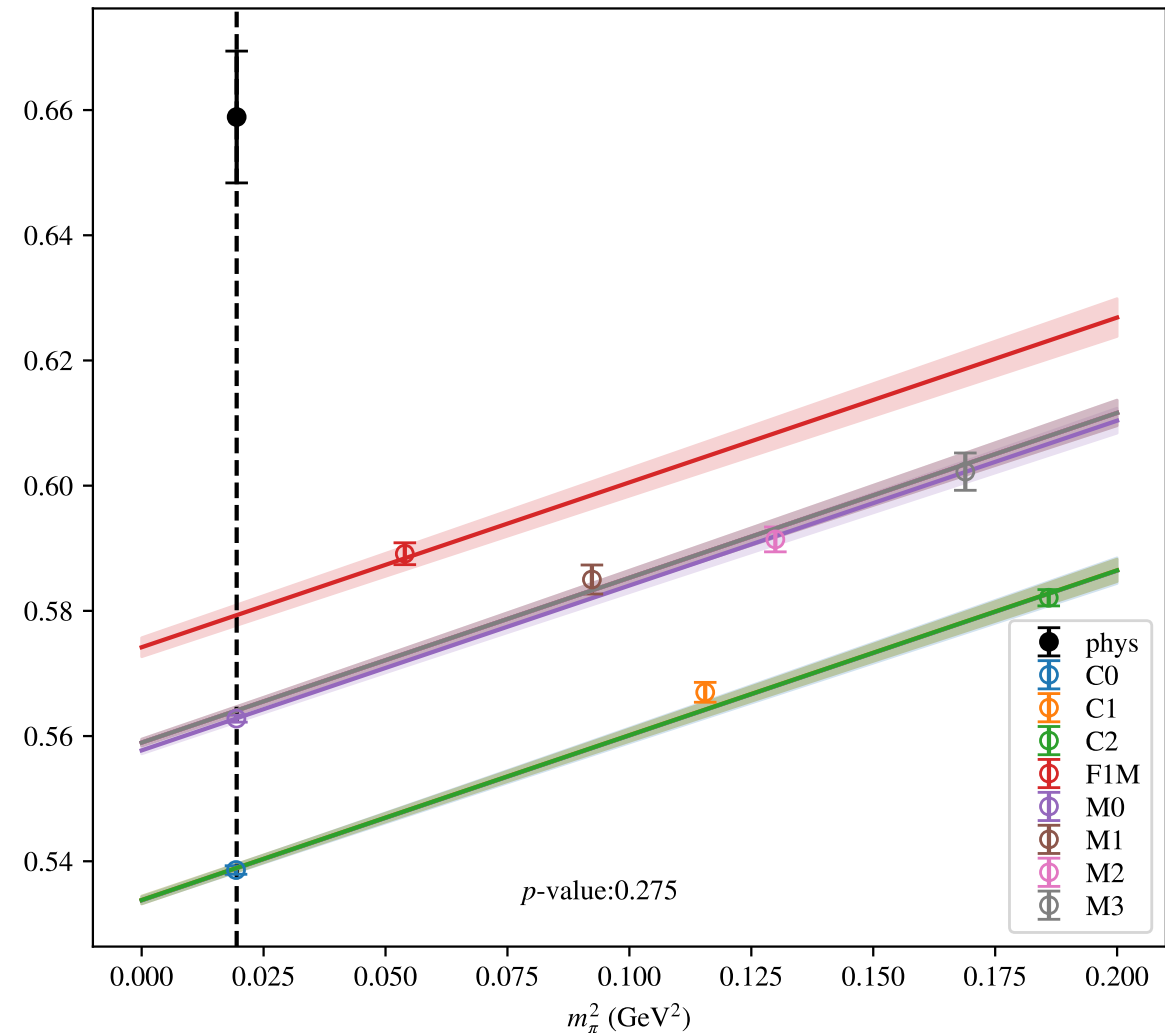


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

