

# Kaon mixing: chiral and continuum extrapolations

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# 1 VVpAA

$\mu$ (GeV)	$a^2, m_\pi^2$	$a^2, m_\pi^2$ (no C)	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$	$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2)$
2.4	<b>0.5213(10)</b> : 2.342 (0.039)	<b>0.5271(46)</b> : 1.22 (0.295)	<b>0.5294(79)</b> : 2.664 (0.031)	<b>0.5233(12)</b> : 0.993 (0.41)	<b>0.51075(99)</b> : 9.014 (0.0)
2.0	<b>0.5264(11)</b> : 1.838 (0.102)	<b>0.5310(48)</b> : 0.864 (0.421)	<b>0.5326(82)</b> : 2.159 (0.071)	<b>0.5282(12)</b> : 0.639 (0.635)	<b>0.5157(10)</b> : 8.193 (0.0)
1.8	<b>0.5293(14)</b> : 1.315 (0.254)	<b>0.5328(53)</b> : 0.478 (0.62)	<b>0.5335(91)</b> : 1.6 (0.171)	<b>0.5310(14)</b> : 0.307 (0.873)	<b>0.5188(13)</b> : 6.778 (0.0)
1.5	<b>0.5338(19)</b> : 0.734 (0.598)	<b>0.5356(64)</b> : 0.159 (0.853)	<b>0.536(11)</b> : 0.909 (0.457)	<b>0.5352(18)</b> : 0.099 (0.983)	<b>0.5237(19)</b> : 4.464 (0.0)

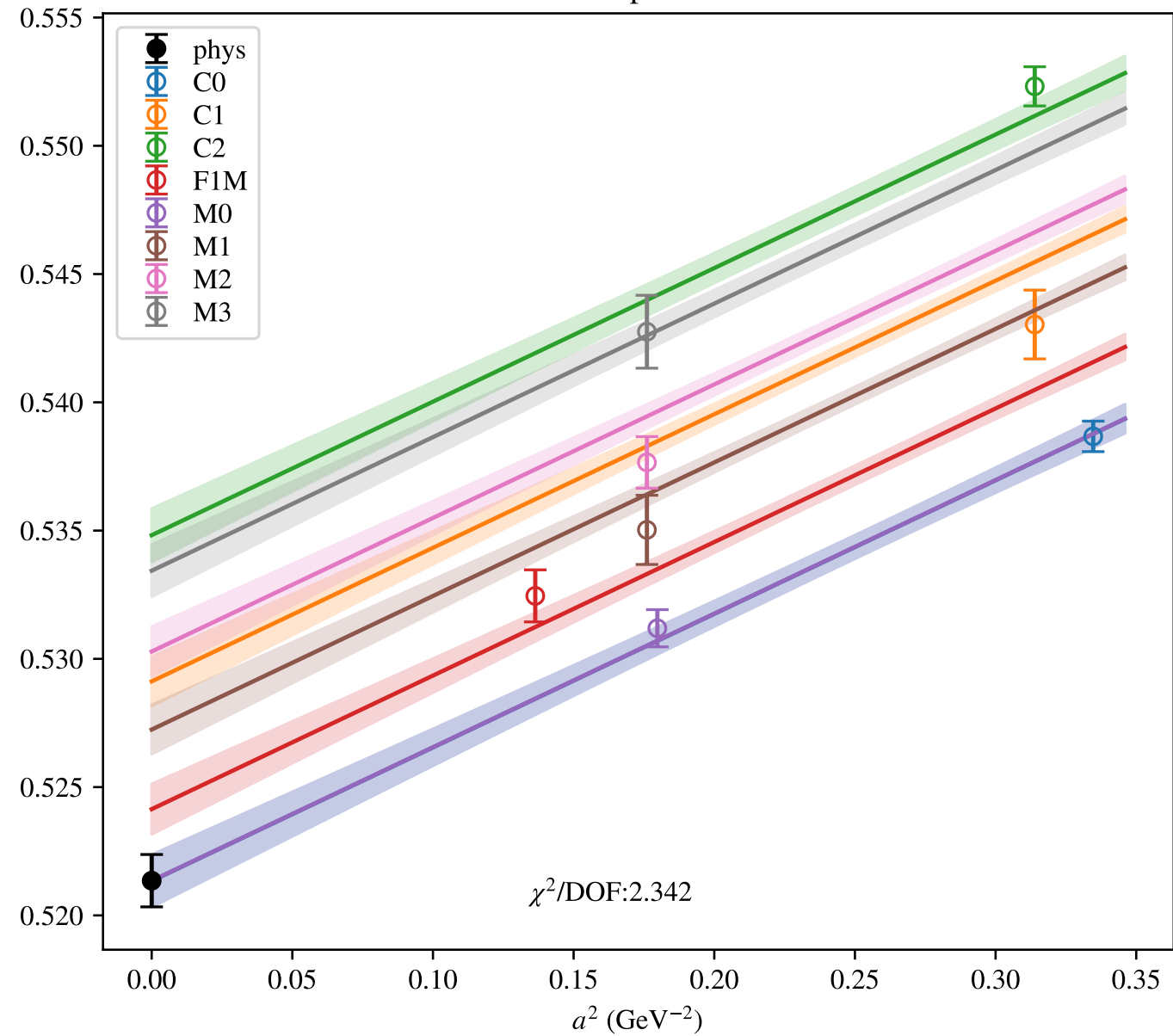
Table 1: Physical point value from chiral and continuum extrapolation at renormalisation scale  $\mu$ . Entries are **value(error)**:  $\chi^2/\text{DOF}$  ( $p$ -value).

$\mu$ (GeV)		$a^2, m_\pi^2$	$a^2, m_\pi^2$ (no C)	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$	$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2)$
2.4	$\alpha$	0.1001(70)	0.040(52)	-0.043	0.0864(83)	0.1218(71)
	$\beta$	0.00264(14)	0.00221(27)	0.00268(15)	0.00016(89)	-0.0019(15)
2.0	$\alpha$	0.0939(73)	0.047(53)	-0.015	0.0813(84)	0.1150(74)
	$\beta$	0.00262(14)	0.00223(28)	0.00265(15)	0.00030(90)	-0.0020(15)
1.8	$\alpha$	0.0894(84)	0.055(57)	0.016	0.0781(89)	0.1088(85)
	$\beta$	0.00262(15)	0.00225(29)	0.00264(15)	0.00039(94)	-0.0020(15)
1.5	$\alpha$	0.081(10)	0.066(65)	0.036	0.0722(99)	0.098(10)
	$\beta$	0.00261(15)	0.00230(34)	0.00261(16)	0.001	-0.0020(15)

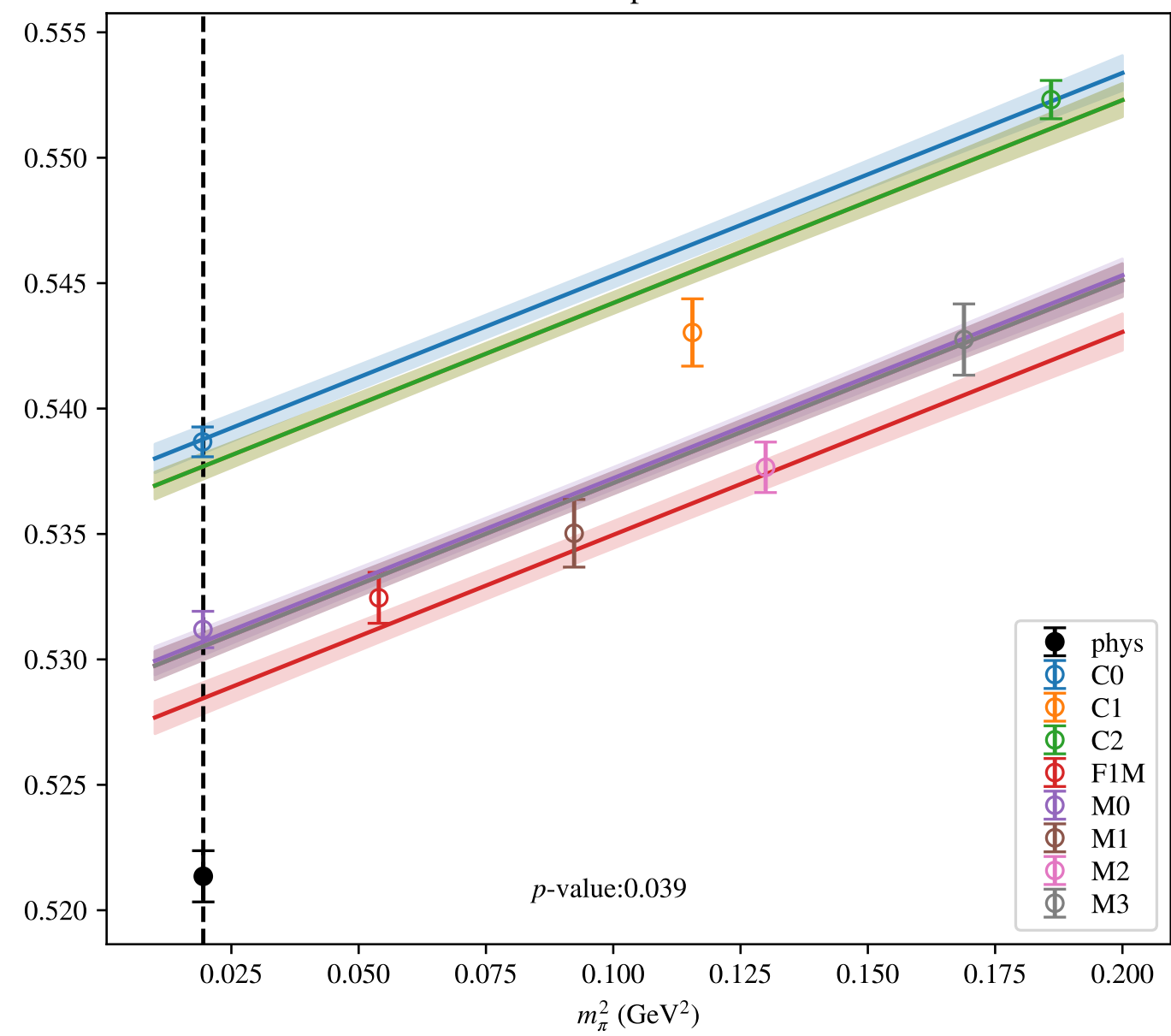
Table 2: Fit values of coefficients in  $B = B_0(1 + \alpha a^2 + \beta \frac{m_\pi^2}{f_\pi^2} + \dots)$ .

$a^2, m_\pi^2, \mu = 2.4 \text{ GeV}$

VVpAA

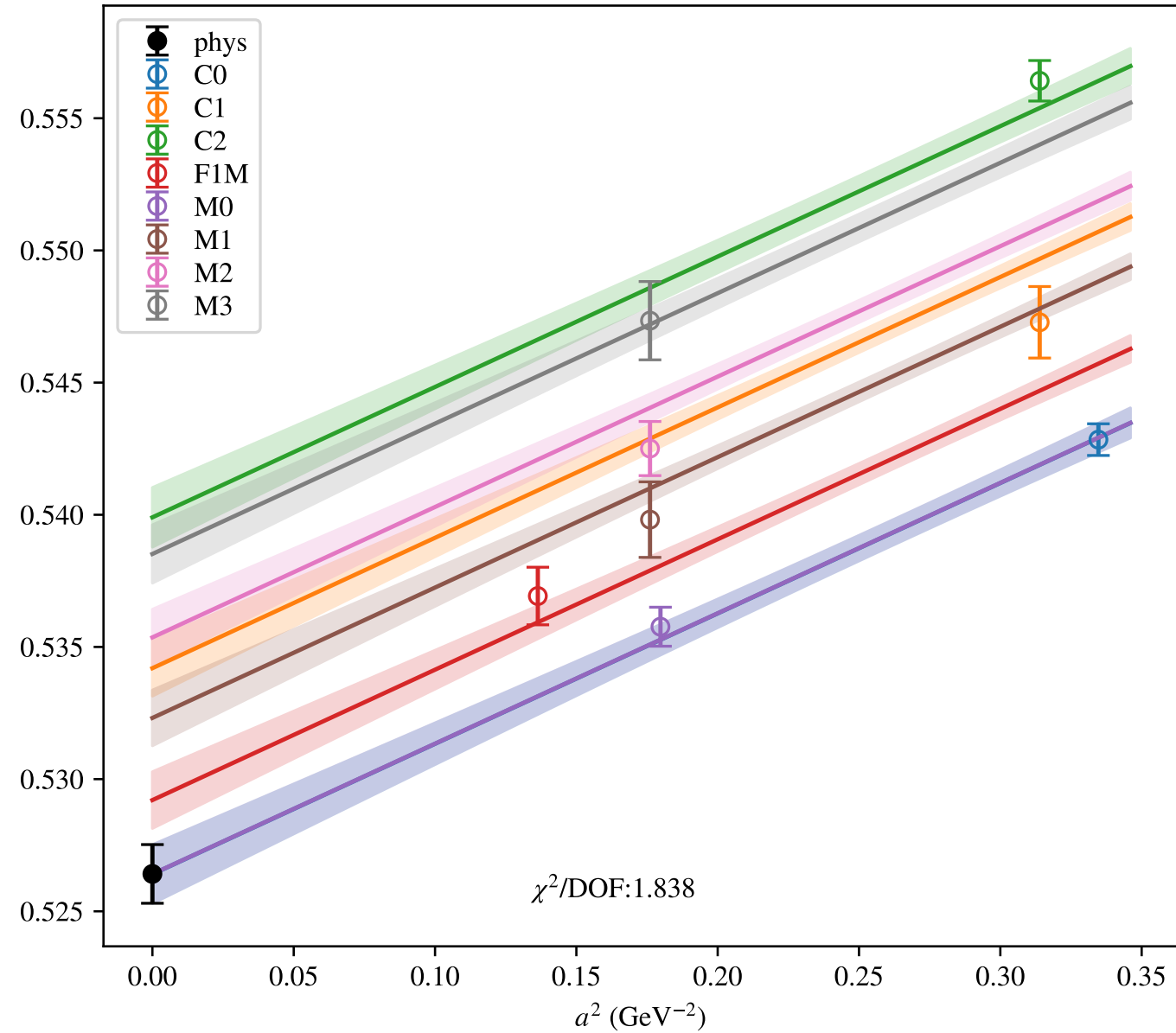


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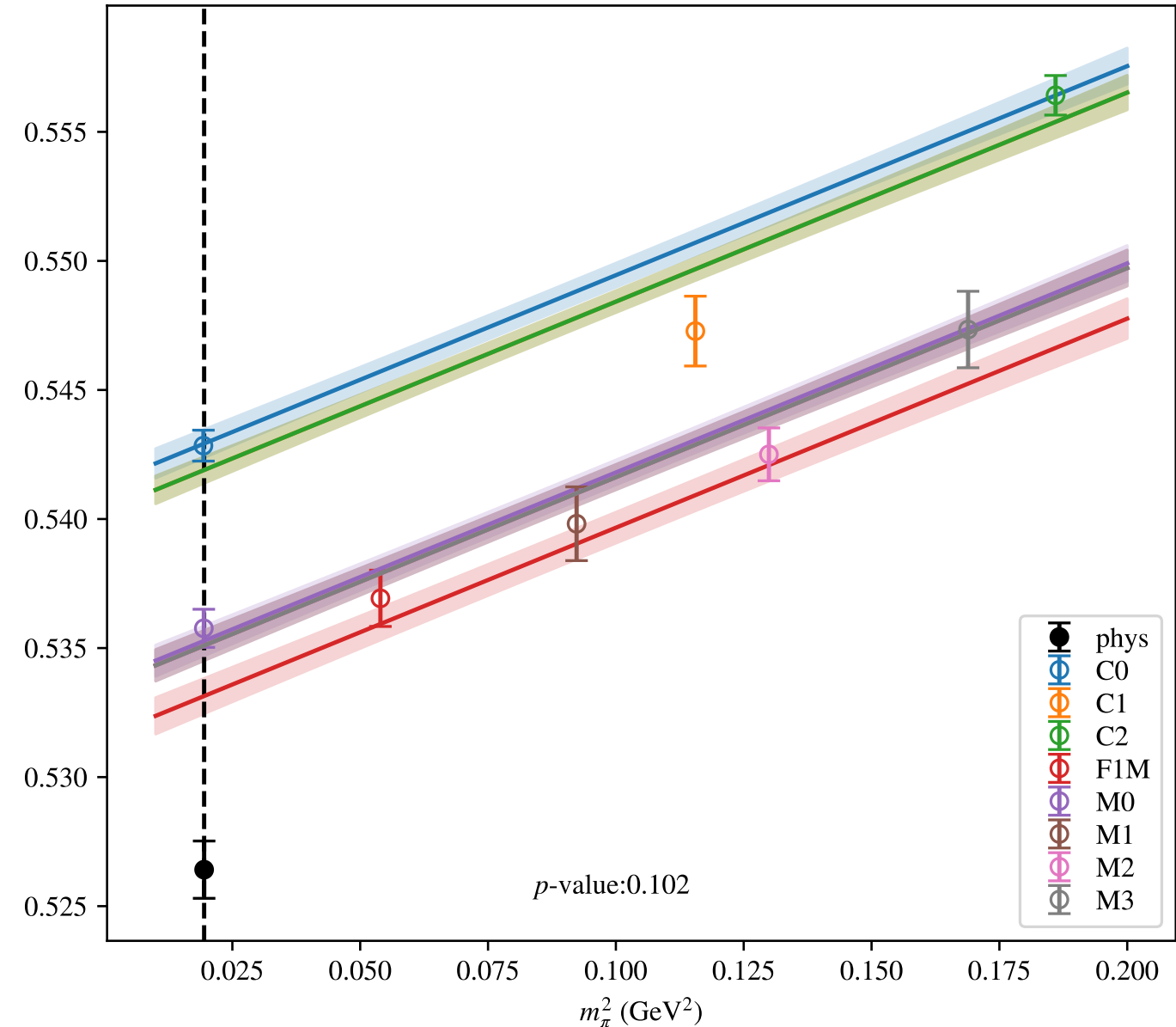


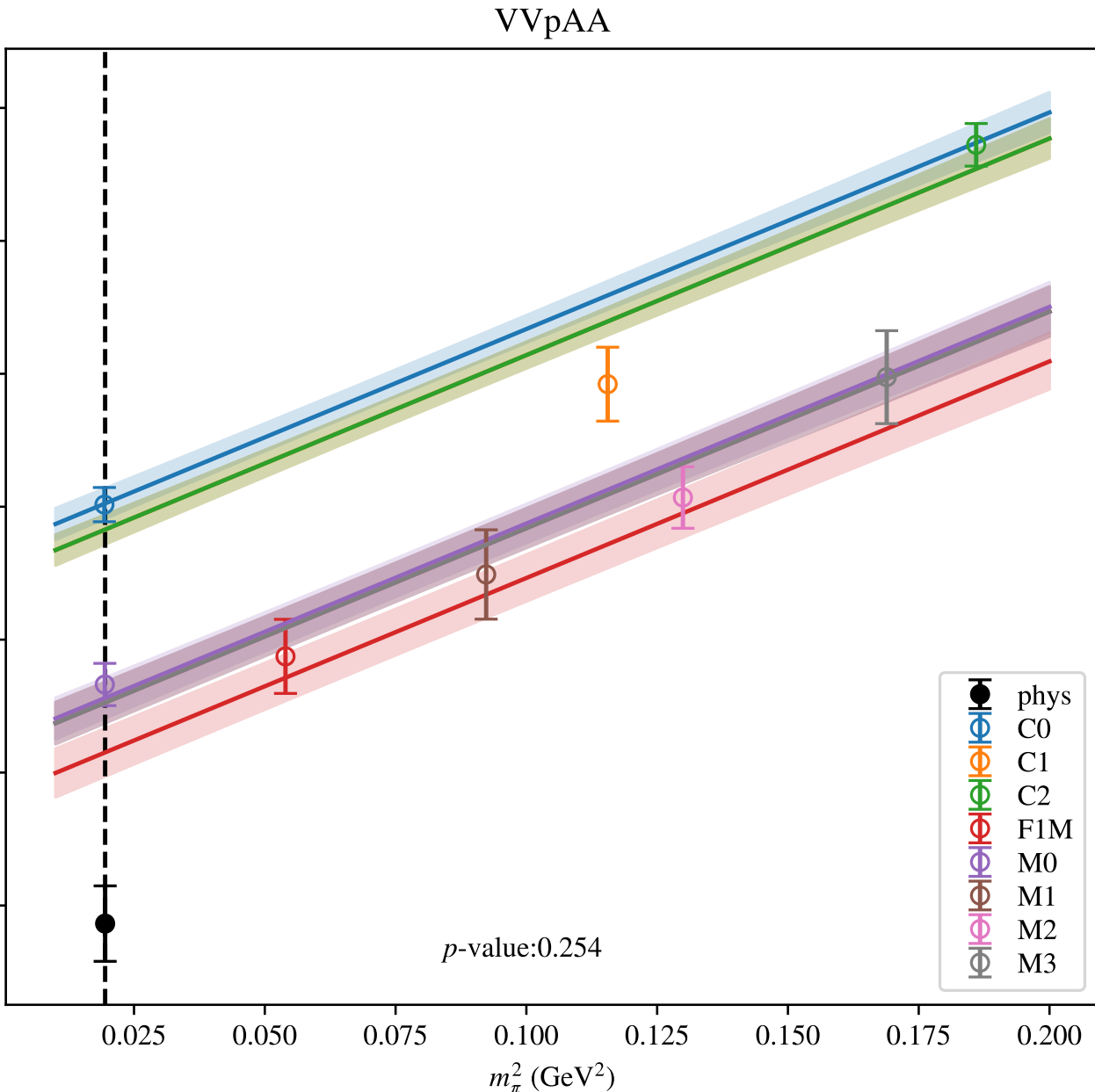
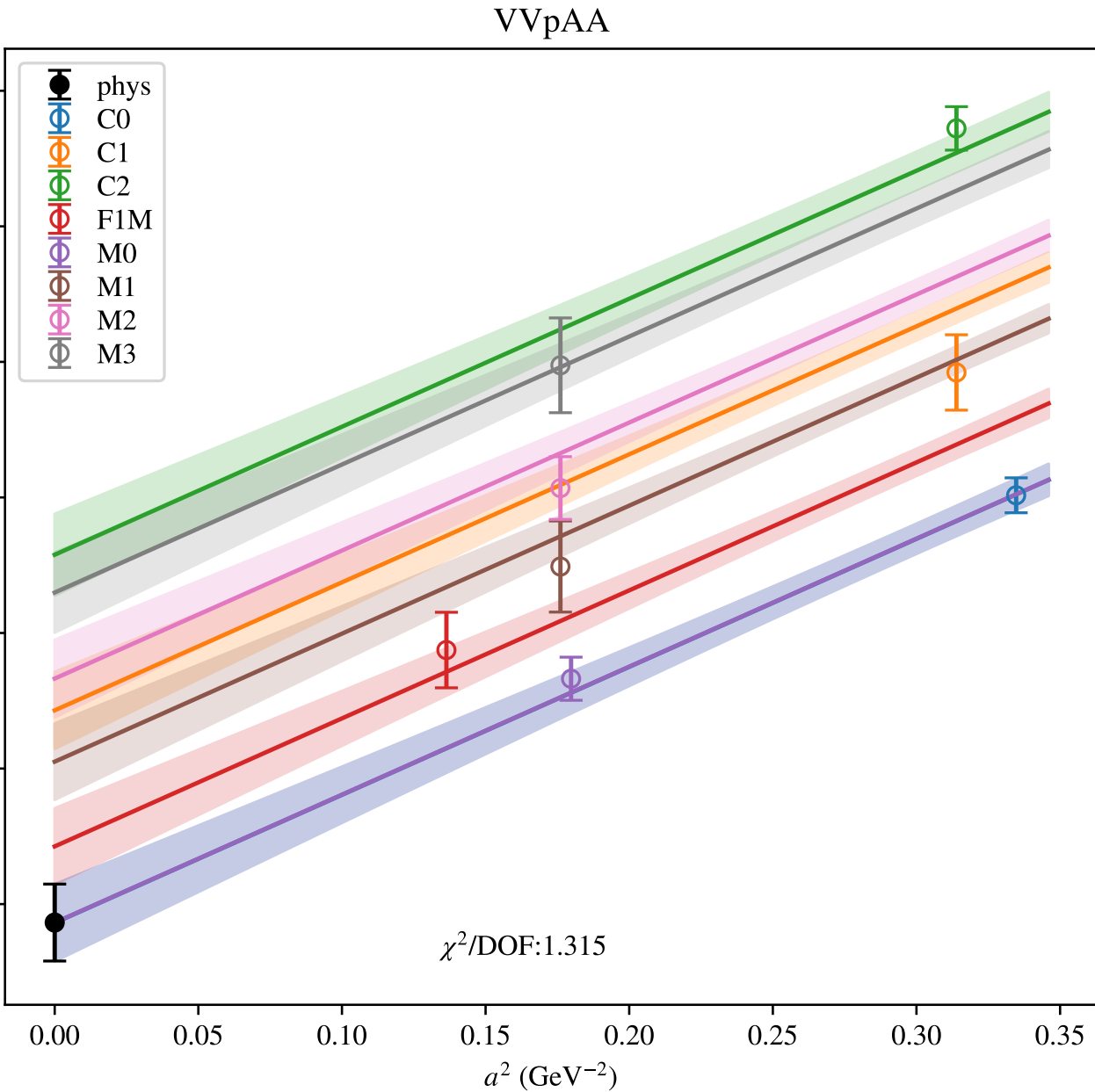
$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$

VVpAA



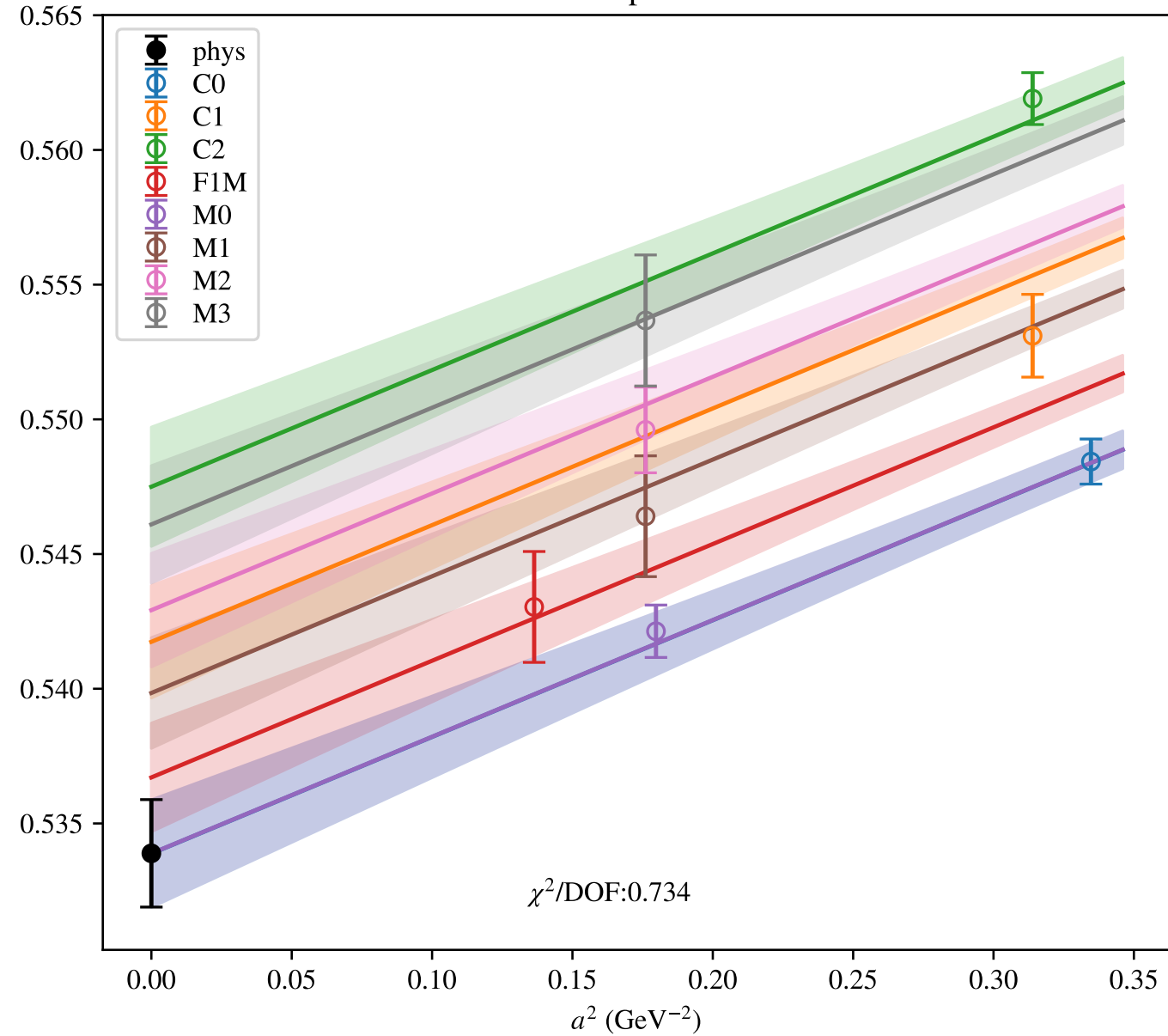
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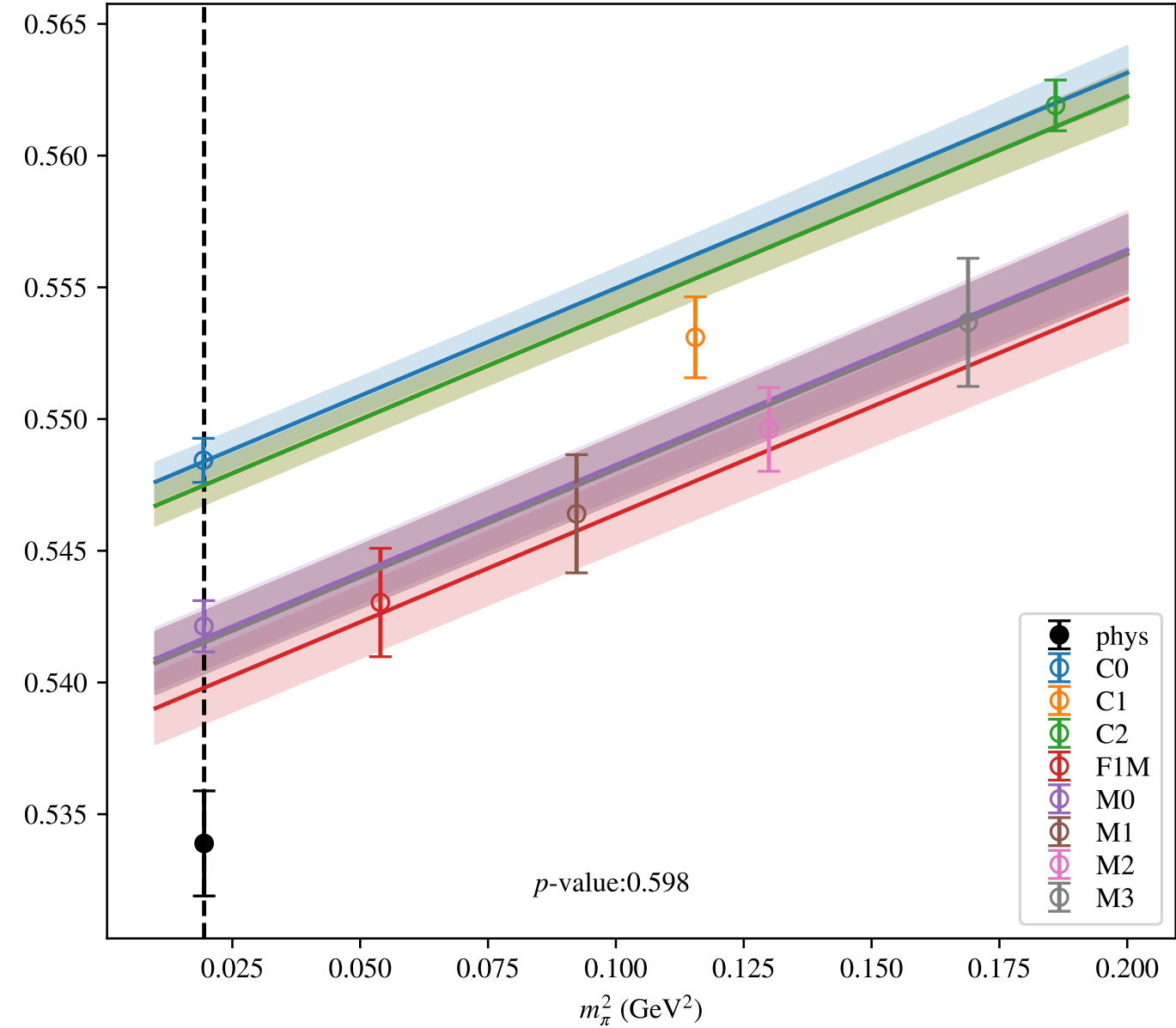
$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$


$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$

VVpAA

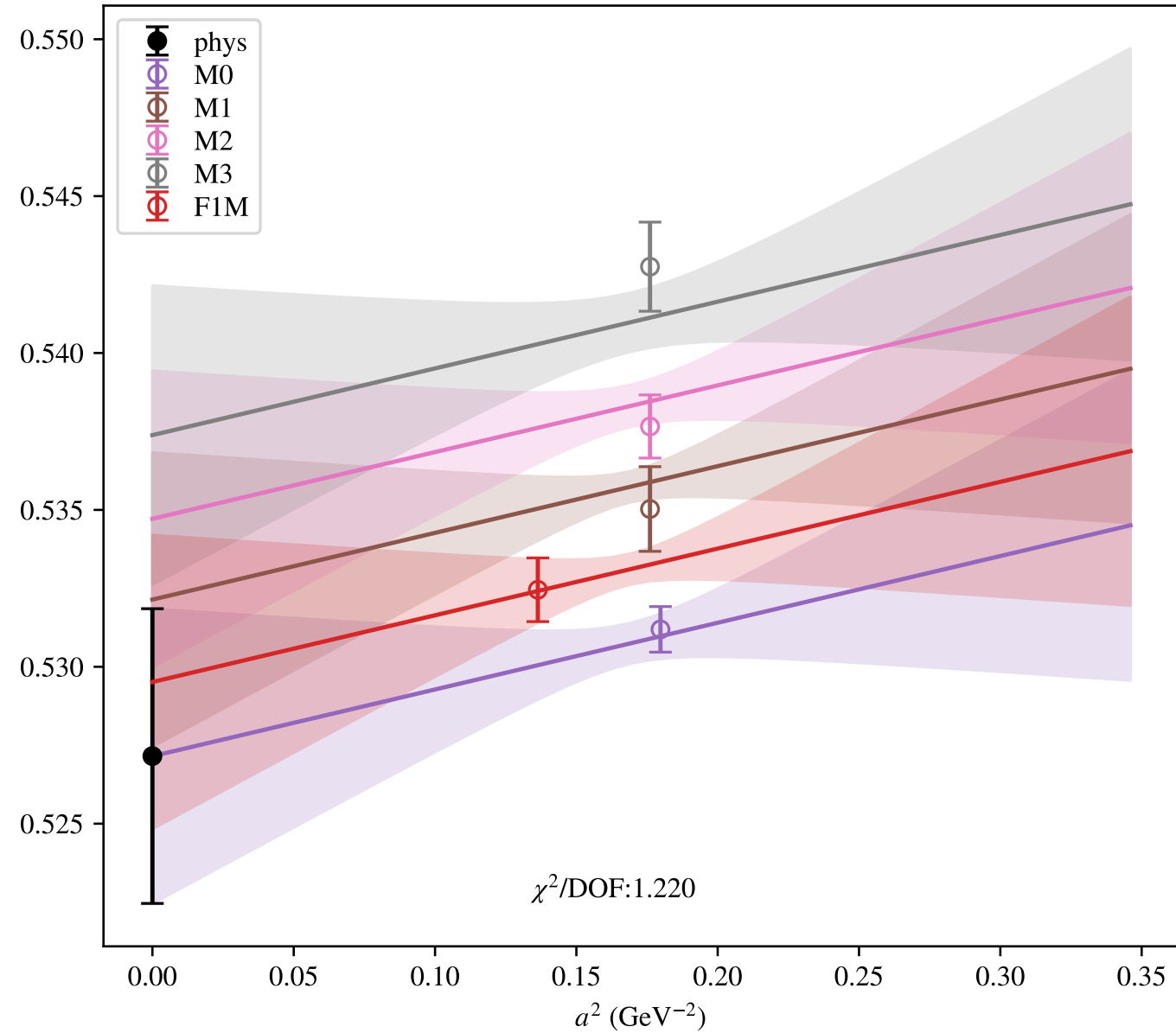


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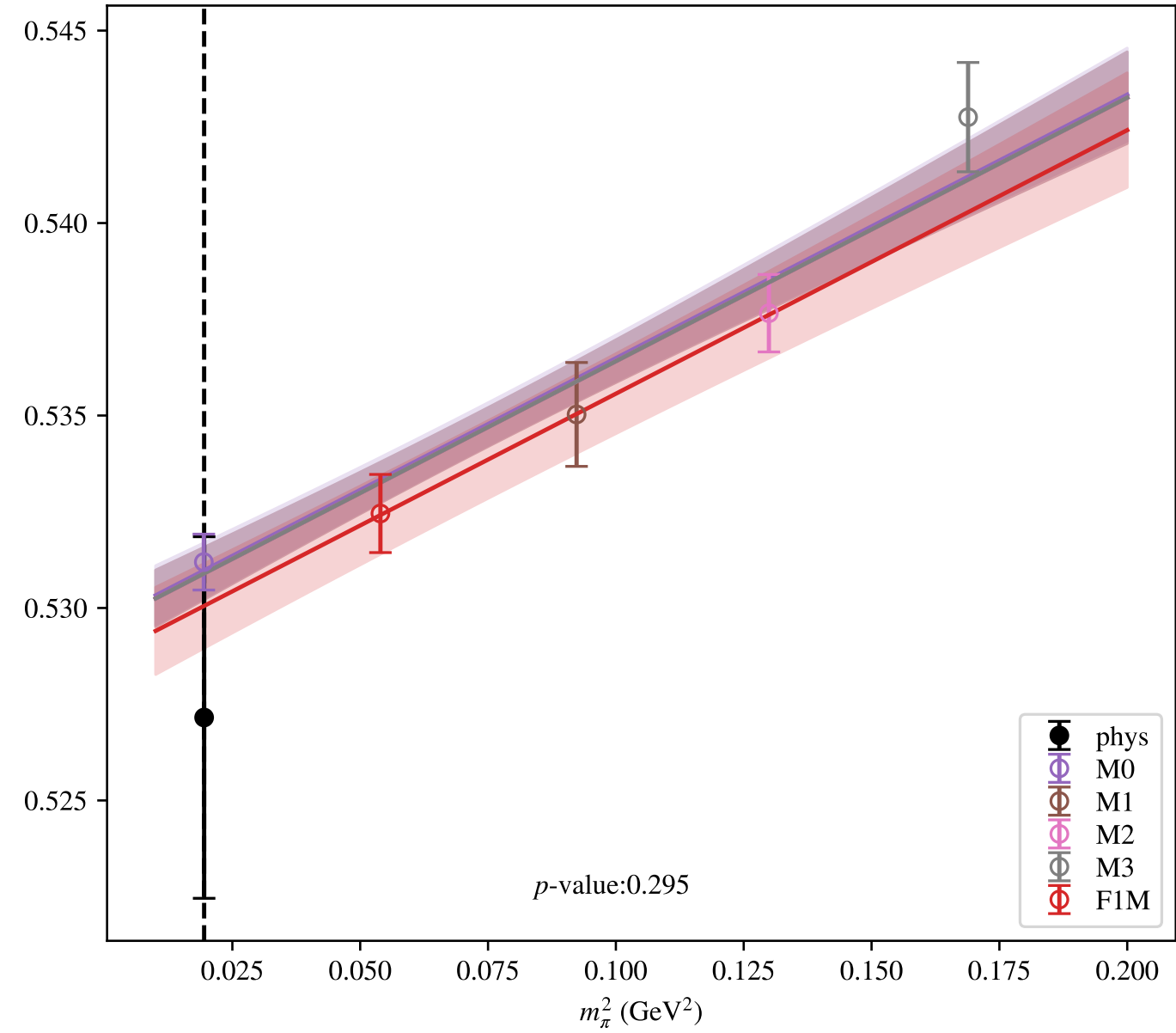


$a^2, m_\pi^2$  (no C),  $\mu = 2.4$  GeV

VVpAA

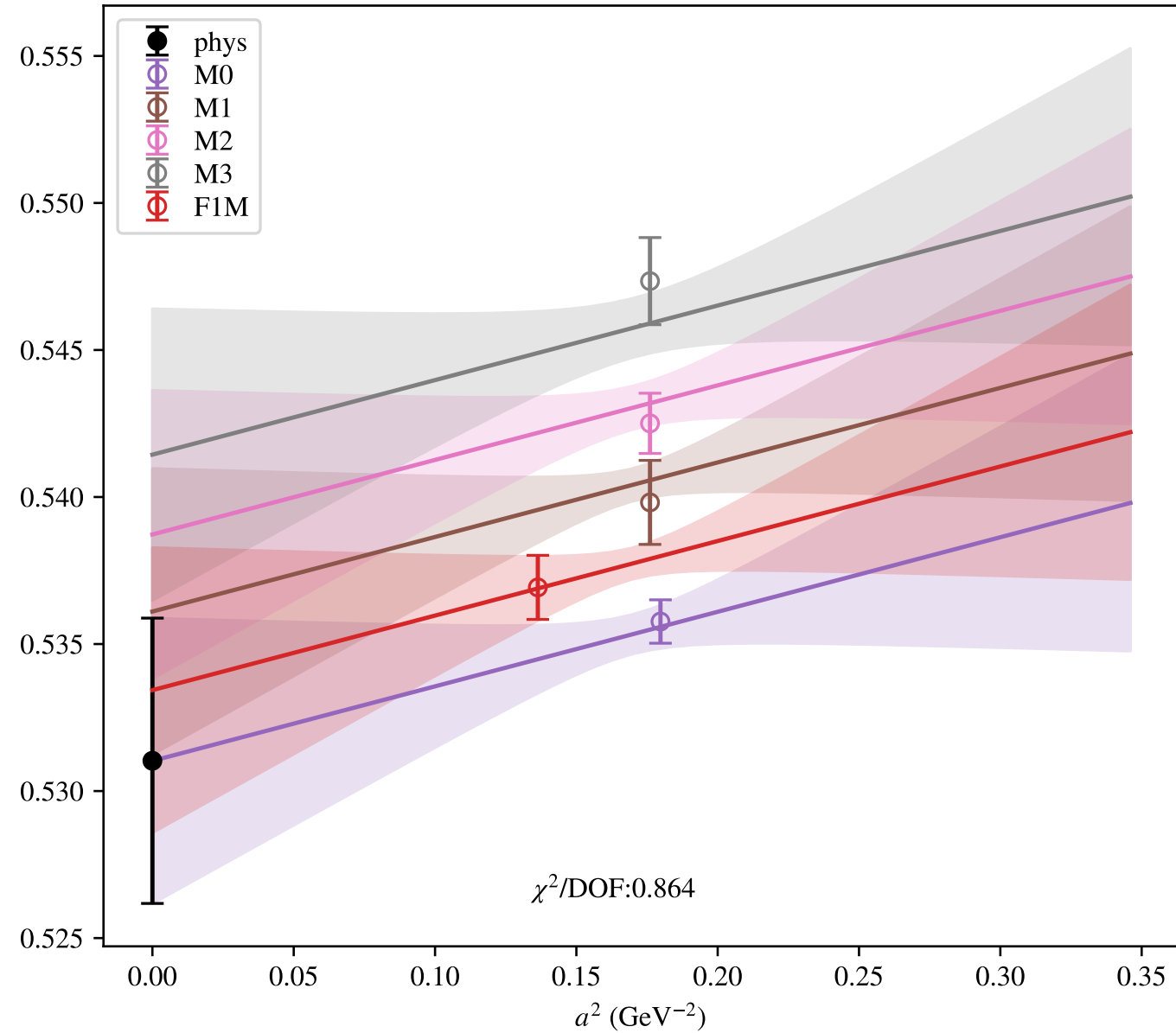


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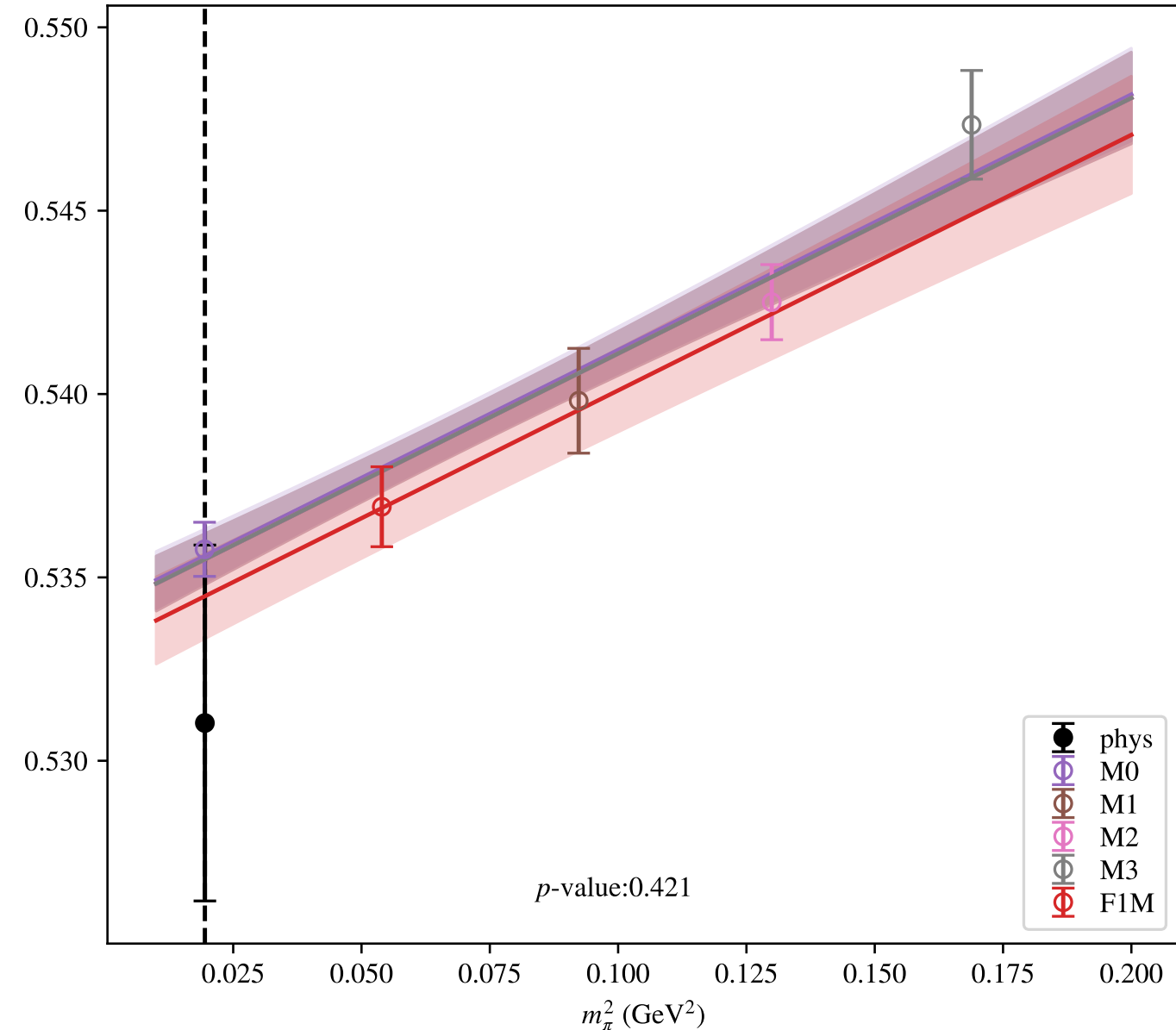


