Kaon mixing: chiral and continuum extrapolations

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Contents

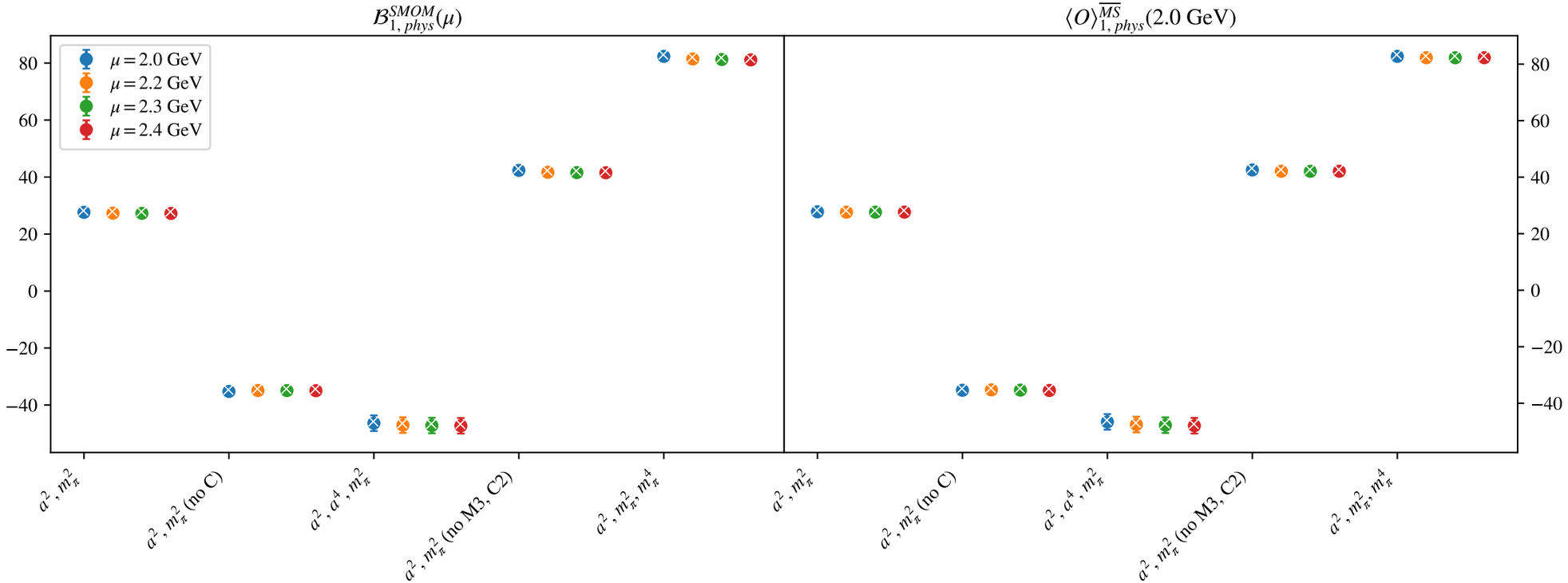


Figure 1: $\langle O\rangle_1$
(left) $\langle O\rangle_{phys}$ in RI/SMOM scheme from fit variations (fits with p-value < 0.05 marked with "$\times$").
(right) $\langle O\rangle_{phys}$ in $\overline{MS}$ computed using $\langle O\rangle^{\overline{MS}} = R^{\overline{MS}\leftarrow SMOM}(2.0)\sigma_{npt}(2.0,\mu)\langle O\rangle^{SMOM}(\mu)$.

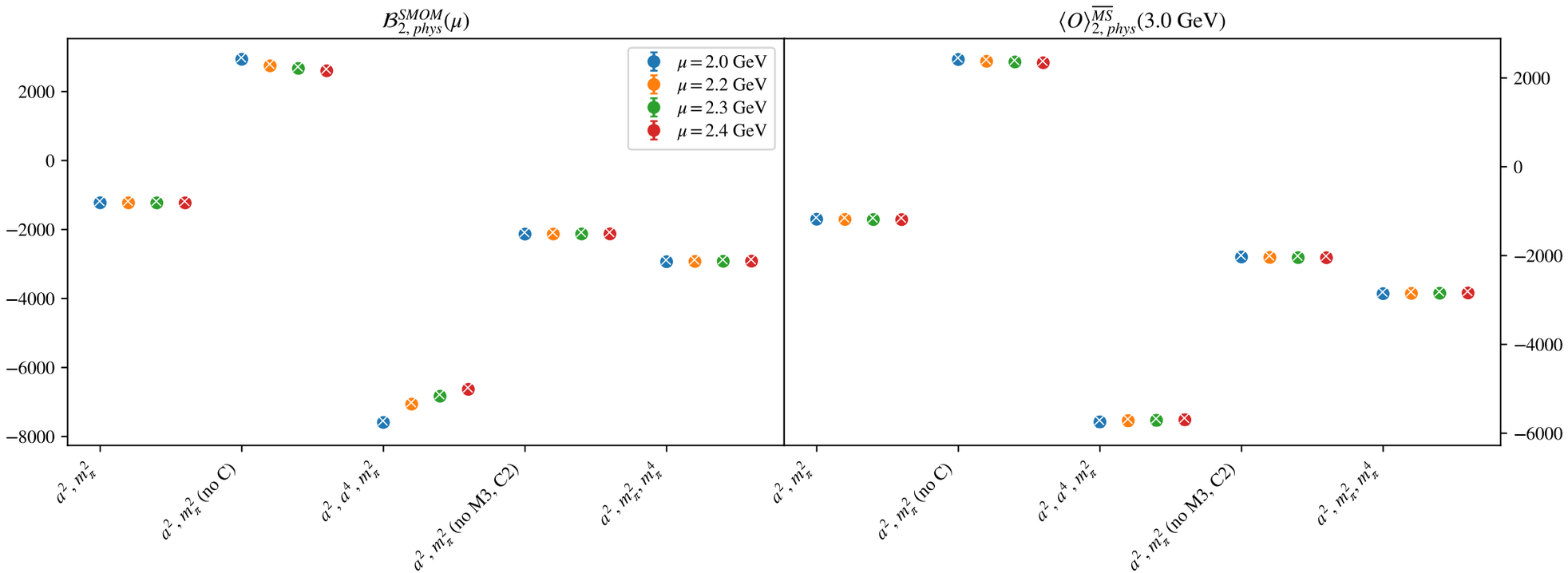


Figure 2: $\langle O\rangle_2$
(left) $\langle O\rangle_{phys}$ in RI/SMOM scheme from fit variations (fits with p-value < 0.05 marked with "$\times$").
(right) $\langle O\rangle_{phys}$ in $\overline{MS}$ computed using $\langle O\rangle^{\overline{MS}} = R^{\overline{MS}\leftarrow SMOM}(3.0)\sigma_{npt}(3.0,\mu)\langle O\rangle^{SMOM}(\mu)$.

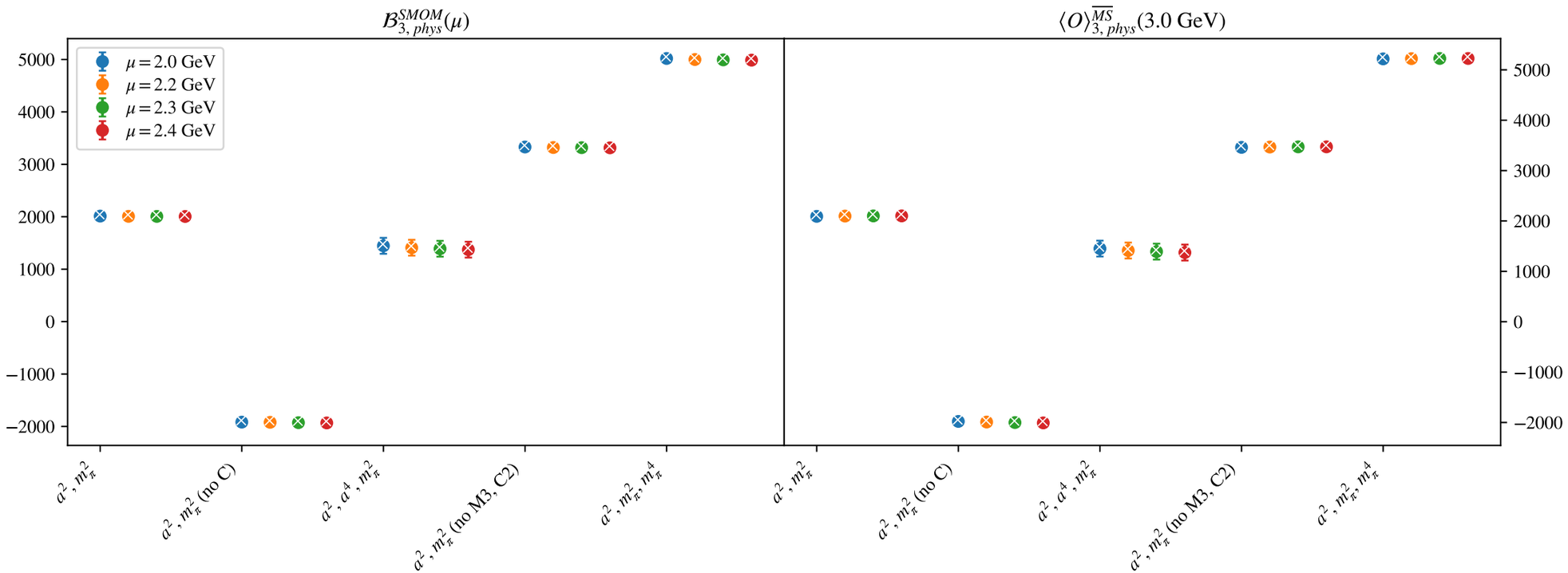


Figure 3: $\langle O\rangle_3$
(left) $\langle O\rangle_{phys}$ in RI/SMOM scheme from fit variations (fits with p-value < 0.05 marked with "$\times$").
(right) $\langle O\rangle_{phys}$ in $\overline{MS}$ computed using $\langle O\rangle^{\overline{MS}} = R^{\overline{MS}\leftarrow SMOM}(3.0)\sigma_{npt}(3.0,\mu)\langle O\rangle^{SMOM}(\mu)$.

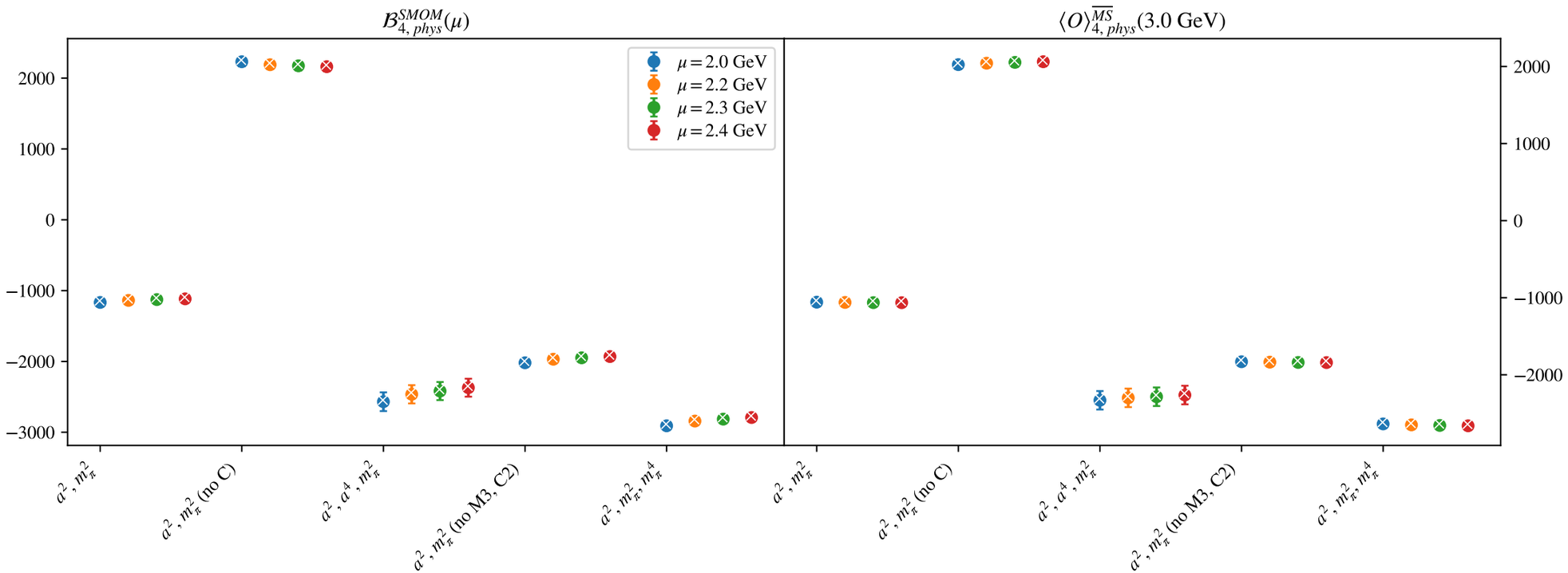


Figure 4: $\langle O\rangle_4$
(left) $\langle O\rangle_{phys}$ in RI/SMOM scheme from fit variations (fits with p-value < 0.05 marked with "$\times$").
(right) $\langle O\rangle_{phys}$ in $\overline{MS}$ computed using $\langle O\rangle^{\overline{MS}} = R^{\overline{MS}\leftarrow SMOM}(3.0)\sigma_{npt}(3.0,\mu)\langle O\rangle^{SMOM}(\mu)$.

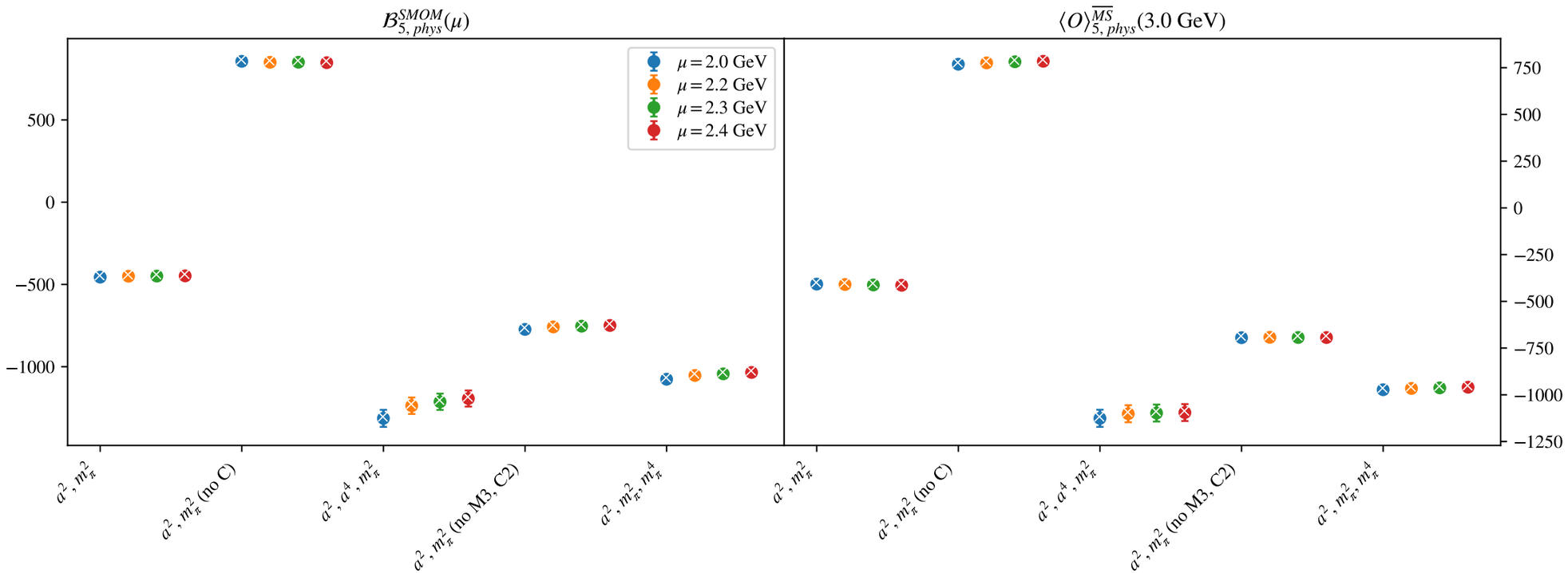


Figure 5: $\langle O\rangle_5$
(left) $\langle O\rangle_{phys}$ in RI/SMOM scheme from fit variations (fits with p-value < 0.05 marked with "$\times$").
(right) $\langle O\rangle_{phys}$ in $\overline{MS}$ computed using $\langle O\rangle^{\overline{MS}} = R^{\overline{MS}\leftarrow SMOM}(3.0)\sigma_{npt}(3.0,\mu)\langle O\rangle^{SMOM}(\mu)$.

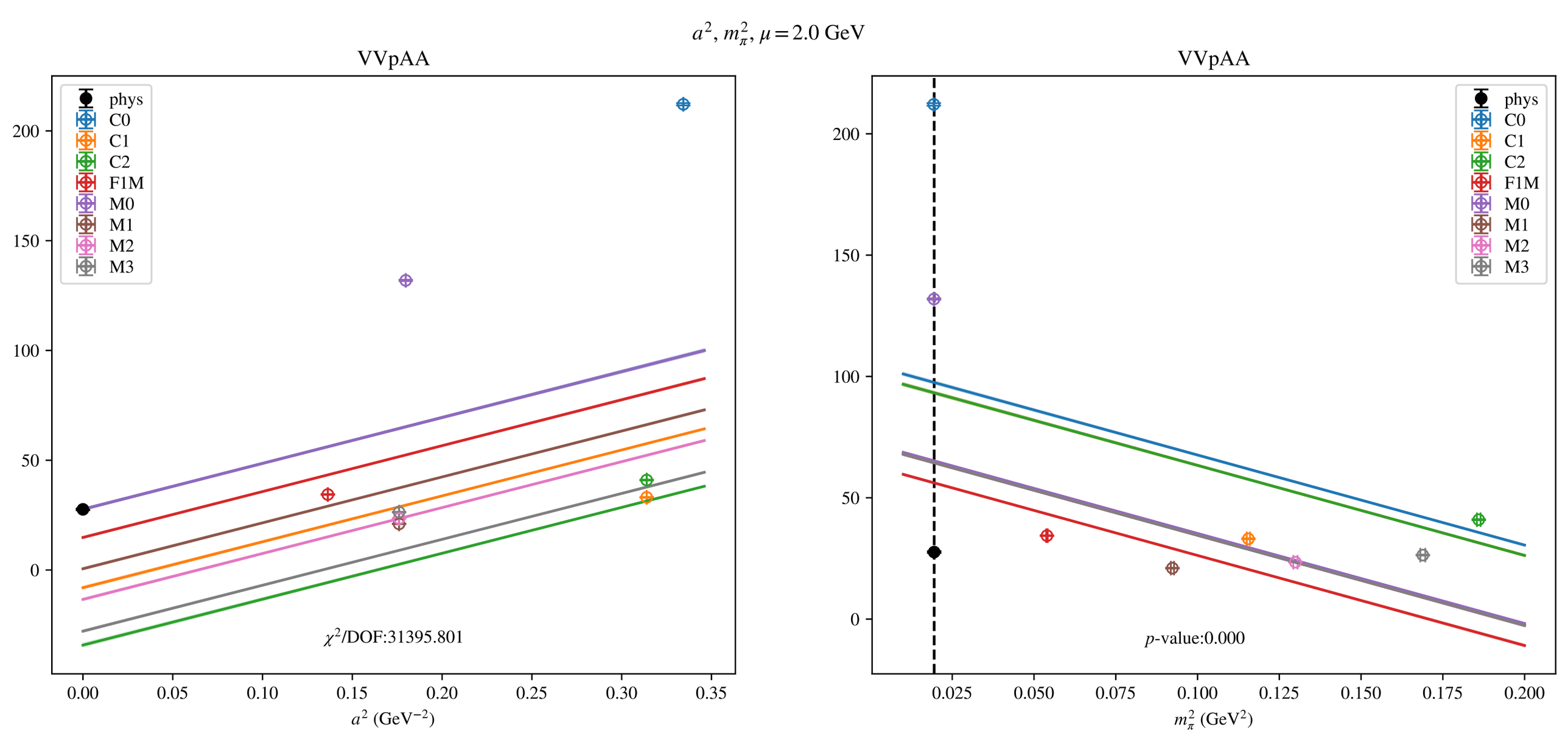
1 $\mathcal{B}_1$

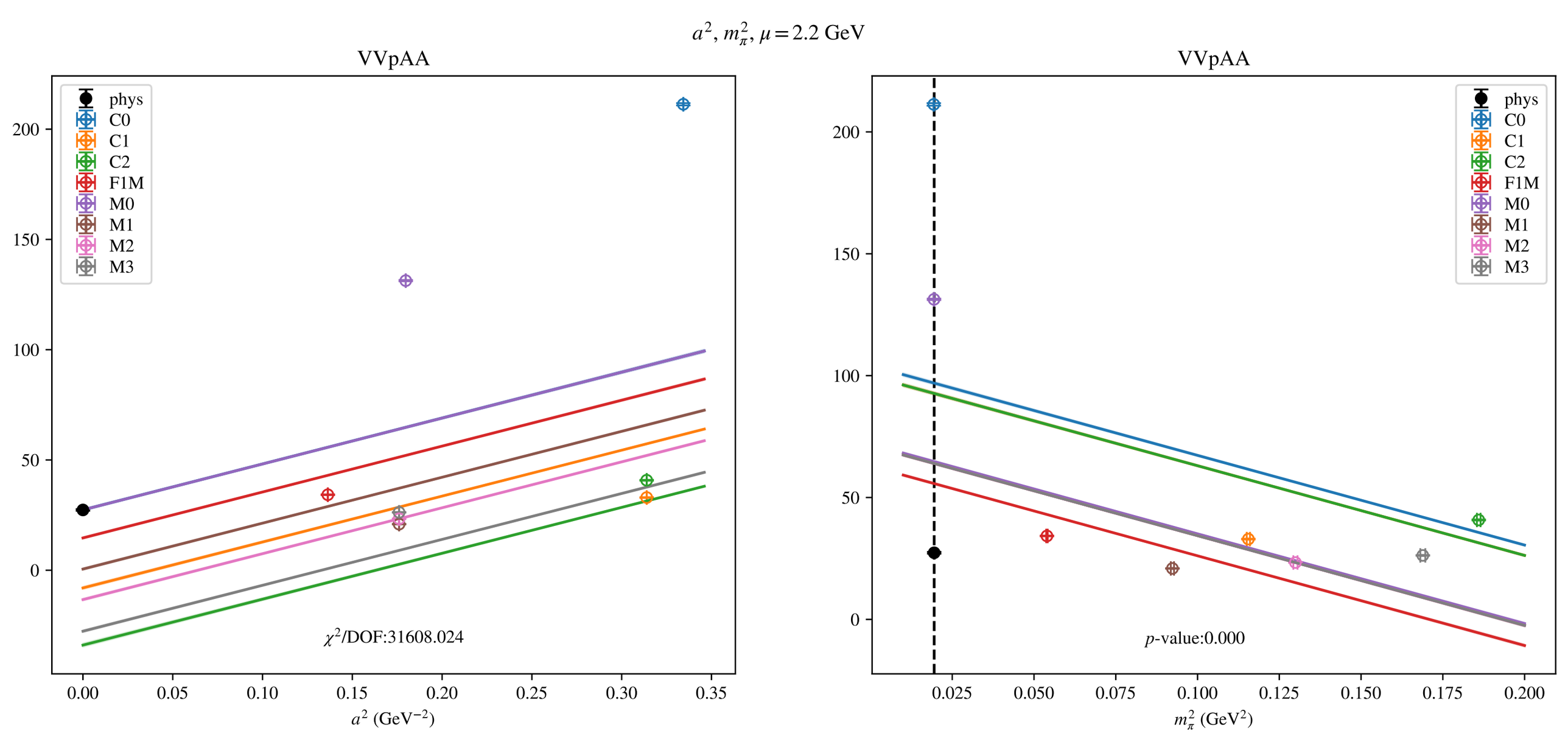
μ (GeV)	a^2, m_π^2	a^2, m_π^2 (no C)	a^2, a^4, m_π^2	a^2, m_π^2 (no M3, C2)	a^2, m_π^2, m_π^4
2.0	**27.6(37)**: 31395.801 (0.0)	**-35(14)**: 38933.302 (0.0)	**-46(27)**: 38585.869 (0.0)	**42.3(64)**: 33587.411 (0.0)	**82.3(60)**: 14300.65 (0.0)
2.2	**27.3(36)**: 31608.024 (0.0)	**-34(14)**: 39223.15 (0.0)	**-47(27)**: 38821.954 (0.0)	**41.6(63)**: 33975.784 (0.0)	**81.4(60)**: 14505.621 (0.0)
2.3	**27.2(36)**: 31654.959 (0.0)	**-34(14)**: 39280.306 (0.0)	**-47(27)**: 38875.092 (0.0)	**41.5(63)**: 34030.531 (0.0)	**81.2(59)**: 14528.355 (0.0)
2.4	**27.2(36)**: 31677.595 (0.0)	**-34(14)**: 39318.403 (0.0)	**-47(27)**: 38899.494 (0.0)	**41.5(62)**: 34047.877 (0.0)	**81.1(59)**: 14530.926 (0.0)

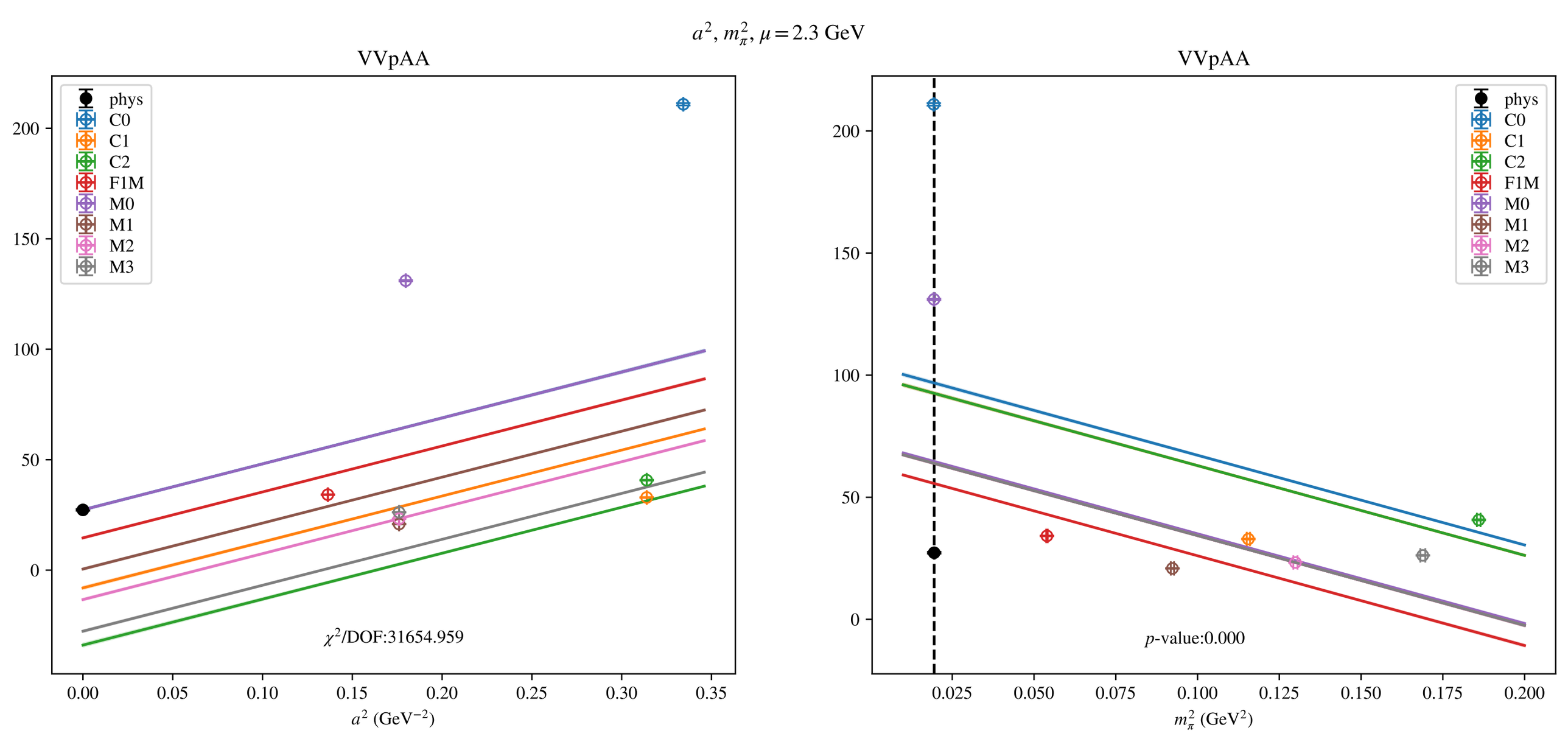
Table 1: Physical point value from chiral and continuum extrapolation at renormalisation scale μ. Entries are **value(error)**: χ^2/DOF (p-value).

μ (GeV)		a^2, m_π^2	a^2, m_π^2 (no C)	a^2, a^4, m_π^2	a^2, m_π^2 (no M3, C2)	a^2, m_π^2, m_π^4
2.0	α	7.57(16)	-17.(39)	-19.(58)	5.95(15)	2.052(34)
	β	-0.228(35)	0.1917(62)	0.1507(79)	-0.301(50)	-0.436(25)
2.2	α	7.61(16)	-17.(39)	-19.(56)	6.03(15)	2.073(35)
	β	-0.229(35)	0.1924(63)	0.1476(75)	-0.303(51)	-0.437(25)
2.3	α	7.62(16)	-17.(39)	-19.(55)	6.04(15)	2.076(35)
	β	-0.229(35)	0.1921(62)	0.1471(75)	-0.303(51)	-0.438(25)
2.4	α	7.62(16)	-17.(39)	-19.(55)	6.04(15)	2.076(35)
	β	-0.229(35)	0.1917(62)	0.1466(74)	-0.303(51)	-0.438(25)

Table 2: Fit values of coefficients in $Q = Q_{phys} + \alpha a^2 + \beta \left(\frac{m_\pi^2}{f_\pi^2} - \frac{m_{\pi,PDG}^2}{f_\pi^2} \right) + \ldots$.


$a^2, m_\pi^2, \mu = 2.0$ GeV
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
χ^2/DOF:31395.801
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
VVpAA
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


$a^2, m_\pi^2, \mu = 2.2$ GeV
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
χ^2/DOF:31608.024
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)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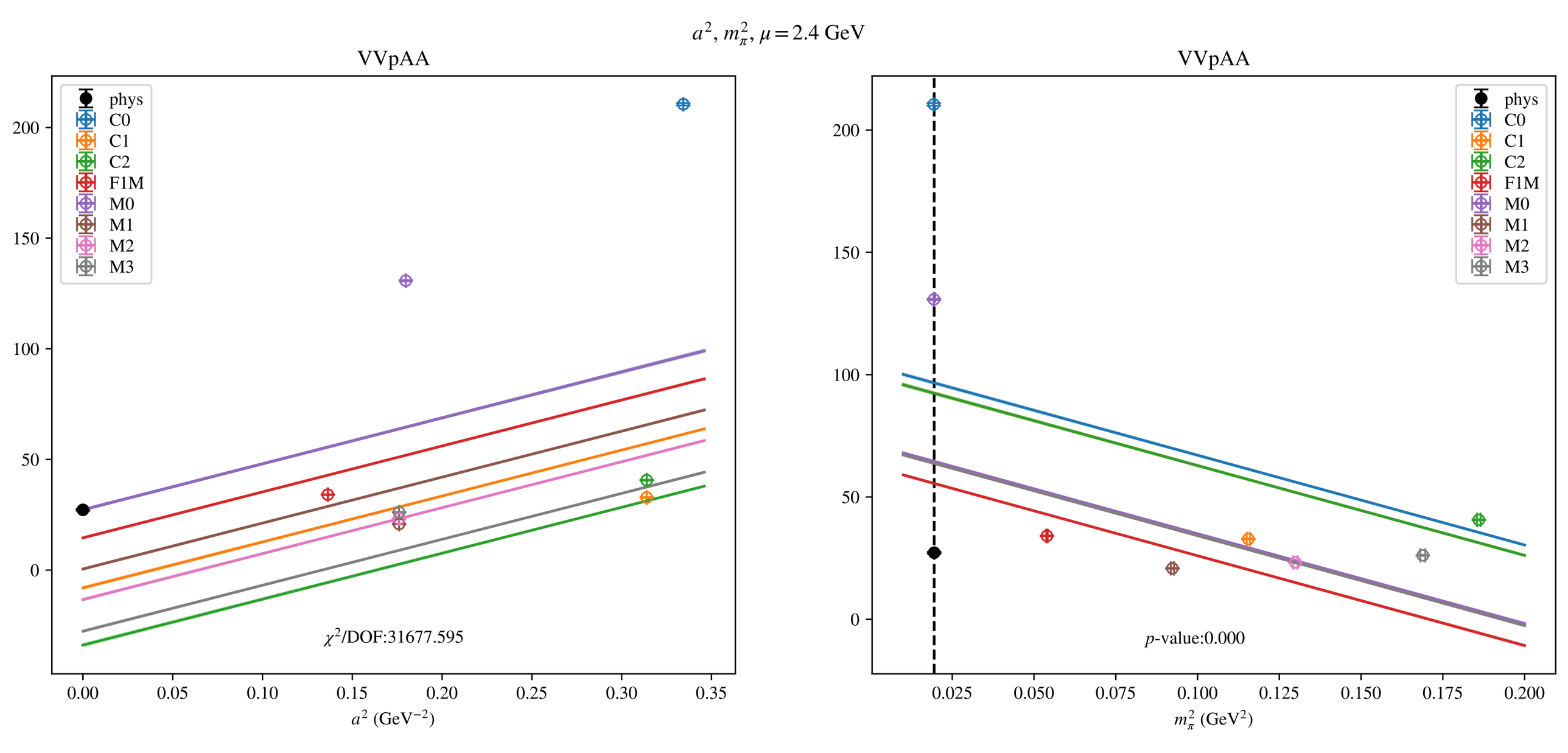
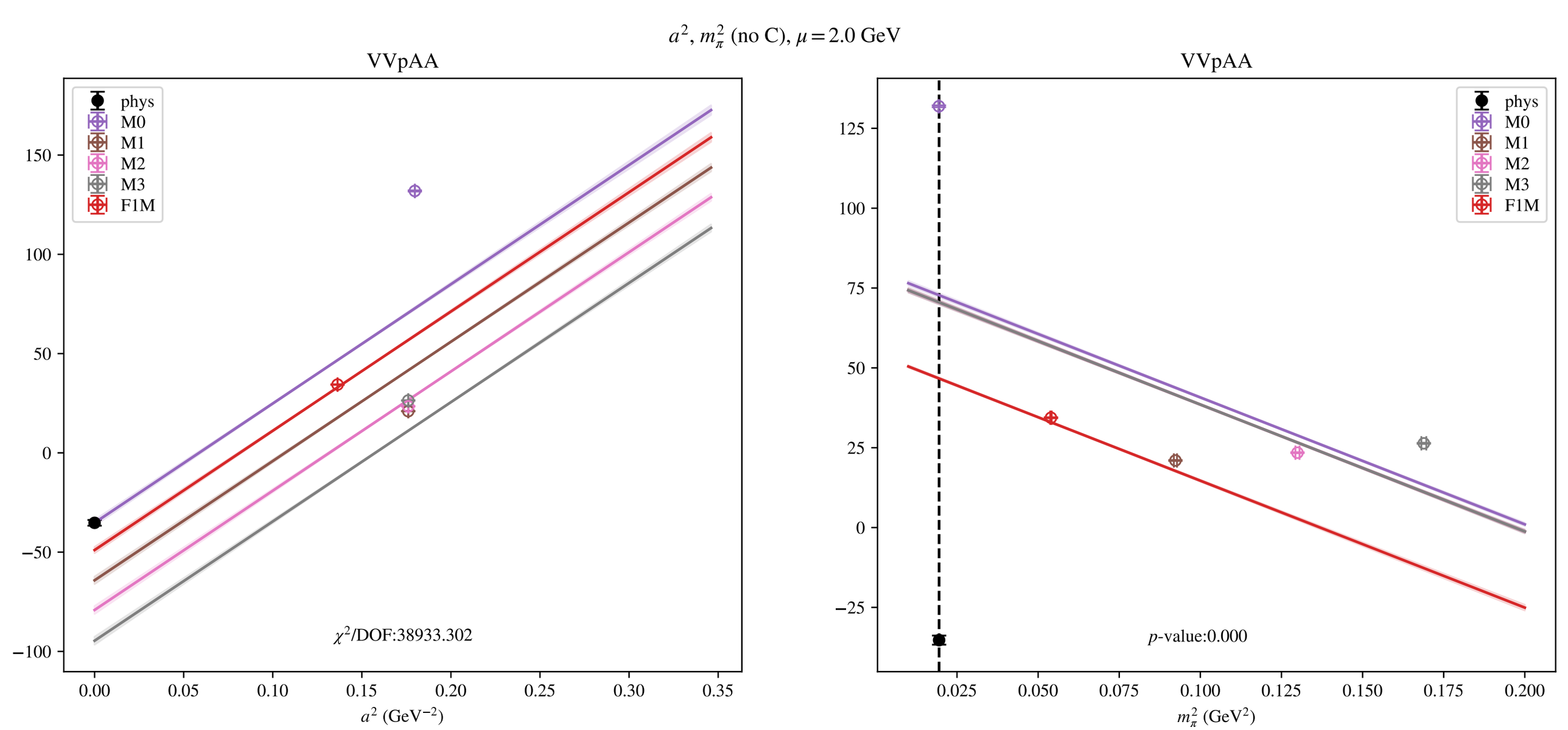
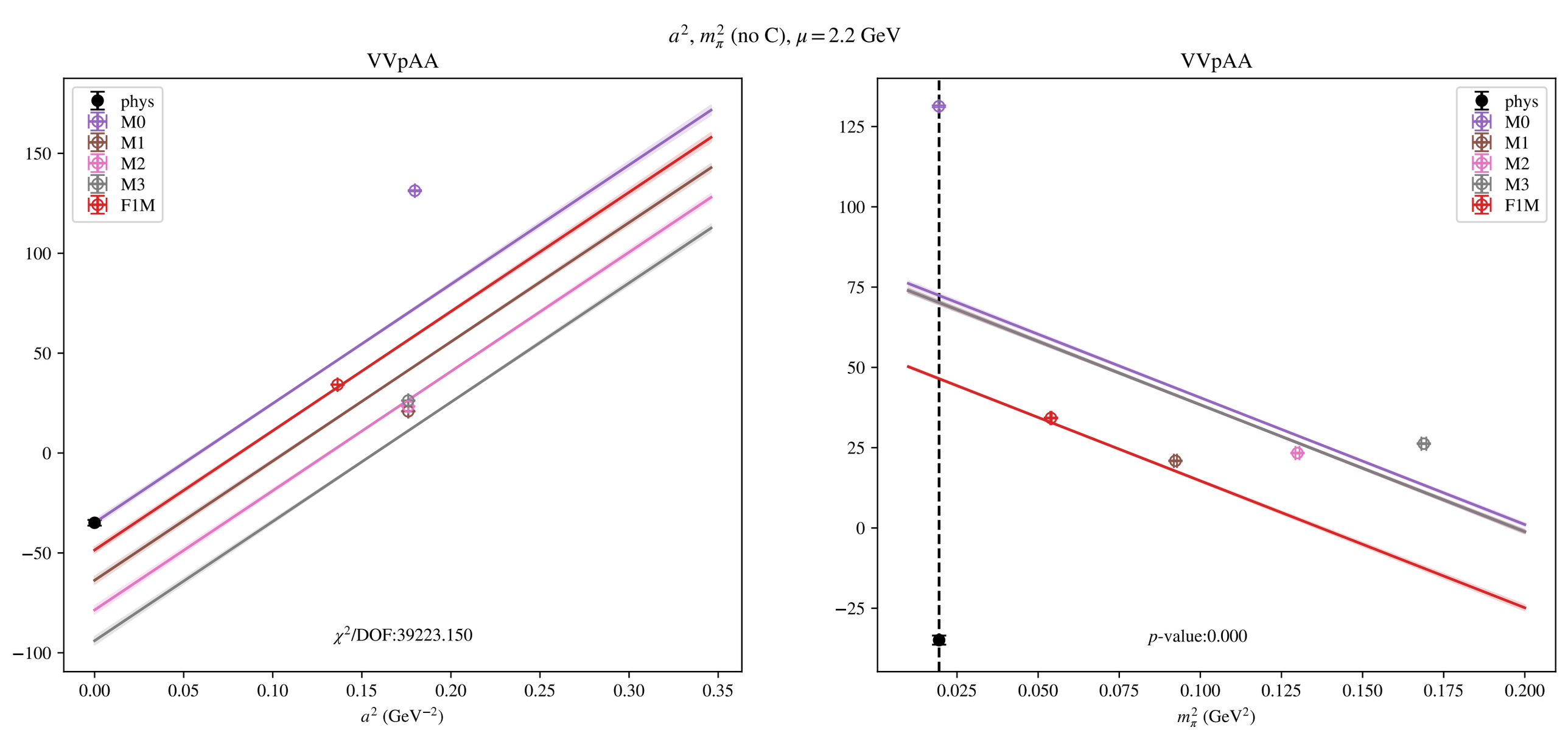
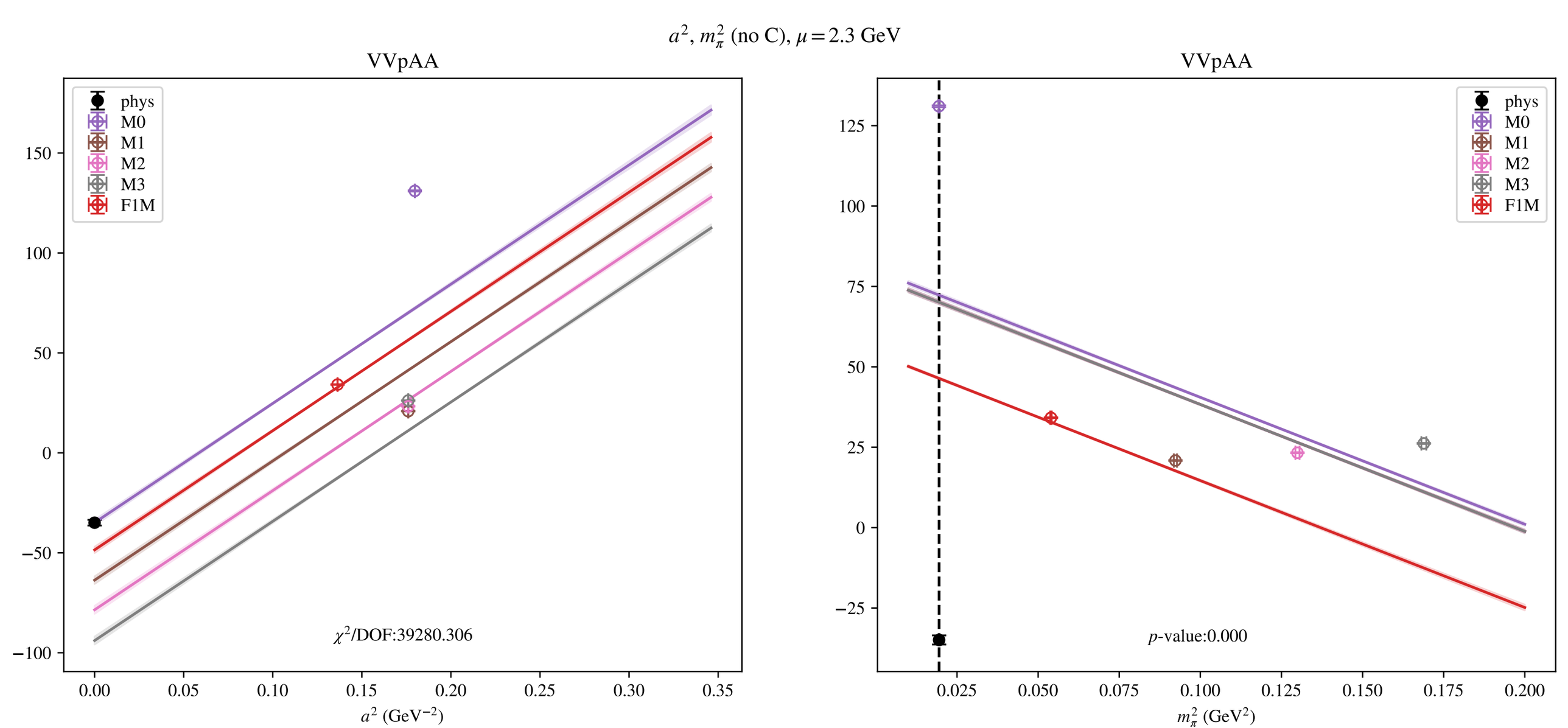
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
χ^2/DOF:31654.959
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