TT

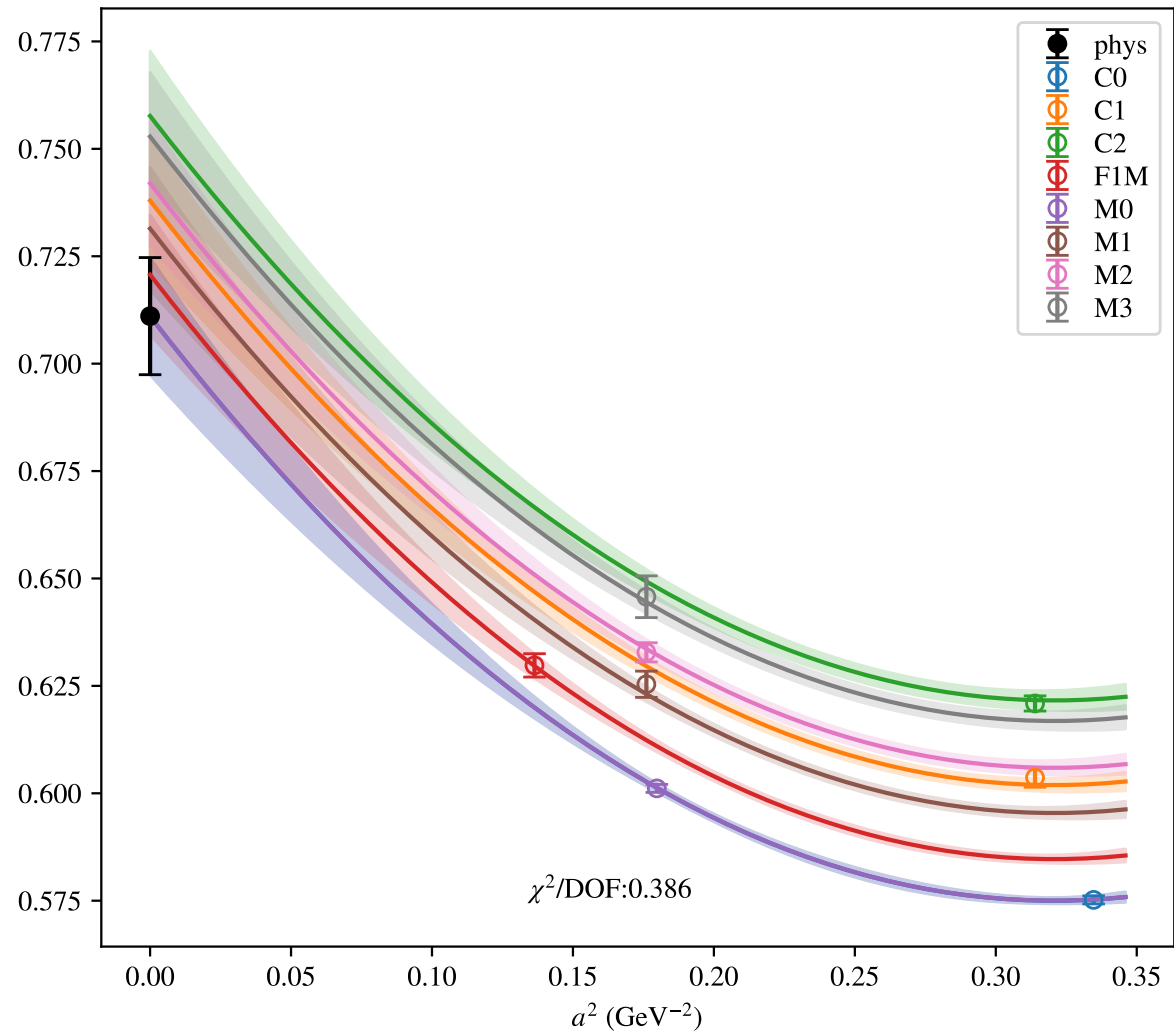


TT