$a^2, m_\pi^2$  (no C),  $\mu = 2.0$  GeV

VVpAA



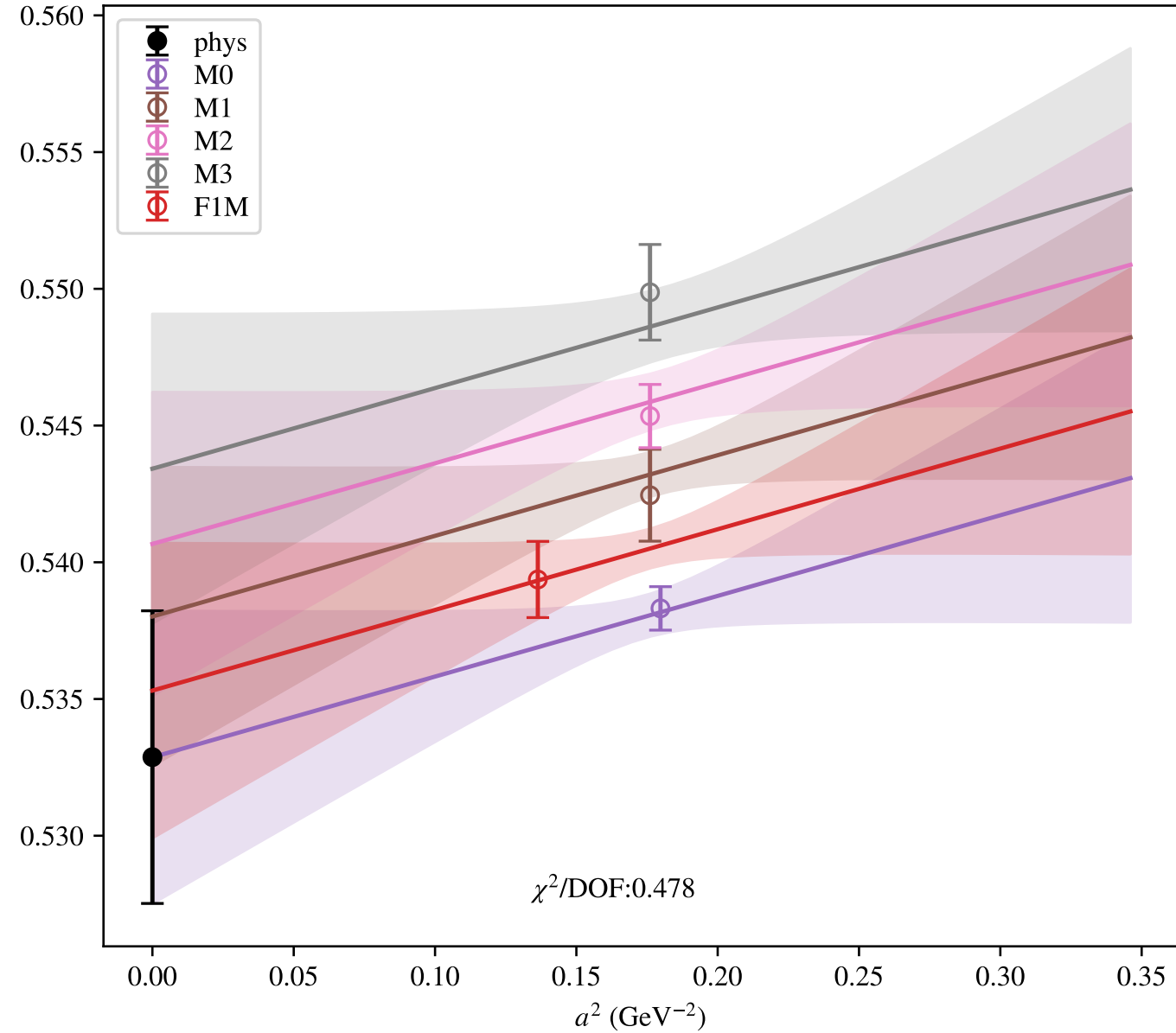
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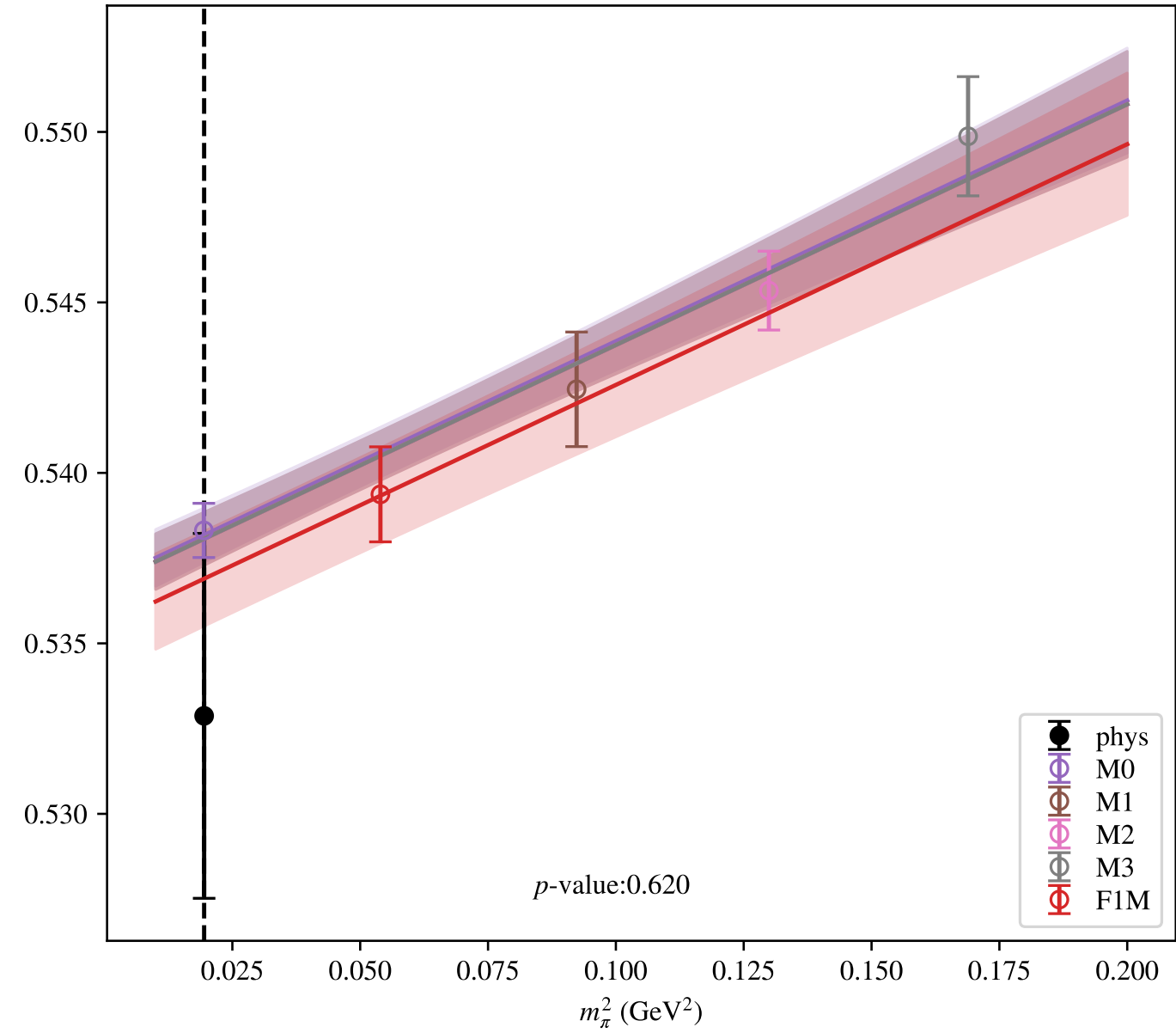


$a^2, m_\pi^2$  (no C),  $\mu = 1.8$  GeV

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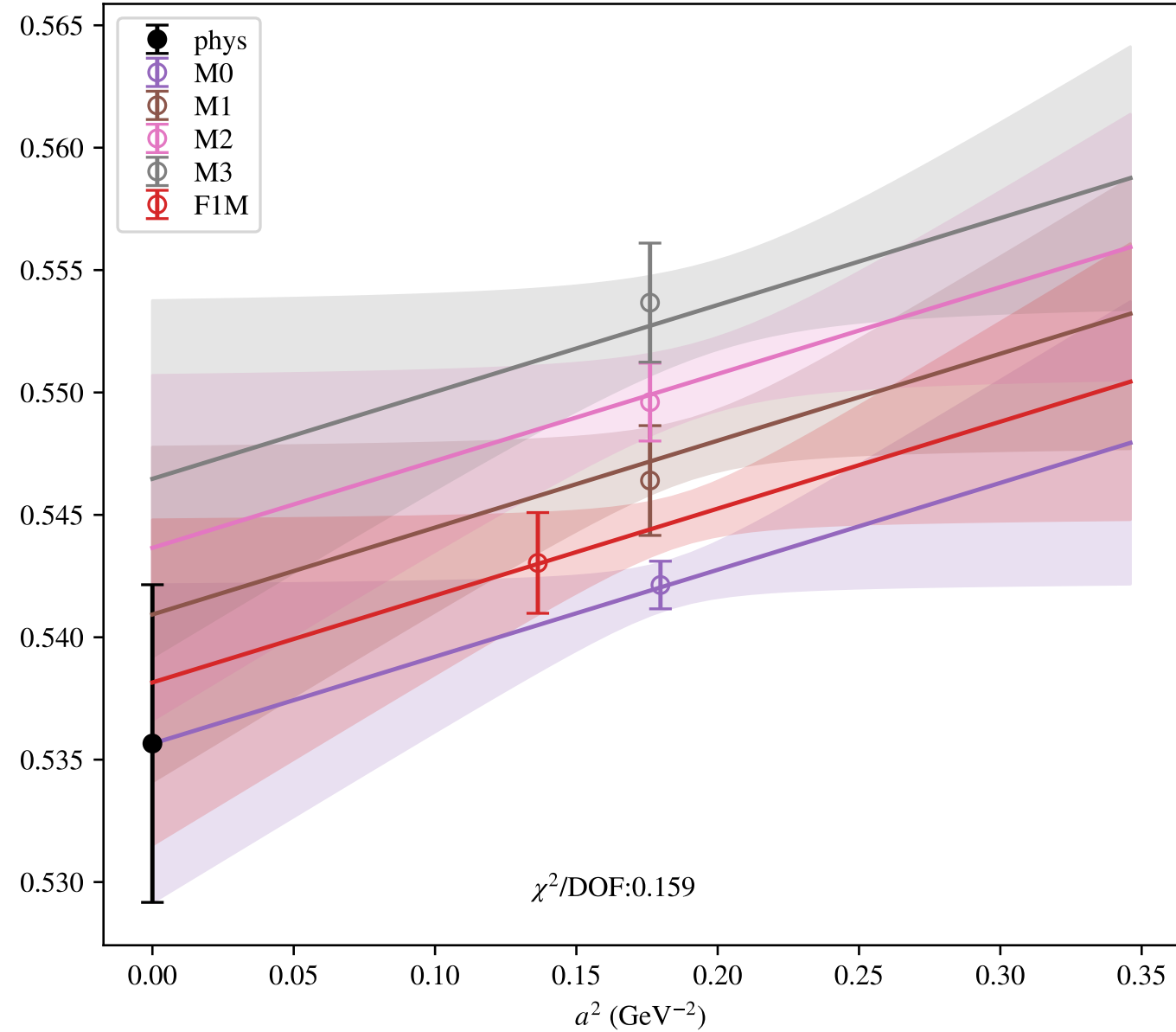


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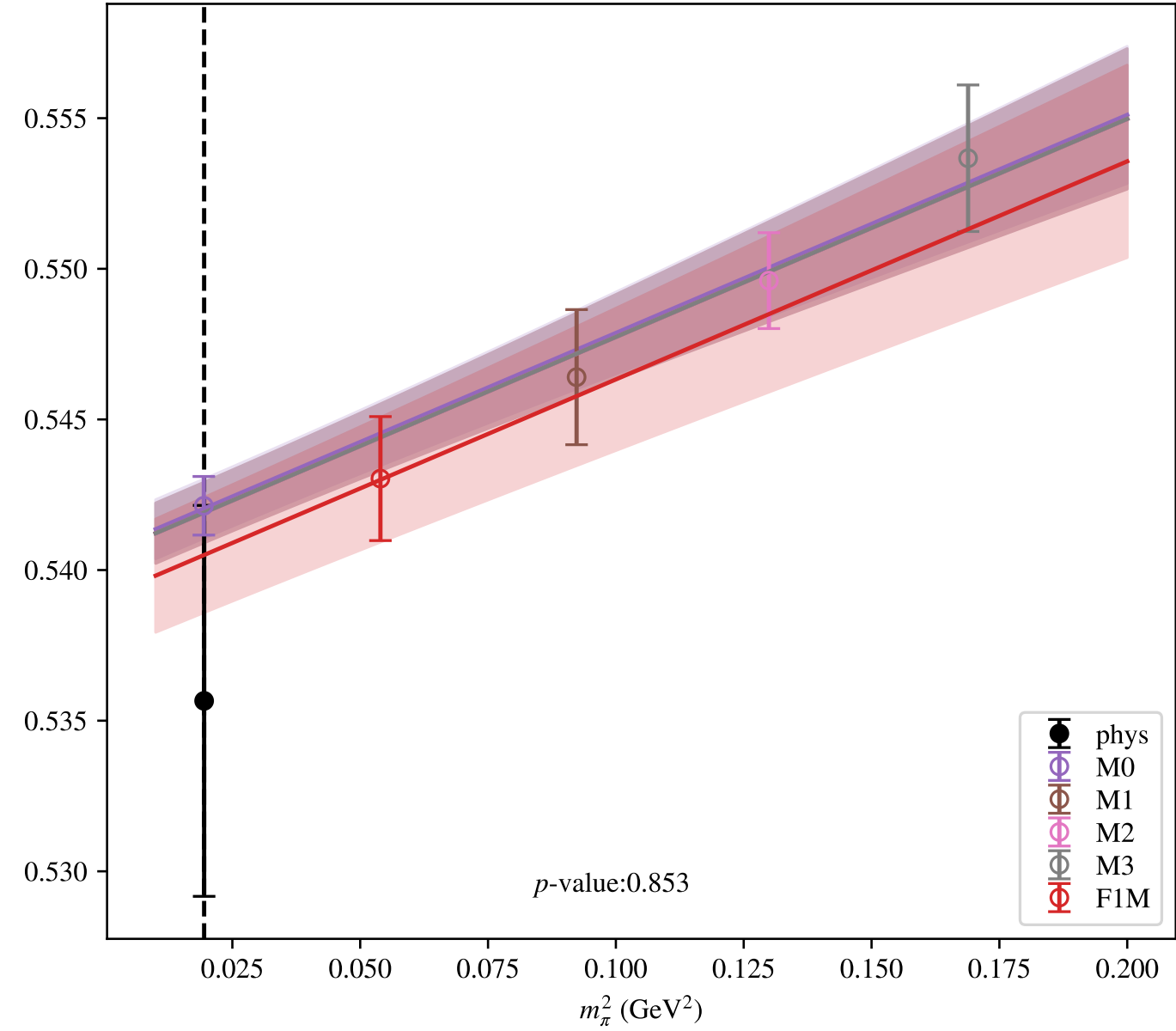


$a^2, m_\pi^2$  (no C),  $\mu = 1.5$  GeV

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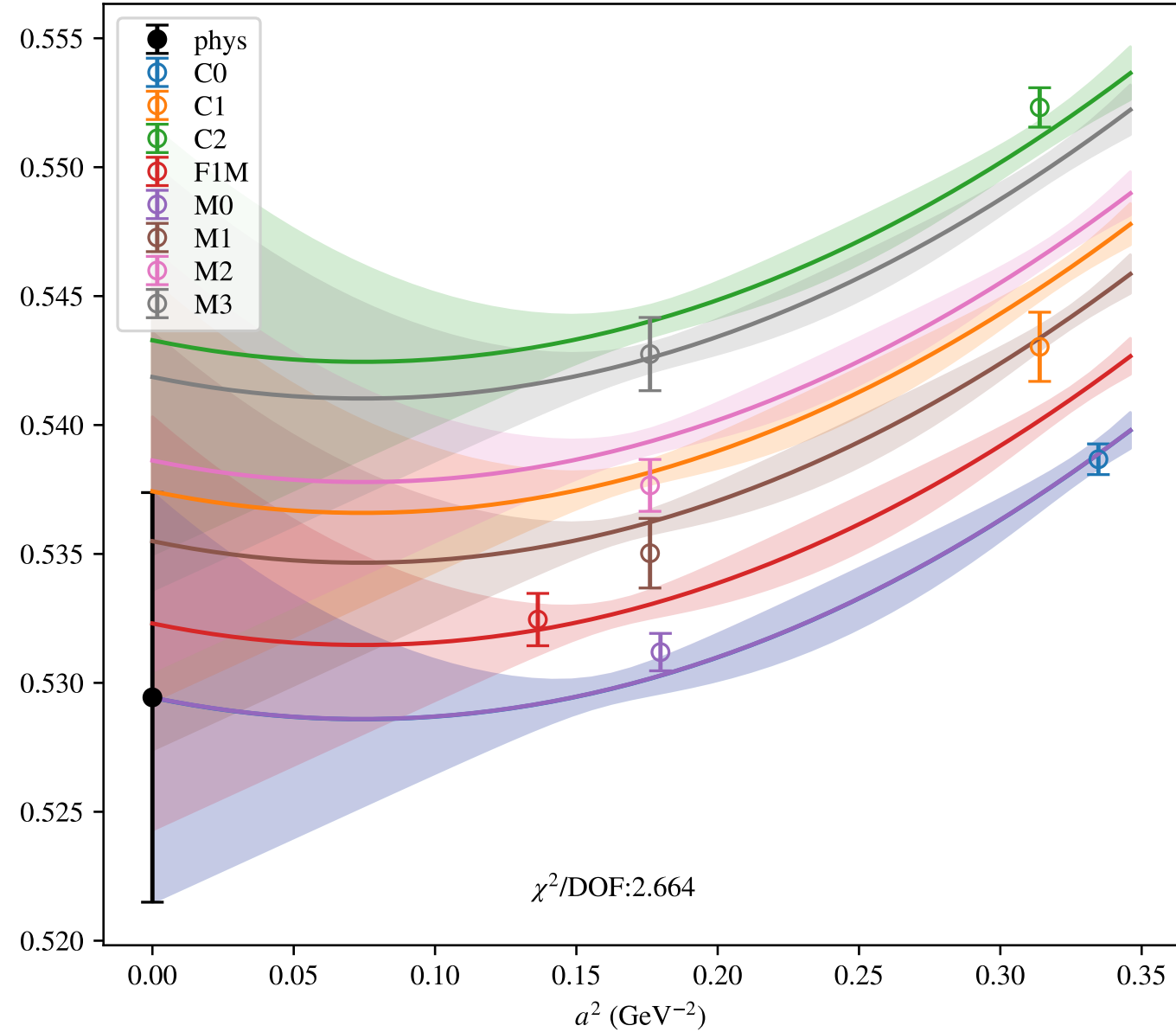


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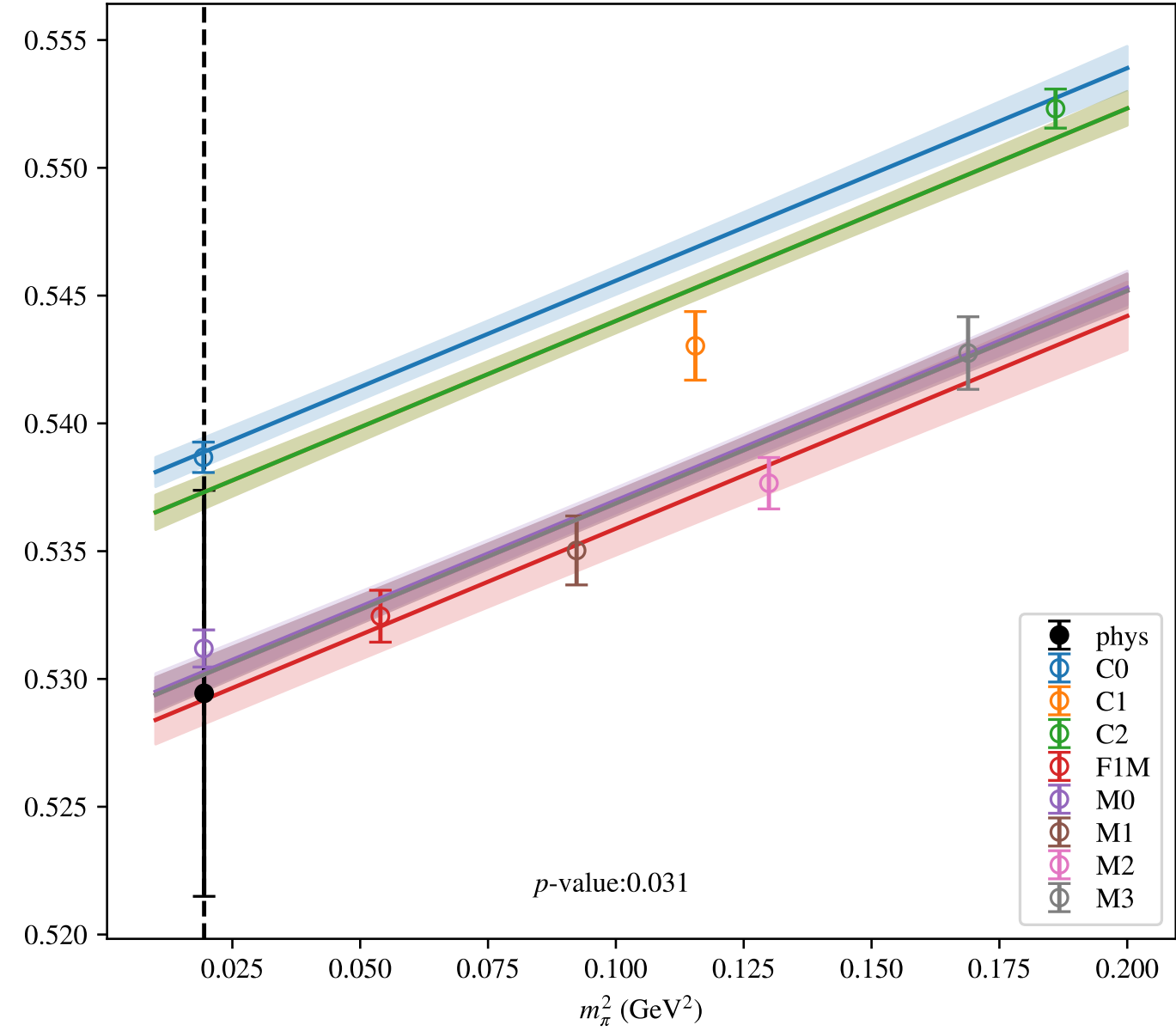


$$a^2, a^4, m_\pi^2, \mu = 2.4 \text{ GeV}$$

VVpAA

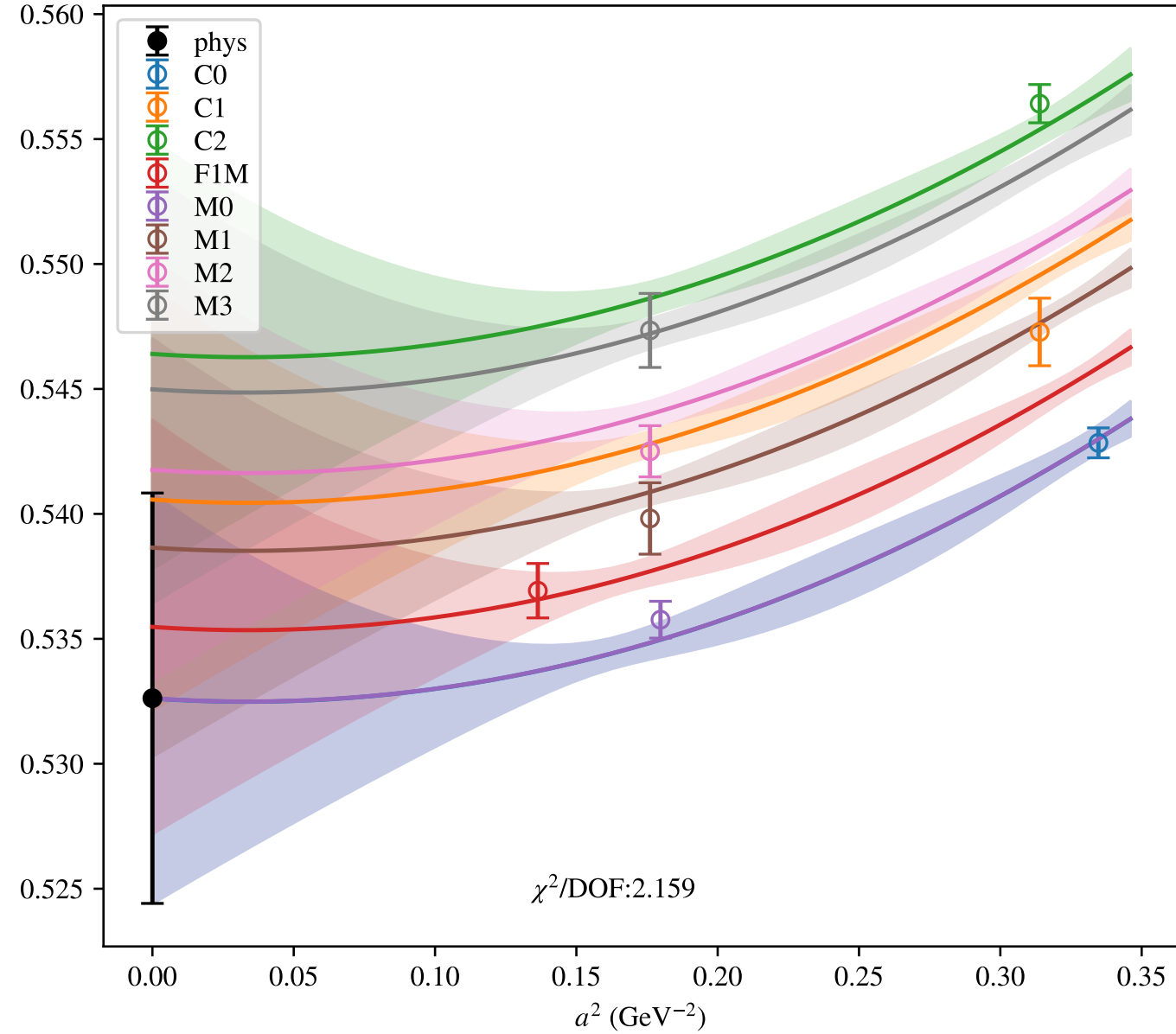


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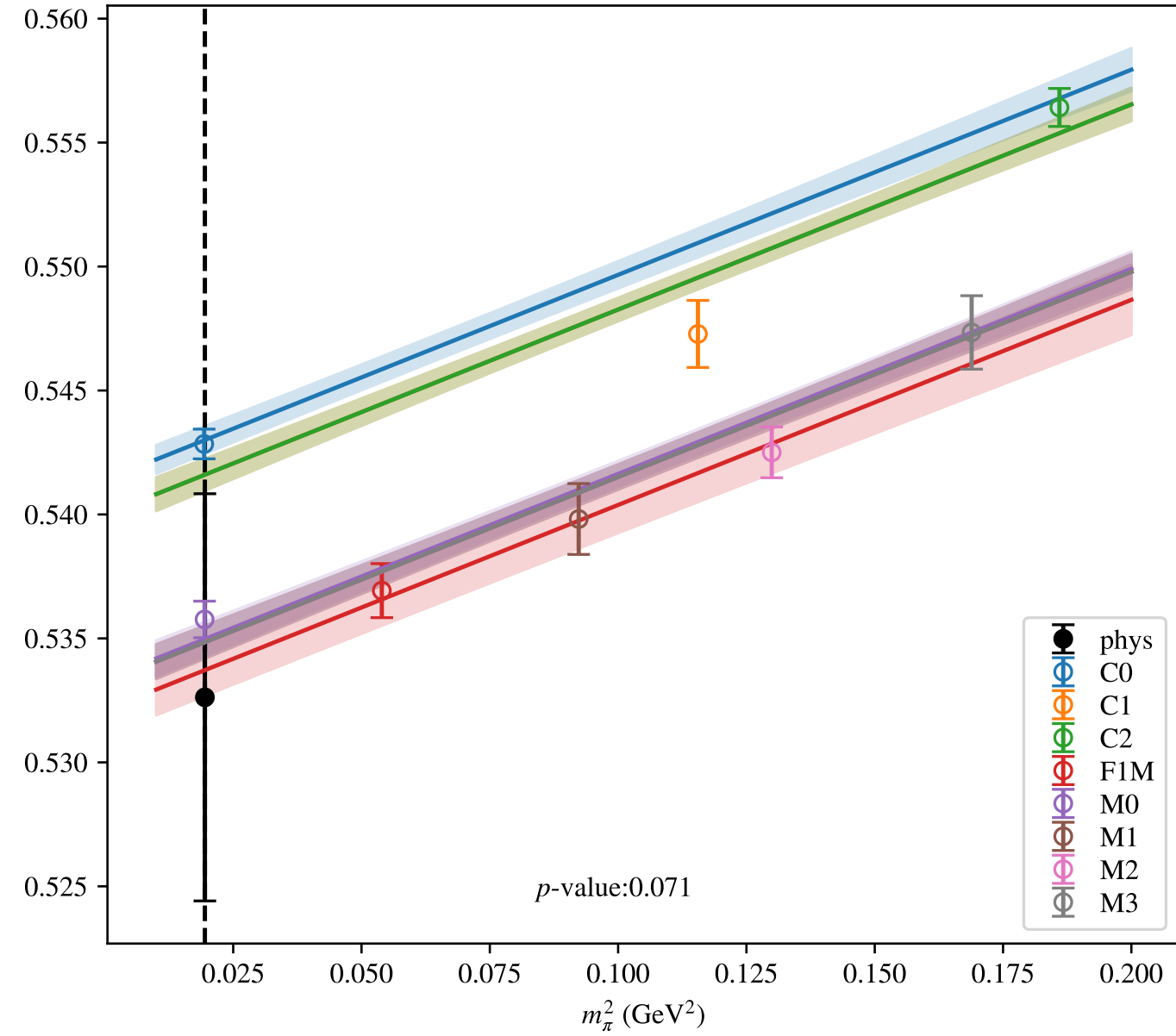


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

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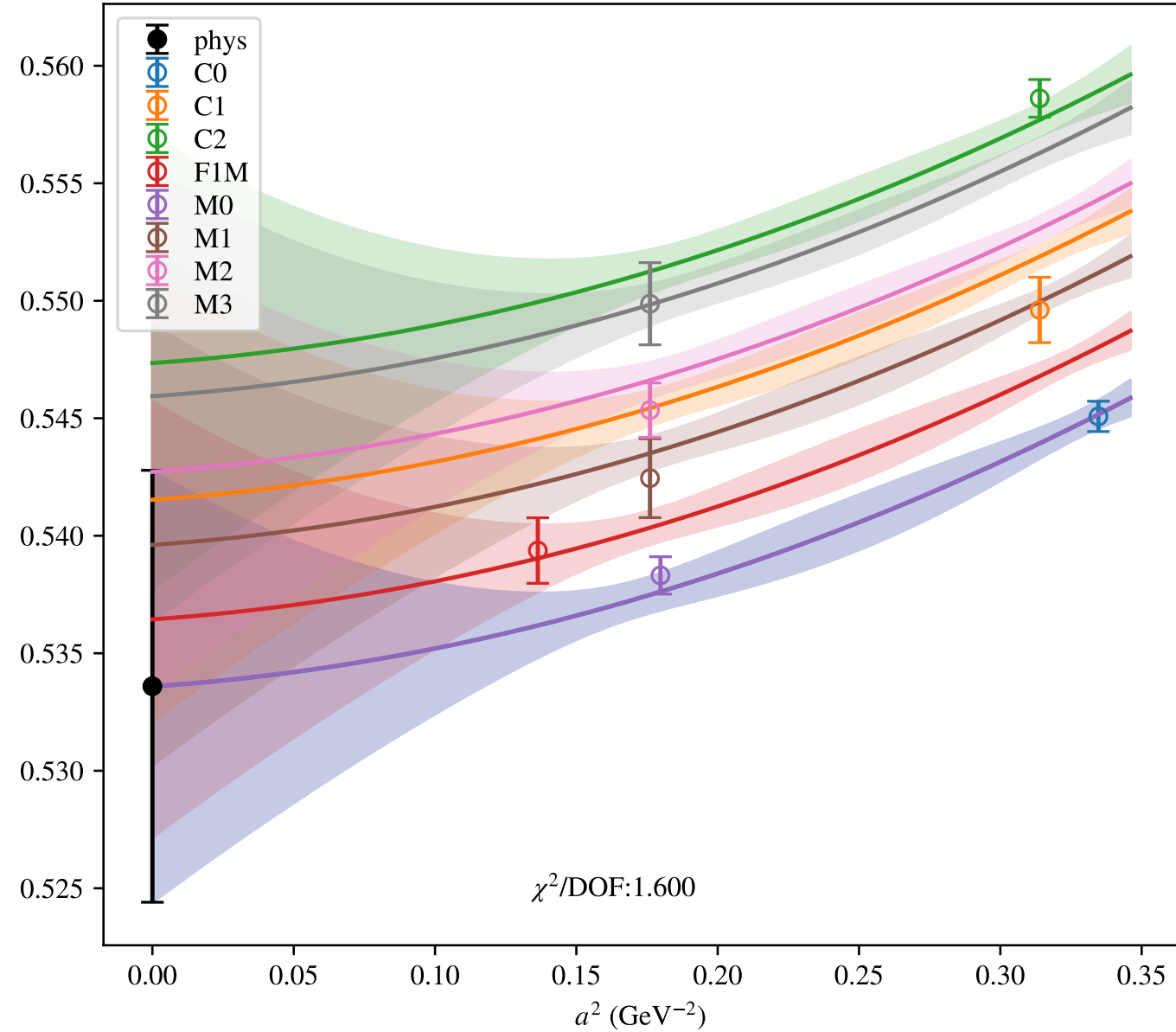


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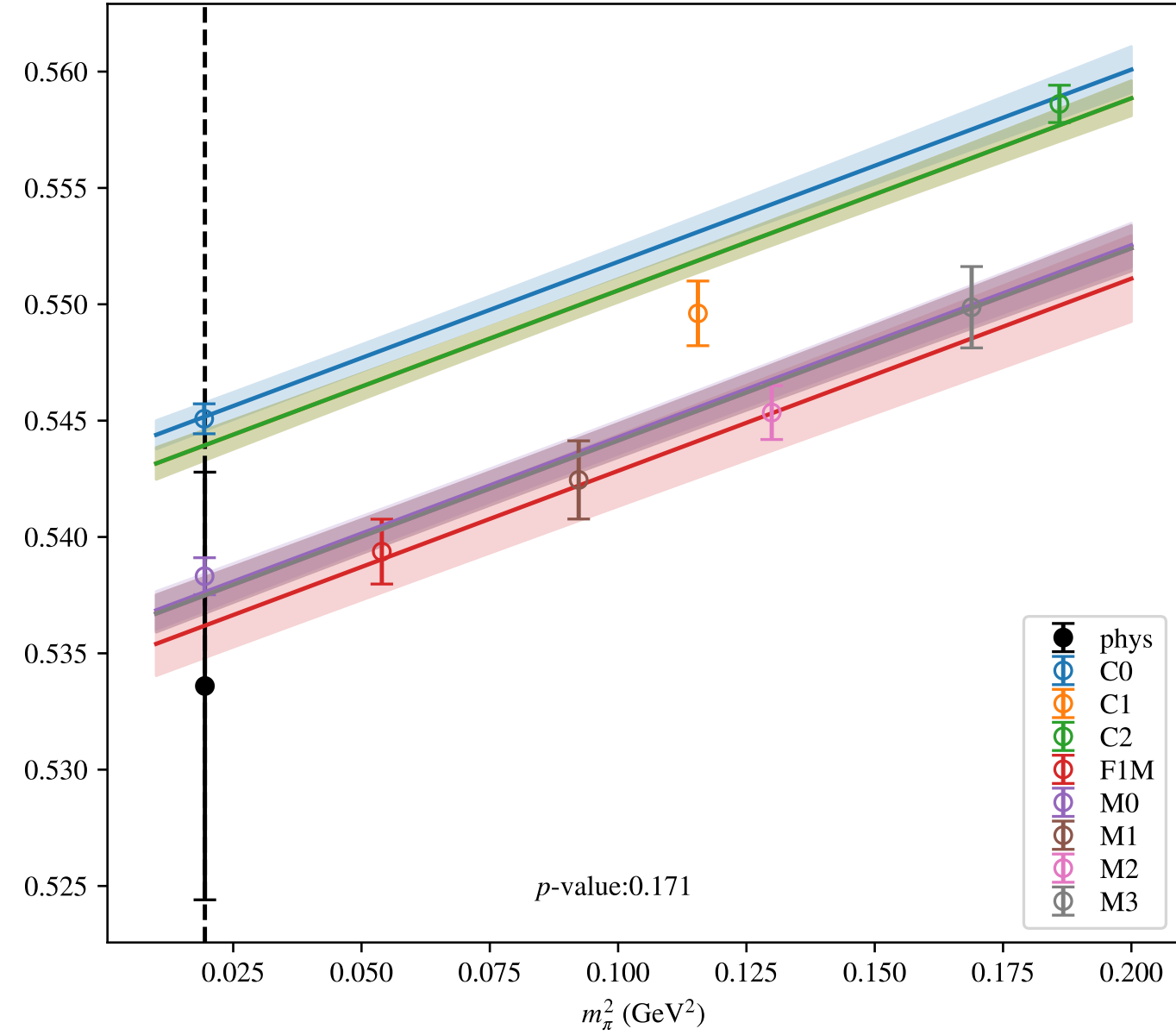


$$a^2, a^4, m_\pi^2, \mu = 1.8 \text{ GeV}$$

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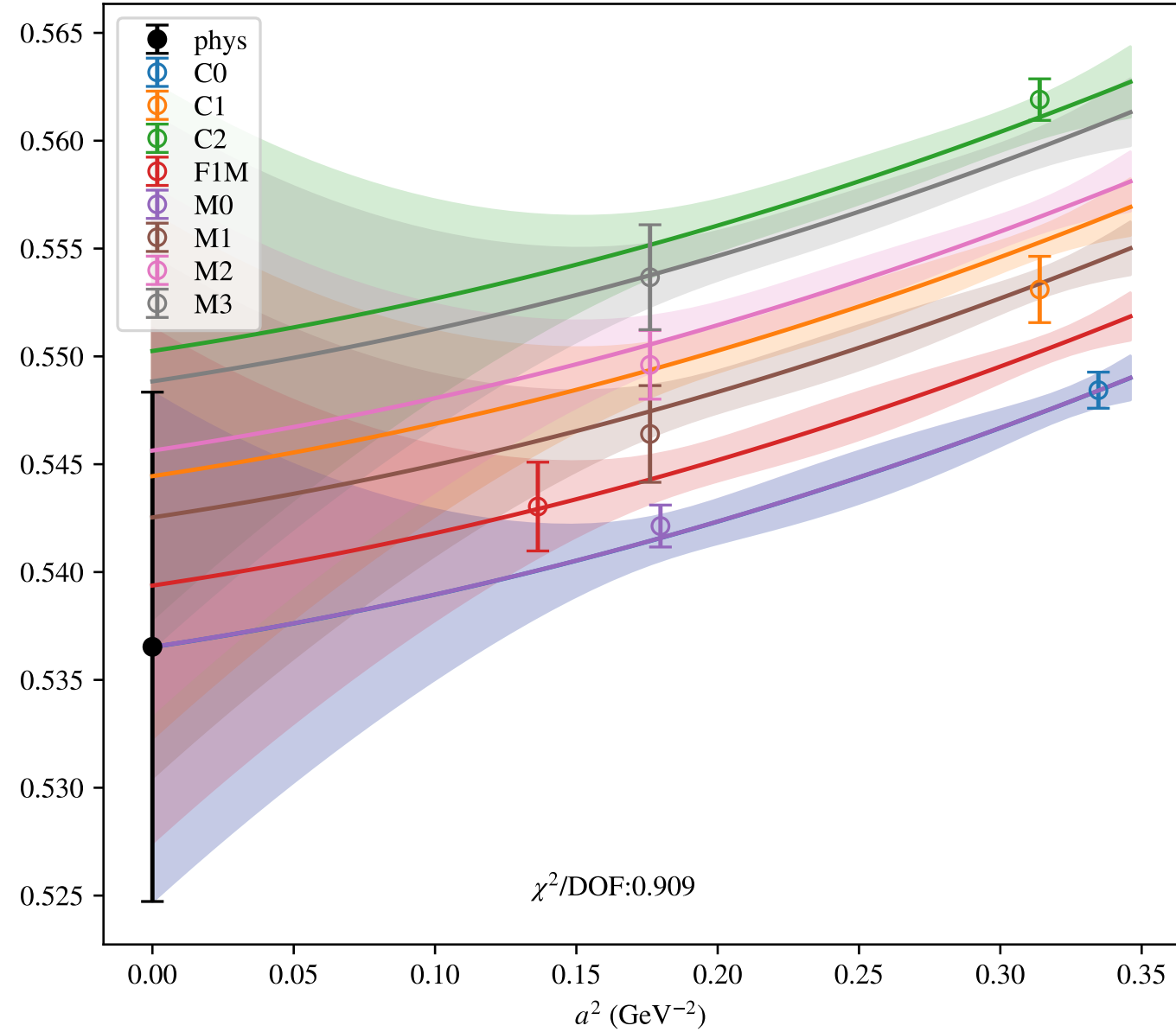