$a^2, m_\pi^2, \mu = 2.4$ GeV
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
χ^2/DOF:31677.595
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


a^2, m_π^2 (no C), $\mu = 2.0$ GeV
VVpAA
phys
M0
M1
M2
M3
F1M
150
100
50
0
−50
−100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:38933.302
VVpAA
phys
M0
M1
M2
M3
F1M
125
100
75
50
25
0
−25
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000


a^2, m_π^2 (no C), $\mu = 2.2$ GeV
VVpAA
phys
M0
M1
M2
M3
F1M
150
100
50
0
−50
−100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:39223.150
VVpAA
phys
M0
M1
M2
M3
F1M
125
100
75
50
25
0
−25
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000


a^2, m_π^2 (no C), $\mu = 2.3$ GeV
VVpAA
phys
M0
M1
M2
M3
F1M
150
100
50
0
−50
−100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:39280.306
VVpAA
phys
M0
M1
M2
M3
F1M
125
100
75
50
25
0
−25
0.025
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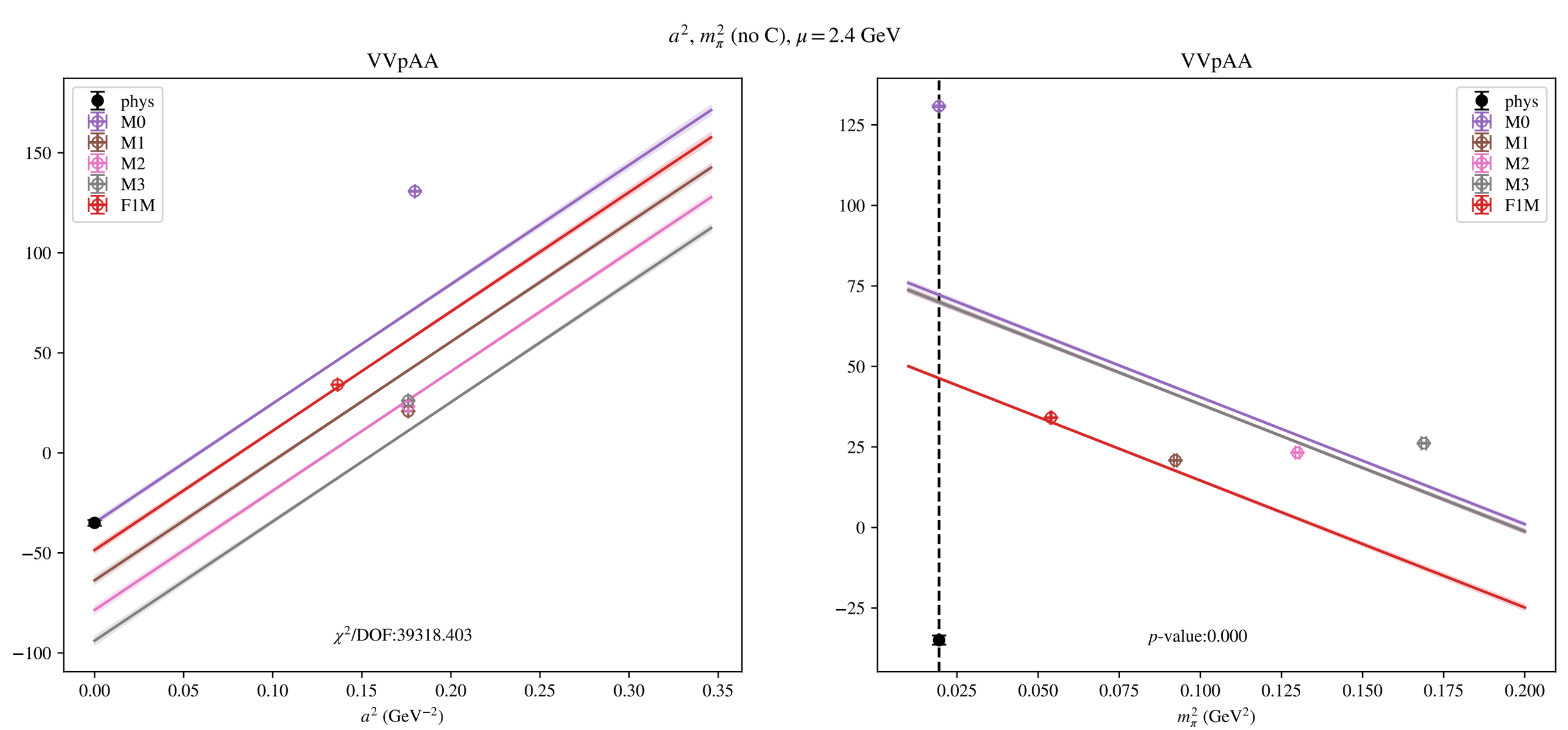
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000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M0
M1
M2
M3
F1M
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100
50
0
−50
−100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:39318.403
VVpAA
phys
M0
M1
M2
M3
F1M
125
100
75
50
25
0
−25
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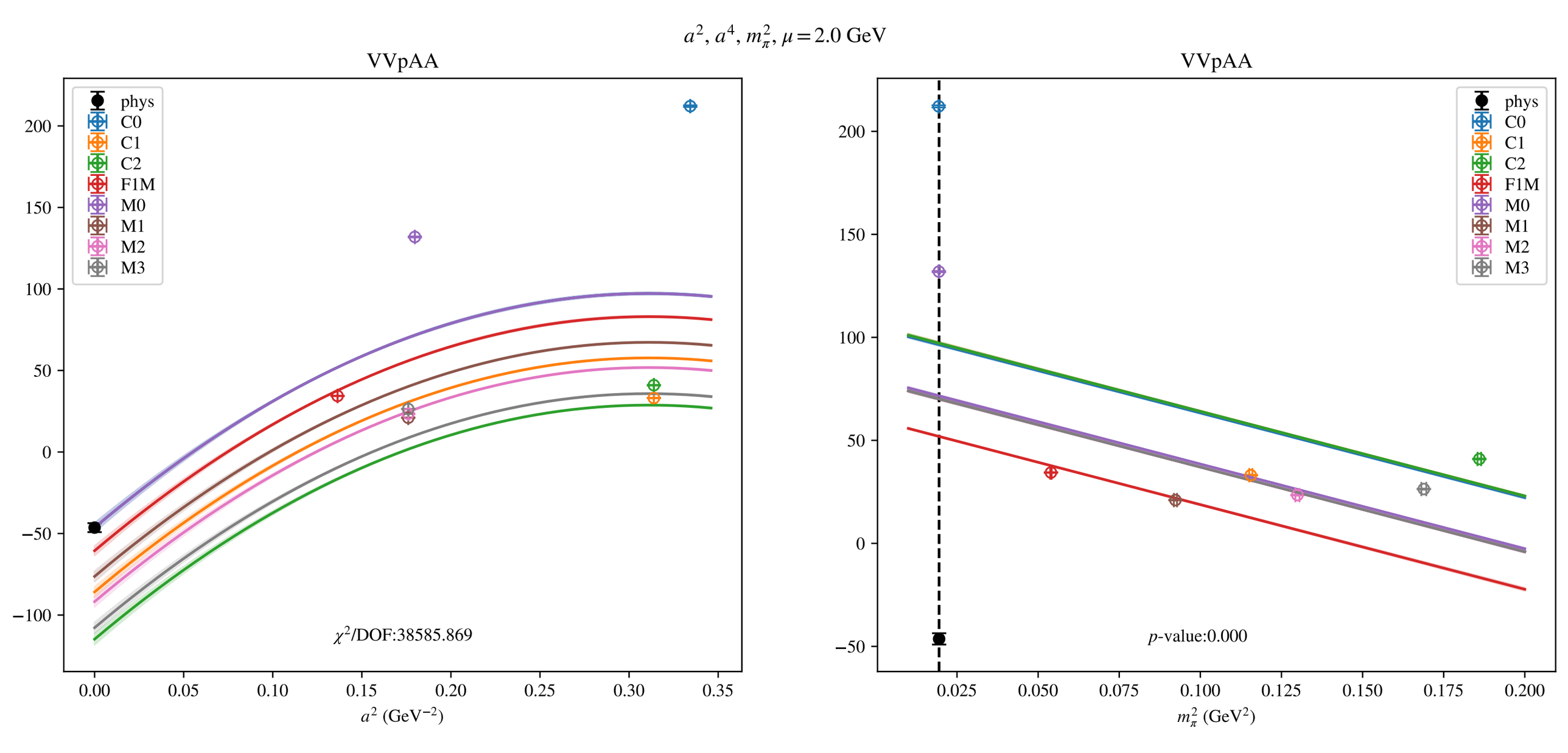
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000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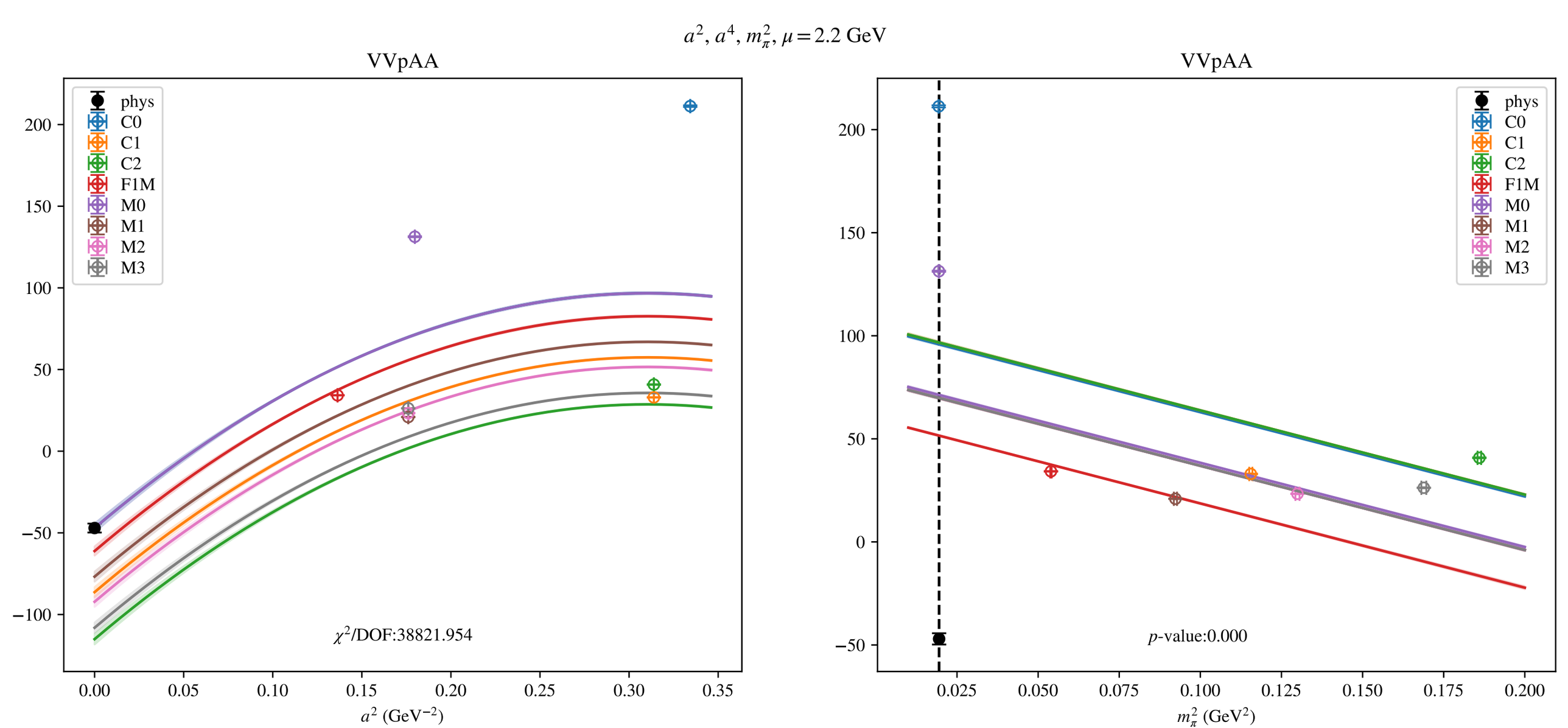
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
−50
−100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:38585.869
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
−50
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000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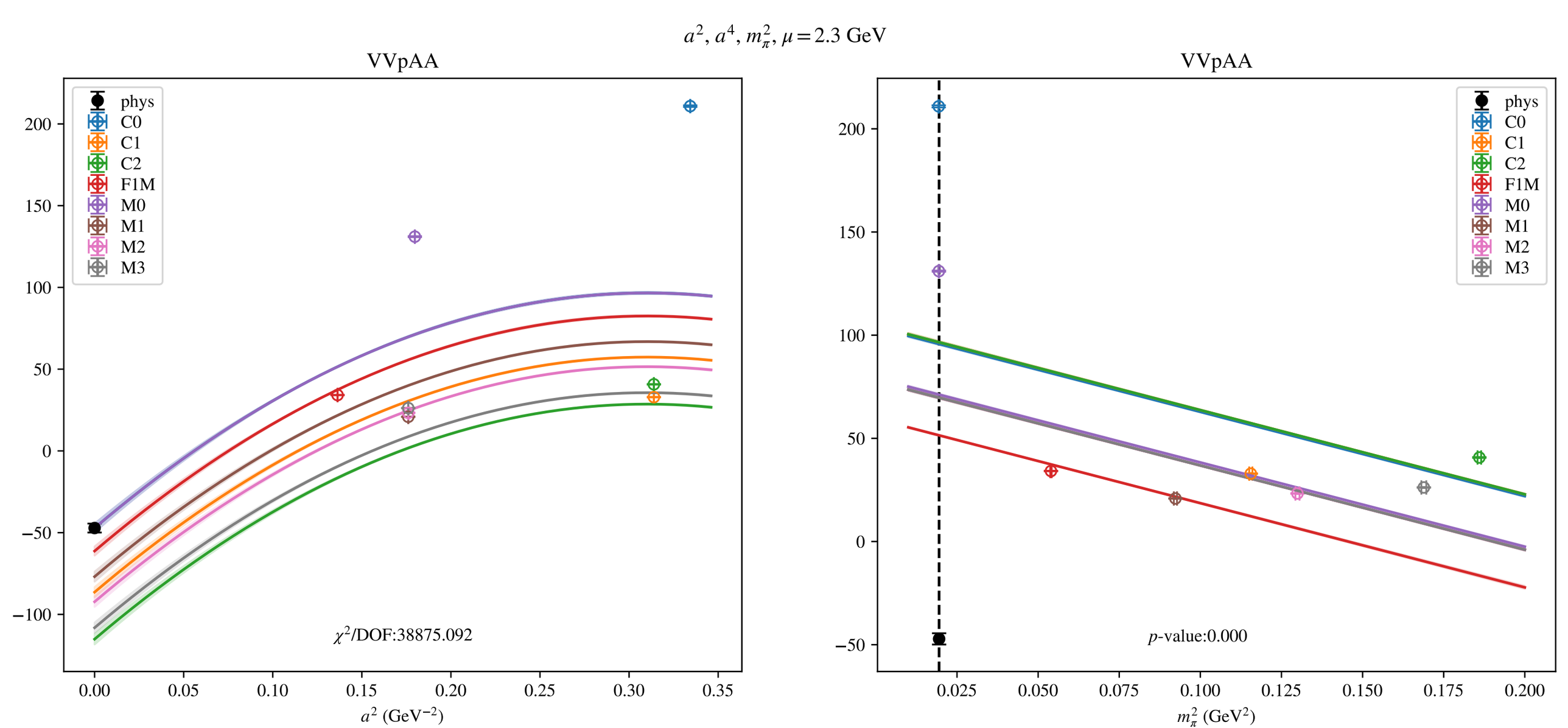
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
−50
−100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:38821.954
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
−50
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000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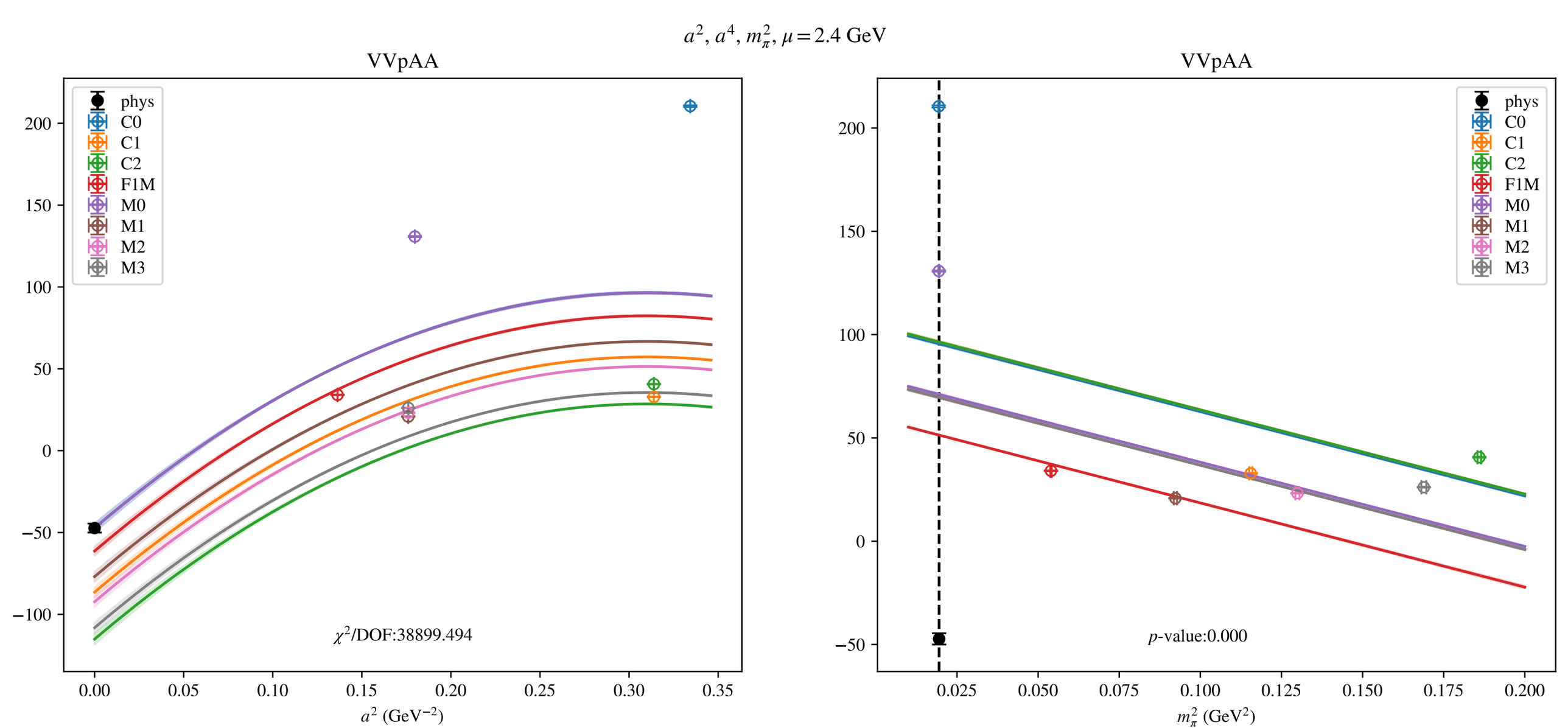
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
−50
−100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:38875.092
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
−50
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000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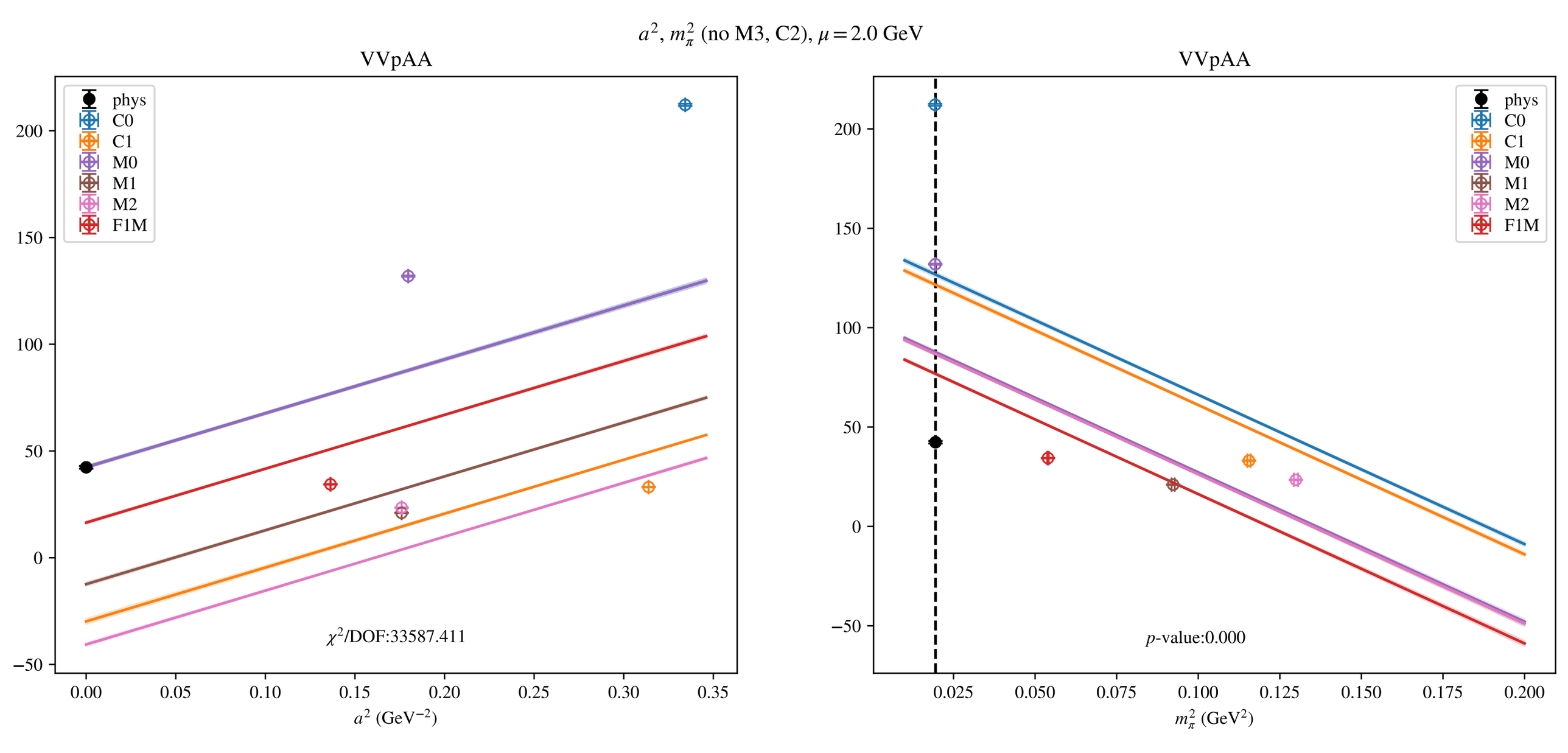
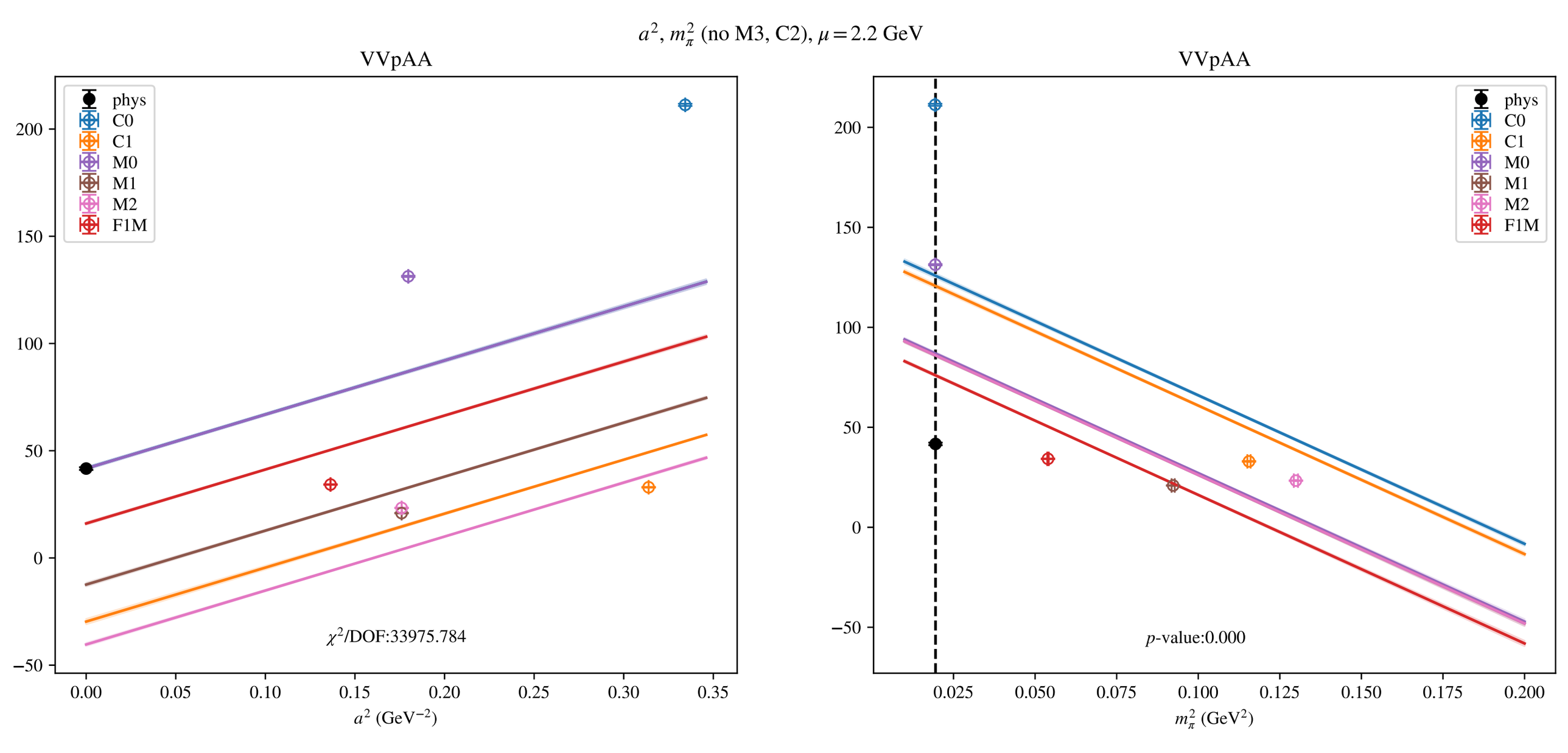
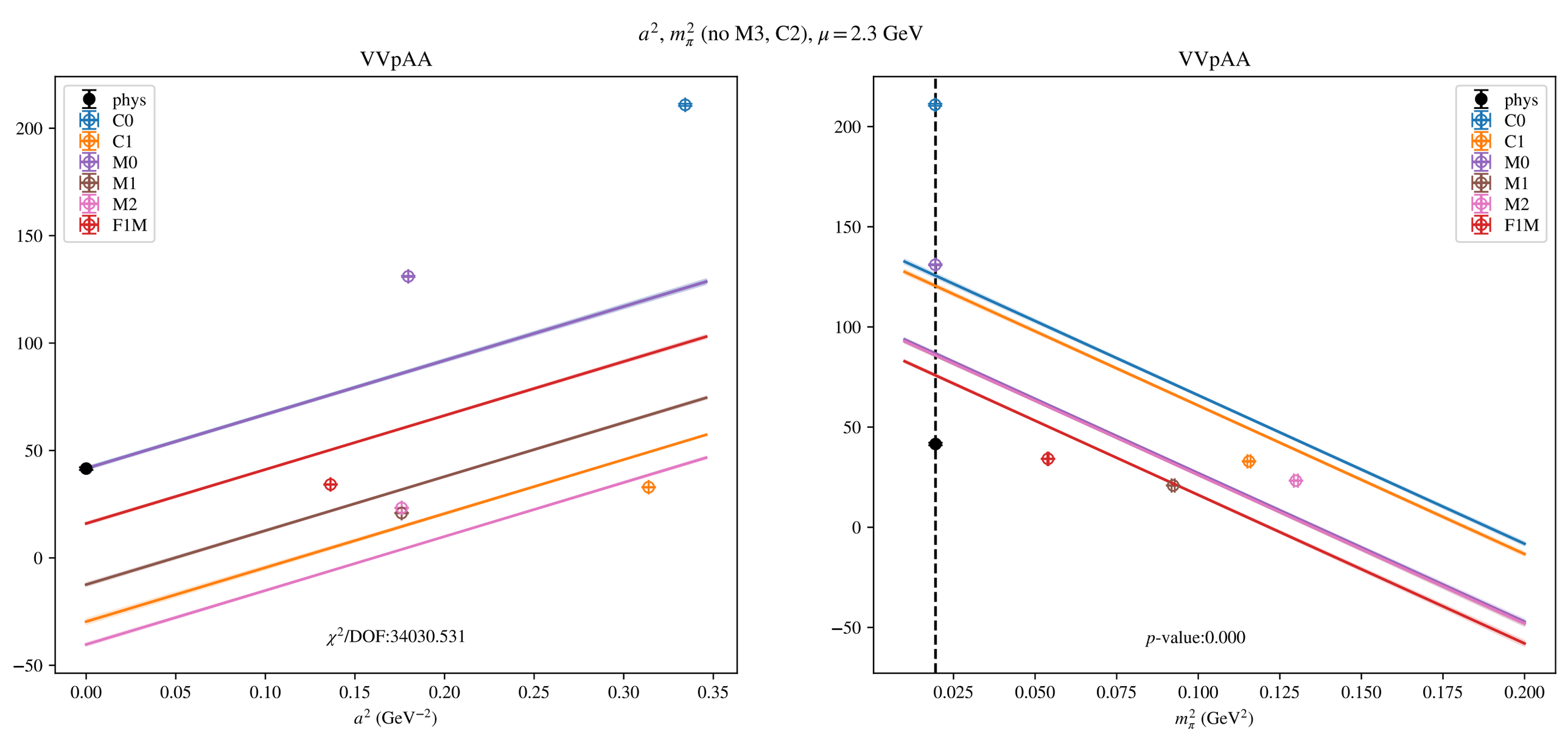
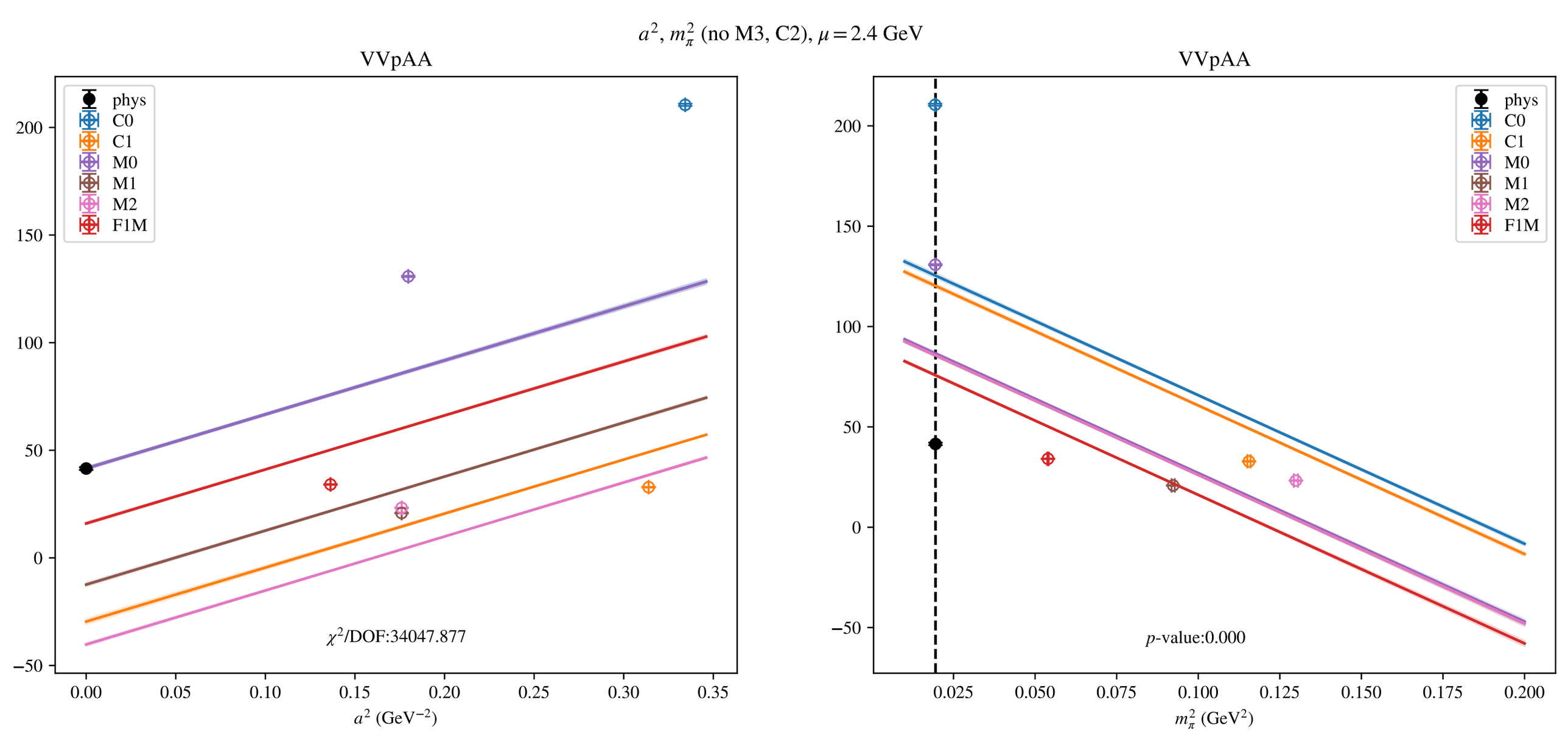
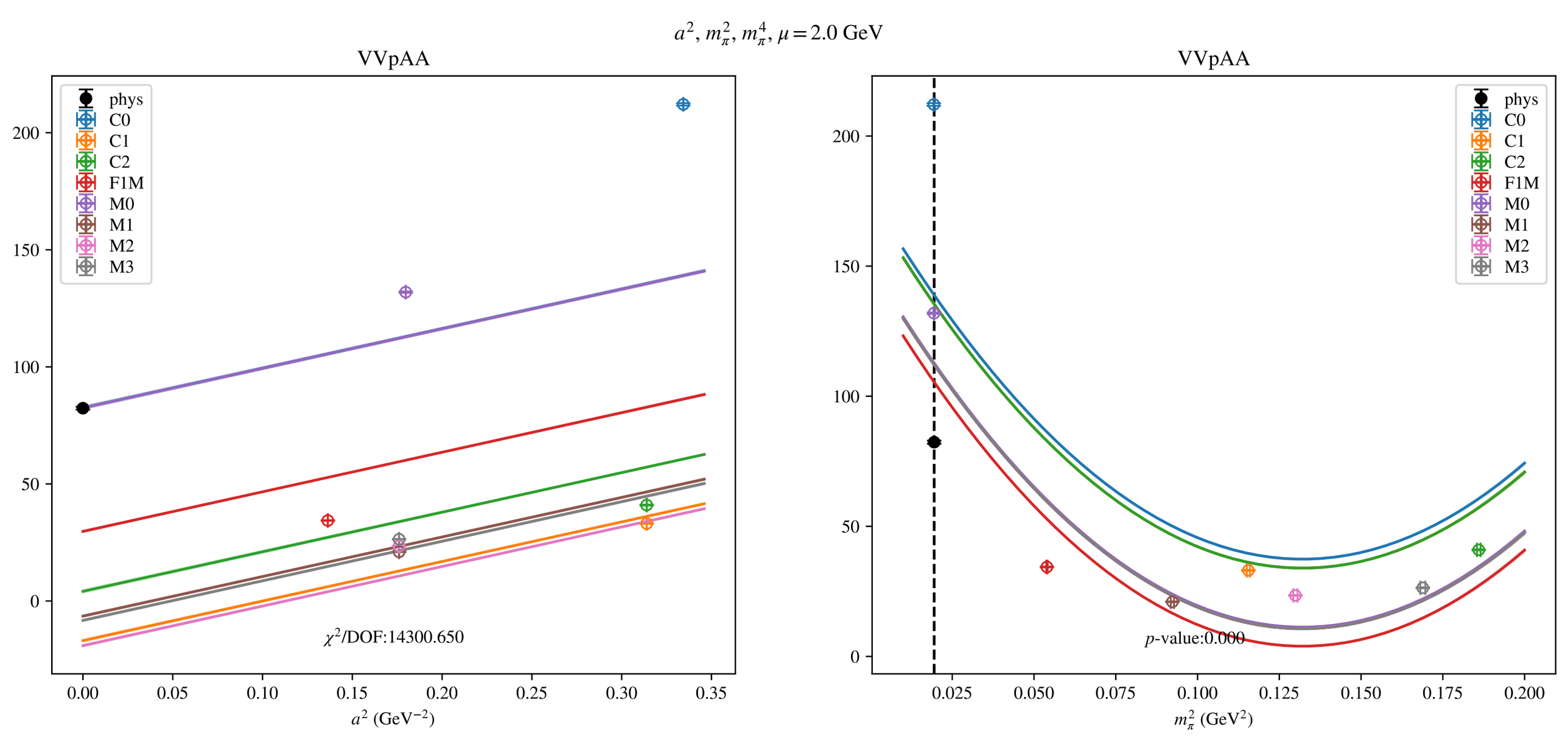
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
−50
−100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:38899.494
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
−50
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

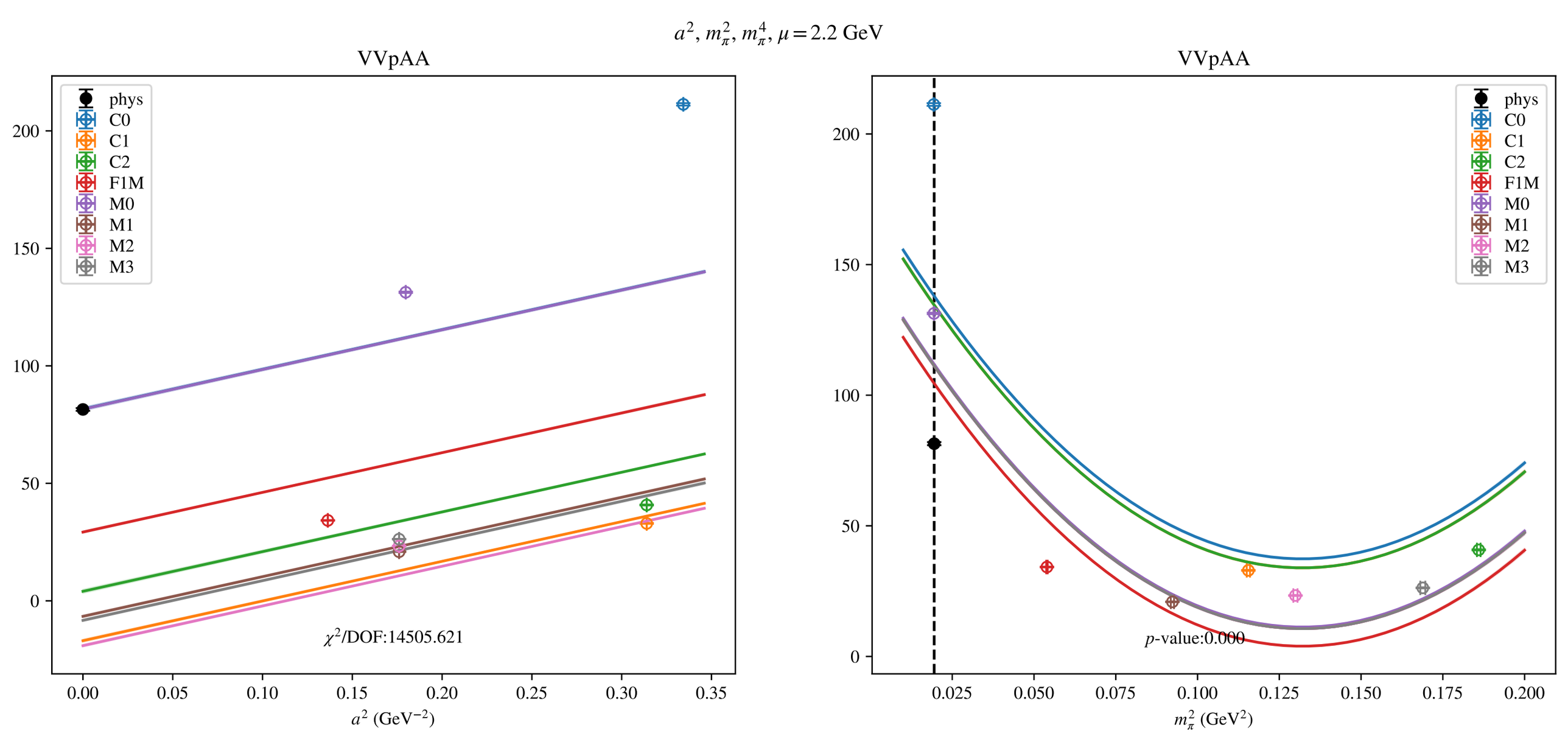

a^2, m_π^2 (no M3, C2), $\mu = 2.0$ GeV
VVpAA
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:33587.411
a^2 (GeV^{-2})
VVpAA
p-value:0.000
m_π^2 (GeV^2)

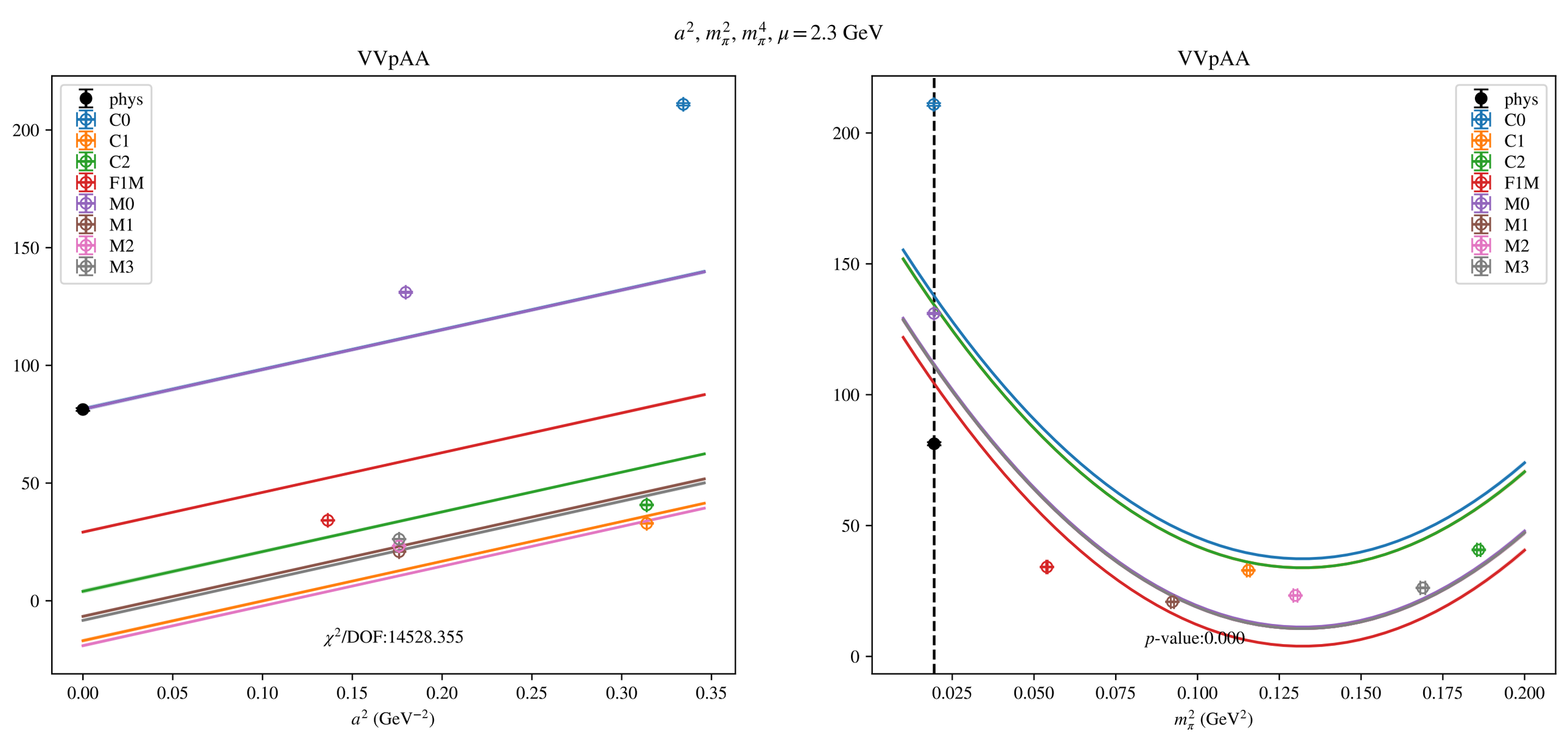

a^2, m_π^2 (no M3, C2), $\mu = 2.2$ GeV
VVpAA
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:33975.784
a^2 (GeV^{-2})
VVpAA
p-value:0.000
m_π^2 (GeV^2)

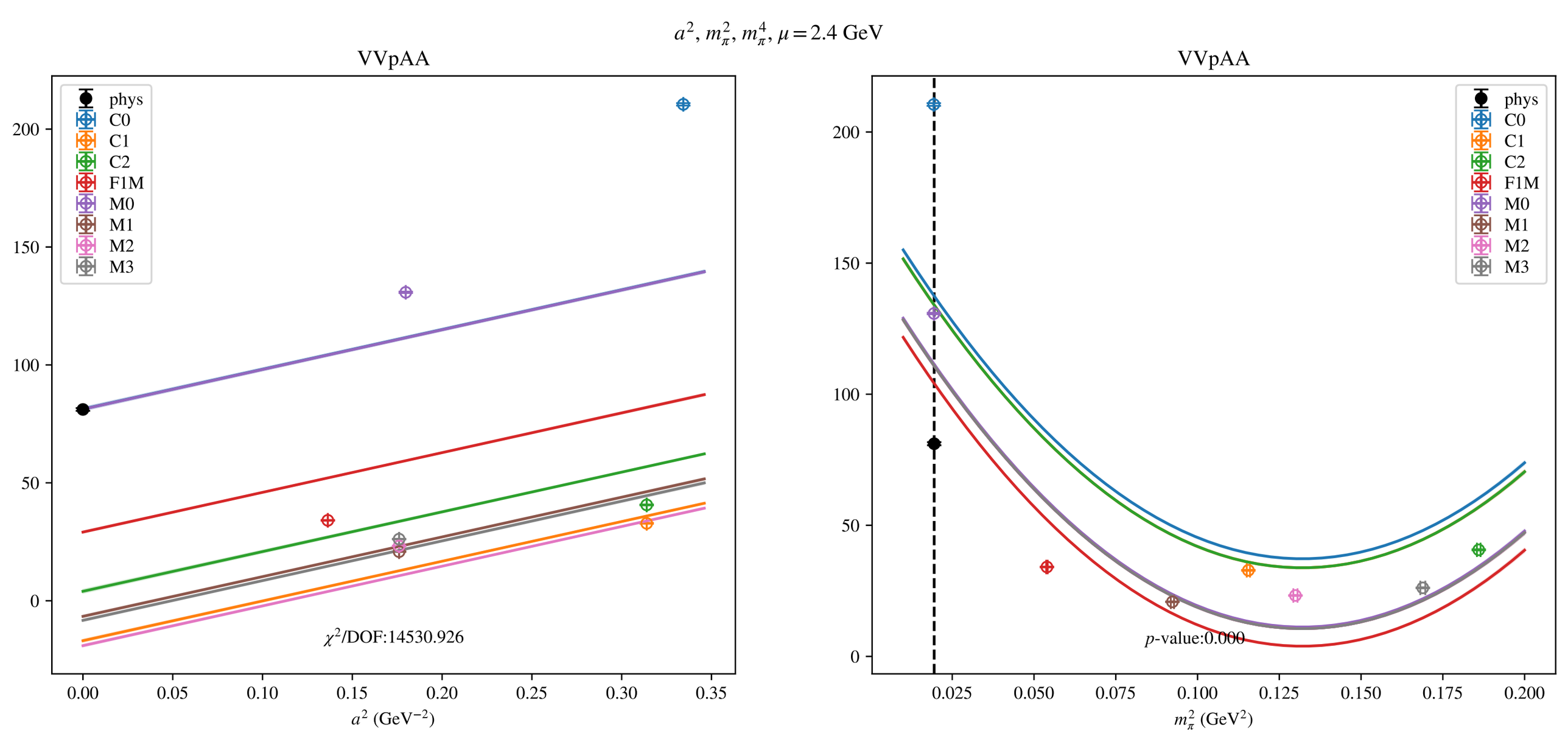

a^2, m_π^2 (no M3, C2), $\mu = 2.3$ GeV
VVpAA
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:34030.531
a^2 (GeV^{-2})
VVpAA
phys
C0
C1
M0
M1
M2
F1M
p-value:0.000
m_π^2 (GeV^2)


a^2, m_π^2 (no M3, C2), $\mu = 2.4$ GeV
VVpAA
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:34047.877
a^2 (GeV^{-2})
VVpAA
phys
C0
C1
M0
M1
M2
F1M
p-value:0.000
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.0$ GeV
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
χ^2/DOF:14300.650
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
VVpAA
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.2$ GeV
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
χ^2/DOF:14505.621
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
VVpAA
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.3$ GeV
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
χ^2/DOF:14528.355
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
VVpAA
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.4$ GeV
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
χ^2/DOF:14530.926
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
VVpAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
200
150
100
50
0
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)

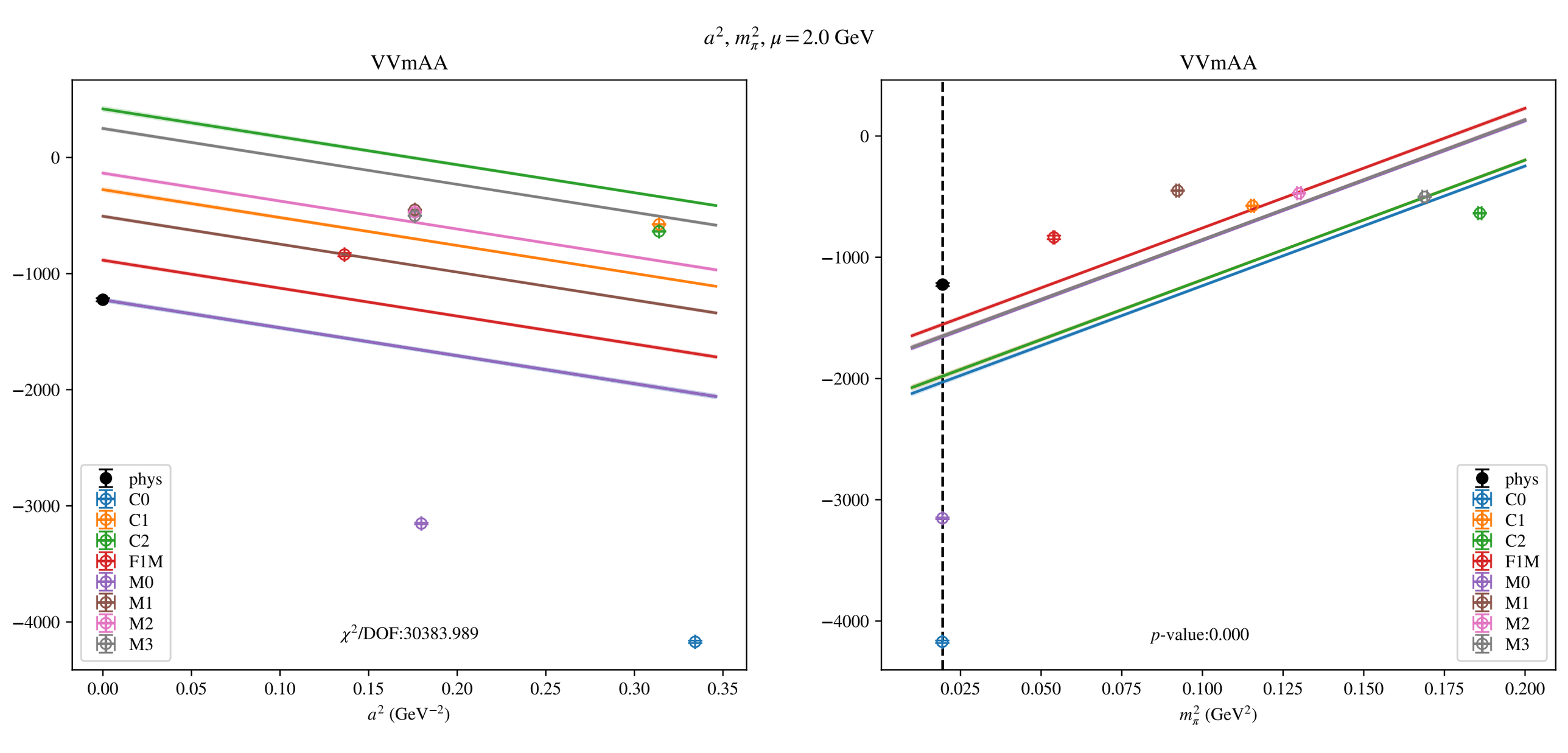
2 $\mathcal{B}_2$

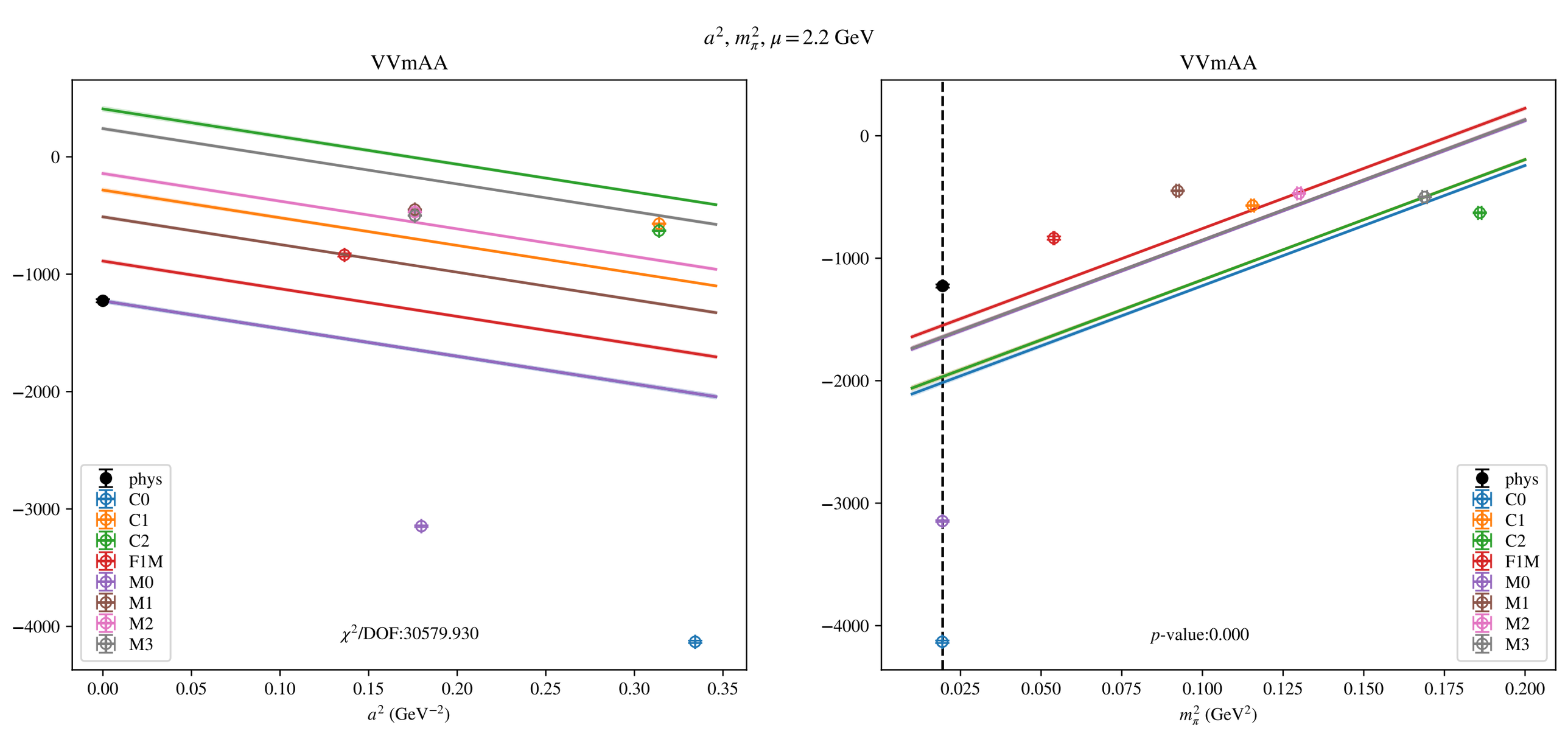
μ (GeV)	a^2, m_π^2	a^2, m_π^2 (no C)	a^2, a^4, m_π^2	a^2, m_π^2 (no M3, C2)	a^2, m_π^2, m_π^4
2.0	**-1(13)**: 30383.989 (0.0)	**29(63)**: 40323.739 (0.0)	**-(12)**: 36955.597 (0.0)	**-2(30)**: 23066.07 (0.0)	**-2(16)**: 6265.725 (0.0)
2.2	**-1(13)**: 30579.93 (0.0)	**27(61)**: 40704.4 (0.0)	**-(12)**: 37303.488 (0.0)	**-2(30)**: 23217.938 (0.0)	**-2(15)**: 6322.07 (0.0)
2.3	**-1(13)**: 30627.412 (0.0)	**26(60)**: 40815.806 (0.0)	**-(12)**: 37408.36 (0.0)	**-2(30)**: 23215.153 (0.0)	**-2(15)**: 6328.523 (0.0)
2.4	**-1(13)**: 30664.362 (0.0)	**26(59)**: 40910.266 (0.0)	**-(12)**: 37493.353 (0.0)	**-2(29)**: 23208.695 (0.0)	**-2(15)**: 6332.774 (0.0)

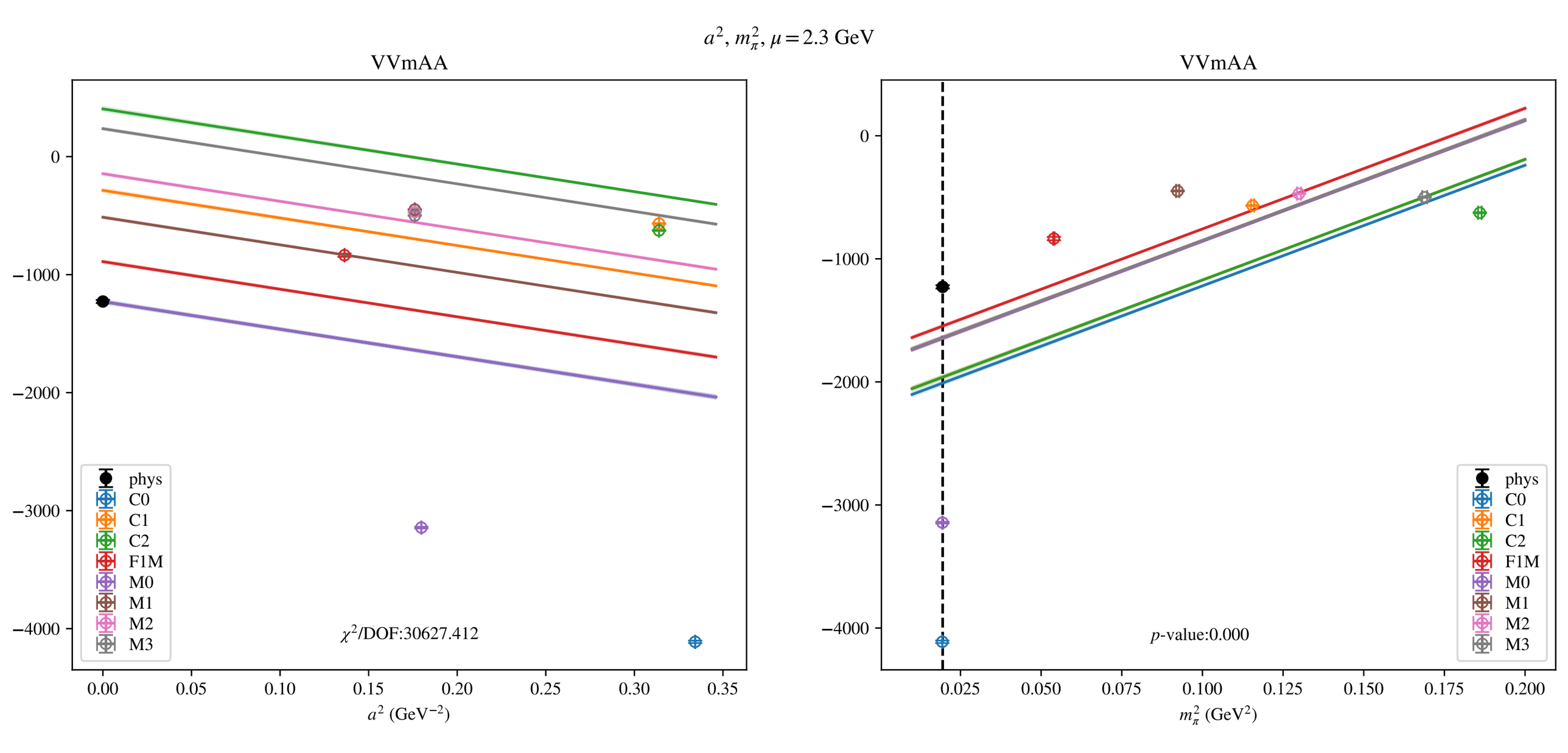
Table 3: Physical point value from chiral and continuum extrapolation at renormalisation scale μ. Entries are **value(error)**: χ^2/DOF (p-value).