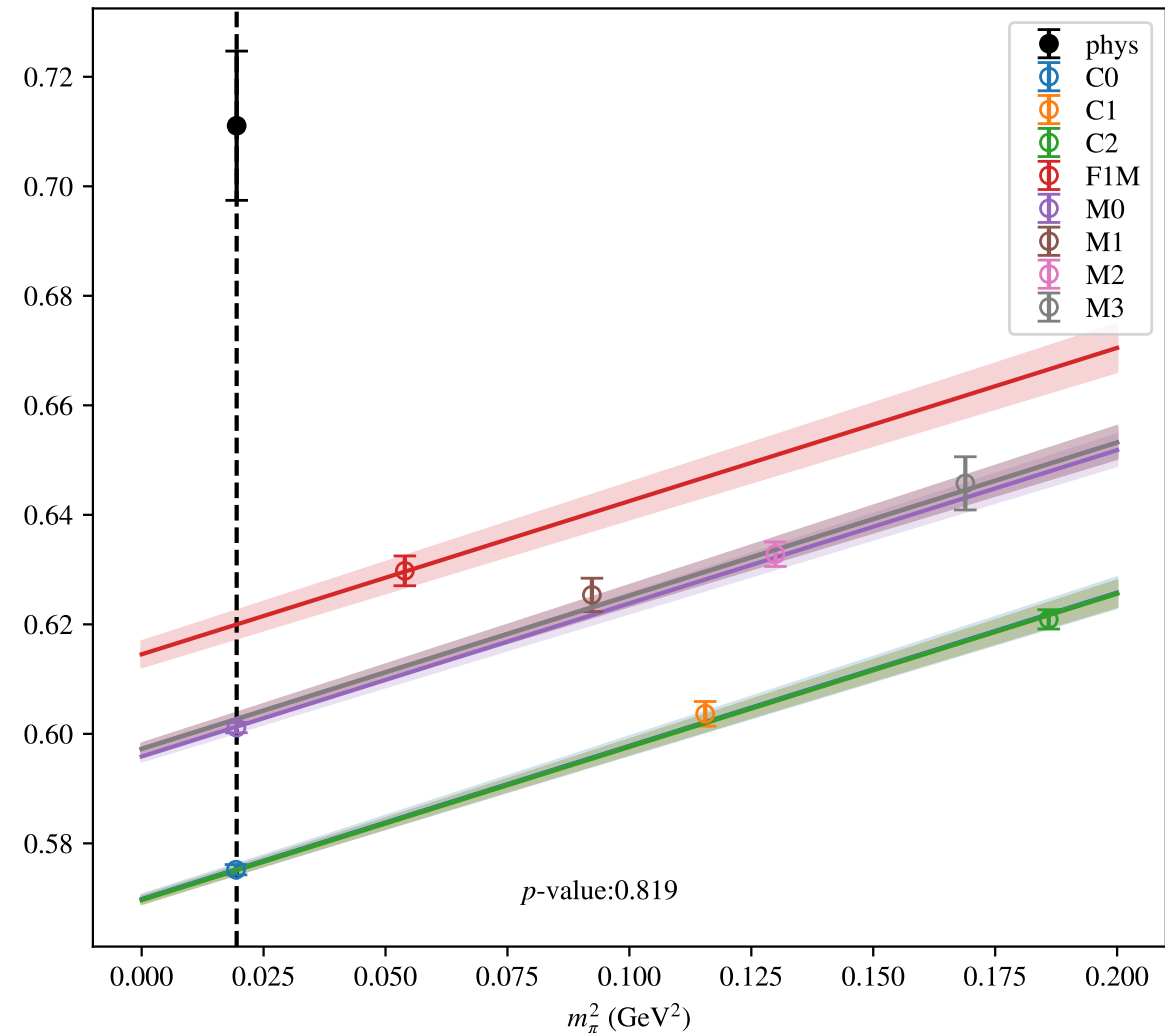


$$a^2, a^4, m_\pi^2, \mu = 1.8 \text{ GeV}$$