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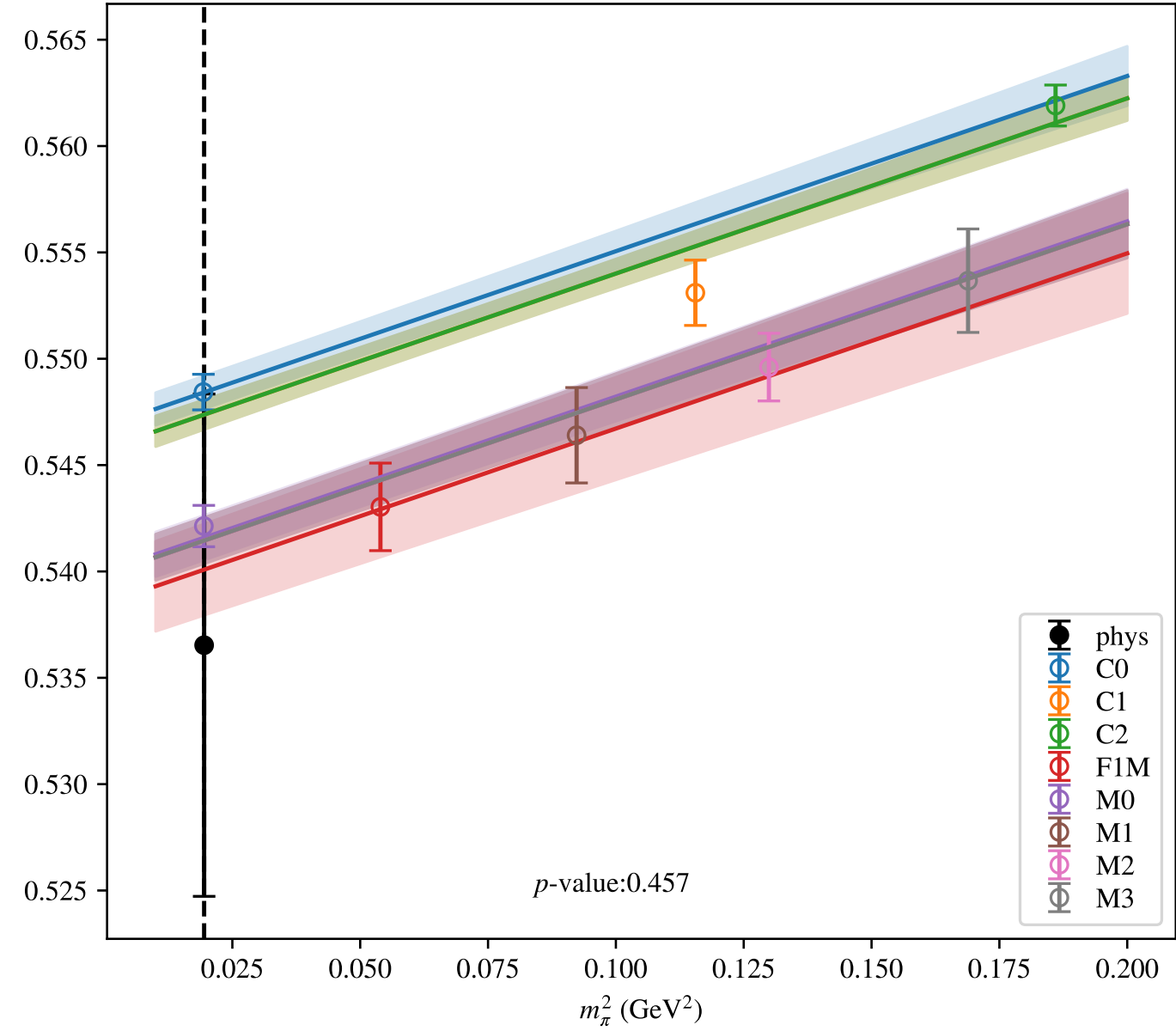


$$a^2, a^4, m_\pi^2, \mu = 1.5 \text{ GeV}$$

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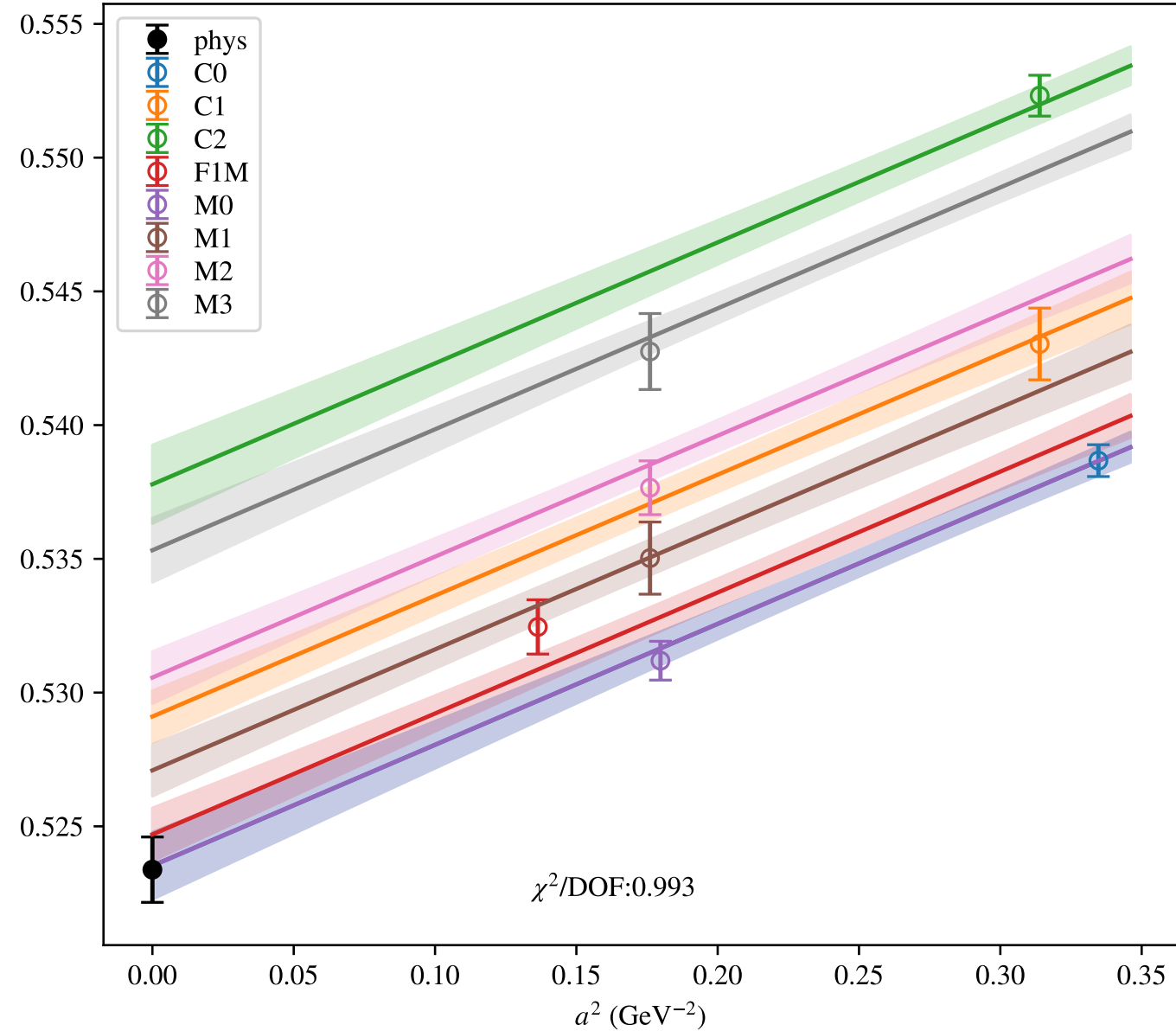


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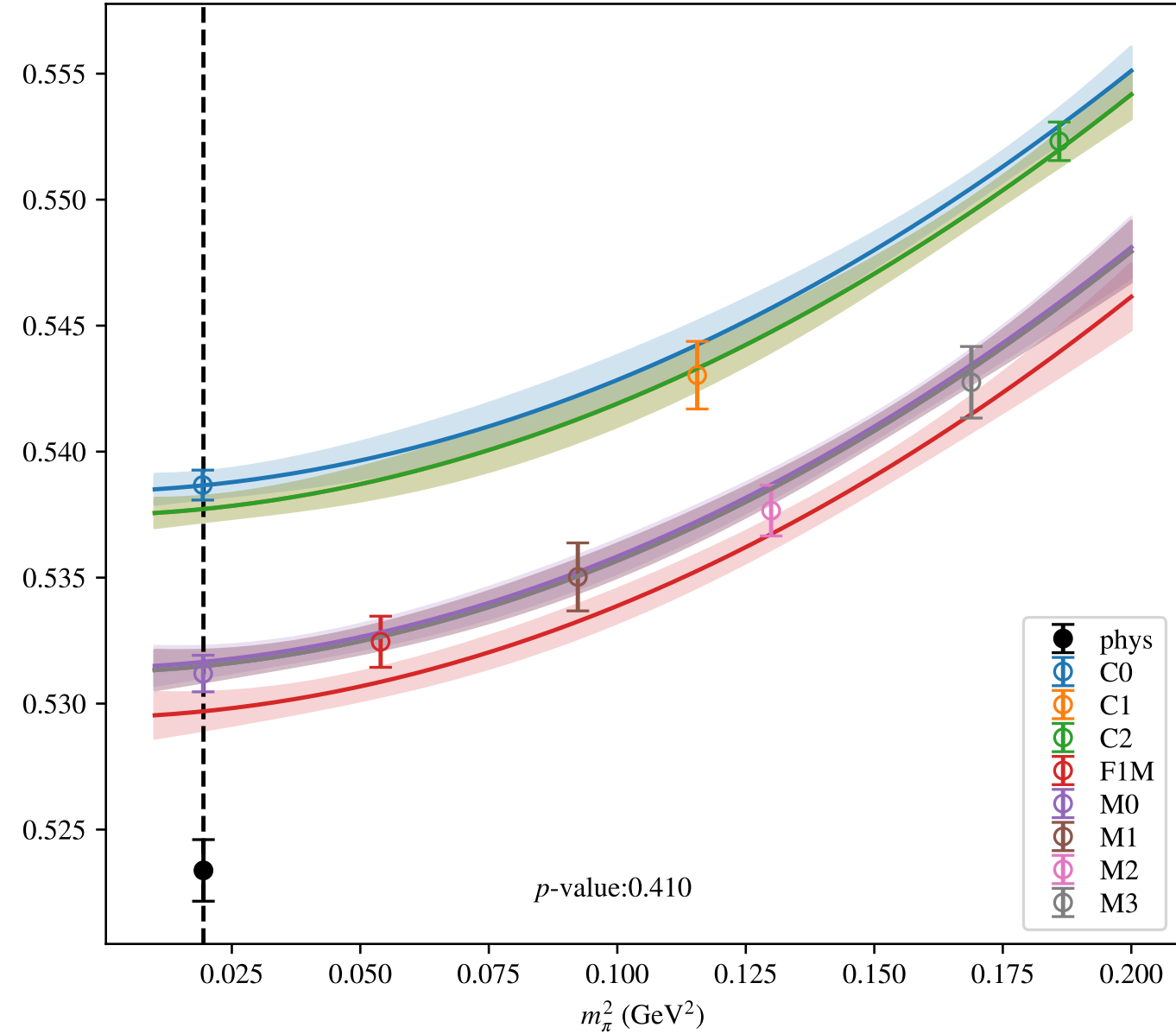


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.4 \text{ GeV}$$

VVpAA

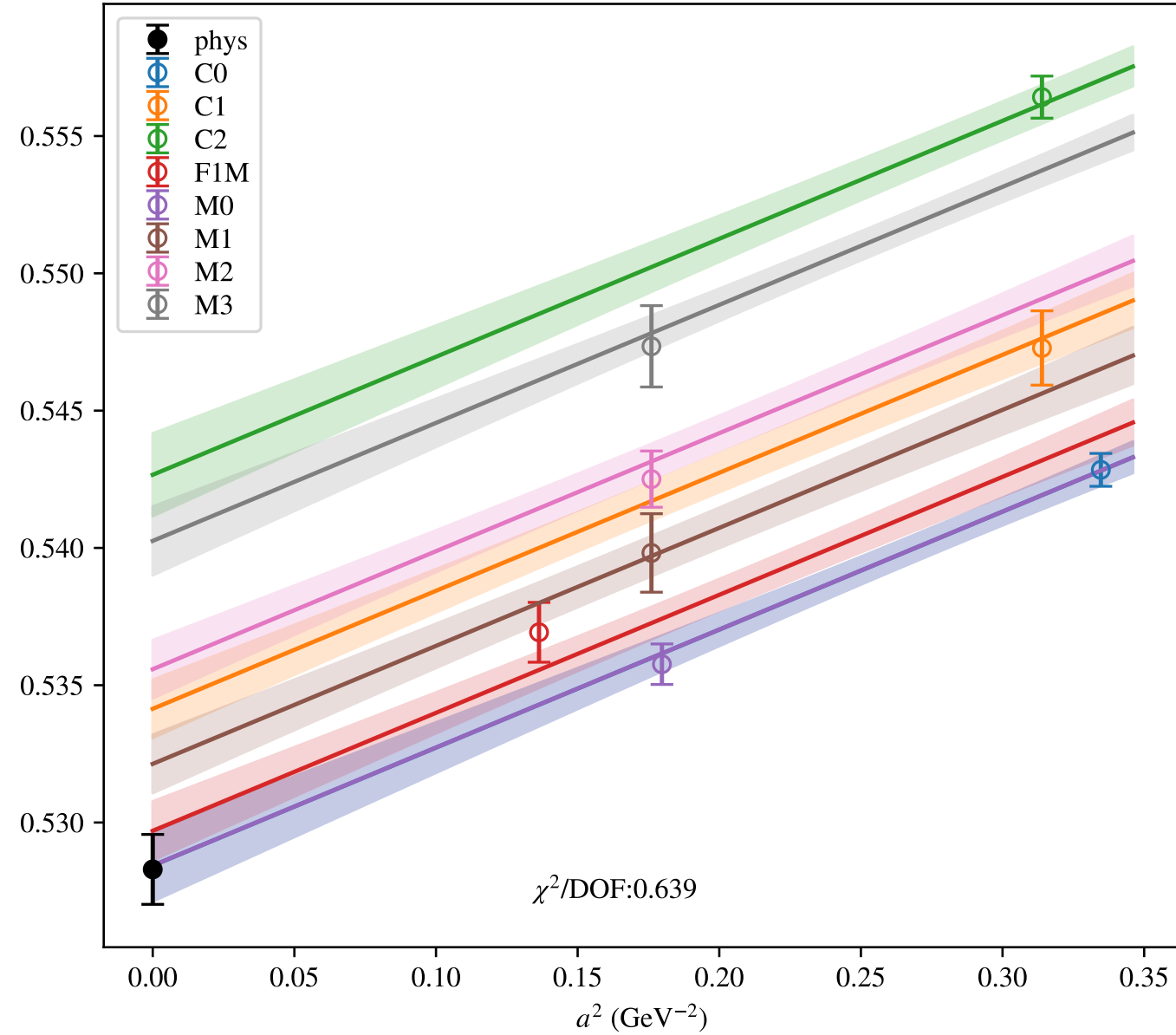


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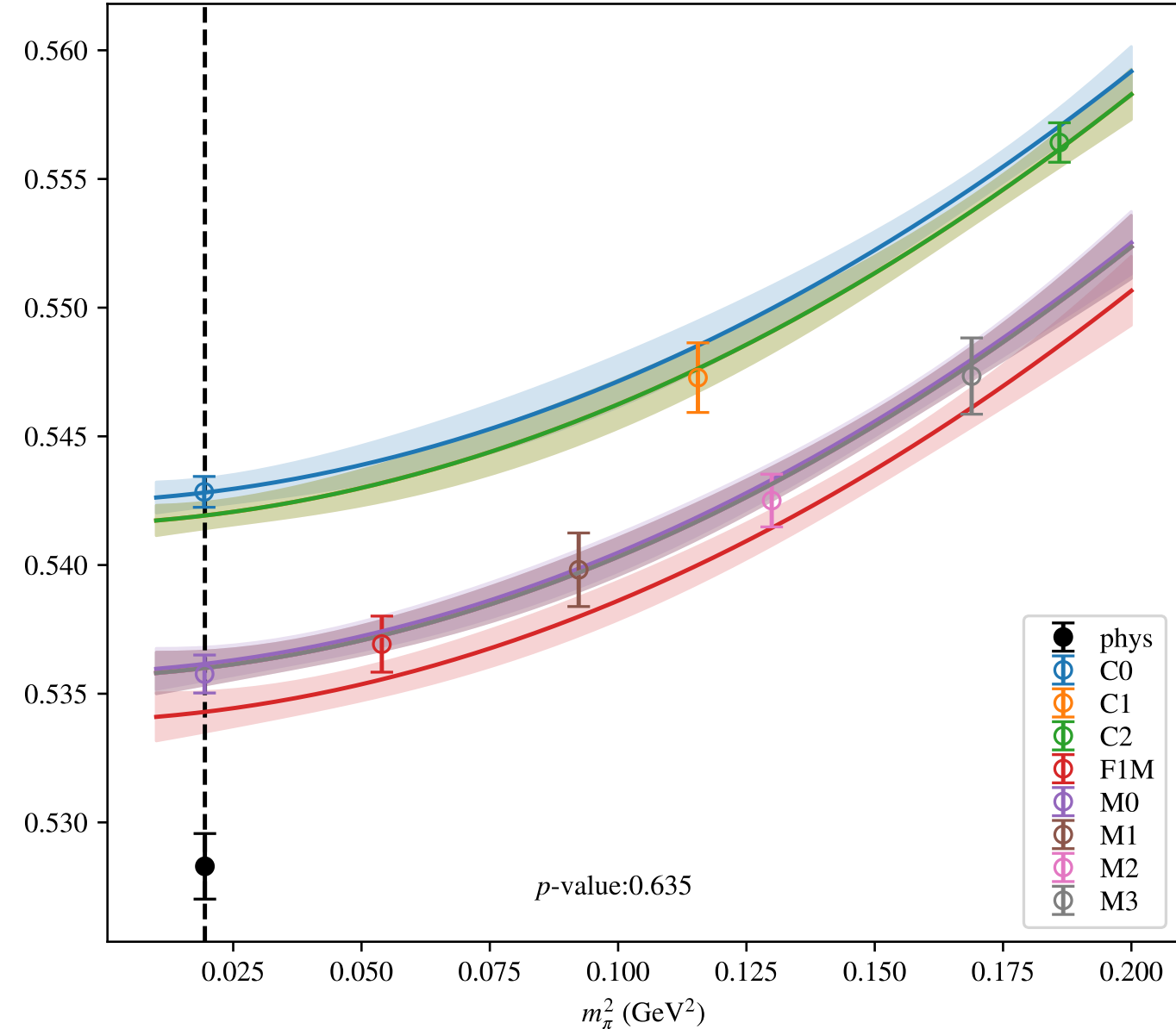


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

VVpAA



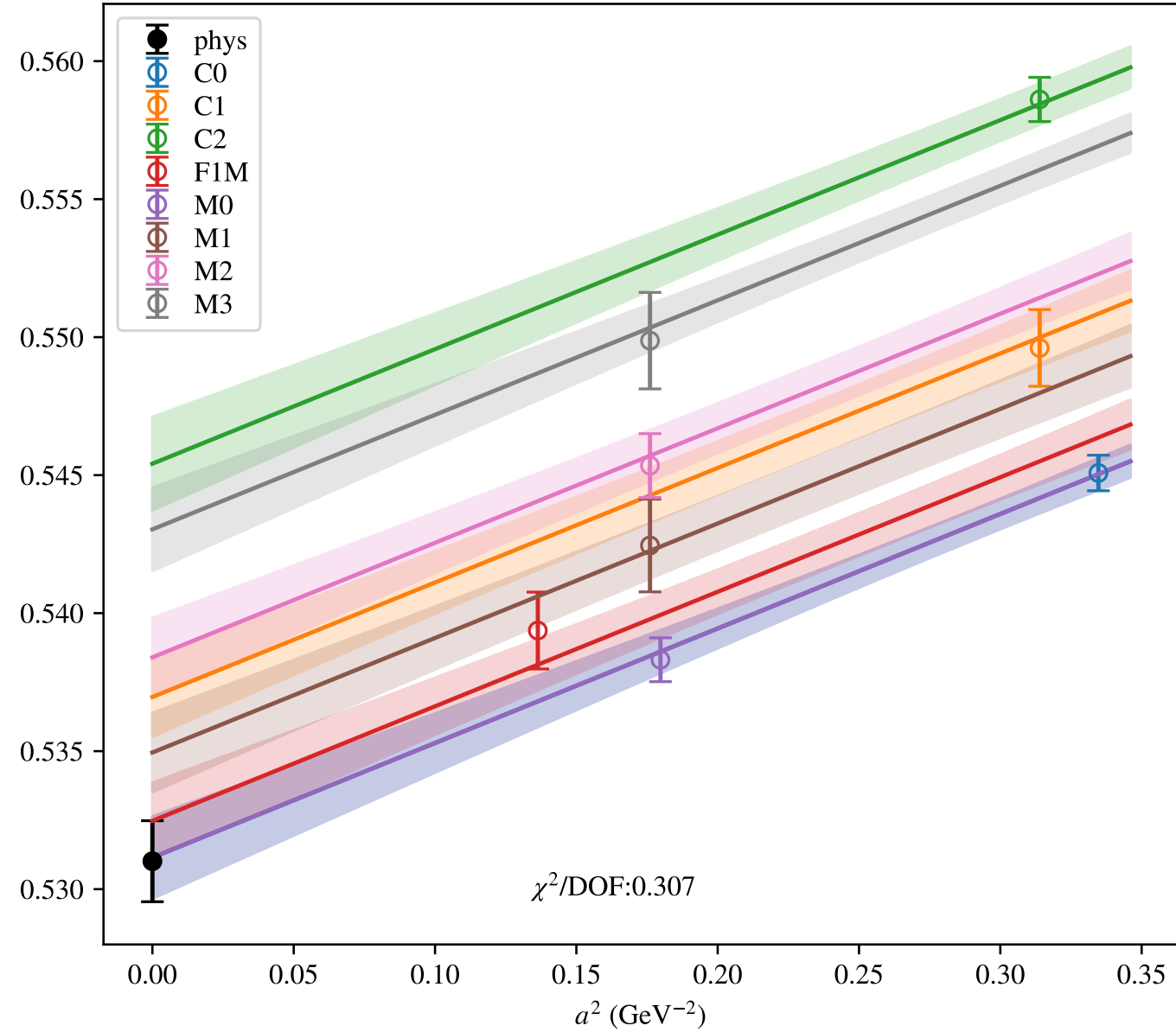
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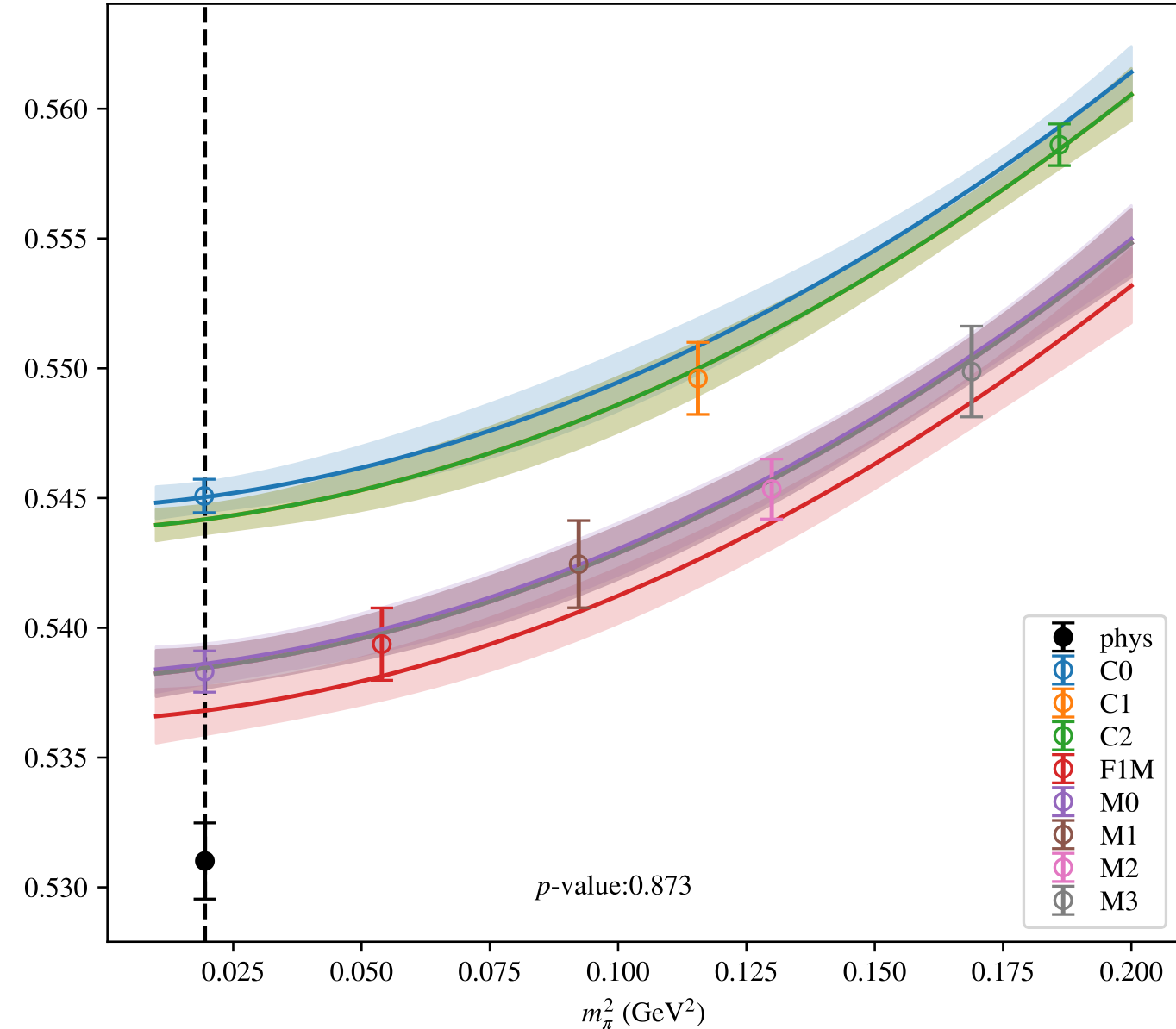


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.8 \text{ GeV}$$

VVpAA

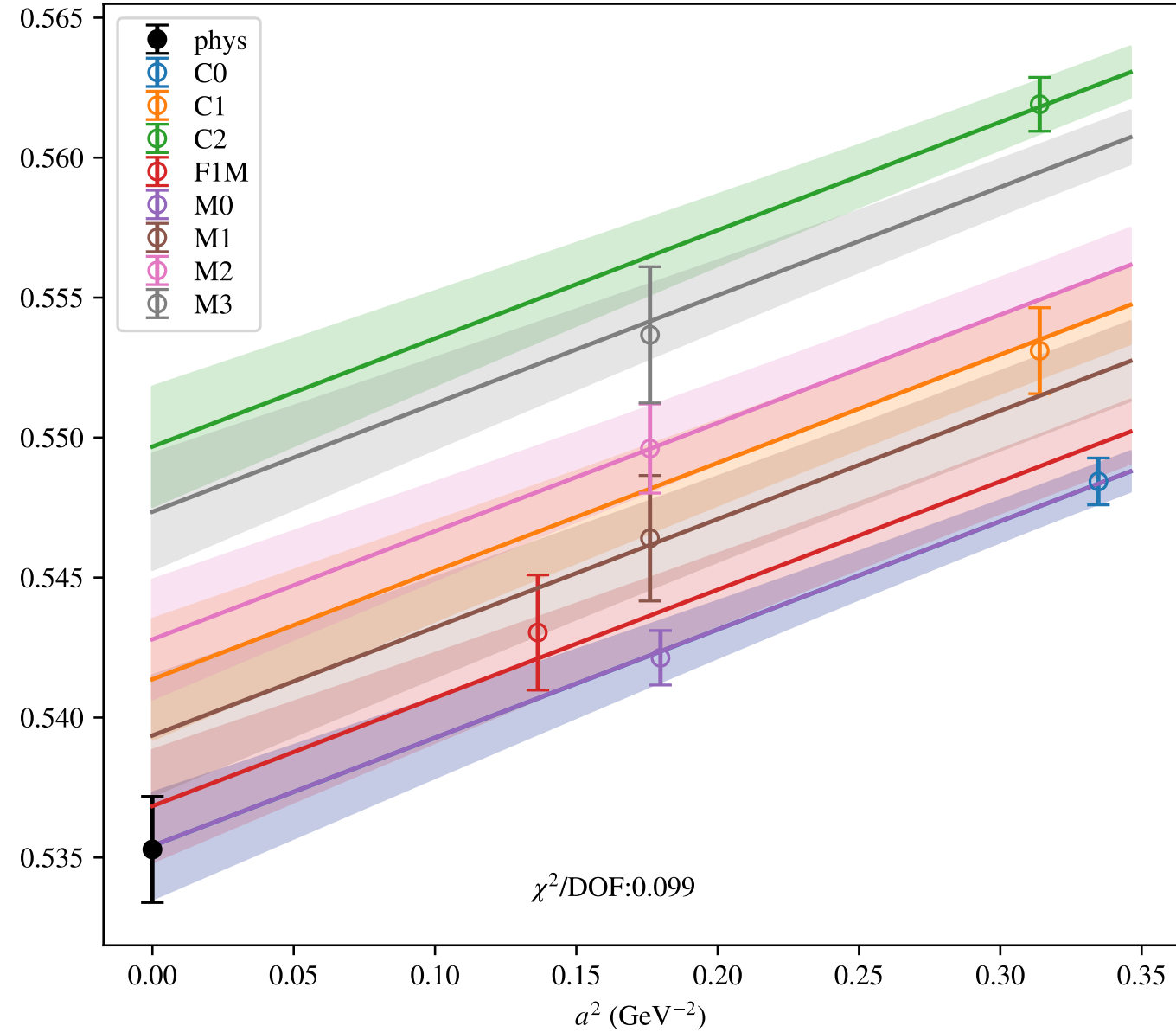


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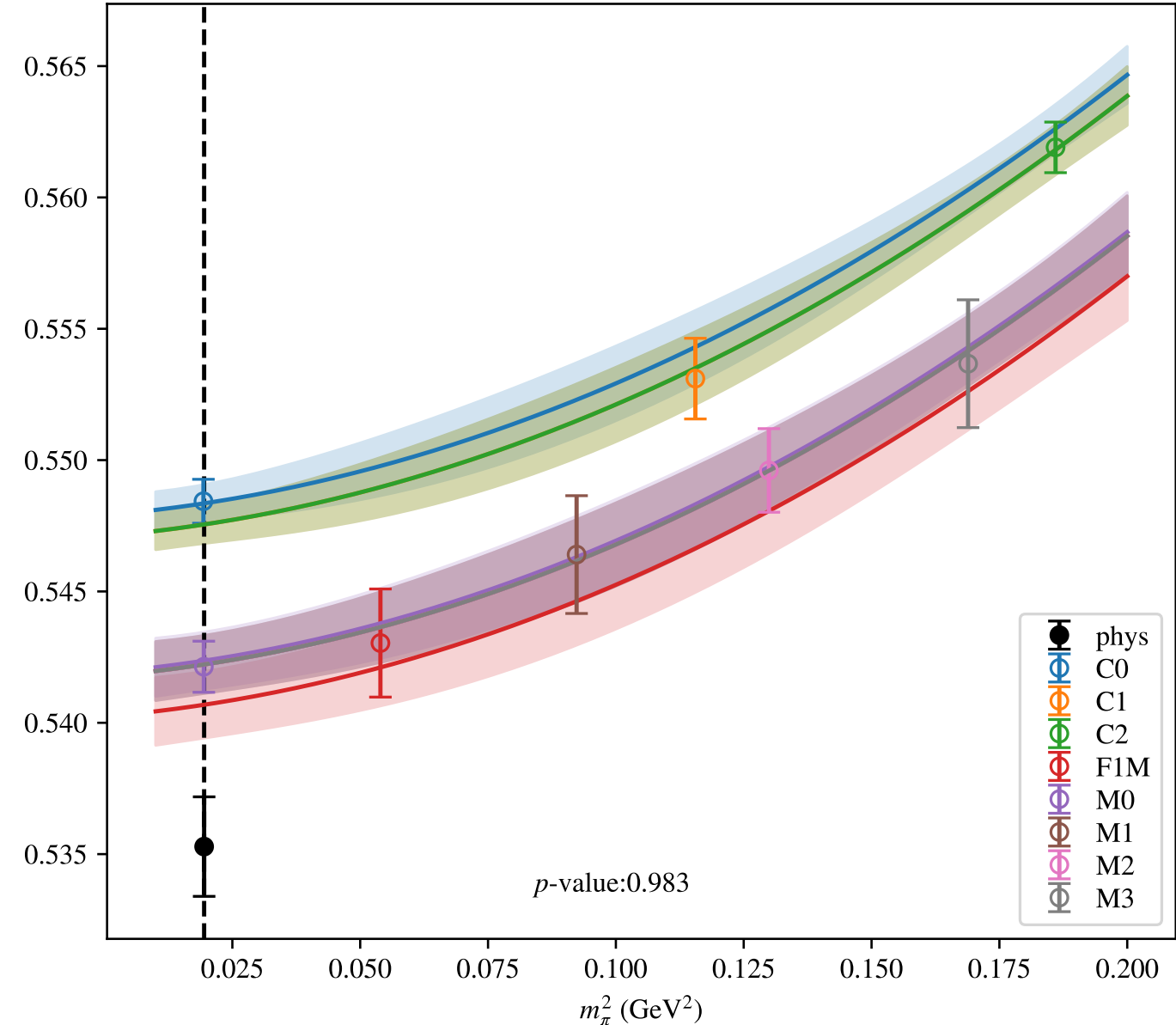


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.5 \text{ GeV}$$

VVpAA

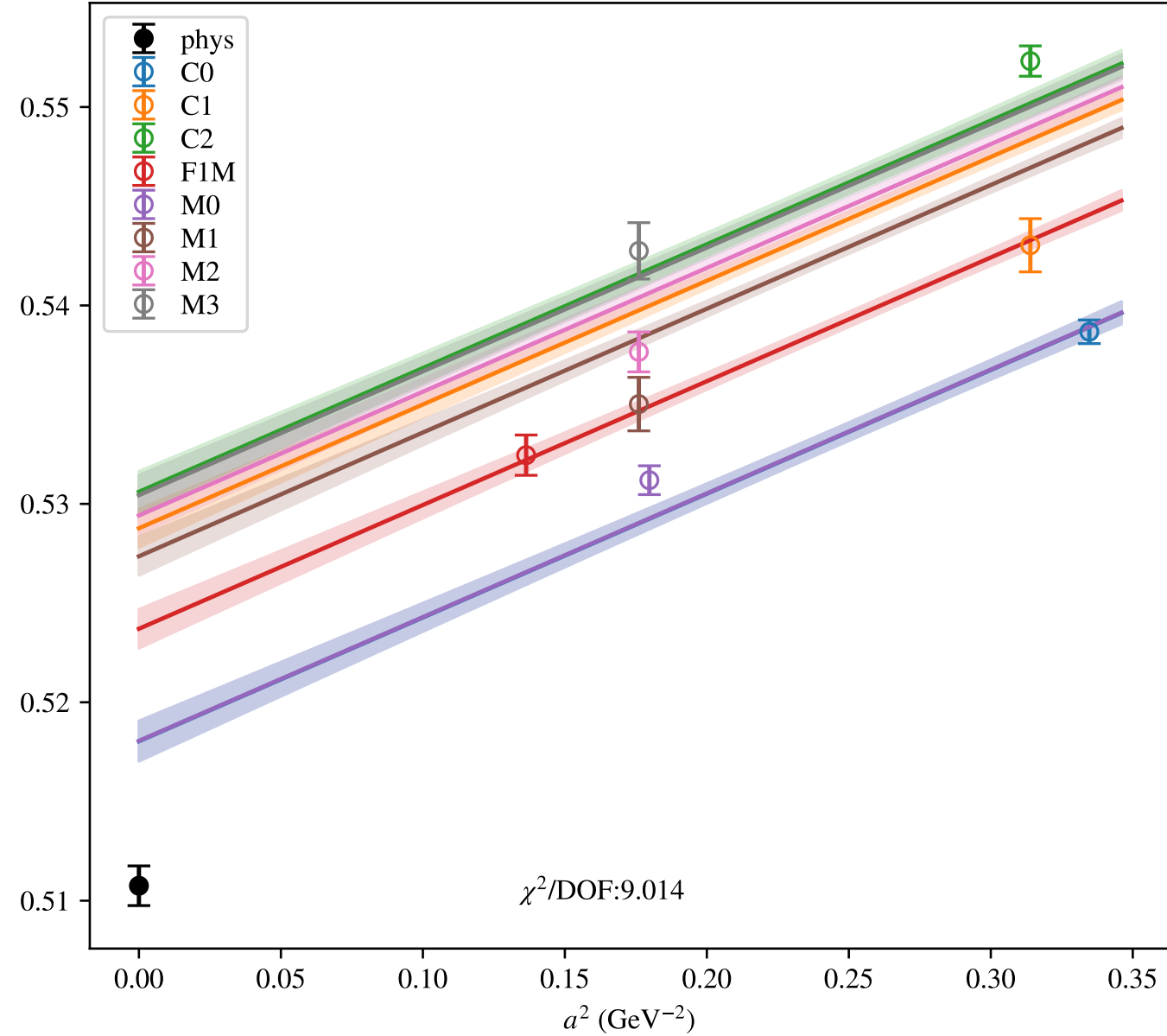


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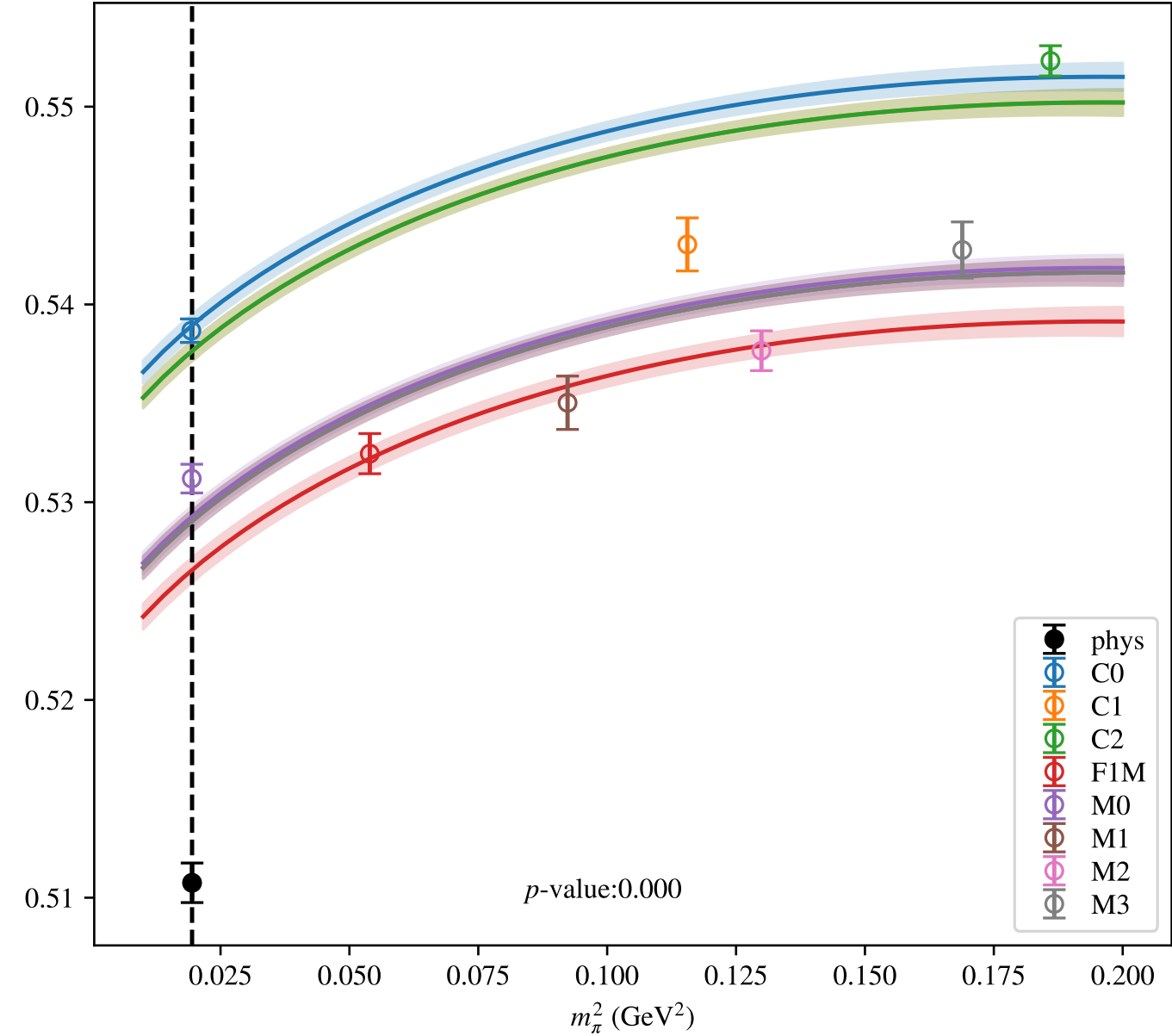


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 2.4 \text{ GeV}$$

VVpAA

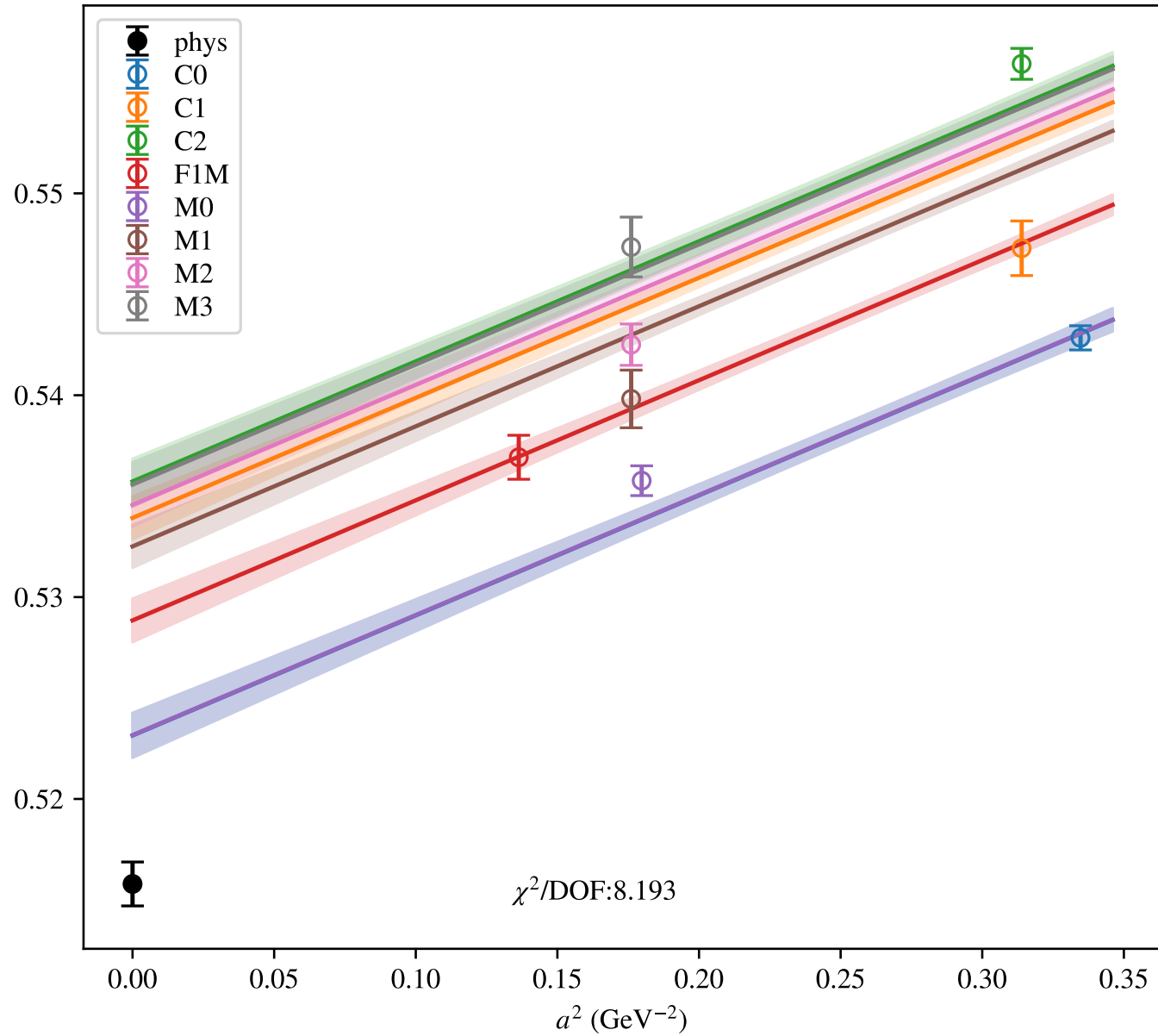


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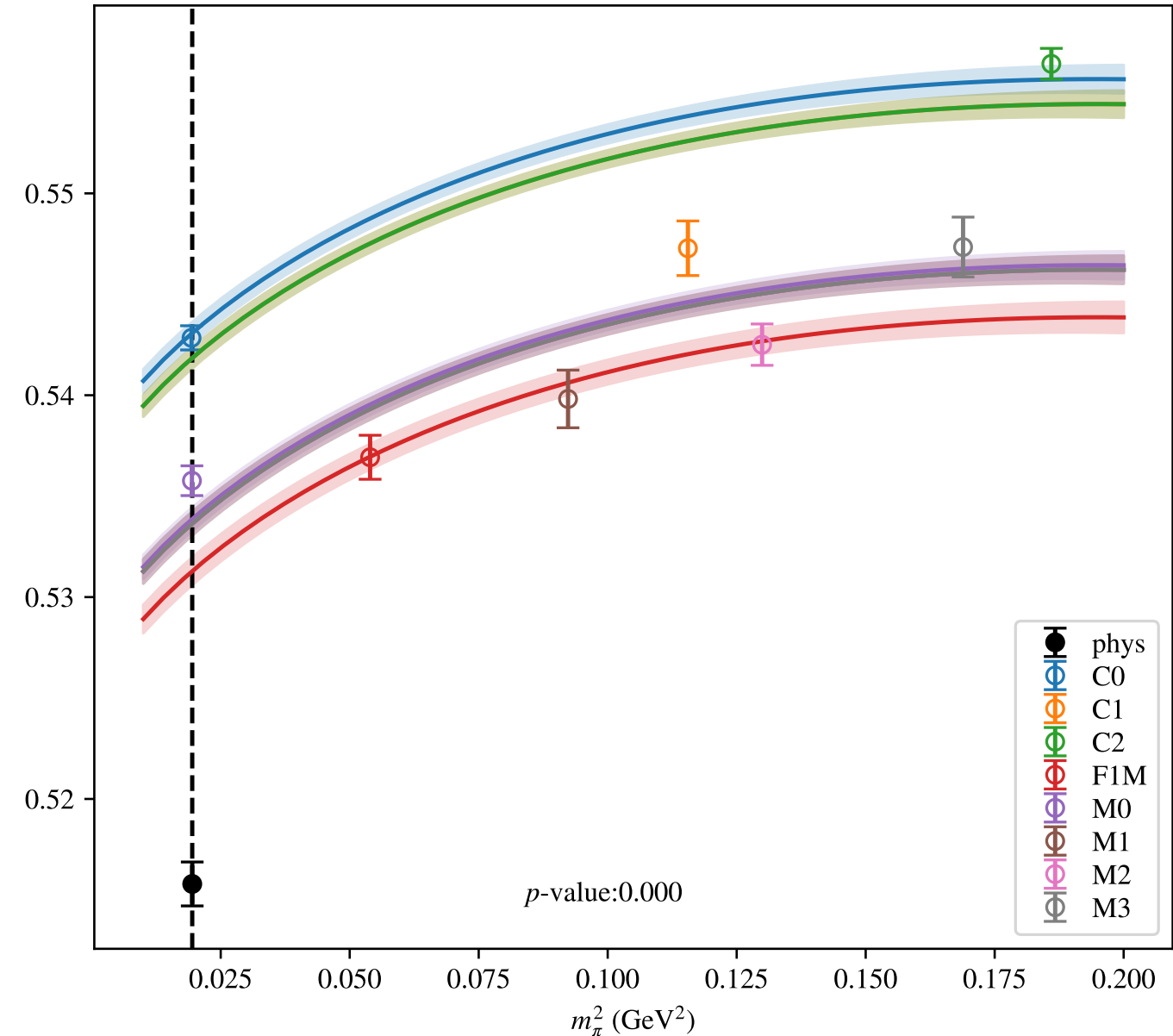