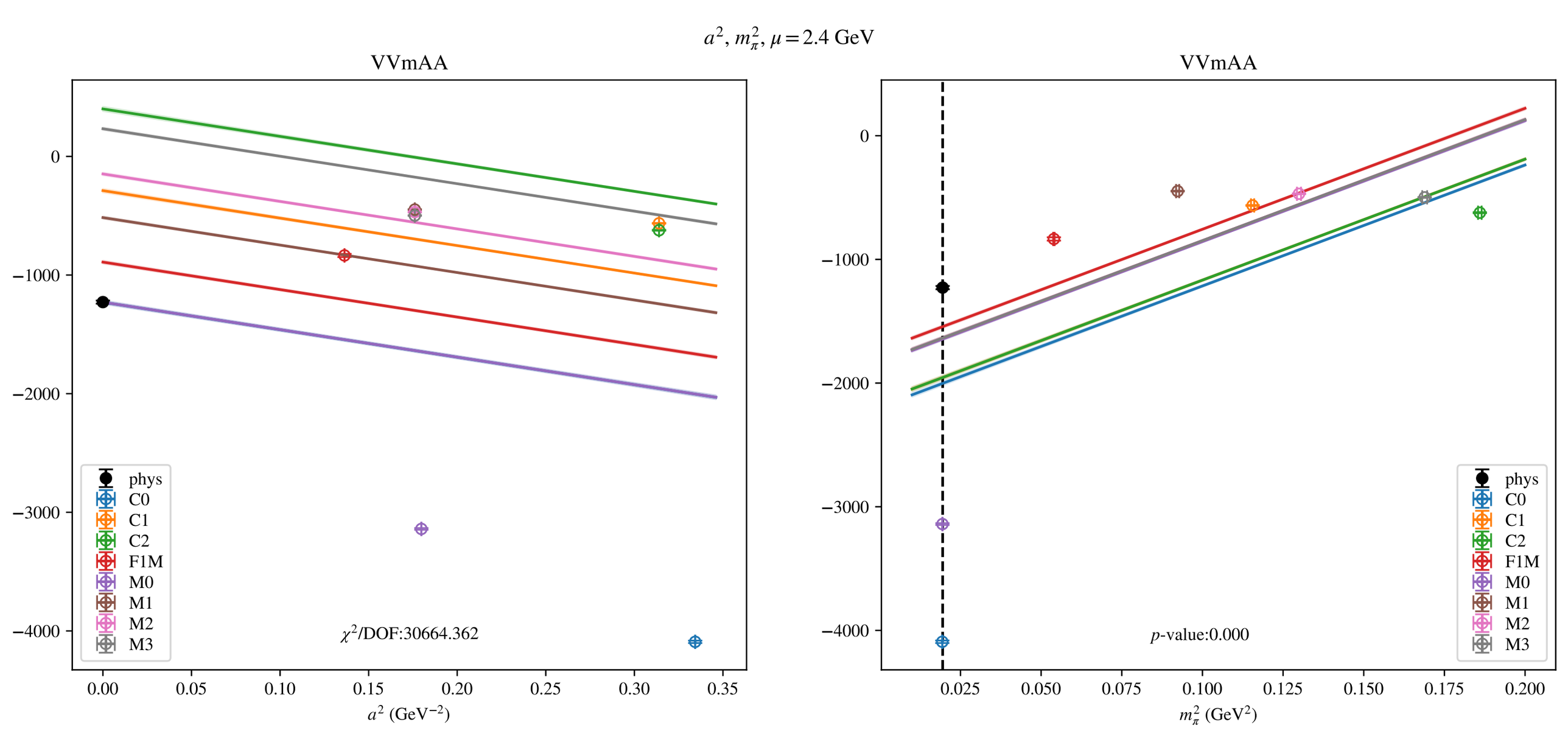
μ (GeV)		a^2, m_π^2	a^2, m_π^2 (no C)	a^2, a^4, m_π^2	a^2, m_π^2 (no M3, C2)	a^2, m_π^2, m_π^4
2.0	α	1.963(57)	-8.78(70)	-7.19(38)	1.165(64)	0.416(18)
	β	-0.137(15)	0.0538(14)	-0.0208(55)	-0.181(23)	-0.326(17)
2.2	α	1.922(56)	-9.00(75)	-7.06(42)	1.139(64)	0.401(18)
	β	-0.136(15)	0.0576(15)	-0.0223(61)	-0.180(23)	-0.325(17)
2.3	α	1.903(56)	-9.10(78)	-7.00(44)	1.130(63)	0.395(18)
	β	-0.135(15)	0.0593(15)	-0.0230(63)	-0.180(23)	-0.325(17)
2.4	α	1.885(55)	-9.19(80)	-6.95(46)	1.121(63)	0.389(18)
	β	-0.135(15)	0.0610(16)	-0.0237(66)	-0.180(23)	-0.324(17)

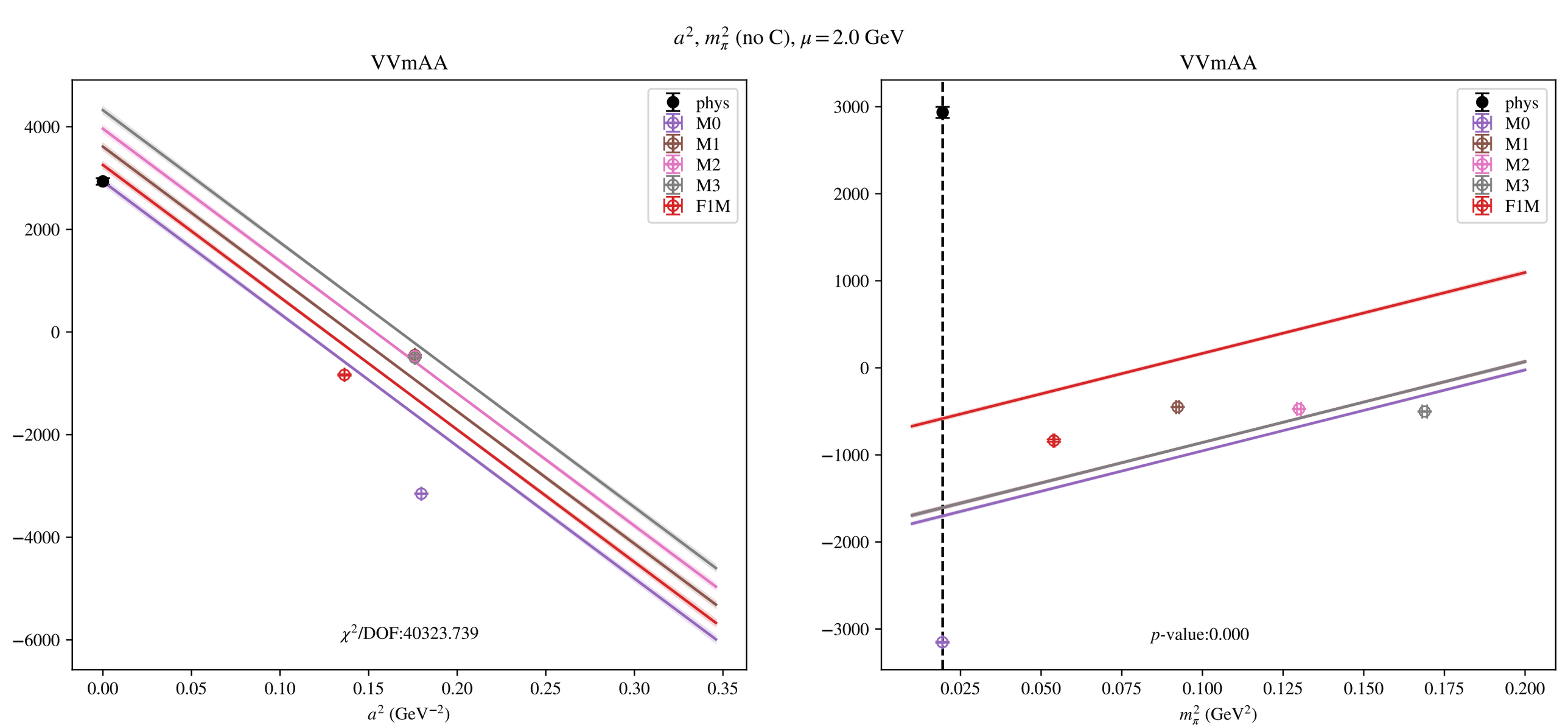
Table 4: Fit values of coefficients in $Q = Q_{phys} + \alpha a^2 + \beta \left(\frac{m_\pi^2}{f_\pi^2} - \frac{m_{\pi,PDG}^2}{f_\pi^2} \right) + \ldots$.

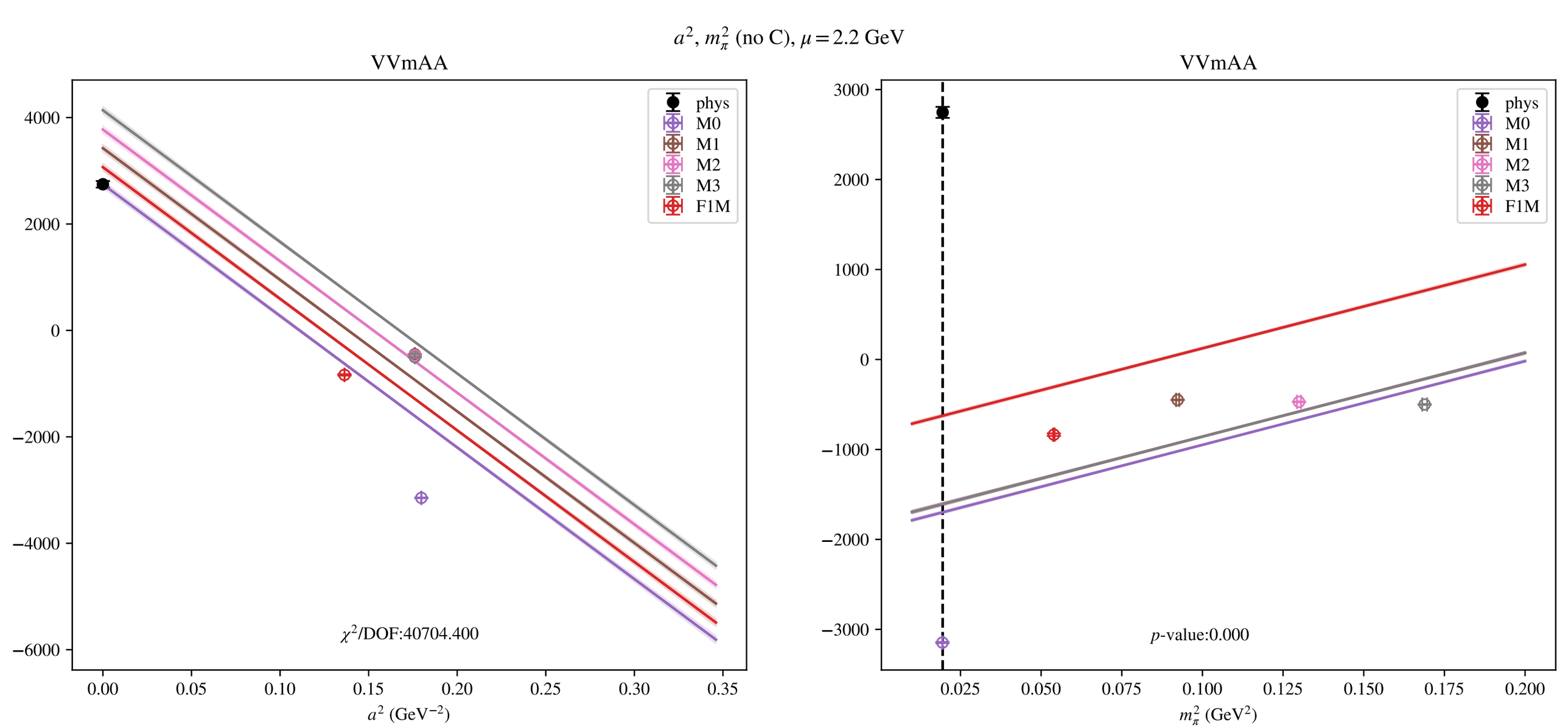

a^2, m_π^2, $\mu = 2.0$ GeV
VVmAA
VVmAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:30383.989
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200


$a^2, m_\pi^2, \mu = 2.2$ GeV
VVmAA
VVmAA
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:30579.930
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

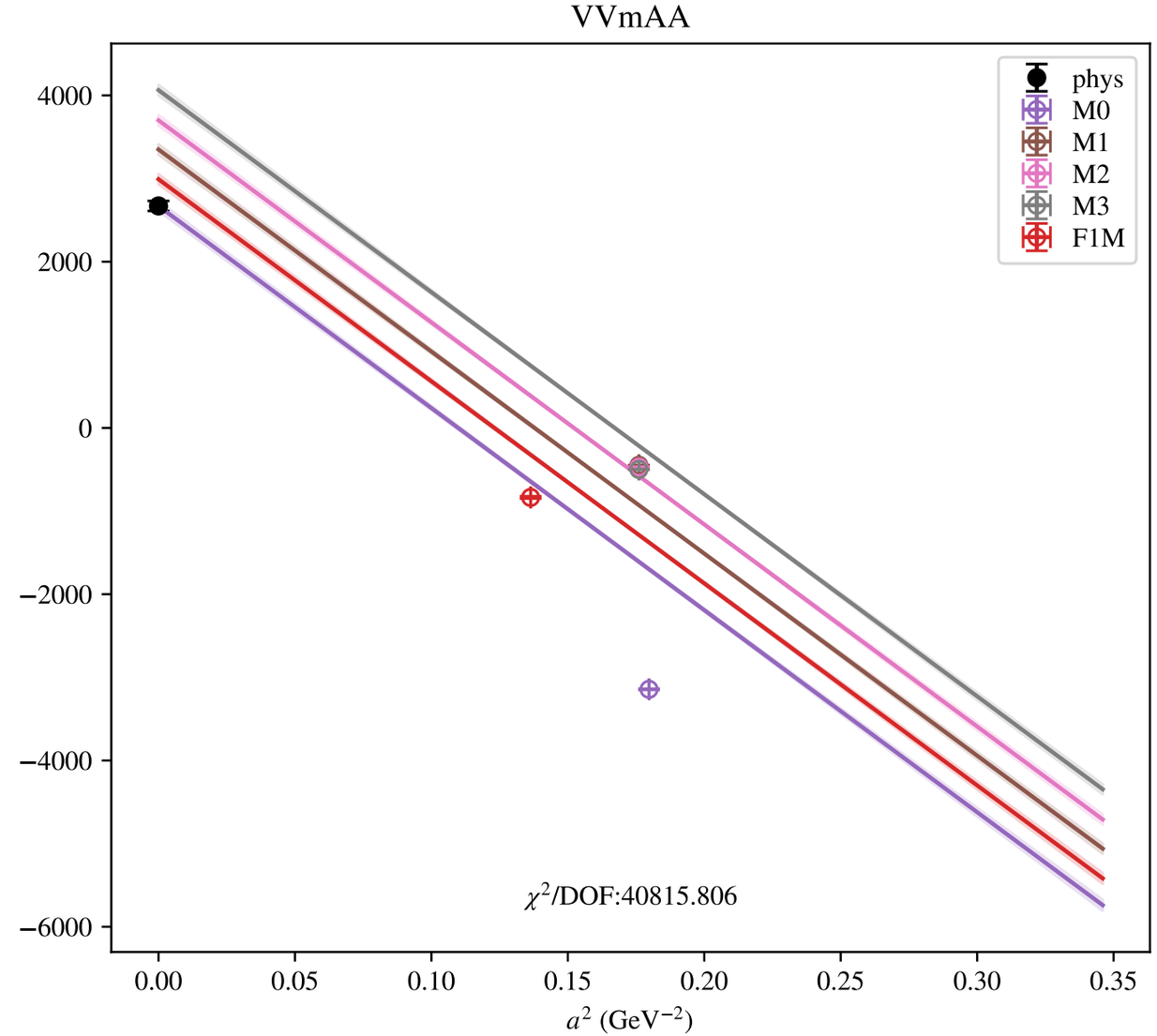

$a^2, m_\pi^2, \mu = 2.3$ GeV
VVmAA
VVmAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:30627.412
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200

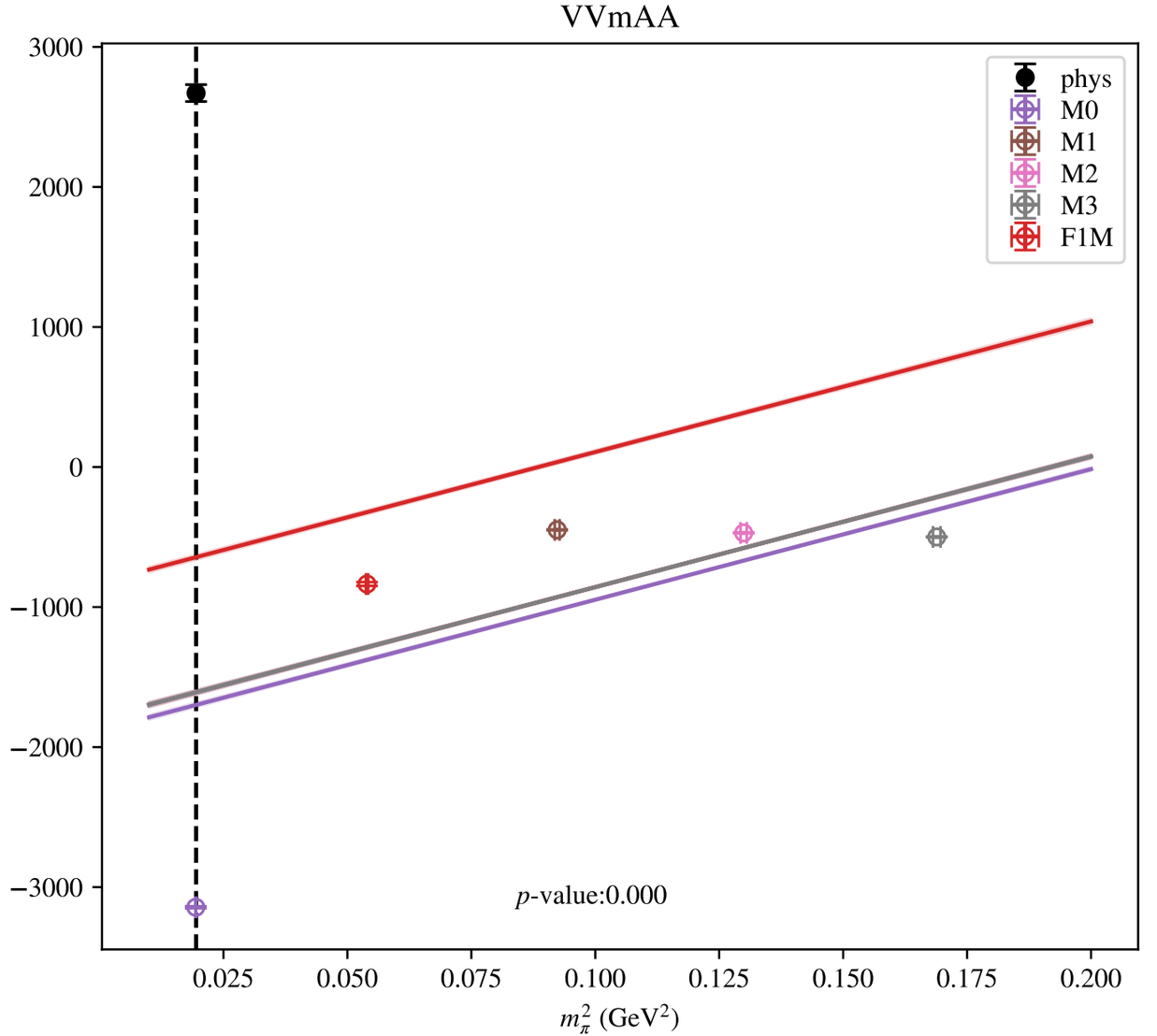

a^2, m_π^2, $\mu = 2.4$ GeV
VVmAA
VVmAA
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:30664.362
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

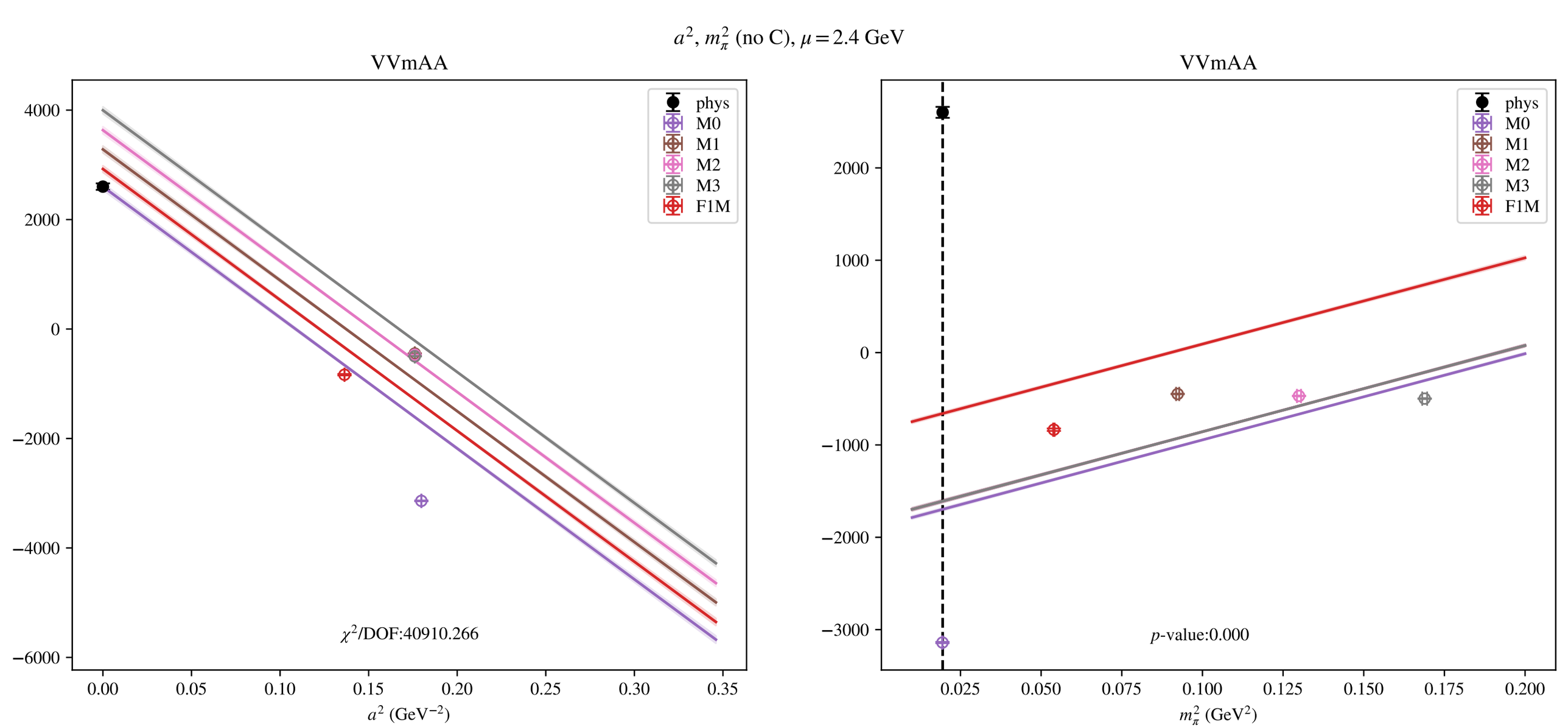

a^2, m_π^2 (no C), $\mu = 2.0$ GeV
VVmAA
phys
M0
M1
M2
M3
F1M
χ^2/DOF:40323.739
a^2 (GeV^{-2})
VVmAA
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

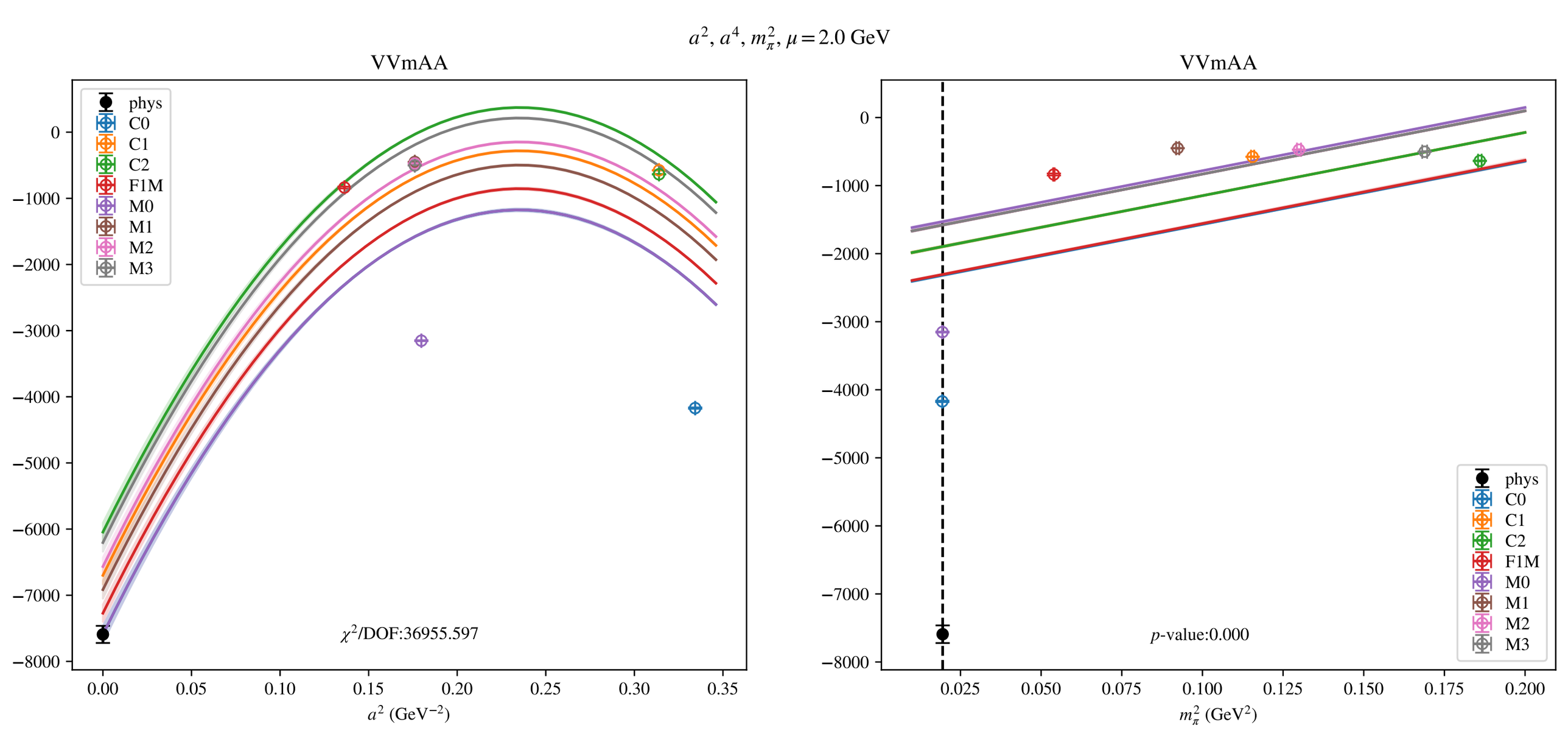

a^2, m_π^2 (no C), $\mu = 2.2$ GeV
VVmAA
phys
M0
M1
M2
M3
F1M
χ^2/DOF:40704.400
a^2 (GeV^{-2})
VVmAA
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

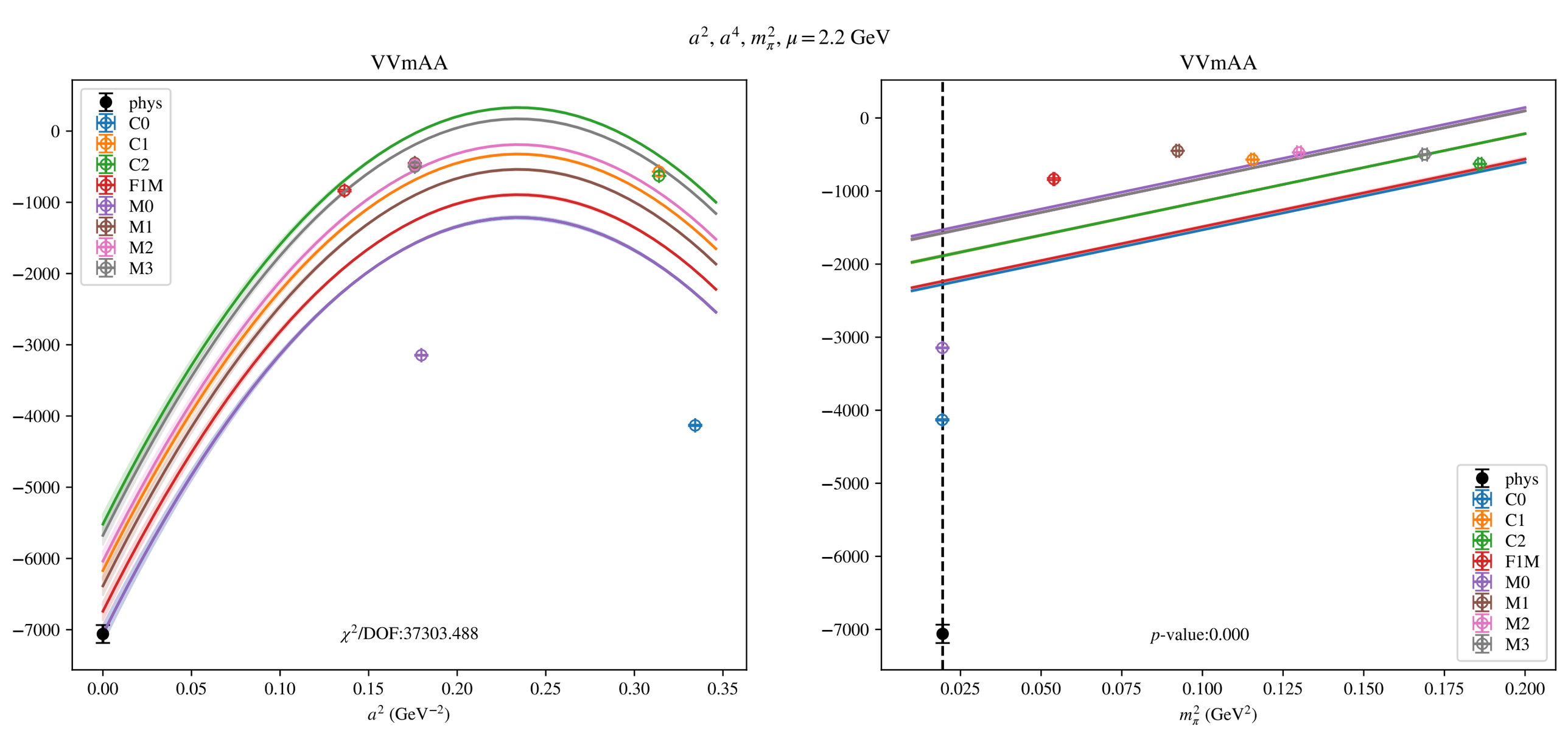
a^2, m_π^2 (no C), $\mu = 2.3$ GeV

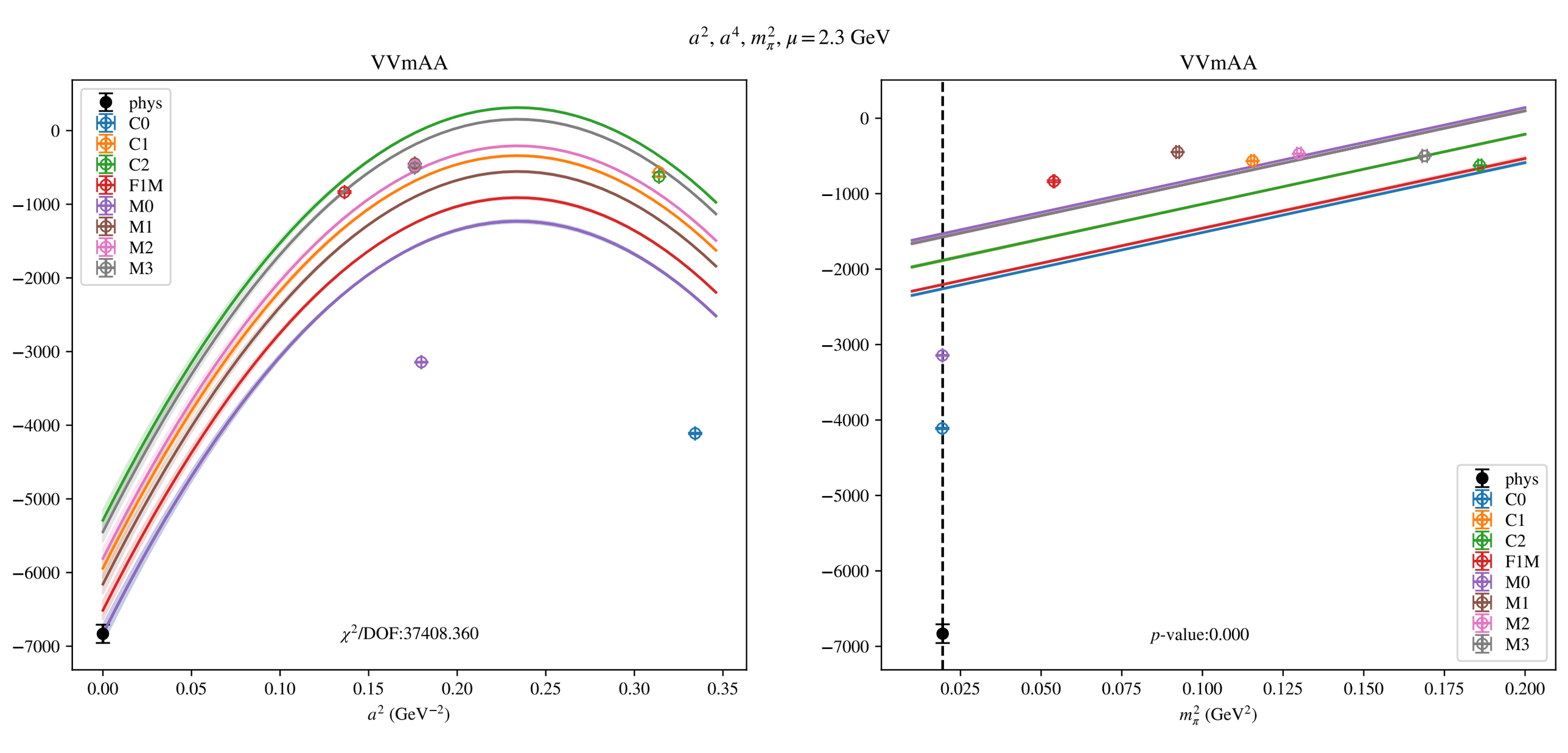




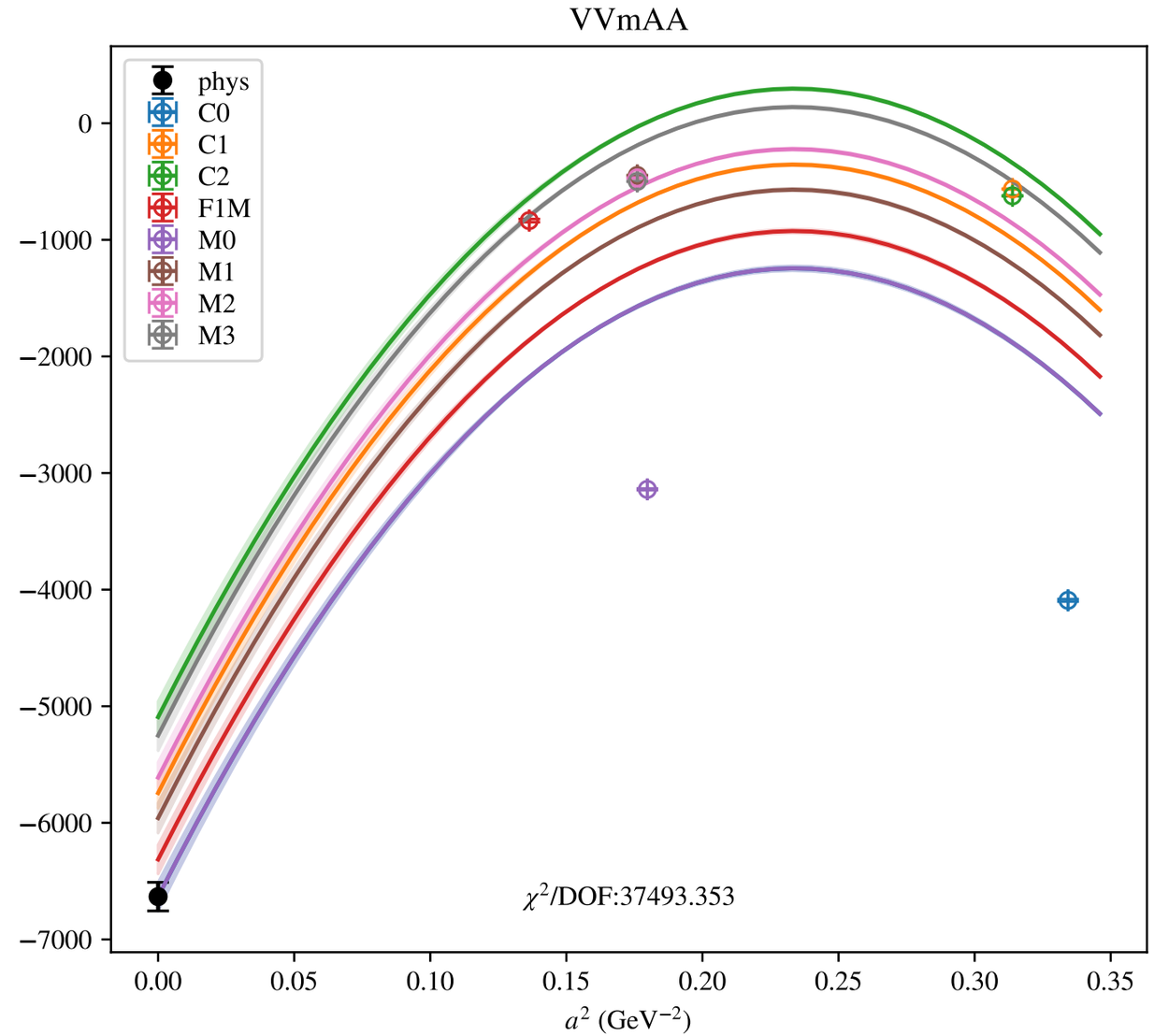

a^2, m_π^2 (no C), $\mu = 2.4$ GeV
VVmAA
phys
M0
M1
M2
M3
F1M
χ^2/DOF:40910.266
a^2 (GeV^{-2})
VVmAA
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

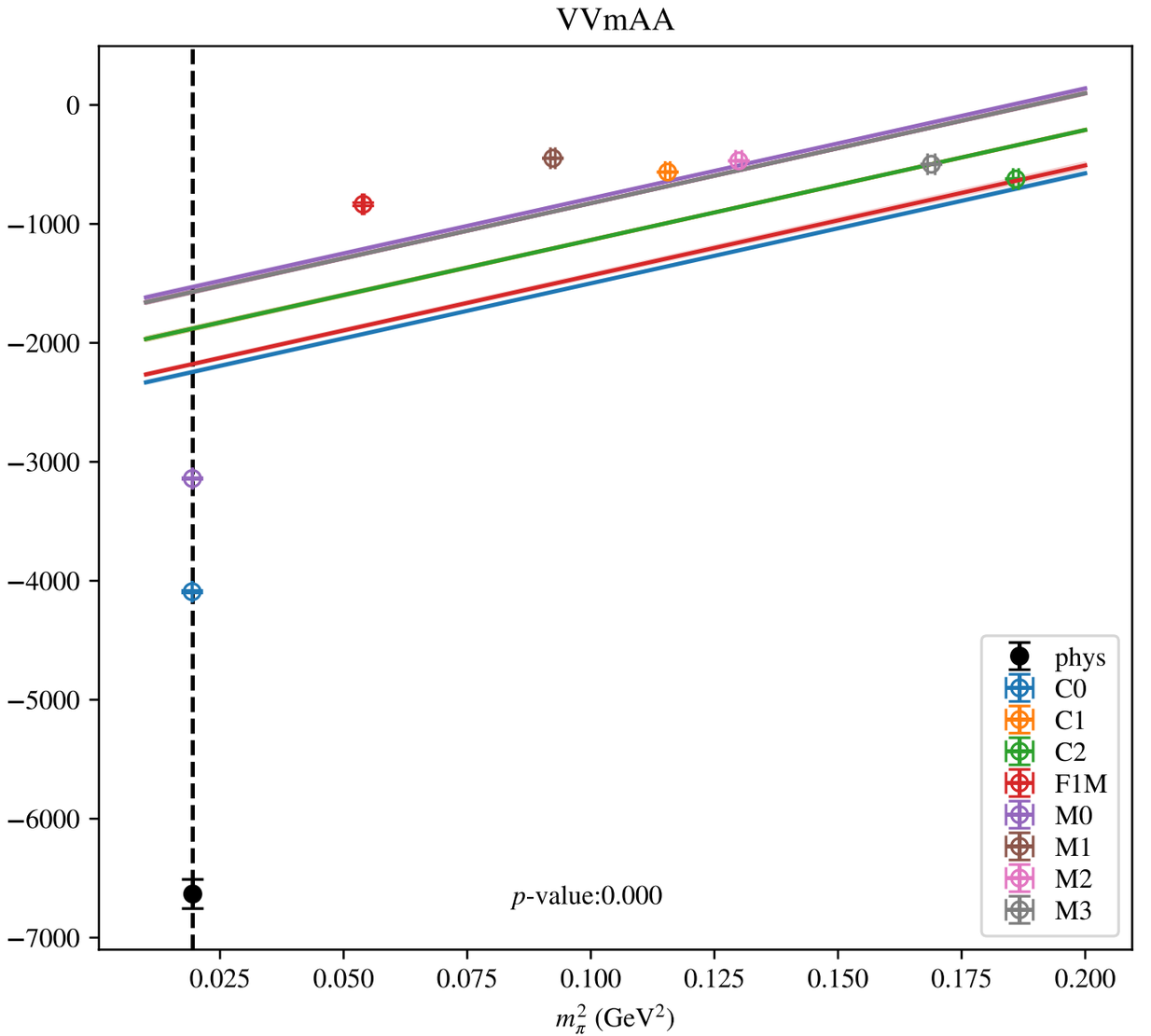

$a^2, a^4, m_\pi^2, \mu = 2.0$ GeV
VVmAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:36955.597
a^2 (GeV^{-2})
VVmAA
p-value:0.000
m_π^2 (GeV^2)

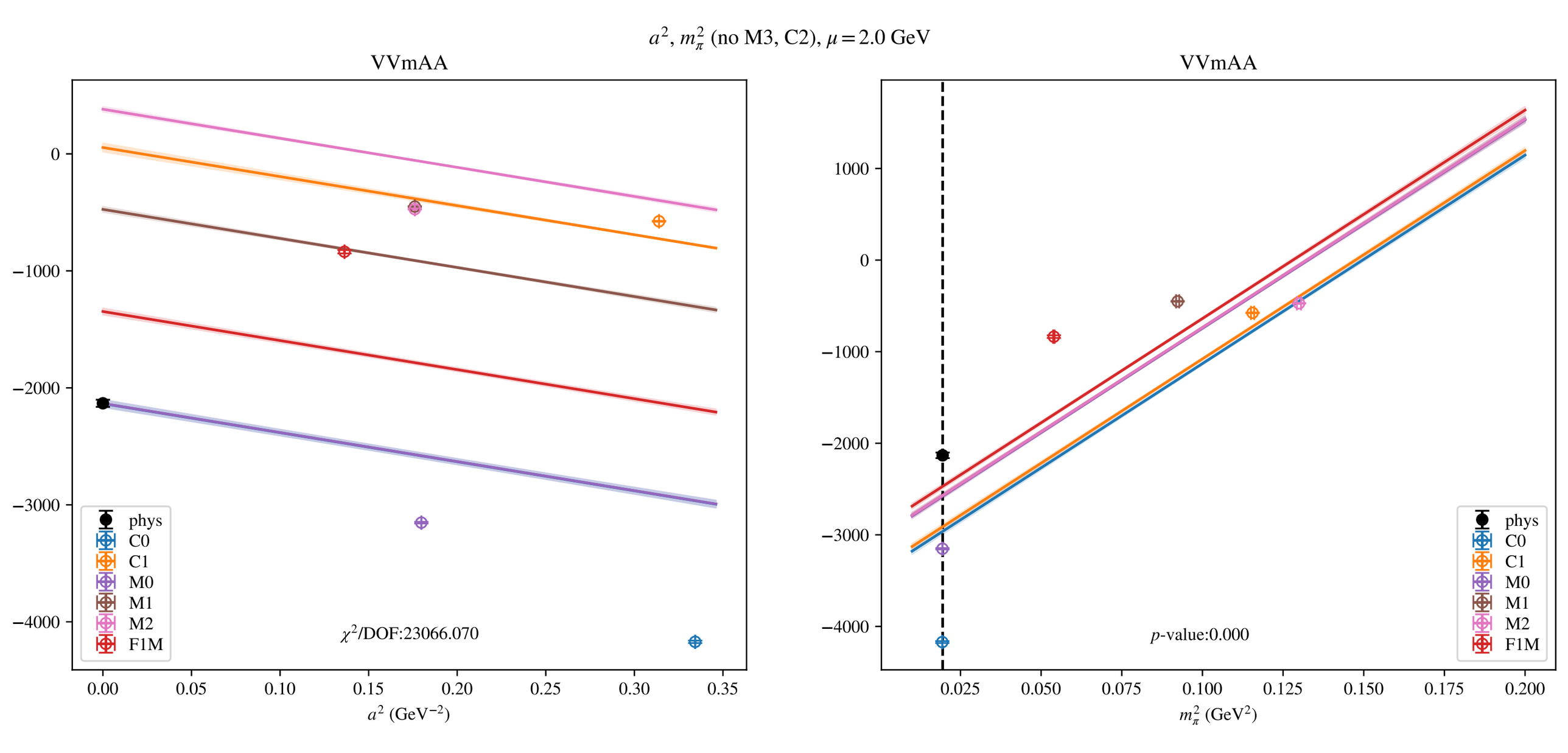

$a^2, a^4, m_\pi^2, \mu = 2.2$ GeV
VVmAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:37303.488
a^2 (GeV^{-2})
VVmAA
p-value:0.000
m_π^2 (GeV^2)

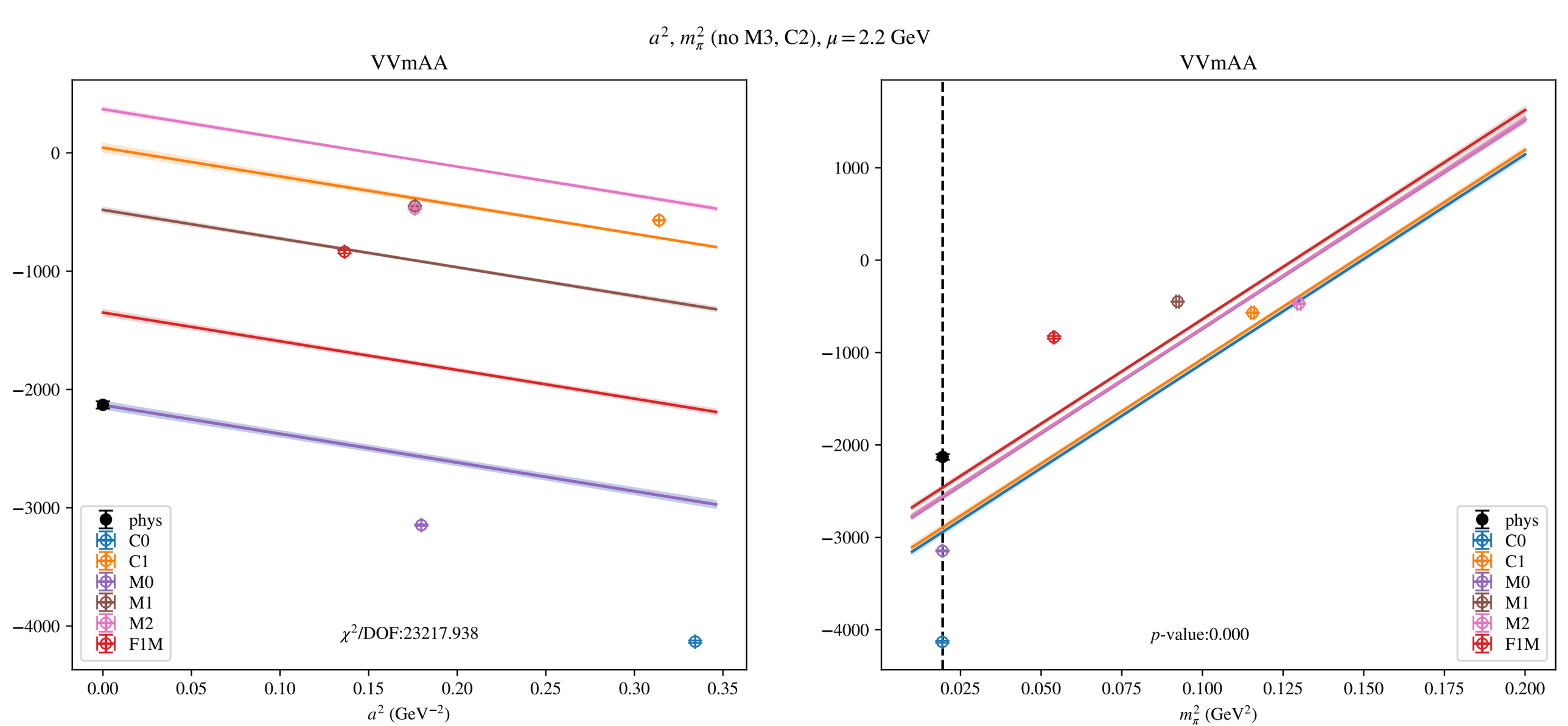

$a^2, a^4, m_\pi^2, \mu = 2.3$ GeV
VVmAA
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:37408.360
a^2 (GeV^{-2})
VVmAA
p-value:0.000
m_π^2 (GeV^2)