TT

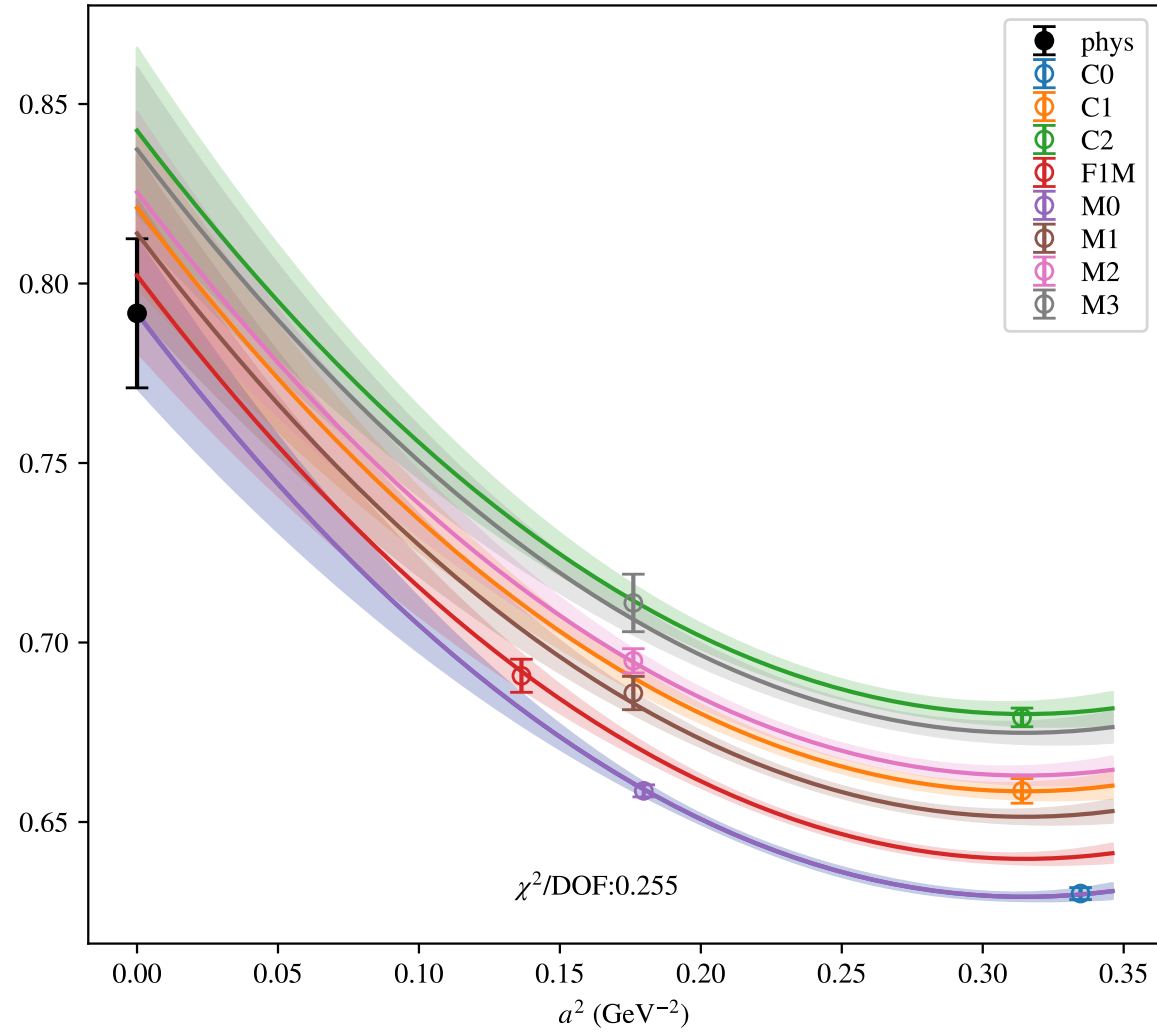


TT

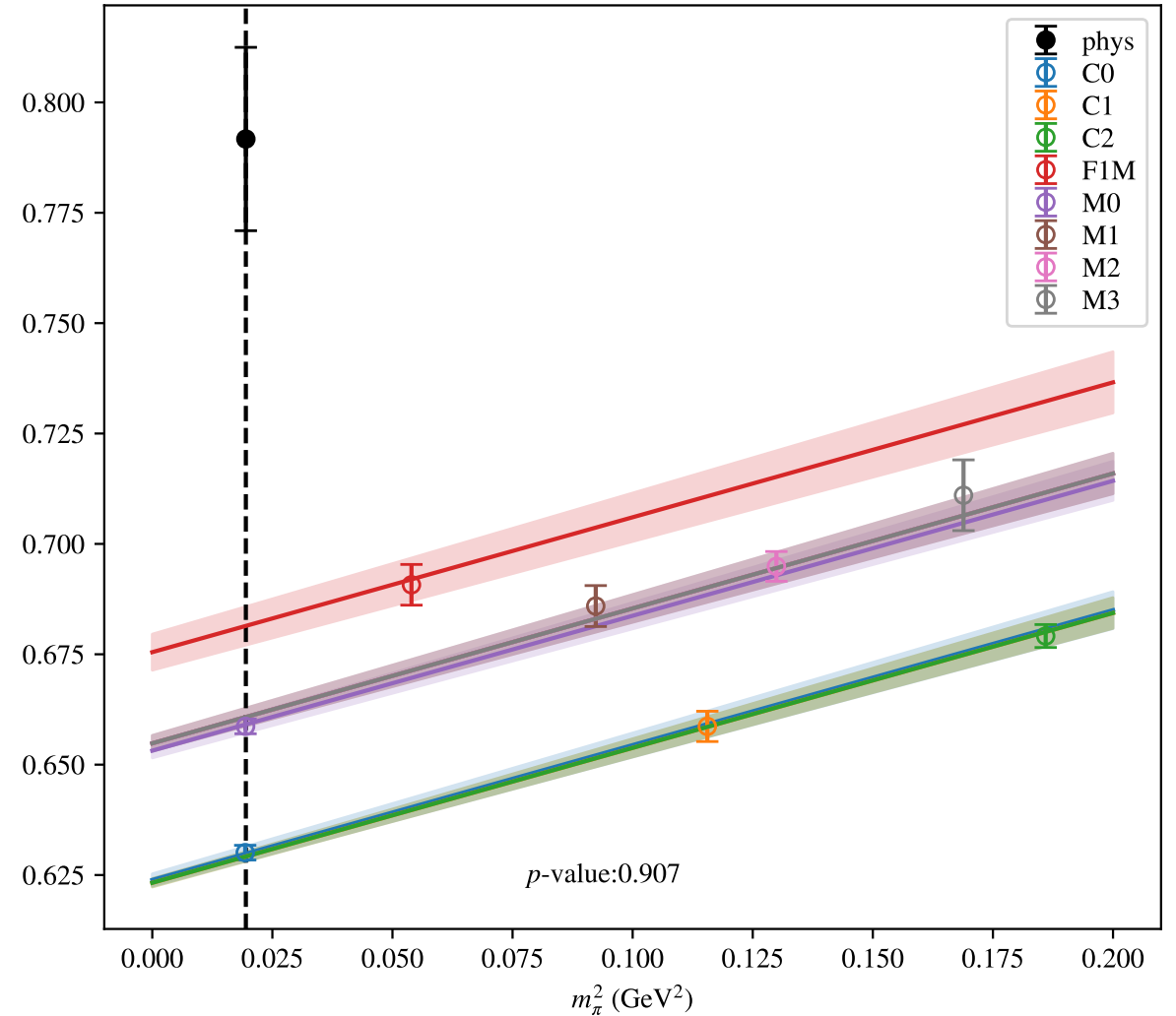