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 2.0 \text{ GeV}$$

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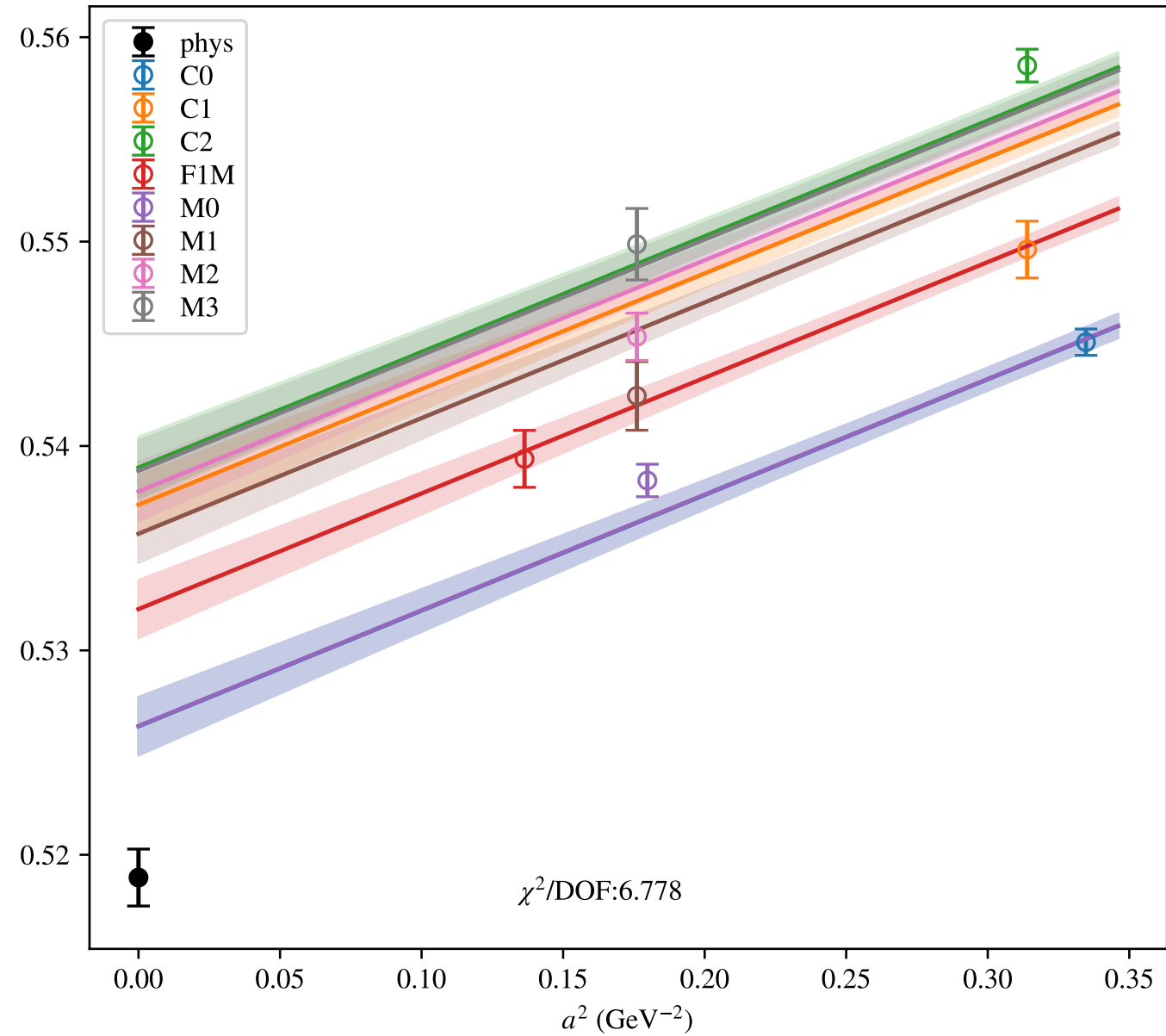


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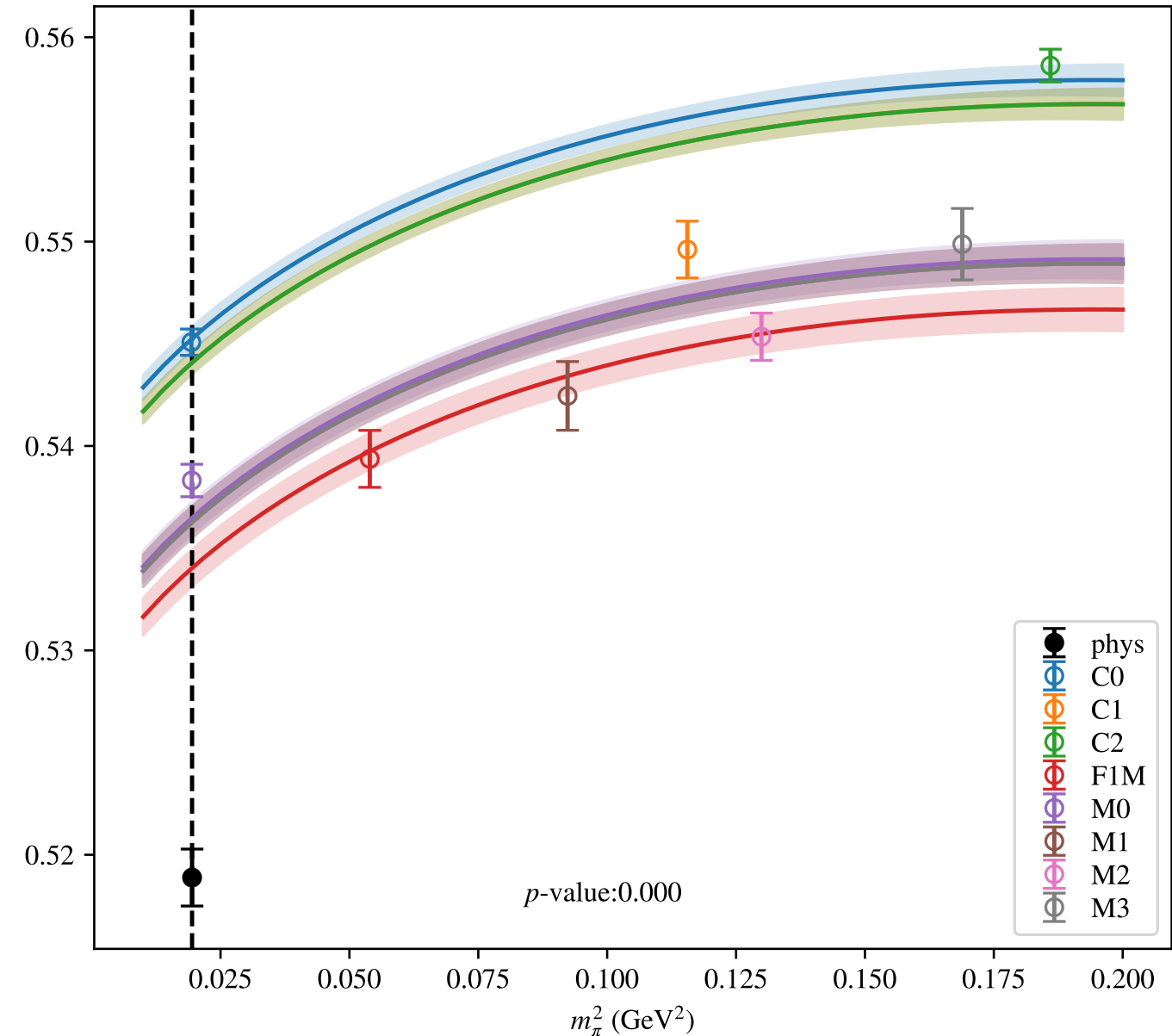


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 1.8 \text{ GeV}$$

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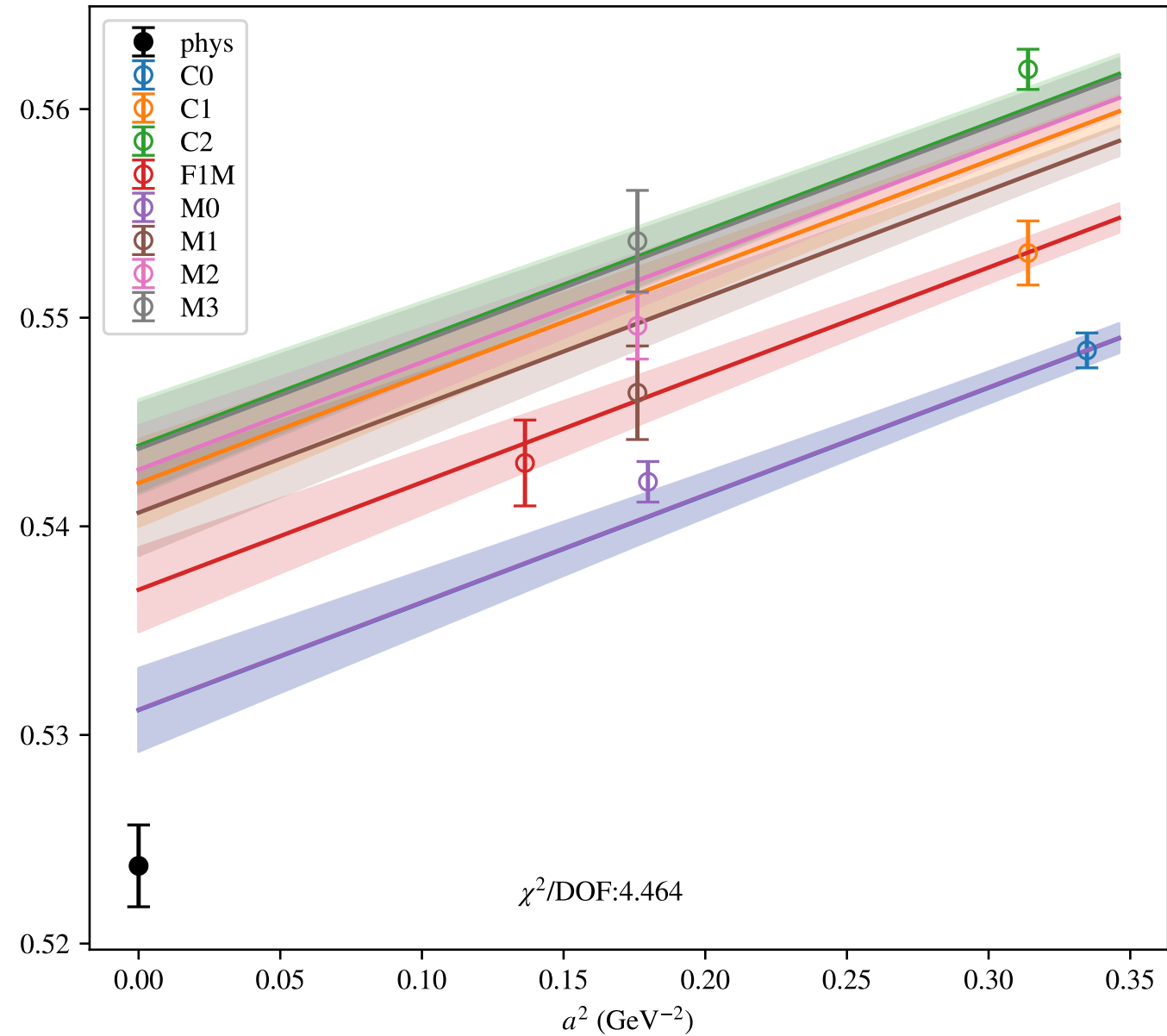


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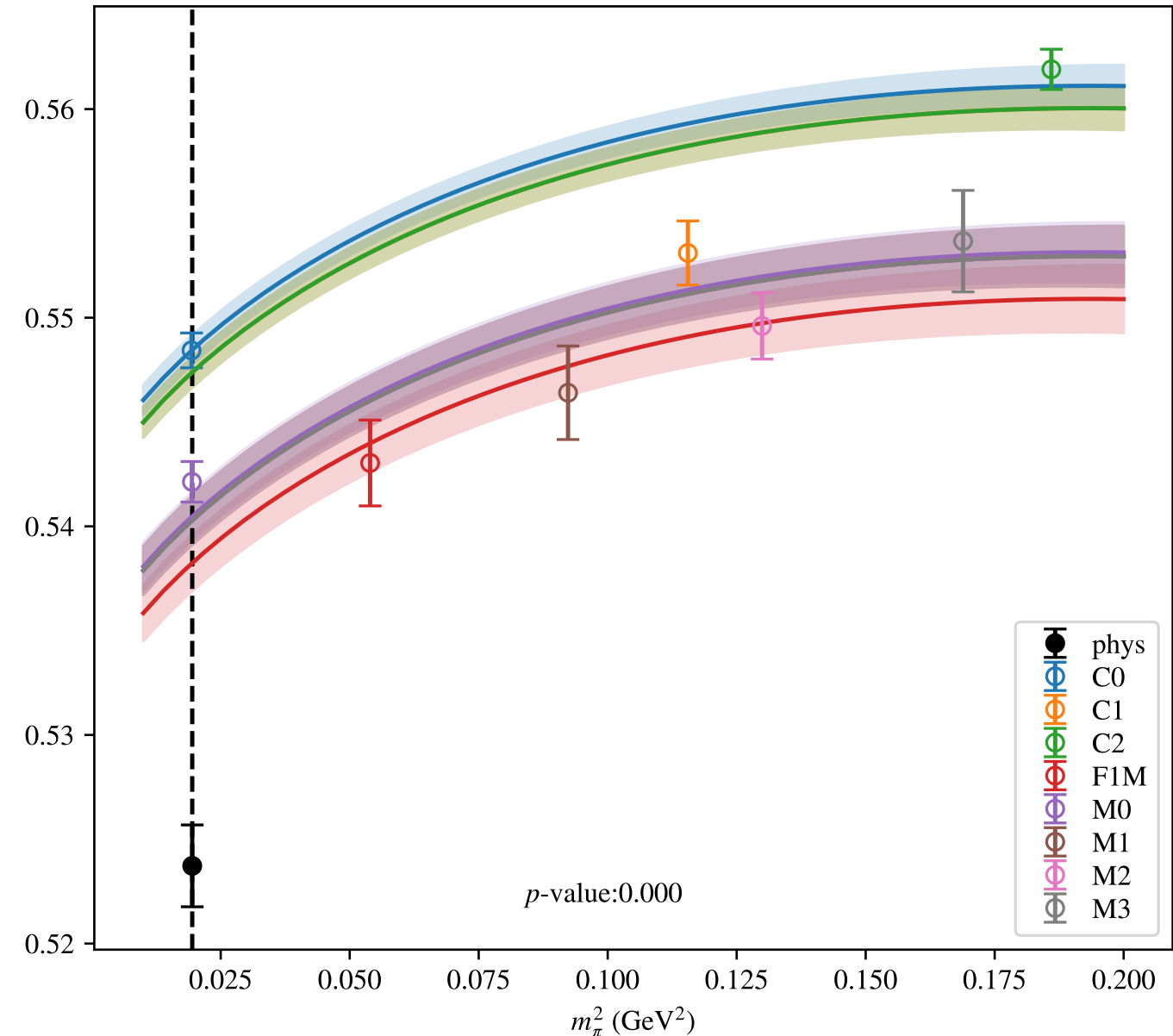


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 1.5 \text{ GeV}$$

VVpAA



VVpAA



## 2 VVmA

$\mu$ (GeV)	$a^2, m_\pi^2$	$a^2, m_\pi^2$ (no C)	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$	$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2)$
2.4	<b>1.9828(25)</b> : 8.464 (0.0)	<b>2.070(13)</b> : 0.597 (0.55)	<b>2.122(21)</b> : 0.86 (0.487)	<b>1.9810(25)</b> : 9.602 (0.0)	<b>1.9481(24)</b> : 9.563 (0.0)
2.0	<b>2.3714(31)</b> : 8.812 (0.0)	<b>2.470(16)</b> : 0.354 (0.702)	<b>2.555(26)</b> : 0.418 (0.796)	<b>2.3688(31)</b> : 9.437 (0.0)	<b>2.3307(31)</b> : 8.041 (0.0)
1.8	<b>2.5968(42)</b> : 8.923 (0.0)	<b>2.692(19)</b> : 0.993 (0.371)	<b>2.821(31)</b> : 2.489 (0.041)	<b>2.5945(40)</b> : 10.51 (0.0)	<b>2.5516(41)</b> : 9.637 (0.0)
1.5	<b>2.9376(65)</b> : 9.386 (0.0)	<b>3.026(26)</b> : 1.469 (0.23)	<b>3.232(43)</b> : 5.497 (0.0)	<b>2.9360(58)</b> : 11.616 (0.0)	<b>2.8854(64)</b> : 10.9 (0.0)

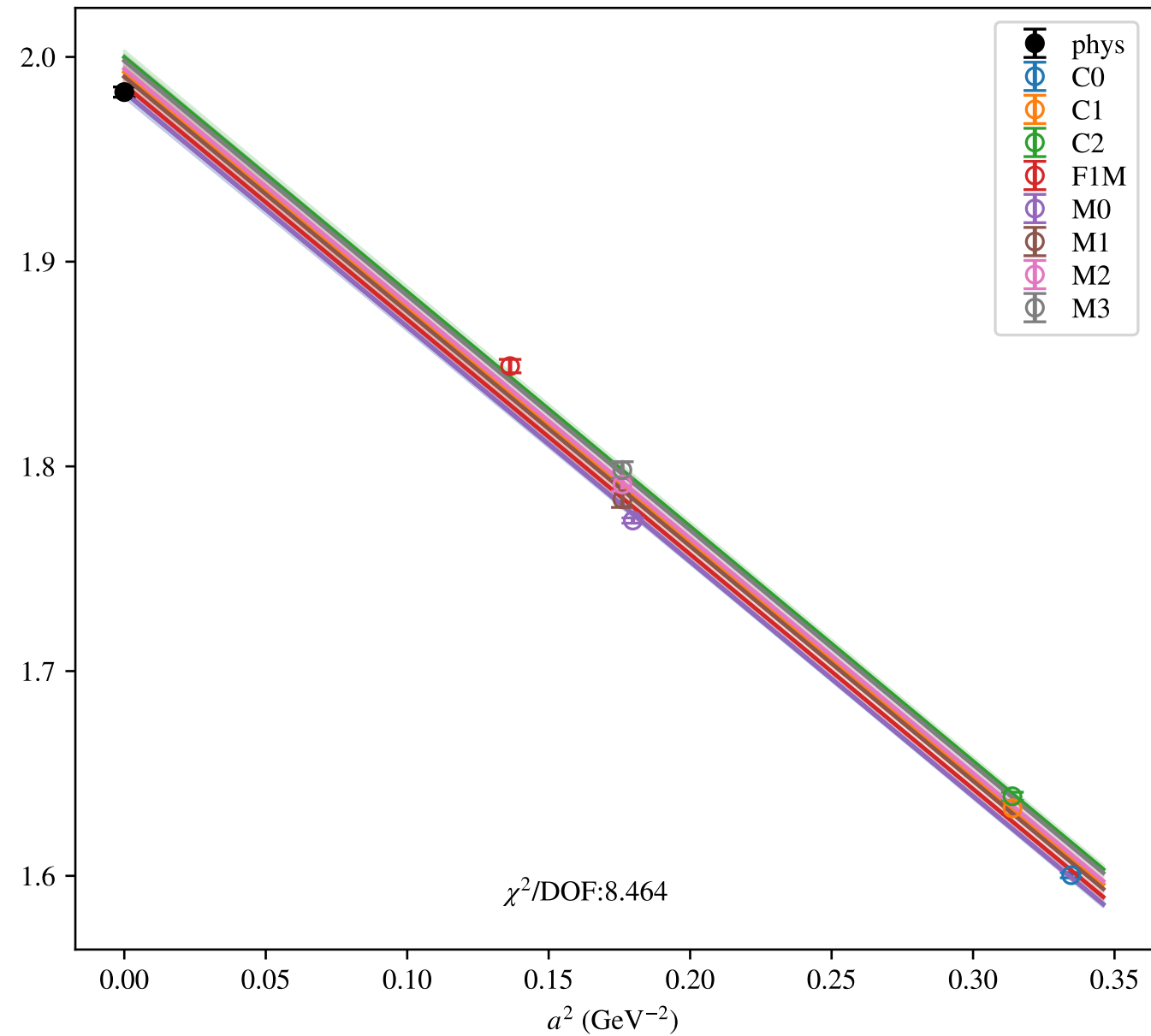
Table 3: Physical point value from chiral and continuum extrapolation at renormalisation scale  $\mu$ . Entries are **value(error)**:  $\chi^2/\text{DOF}$  ( $p$ -value).

$\mu$ (GeV)		$a^2, m_\pi^2$	$a^2, m_\pi^2$ (no C)	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$	$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2)$
2.4	$\alpha$	-0.578(37)	-0.79(32)	-1.12(78)	-0.576(39)	-0.574(38)
	$\beta$	0.00087(10)	0.00090(18)	0.001053(99)	0.00198(55)	-0.0038(10)
2.0	$\alpha$	-0.681(36)	-0.88(31)	-1.27(76)	-0.679(36)	-0.679(36)
	$\beta$	0.00112(12)	0.00147(23)	0.00129(12)	0.00267(59)	-0.0036(12)
1.8	$\alpha$	-0.703(37)	-0.89(33)	-1.35(80)	-0.701(37)	-0.701(38)
	$\beta$	0.00087(13)	0.00166(24)	0.00109(13)	0.00199(61)	-0.0039(13)
1.5	$\alpha$	-0.733(40)	-0.89(38)	-1.47(91)	-0.732(38)	-0.730(40)
	$\beta$	0.00056(14)	0.00188(26)	0.00086(14)	0.00115(67)	-0.0042(14)

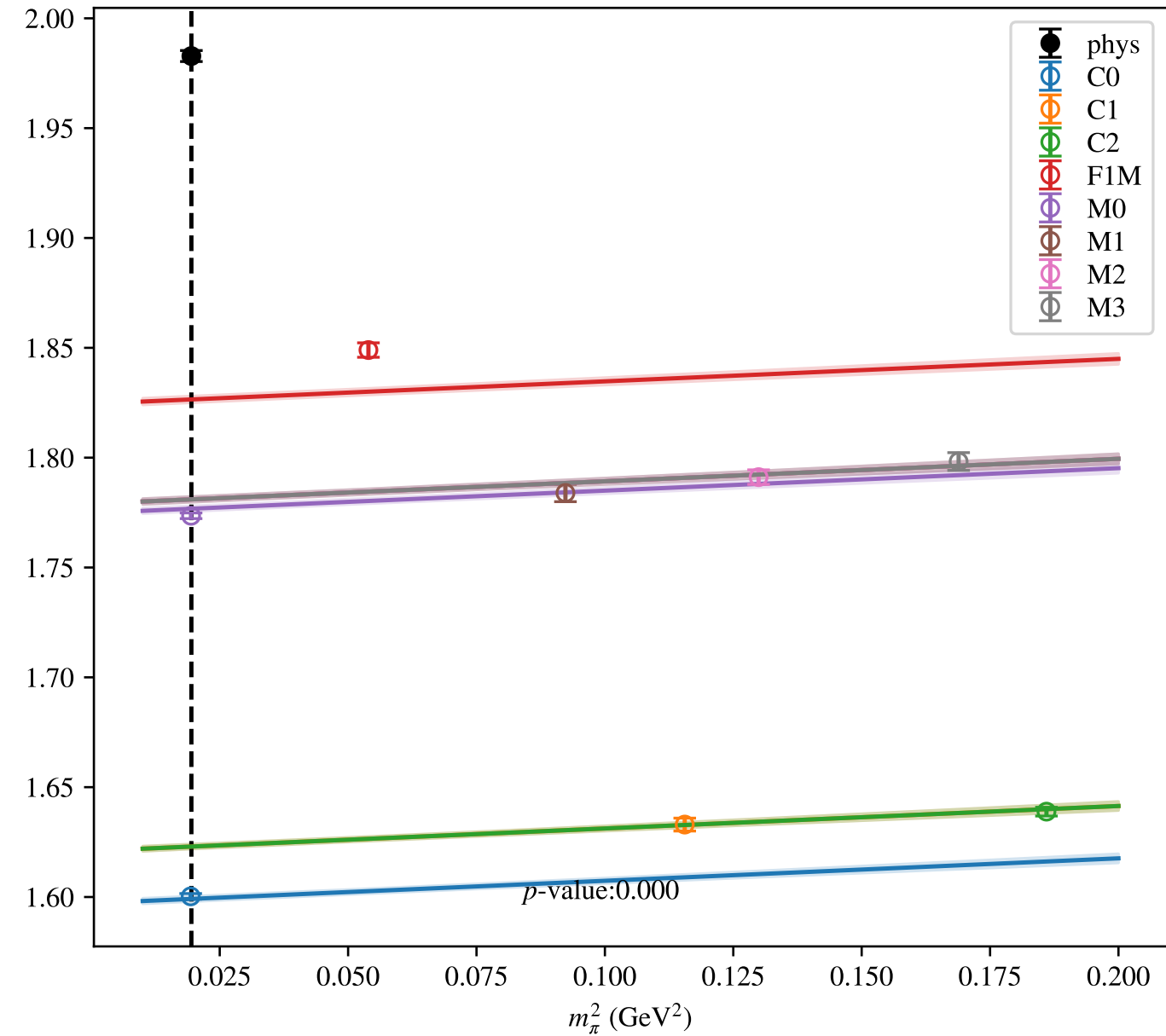
Table 4: Fit values of coefficients in  $B = B_0(1 + \alpha a^2 + \beta \frac{m_\pi^2}{f_\pi^2} + \dots)$ .

$a^2, m_\pi^2, \mu = 2.4 \text{ GeV}$

VVmAA



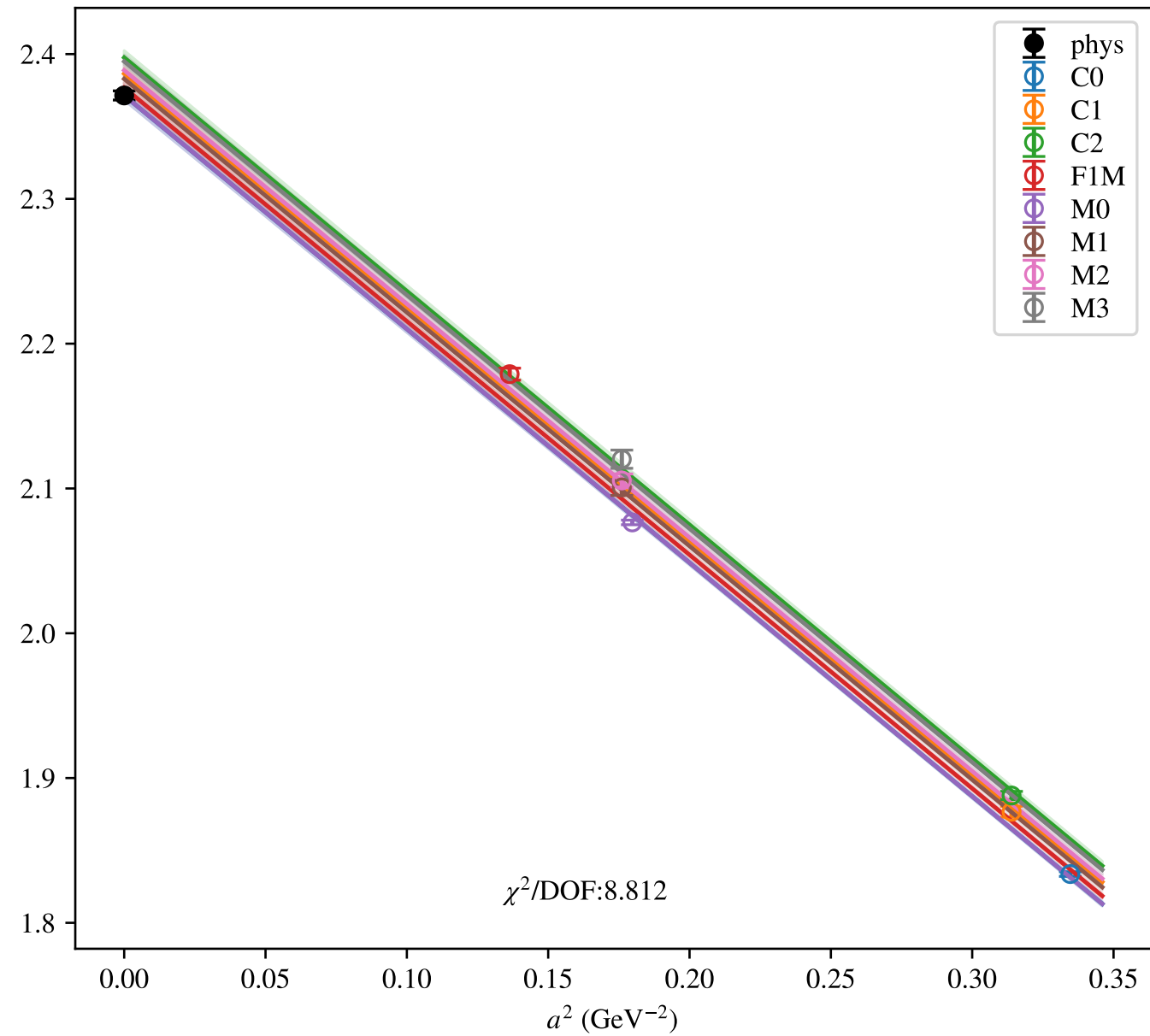
VVmAA



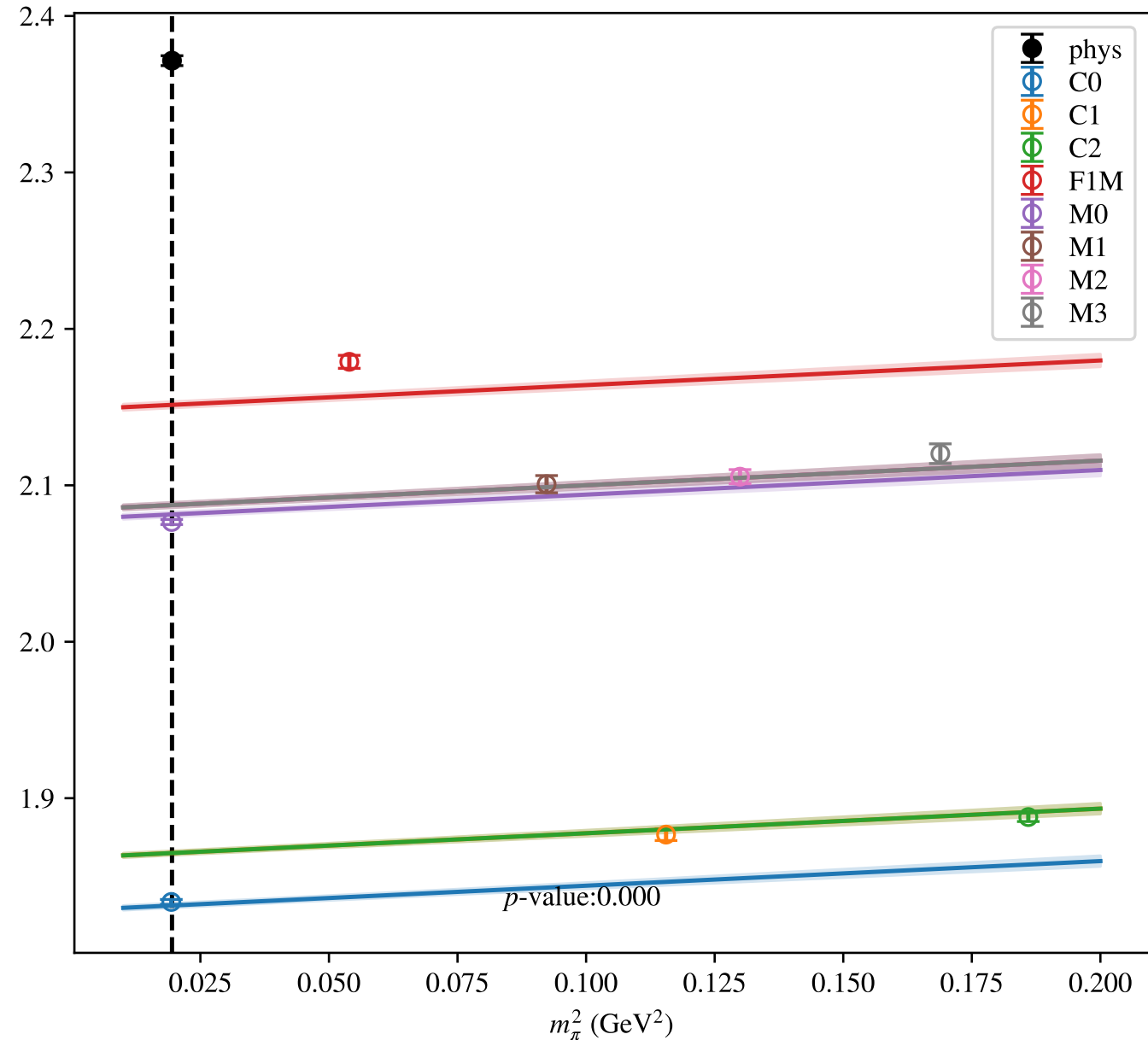


$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$

VVmAA

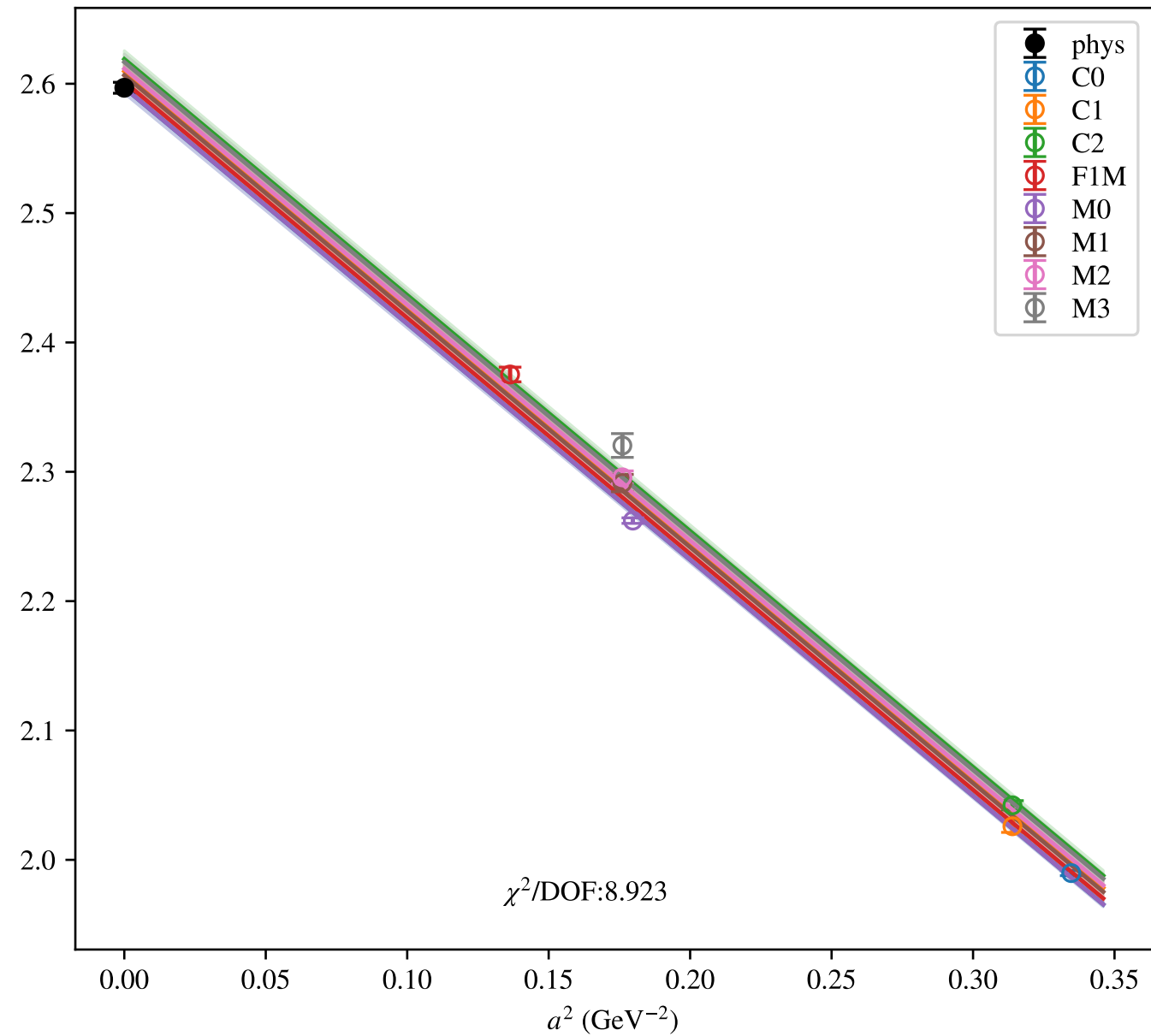


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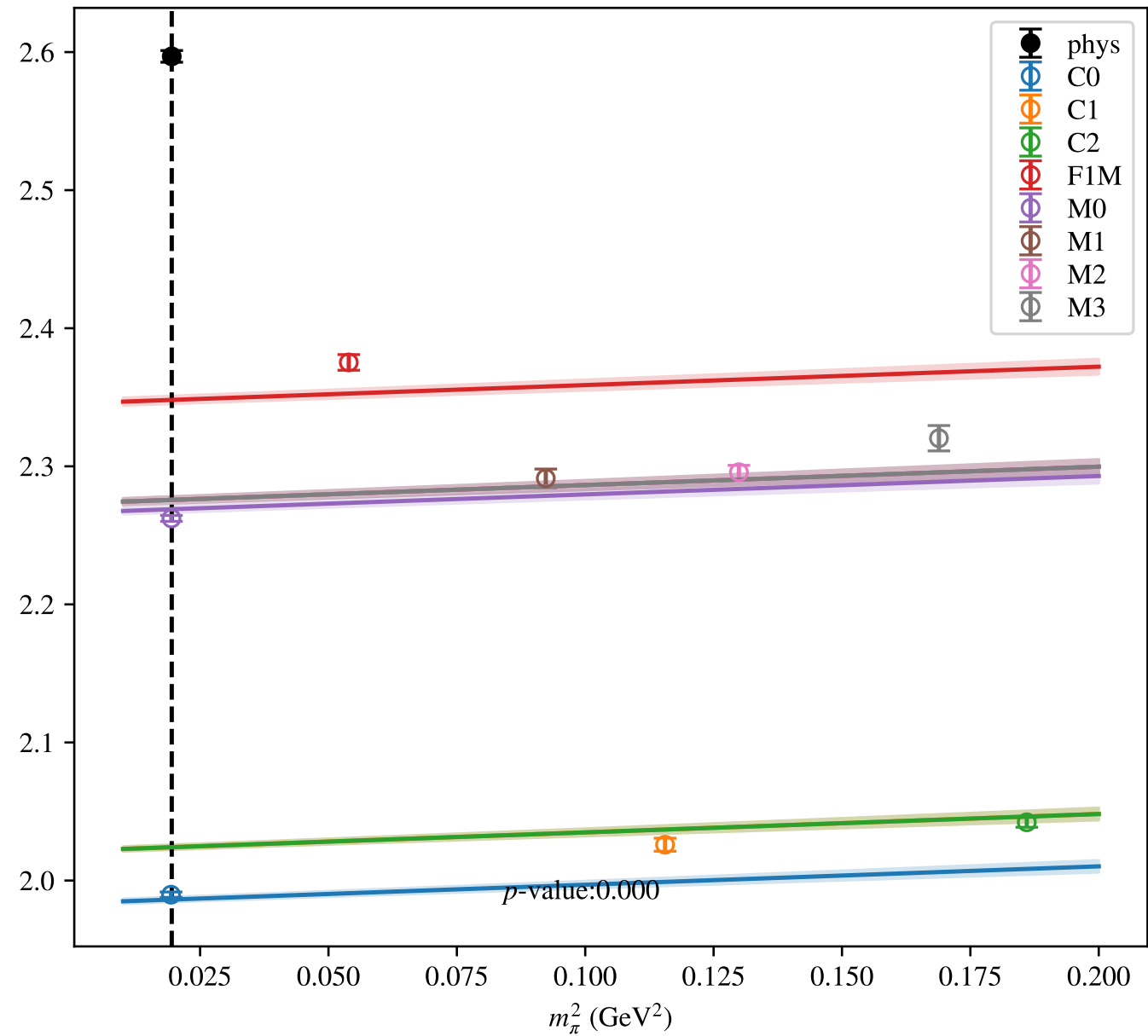


$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$

VVmAA

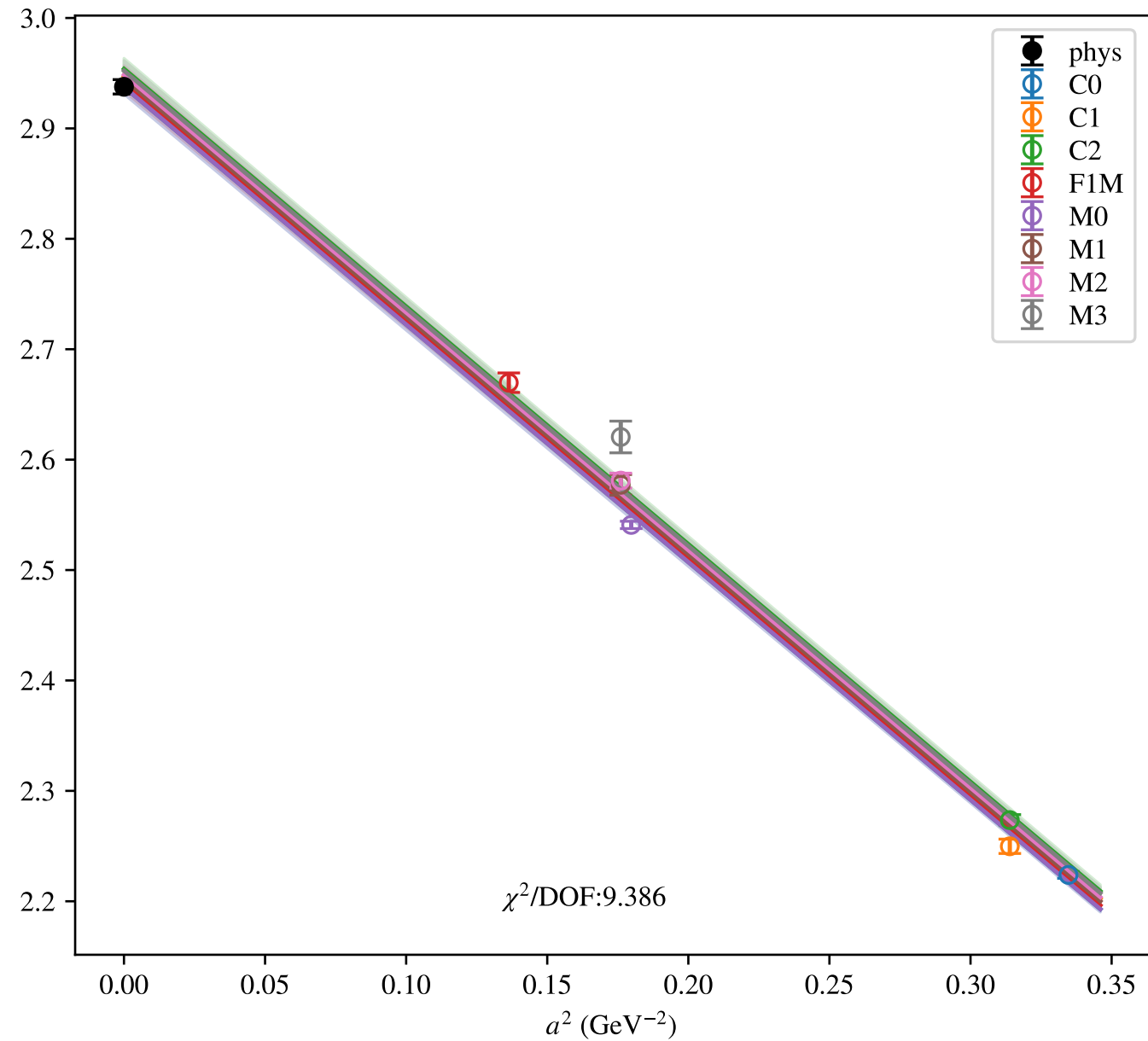


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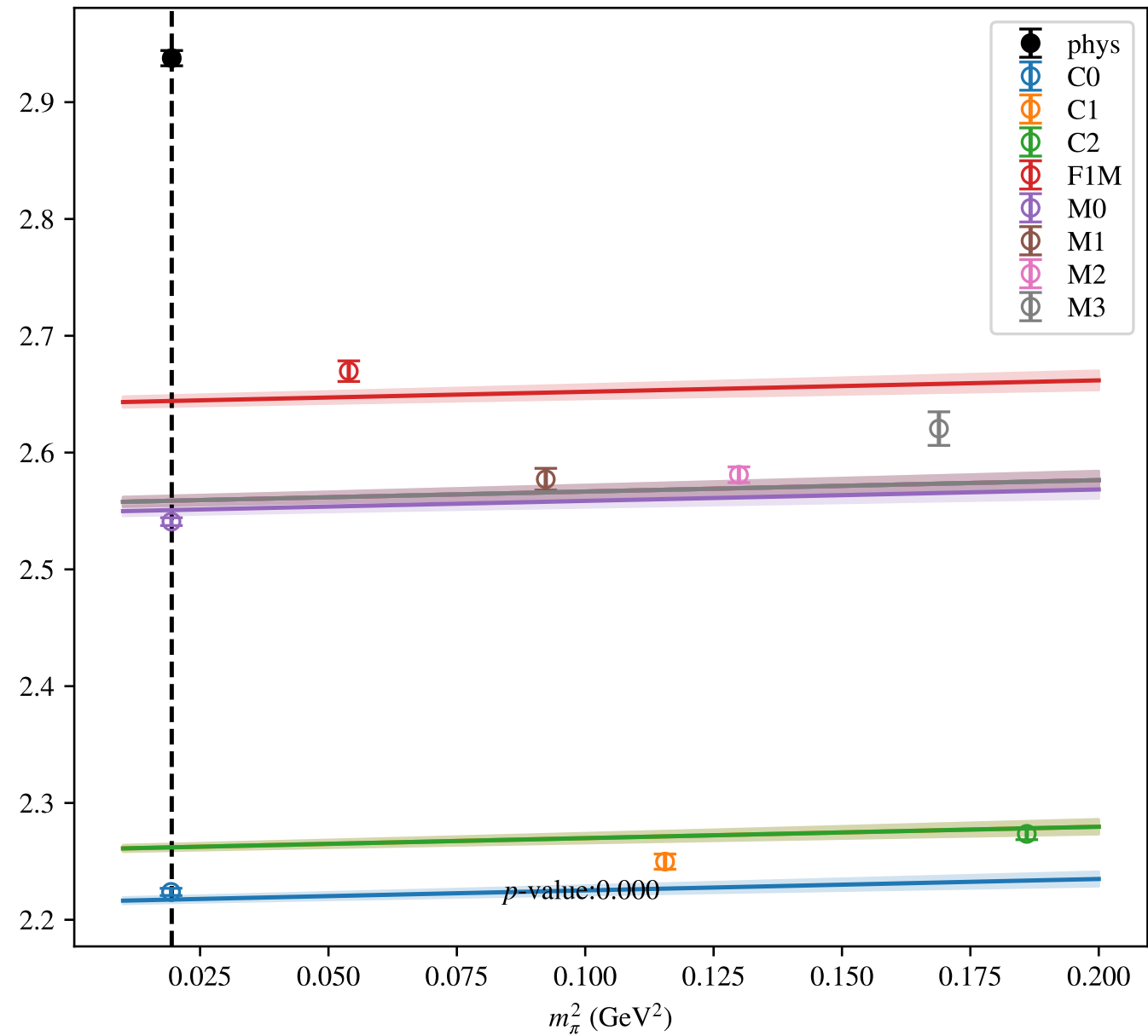