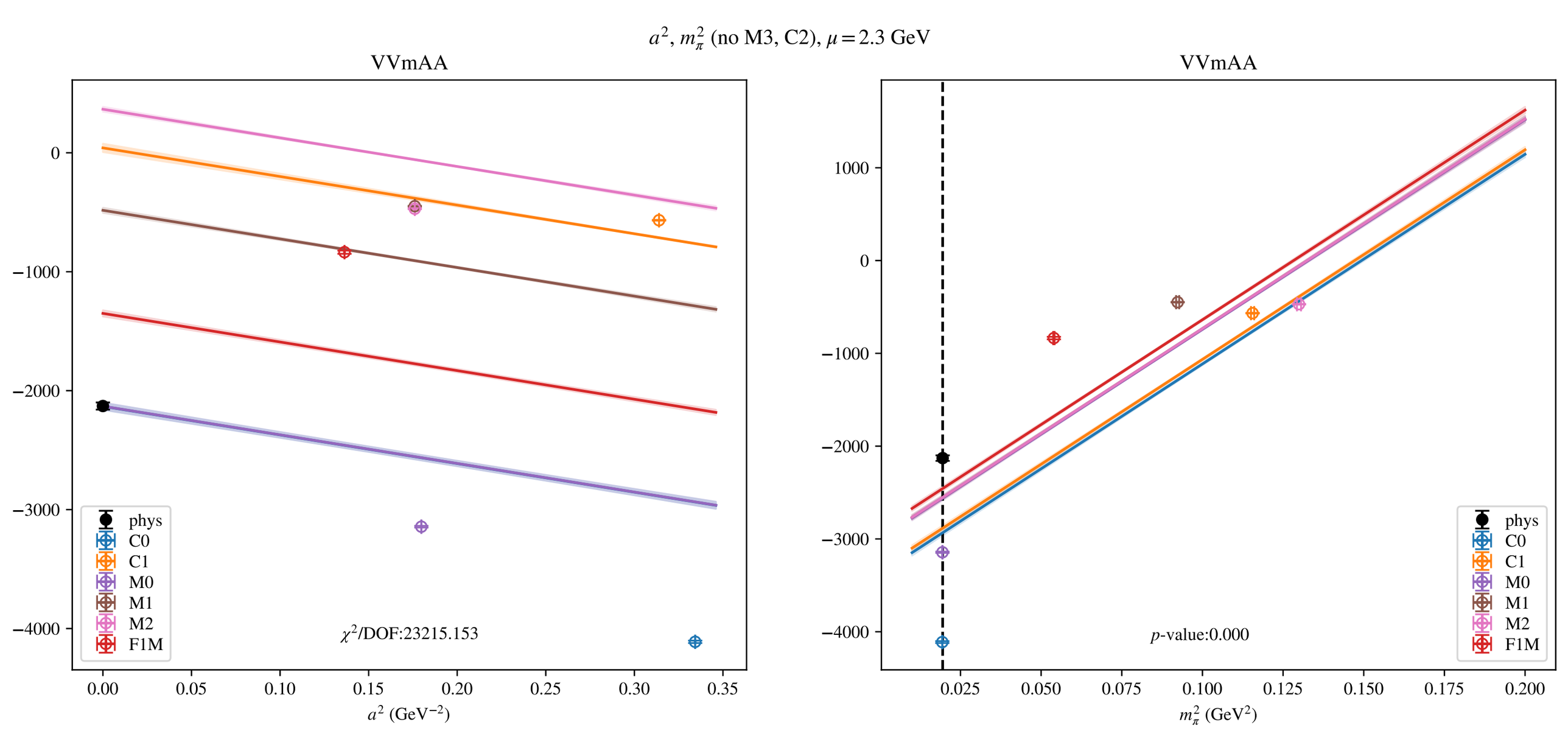
$a^2, a^4, m_\pi^2, \mu = 2.4$ GeV

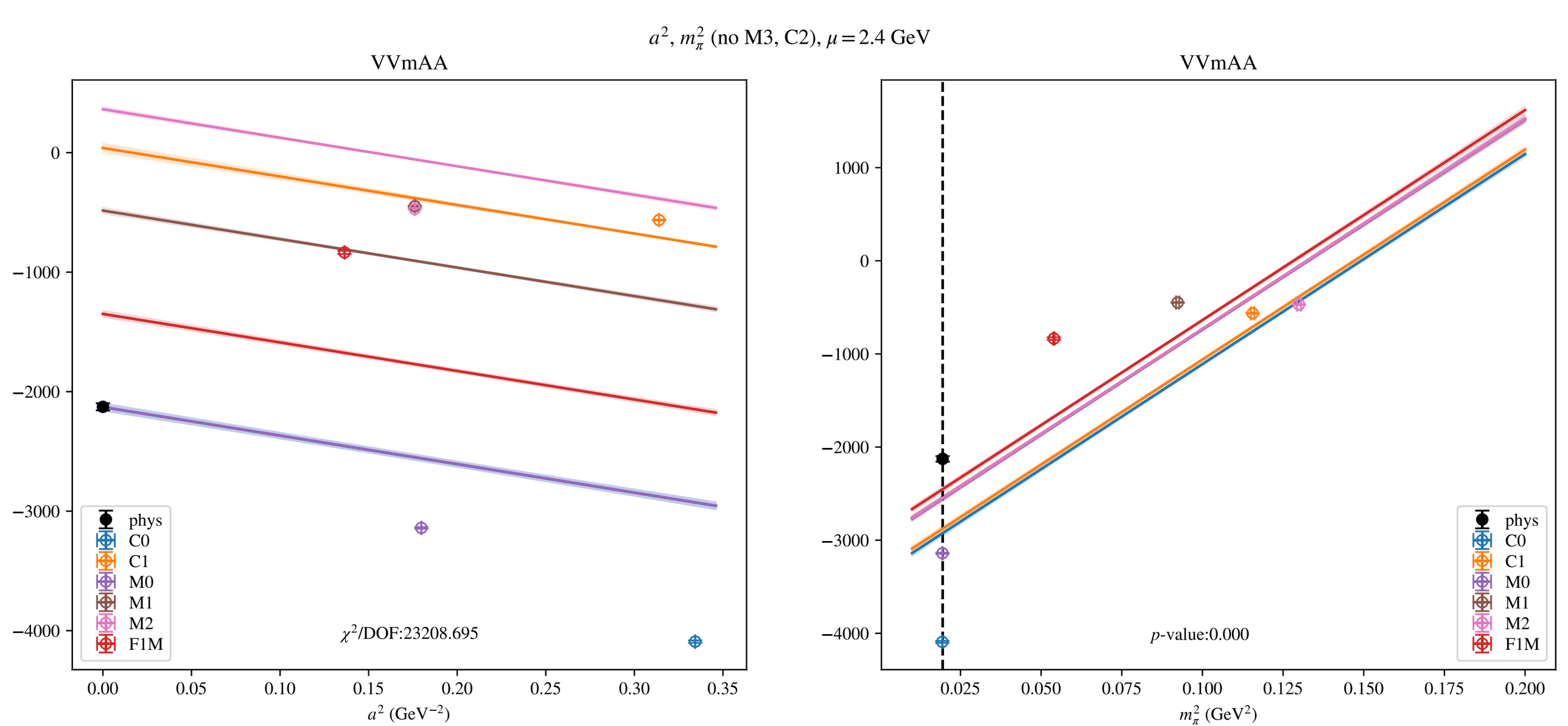


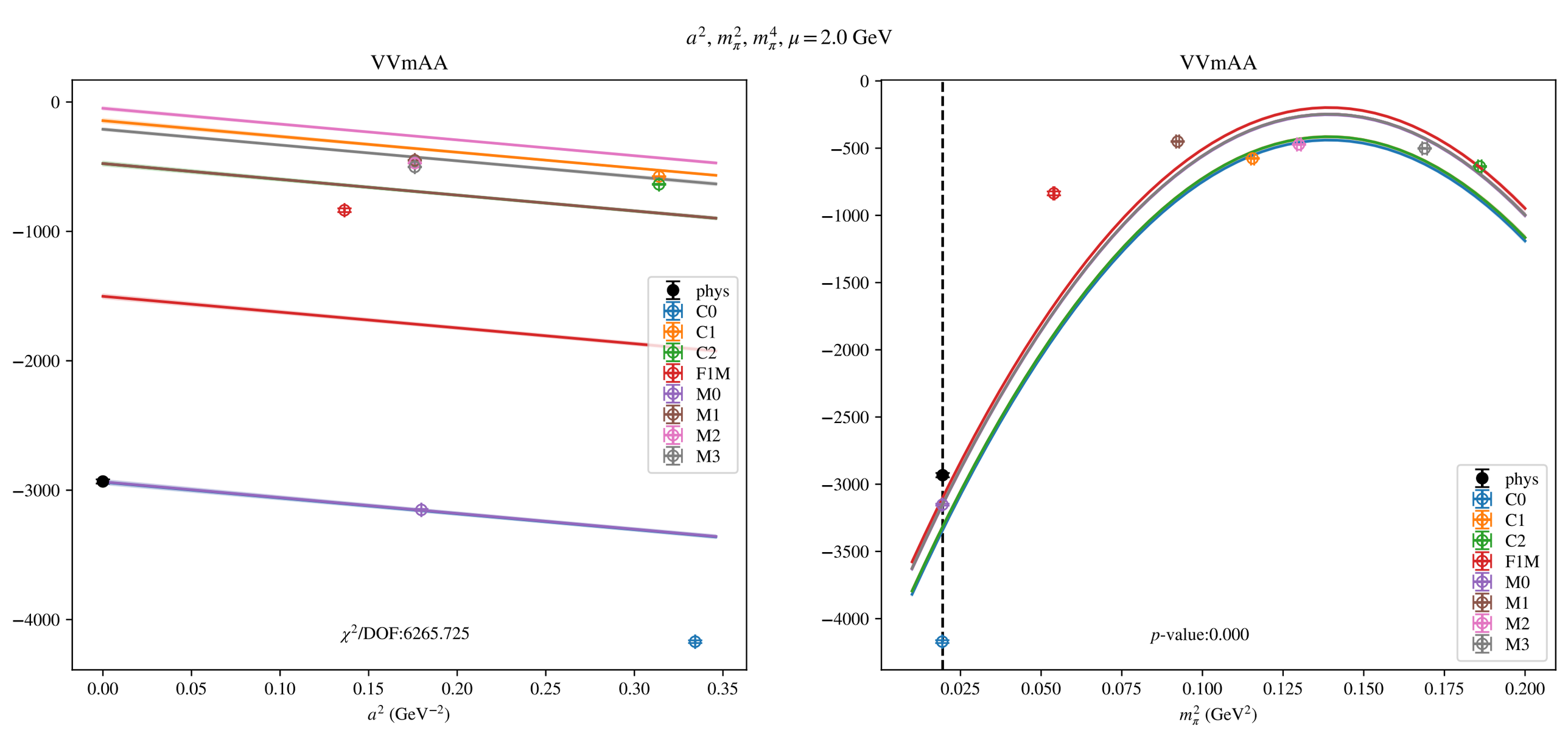


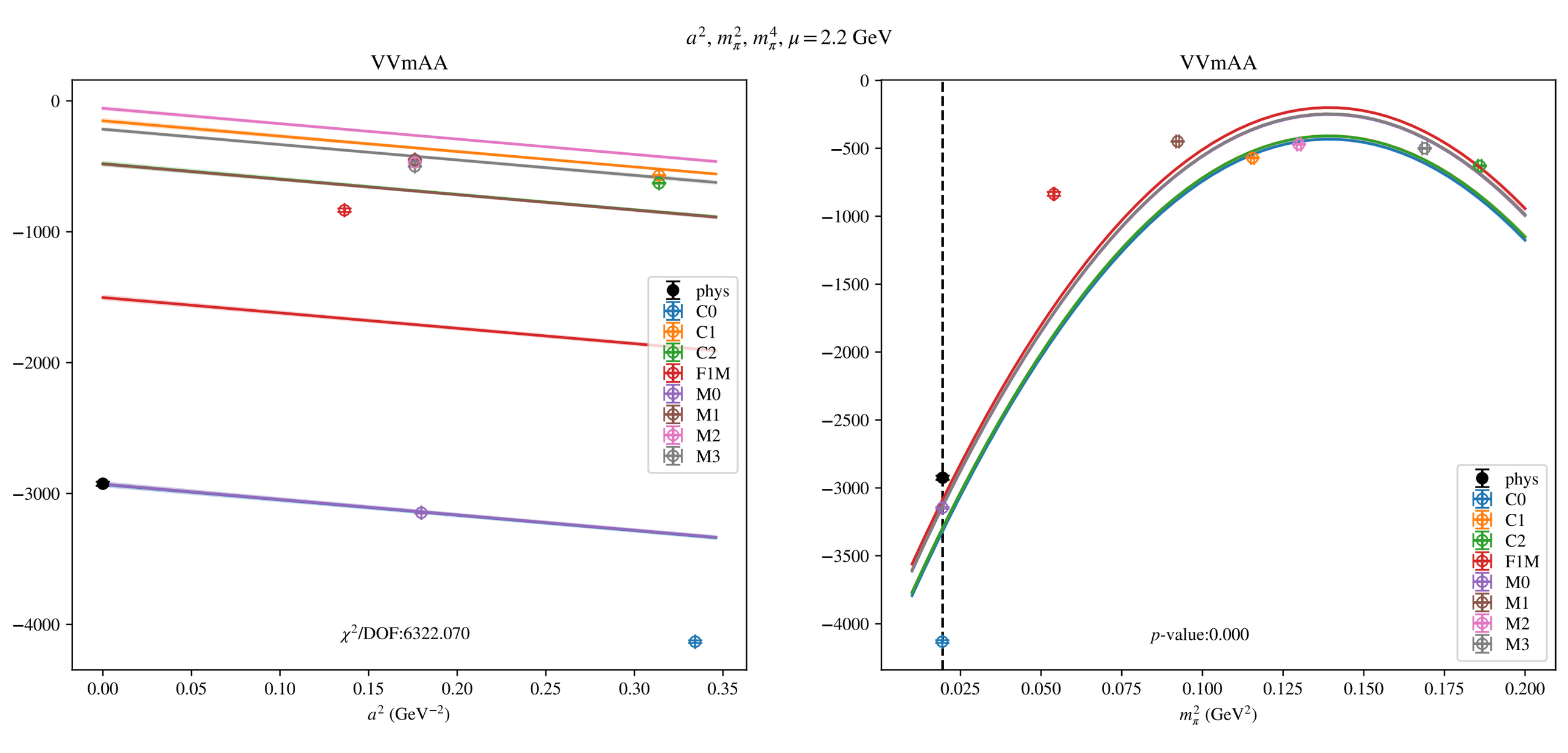

a^2, m_π^2 (no M3, C2), $\mu = 2.0$ GeV
VVmAA
VVmAA
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:23066.070
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)

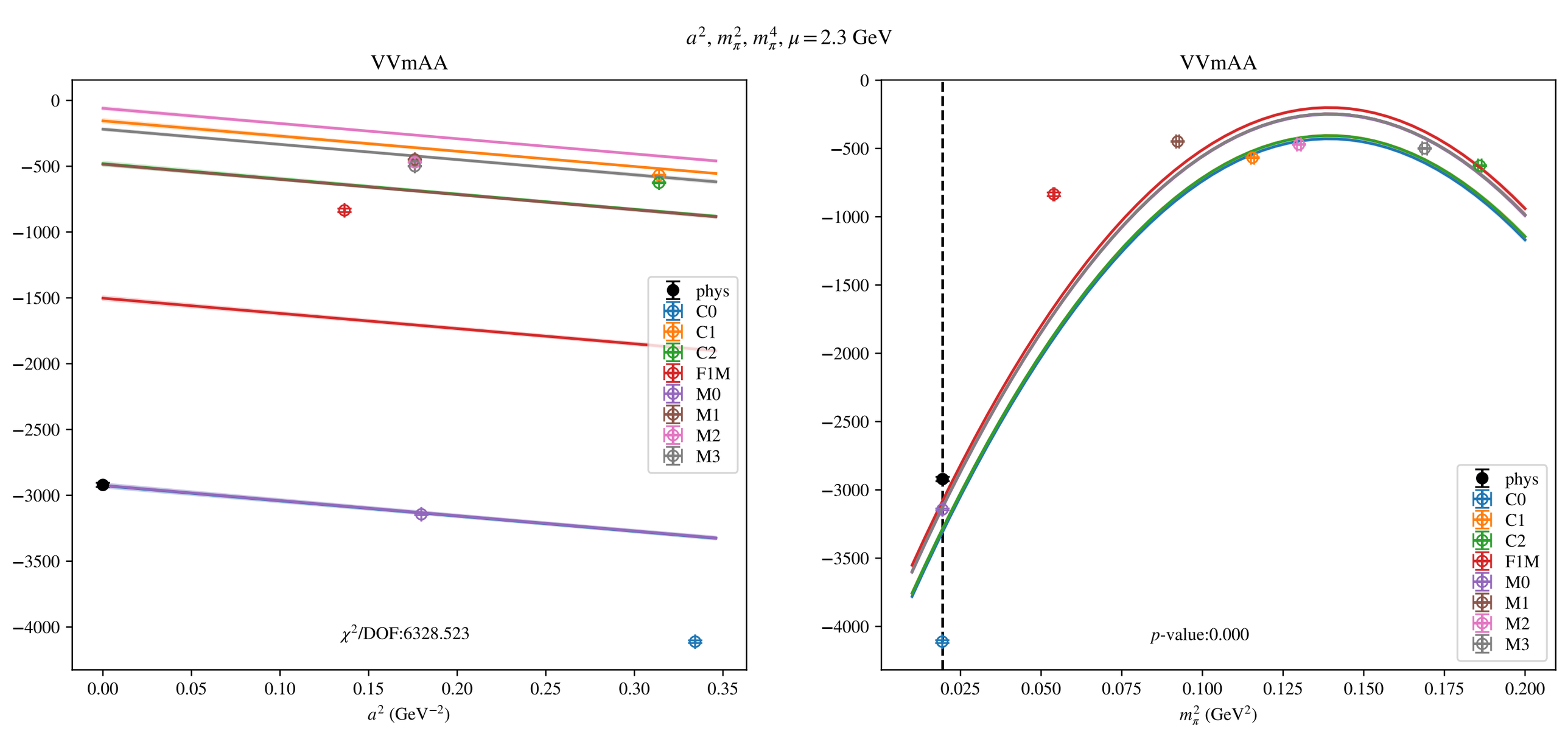

a^2, m_π^2 (no M3, C2), $\mu = 2.2$ GeV
VVmAA
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:23217.938
VVmAA
1000
0
−1000
−2000
−3000
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000
phys
C0
C1
M0
M1
M2
F1M

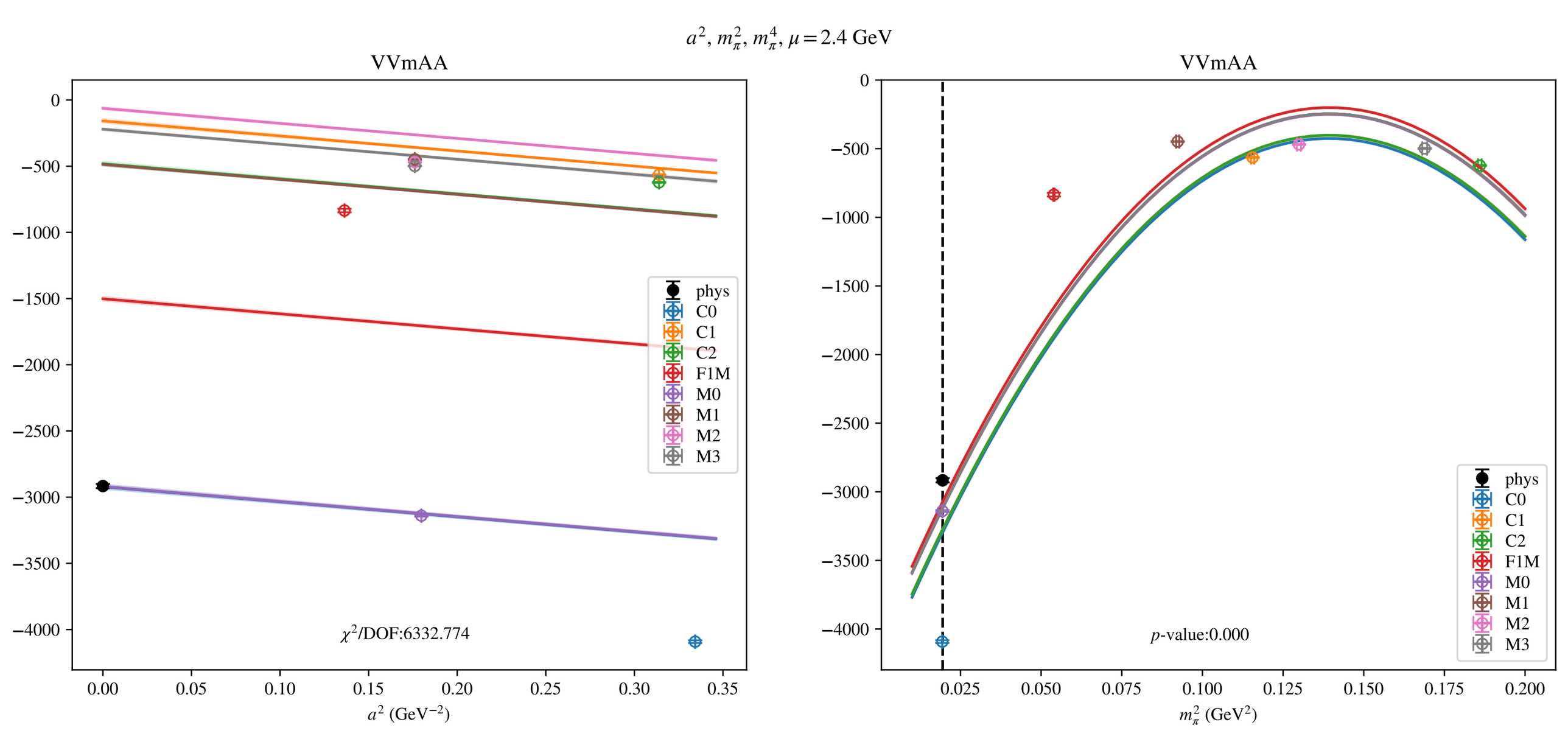

a^2, m_π^2 (no M3, C2), $\mu = 2.3$ GeV
VVmAA
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:23215.153
VVmAA
1000
0
−1000
−2000
−3000
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000
phys
C0
C1
M0
M1
M2
F1M


a^2, m_π^2 (no M3, C2), $\mu = 2.4$ GeV
VVmAA
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:23208.695
VVmAA
1000
0
−1000
−2000
−3000
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000
phys
C0
C1
M0
M1
M2
F1M


$a^2, m_\pi^2, m_\pi^4, \mu = 2.0$ GeV
VVmAA
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:6265.725
VVmAA
0
−500
−1000
−1500
−2000
−2500
−3000
−3500
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
phys
C0
C1
C2
F1M
M0
M1
M2
M3
p-value:0.000


$a^2, m_\pi^2, m_\pi^4, \mu = 2.2$ GeV
VVmAA
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:6322.070
VVmAA
0
−500
−1000
−1500
−2000
−2500
−3000
−3500
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
phys
C0
C1
C2
F1M
M0
M1
M2
M3
p-value:0.000


$a^2, m_\pi^2, m_\pi^4, \mu = 2.3$ GeV
VVmAA
0
−500
−1000
−1500
−2000
−2500
−3000
−3500
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:6328.523
phys
C0
C1
C2
F1M
M0
M1
M2
M3
VVmAA
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000


$a^2, m_\pi^2, m_\pi^4, \mu = 2.4$ GeV
VVmAA
0
−500
−1000
−1500
−2000
−2500
−3000
−3500
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:6332.774
phys
C0
C1
C2
F1M
M0
M1
M2
M3
VVmAA
0
−500
−1000
−1500
−2000
−2500
−3000
−3500
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000
phys
C0
C1
C2
F1M
M0
M1
M2
M3

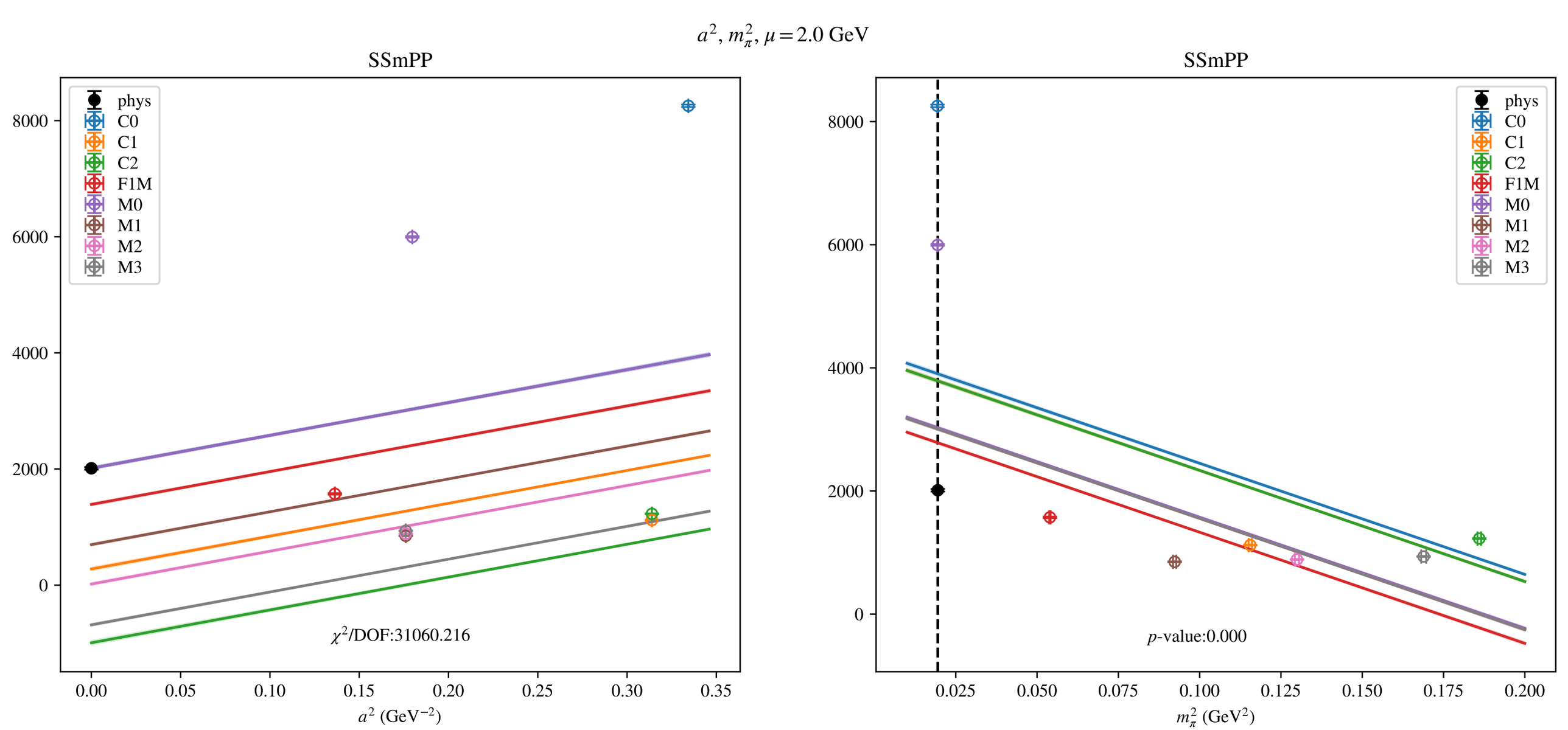
3 $\mathcal{B}_3$

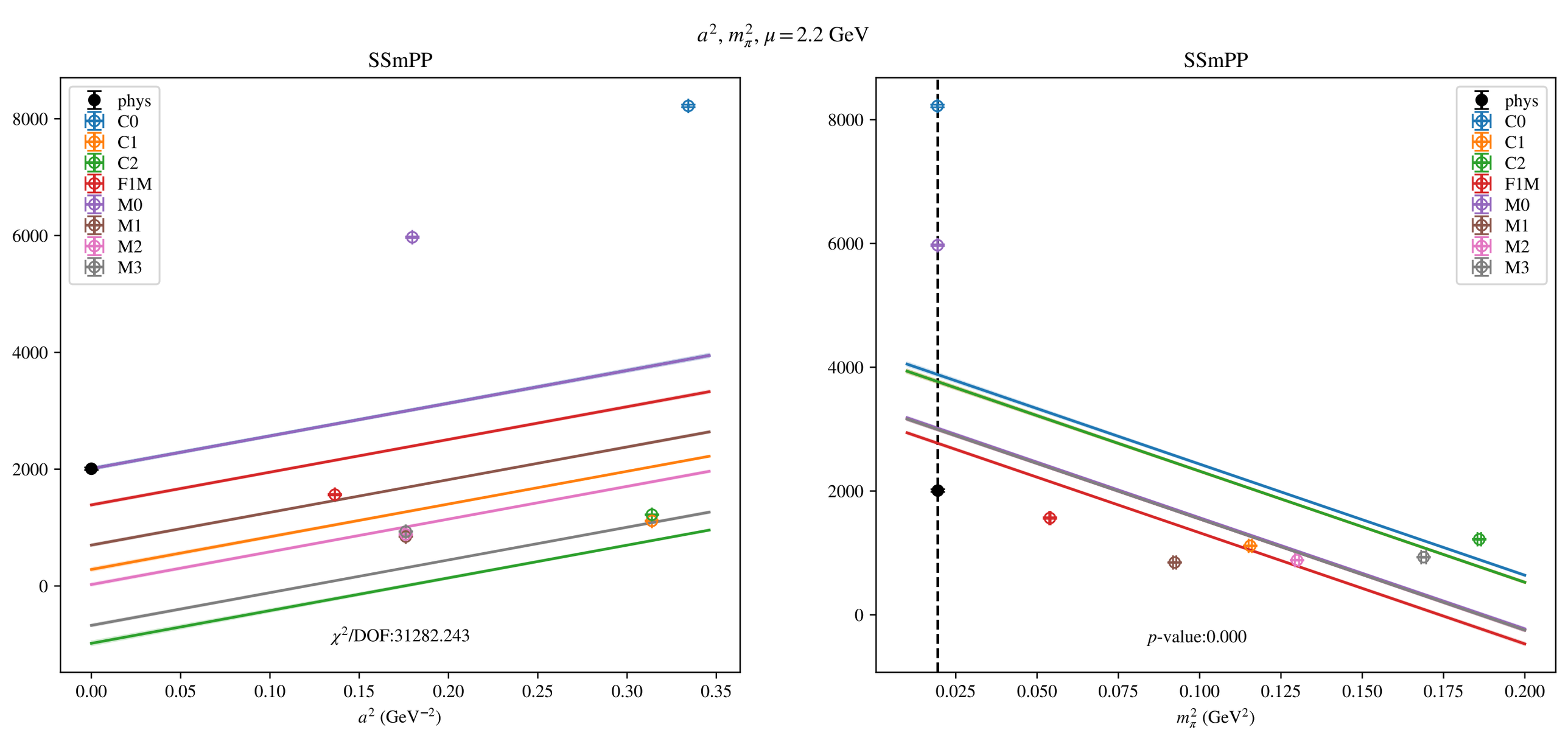
μ (GeV)	a^2, m_π^2	a^2, m_π^2 (no C)	a^2, a^4, m_π^2	a^2, m_π^2 (no M3, C2)	a^2, m_π^2, m_π^4
2.0	**20(21)**: 31060.216 (0.0)	**-1(76)**: 41155.309 (0.0)	**1(15)**: 38817.401 (0.0)	**33(44)**: 27003.395 (0.0)	**50(26)**: 9500.047 (0.0)
2.2	**20(21)**: 31282.243 (0.0)	**-1(76)**: 41492.919 (0.0)	**1(15)**: 39093.899 (0.0)	**33(44)**: 27257.96 (0.0)	**49(26)**: 9581.59 (0.0)
2.3	**20(21)**: 31350.748 (0.0)	**-1(75)**: 41596.728 (0.0)	**1(15)**: 39178.917 (0.0)	**33(44)**: 27321.407 (0.0)	**49(26)**: 9602.315 (0.0)
2.4	**20(21)**: 31392.56 (0.0)	**-1(75)**: 41674.24 (0.0)	**1(15)**: 39230.658 (0.0)	**33(44)**: 27358.42 (0.0)	**49(26)**: 9613.078 (0.0)

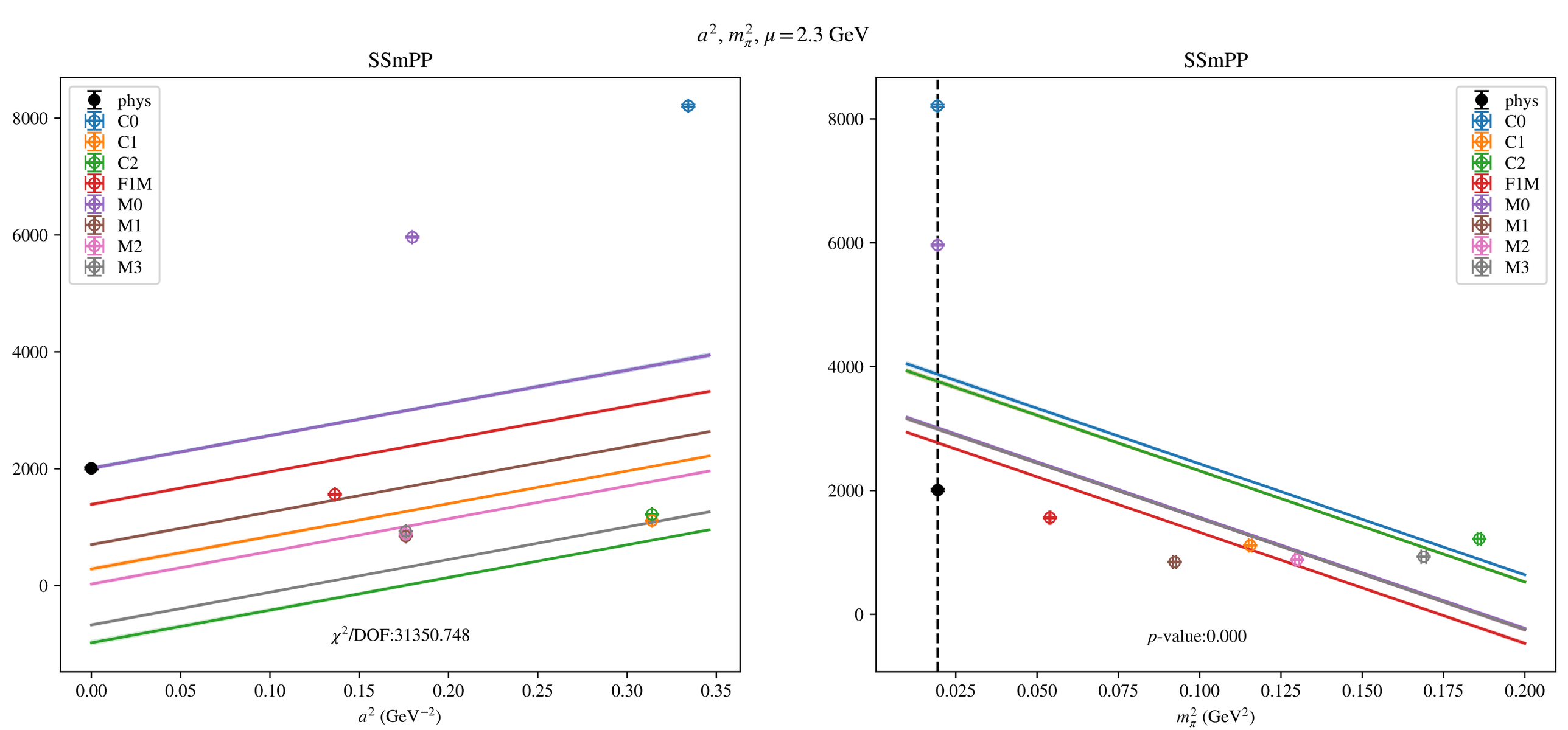
Table 5: Physical point value from chiral and continuum extrapolation at renormalisation scale μ. Entries are **value(error)**: χ^2/DOF (p-value).

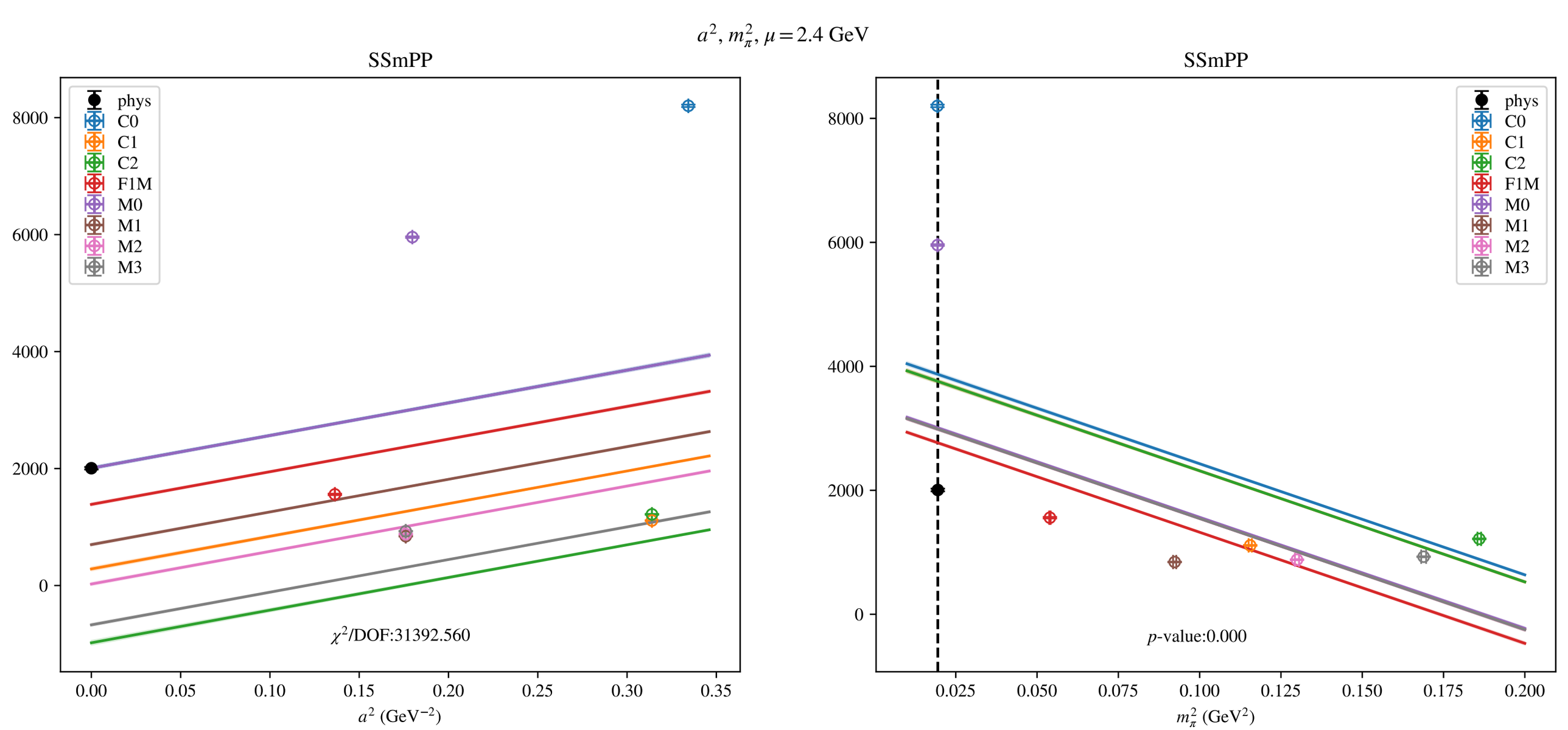
μ (GeV)		a^2, m_π^2	a^2, m_π^2 (no C)	a^2, a^4, m_π^2	a^2, m_π^2 (no M3, C2)	a^2, m_π^2, m_π^4
2.0	α	2.810(70)	-14.(31)	7.5(20)	1.976(78)	0.667(19)
	β	-0.152(18)	0.1623(54)	-0.21(26)	-0.205(28)	-0.344(18)
2.2	α	2.791(69)	-14.(31)	7.8(21)	1.961(77)	0.663(19)
	β	-0.152(18)	0.1613(54)	-0.21(28)	-0.204(28)	-0.344(18)
2.3	α	2.784(69)	-14.(31)	8.0(21)	1.957(77)	0.662(19)
	β	-0.152(18)	0.1607(53)	-0.22(28)	-0.204(28)	-0.343(18)
2.4	α	2.784(69)	-14.(30)	8.2(22)	1.956(77)	0.663(19)
	β	-0.152(18)	0.1603(53)	-0.22(29)	-0.204(28)	-0.344(18)

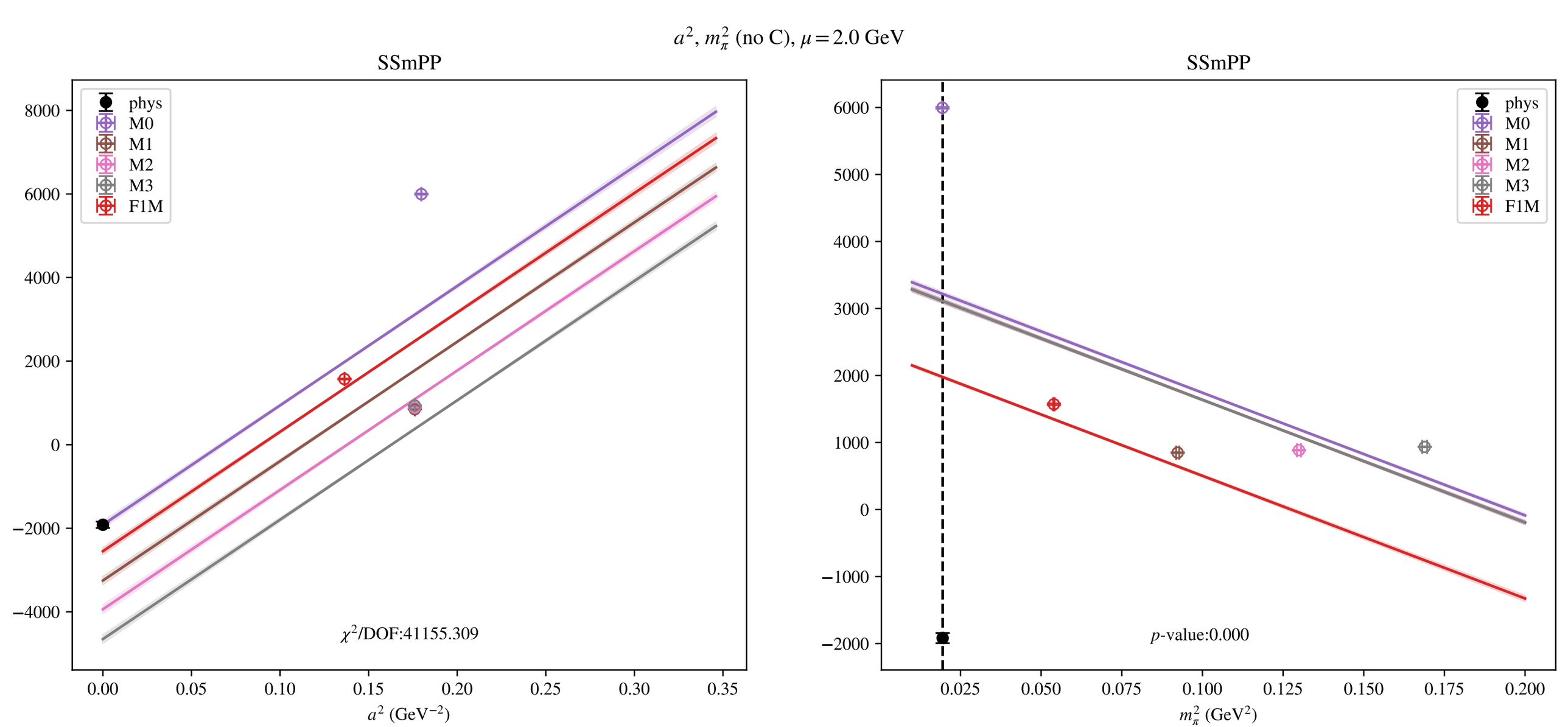
Table 6: Fit values of coefficients in $Q = Q_{phys} + \alpha a^2 + \beta \left(\frac{m_\pi^2}{f_\pi^2} - \frac{m_{\pi,PDG}^2}{f_\pi^2} \right) + \ldots$.

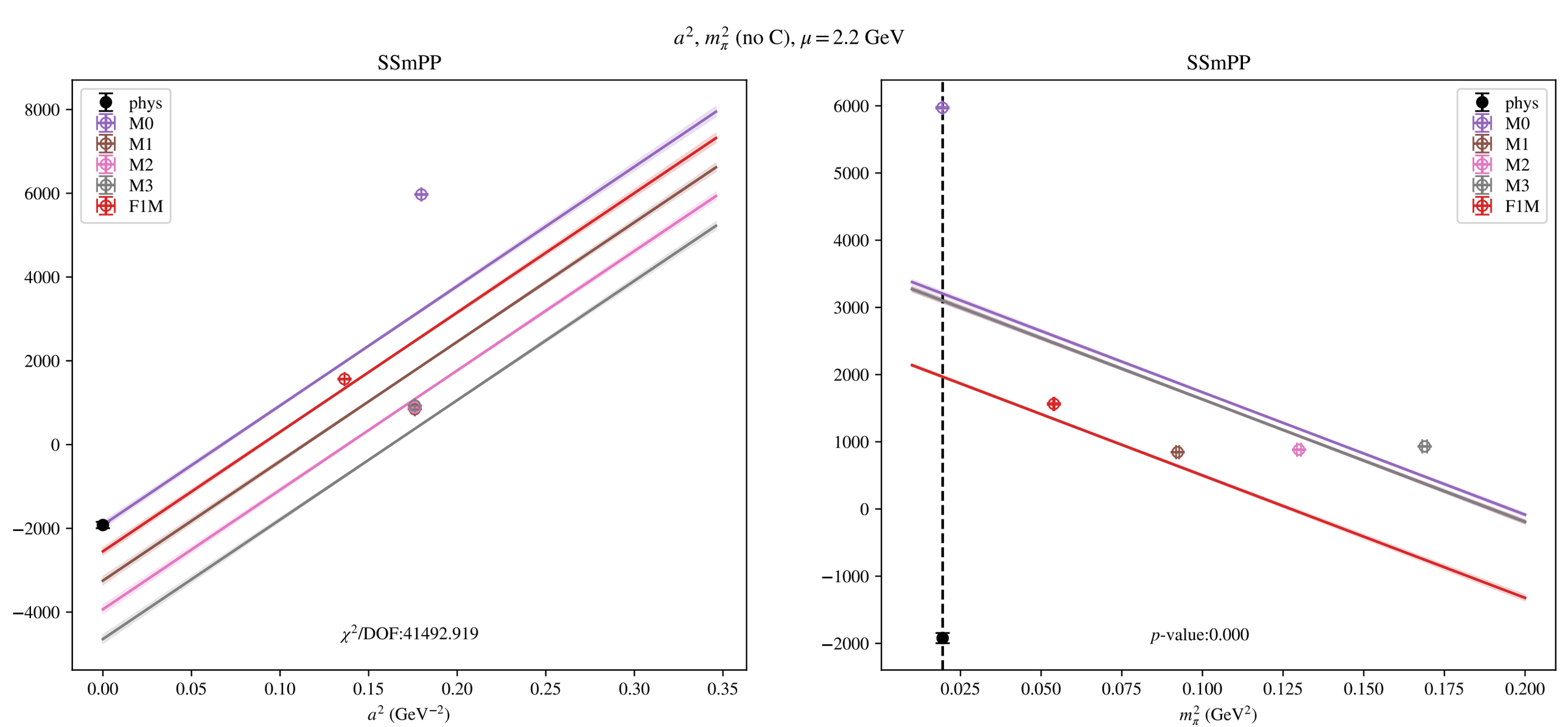

a^2, m_π^2, $\mu = 2.0$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
χ^2/DOF:31060.216
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSmPP
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)

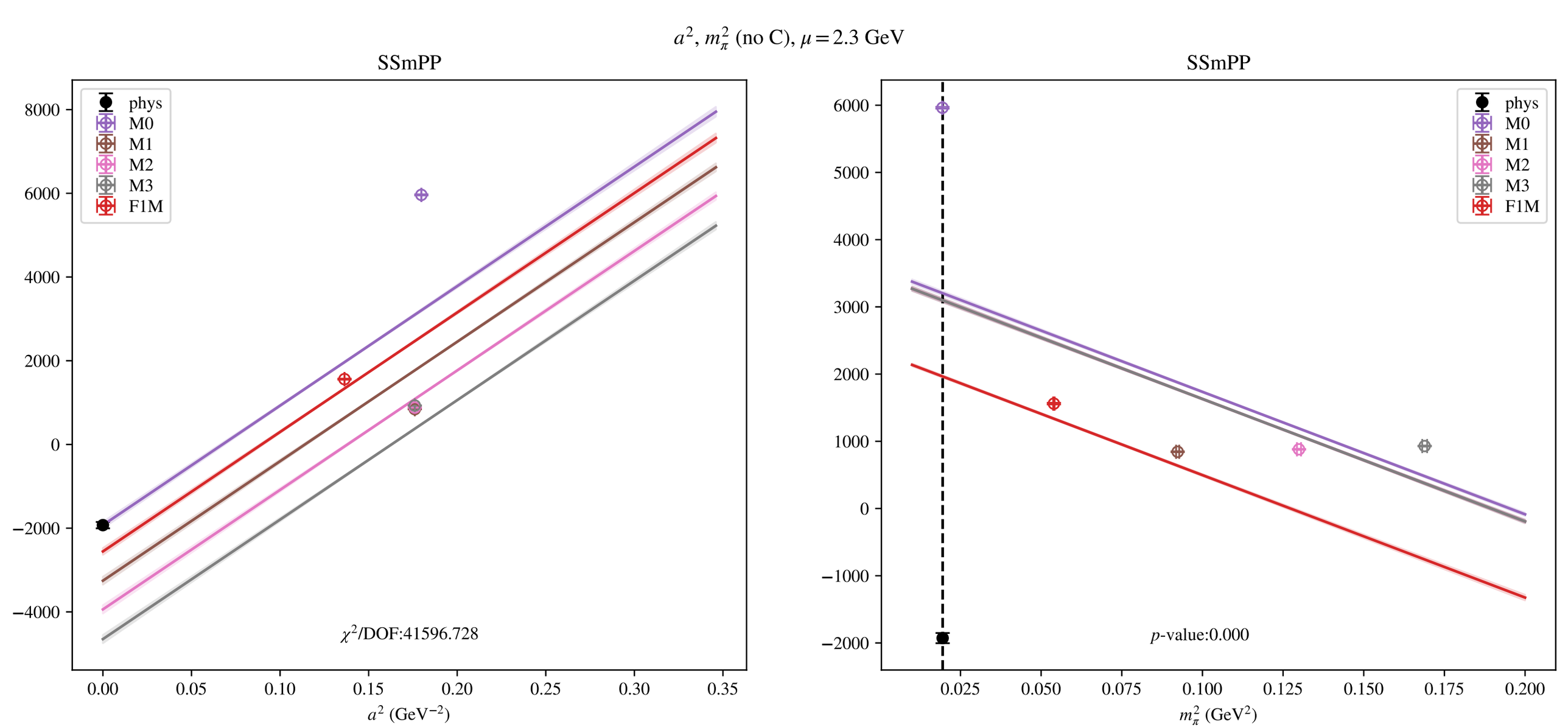

a^2, m_π^2, $\mu = 2.2$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
χ^2/DOF:31282.243
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSmPP
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)