$$a^2, a^4, m_\pi^2, \mu = 1.5 \text{ GeV}$$

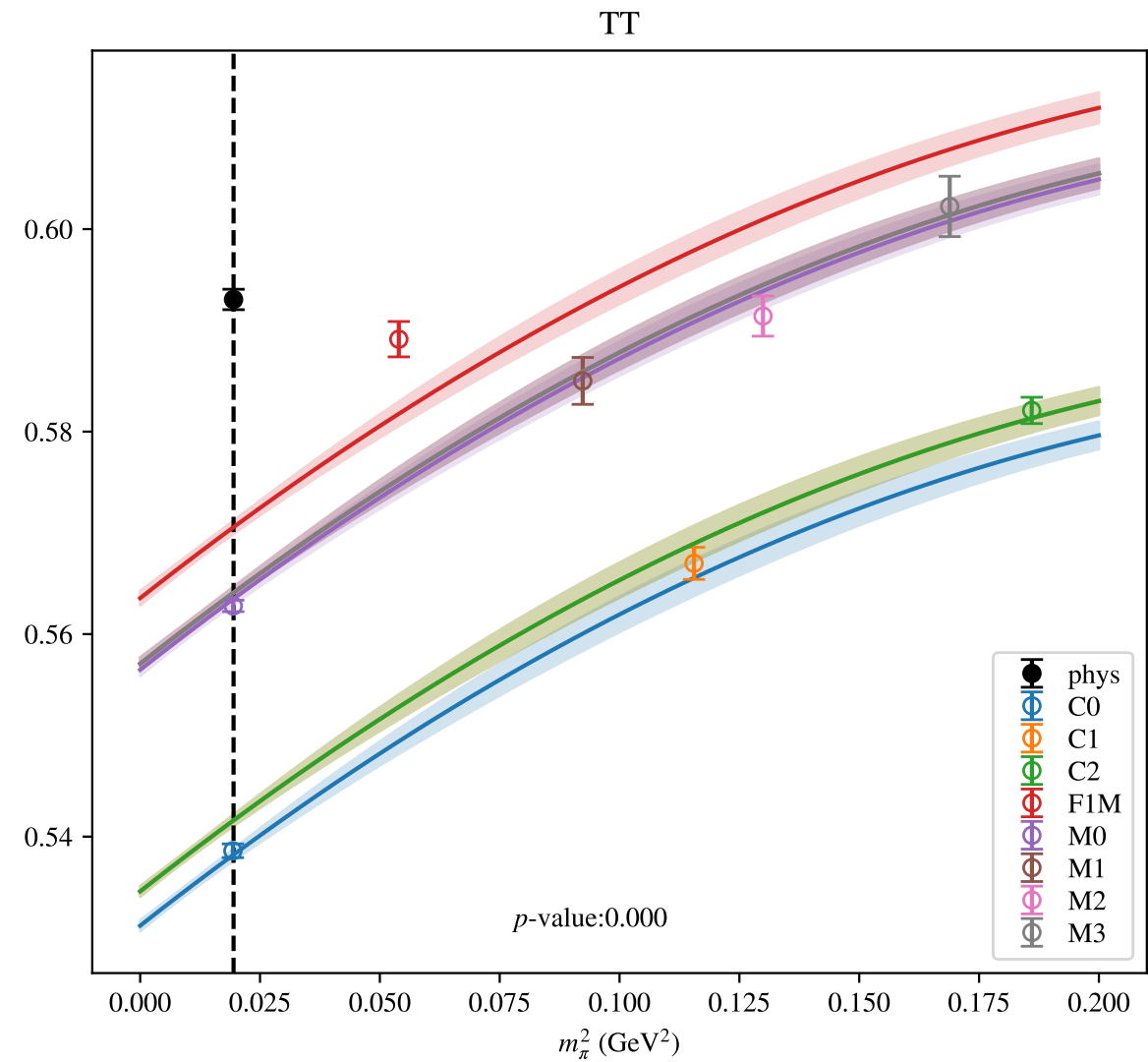
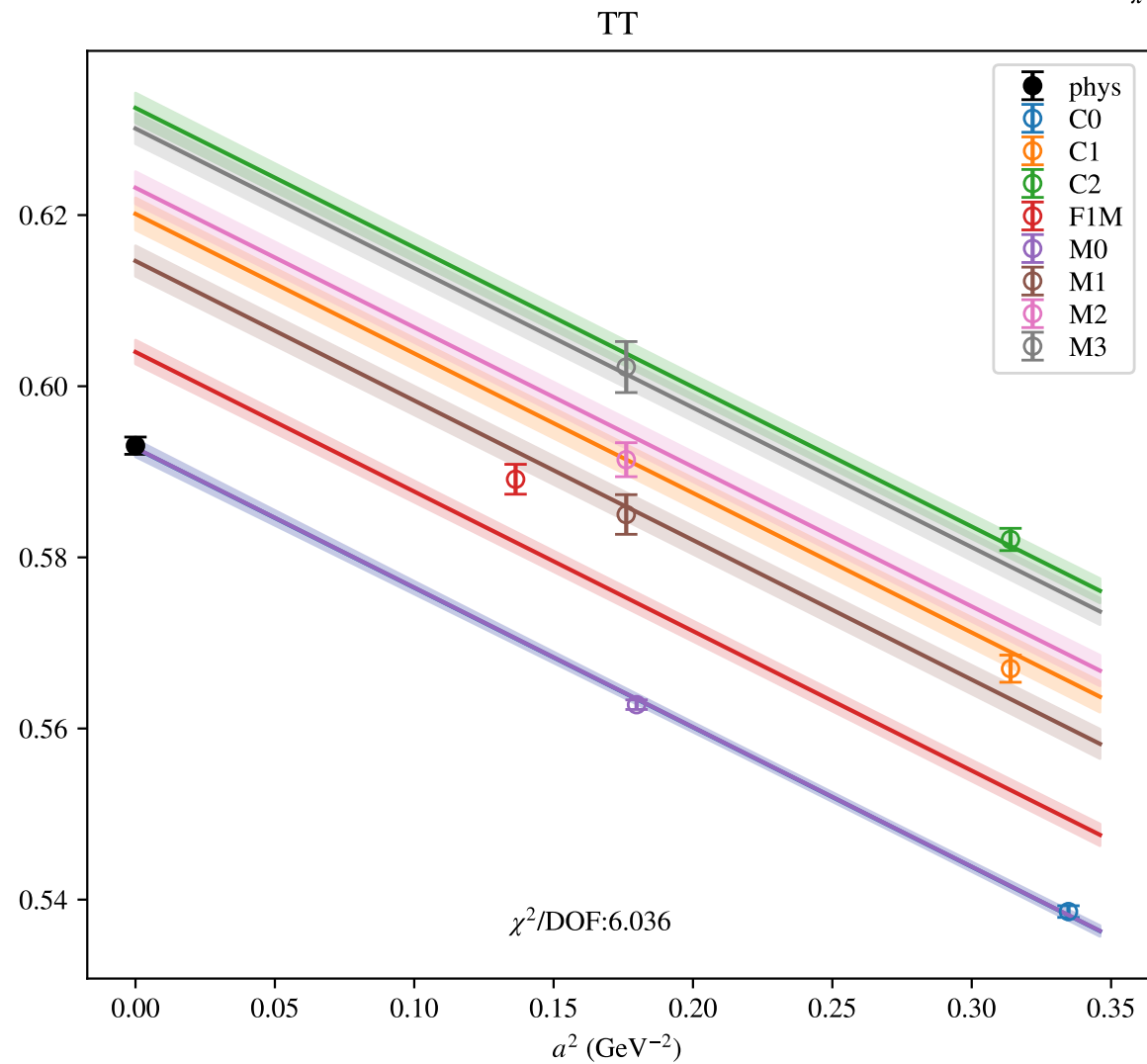
TT