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

VVmAA

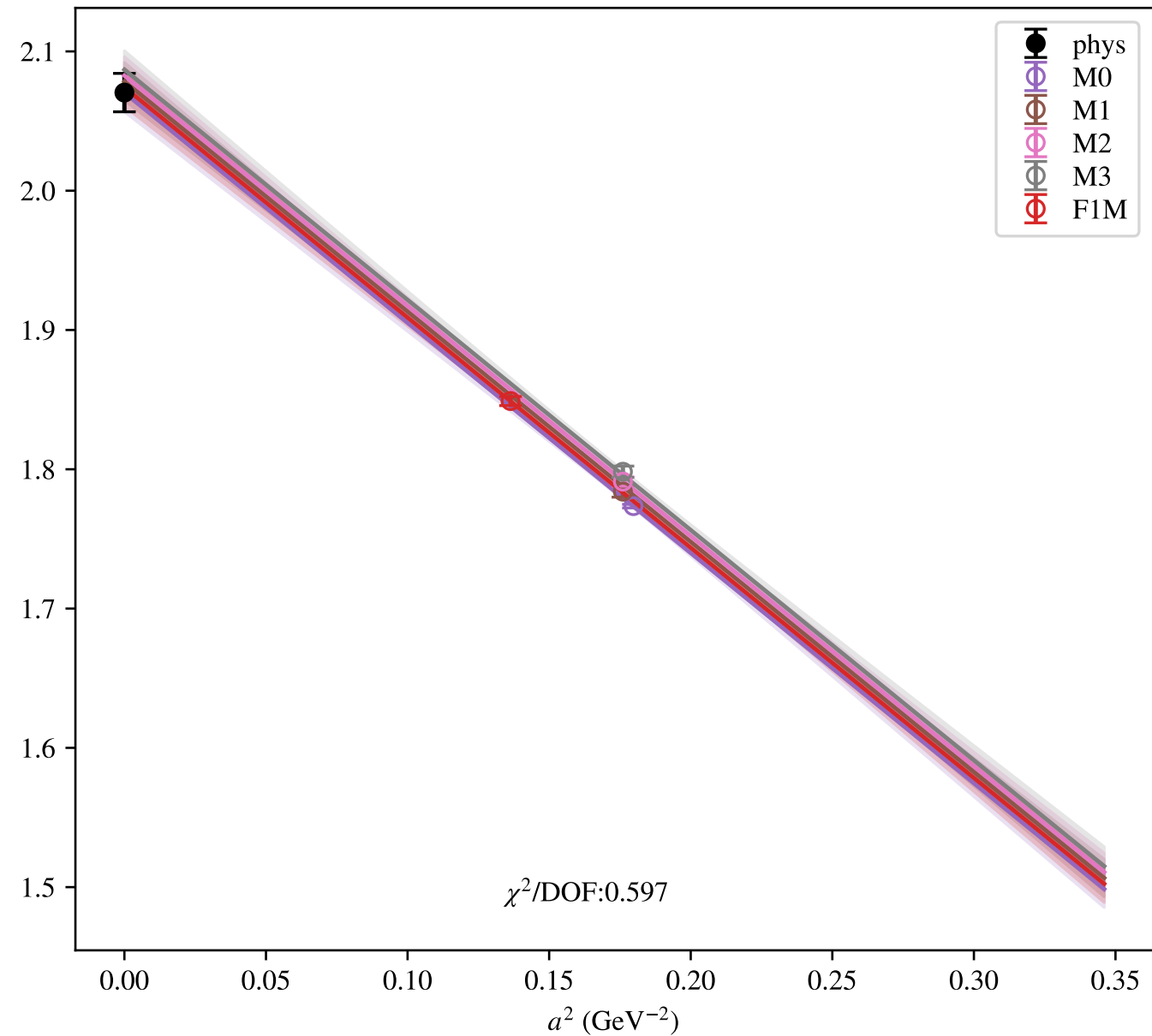


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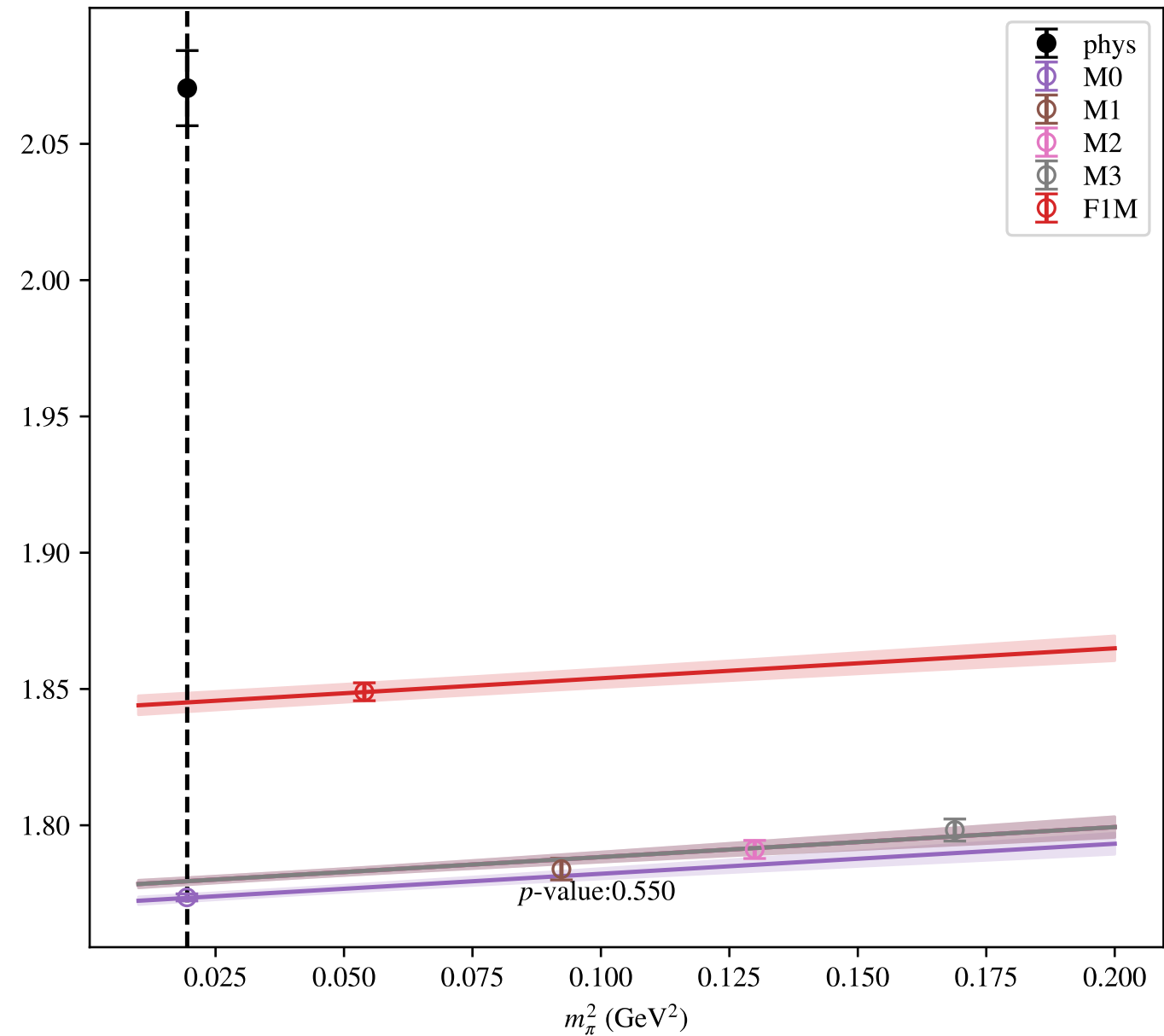


$a^2, m_\pi^2$  (no C),  $\mu = 2.4$  GeV

VVmAA

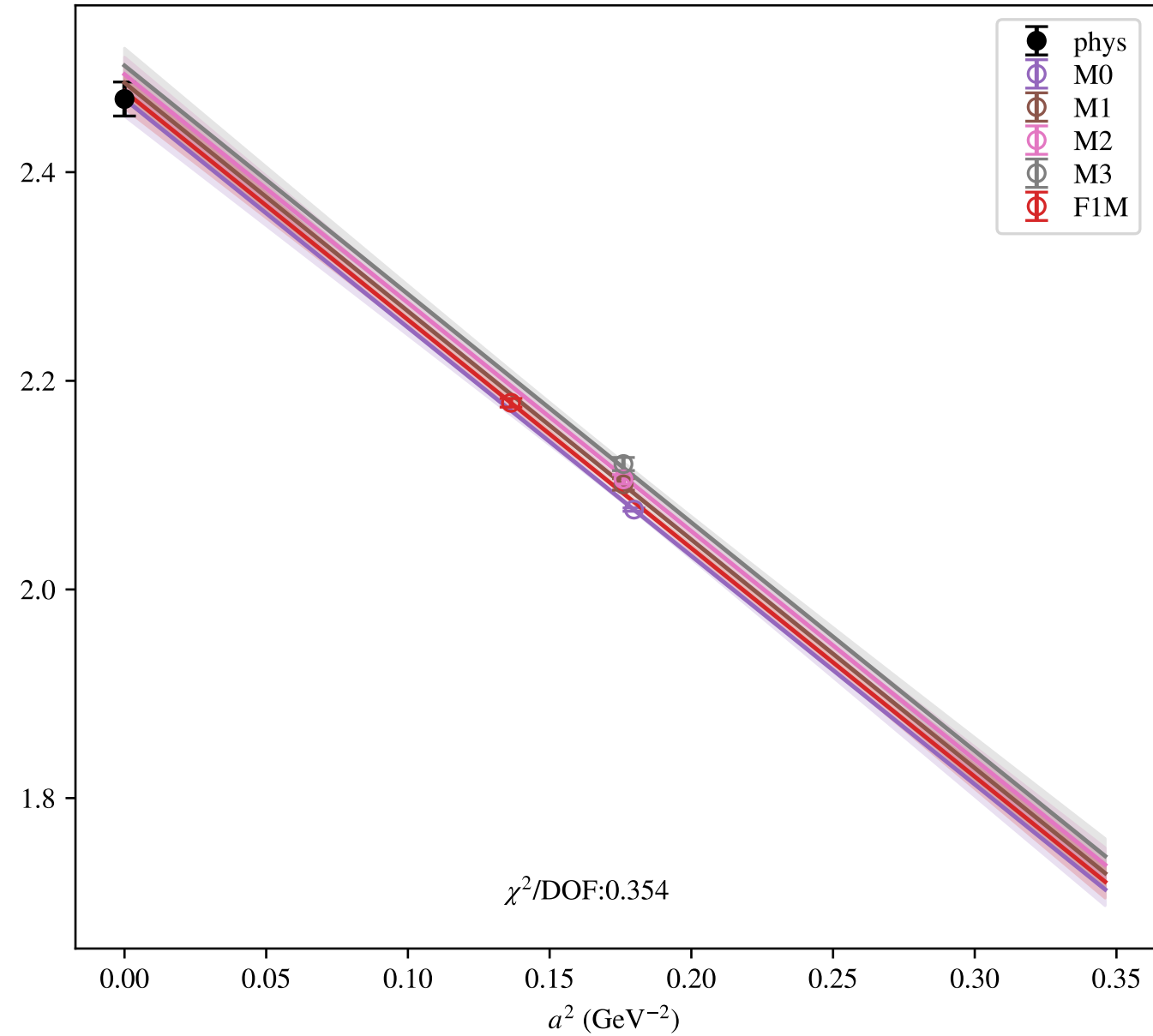


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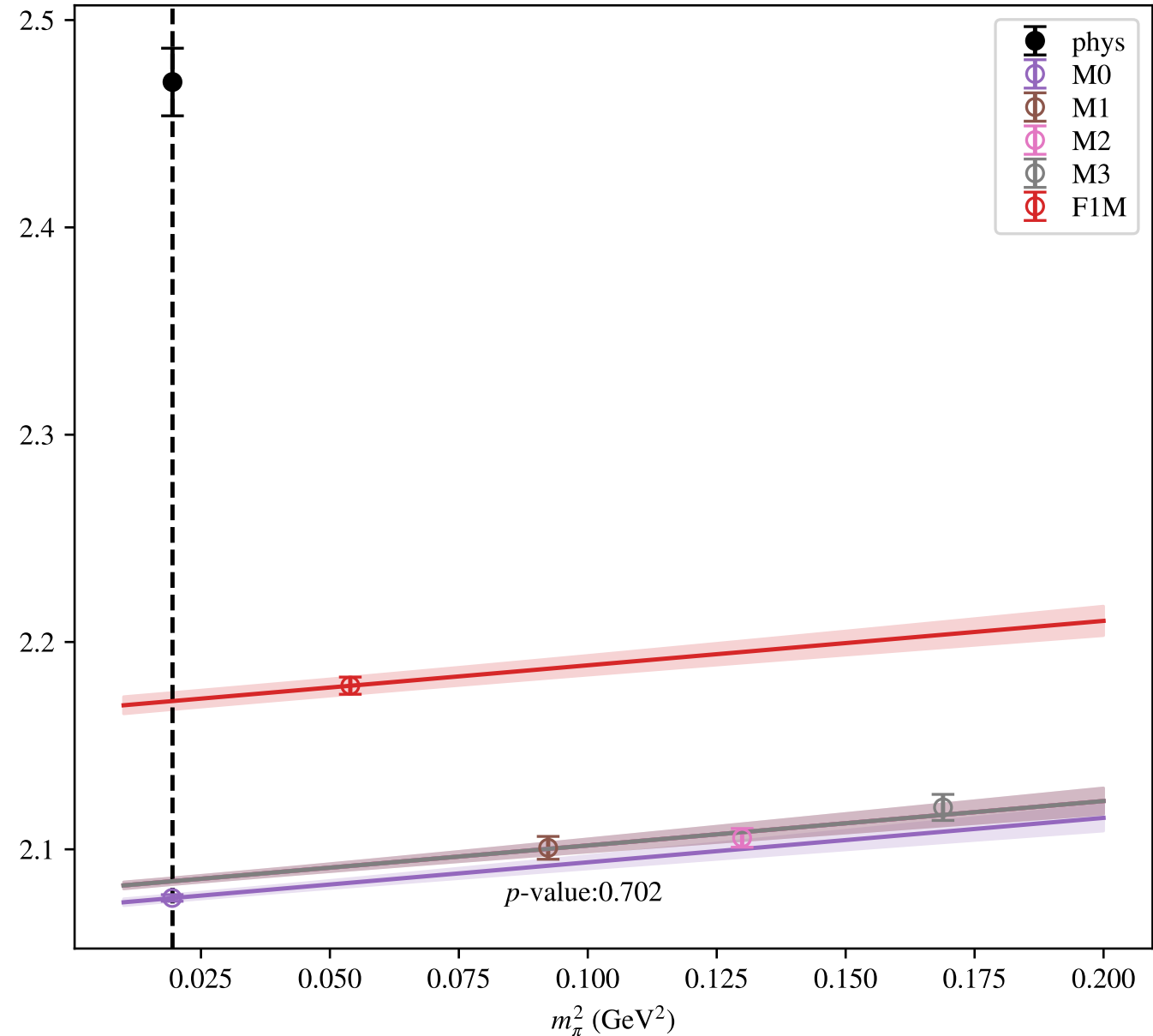


$a^2, m_\pi^2$  (no C),  $\mu = 2.0$  GeV

VVmAA

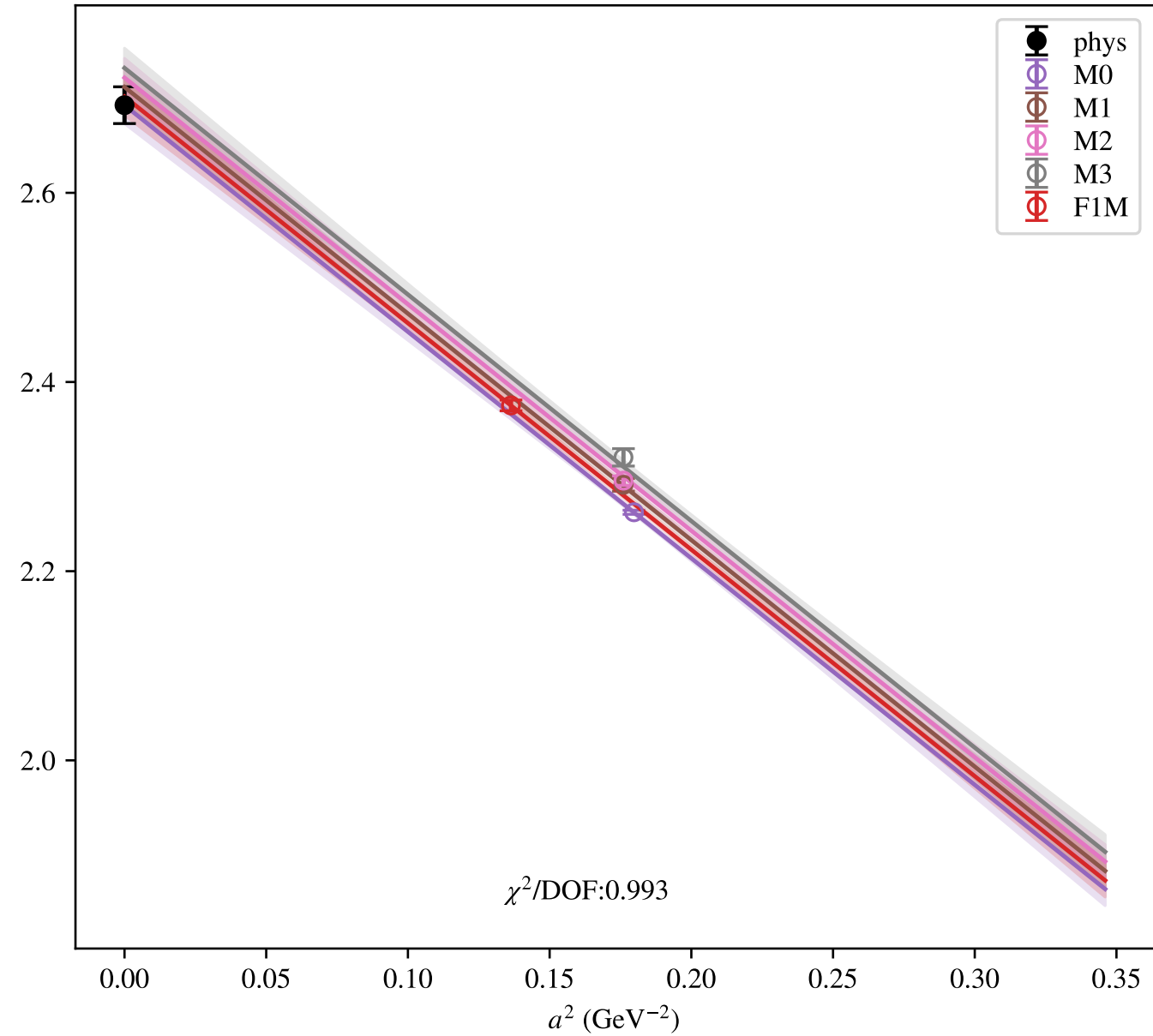


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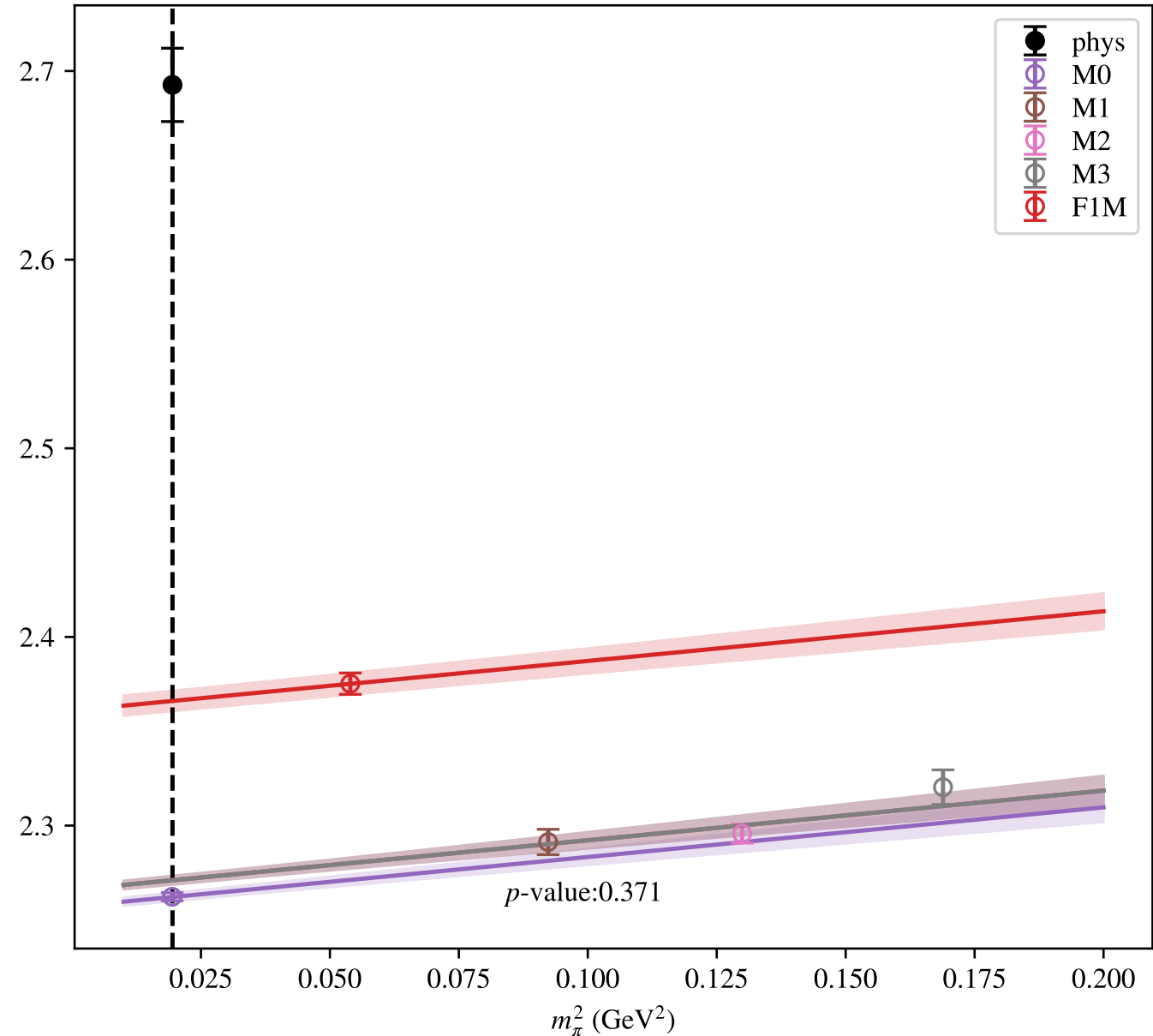


$a^2, m_\pi^2$  (no C),  $\mu = 1.8$  GeV

VVmAA

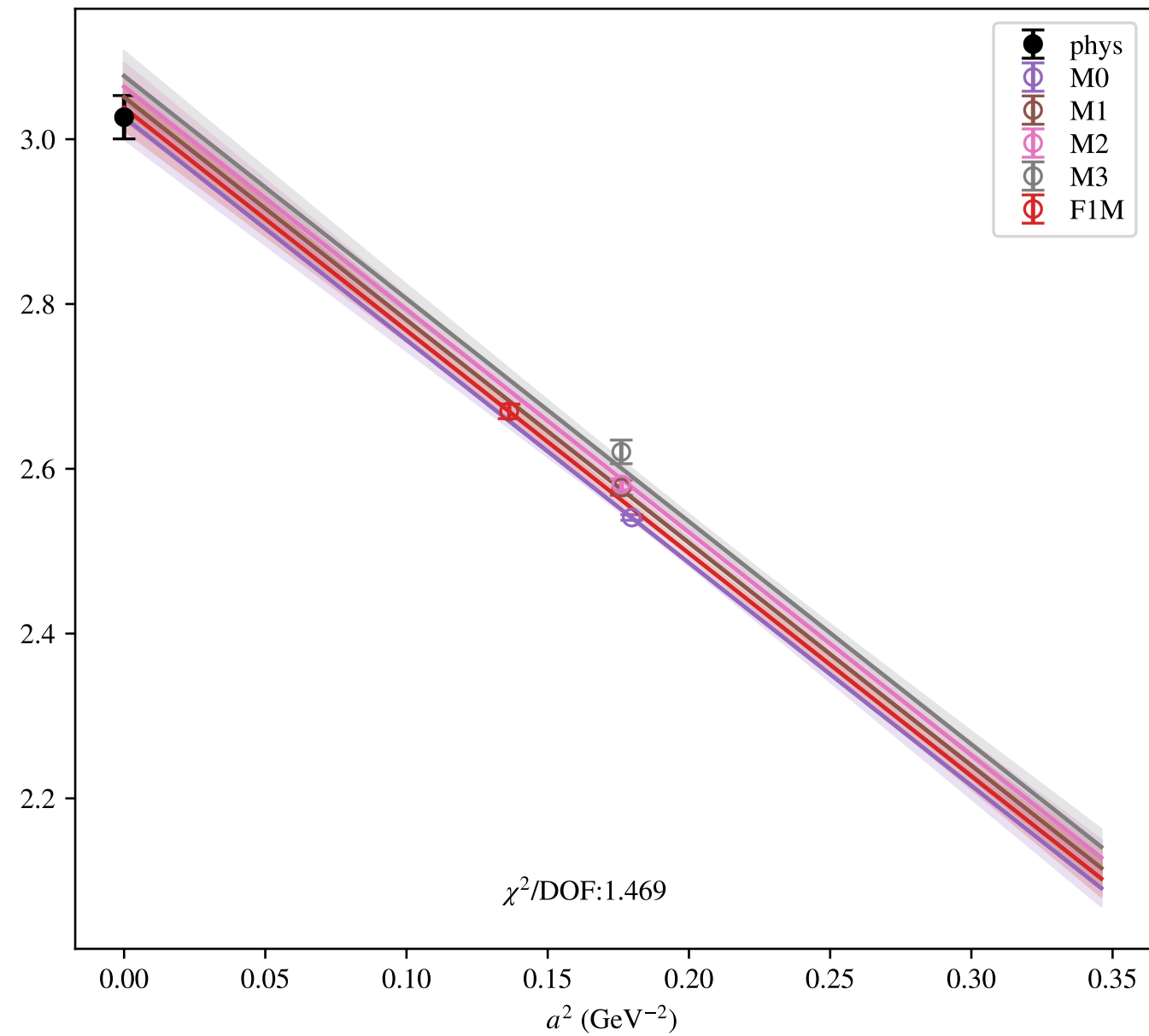


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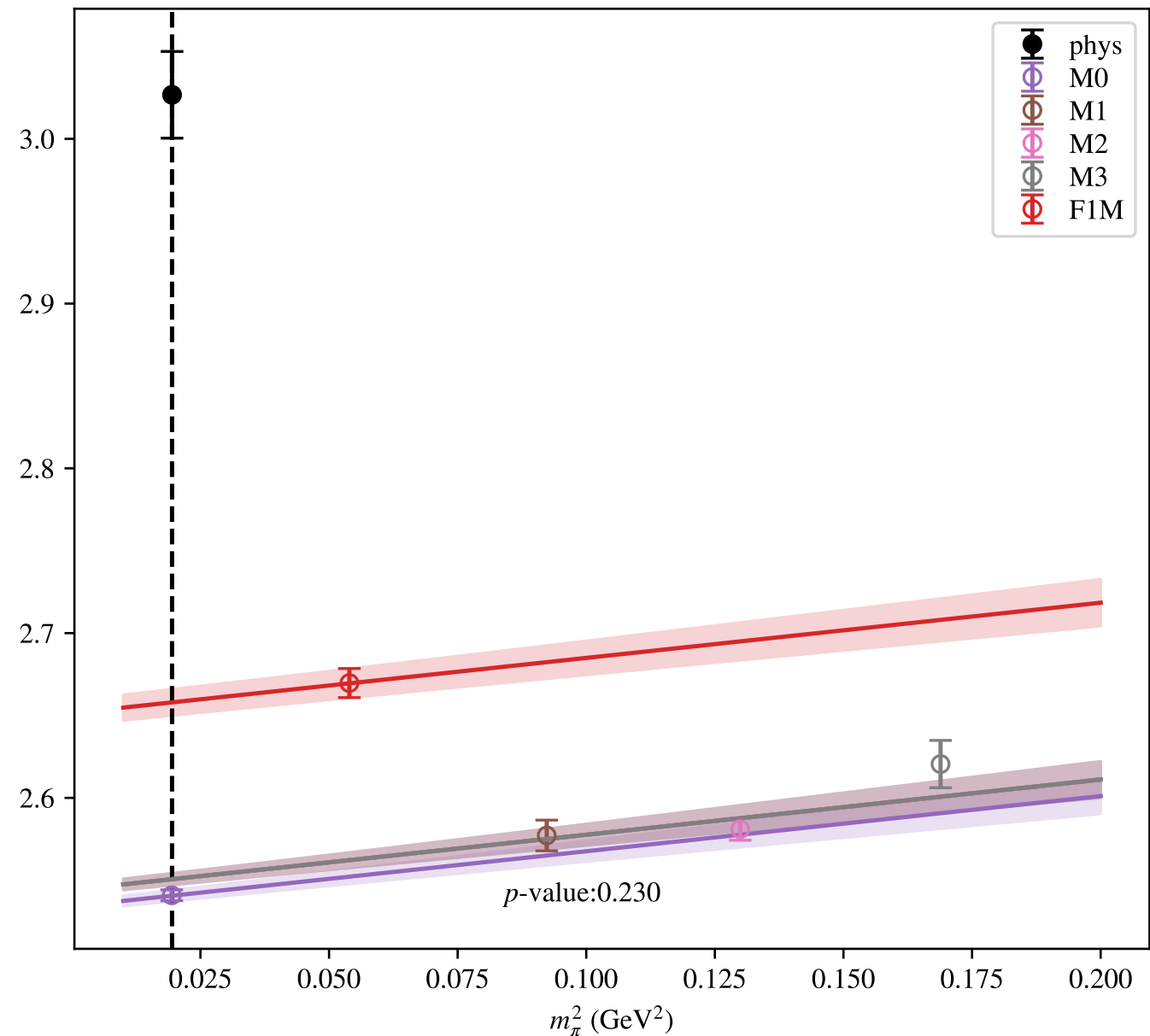


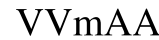
$a^2, m_\pi^2$  (no C),  $\mu = 1.5$  GeV