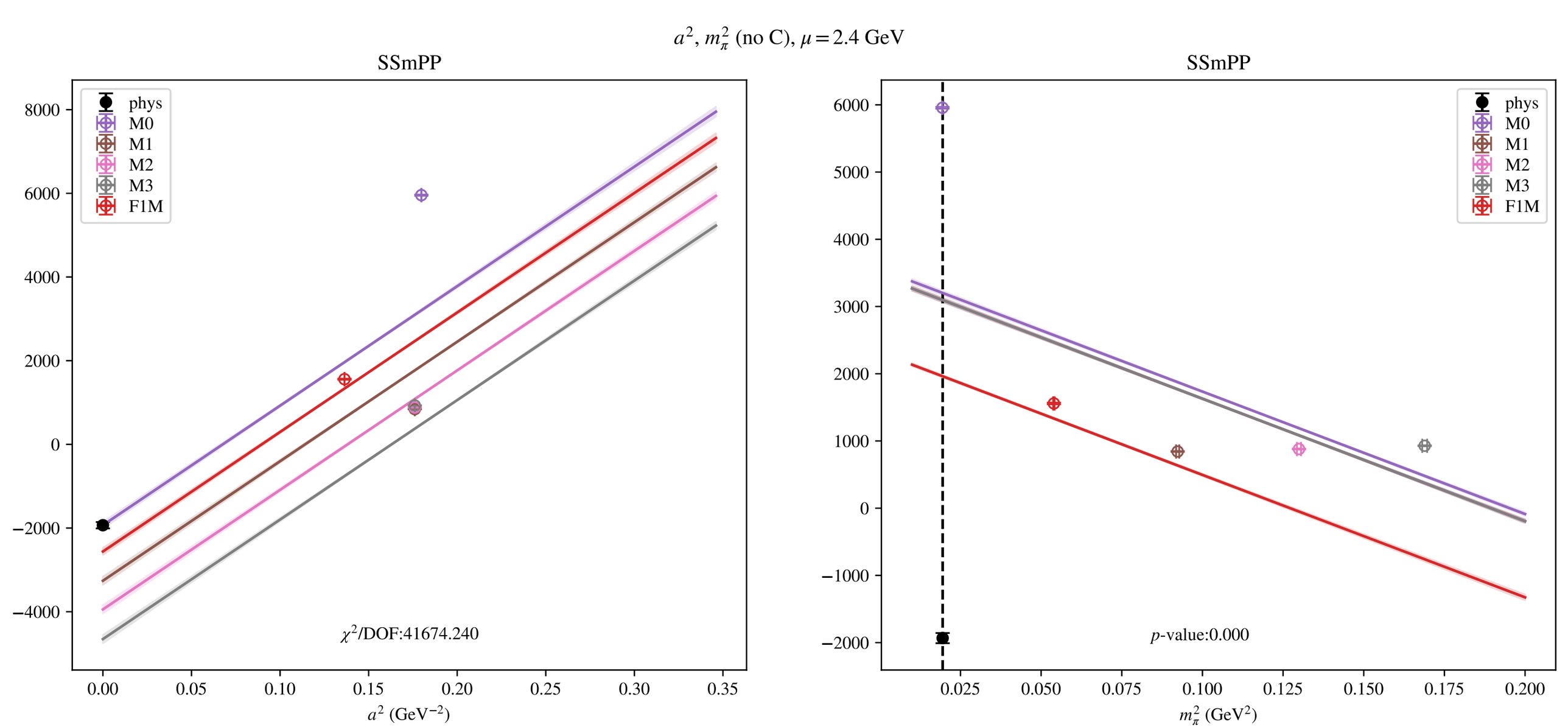

a^2, m_π^2, $\mu = 2.3$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
χ^2/DOF:31350.748
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSmPP
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)

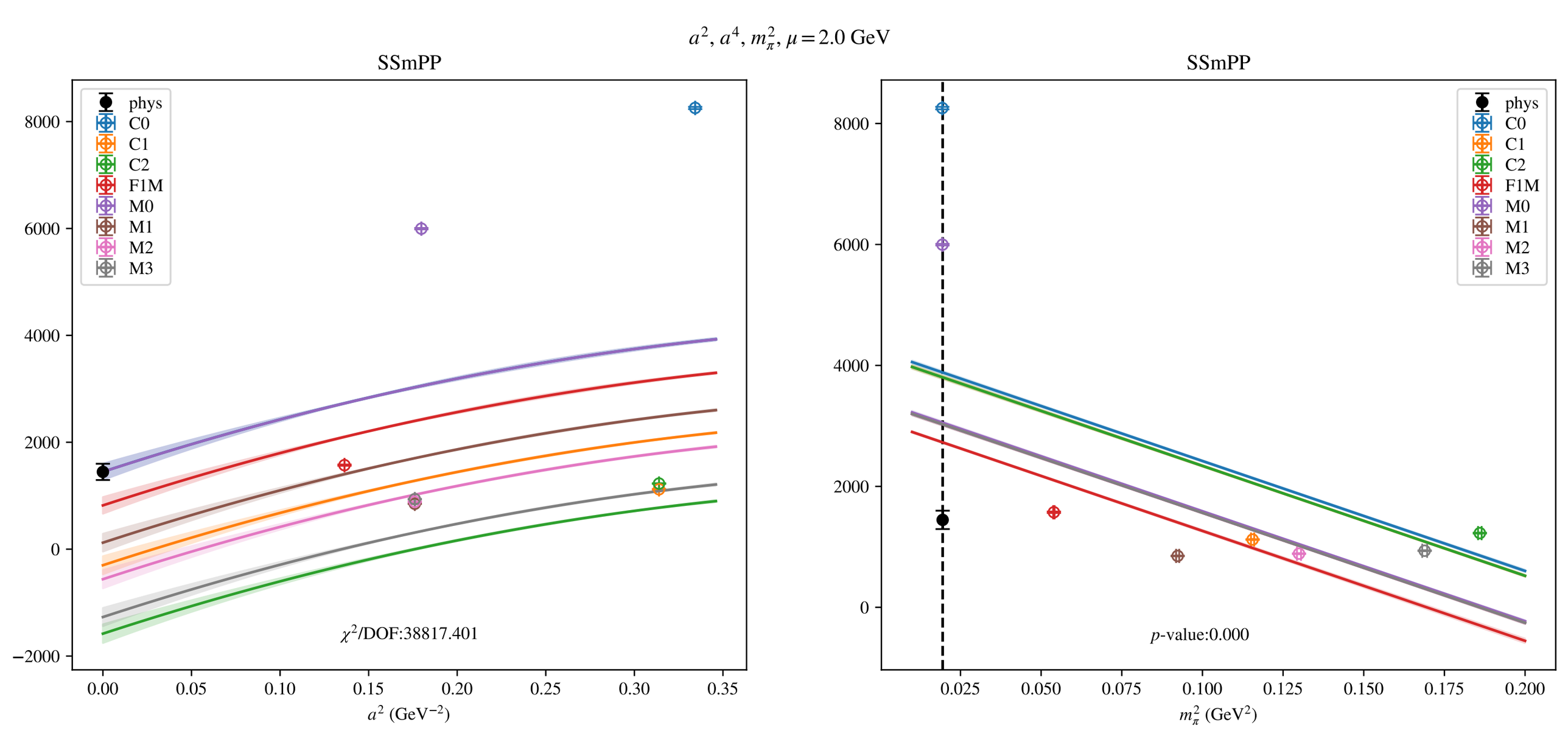

a^2, m_π^2, $\mu = 2.4$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
χ^2/DOF:31392.560
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSmPP
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)

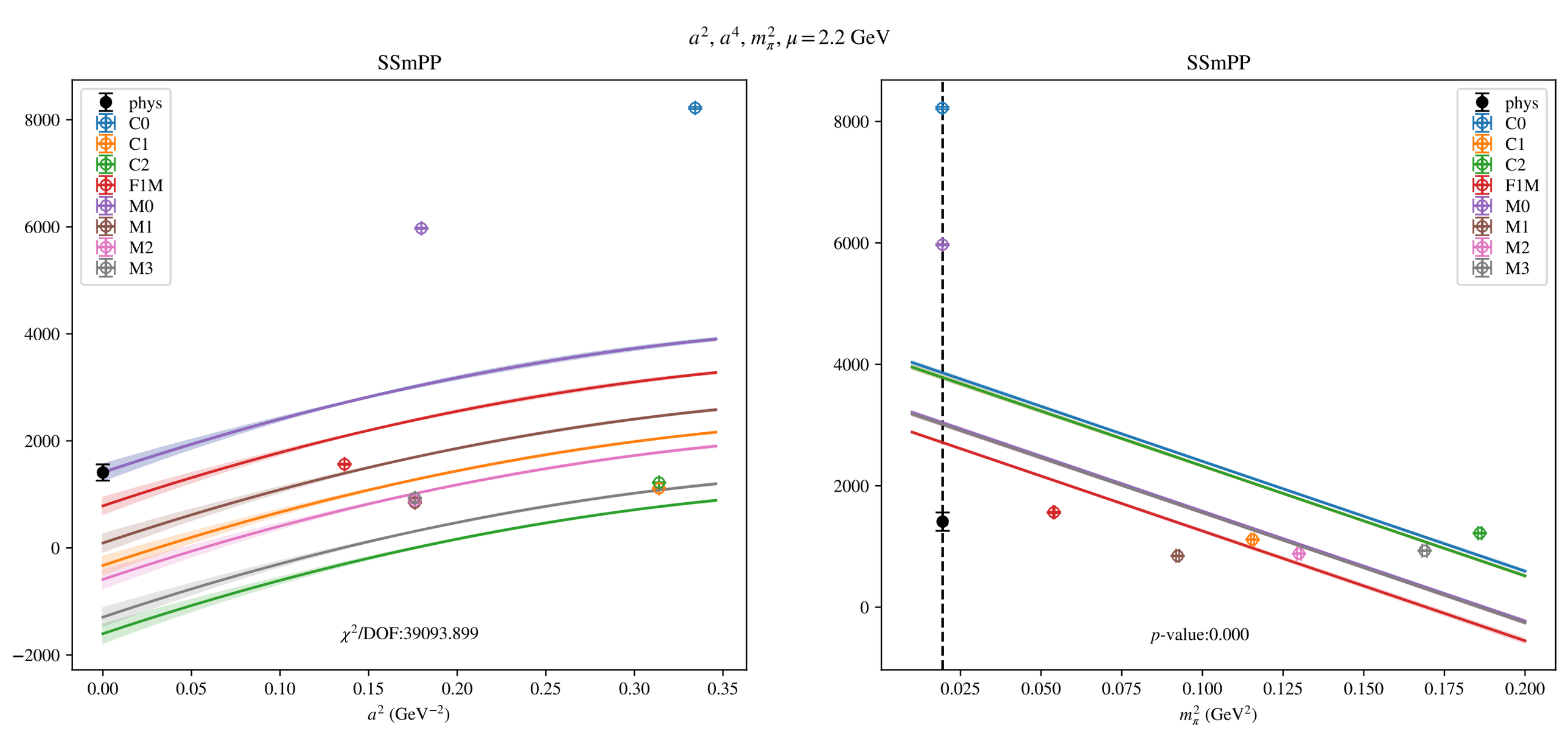

a^2, m_π^2 (no C), $\mu = 2.0$ GeV
SSmPP
phys
M0
M1
M2
M3
F1M
8000
6000
4000
2000
0
−2000
−4000
χ^2/DOF:41155.309
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSmPP
phys
M0
M1
M2
M3
F1M
6000
5000
4000
3000
2000
1000
0
−1000
−2000
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)

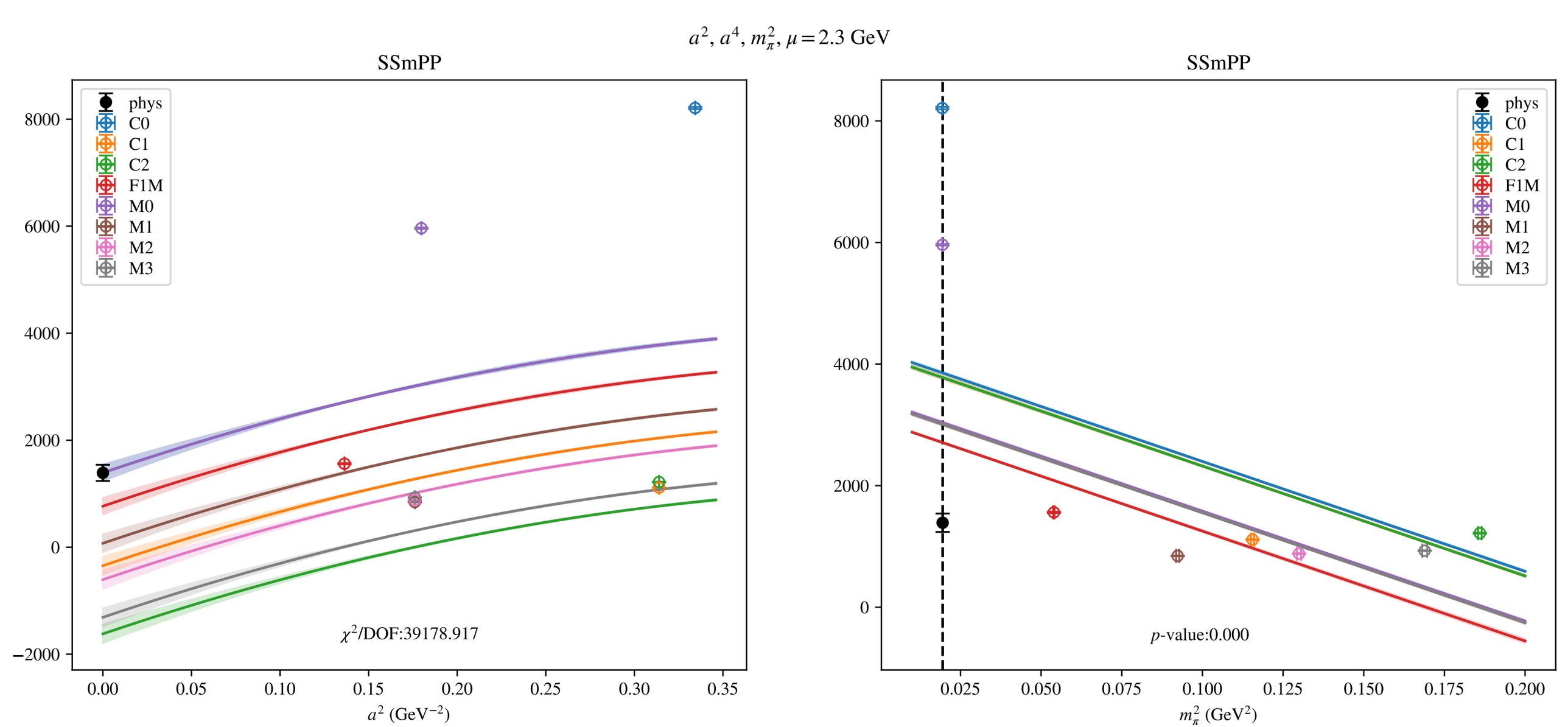

a^2, m_π^2 (no C), $\mu = 2.2$ GeV
SSmPP
phys
M0
M1
M2
M3
F1M
χ^2/DOF:41492.919
a^2 (GeV^{-2})
SSmPP
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

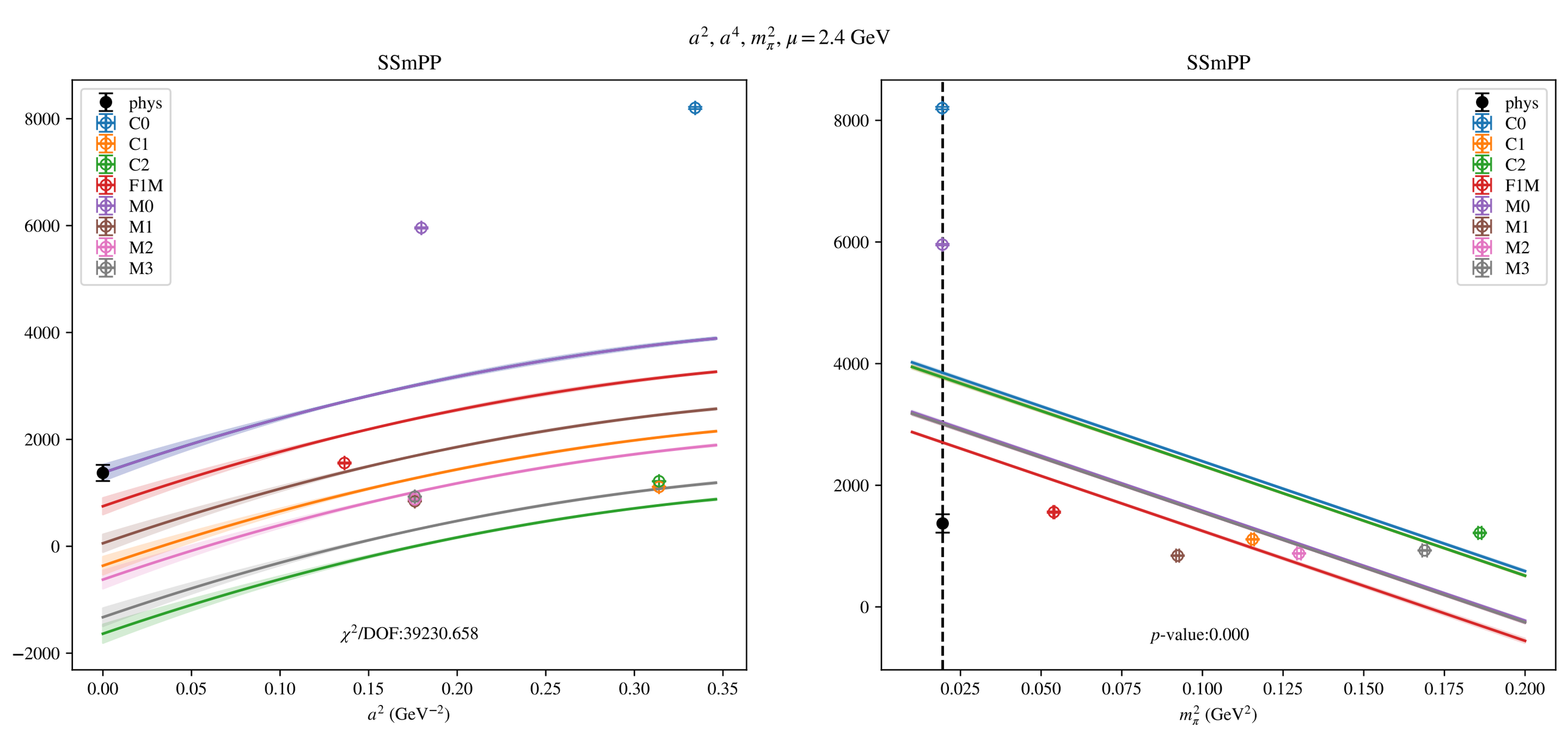

a^2, m_π^2 (no C), $\mu = 2.3$ GeV
SSmPP
phys
M0
M1
M2
M3
F1M
8000
6000
4000
2000
0
−2000
−4000
χ^2/DOF:41596.728
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSmPP
phys
M0
M1
M2
M3
F1M
6000
5000
4000
3000
2000
1000
0
−1000
−2000
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)

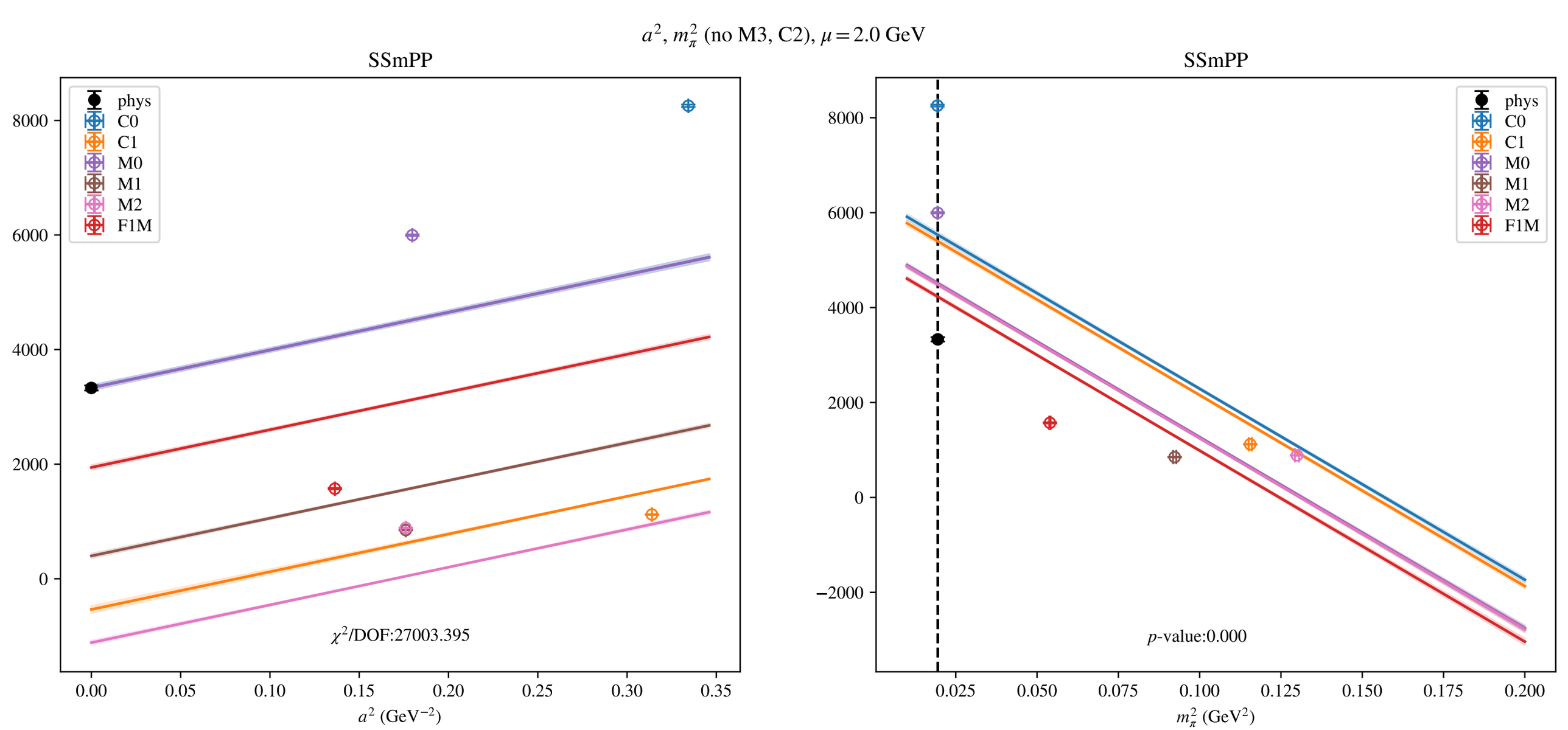

a^2, m_π^2 (no C), $\mu = 2.4$ GeV
SSmPP
phys
M0
M1
M2
M3
F1M
χ^2/DOF:41674.240
a^2 (GeV^{-2})
SSmPP
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

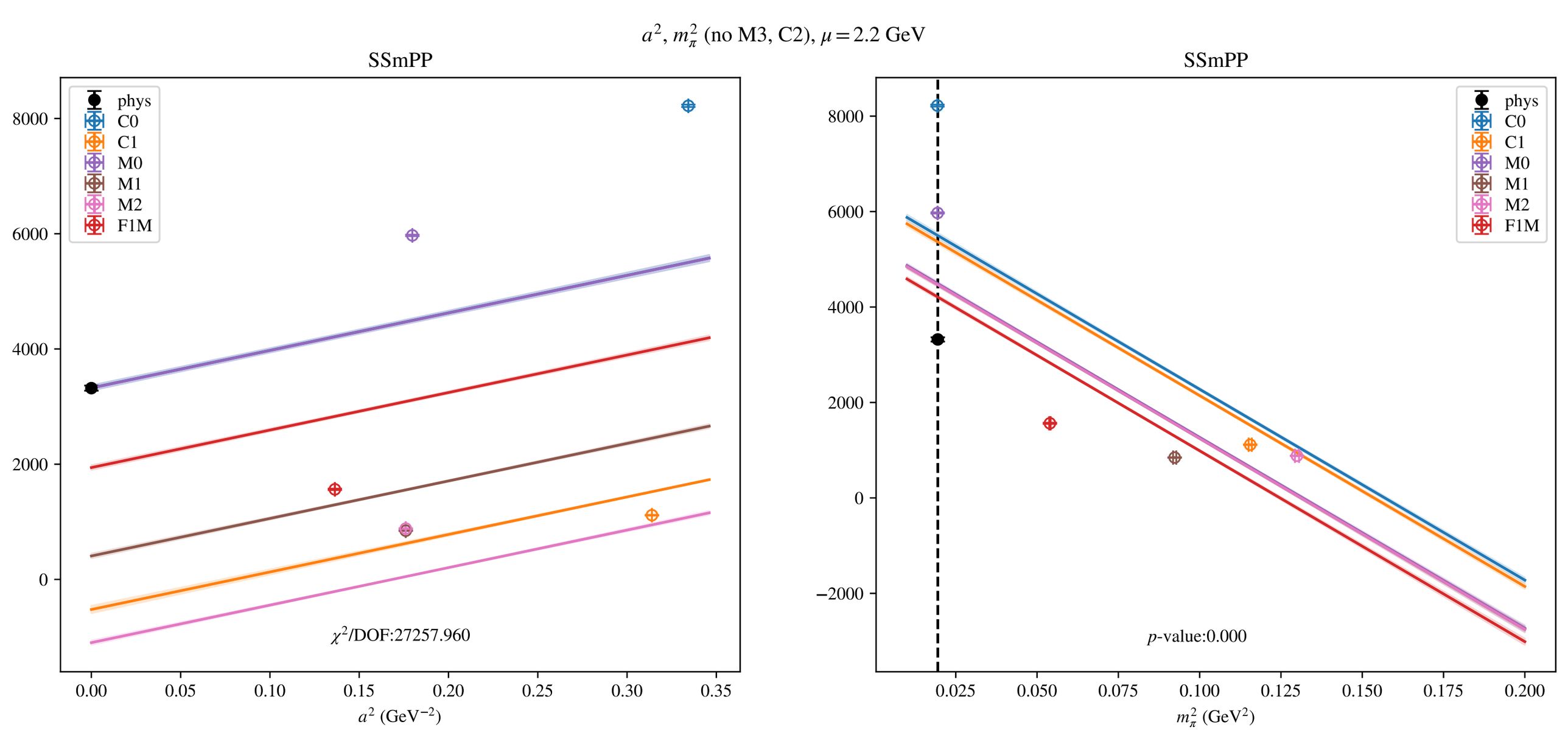

$a^2, a^4, m_\pi^2, \mu = 2.0$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
−2000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:38817.401
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

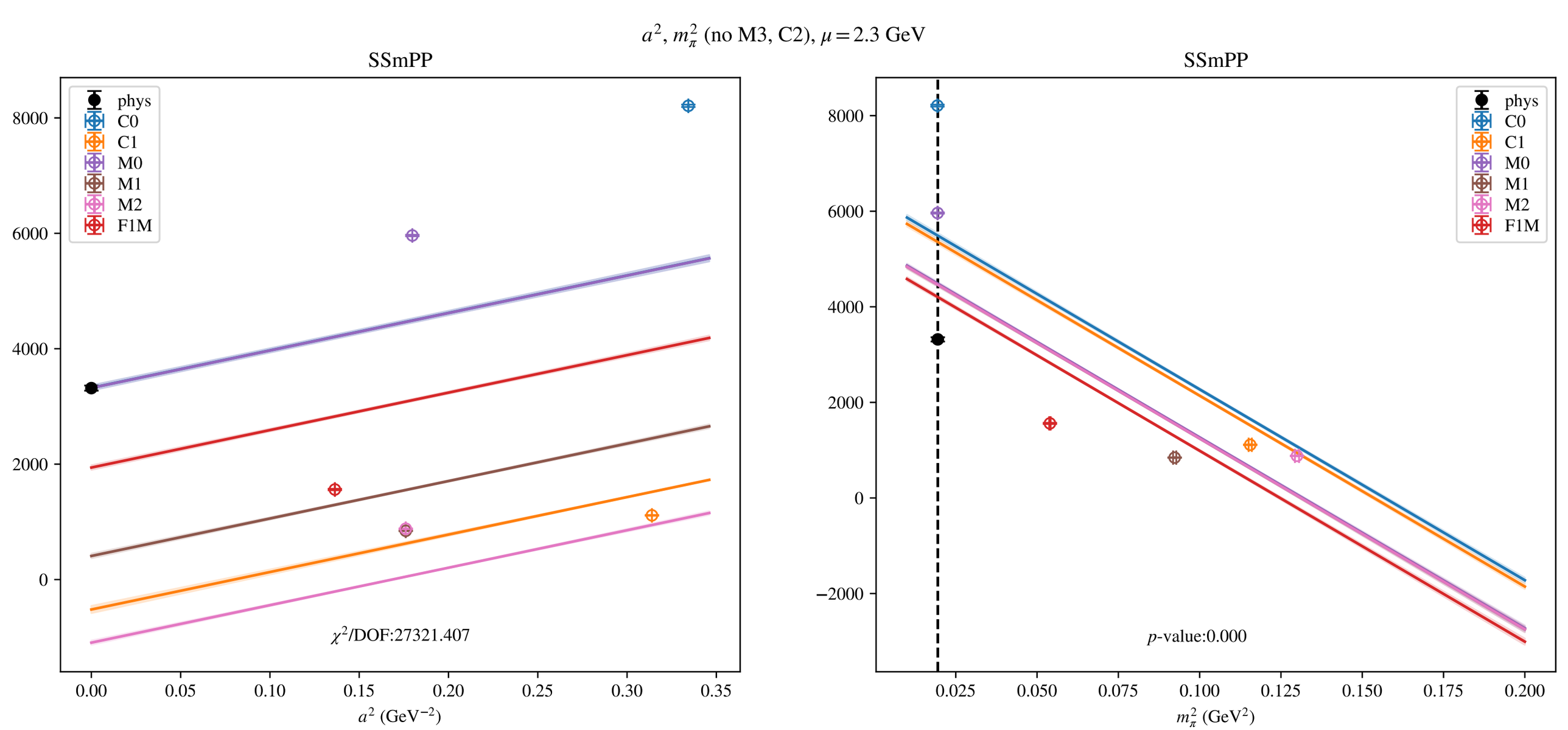

$a^2, a^4, m_\pi^2, \mu = 2.2$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
−2000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:39093.899
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

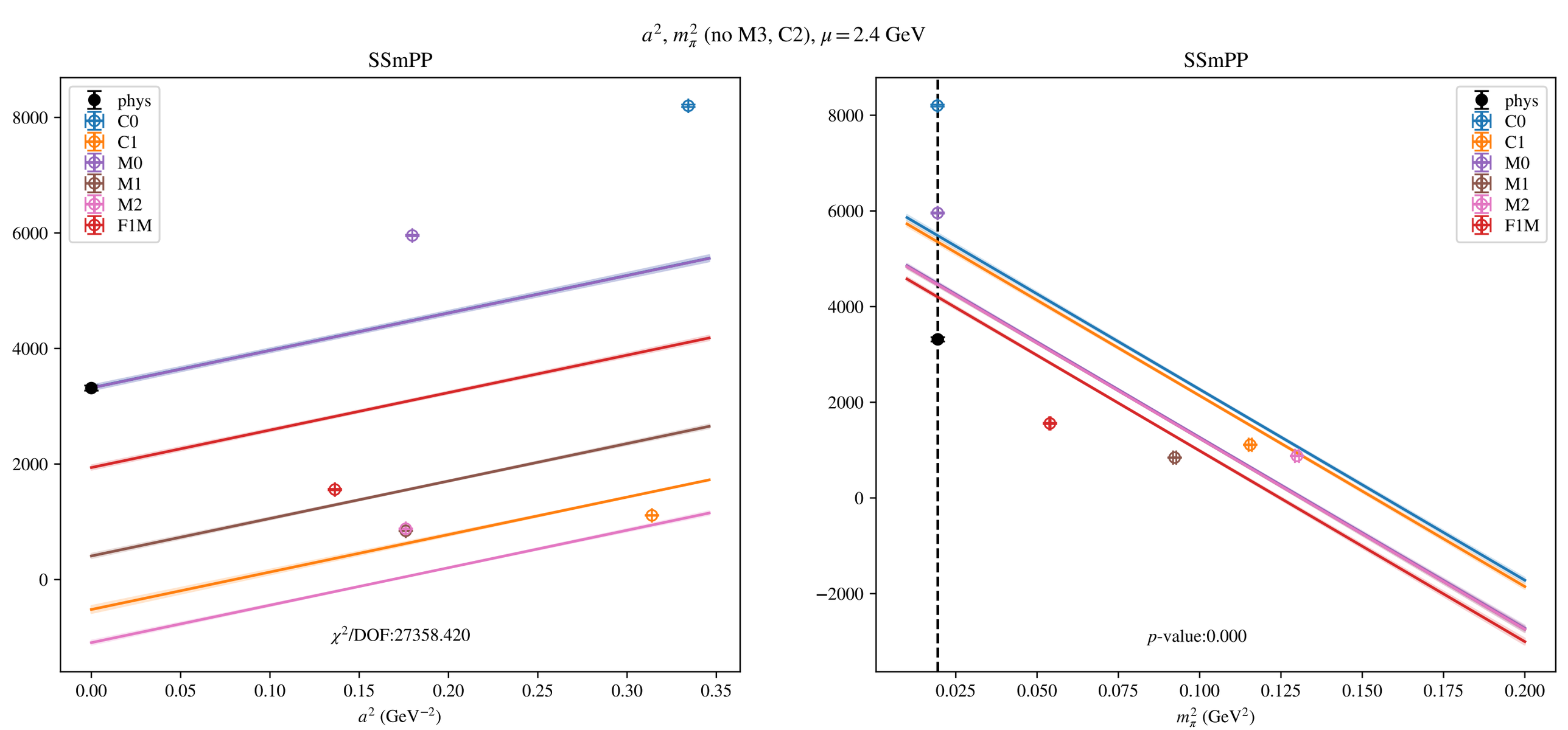

$a^2, a^4, m_\pi^2, \mu = 2.3$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
−2000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:39178.917
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

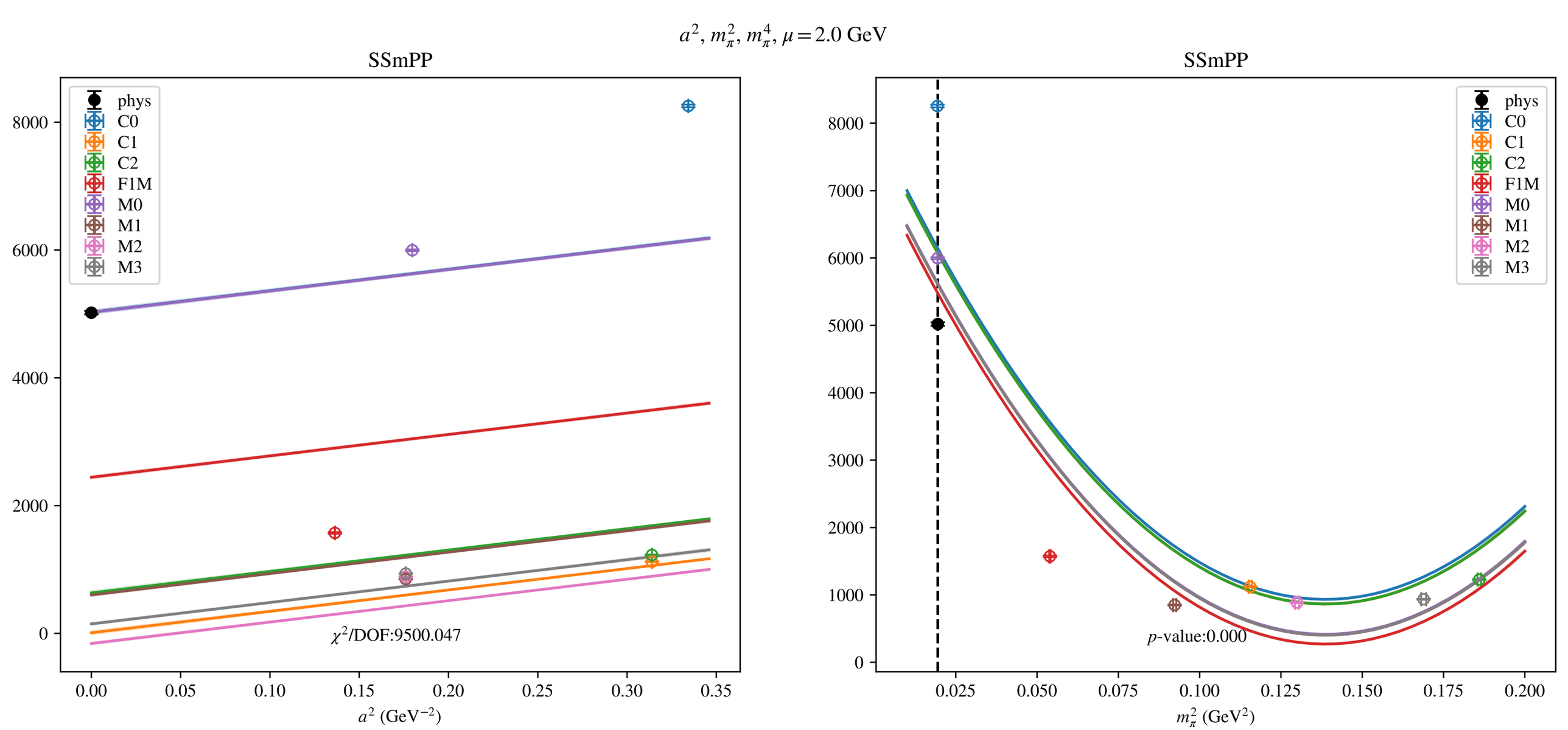

$a^2, a^4, m_\pi^2, \mu = 2.4$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
−2000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:39230.658
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

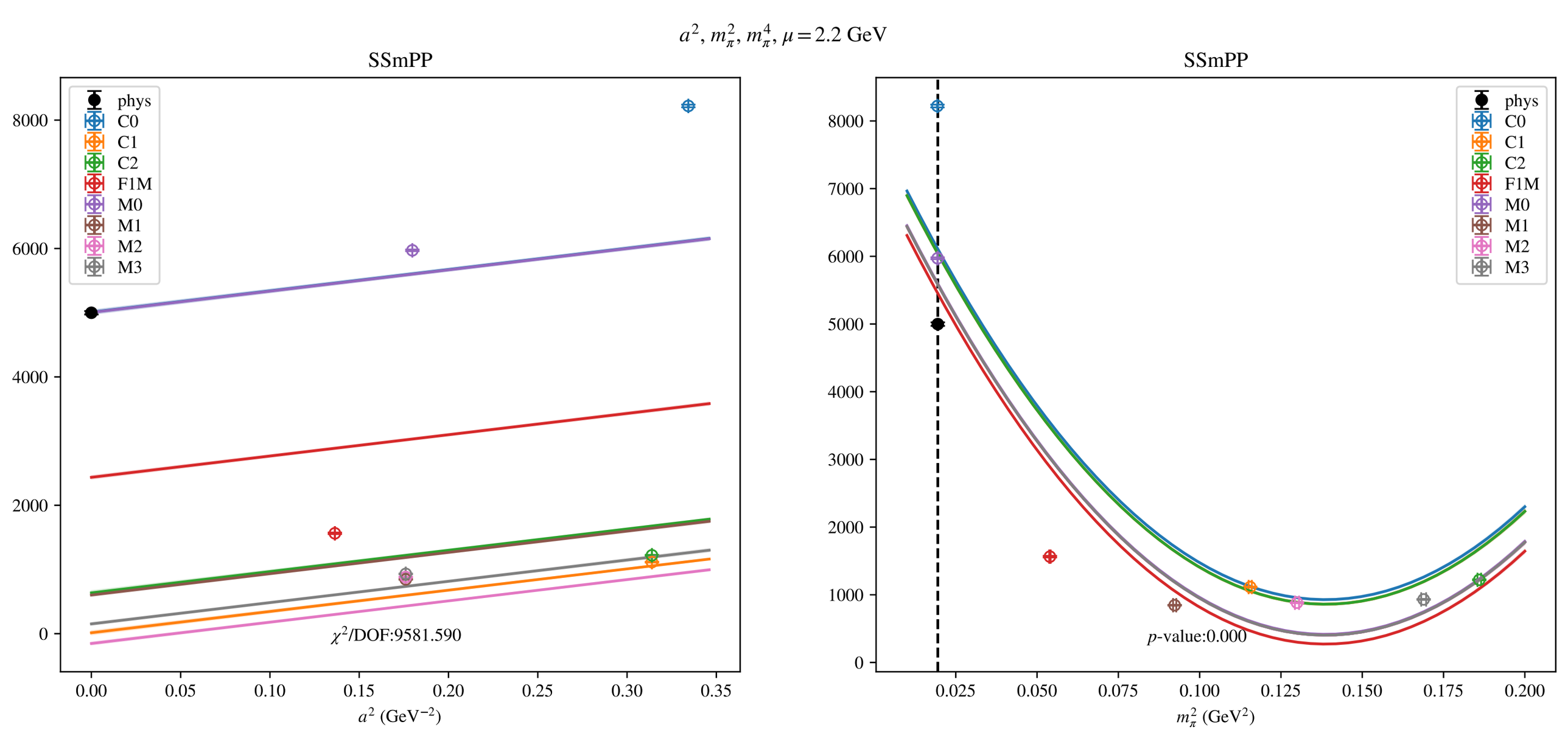

a^2, m_π^2 (no M3, C2), $\mu = 2.0$ GeV
SSmPP
phys
C0
C1
M0
M1
M2
F1M
8000
6000
4000
2000
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:27003.395
SSmPP
phys
C0
C1
M0
M1
M2
F1M
8000
6000
4000
2000
0
−2000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

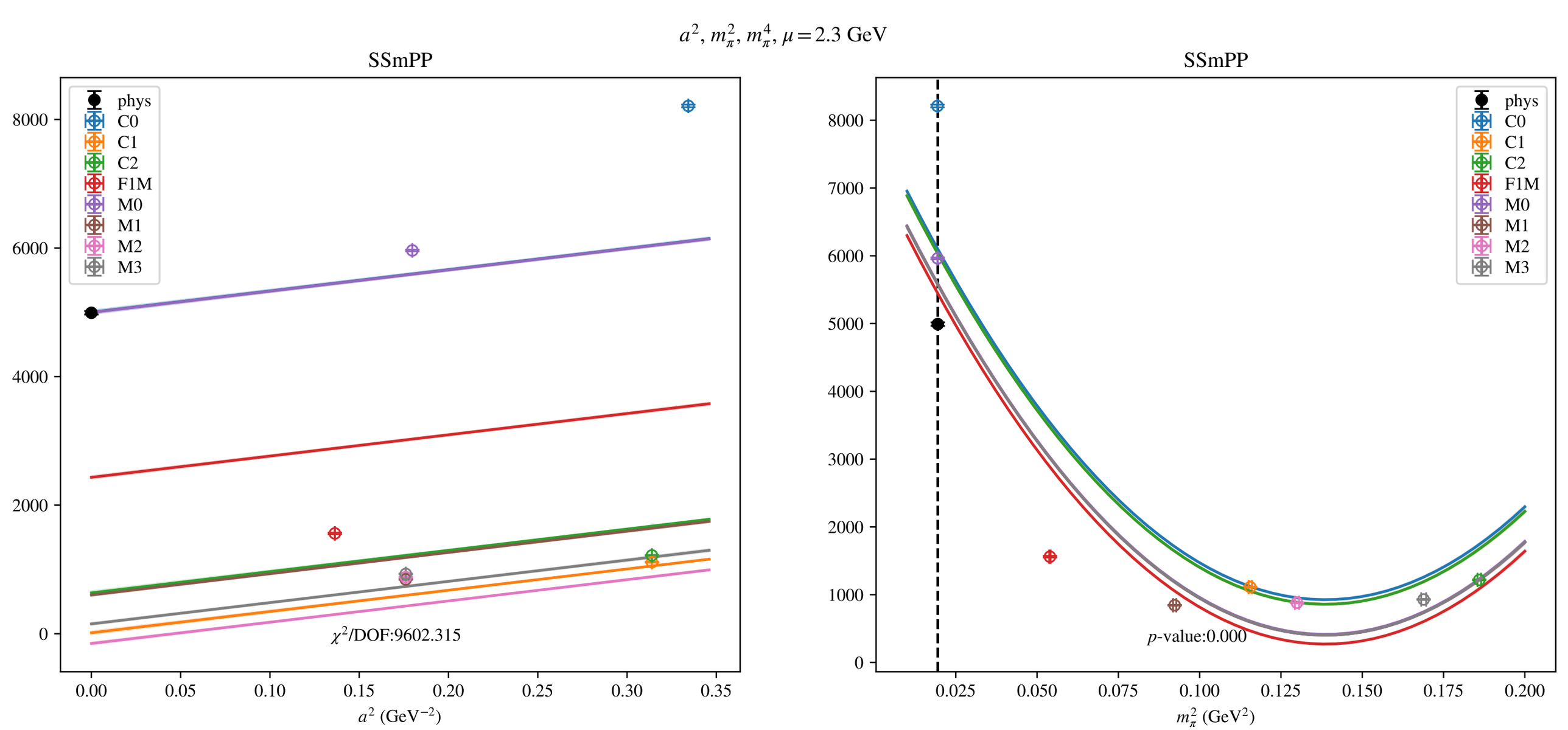

a^2, m_π^2 (no M3, C2), $\mu = 2.2$ GeV
SSmPP
phys
C0
C1
M0
M1
M2
F1M
8000
6000
4000
2000
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:27257.960
SSmPP
phys
C0
C1
M0
M1
M2
F1M
8000
6000
4000
2000
0
−2000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

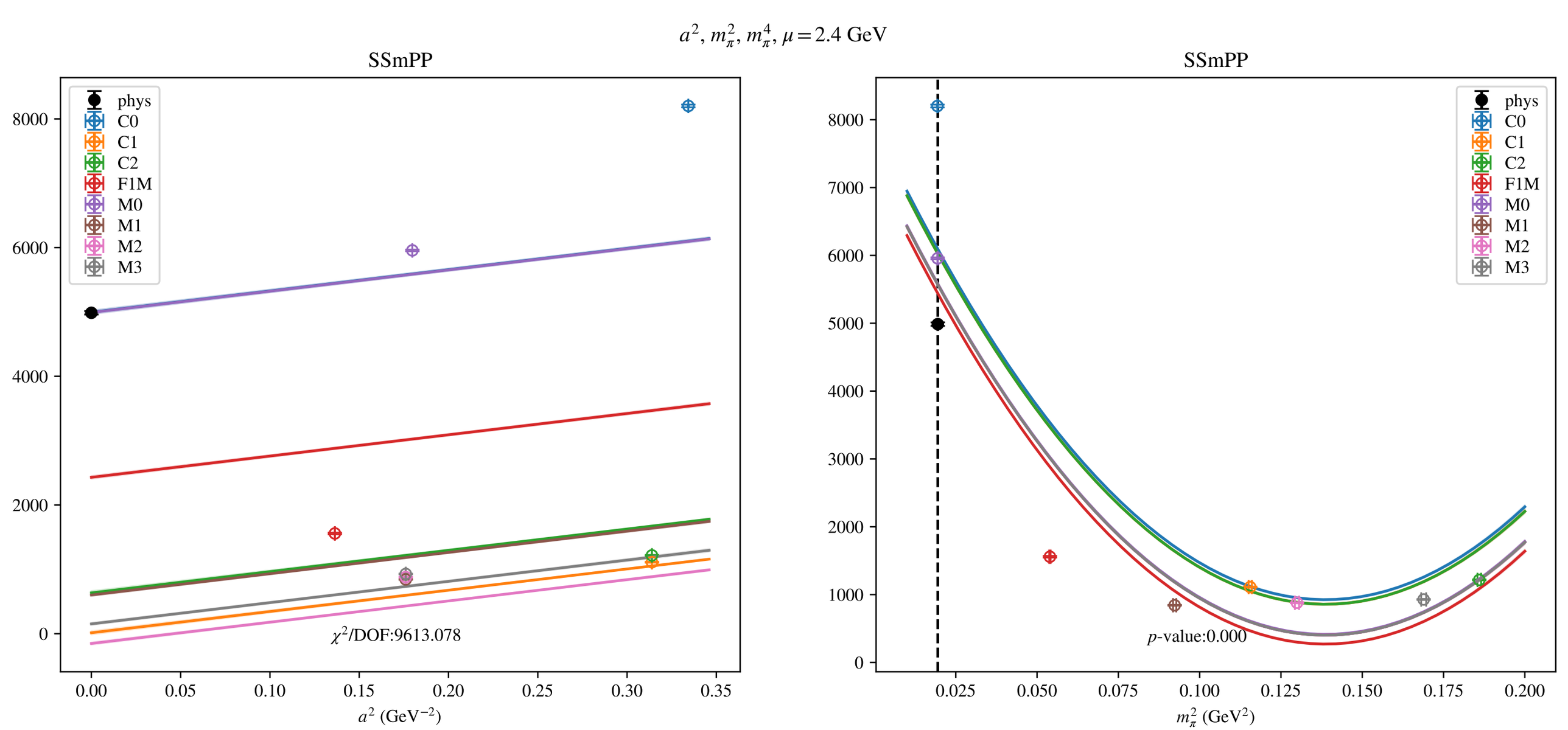

a^2, m_π^2 (no M3, C2), $\mu = 2.3$ GeV
SSmPP
phys
C0
C1
M0
M1
M2
F1M
8000
6000
4000
2000
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:27321.407
SSmPP
phys
C0
C1
M0
M1
M2
F1M
8000
6000
4000
2000
0
−2000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000


a^2, m_π^2 (no M3, C2), $\mu = 2.4$ GeV
SSmPP
phys
C0
C1
M0
M1
M2
F1M
8000
6000
4000
2000
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
χ^2/DOF:27358.420
SSmPP
phys
C0
C1
M0
M1
M2
F1M
8000
6000
4000
2000
0
−2000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000


$a^2, m_\pi^2, m_\pi^4, \mu = 2.0$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
χ^2/DOF:9500.047
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
7000
6000
5000
4000
3000
2000
1000
0
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.2$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
χ^2/DOF:9581.590
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
7000
6000
5000
4000
3000
2000
1000
0
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.3$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
6000
4000
2000
0
χ^2/DOF:9602.315
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
8000
7000
6000
5000
4000
3000
2000
1000
0
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.4$ GeV
SSmPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:9613.078
a^2 (GeV^{-2})
SSmPP
p-value:0.000
m_π^2 (GeV^2)