TT

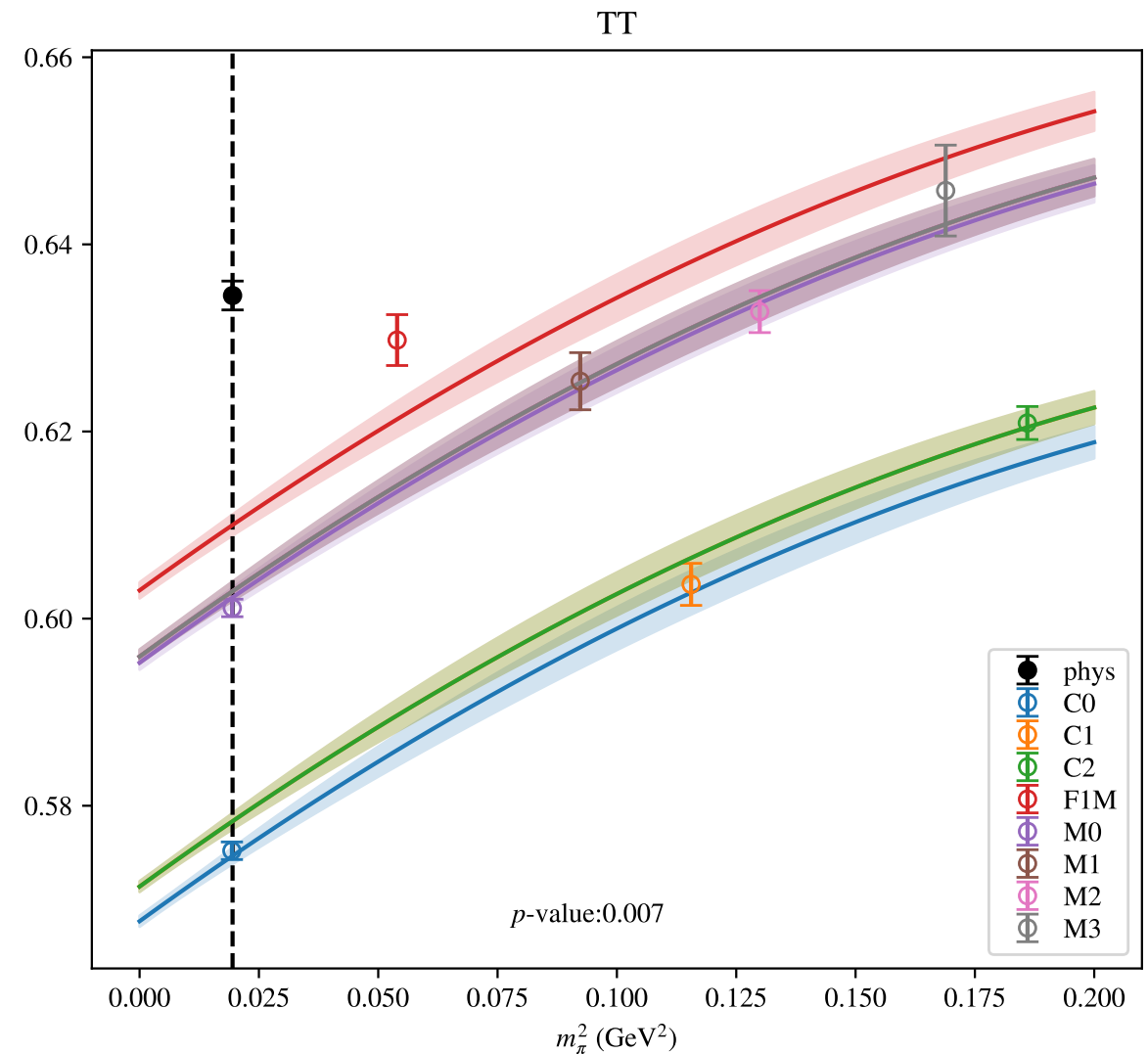
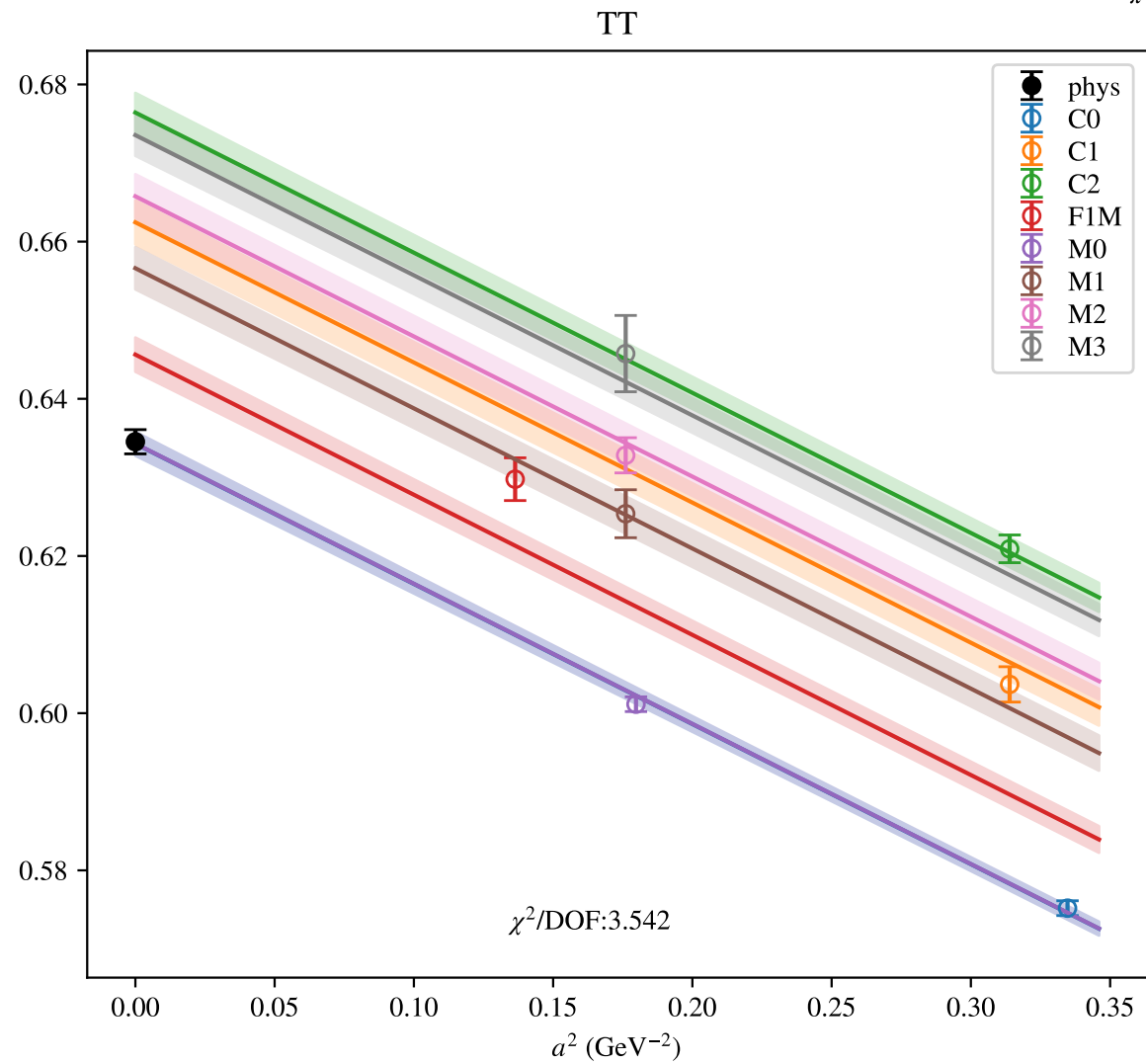