VVmAA



VVmAA

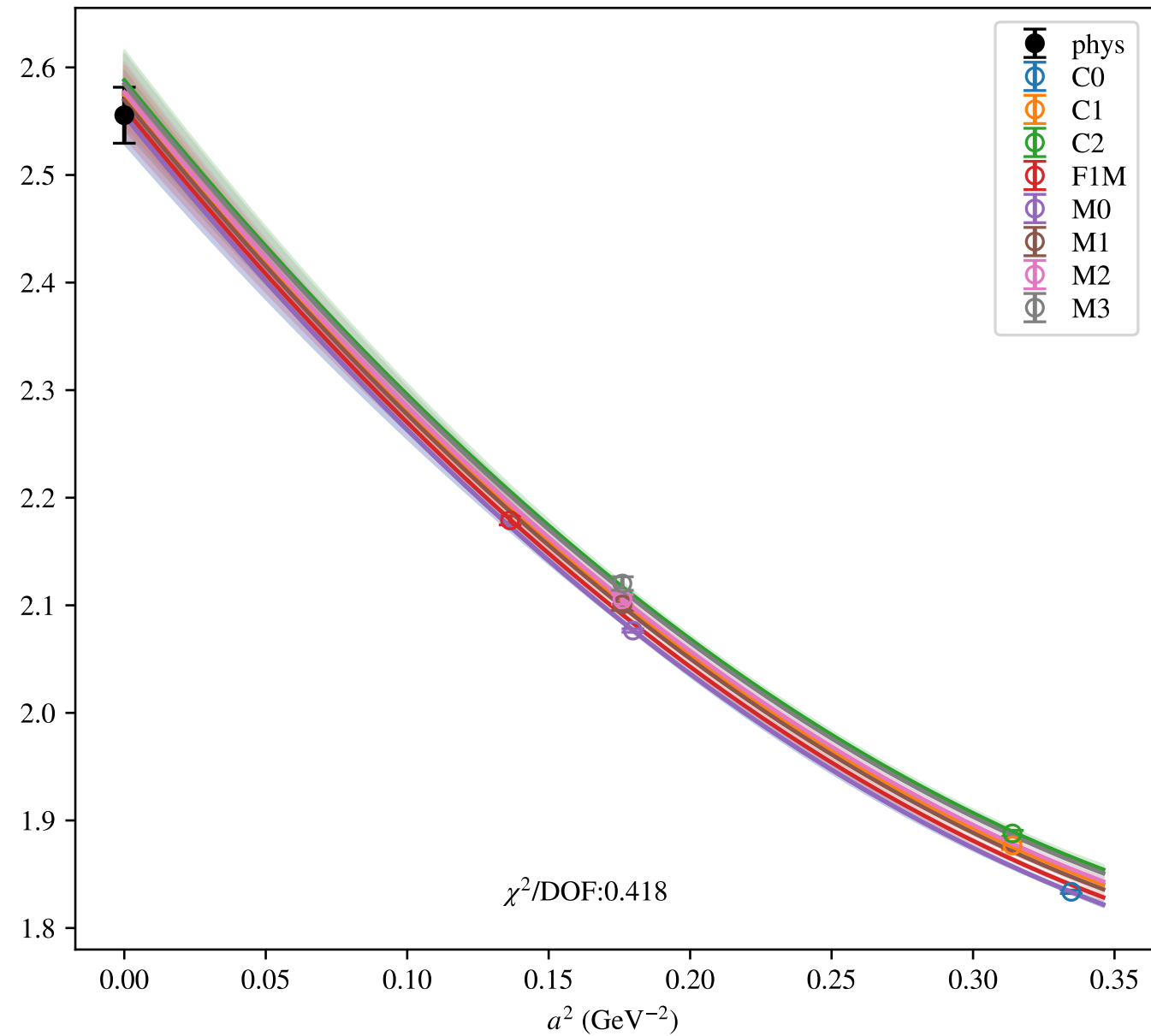


$a^2, a^4, m_\pi^2, \mu = 2.4 \text{ GeV}$ 

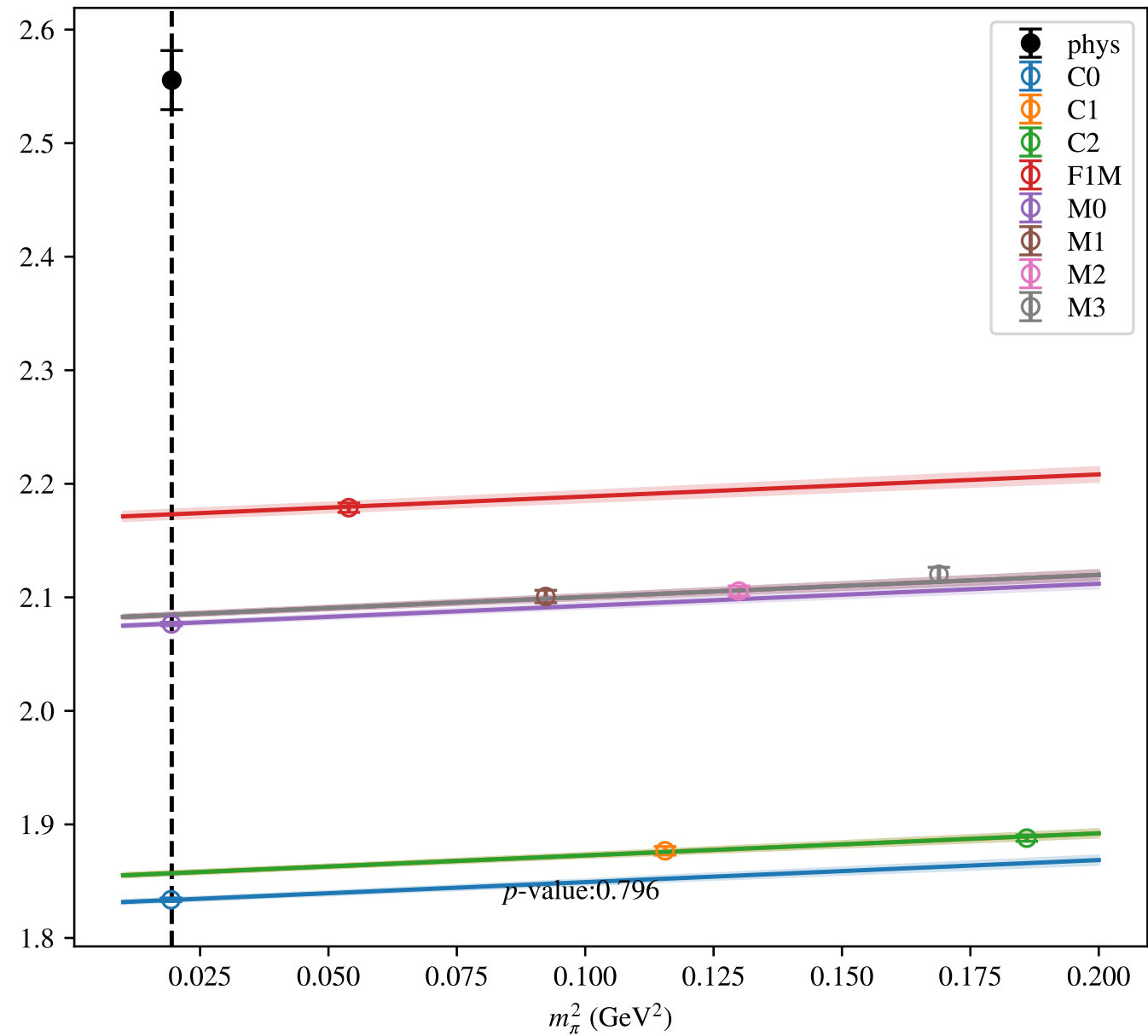


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

VVmAA

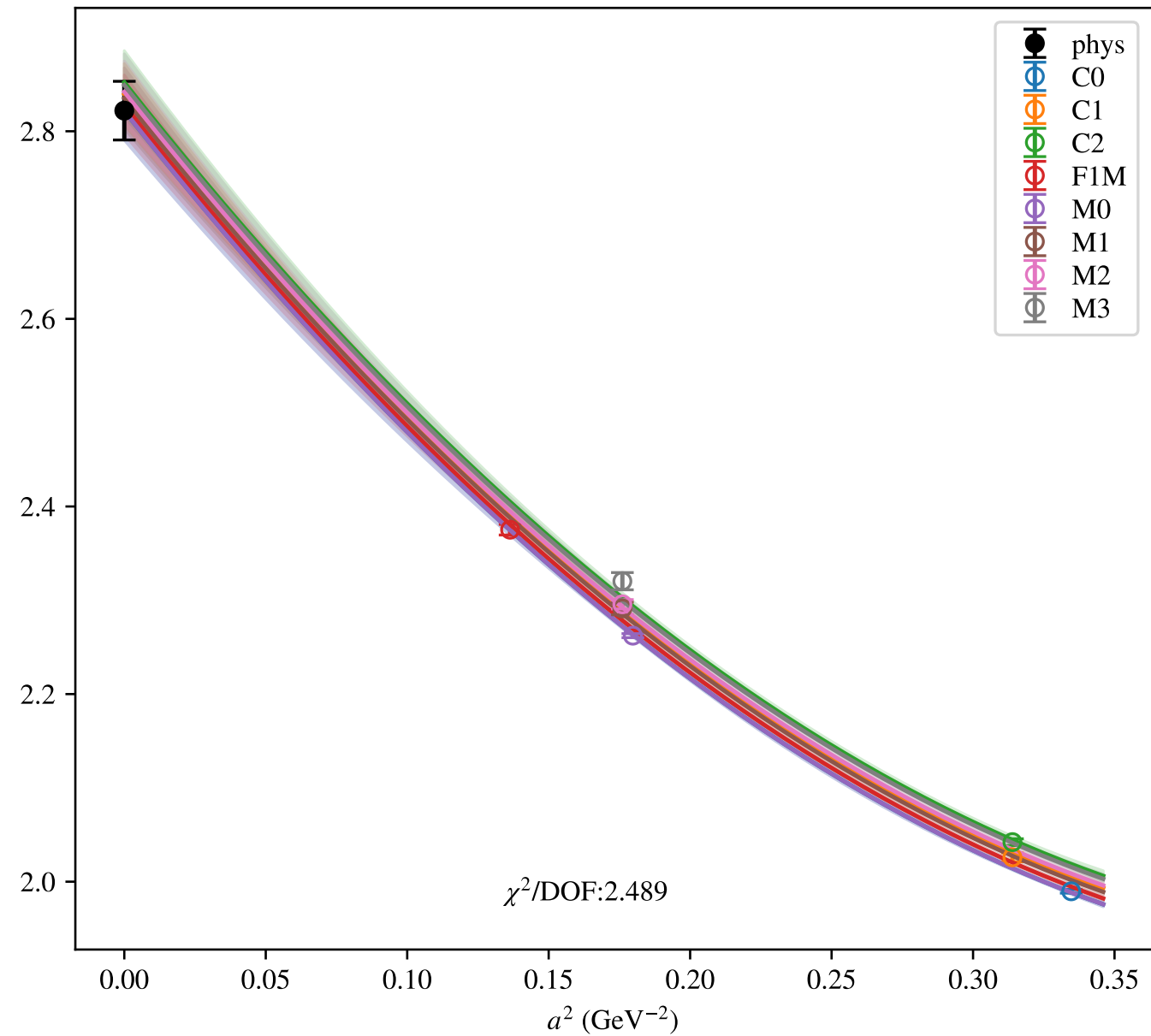


VVmAA

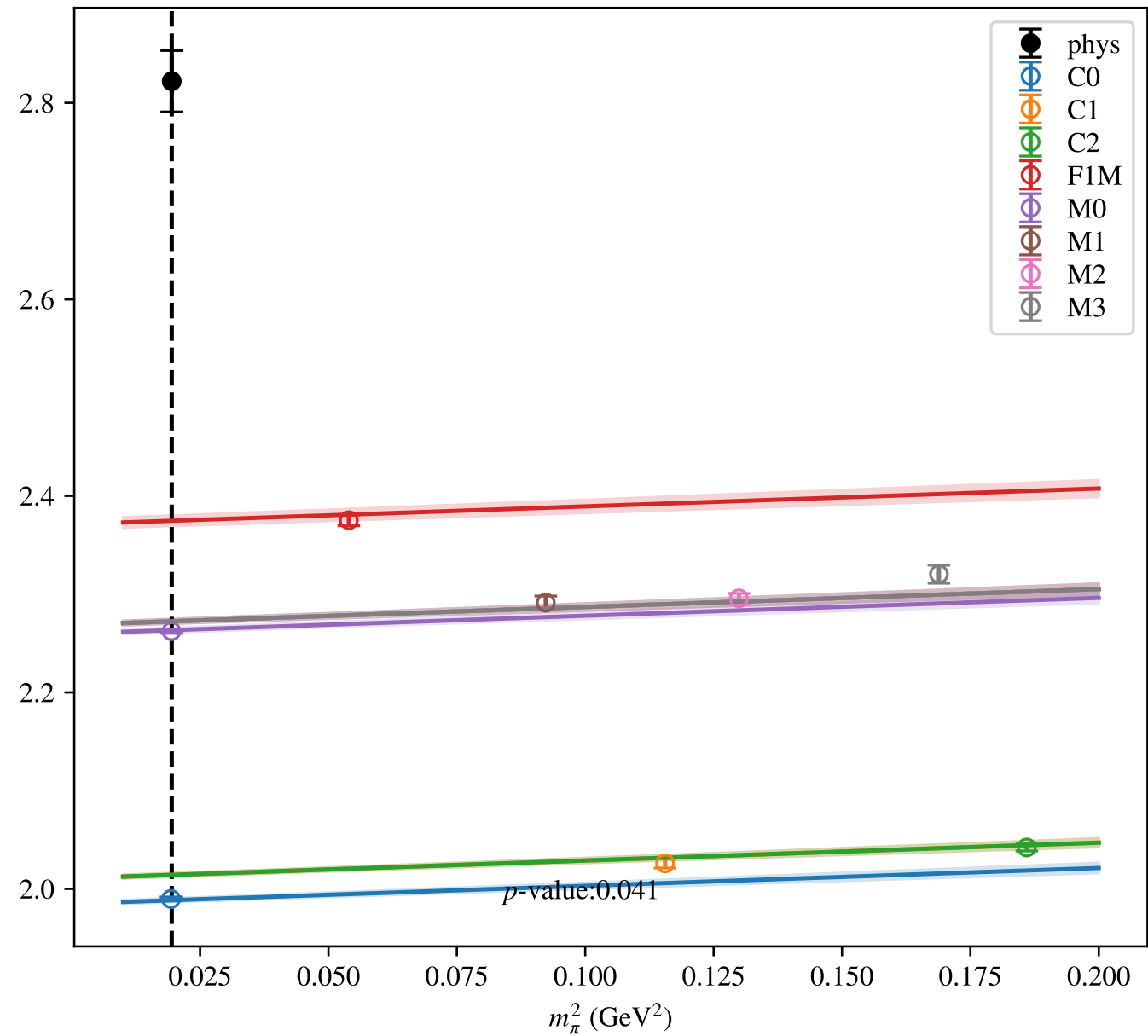


$$a^2, a^4, m_\pi^2, \mu = 1.8 \text{ GeV}$$

VVmAA

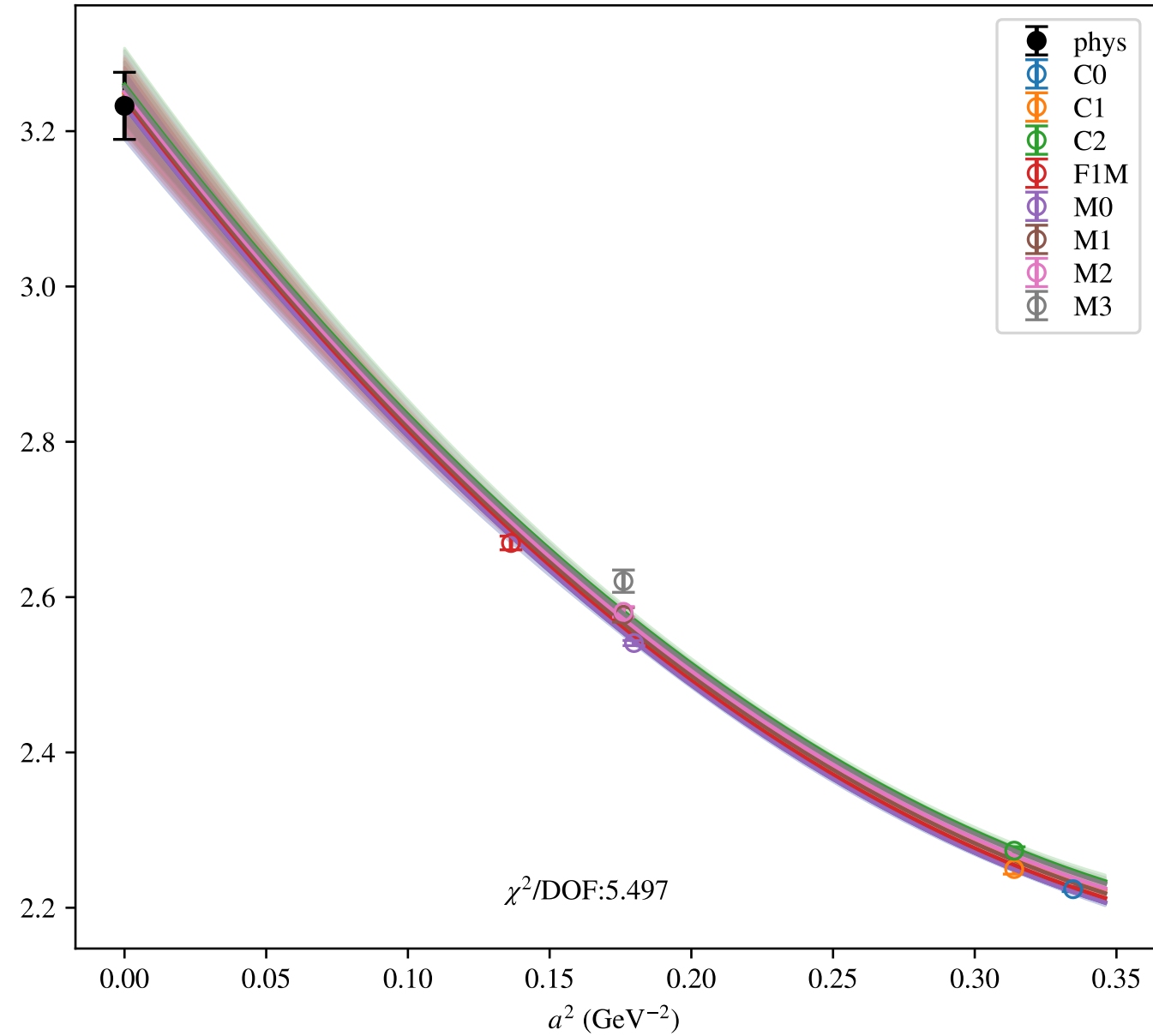


VVmAA

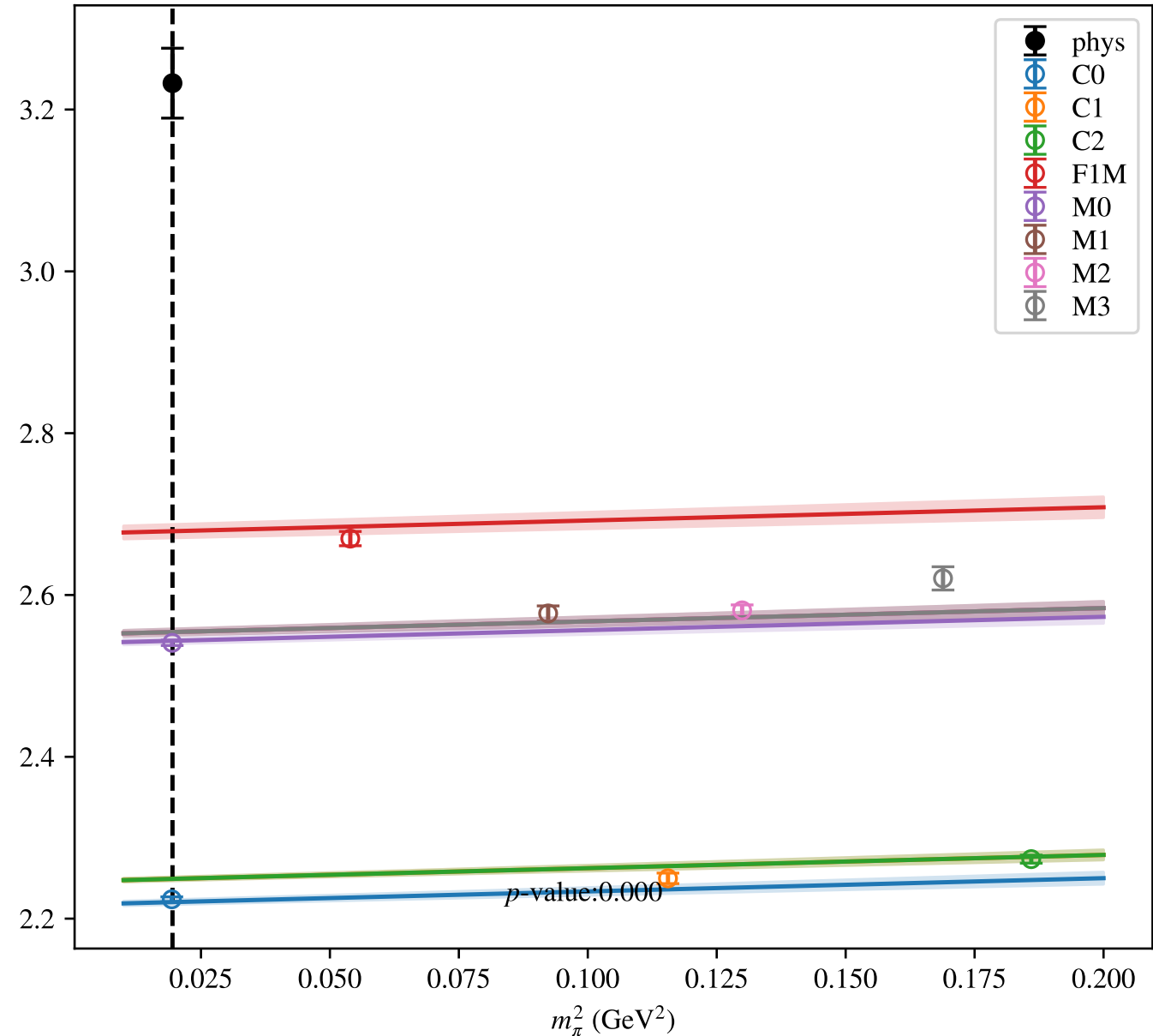


$$a^2, a^4, m_\pi^2, \mu = 1.5 \text{ GeV}$$

VVmAA

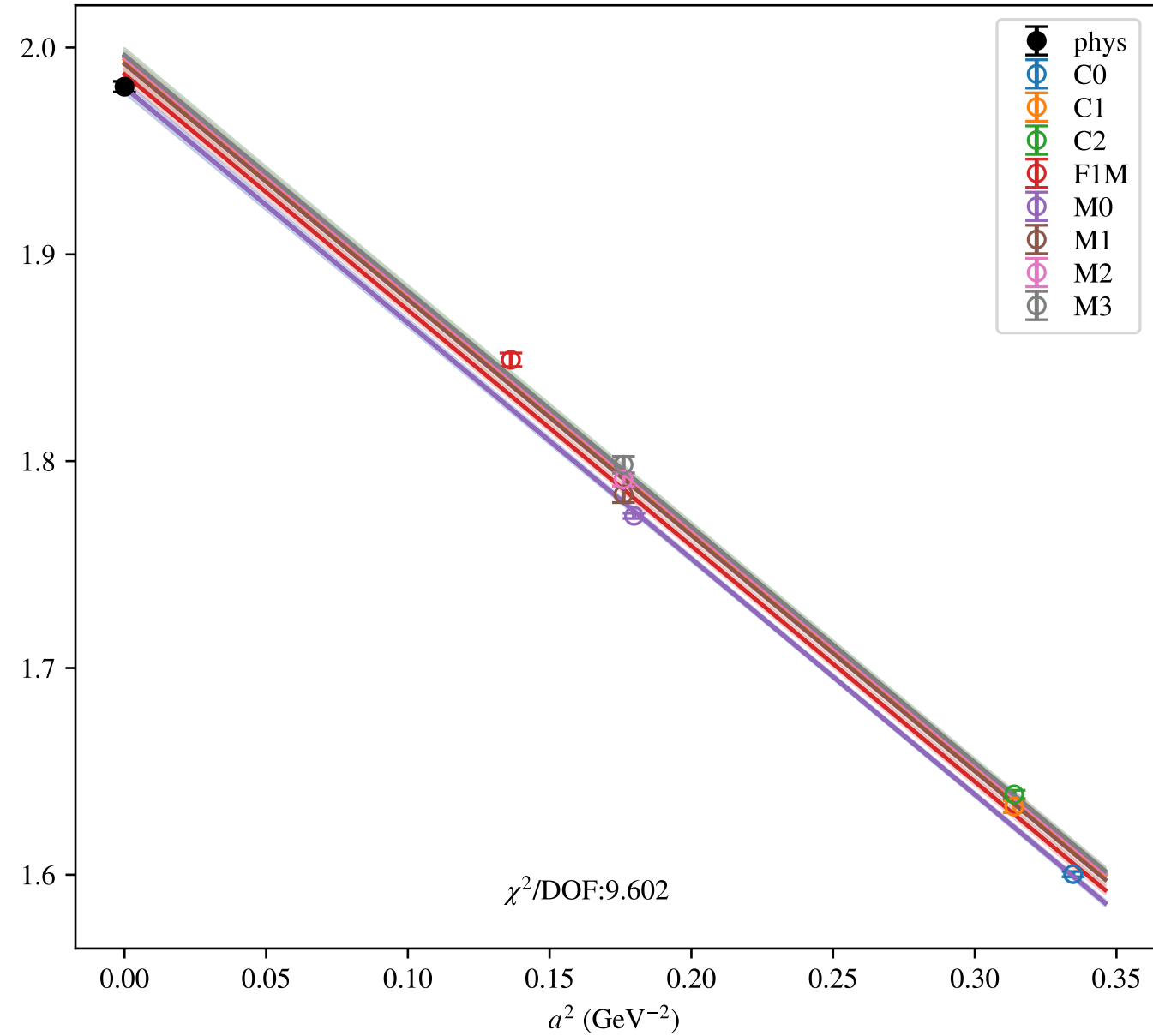


VVmAA

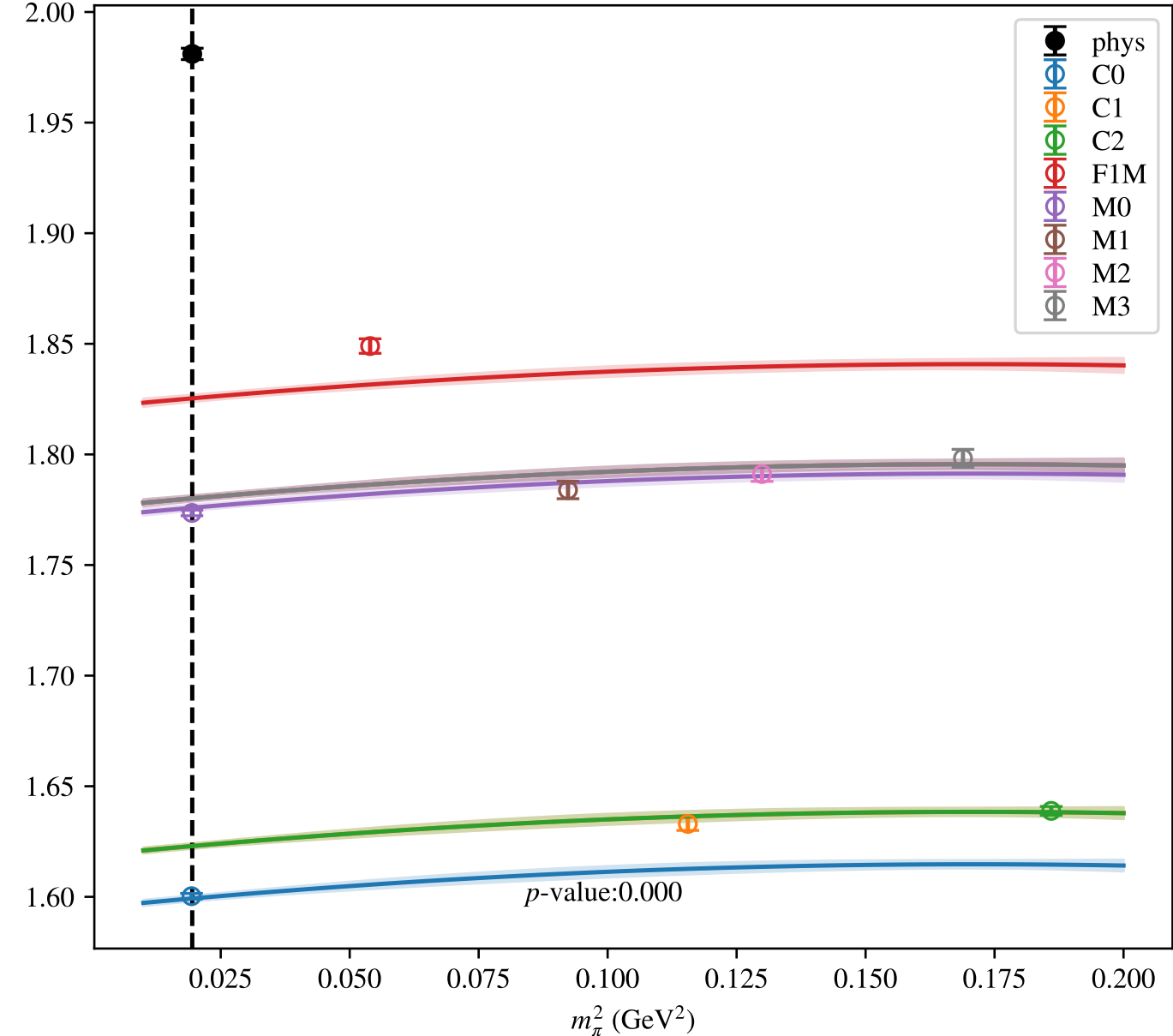


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.4 \text{ GeV}$$

VVmAA

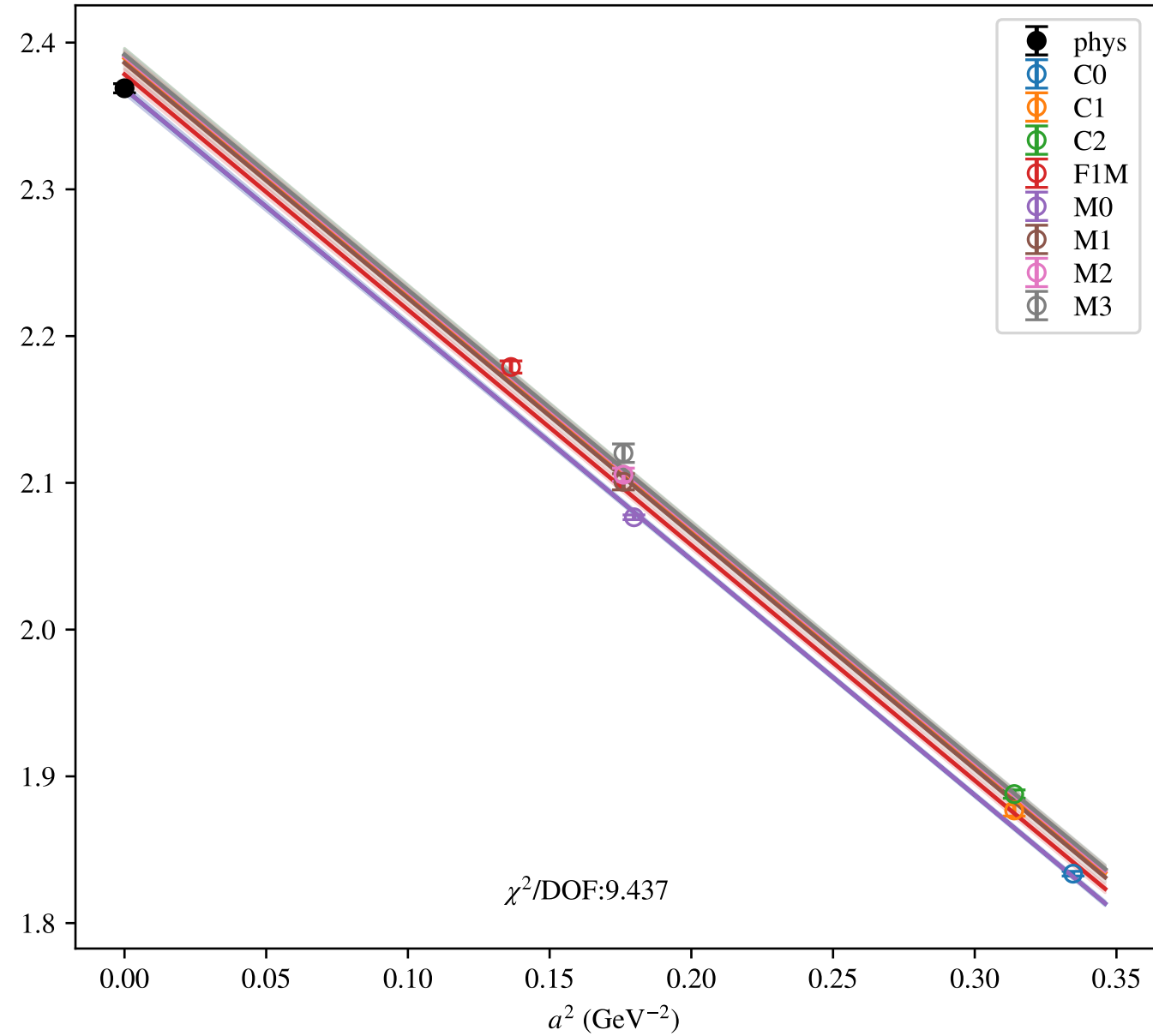


VVmAA

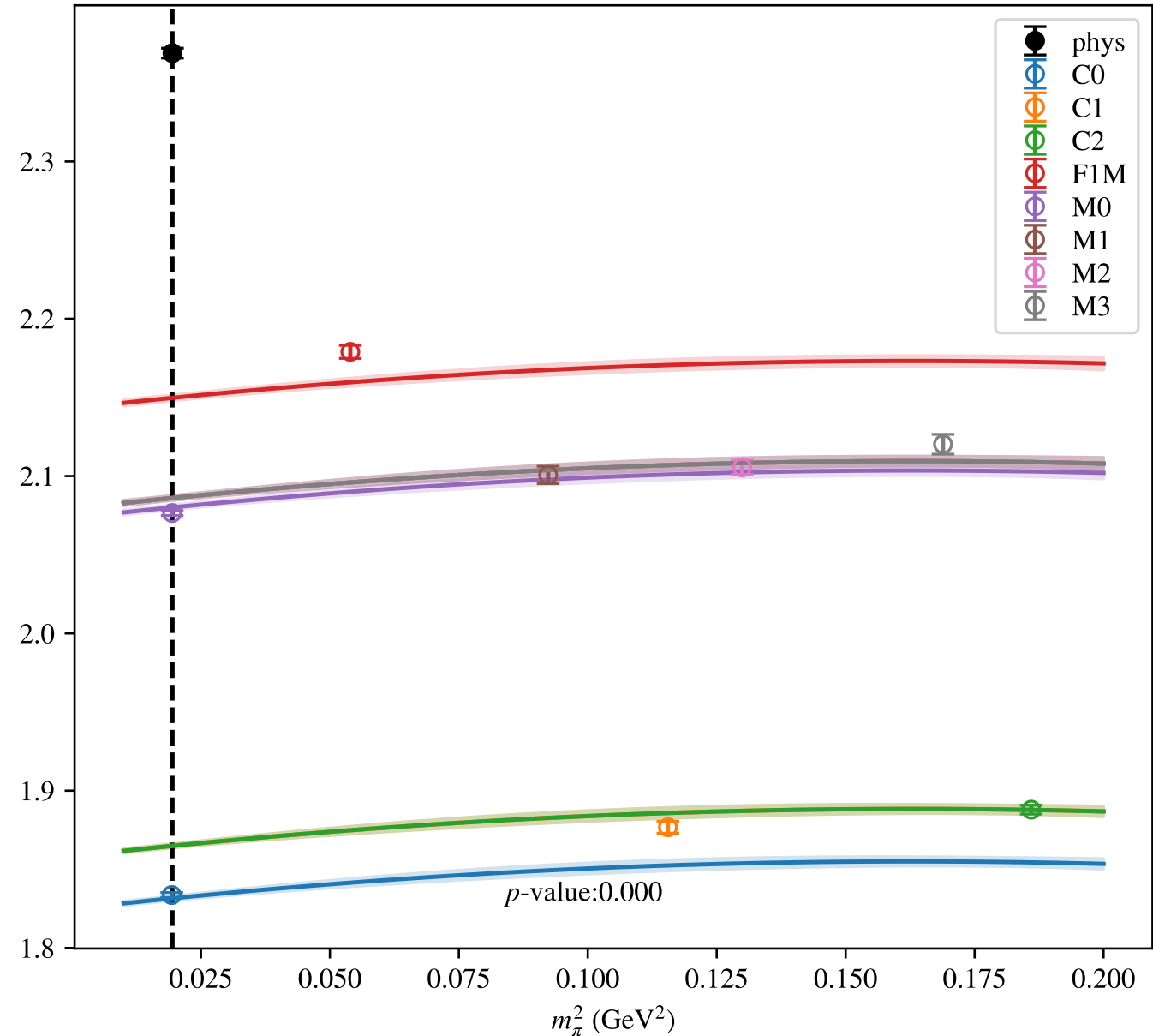


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

VVmAA

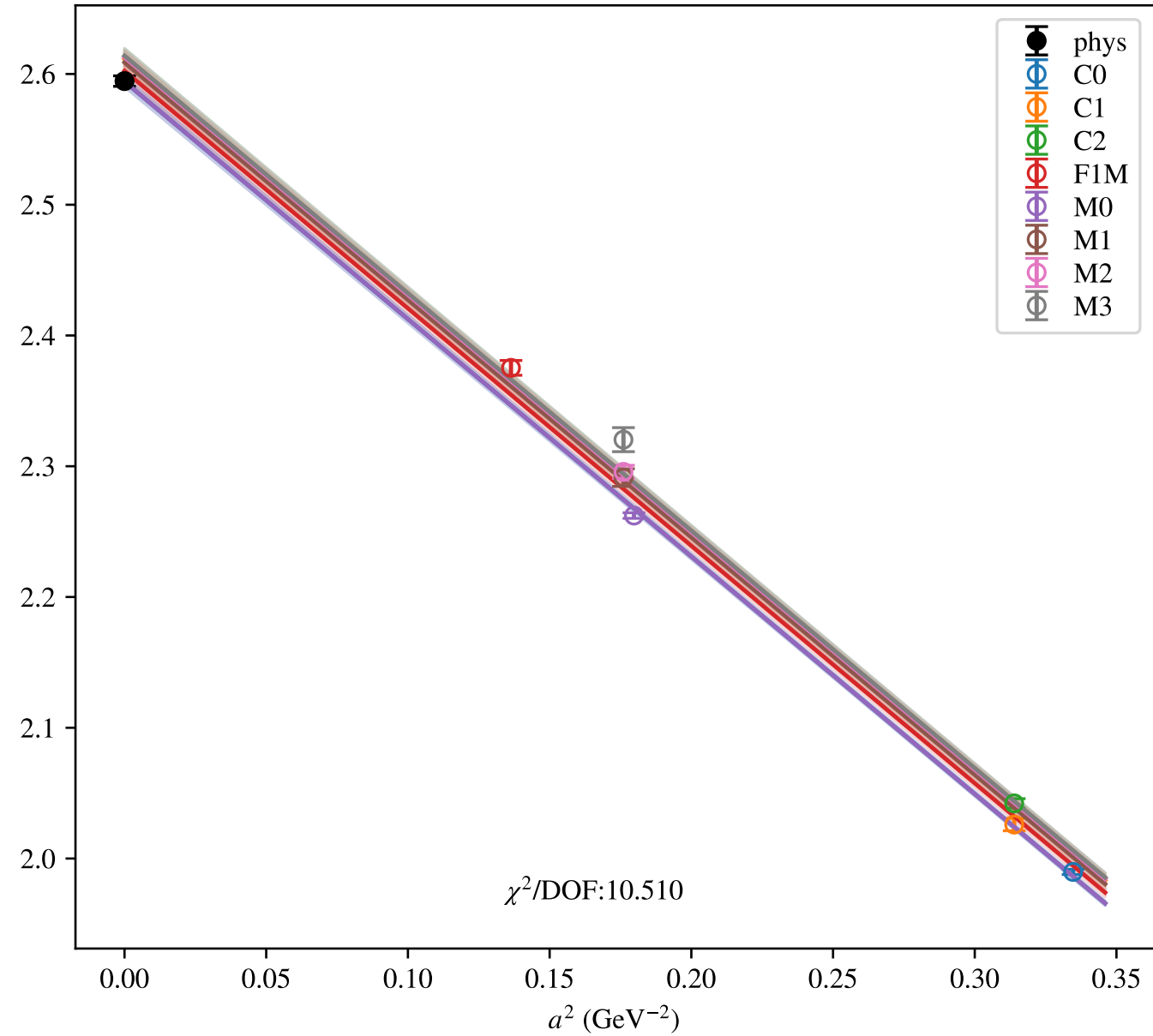


VVmAA

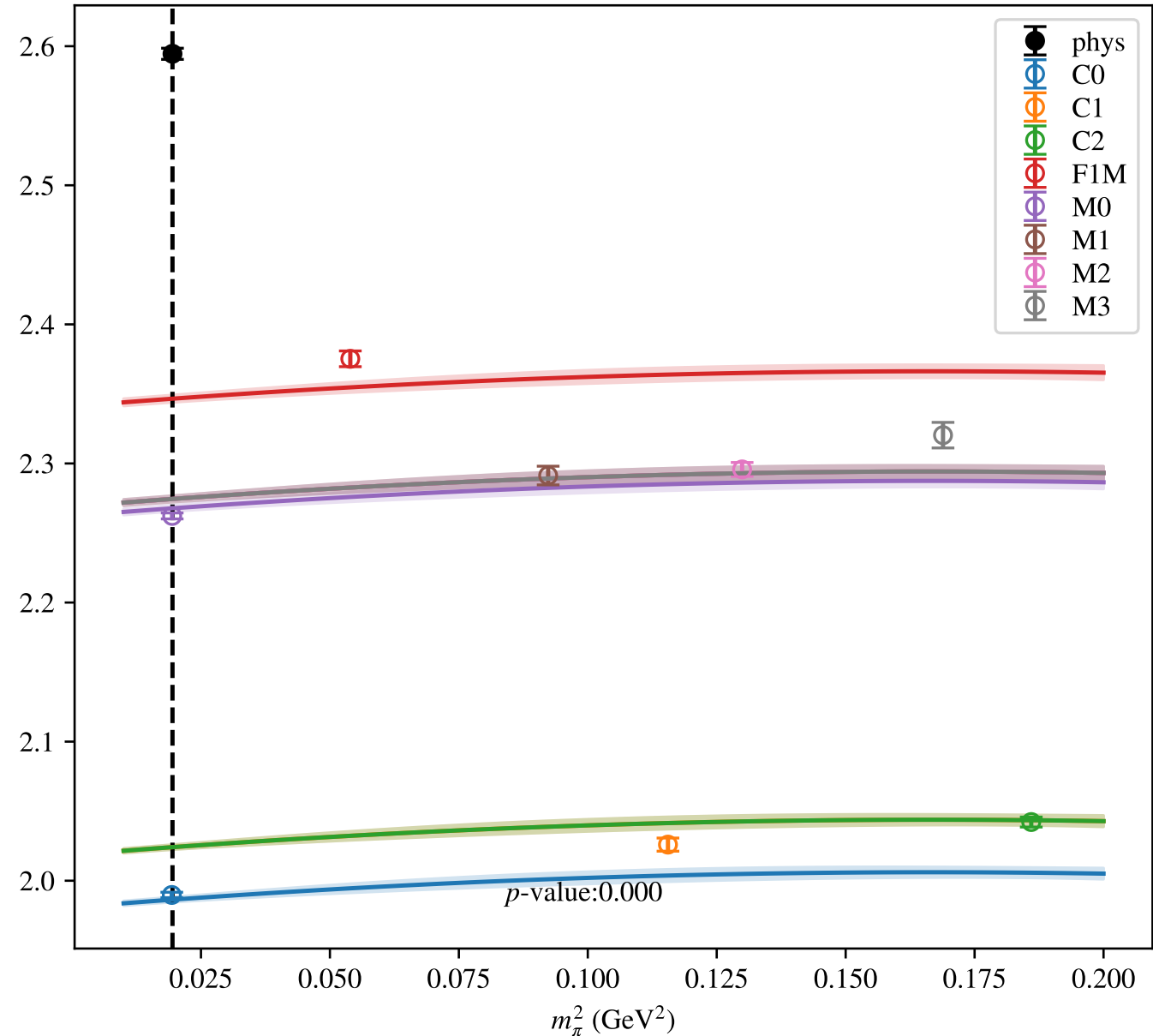


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.8 \text{ GeV}$$

VVmAA

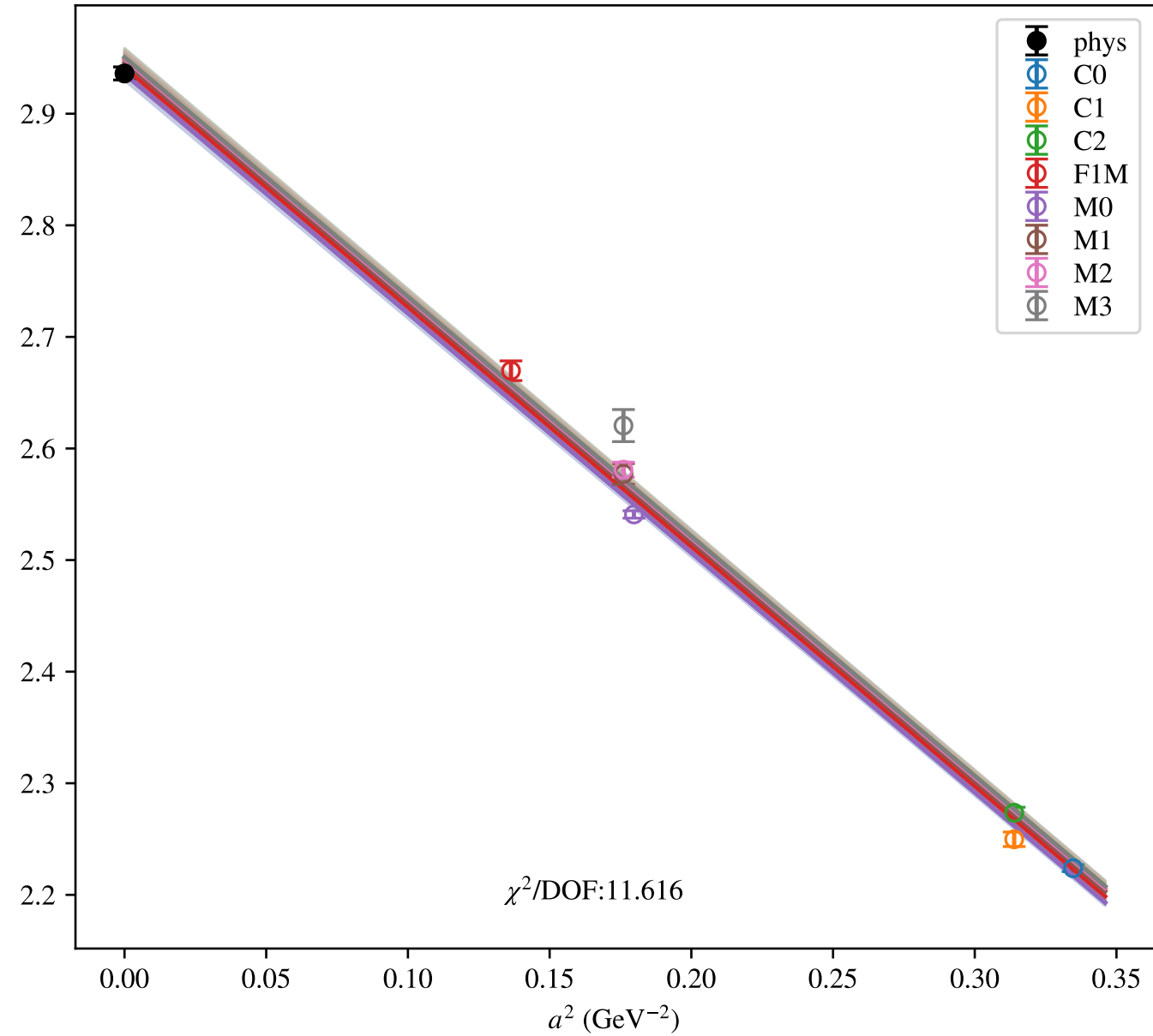


VVmAA

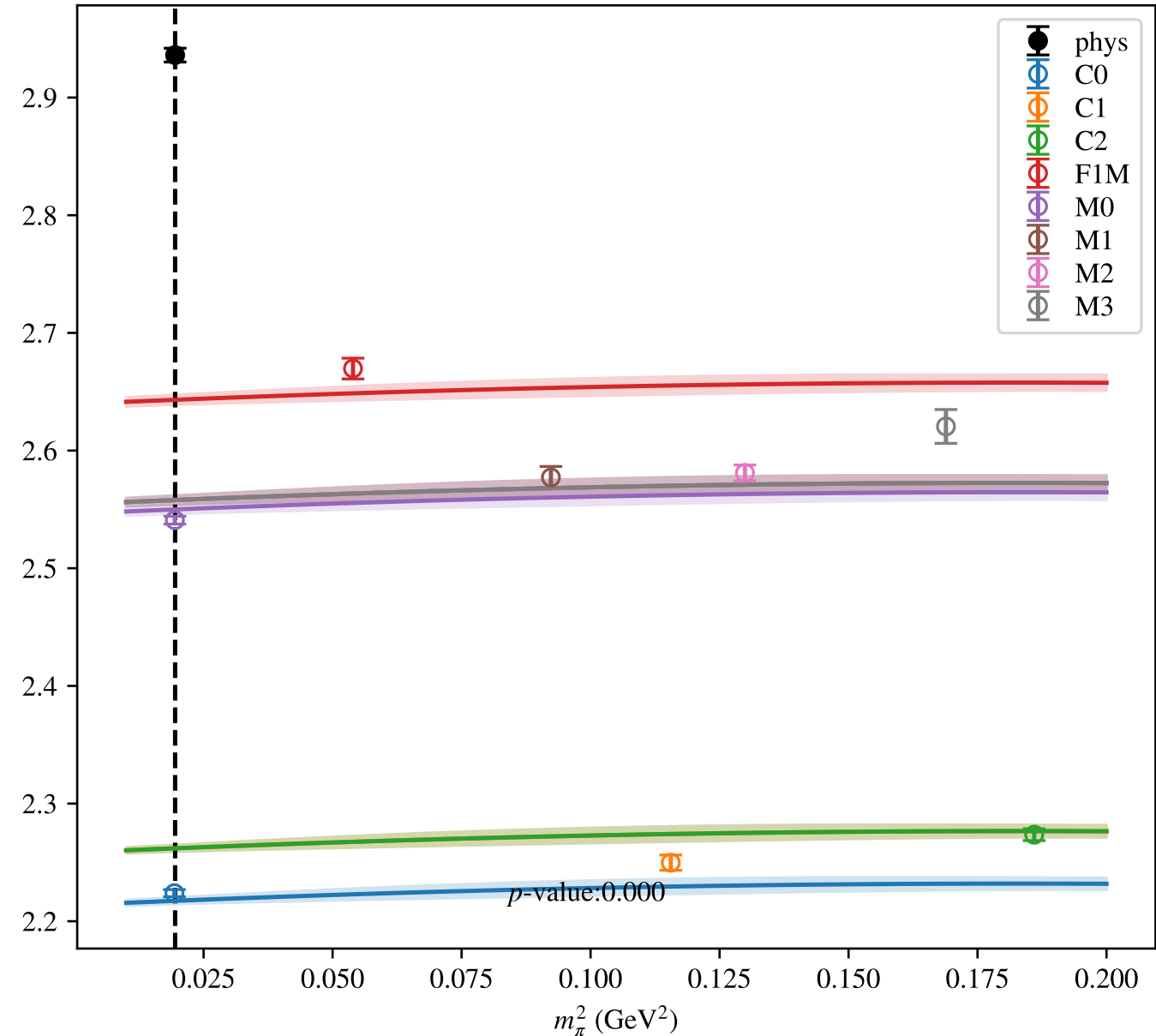


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.5 \text{ GeV}$$

VVmAA

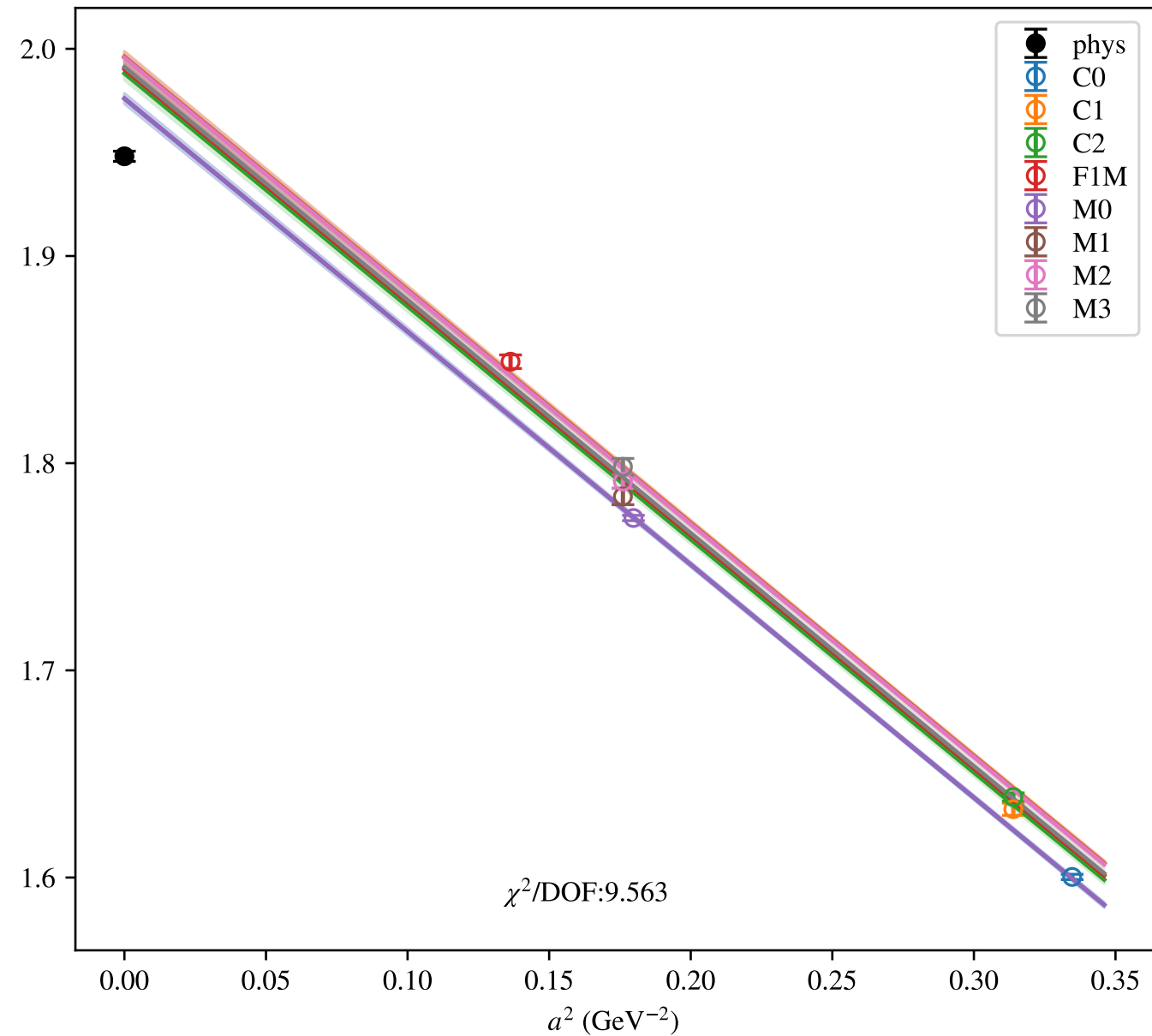


VVmAA

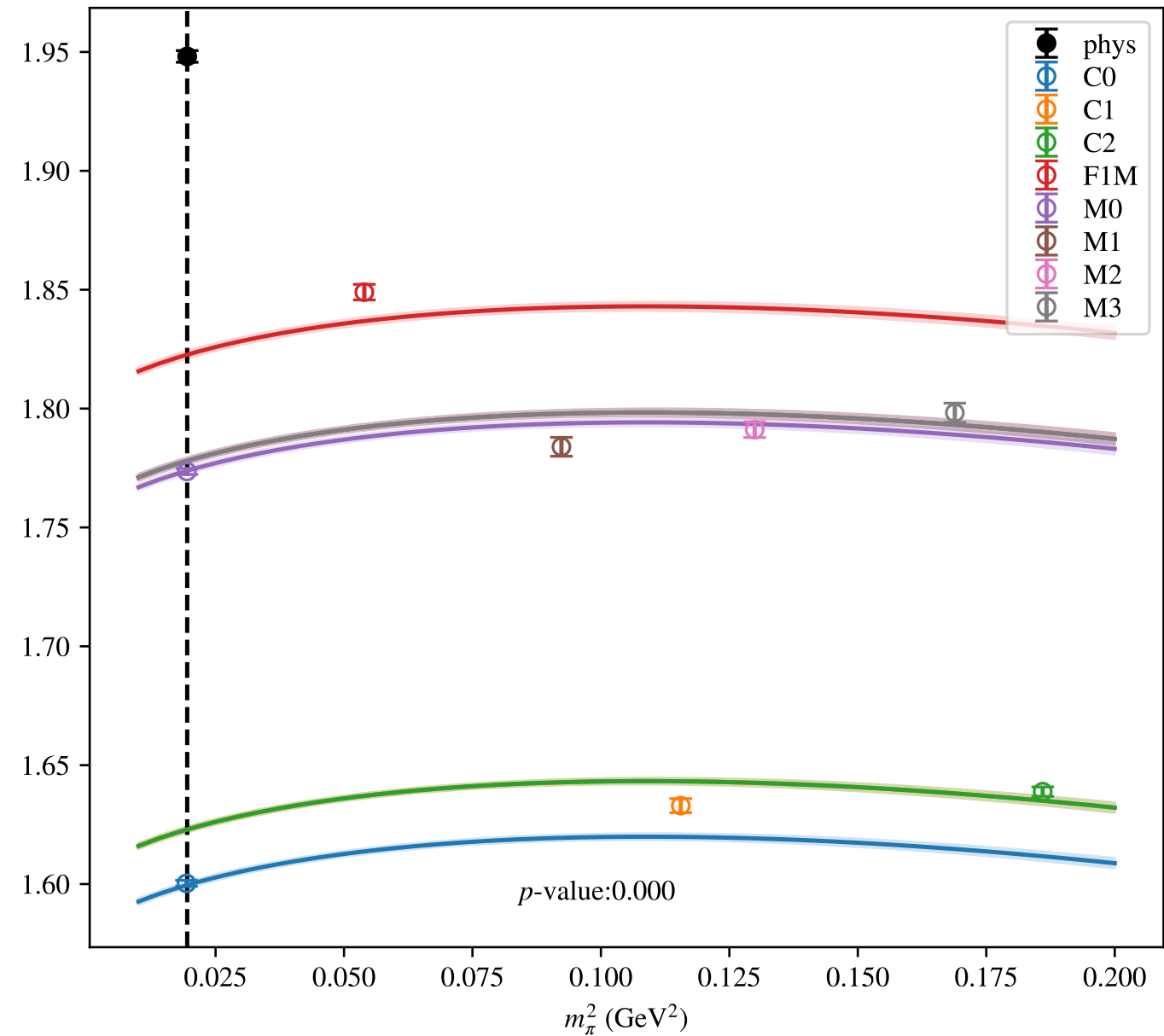


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 2.4 \text{ GeV}$$

VVmAA



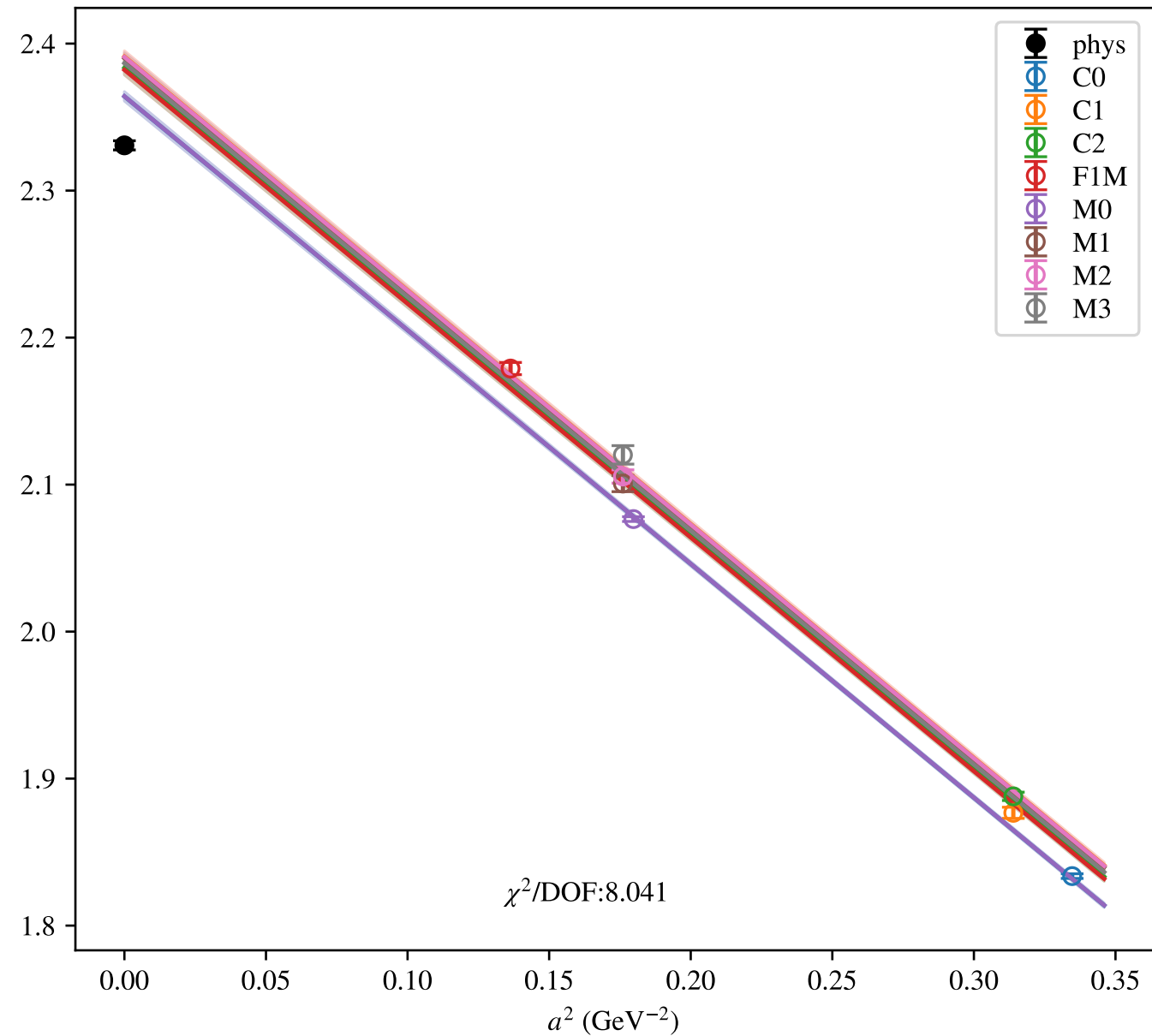
VVmAA



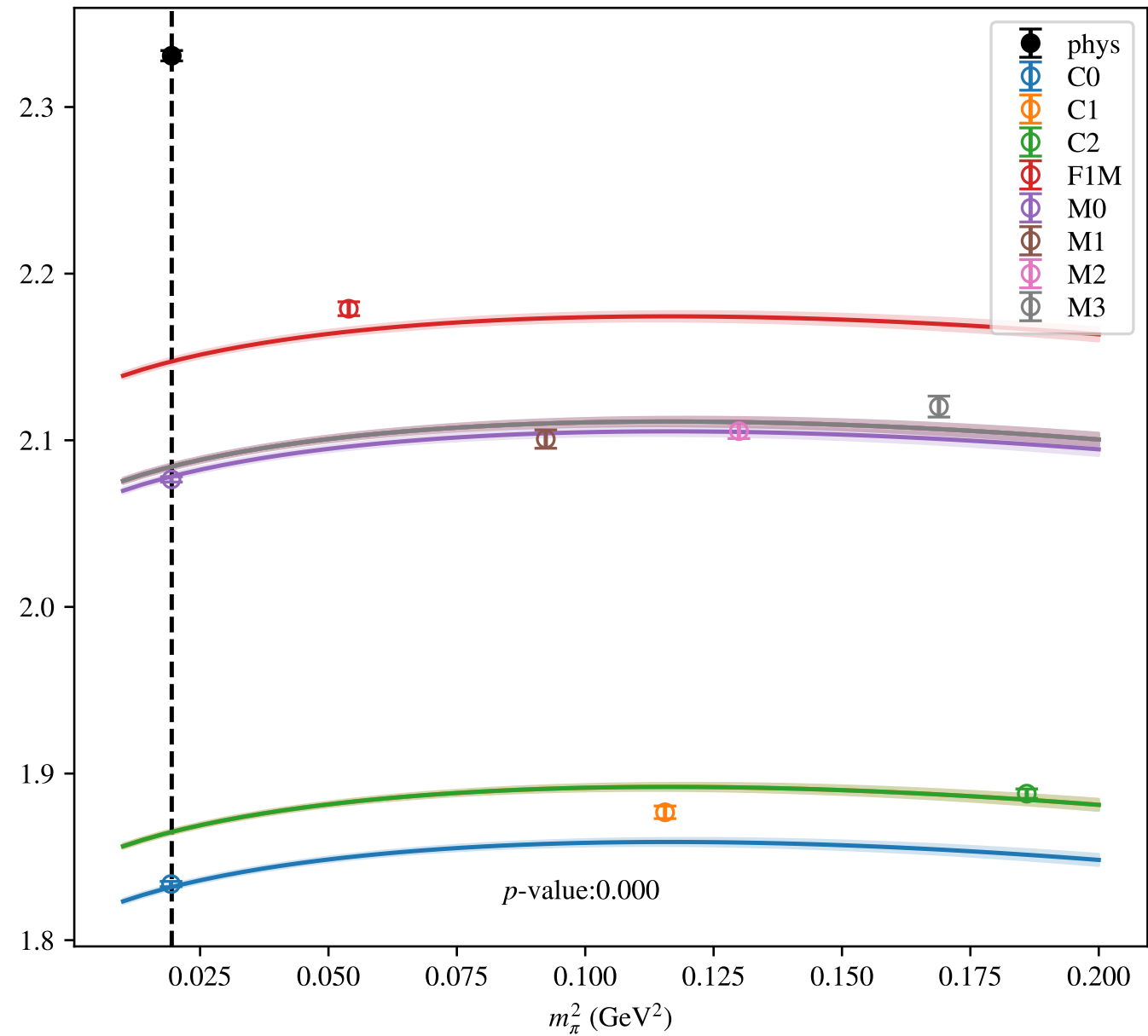


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 2.0 \text{ GeV}$$

VVmAA

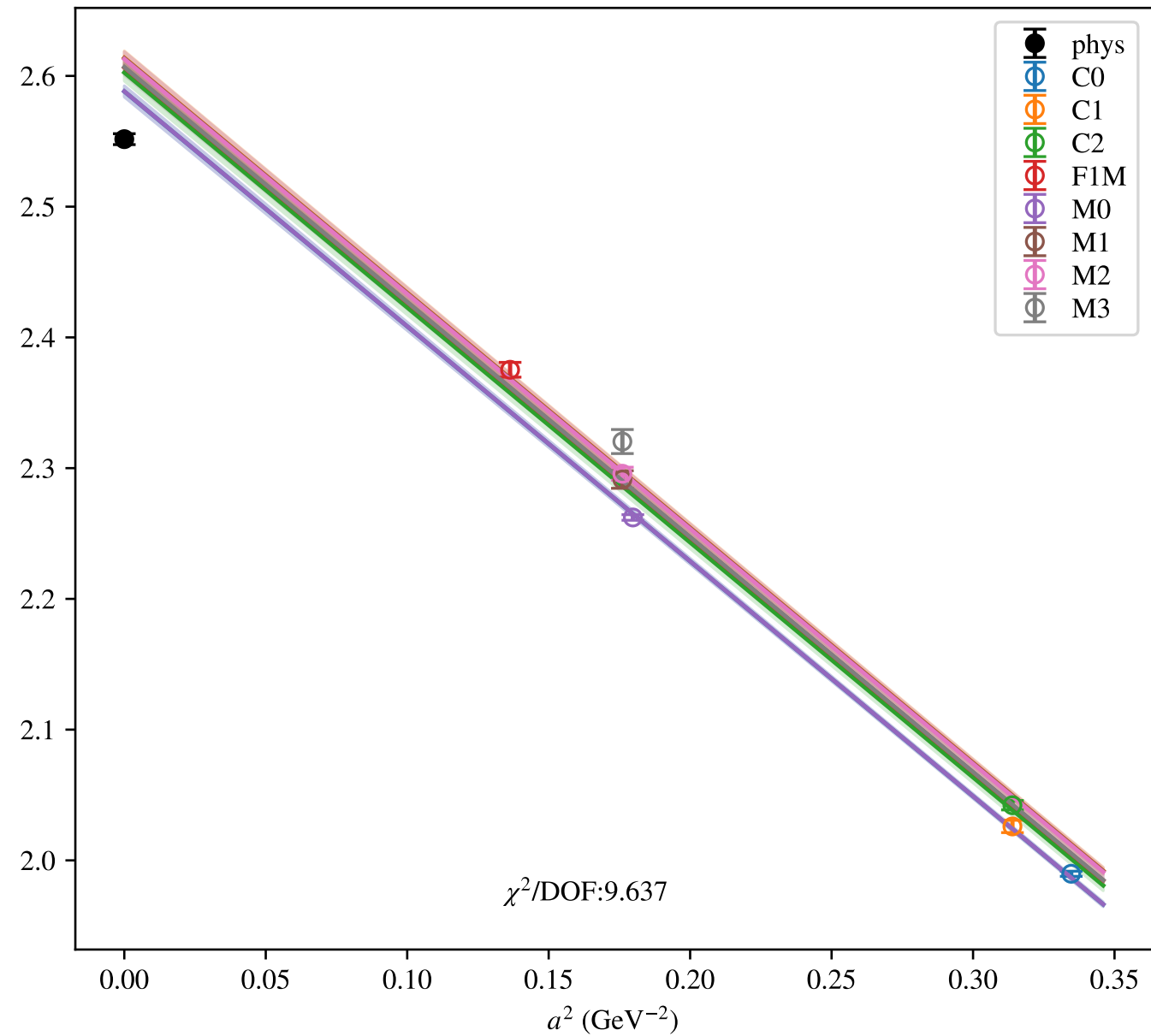


VVmAA

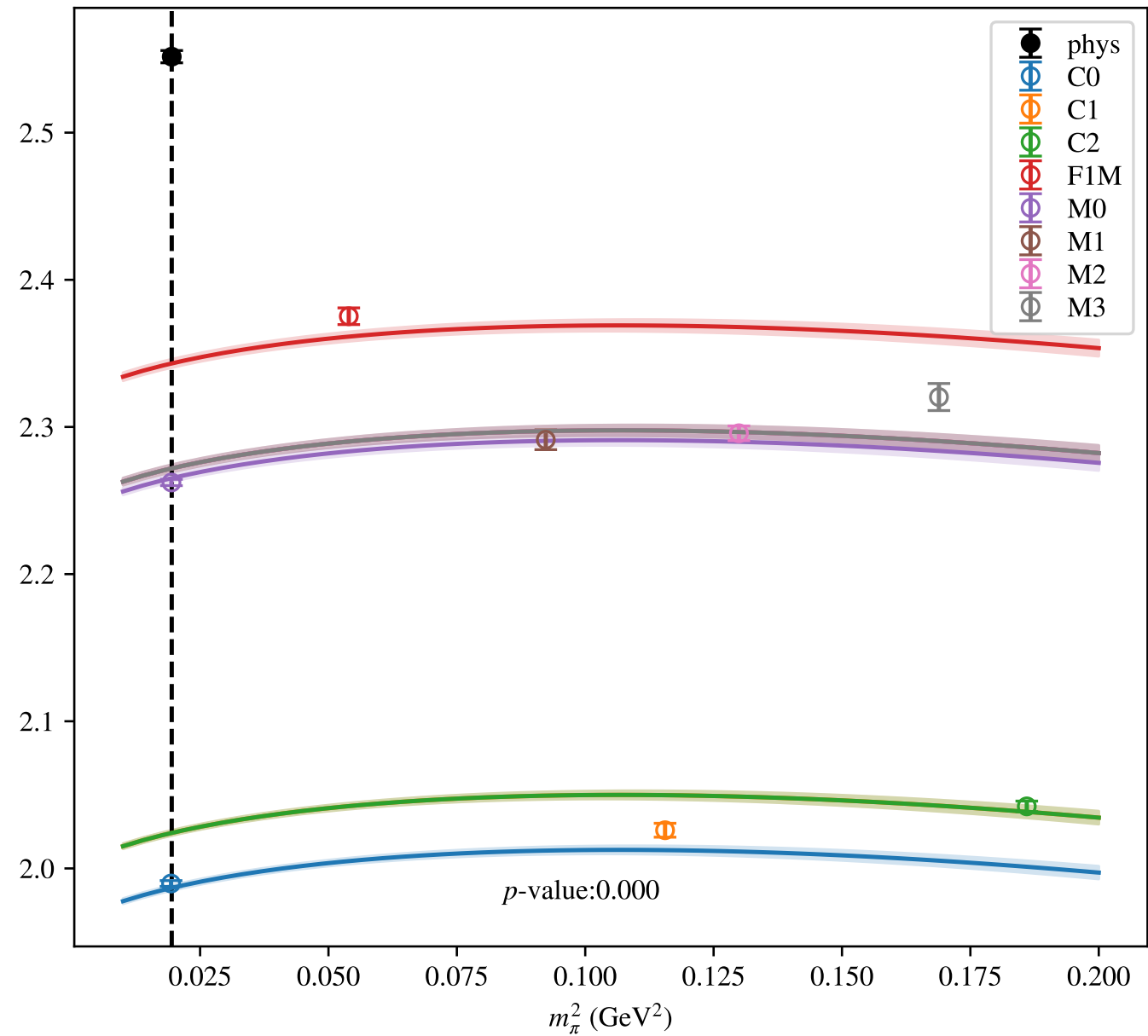


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 1.8 \text{ GeV}$$

VVmAA

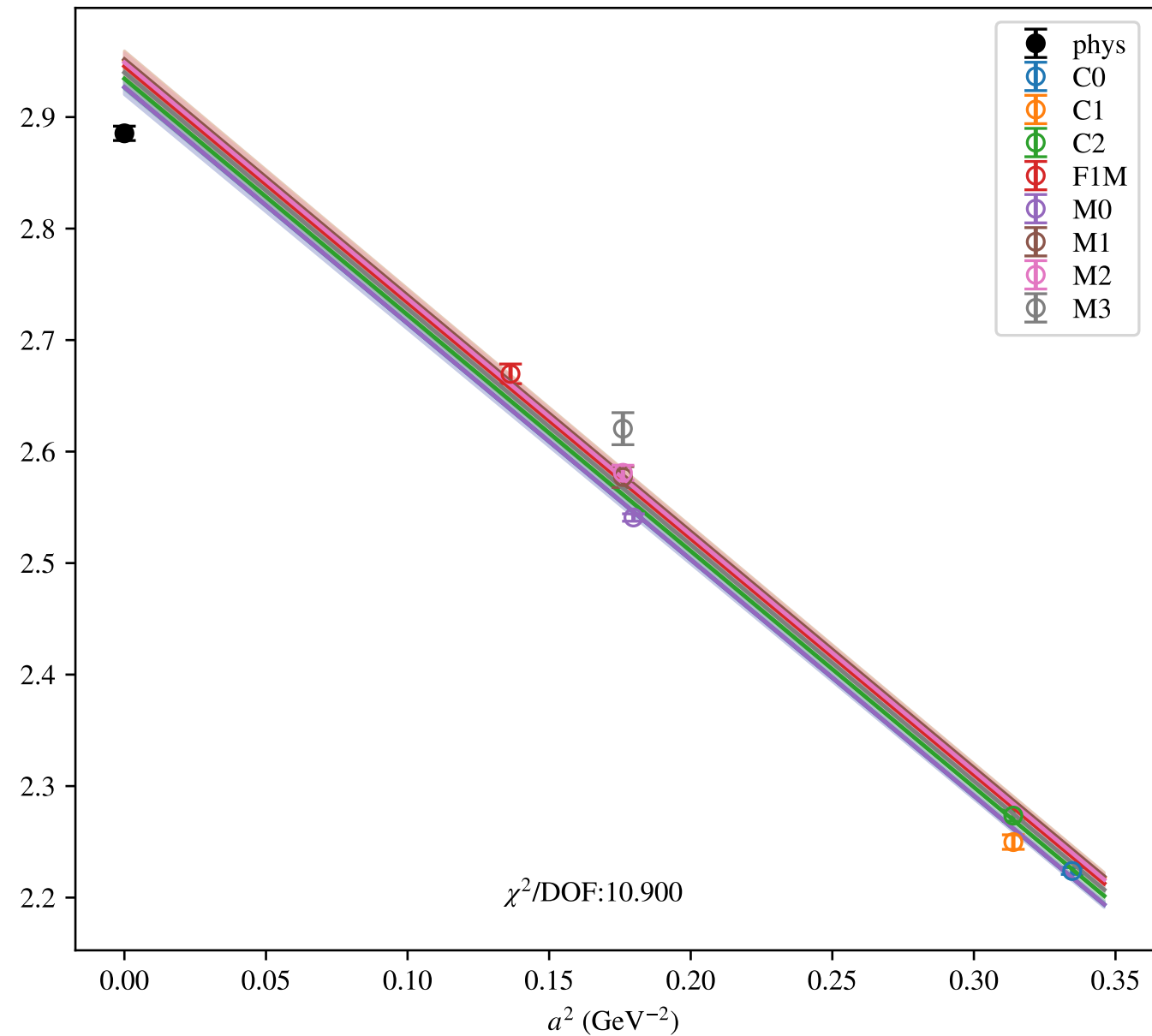


VVmAA

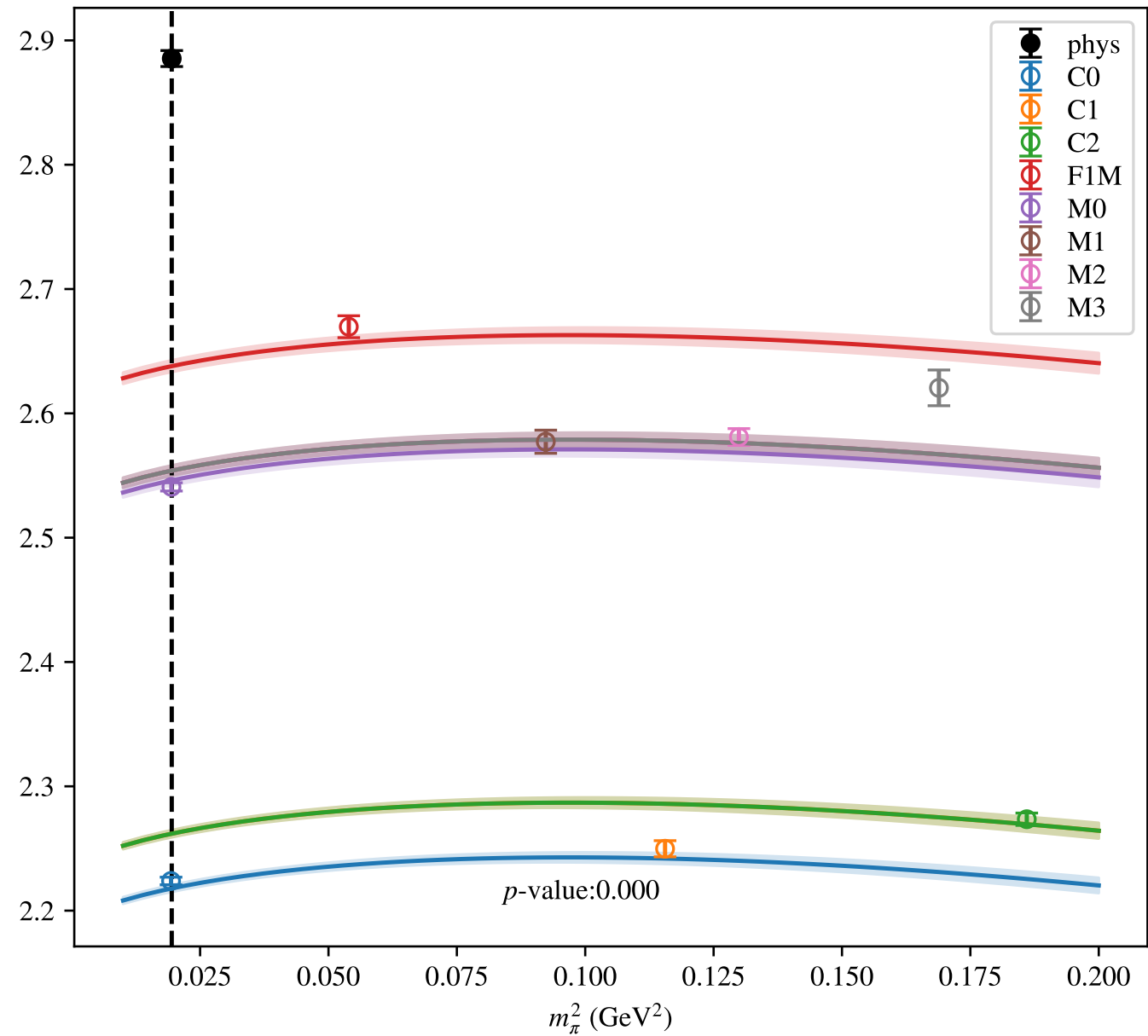


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 1.5 \text{ GeV}$$

VVmAA



VVmAA