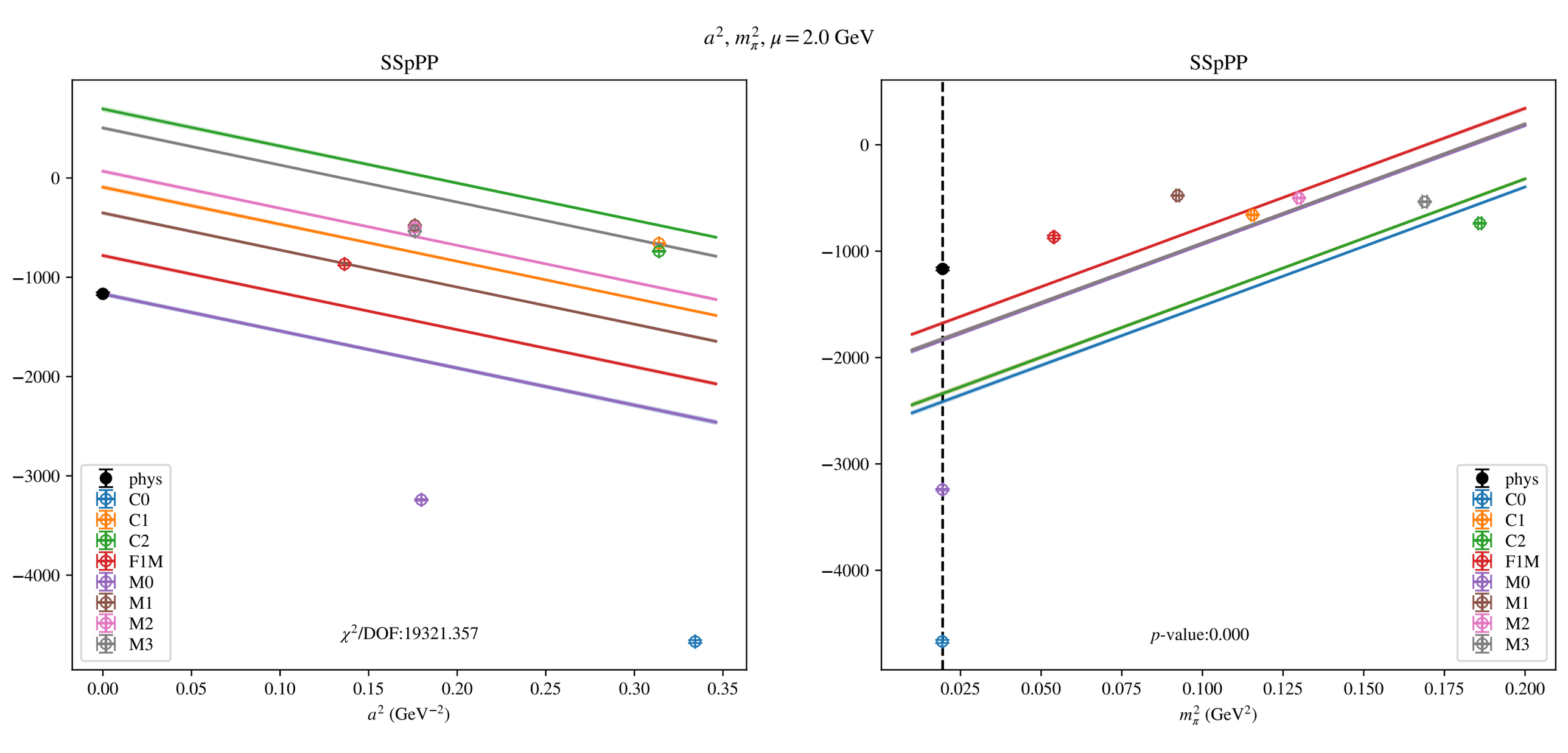
4 $\mathcal{B}_4$

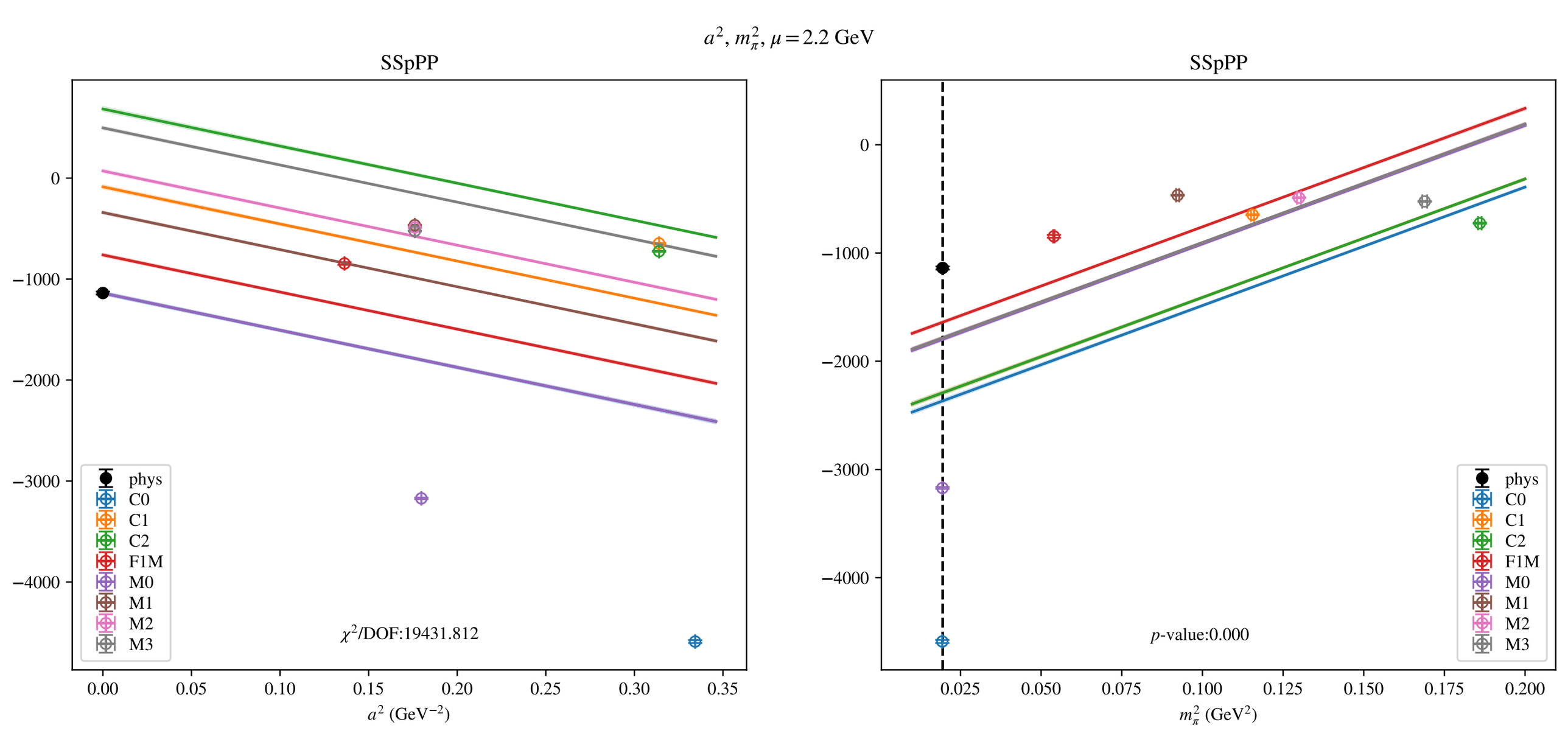
μ (GeV)	a^2, m_π^2	a^2, m_π^2 (no C)	a^2, a^4, m_π^2	a^2, m_π^2 (no M3, C2)	a^2, m_π^2, m_π^4
2.0	**-1(14)**: 19321.357 (0.0)	**22(65)**: 26585.047 (0.0)	**-(13)**: 24104.429 (0.0)	**-2(30)**: 14722.37 (0.0)	**-2(15)**: 4633.002 (0.0)
2.2	**-1(14)**: 19431.812 (0.0)	**21(64)**: 26747.963 (0.0)	**-(12)**: 24245.309 (0.0)	**-1(29)**: 14867.319 (0.0)	**-2(15)**: 4691.512 (0.0)
2.3	**-1(14)**: 19470.107 (0.0)	**21(63)**: 26794.189 (0.0)	**-(12)**: 24294.592 (0.0)	**-1(29)**: 14919.225 (0.0)	**-2(15)**: 4717.694 (0.0)
2.4	**-1(14)**: 19494.308 (0.0)	**21(62)**: 26842.422 (0.0)	**-(12)**: 24326.566 (0.0)	**-1(28)**: 14959.564 (0.0)	**-2(14)**: 4734.91 (0.0)

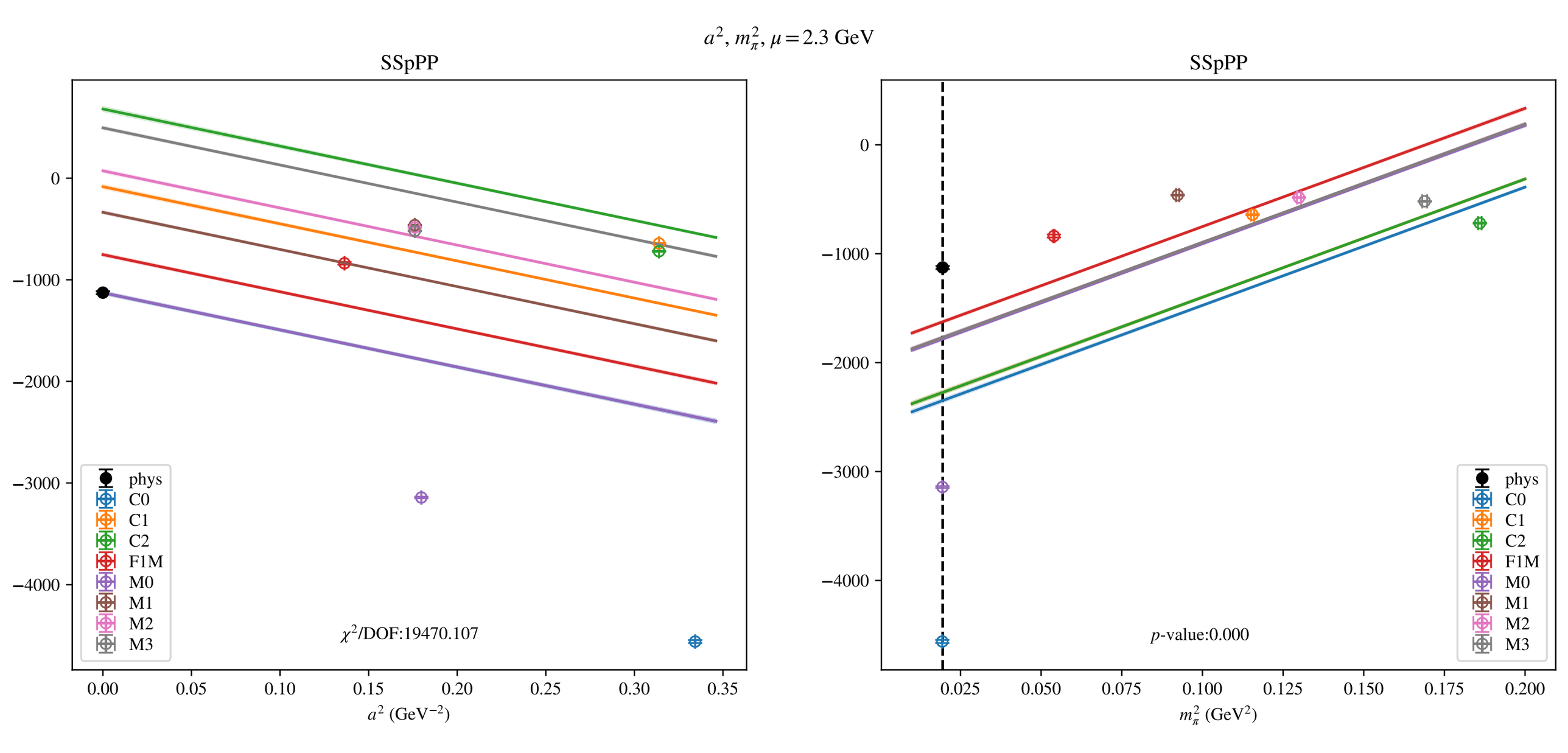
Table 7: Physical point value from chiral and continuum extrapolation at renormalisation scale μ. Entries are **value(error)**: χ^2/DOF (p-value).

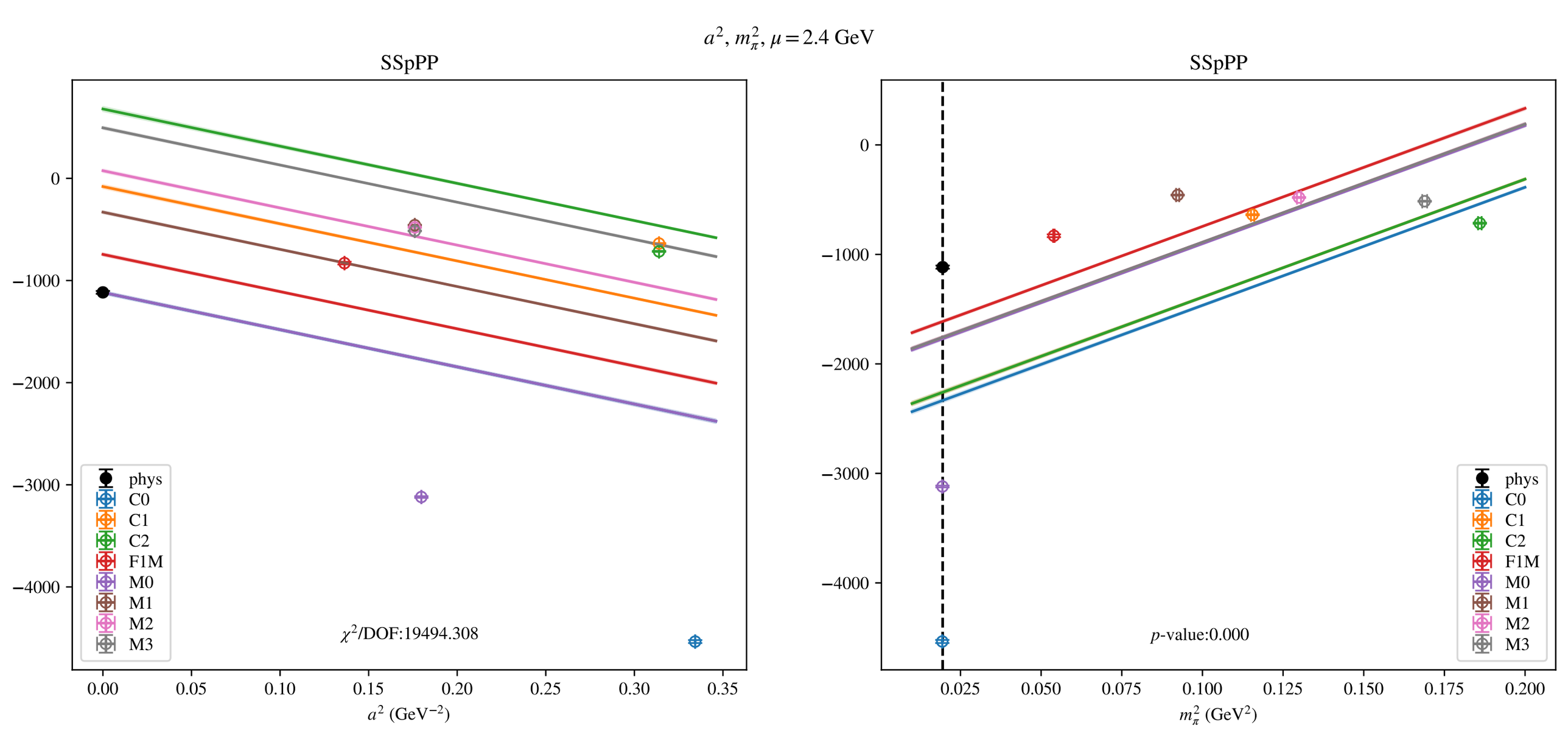
μ (GeV)		a^2, m_π^2	a^2, m_π^2 (no C)	a^2, a^4, m_π^2	a^2, m_π^2 (no M3, C2)	a^2, m_π^2, m_π^4
2.0	α	3.199(86)	-10.(12)	-3.4(32)	1.897(83)	0.636(20)
	β	-0.162(21)	0.0841(24)	-0.072(46)	-0.205(29)	-0.341(19)
2.2	α	3.228(86)	-10.(12)	-3.3(34)	1.918(84)	0.645(20)
	β	-0.163(22)	0.0838(23)	-0.074(48)	-0.205(29)	-0.342(19)
2.3	α	3.243(87)	-10.(12)	-3.2(34)	1.929(84)	0.650(20)
	β	-0.163(22)	0.0837(23)	-0.075(49)	-0.206(29)	-0.342(19)
2.4	α	3.262(87)	-10.(12)	-3.2(35)	1.941(84)	0.656(20)
	β	-0.164(22)	0.0836(23)	-0.076(50)	-0.206(29)	-0.343(19)

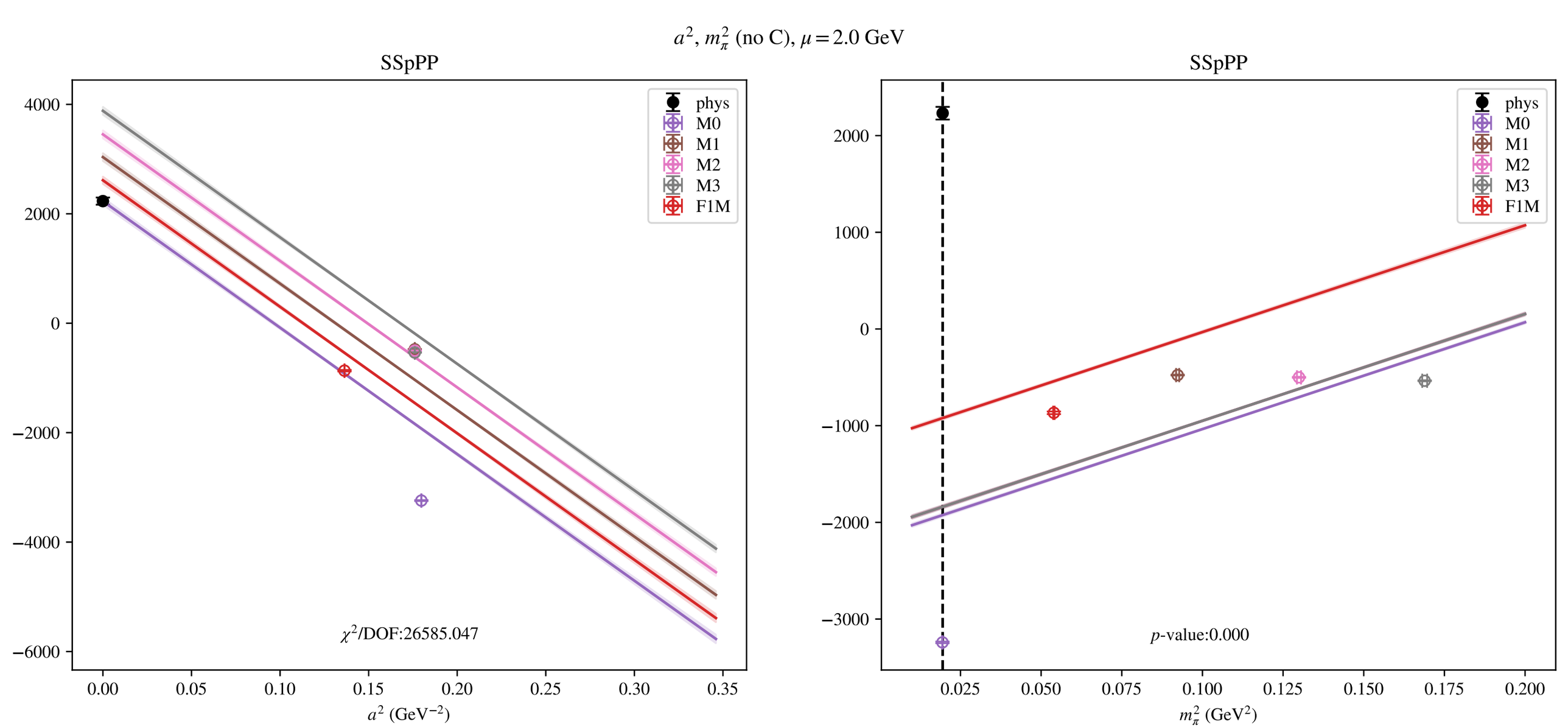
Table 8: Fit values of coefficients in $Q = Q_{phys} + \alpha a^2 + \beta \left(\frac{m_\pi^2}{f_\pi^2} - \frac{m_{\pi,PDG}^2}{f_\pi^2} \right) + \ldots$.

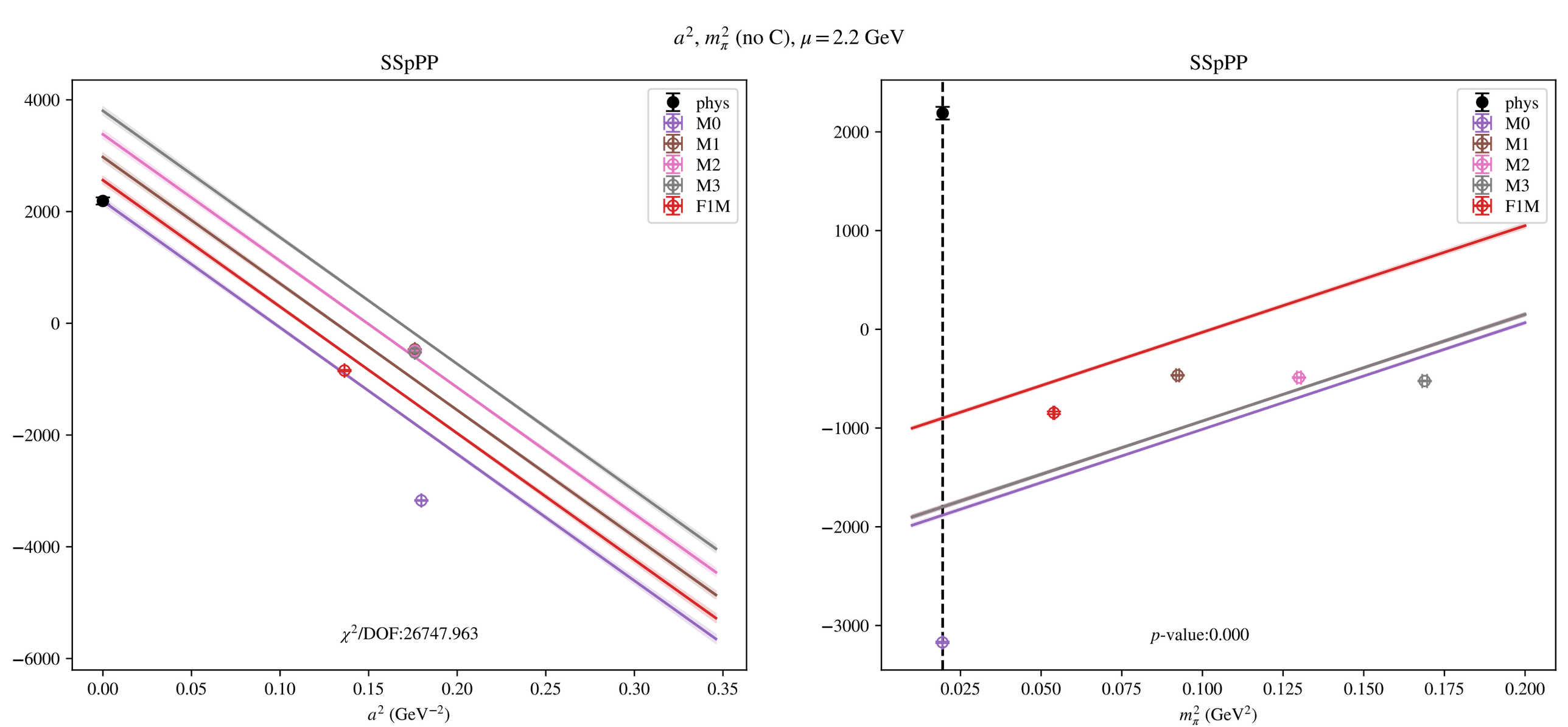

a^2, m_π^2, $\mu = 2.0$ GeV
SSpPP
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:19321.357
SSpPP
0
−1000
−2000
−3000
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
phys
C0
C1
C2
F1M
M0
M1
M2
M3
p-value:0.000

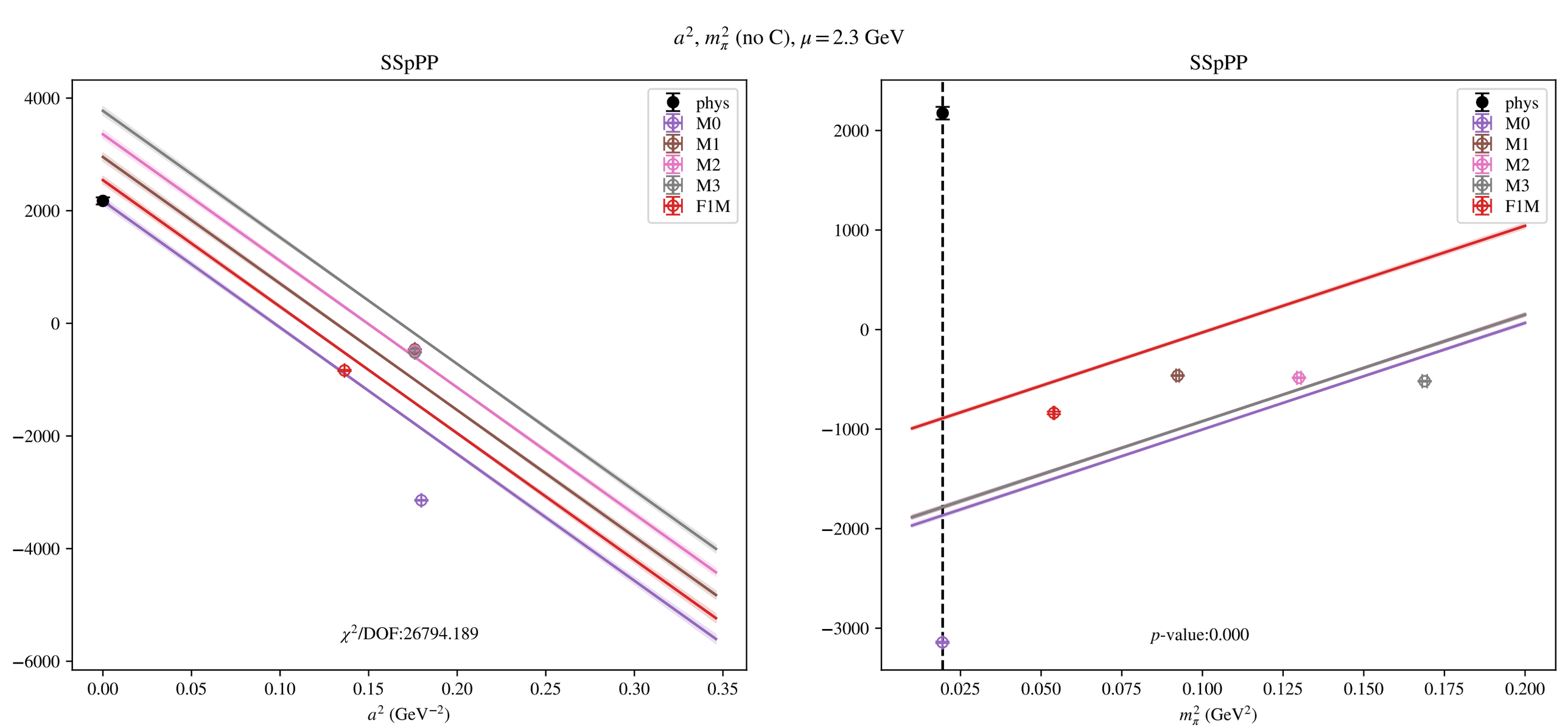

a^2, m_π^2, $\mu = 2.2$ GeV
SSpPP
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:19431.812
SSpPP
0
−1000
−2000
−3000
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
phys
C0
C1
C2
F1M
M0
M1
M2
M3
p-value:0.000

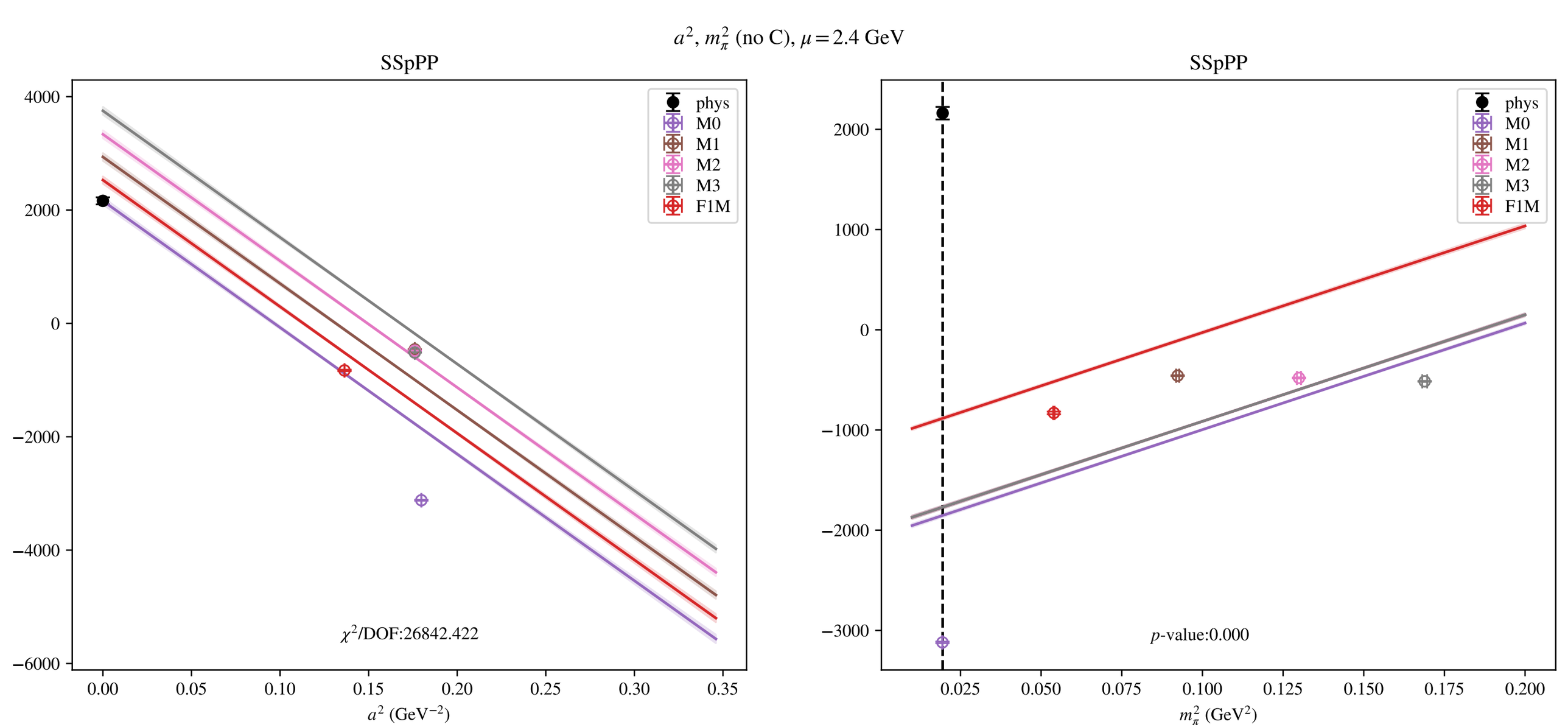

a^2, m_π^2, $\mu = 2.3$ GeV
SSpPP
0
−1000
−2000
−3000
−4000
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:19470.107
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSpPP
0
−1000
−2000
−3000
−4000
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)

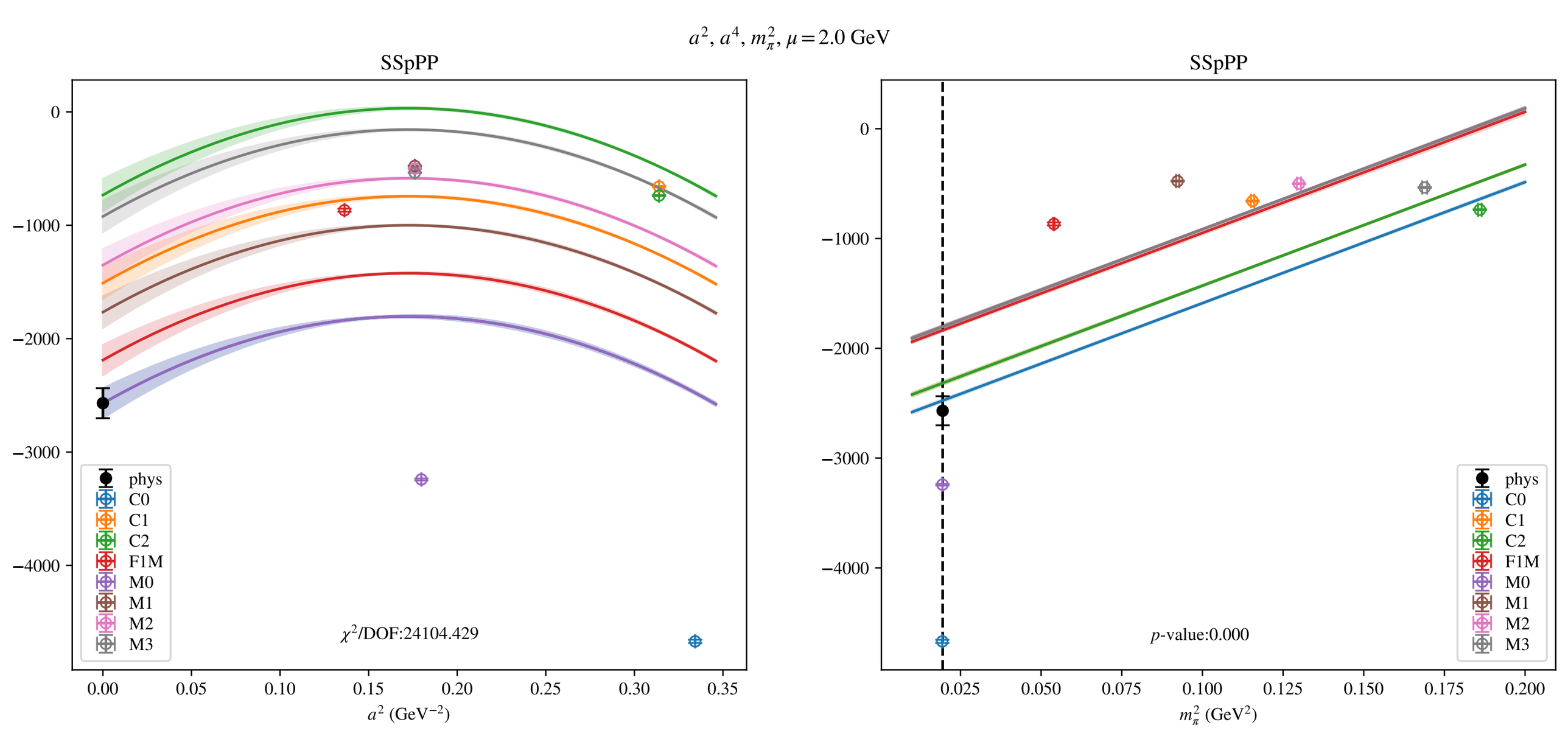

a^2, m_π^2, $\mu = 2.4$ GeV
SSpPP
SSpPP
0
−1000
−2000
−3000
−4000
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:19494.308
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)

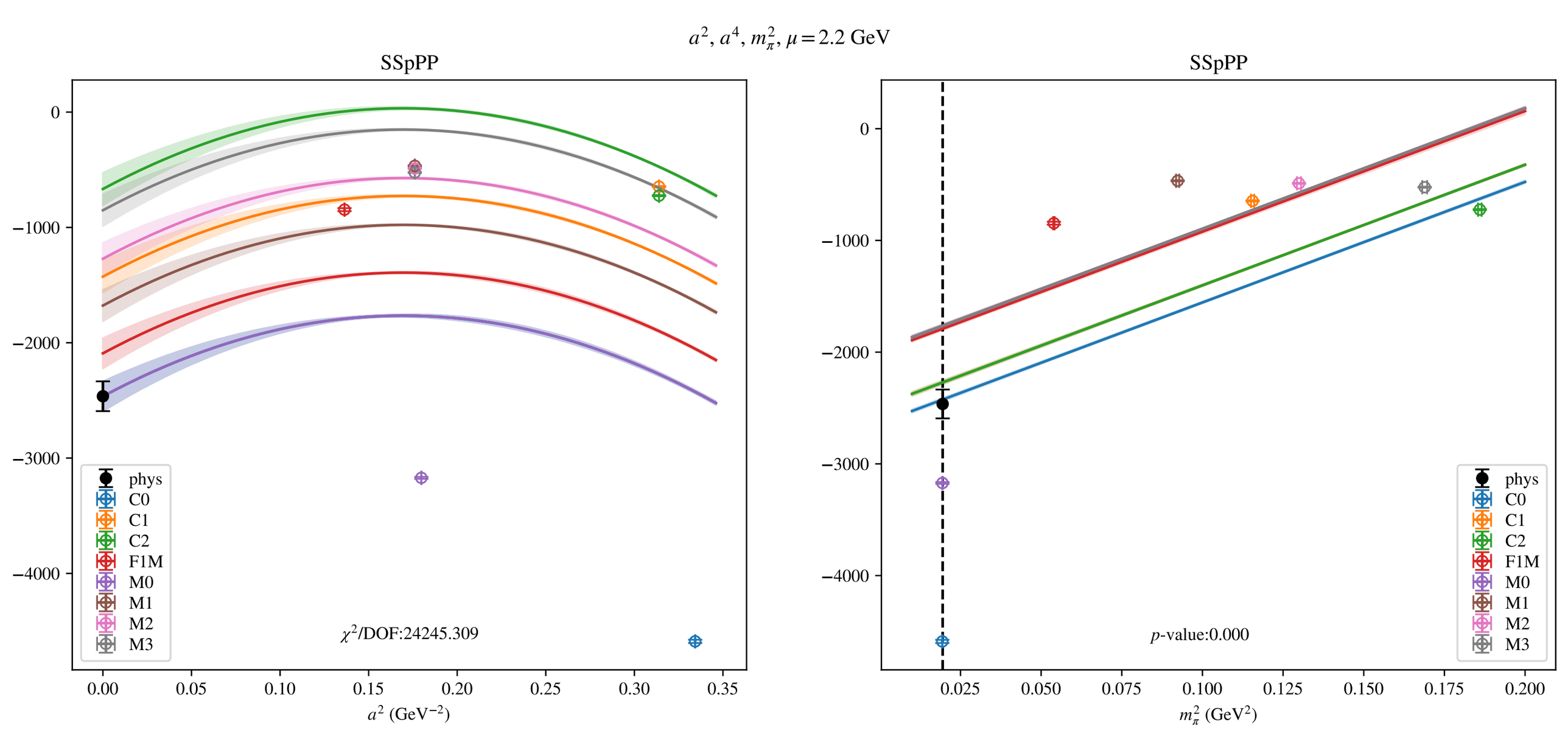

a^2, m_π^2 (no C), $\mu = 2.0$ GeV
SSpPP
phys
M0
M1
M2
M3
F1M
χ^2/DOF:26585.047
a^2 (GeV^{-2})
SSpPP
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

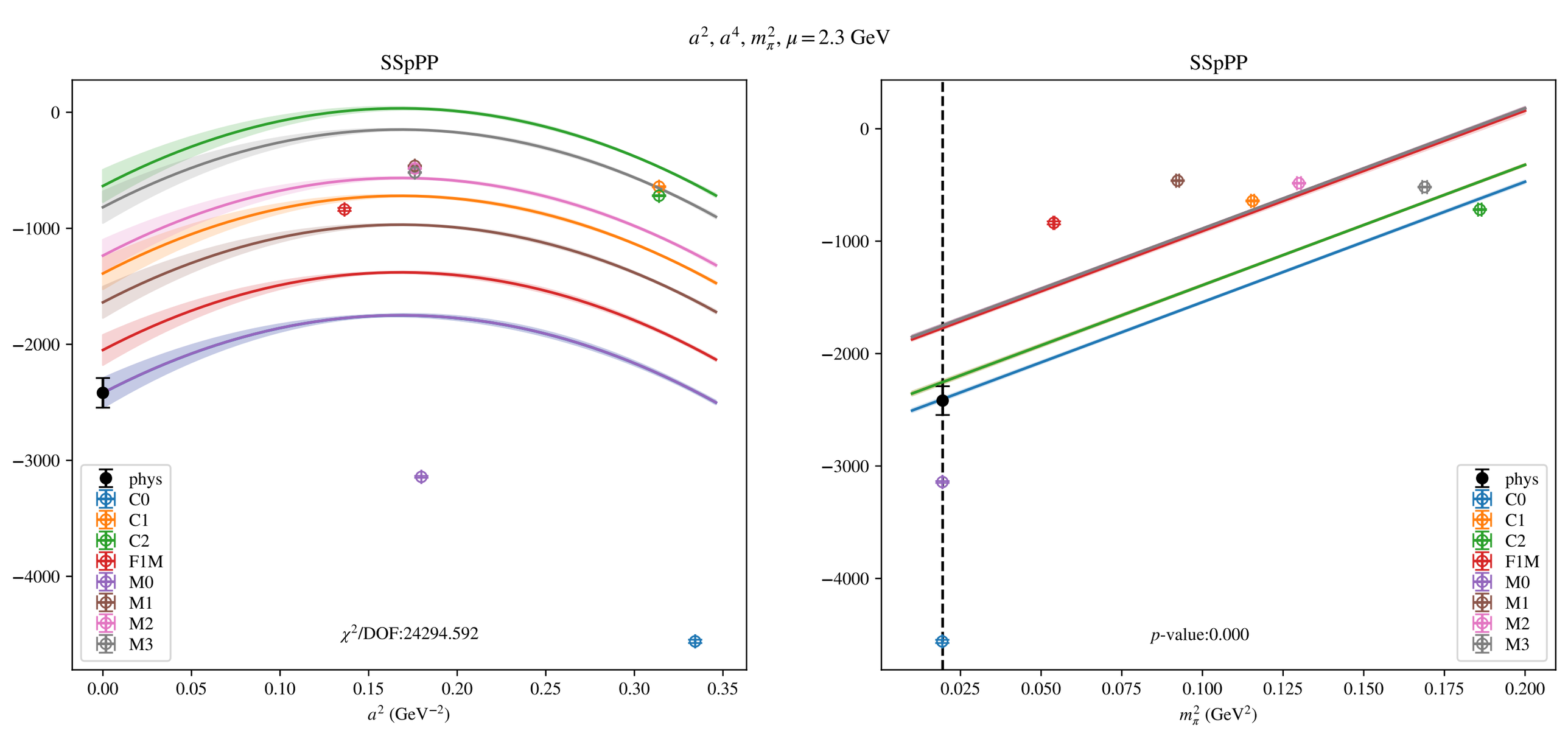

a^2, m_π^2 (no C), $\mu = 2.2$ GeV
SSpPP
phys
M0
M1
M2
M3
F1M
χ^2/DOF:26747.963
a^2 (GeV^{-2})
SSpPP
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)


a^2, m_π^2 (no C), $\mu = 2.3$ GeV
SSpPP
phys
M0
M1
M2
M3
F1M
χ^2/DOF:26794.189
a^2 (GeV^{-2})
SSpPP
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

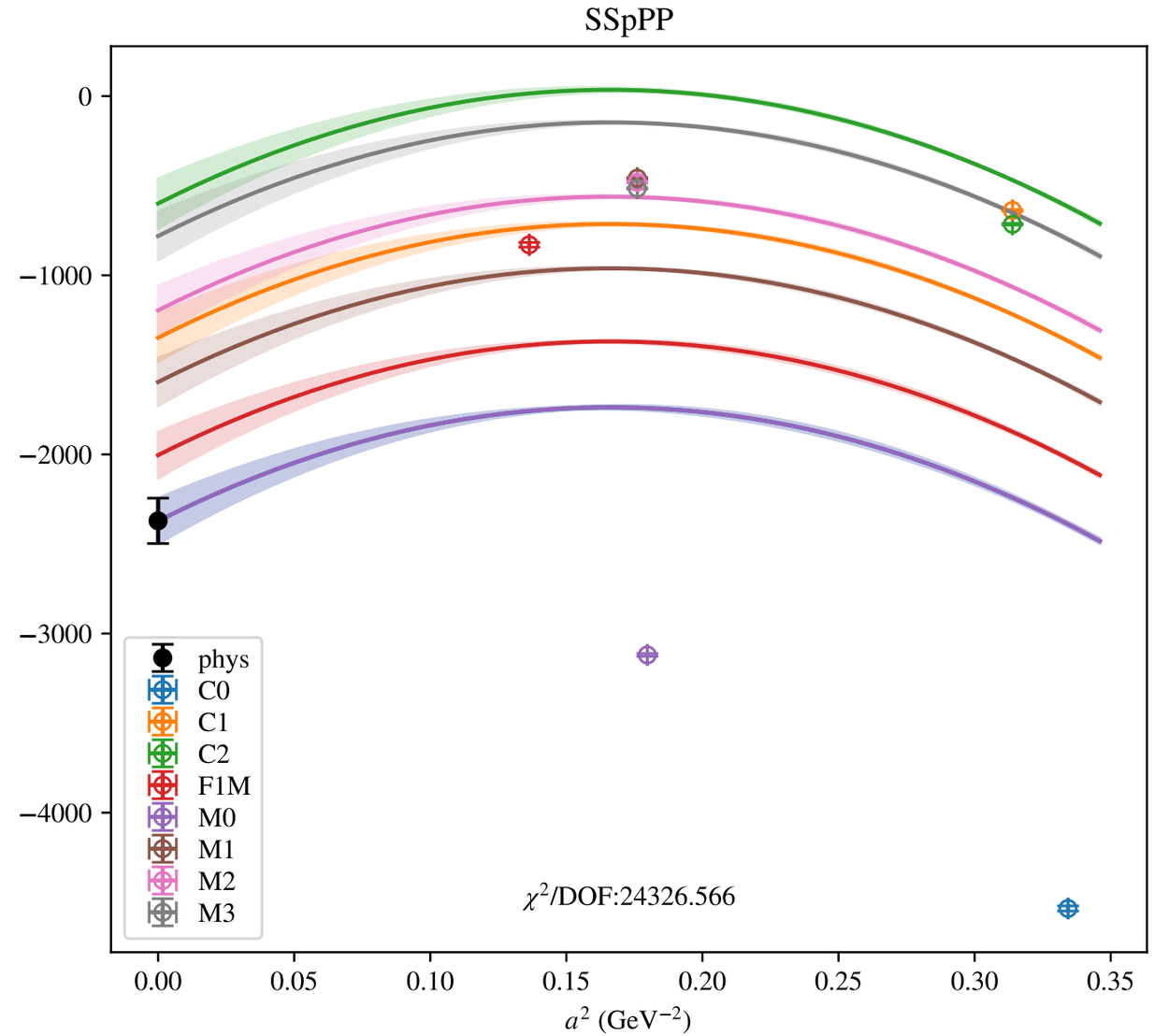

a^2, m_π^2 (no C), $\mu = 2.4$ GeV
SSpPP
phys
M0
M1
M2
M3
F1M
χ^2/DOF:26842.422
a^2 (GeV^{-2})
SSpPP
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

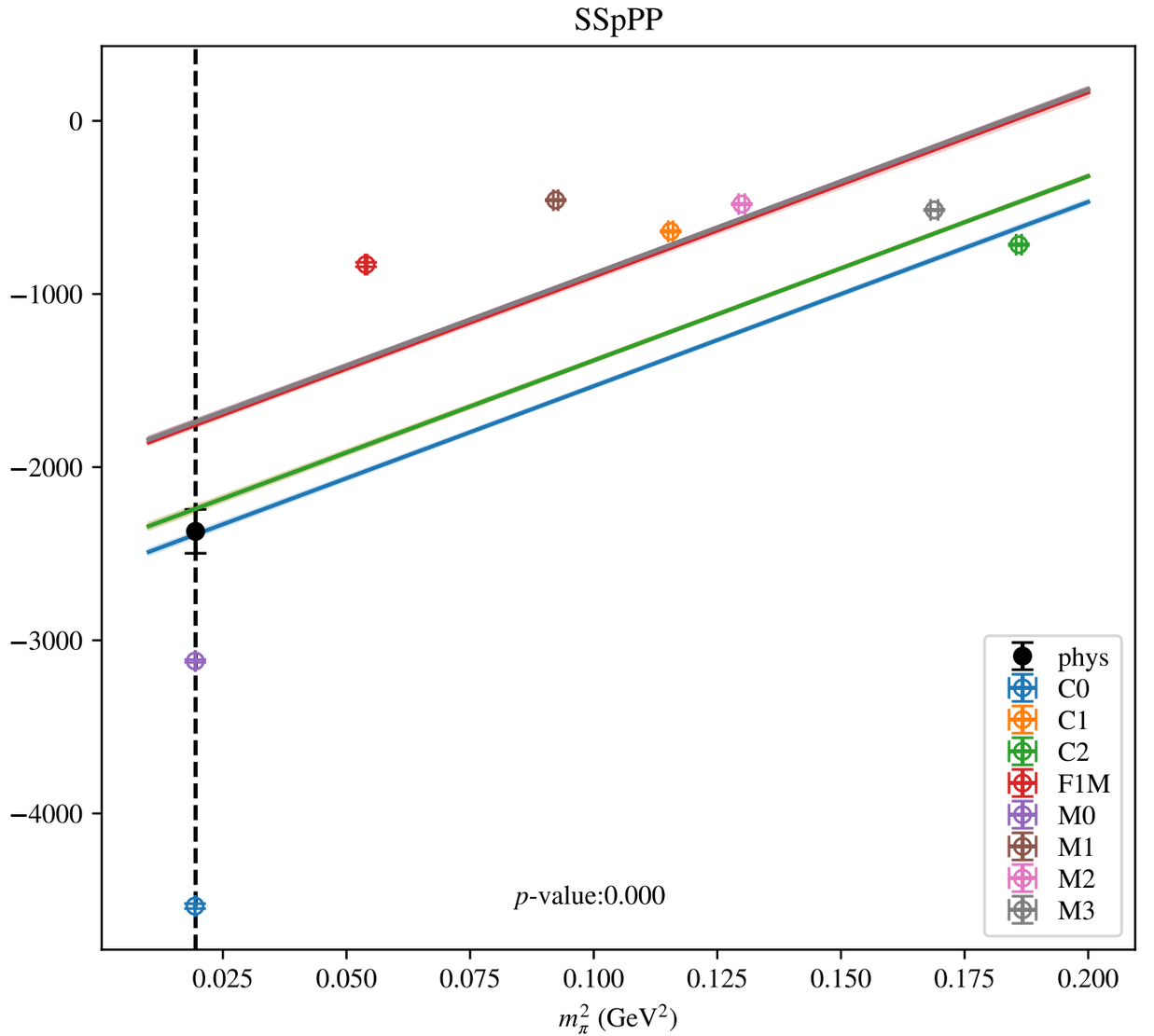

$a^2, a^4, m_\pi^2, \mu = 2.0$ GeV
SSpPP
SSpPP
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:24104.429
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

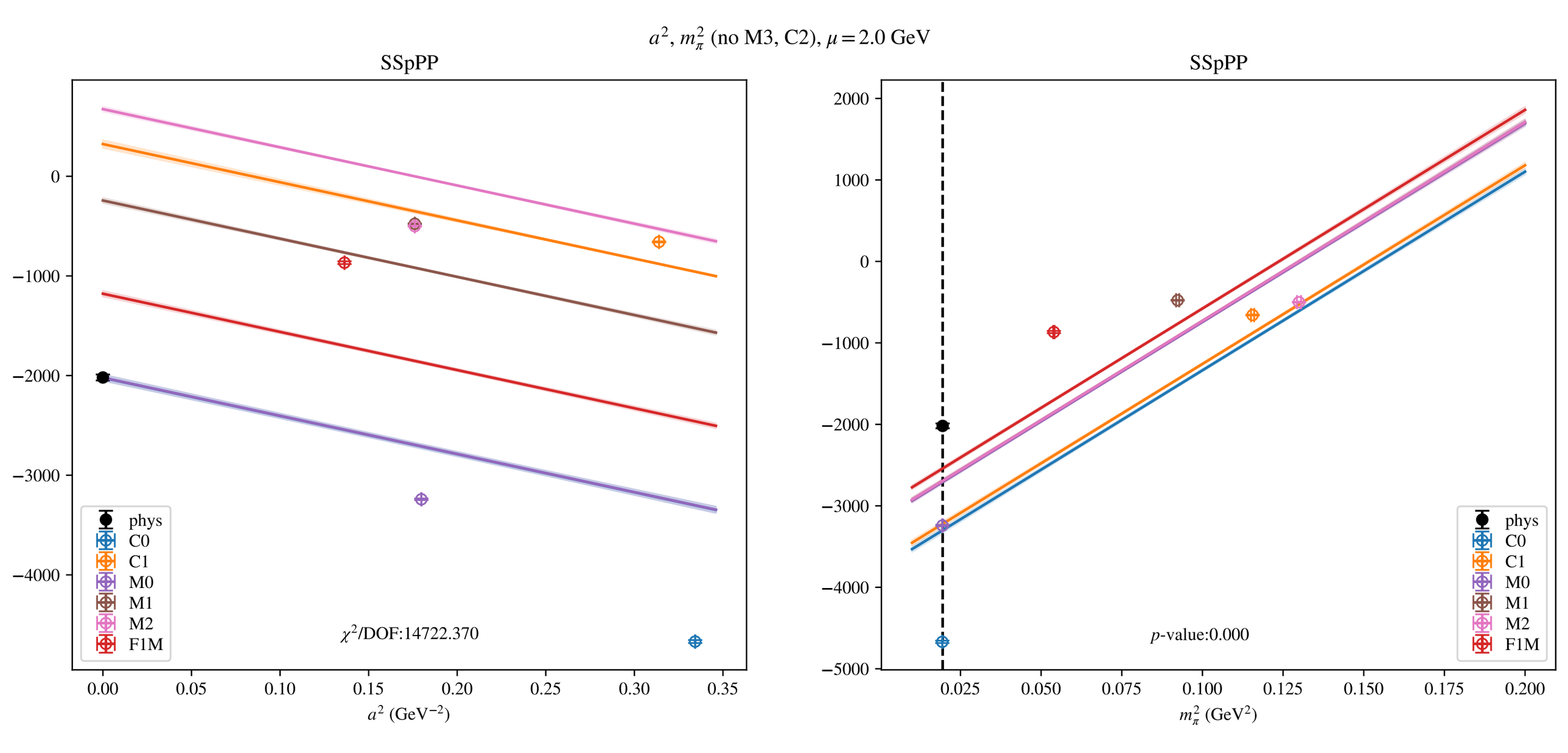

$a^2, a^4, m_\pi^2, \mu = 2.2$ GeV
SSpPP
SSpPP
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:24245.309
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