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$



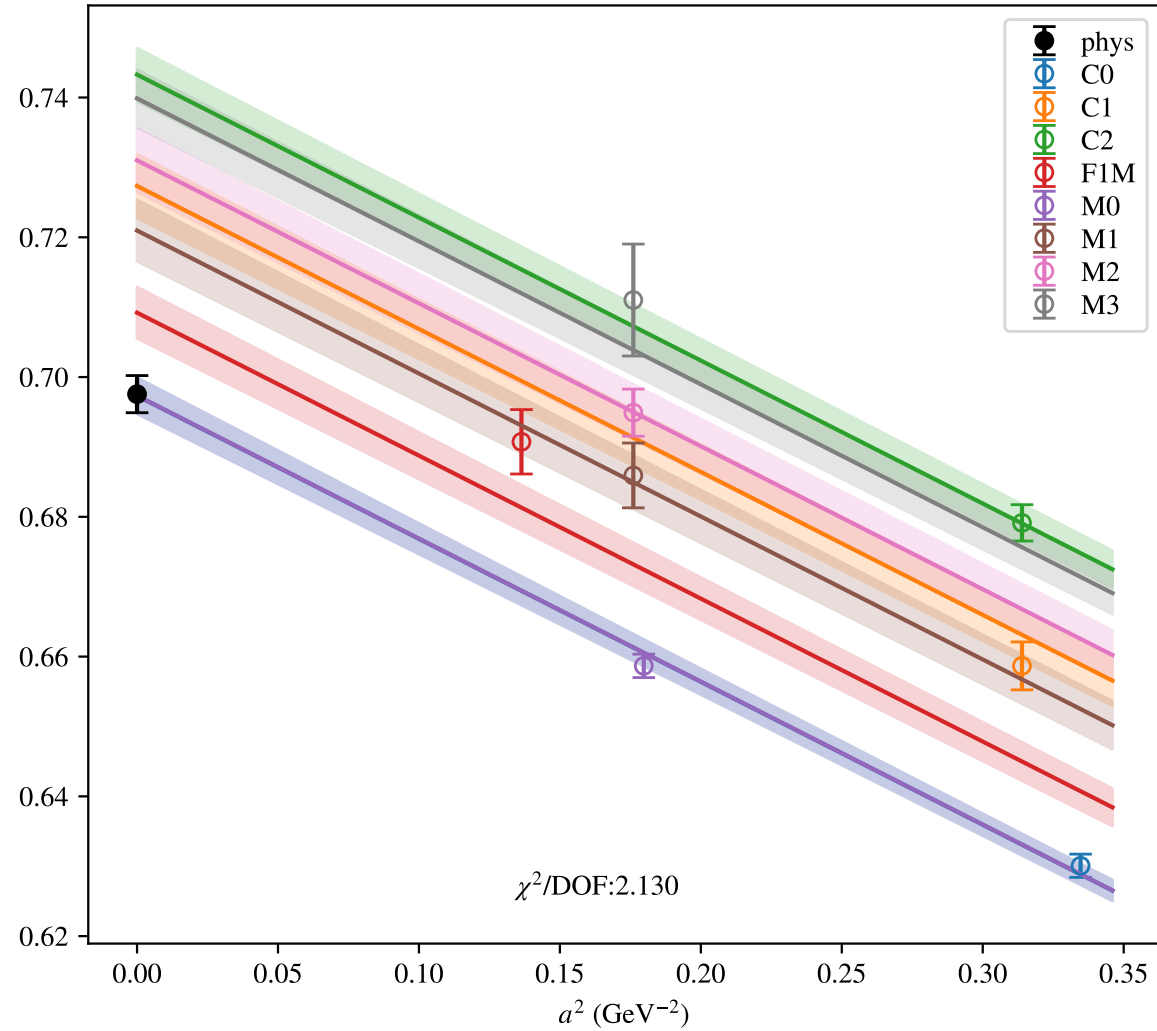


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.8 \text{ GeV}$$



$$a^2, m_\pi^2, m_\pi^4, \mu = 1.5 \text{ GeV}$$

TT



TT