### 3 SSmPP

$\mu$ (GeV)	$a^2, m_\pi^2$	$a^2, m_\pi^2$ (no C)	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$	$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2)$
2.4	<b>0.9436(13)</b> : 6.273 (0.0)	<b>0.9086(66)</b> : 1.786 (0.168)	<b>0.885(10)</b> : 1.117 (0.346)	<b>0.9453(14)</b> : 5.797 (0.0)	<b>0.9266(13)</b> : 18.643 (0.0)
2.0	<b>0.9557(14)</b> : 4.069 (0.001)	<b>0.9259(67)</b> : 1.711 (0.181)	<b>0.906(11)</b> : 1.012 (0.4)	<b>0.9572(15)</b> : 3.563 (0.007)	<b>0.9388(14)</b> : 13.53 (0.0)
1.8	<b>0.9631(18)</b> : 2.42 (0.033)	<b>0.9365(74)</b> : 2.117 (0.12)	<b>0.922(12)</b> : 1.336 (0.254)	<b>0.9647(17)</b> : 1.96 (0.098)	<b>0.9460(18)</b> : 8.945 (0.0)
1.5	<b>0.9741(27)</b> : 1.372 (0.231)	<b>0.9523(94)</b> : 2.007 (0.134)	<b>0.947(15)</b> : 1.41 (0.228)	<b>0.9759(24)</b> : 1.054 (0.378)	<b>0.9565(27)</b> : 4.87 (0.0)

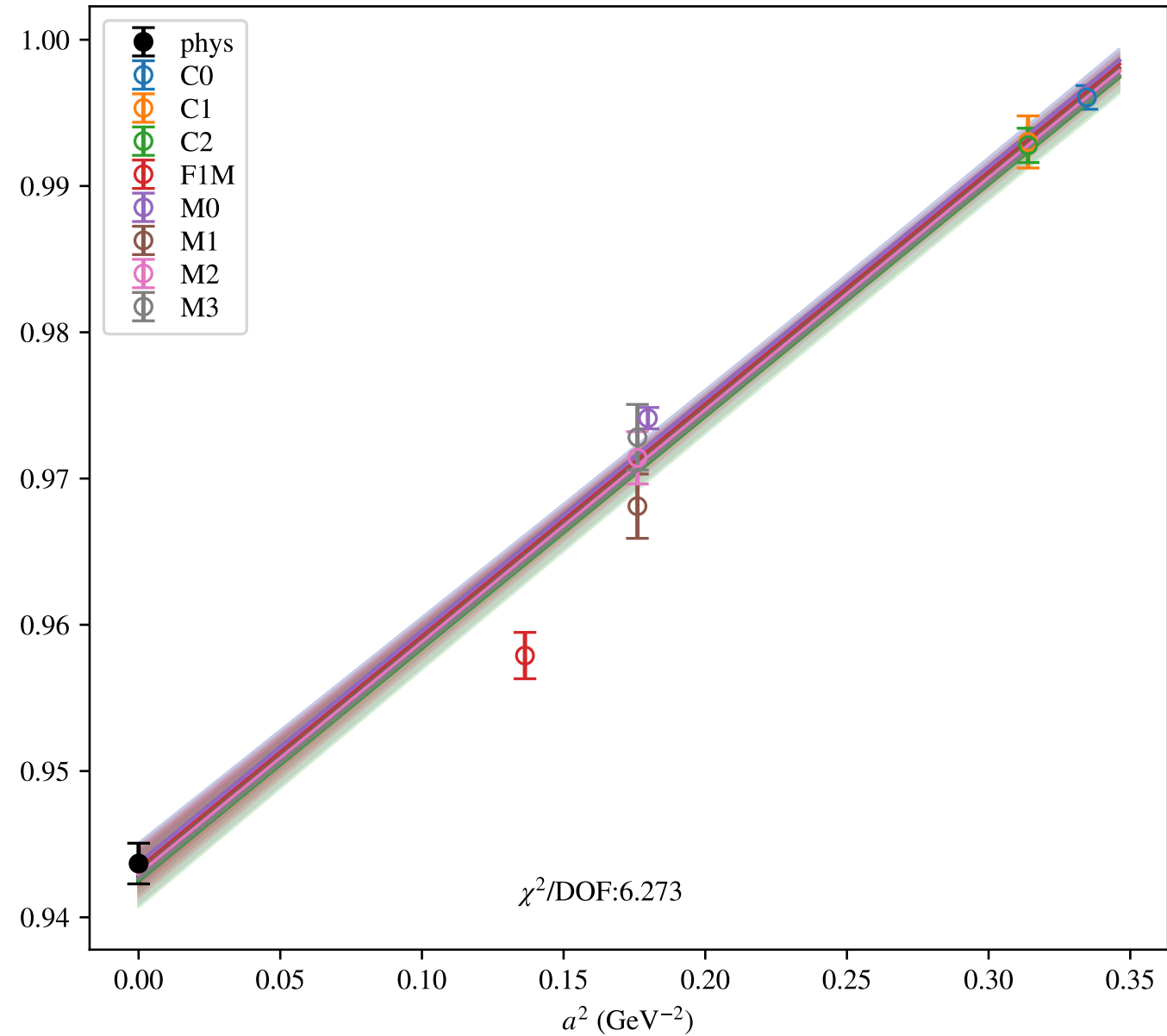
Table 5: Physical point value from chiral and continuum extrapolation at renormalisation scale  $\mu$ . Entries are **value(error)**:  $\chi^2/\text{DOF}$  ( $p$ -value).

$\mu$ (GeV)		$a^2, m_\pi^2$	$a^2, m_\pi^2$ (no C)	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$	$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2)$
2.4	$\alpha$	0.1681(56)	0.399(44)	0.76(11)	0.1609(60)	0.1834(56)
	$\beta$	-0.0001(12)	-0.0001(22)	-0.0003(14)	-0.0020(64)	-0.0048(12)
2.0	$\alpha$	0.1378(56)	0.330(43)	0.63(11)	0.1316(60)	0.1515(56)
	$\beta$	0.00023(15)	0.00022(29)	0.0	-0.0016(71)	-0.0045(16)
1.8	$\alpha$	0.1270(59)	0.295(46)	0.52(12)	0.1206(60)	0.1410(59)
	$\beta$	0.00049(17)	0.00053(30)	0.00036(19)	-0.0013(74)	-0.0043(17)
1.5	$\alpha$	0.1119(69)	0.245(53)	0.36(14)	0.1047(64)	0.1262(70)
	$\beta$	0.00086(20)	0.00102(36)	0.00078(23)	-0.0010(90)	-0.0039(20)

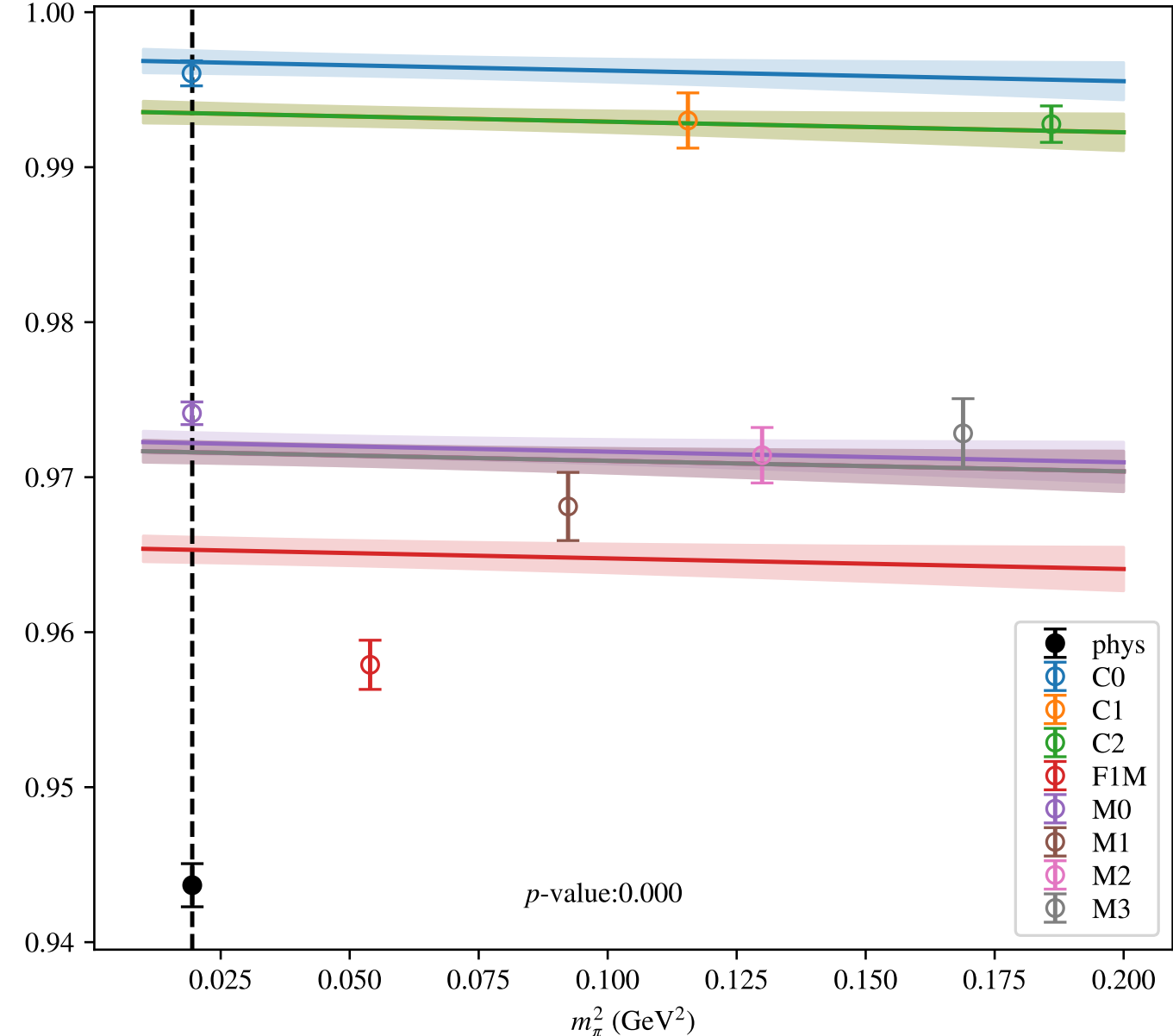
Table 6: Fit values of coefficients in  $B = B_0(1 + \alpha a^2 + \beta \frac{m_\pi^2}{f_\pi^2} + \dots)$ .