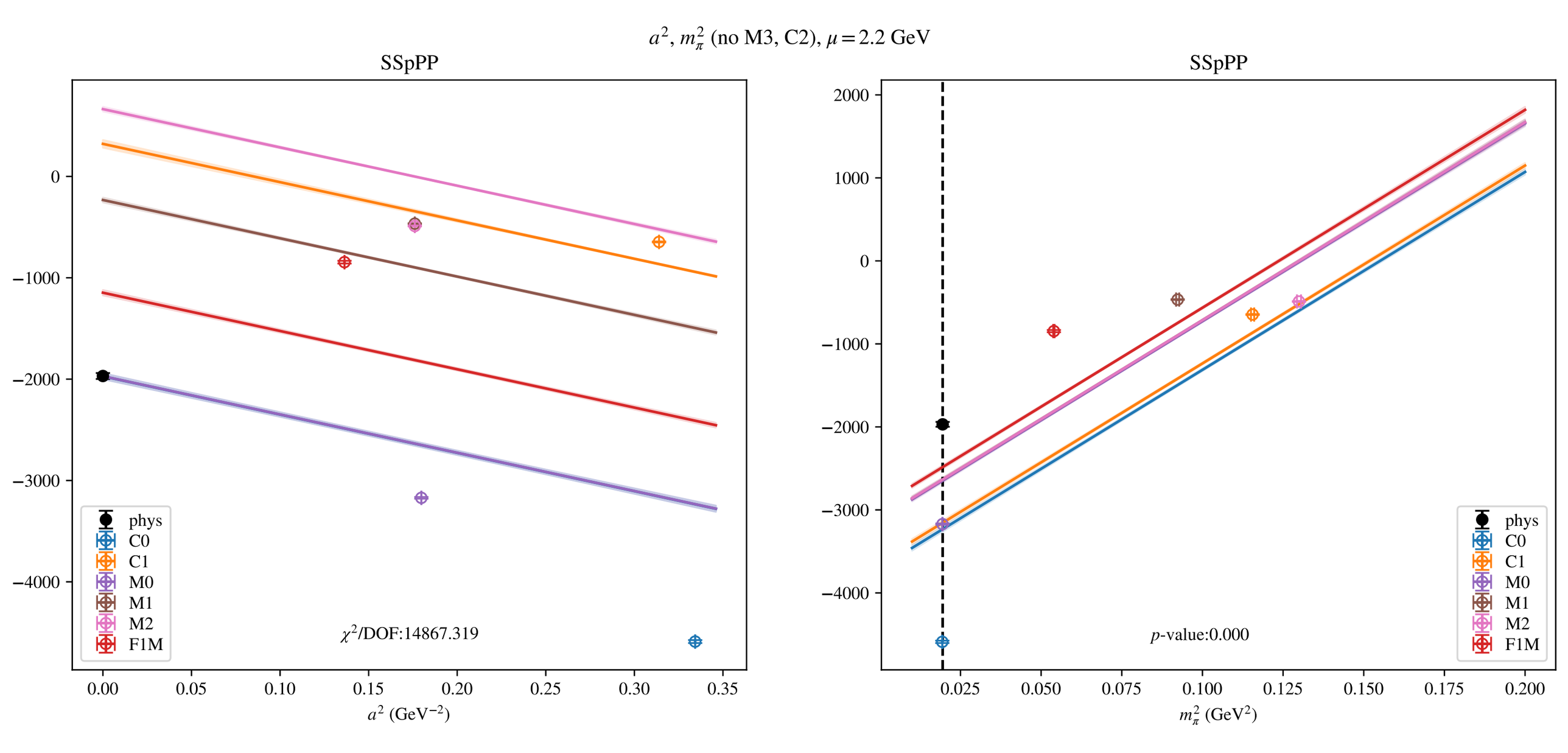

$a^2, a^4, m_\pi^2, \mu = 2.3$ GeV
SSpPP
SSpPP
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:24294.592
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

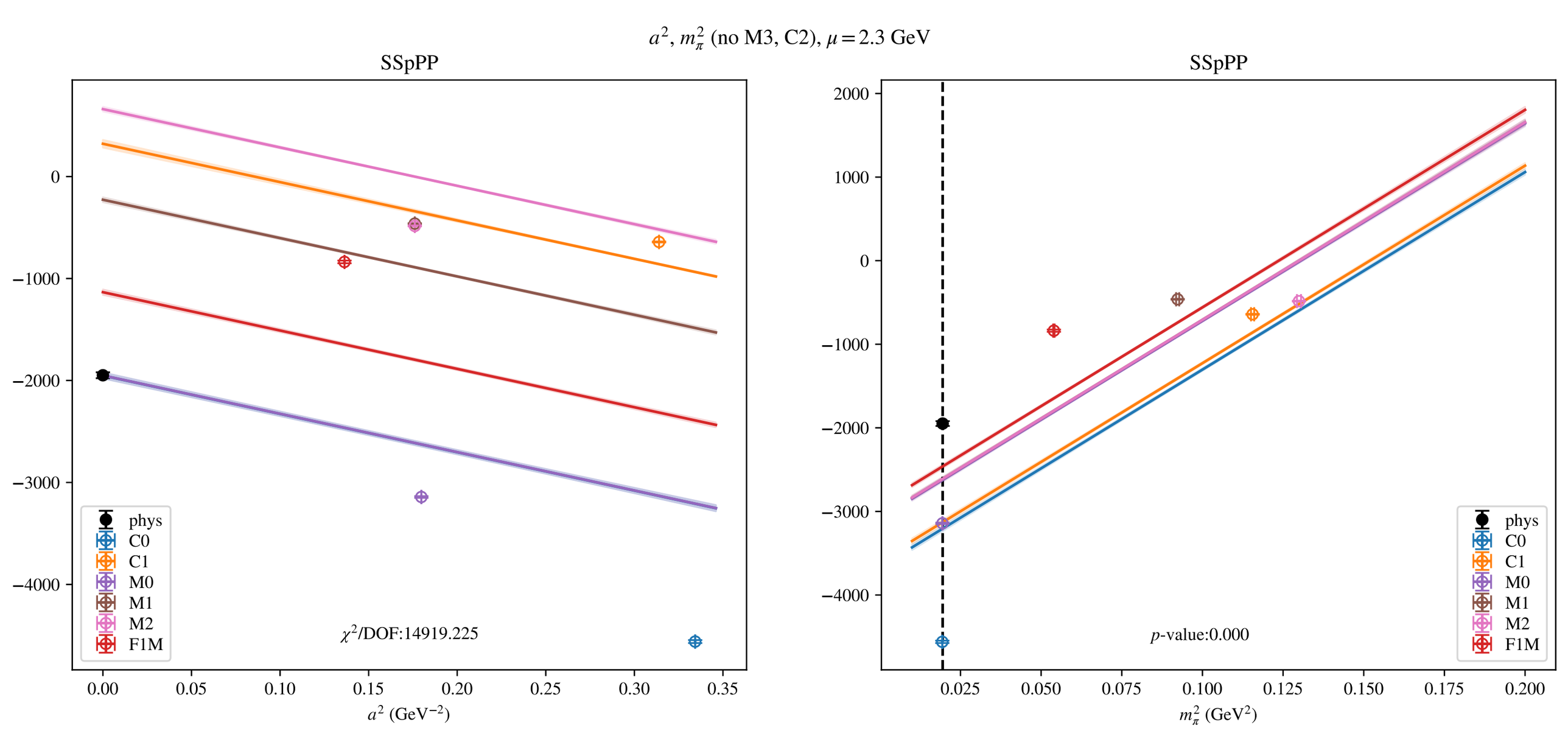
$a^2, a^4, m_\pi^2, \mu = 2.4$ GeV

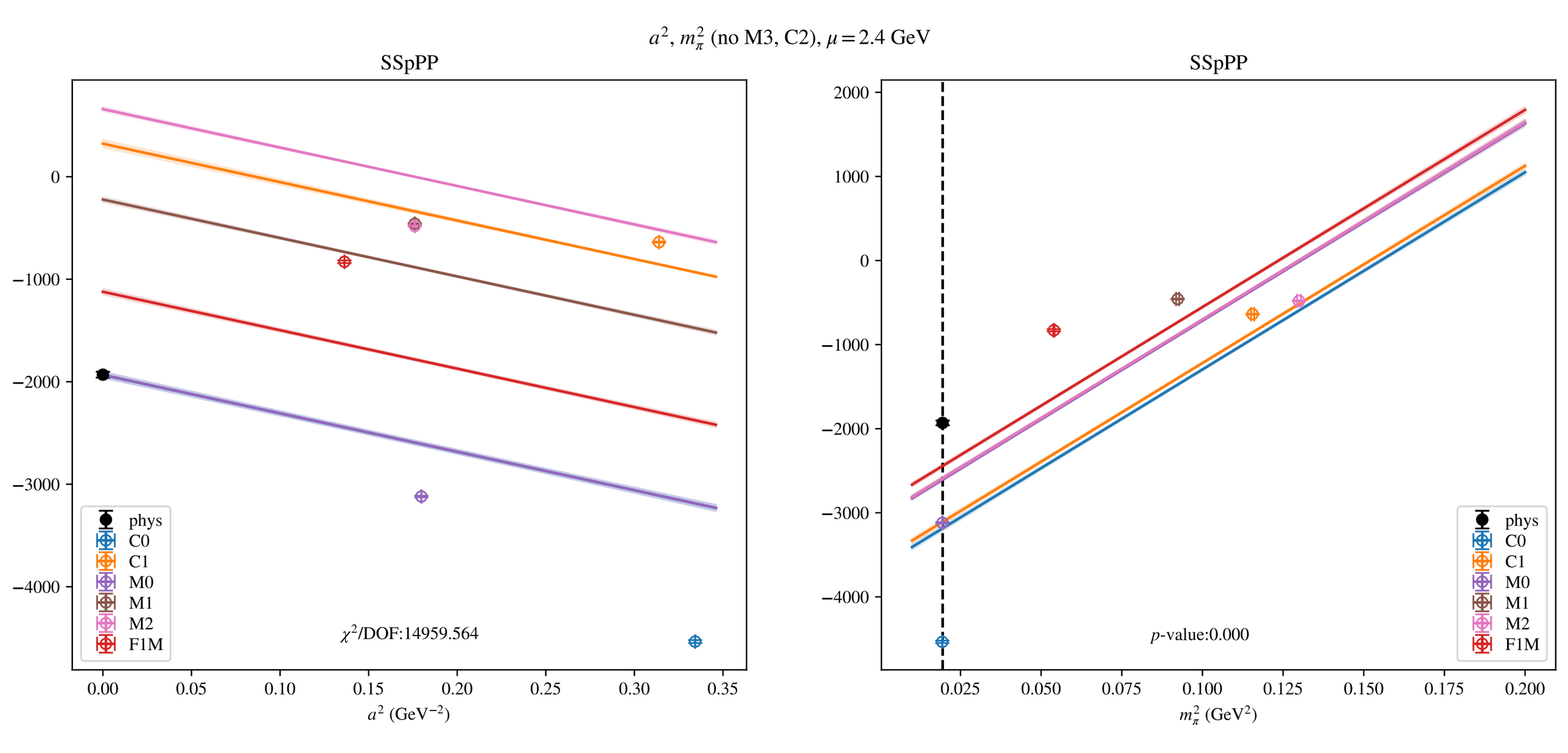


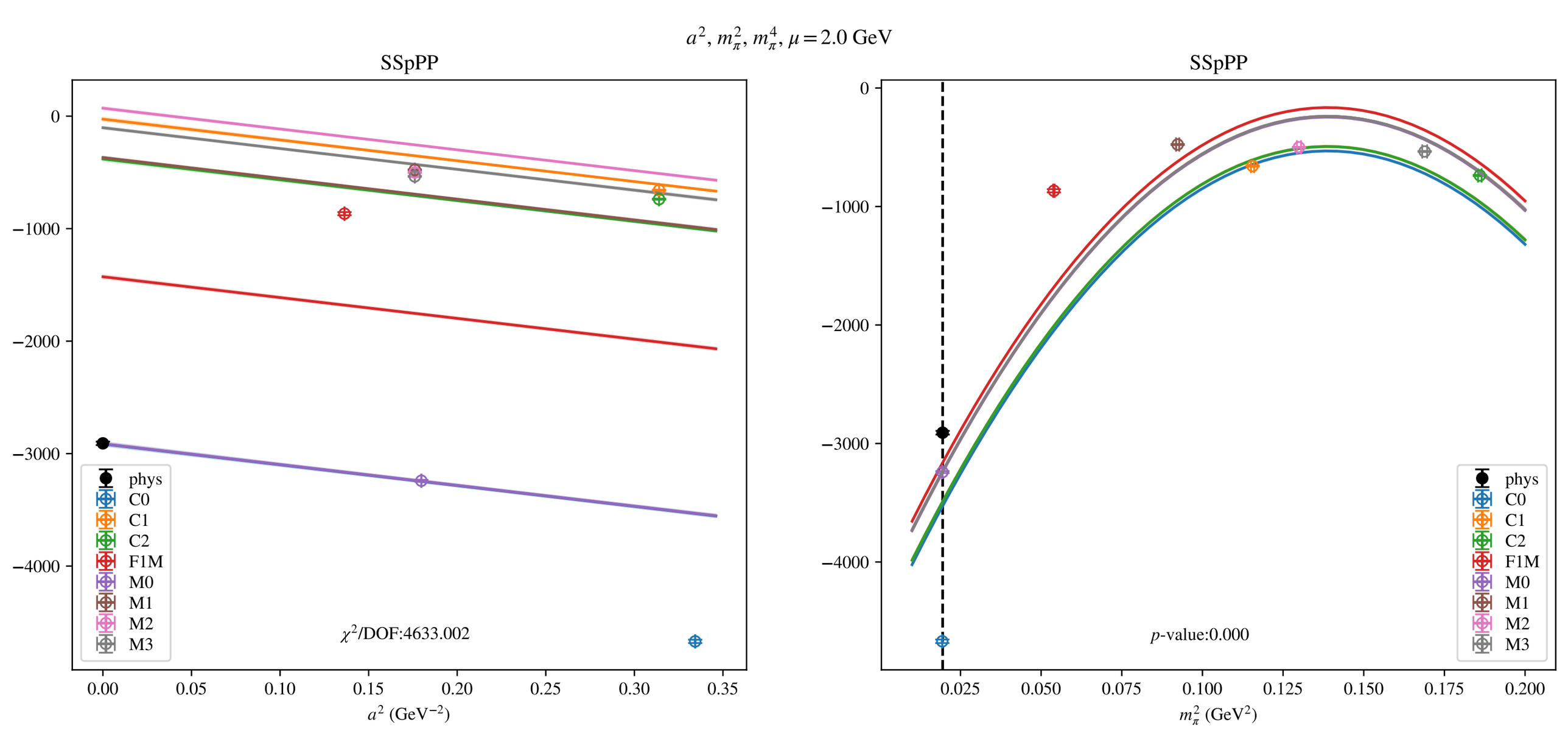


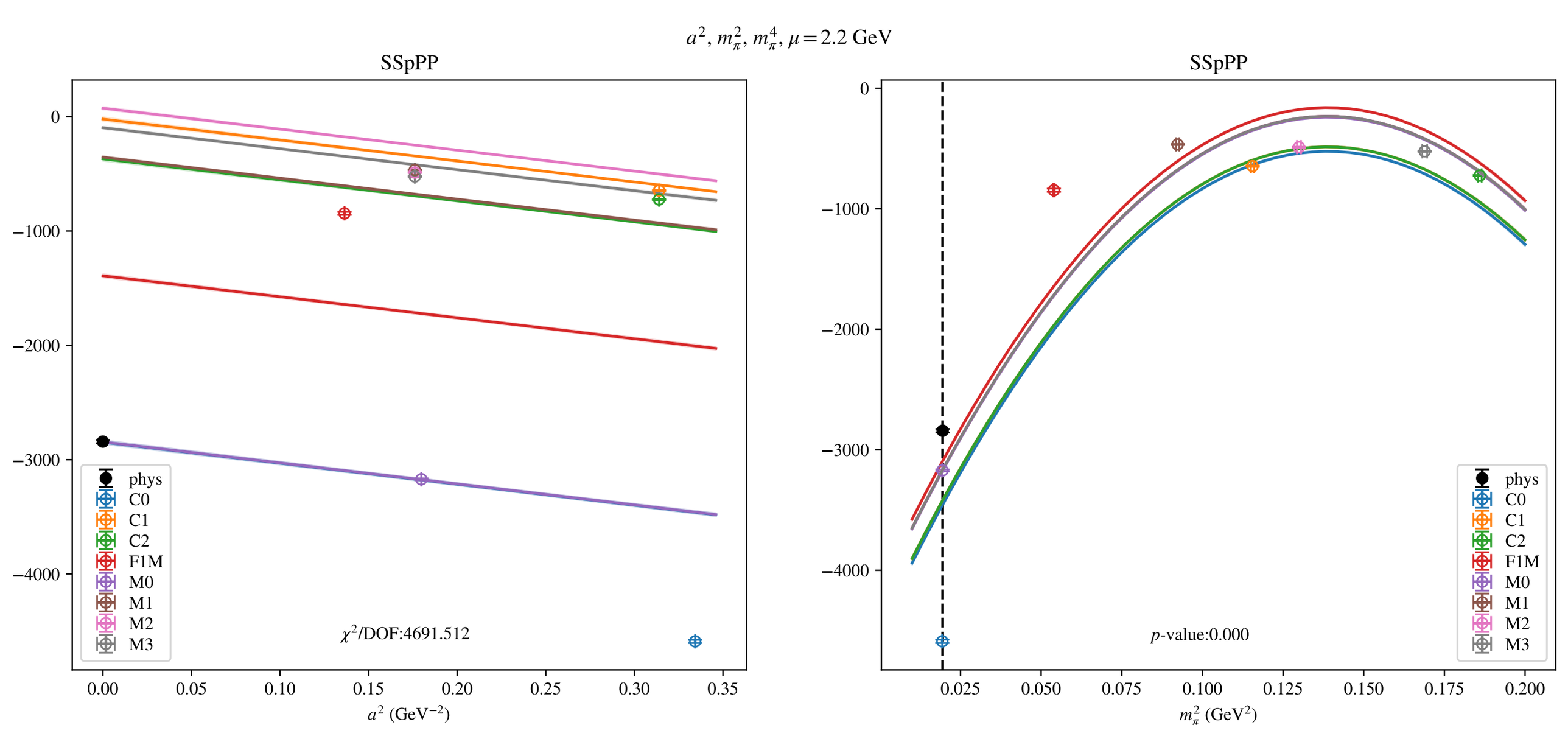

a^2, m_π^2 (no M3, C2), $\mu = 2.0$ GeV
SSpPP
0
−1000
−2000
−3000
−4000
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:14722.370
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSpPP
2000
1000
0
−1000
−2000
−3000
−4000
−5000
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)

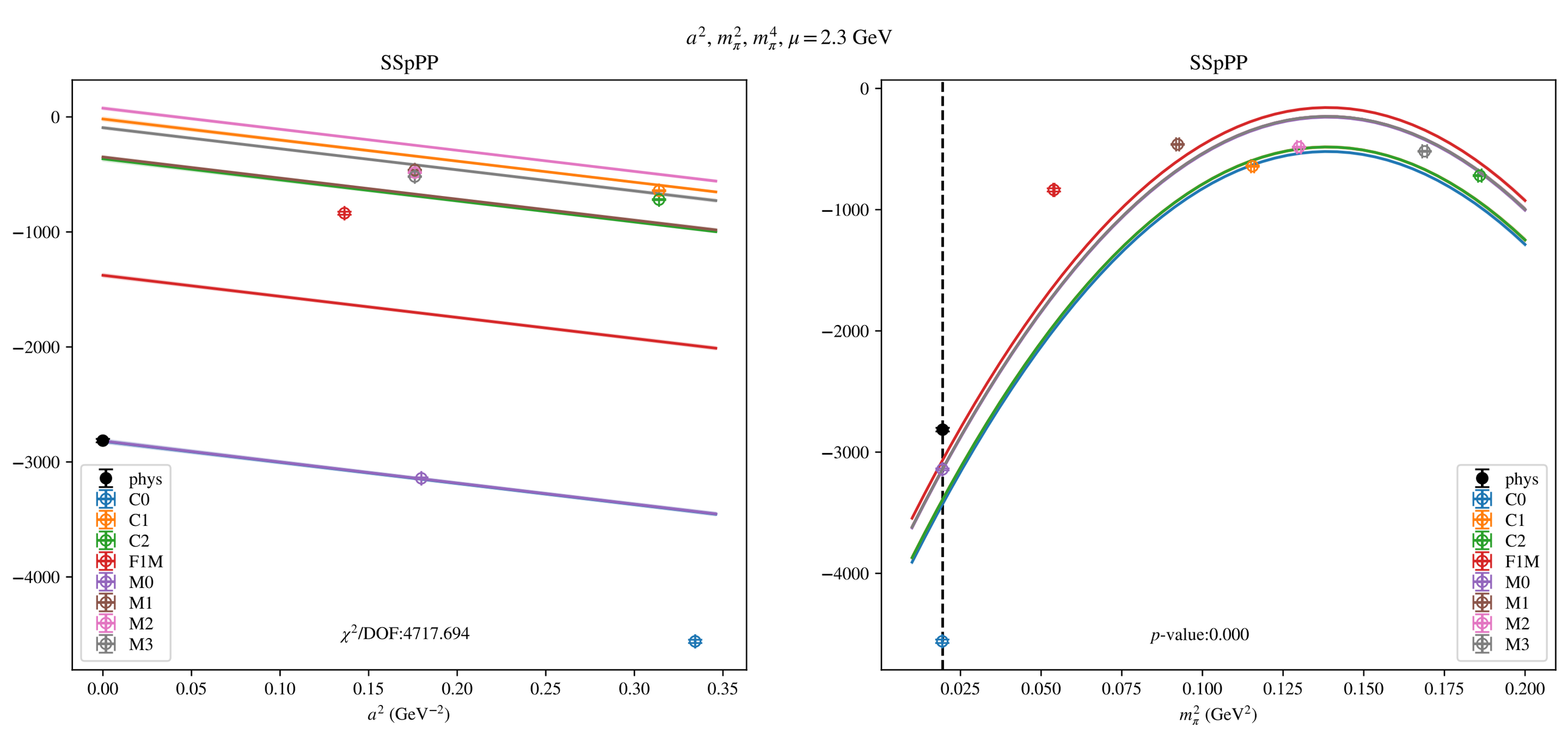

a^2, m_π^2 (no M3, C2), $\mu = 2.2$ GeV
SSpPP
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:14867.319
SSpPP
2000
1000
0
−1000
−2000
−3000
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
phys
C0
C1
M0
M1
M2
F1M
p-value:0.000

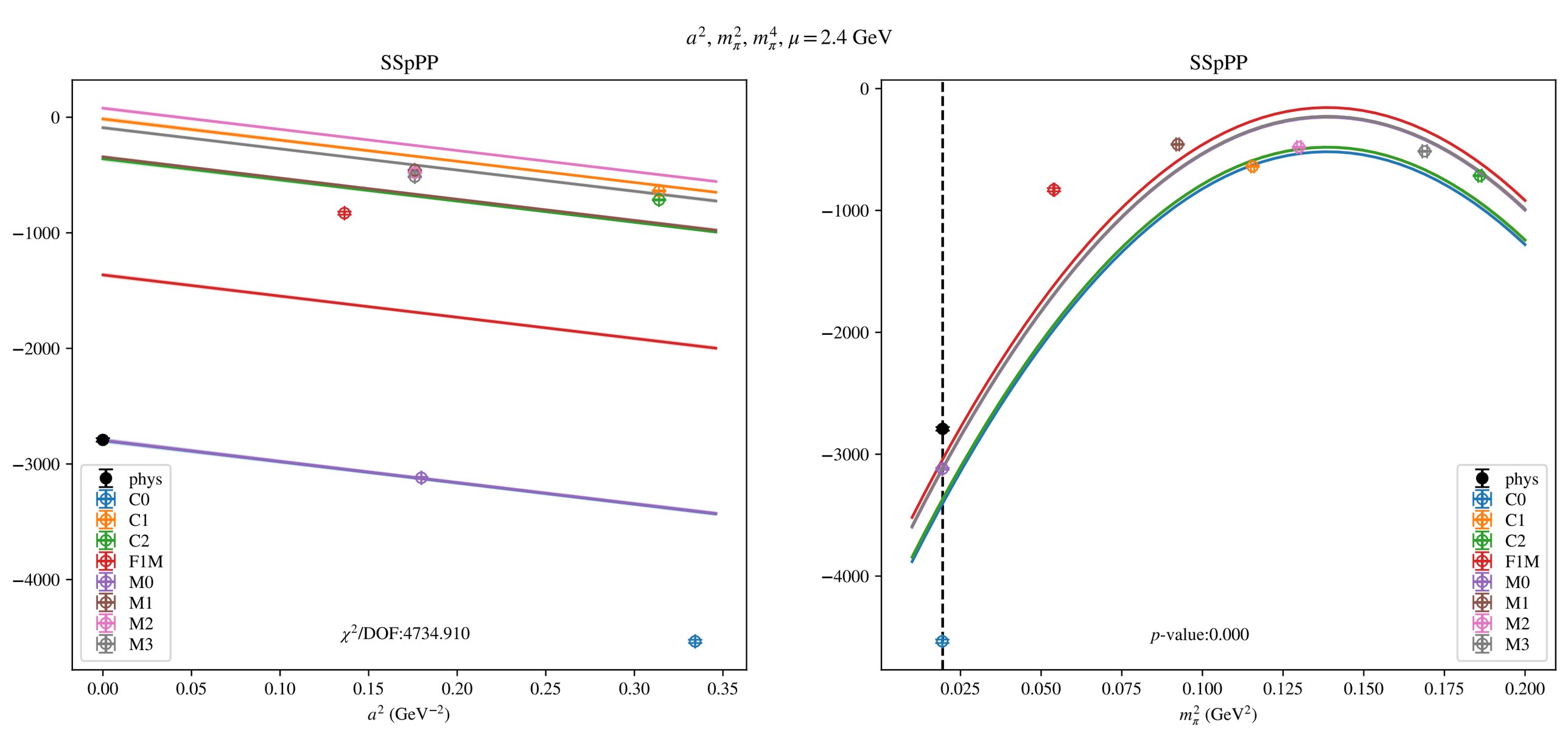

a^2, m_π^2 (no M3, C2), $\mu = 2.3$ GeV
SSpPP
0
−1000
−2000
−3000
−4000
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:14919.225
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSpPP
2000
1000
0
−1000
−2000
−3000
−4000
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


a^2, m_π^2 (no M3, C2), $\mu = 2.4$ GeV
SSpPP
0
−1000
−2000
−3000
−4000
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:14959.564
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
SSpPP
2000
1000
0
−1000
−2000
−3000
−4000
p-value:0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.0$ GeV
SSpPP
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:4633.002
SSpPP
0
−1000
−2000
−3000
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
phys
C0
C1
C2
F1M
M0
M1
M2
M3
p-value:0.000


$a^2, m_\pi^2, m_\pi^4, \mu = 2.2$ GeV
SSpPP
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:4691.512
SSpPP
0
−1000
−2000
−3000
−4000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
phys
C0
C1
C2
F1M
M0
M1
M2
M3
p-value:0.000


$a^2, m_\pi^2, m_\pi^4, \mu = 2.3$ GeV
SSpPP
SSpPP
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:4717.694
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200


$a^2, m_\pi^2, m_\pi^4, \mu = 2.4$ GeV
SSpPP
SSpPP
0
−1000
−2000
−3000
−4000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:4734.910
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
p-value:0.000

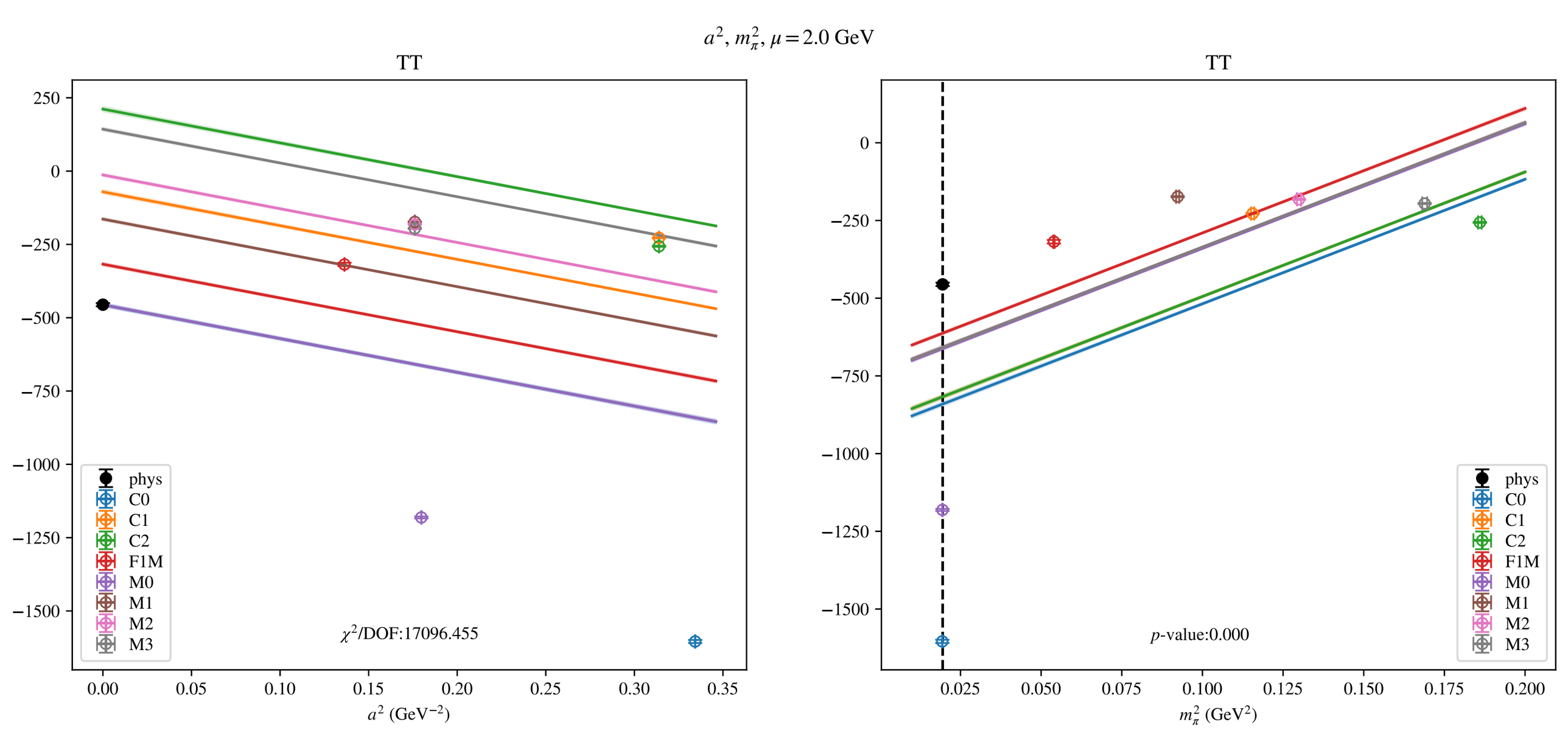
5 $\mathcal{B}_5$

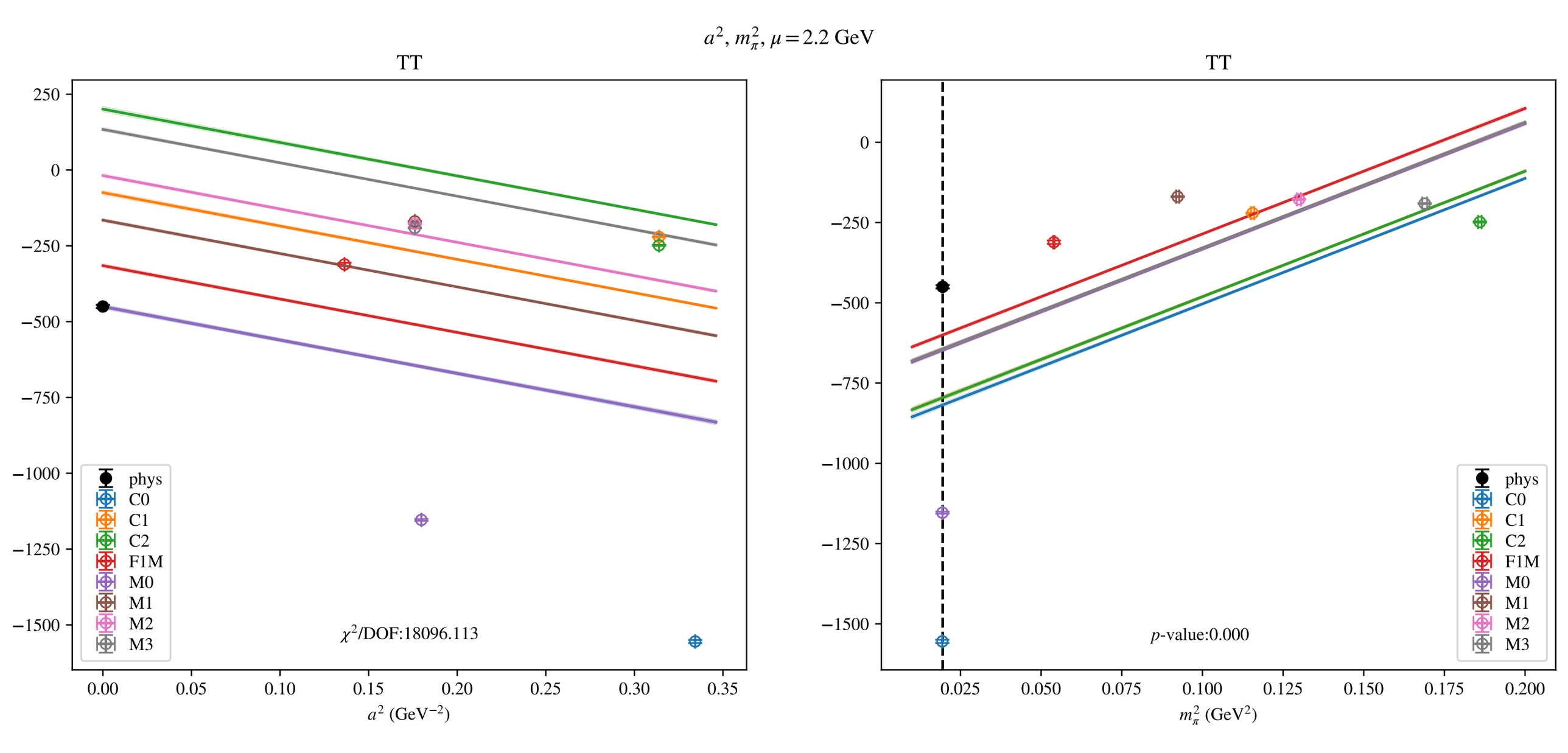
μ (GeV)	a^2, m_π^2	a^2, m_π^2 (no C)	a^2, a^4, m_π^2	a^2, m_π^2 (no M3, C2)	a^2, m_π^2, m_π^4
2.0	**-45(55)**: 17096.455 (0.0)	**85(25)**: 23134.53 (0.0)	**-1(51)**: 21262.08 (0.0)	**-7(11)**: 12246.406 (0.0)	**-10(60)**: 3655.794 (0.0)
2.2	**-45(53)**: 18096.113 (0.0)	**85(24)**: 24856.421 (0.0)	**-1(50)**: 22519.23 (0.0)	**-7(10)**: 12959.163 (0.0)	**-10(57)**: 3819.876 (0.0)
2.3	**-44(52)**: 18351.48 (0.0)	**85(24)**: 25228.853 (0.0)	**-1(49)**: 22841.031 (0.0)	**-7(10)**: 13074.692 (0.0)	**-10(56)**: 3844.174 (0.0)
2.4	**-44(52)**: 18521.422 (0.0)	**84(24)**: 25482.221 (0.0)	**-1(49)**: 23055.41 (0.0)	**-7(10)**: 13138.571 (0.0)	**-10(55)**: 3851.878 (0.0)

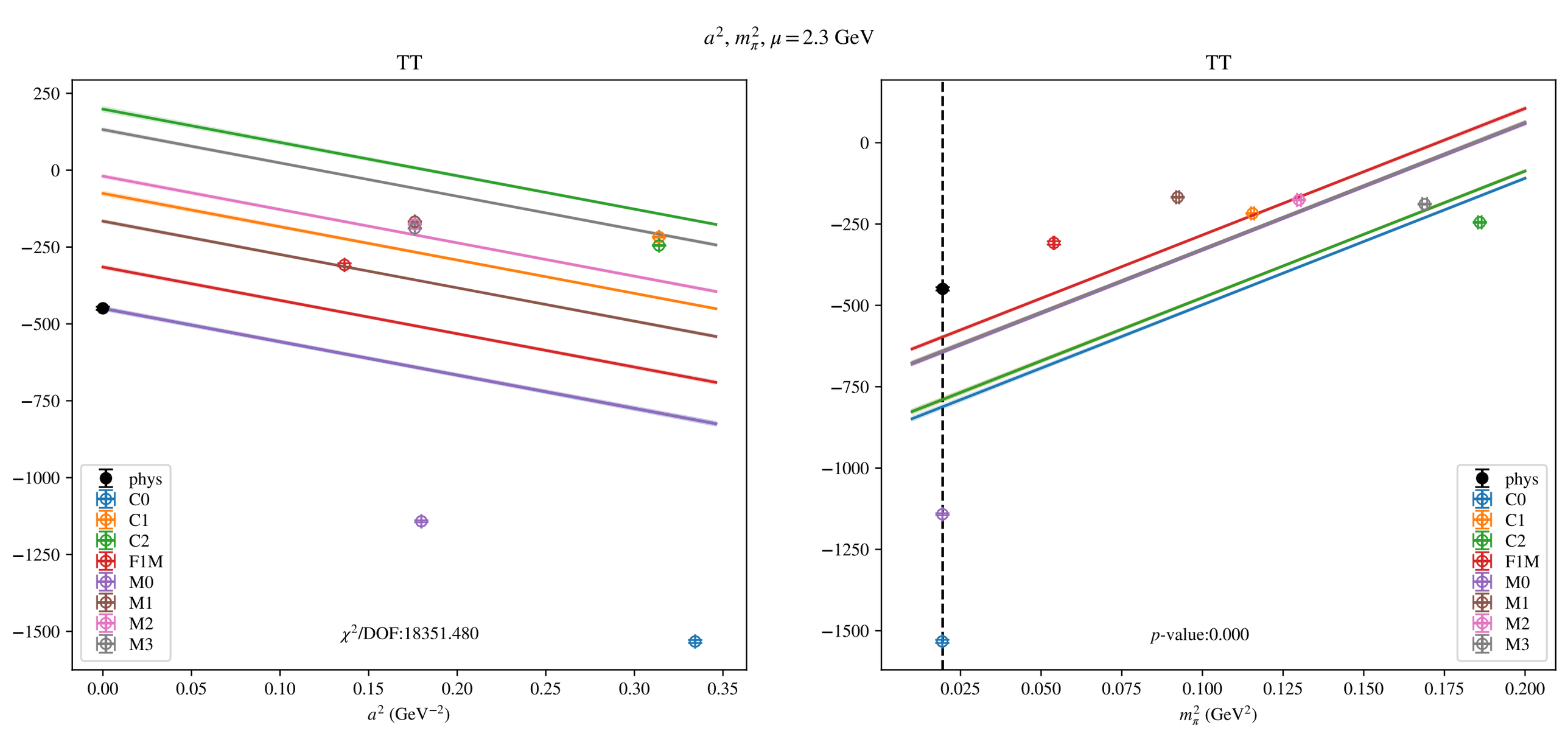
Table 9: Physical point value from chiral and continuum extrapolation at renormalisation scale μ. Entries are **value(error)**: χ^2/DOF (p-value).

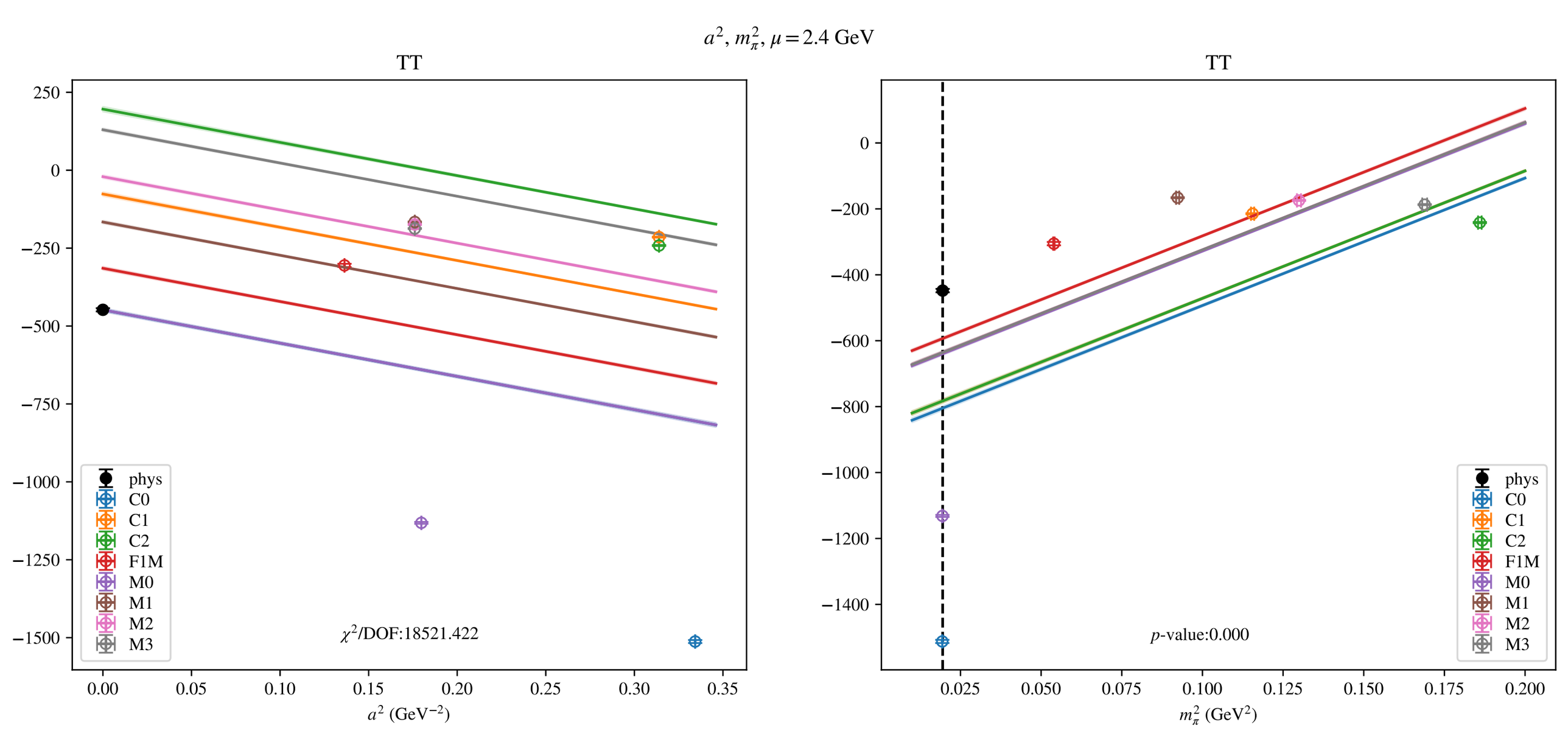
μ (GeV)		a^2, m_π^2	a^2, m_π^2 (no C)	a^2, a^4, m_π^2	a^2, m_π^2 (no M3, C2)	a^2, m_π^2, m_π^4
2.0	α	2.528(74)	-10.(12)	-4.9(17)	1.568(77)	0.529(20)
	β	-0.149(19)	0.0792(24)	-0.050(24)	-0.197(27)	-0.334(19)
2.2	α	2.446(71)	-10.(12)	-4.8(18)	1.528(75)	0.506(20)
	β	-0.147(18)	0.0779(23)	-0.052(26)	-0.195(27)	-0.333(18)
2.3	α	2.412(71)	-10.(11)	-4.7(19)	1.510(75)	0.498(19)
	β	-0.147(18)	0.0776(23)	-0.053(26)	-0.195(27)	-0.332(18)
2.4	α	2.381(70)	-10.(11)	-4.7(19)	1.492(74)	0.491(19)
	β	-0.146(18)	0.0775(22)	-0.053(27)	-0.194(26)	-0.332(18)

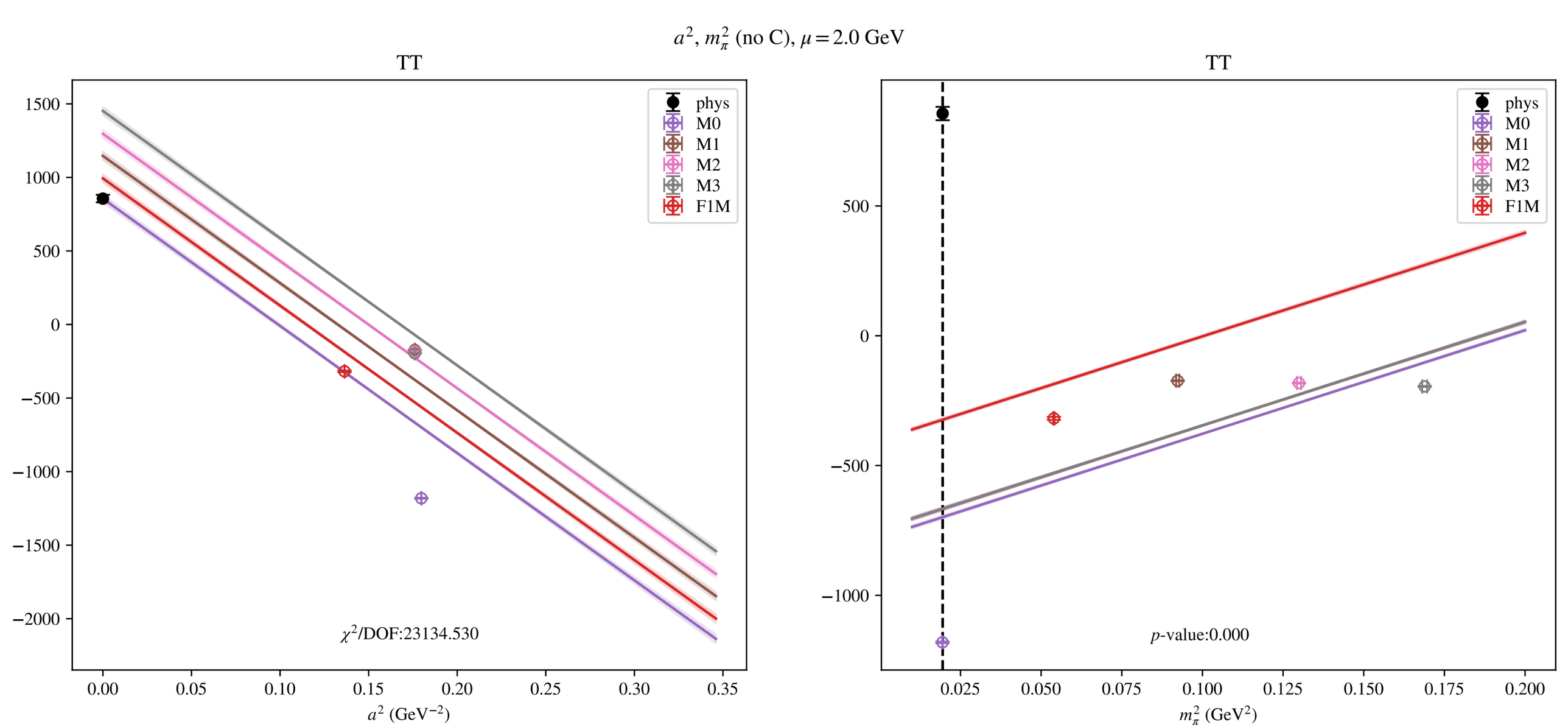
Table 10: Fit values of coefficients in $Q = Q_{phys} + \alpha a^2 + \beta \left(\frac{m_\pi^2}{f_\pi^2} - \frac{m_{\pi,PDG}^2}{f_\pi^2} \right) + \ldots$.

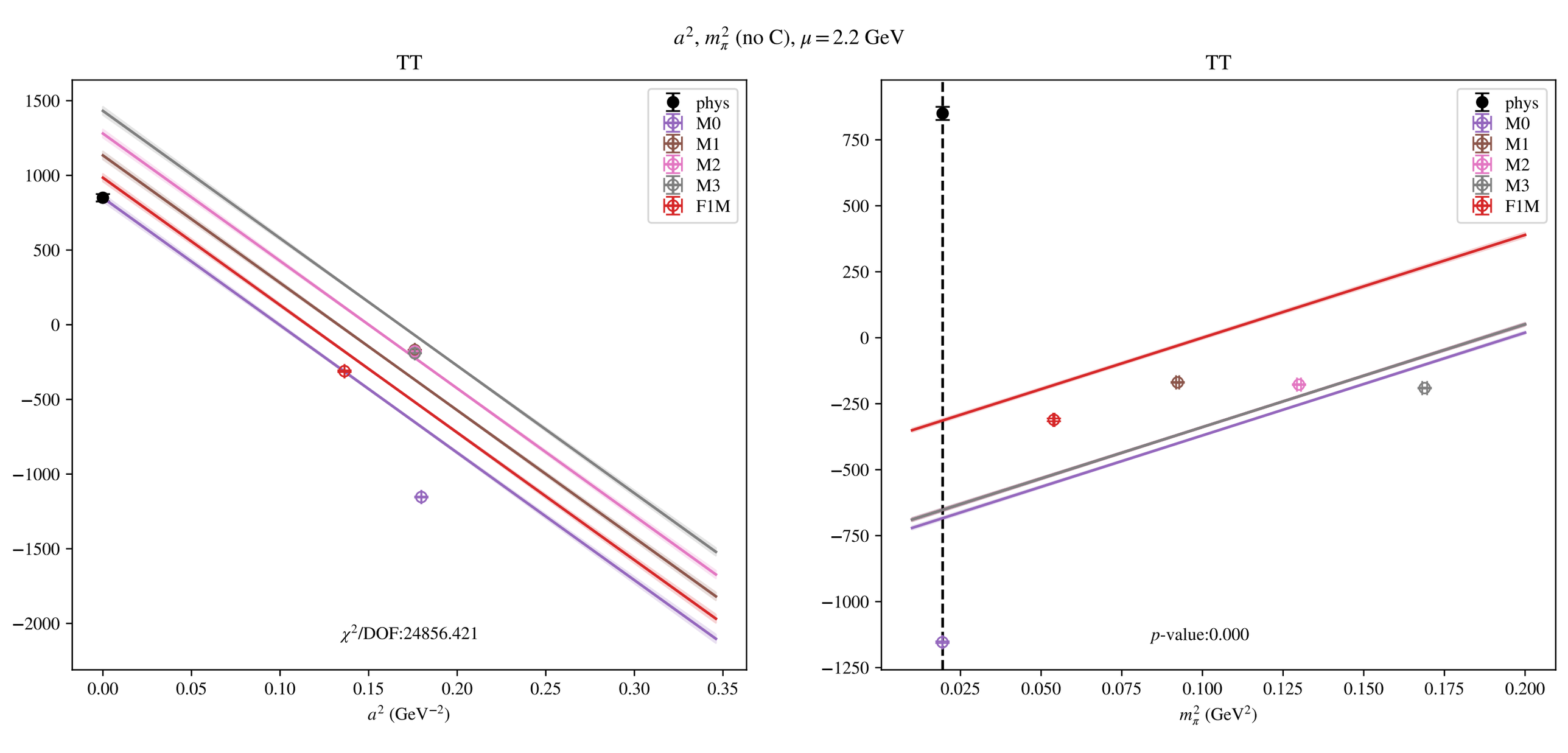

$a^2, m_\pi^2, \mu = 2.0$ GeV
TT
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:17096.455
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)

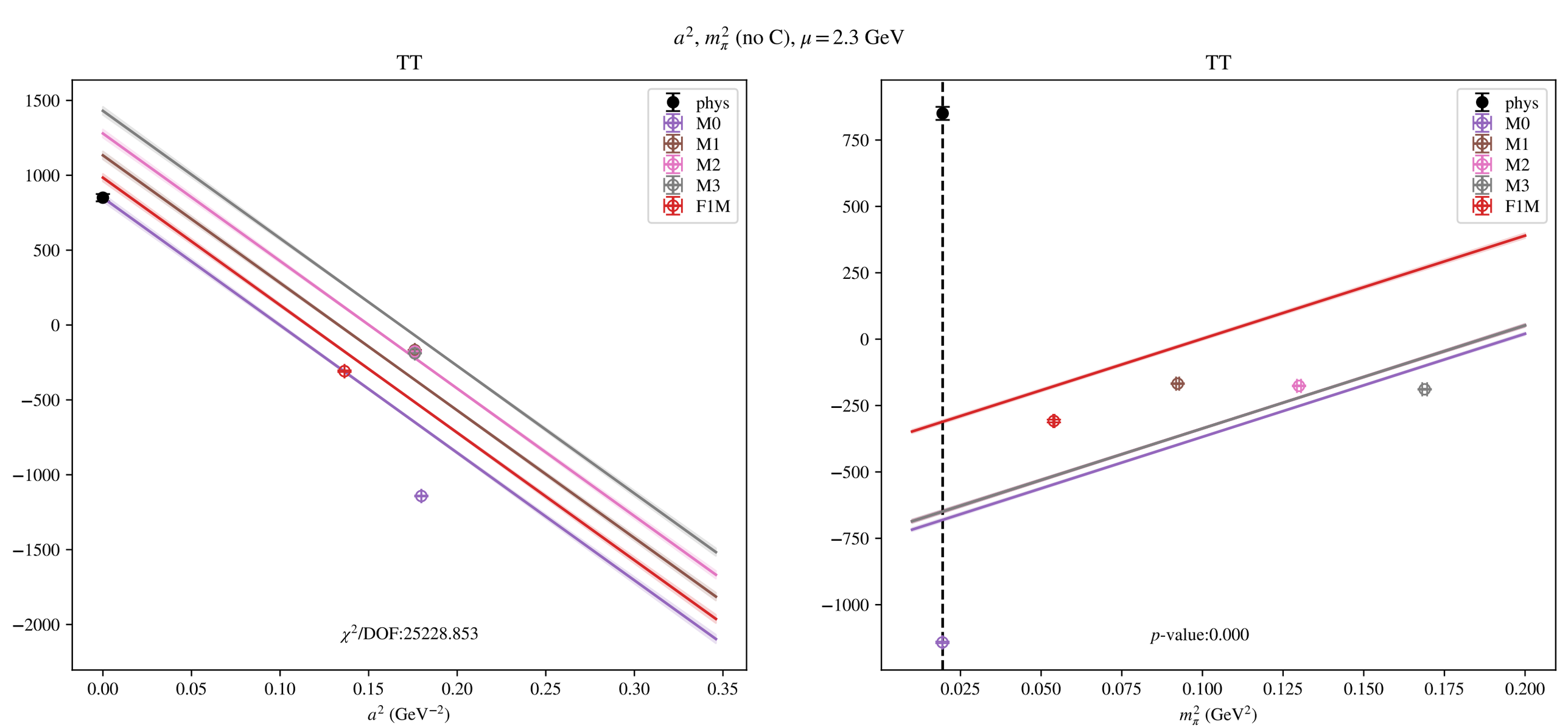

$a^2, m_\pi^2, \mu = 2.2$ GeV
TT
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:18096.113
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)

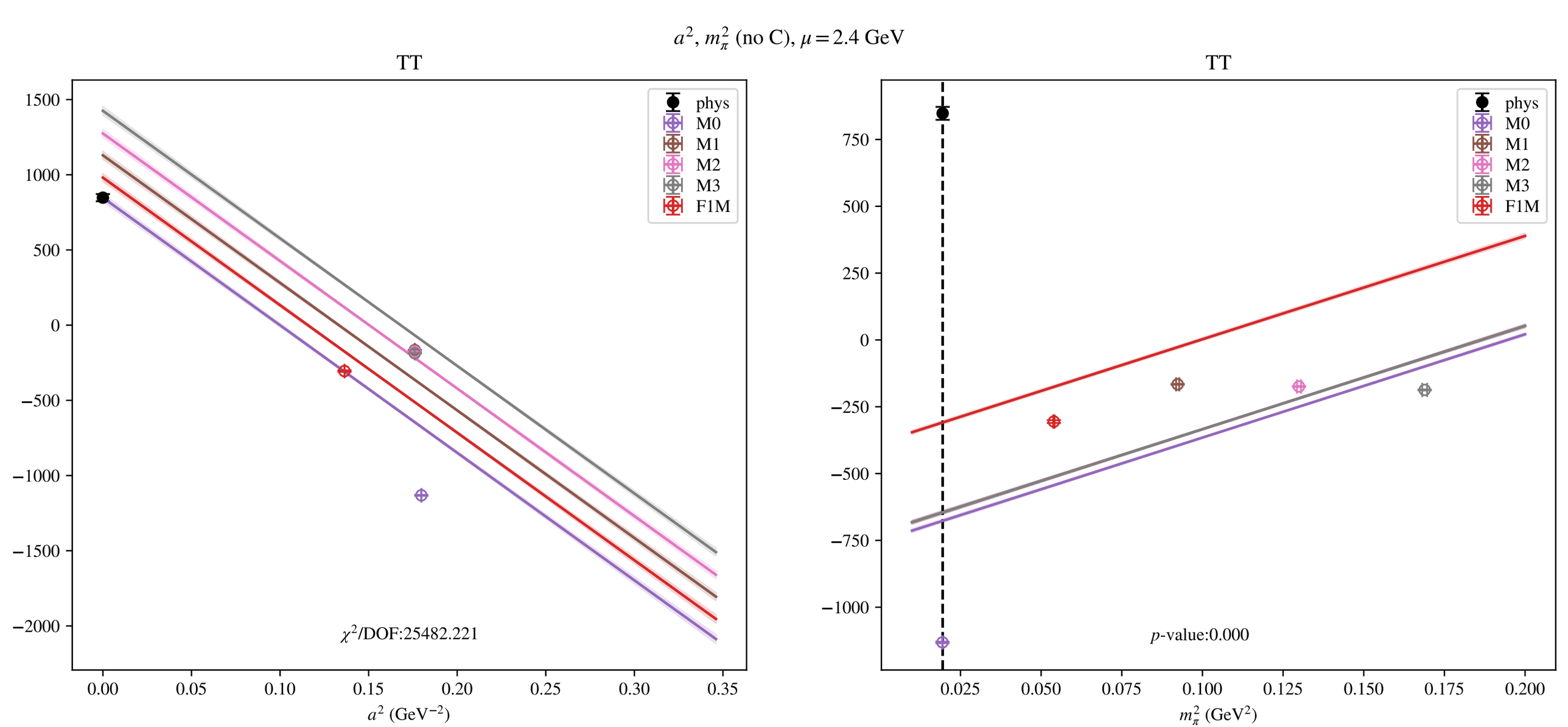

$a^2, m_\pi^2, \mu = 2.3$ GeV
TT
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:18351.480
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)

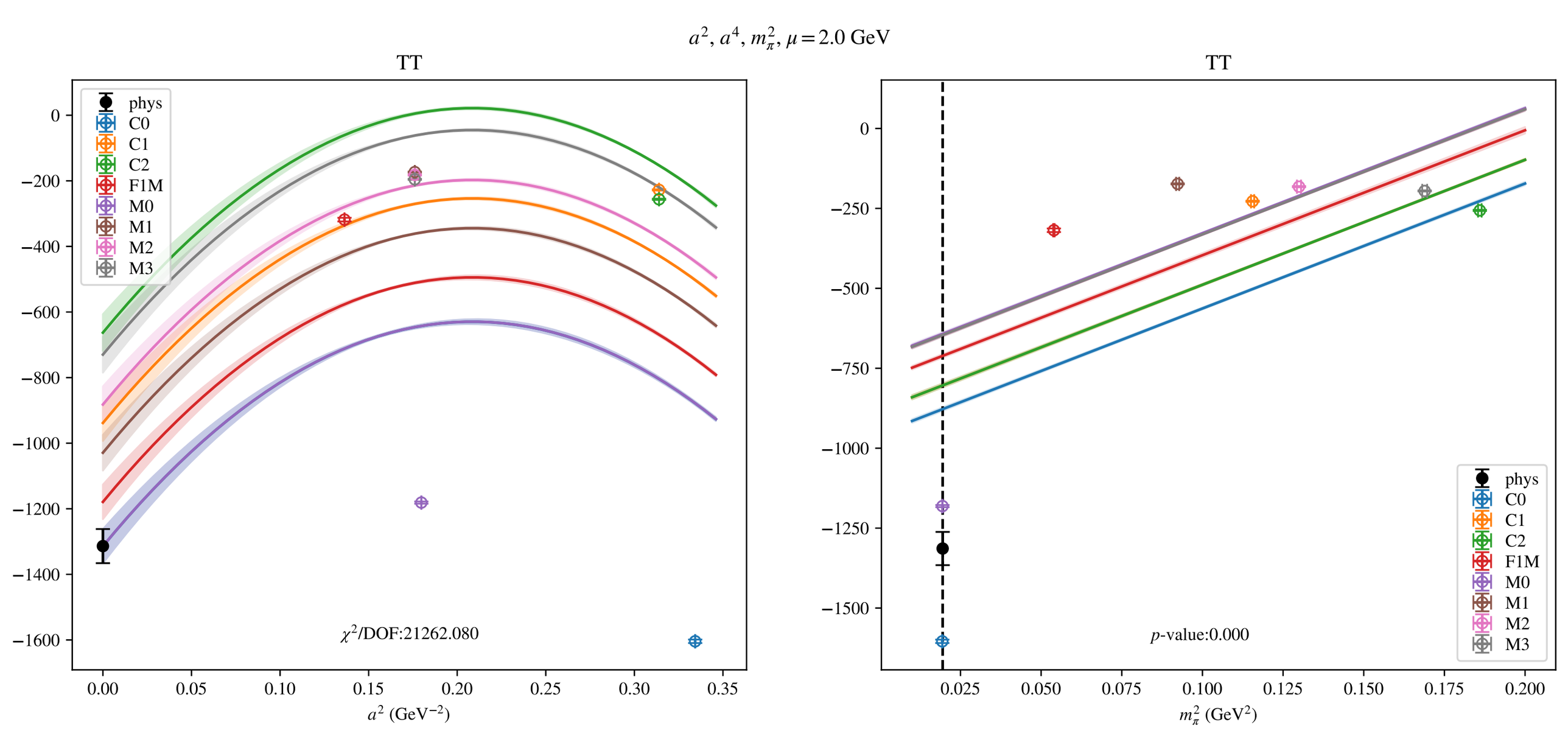

$a^2, m_\pi^2, \mu = 2.4$ GeV
TT
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:18521.422
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)

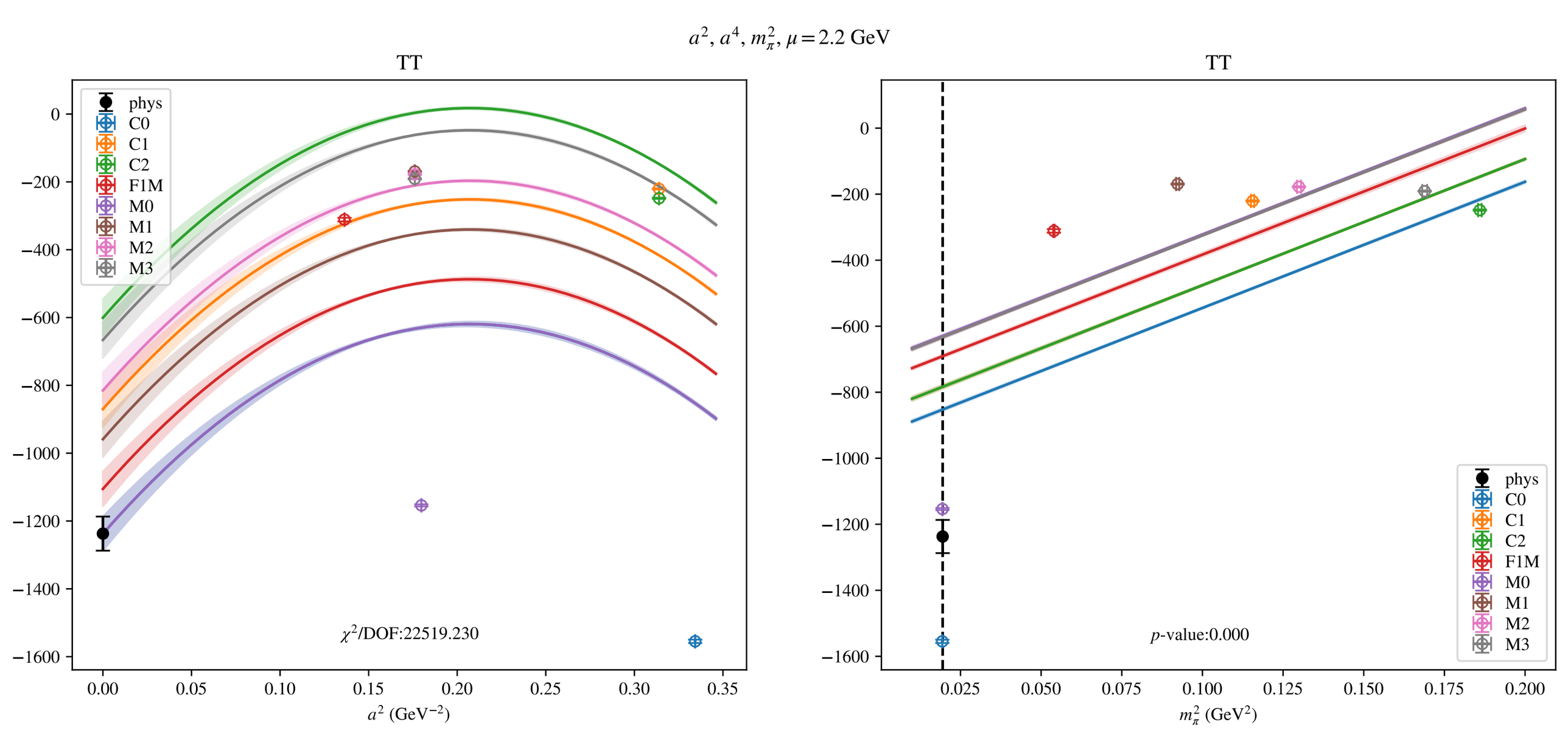

a^2, m_π^2 (no C), $\mu = 2.0$ GeV
TT
phys
M0
M1
M2
M3
F1M
χ^2/DOF:23134.530
a^2 (GeV^{-2})
TT
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

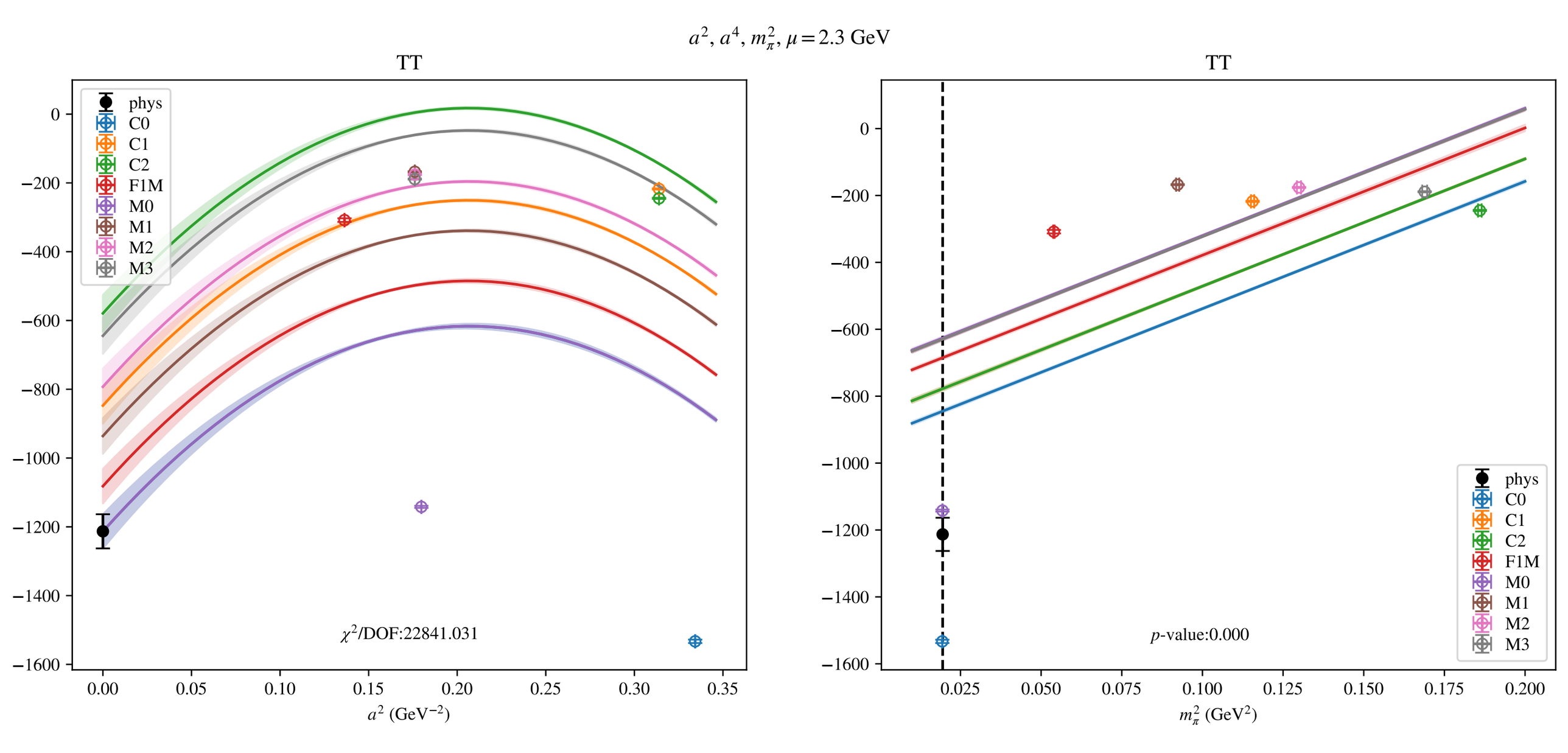

a^2, m_π^2 (no C), $\mu = 2.2$ GeV
TT
phys
M0
M1
M2
M3
F1M
χ^2/DOF:24856.421
a^2 (GeV^{-2})
TT
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

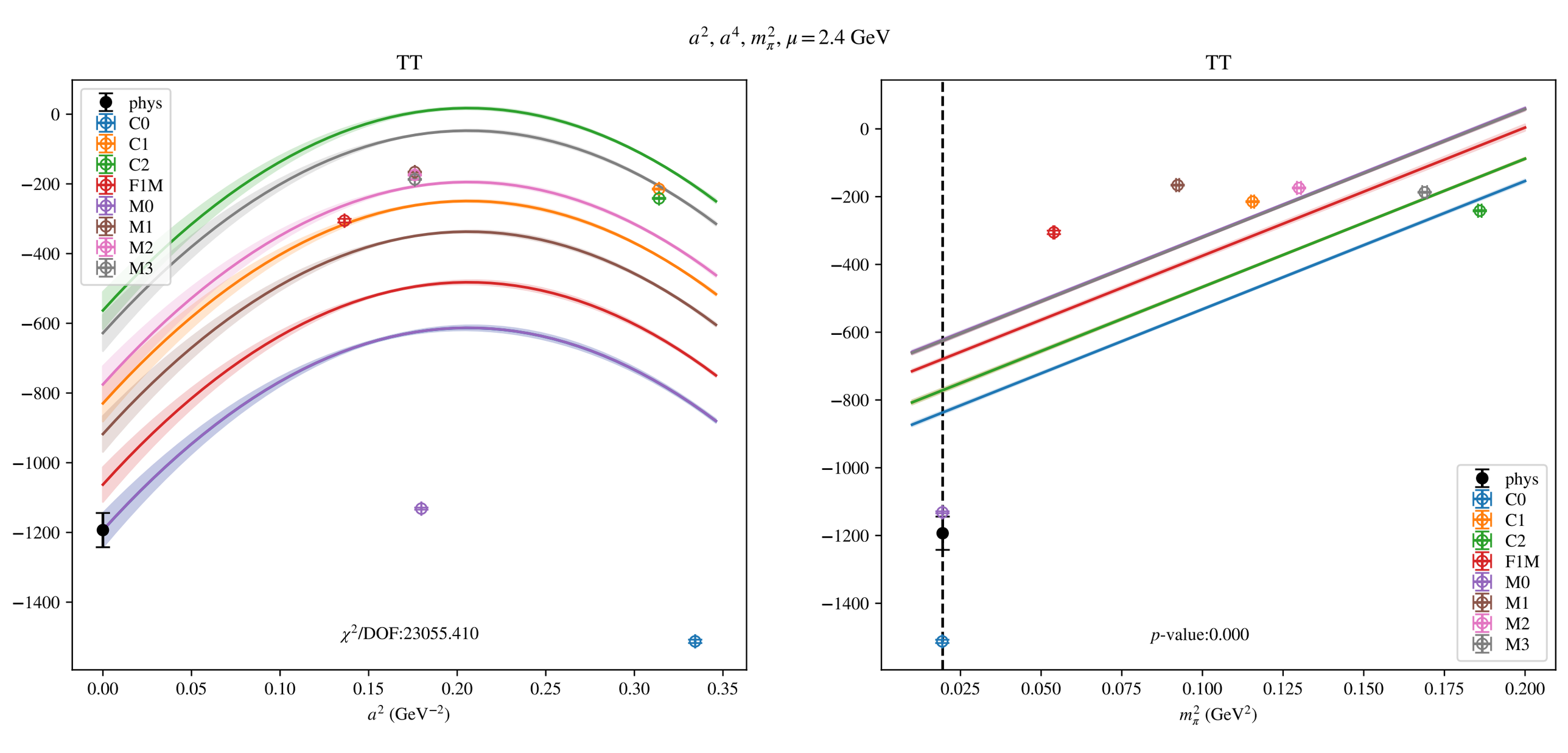

a^2, m_π^2 (no C), $\mu = 2.3$ GeV
TT
phys
M0
M1
M2
M3
F1M
χ^2/DOF:25228.853
a^2 (GeV^{-2})
TT
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

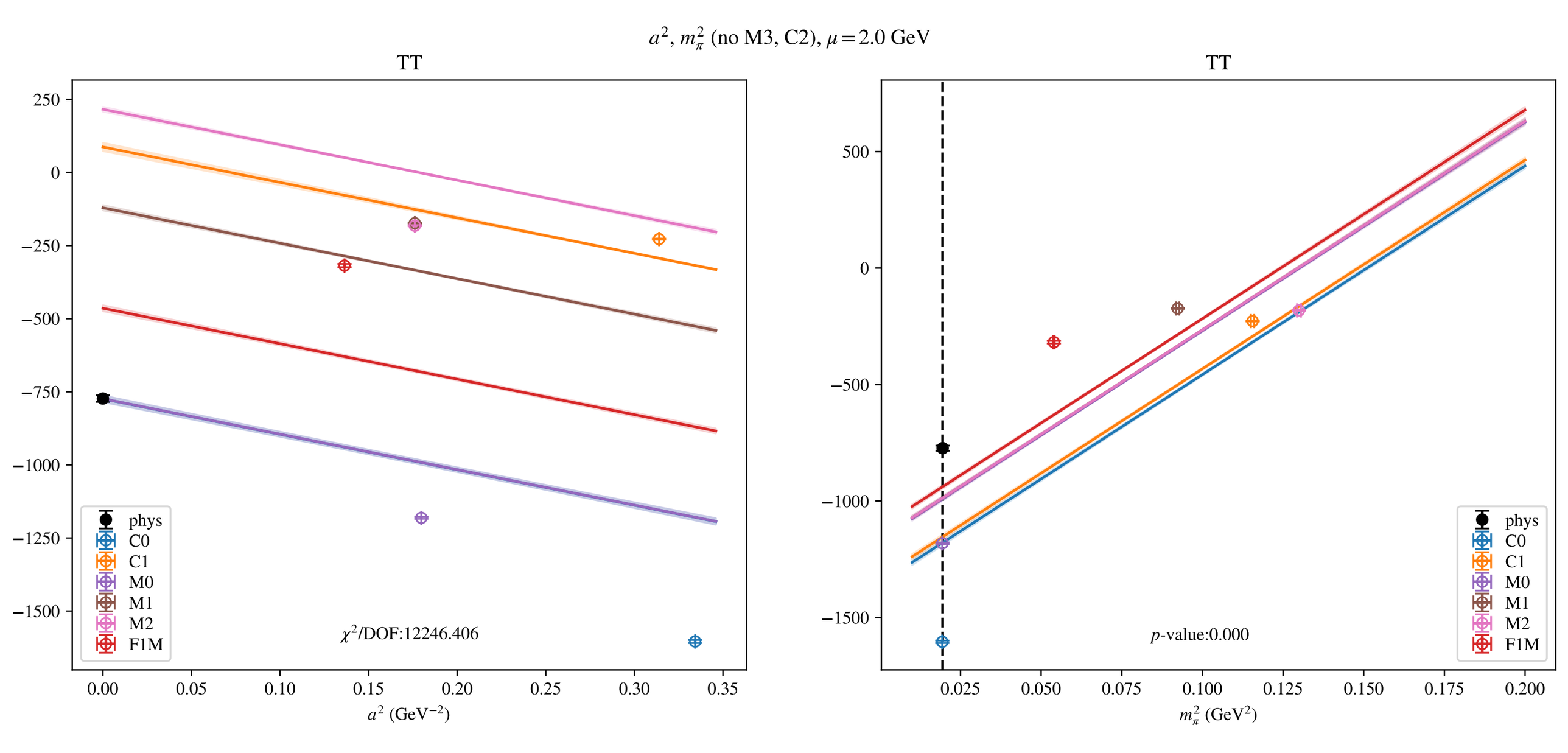

a^2, m_π^2 (no C), $\mu = 2.4$ GeV
TT
phys
M0
M1
M2
M3
F1M
χ^2/DOF:25482.221
a^2 (GeV^{-2})
TT
phys
M0
M1
M2
M3
F1M
p-value:0.000
m_π^2 (GeV^2)

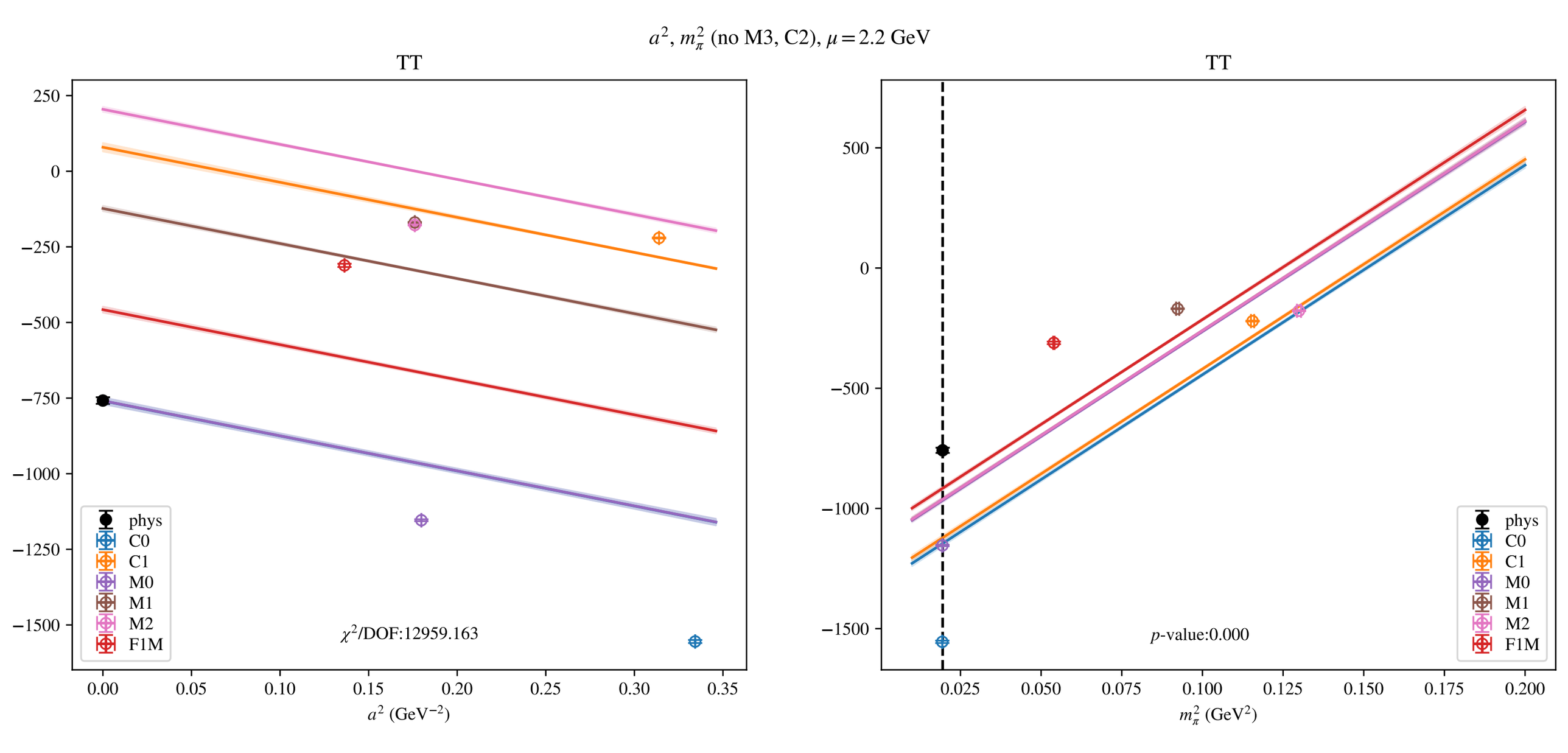

$a^2, a^4, m_\pi^2, \mu = 2.0$ GeV
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:21262.080
a^2 (GeV^{-2})
TT
p-value:0.000
m_π^2 (GeV^2)

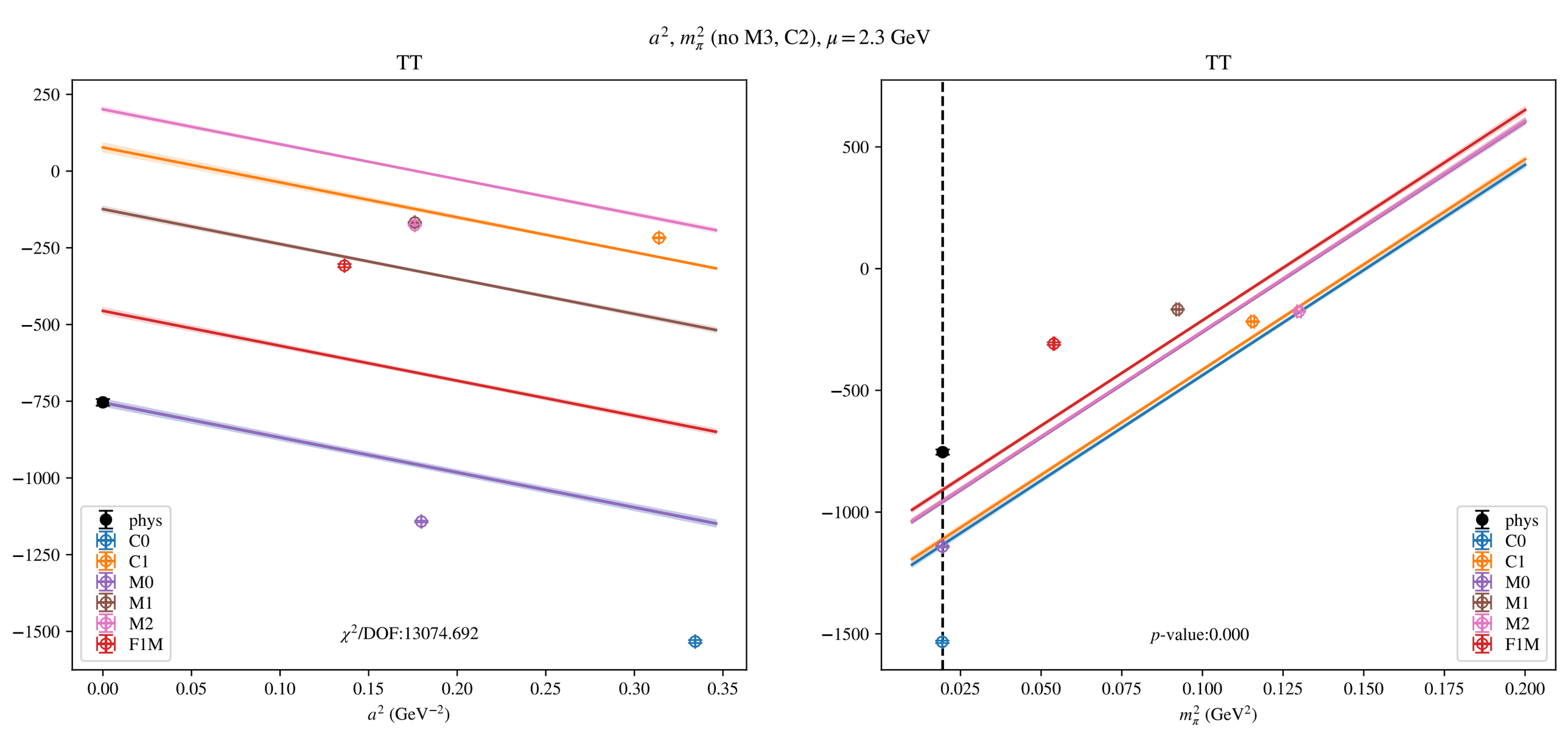

$a^2, a^4, m_\pi^2, \mu = 2.2$ GeV
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:22519.230
a^2 (GeV^{-2})
TT
p-value:0.000
m_π^2 (GeV^2)

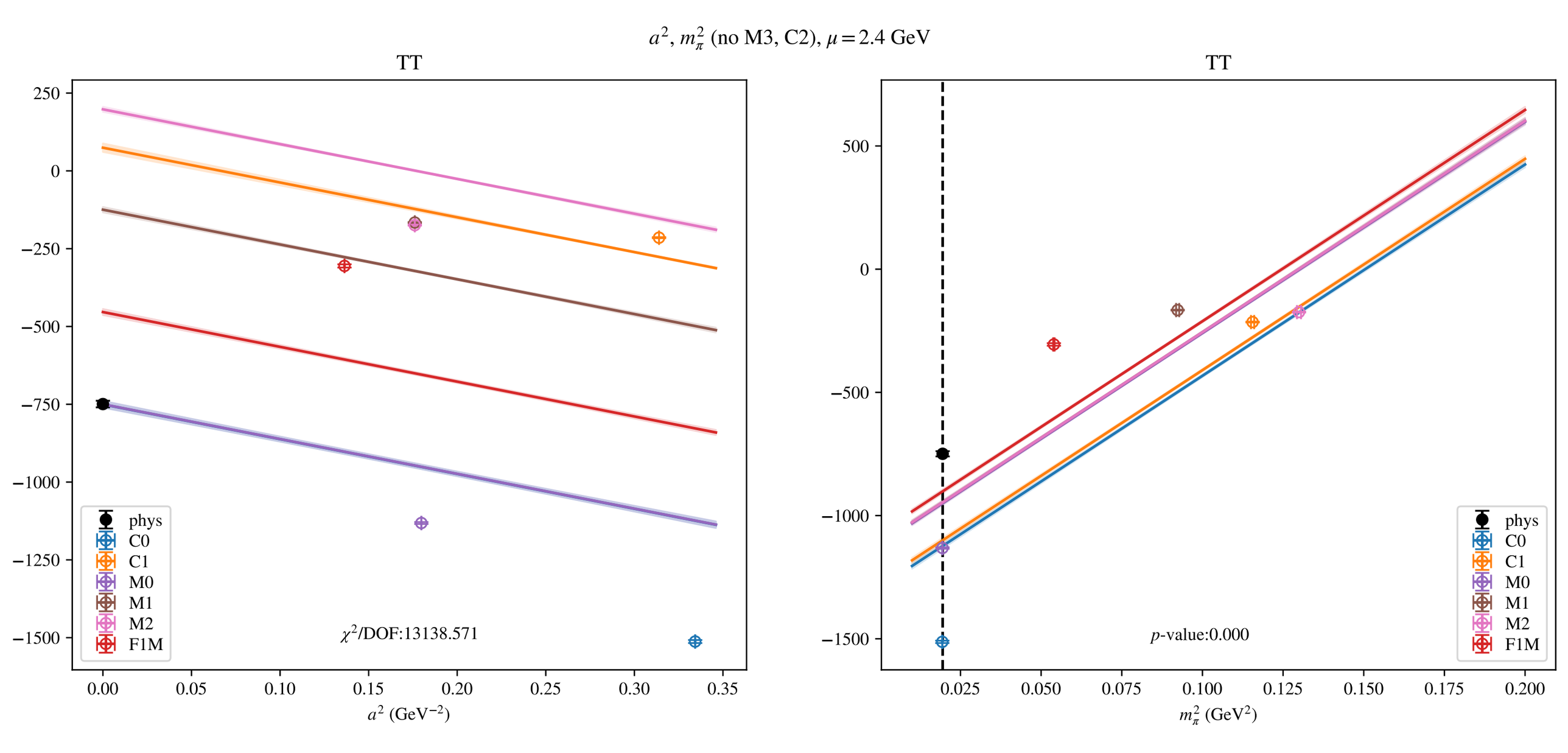

$a^2, a^4, m_\pi^2, \mu = 2.3$ GeV
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:22841.031
a^2 (GeV^{-2})
TT
p-value:0.000
m_π^2 (GeV^2)

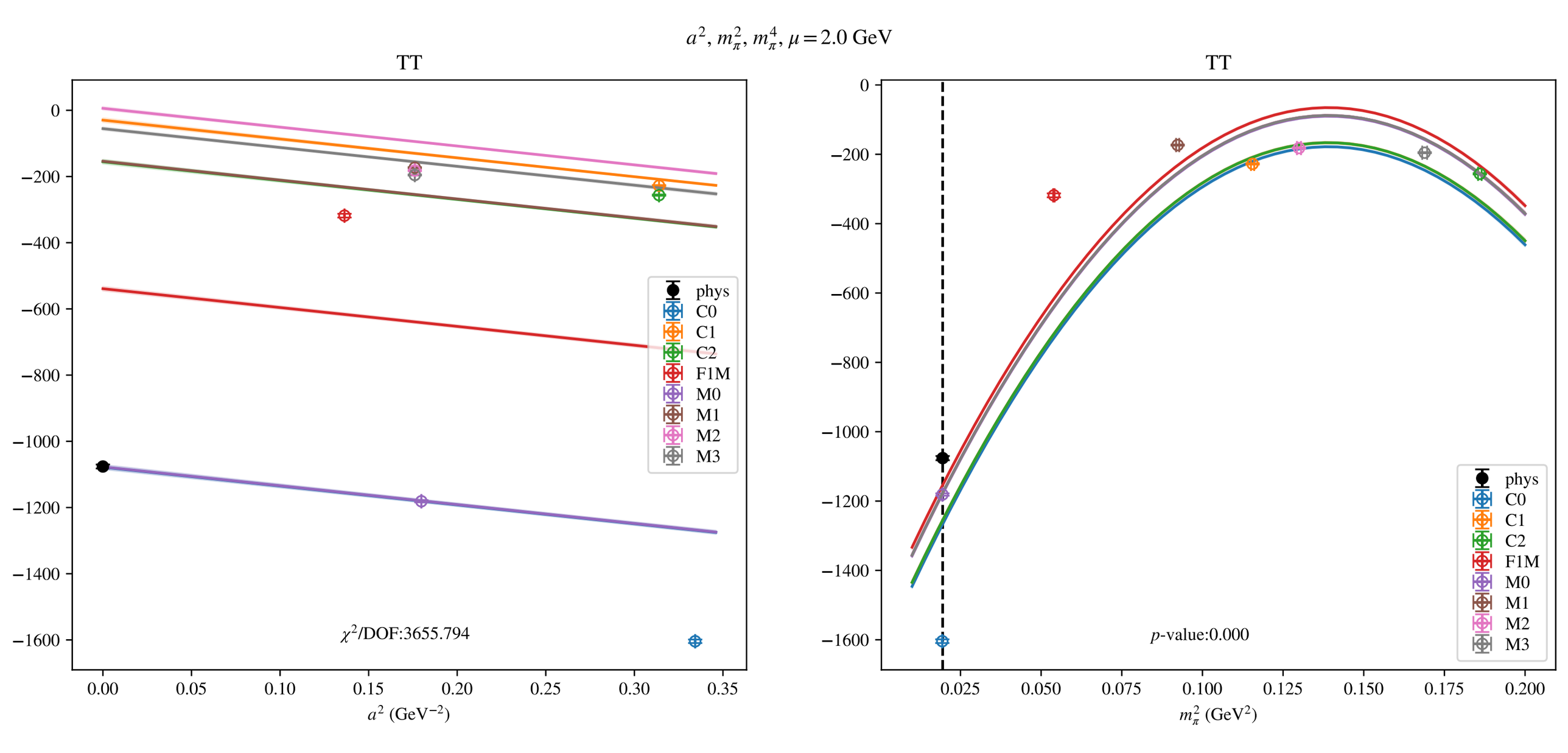

$a^2, a^4, m_\pi^2, \mu = 2.4$ GeV
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:23055.410
a^2 (GeV^{-2})
TT
p-value:0.000
m_π^2 (GeV^2)

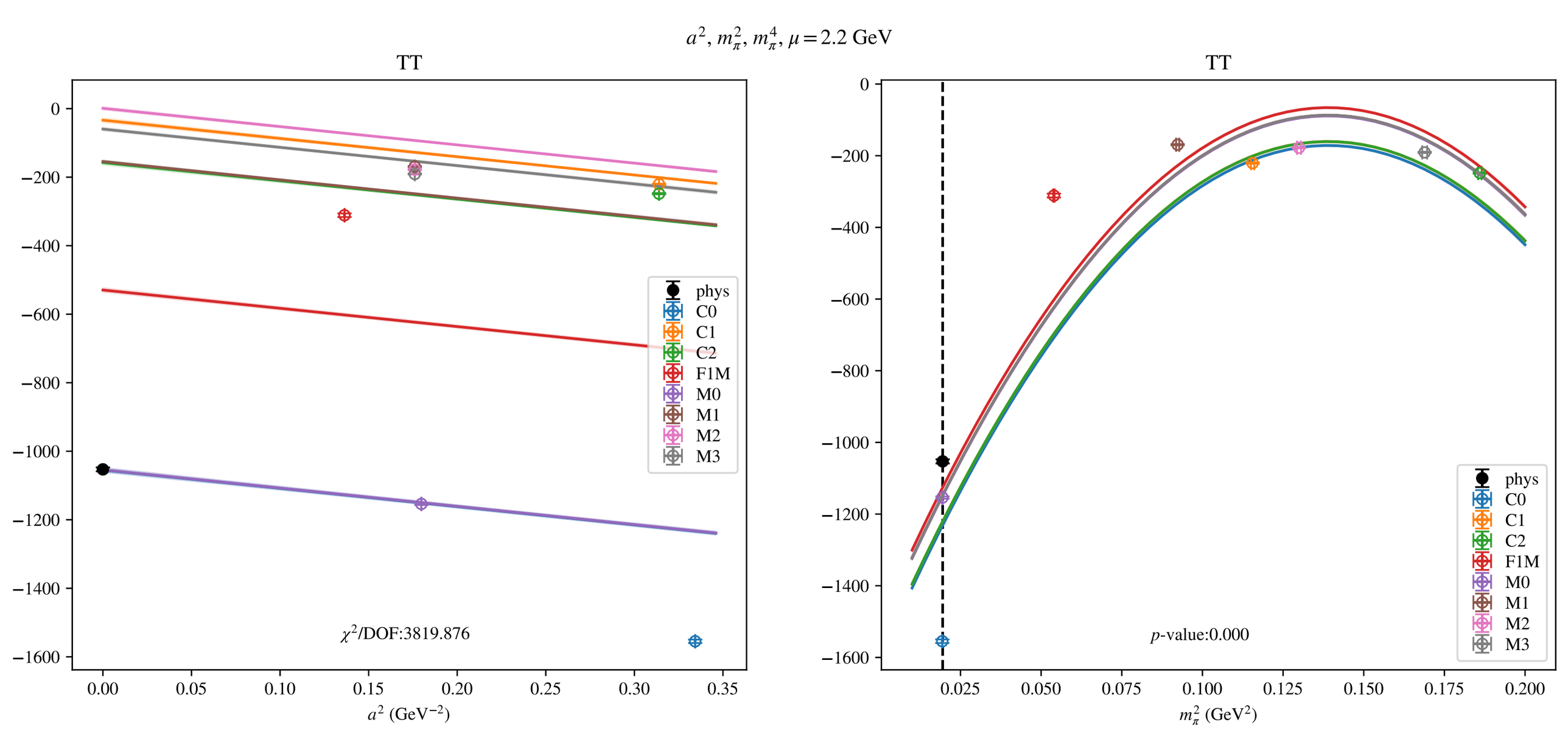

a^2, m_π^2 (no M3, C2), $\mu = 2.0$ GeV
TT
TT
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:12246.406
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)

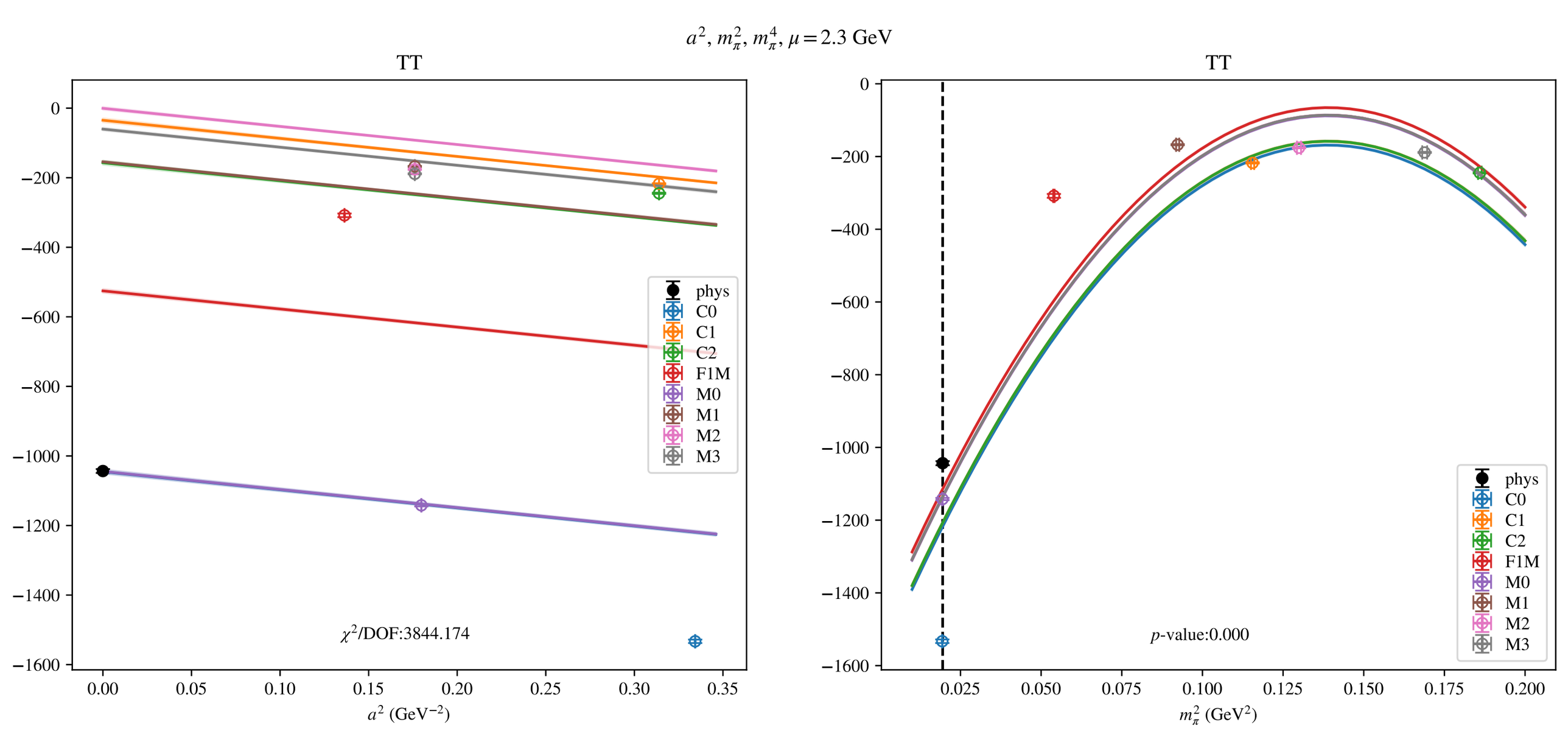

a^2, m_π^2 (no M3, C2), $\mu = 2.2$ GeV
TT
250
0
−250
−500
−750
−1000
−1250
−1500
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:12959.163
TT
500
0
−500
−1000
−1500
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
phys
C0
C1
M0
M1
M2
F1M
p-value:0.000

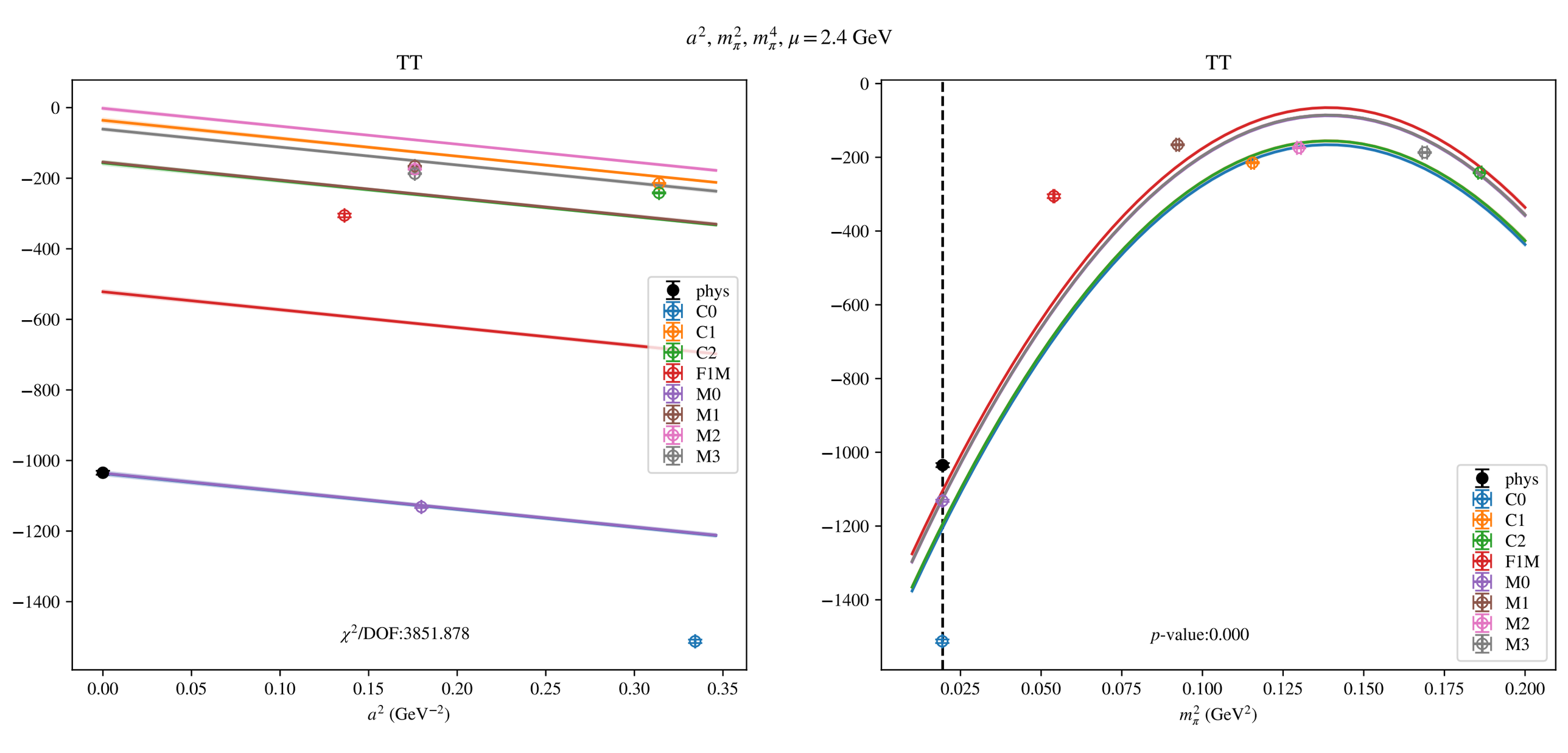

a^2, m_π^2 (no M3, C2), $\mu = 2.3$ GeV
TT
TT
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:13074.692
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)


a^2, m_π^2 (no M3, C2), $\mu = 2.4$ GeV
TT
TT
phys
C0
C1
M0
M1
M2
F1M
χ^2/DOF:13138.571
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.0$ GeV
TT
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:3655.794
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.2$ GeV
TT
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:3819.876
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.3$ GeV
TT
TT
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:3844.174
p-value:0.000
a^2 (GeV^{-2})
m_π^2 (GeV^2)


$a^2, m_\pi^2, m_\pi^4, \mu = 2.4$ GeV
TT
0
−200
−400
−600
−800
−1000
−1200
−1400
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
a^2 (GeV^{-2})
phys
C0
C1
C2
F1M
M0
M1
M2
M3
χ^2/DOF:3851.878
TT
0
−200
−400
−600
−800
−1000
−1200
−1400
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
m_π^2 (GeV^2)
phys
C0
C1
C2
F1M
M0
M1
M2
M3
p-value:0.000