$$a^2, m_\pi^2, \mu = 2.4 \text{ GeV}$$

SSmPP

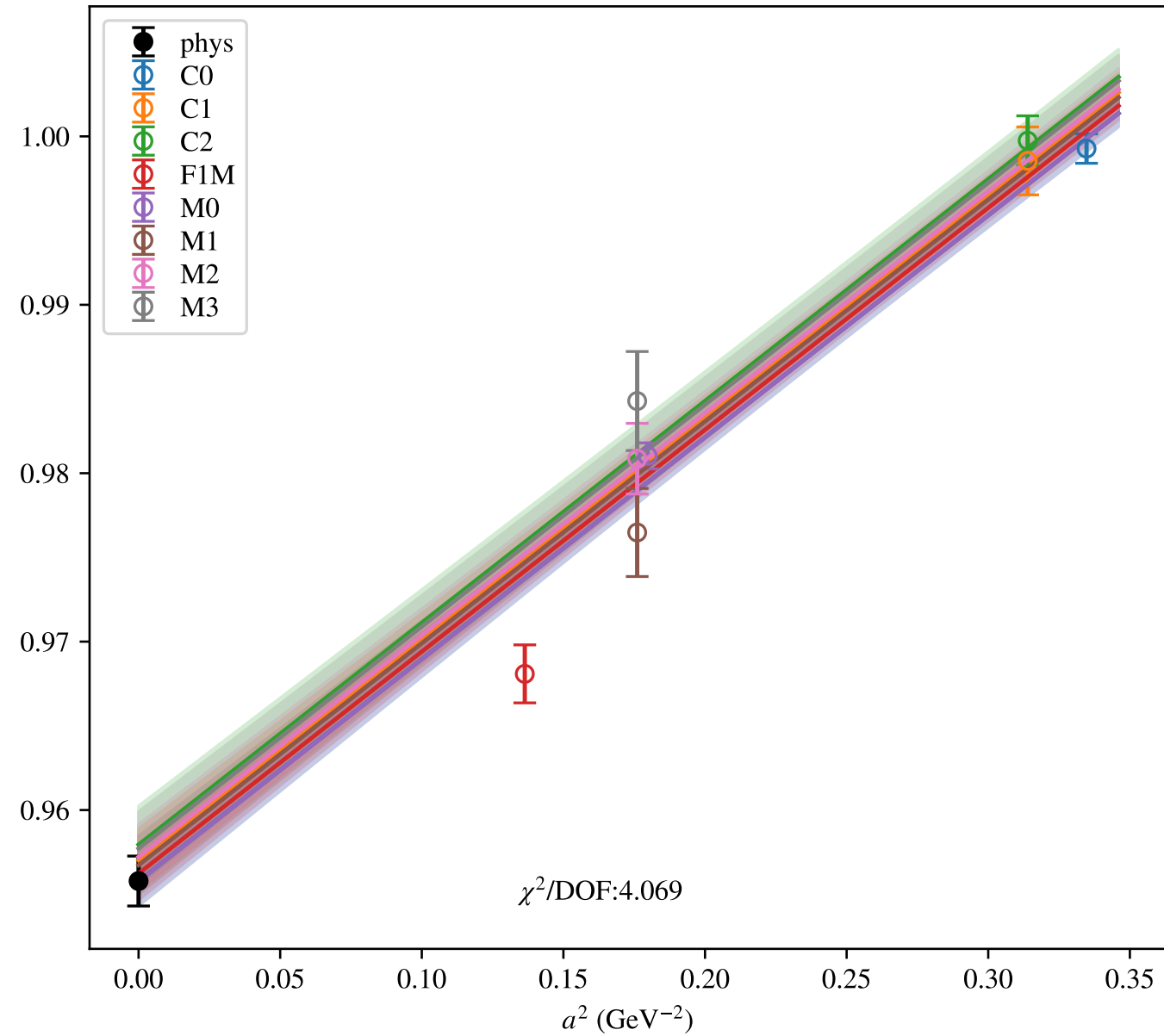


SSmPP

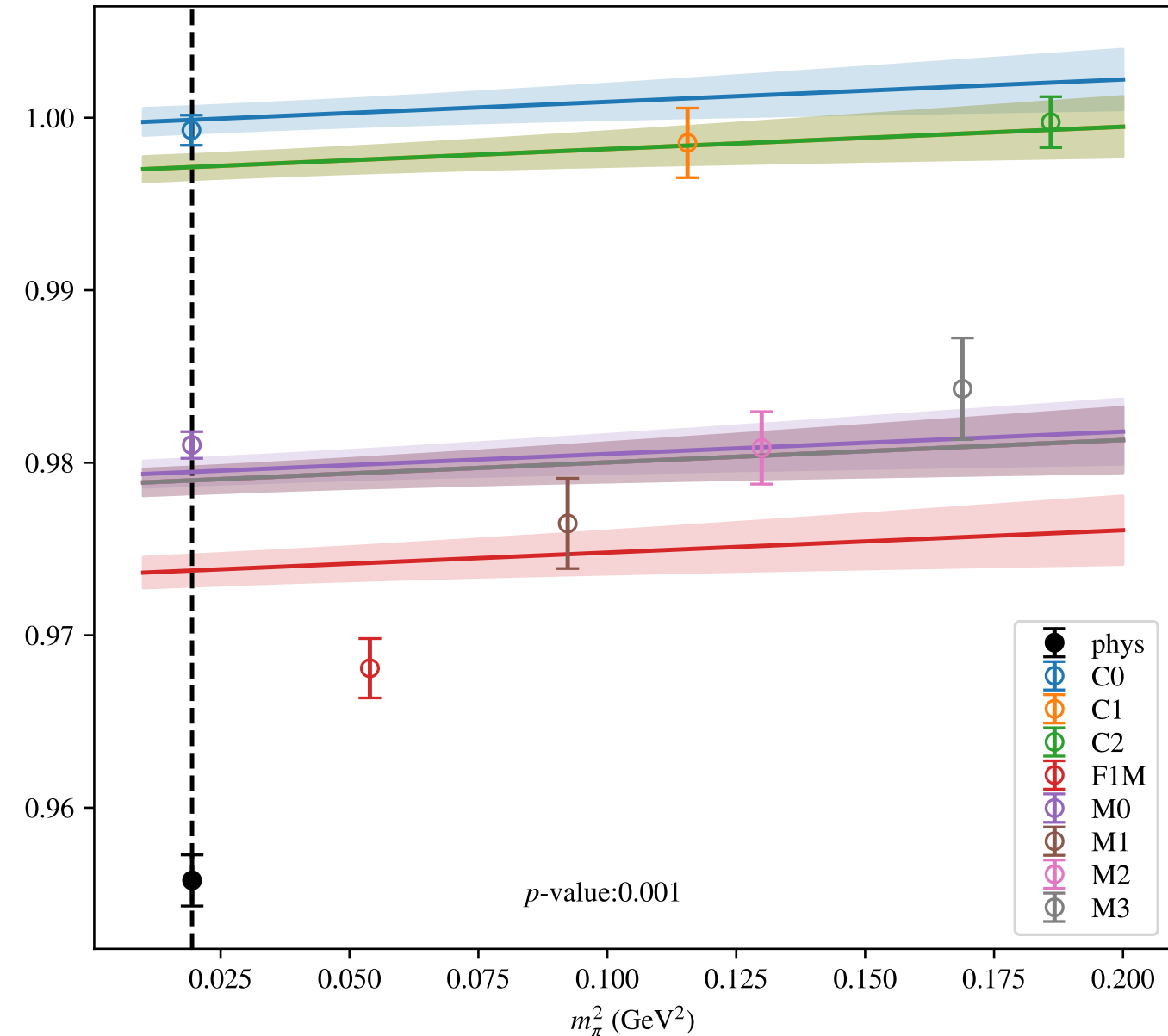


$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

SSmPP

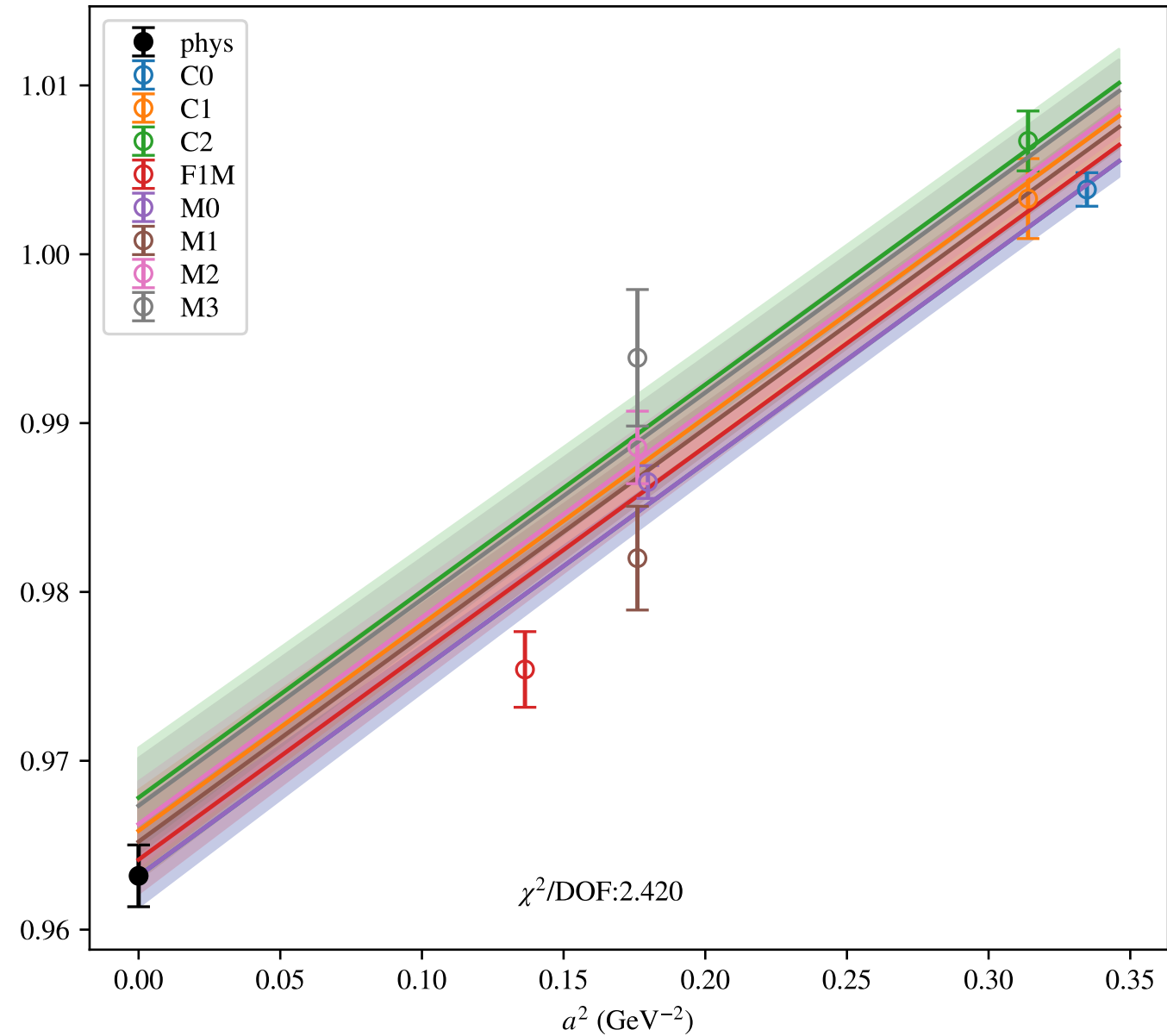


SSmPP

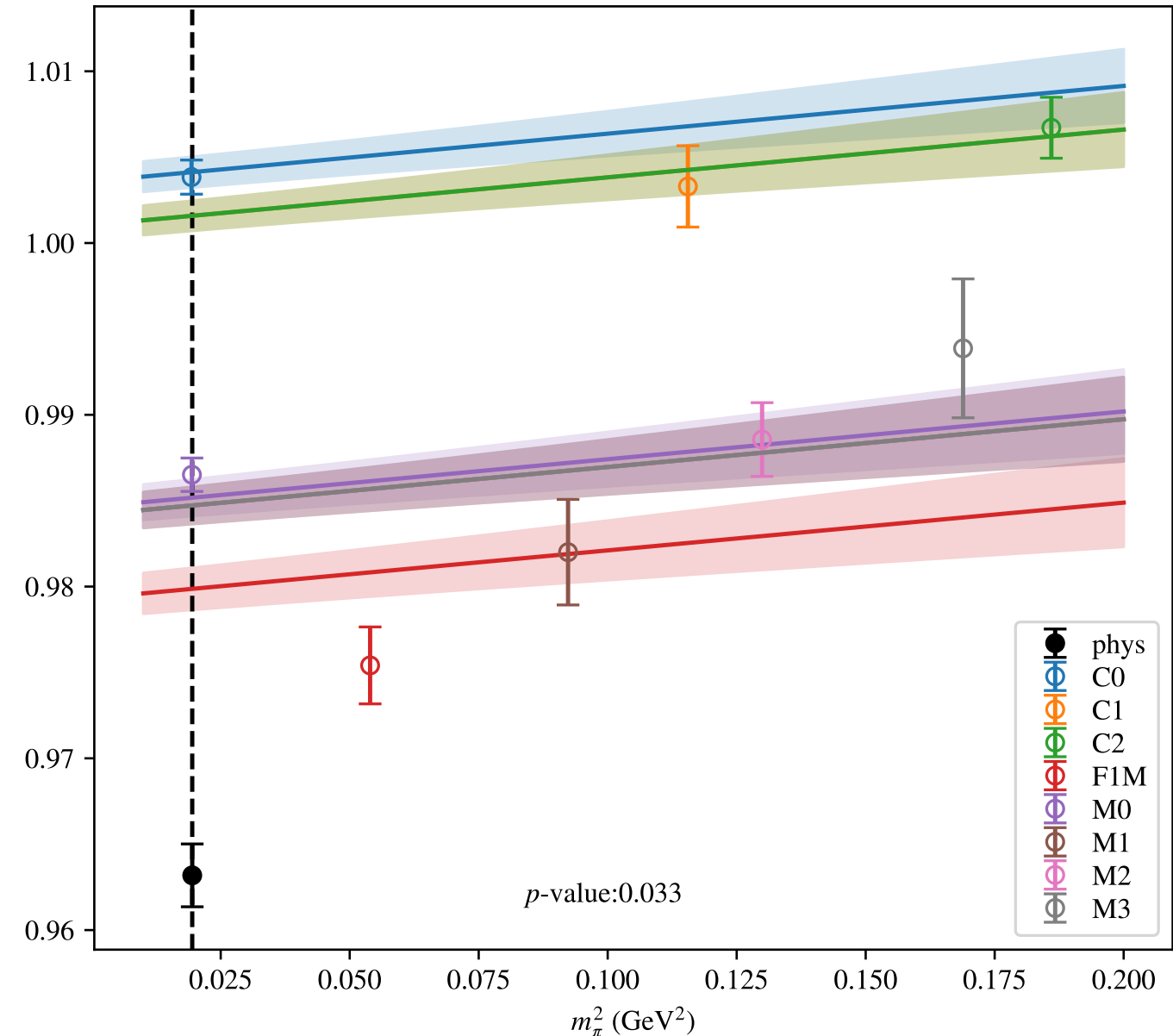


$$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$$

SSmPP

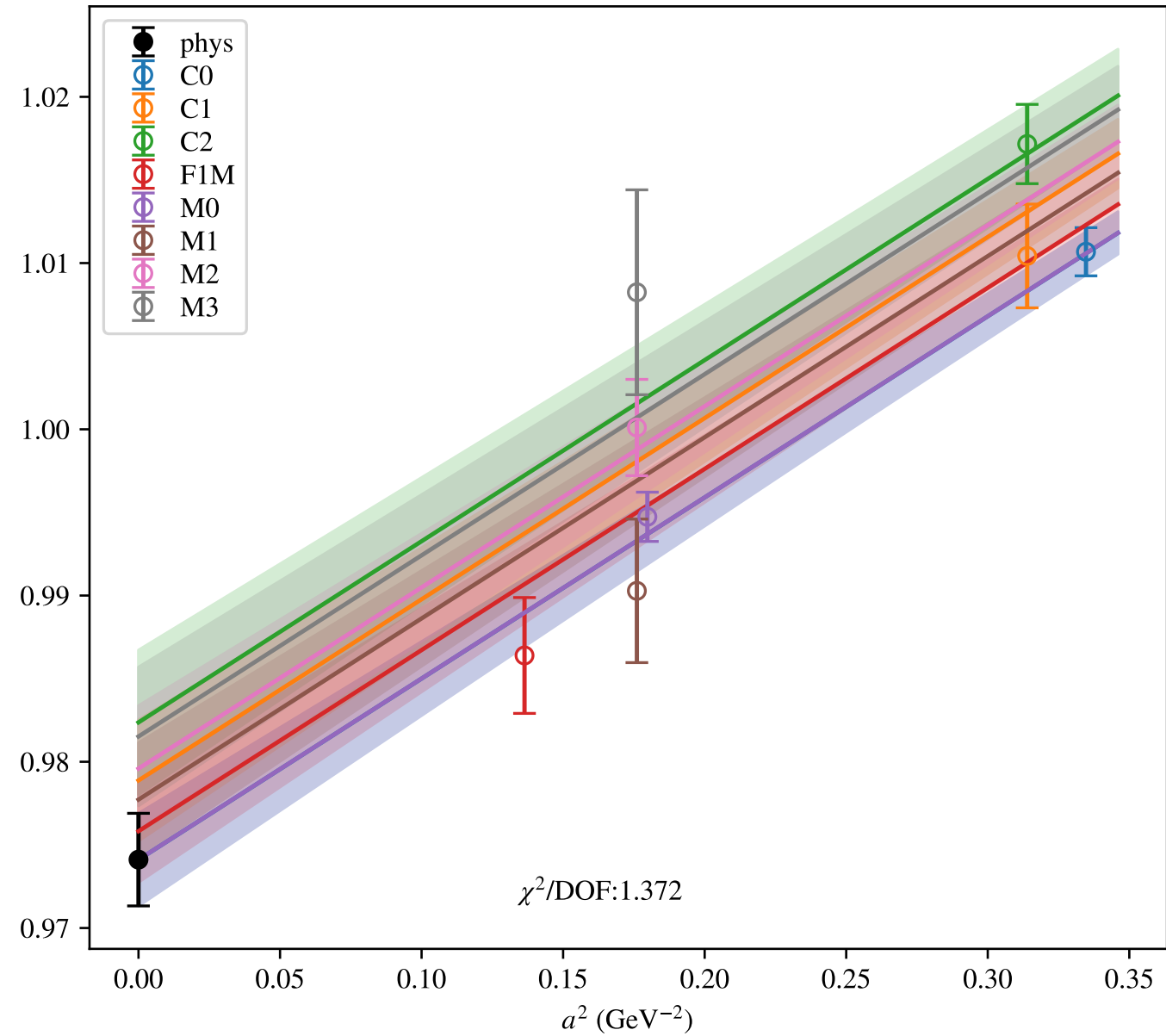


SSmPP

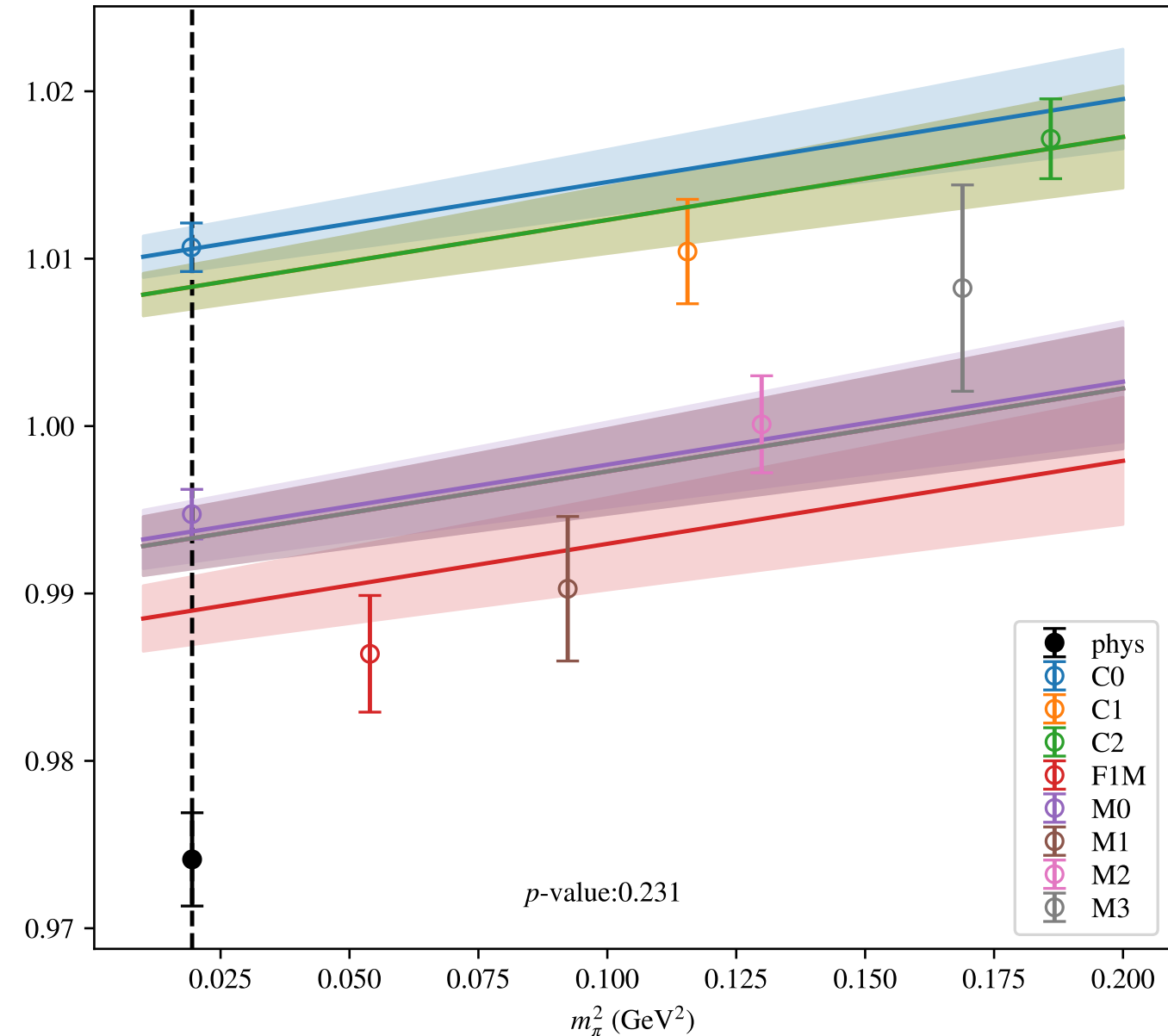


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

SSmPP



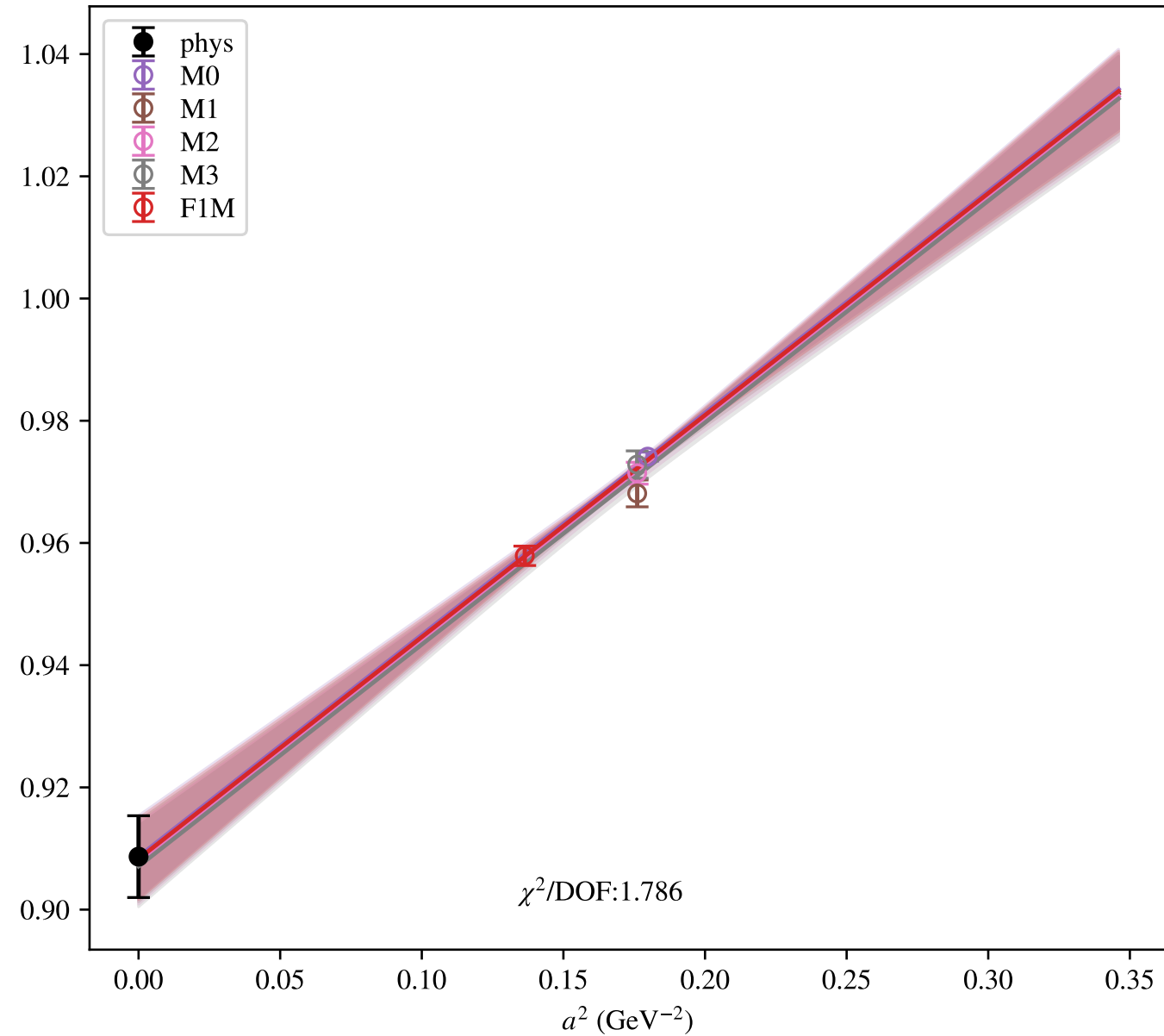
SSmPP



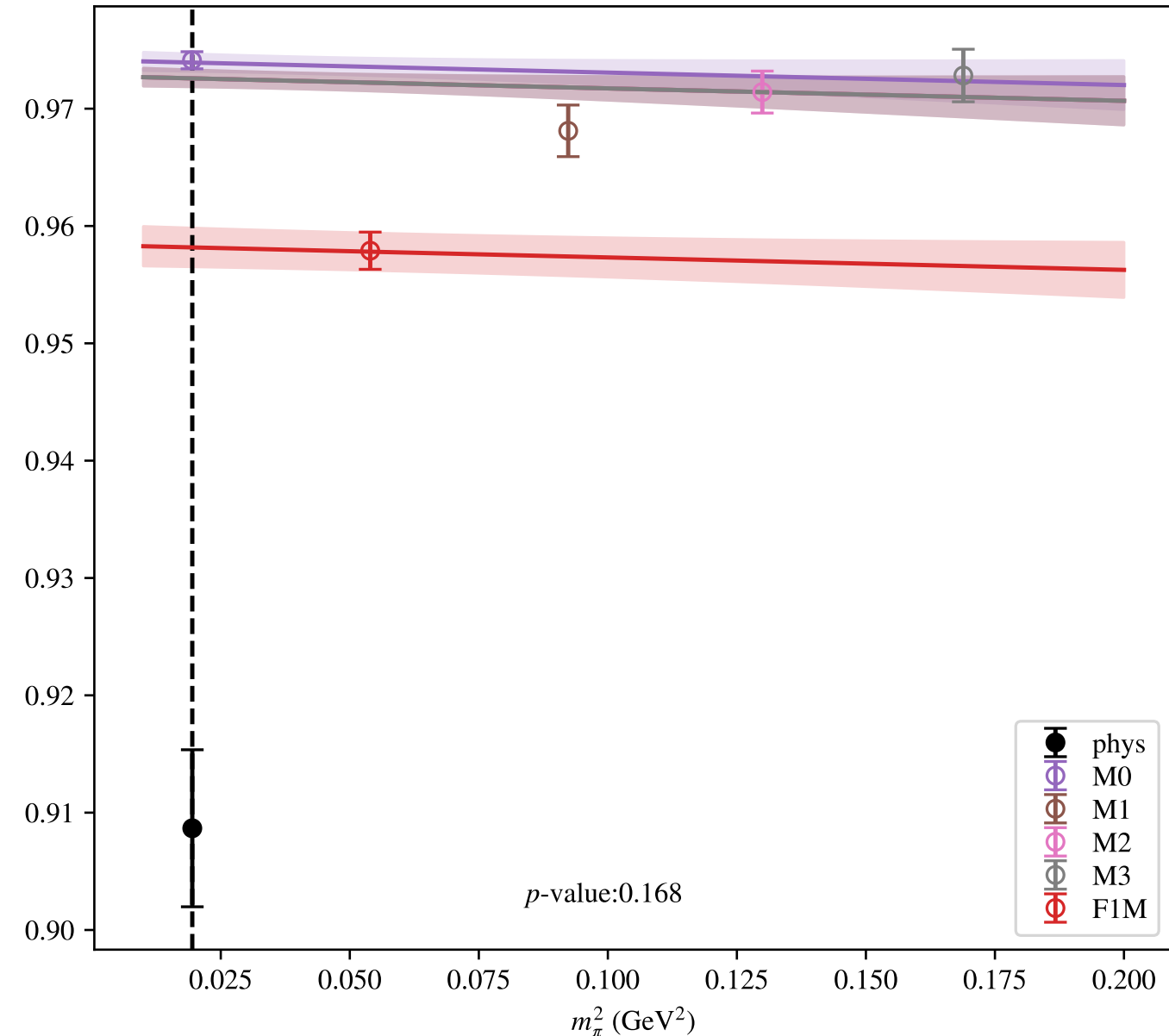


$a^2, m_\pi^2$  (no C),  $\mu = 2.4$  GeV

SSmPP

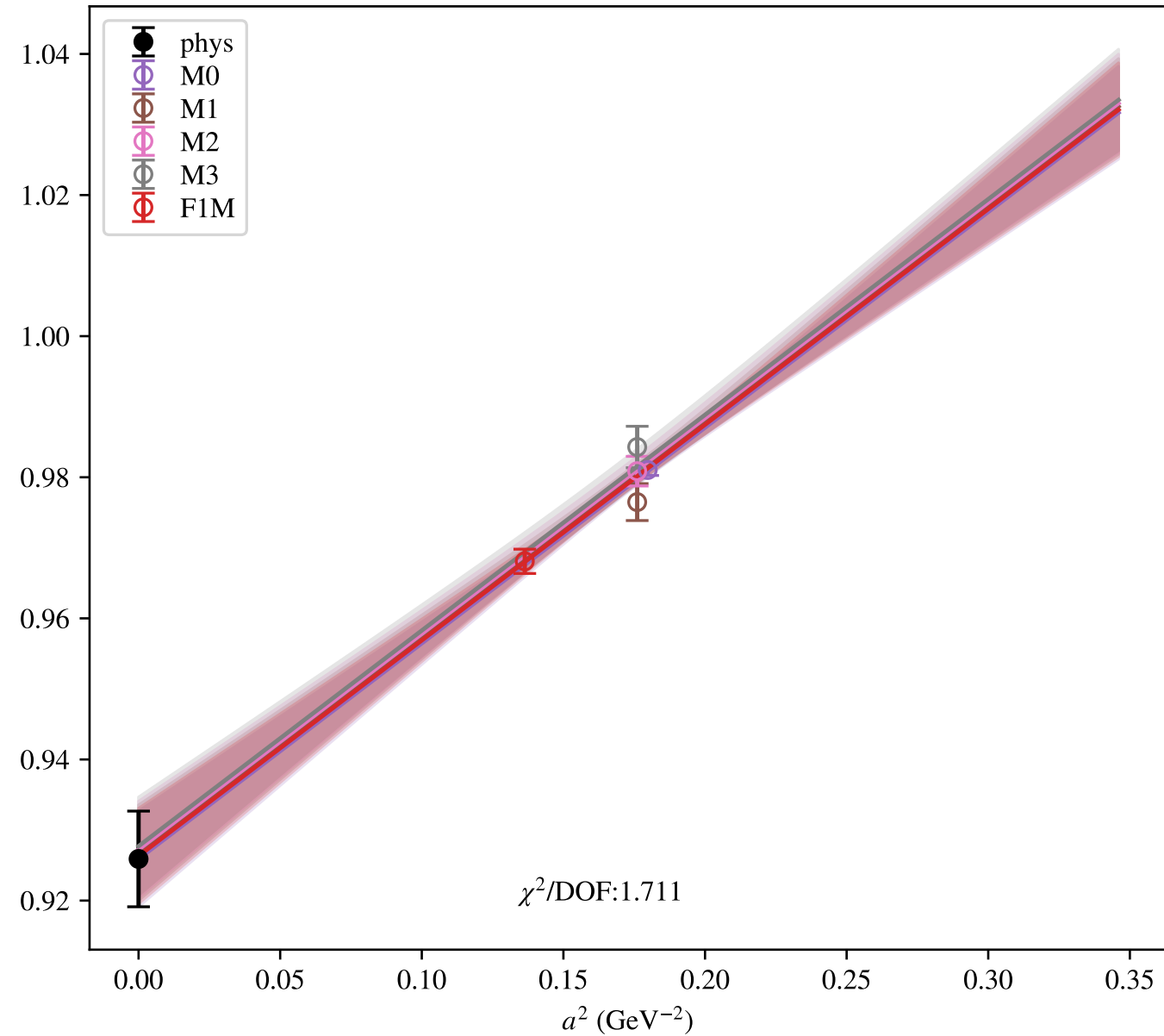


SSmPP

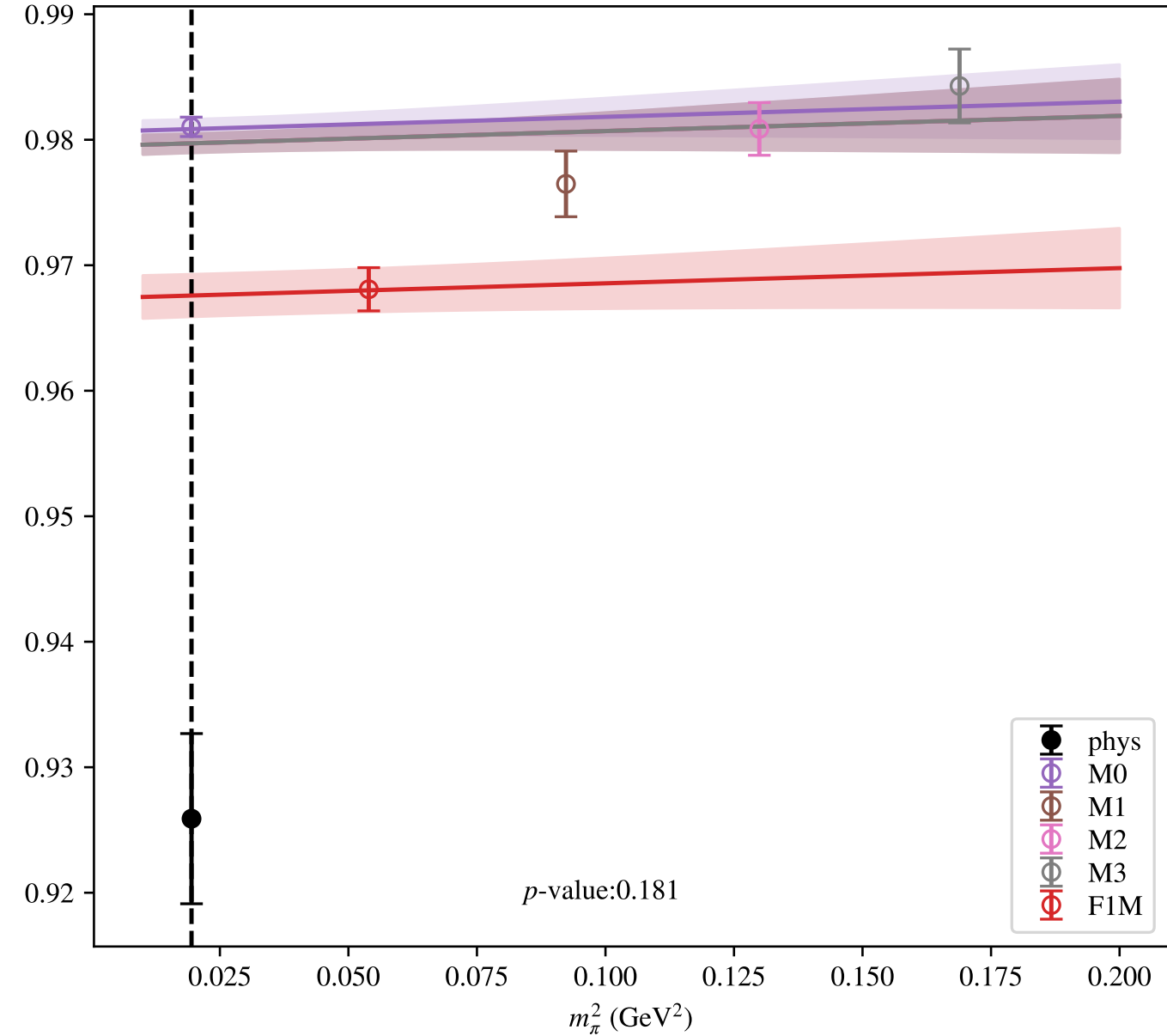


$a^2, m_\pi^2$  (no C),  $\mu = 2.0$  GeV

SSmPP

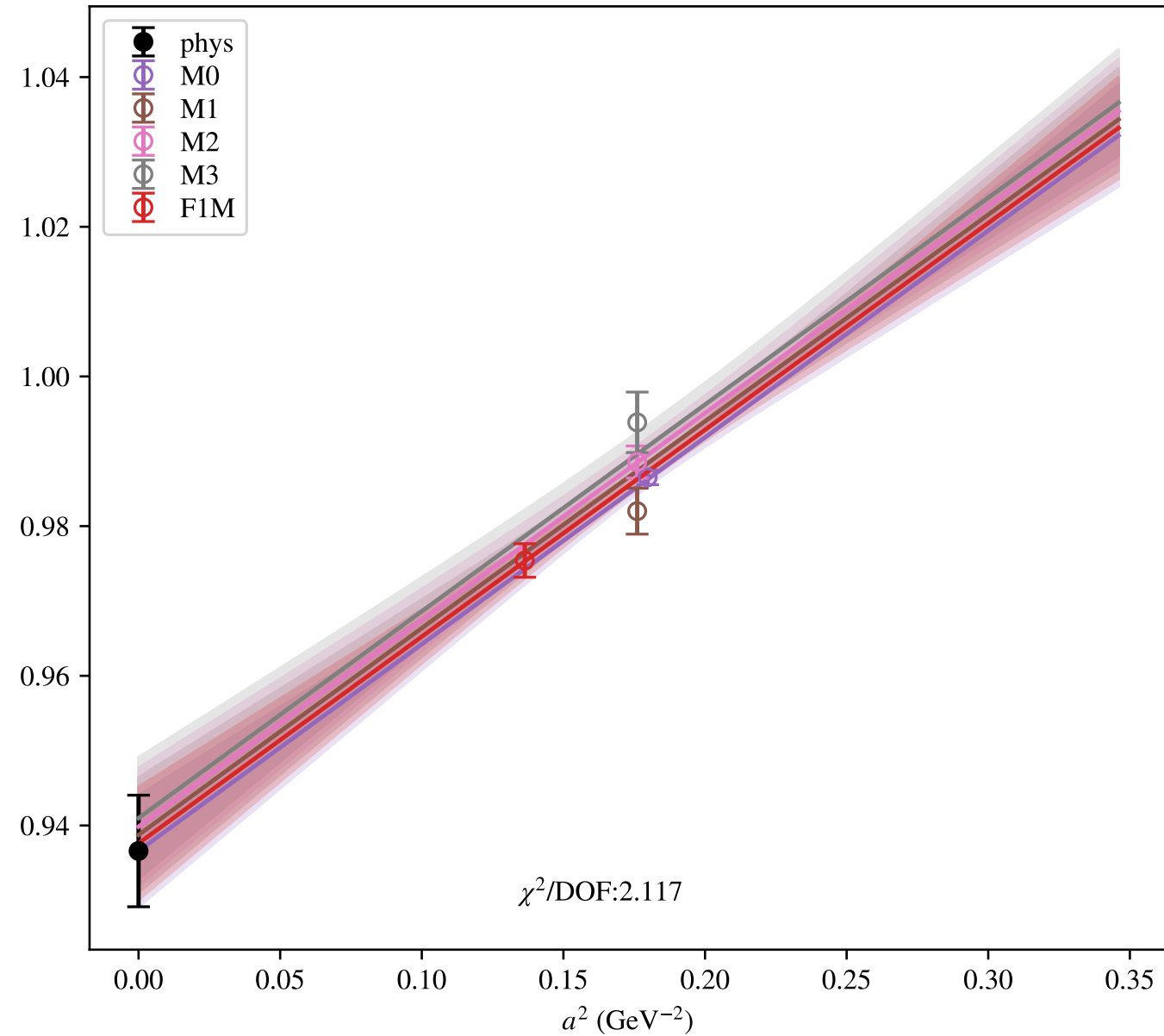


SSmPP

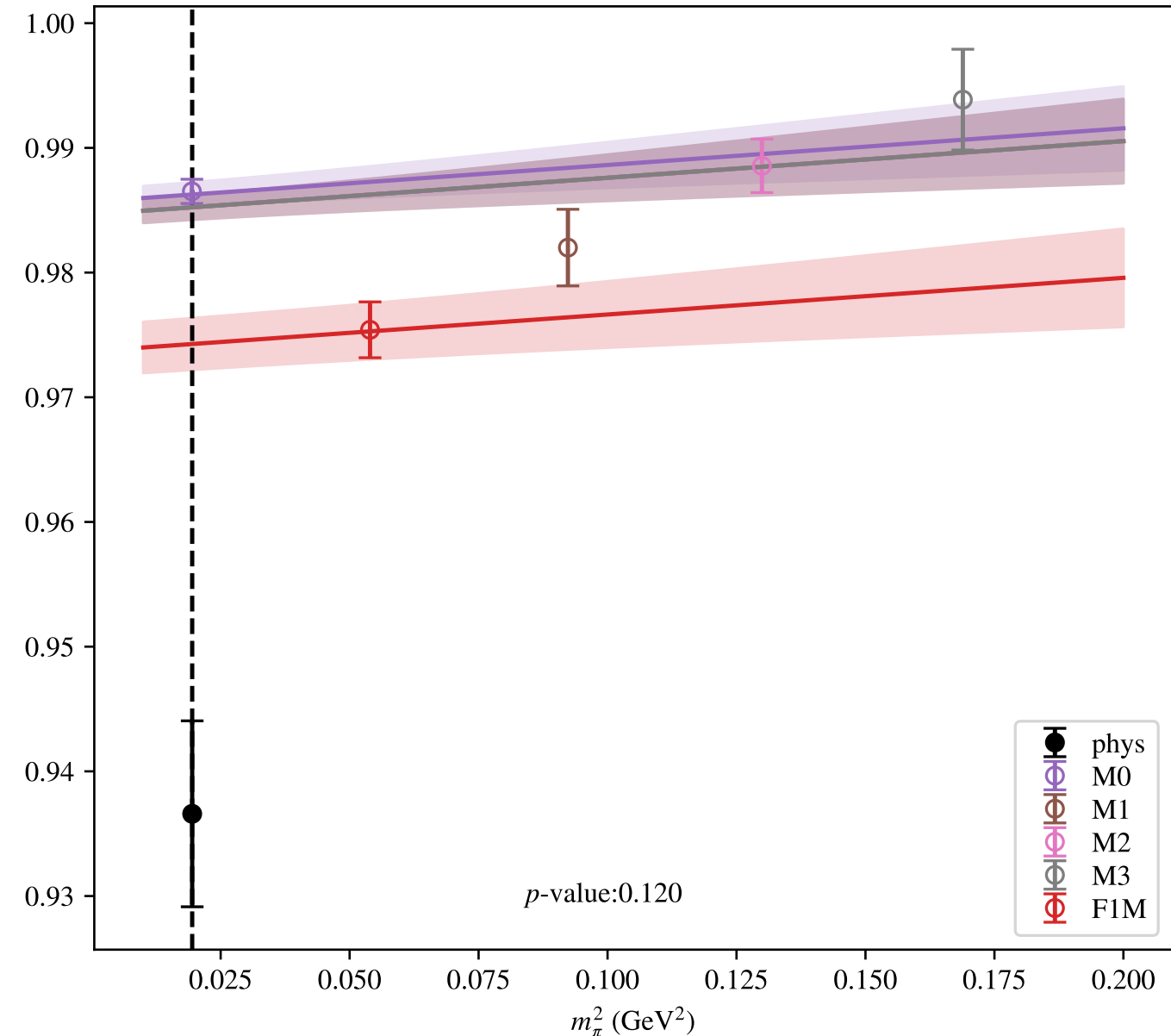


$a^2, m_\pi^2$  (no C),  $\mu = 1.8$  GeV

SSmPP

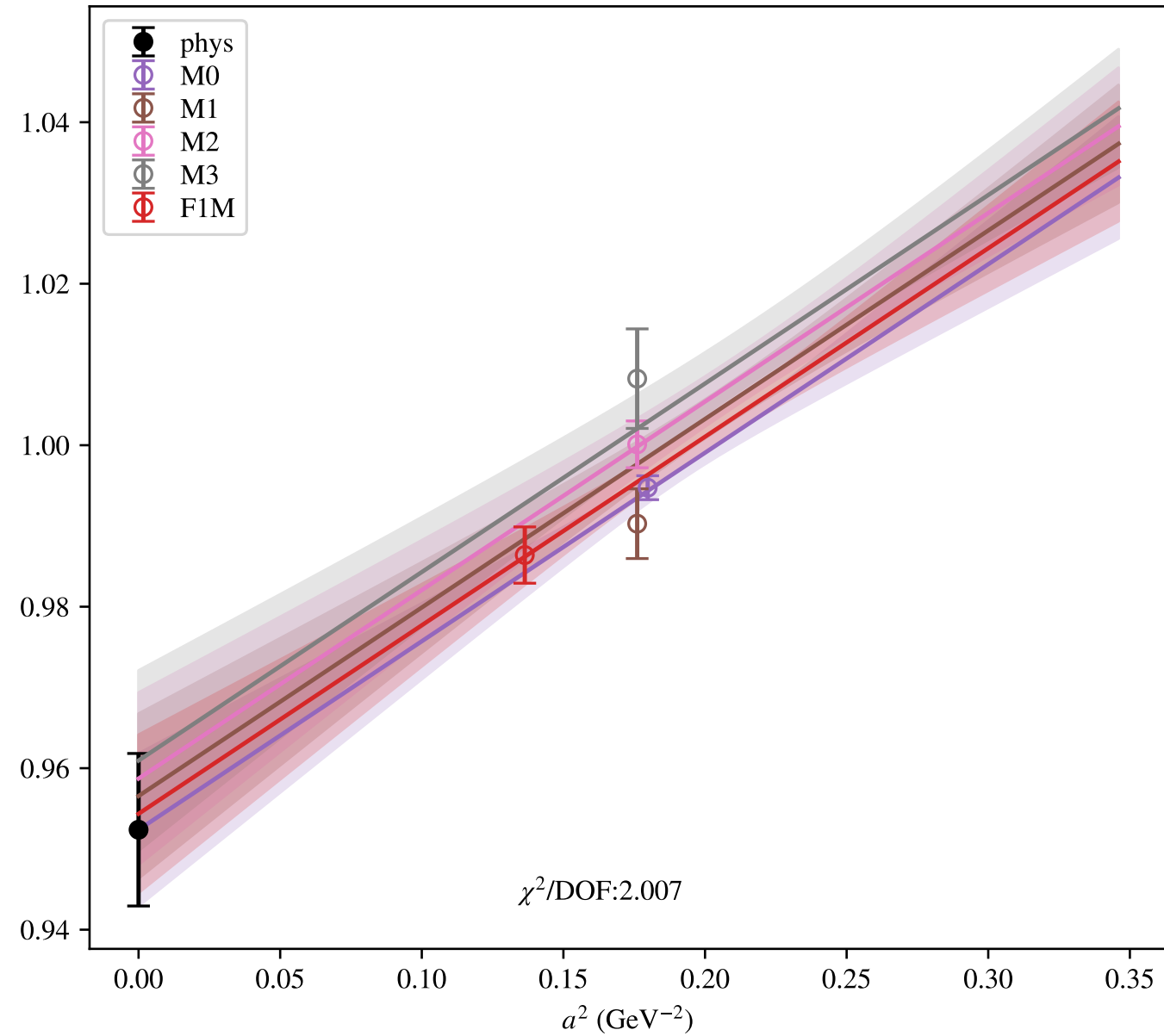


SSmPP

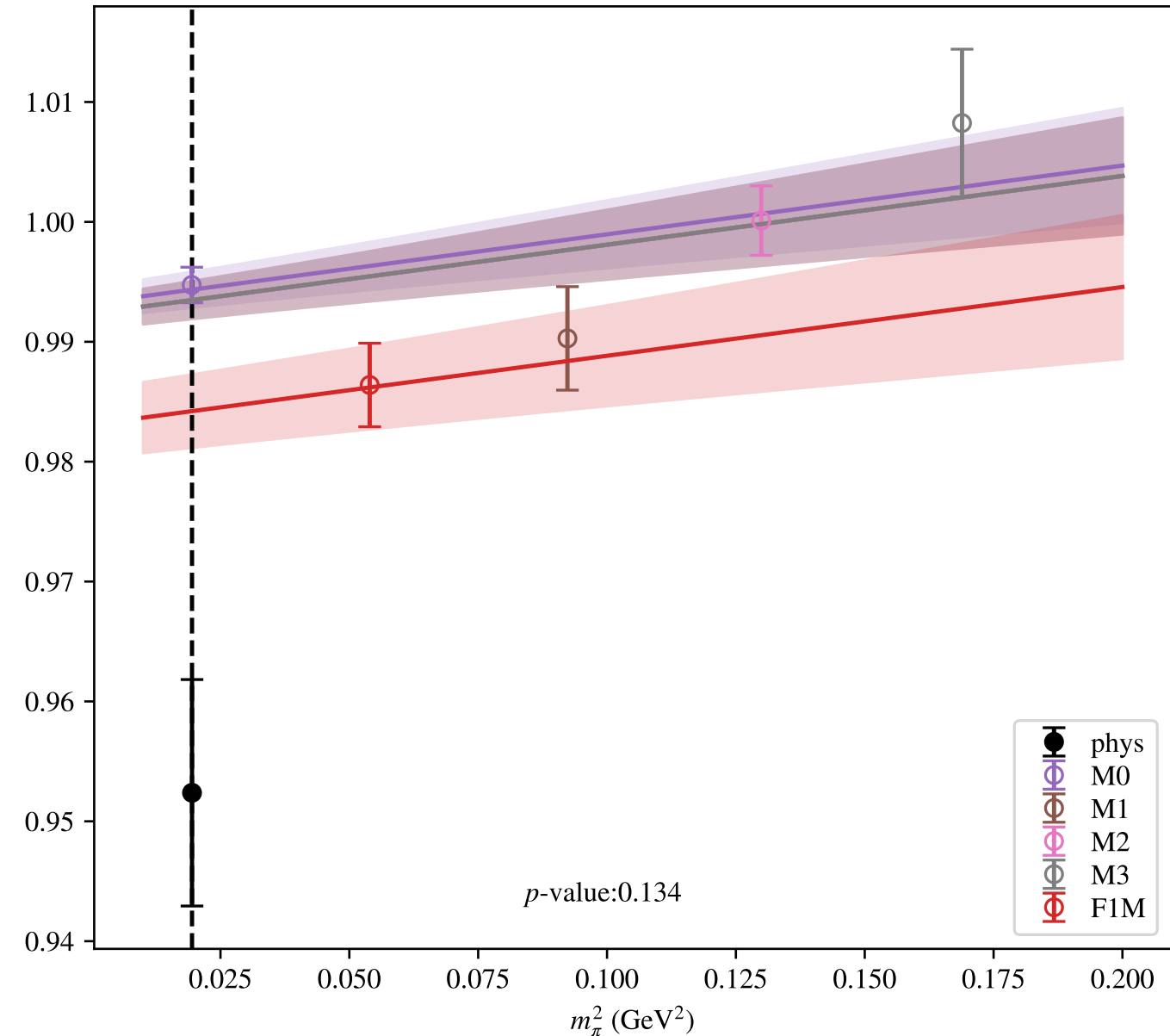


$a^2, m_\pi^2$  (no C),  $\mu = 1.5$  GeV

SSmPP

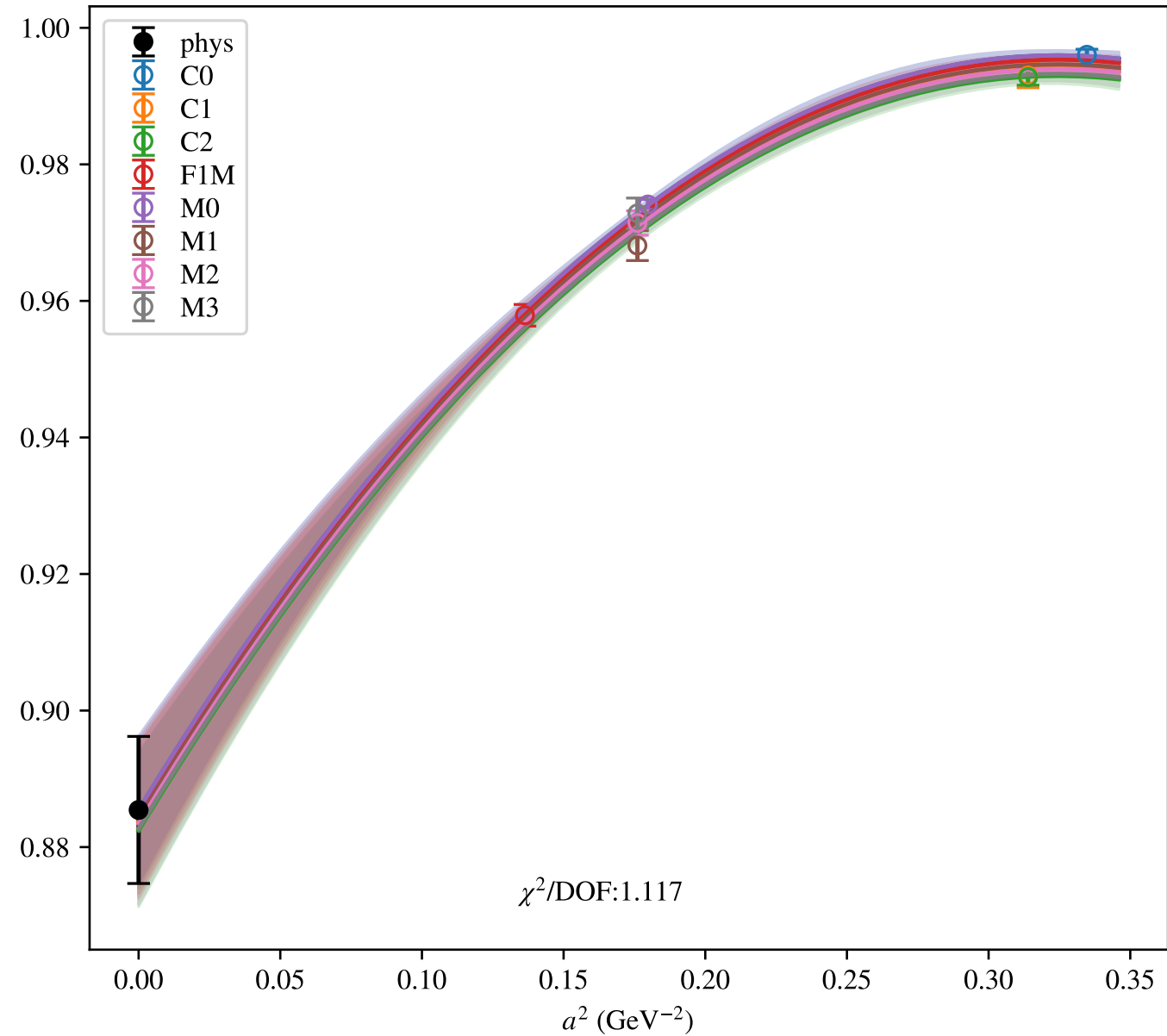


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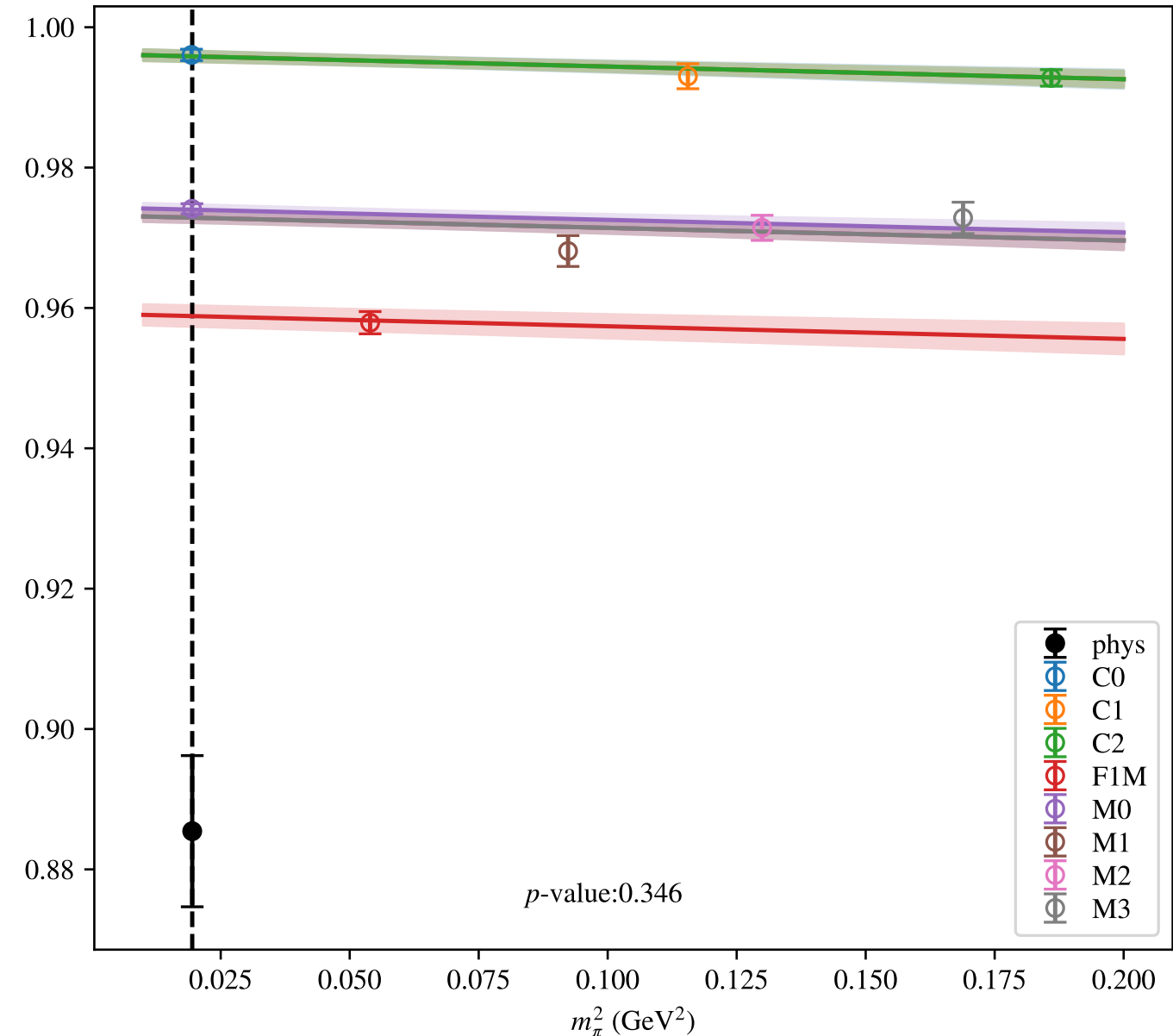


$$a^2, a^4, m_\pi^2, \mu = 2.4 \text{ GeV}$$

SSmPP

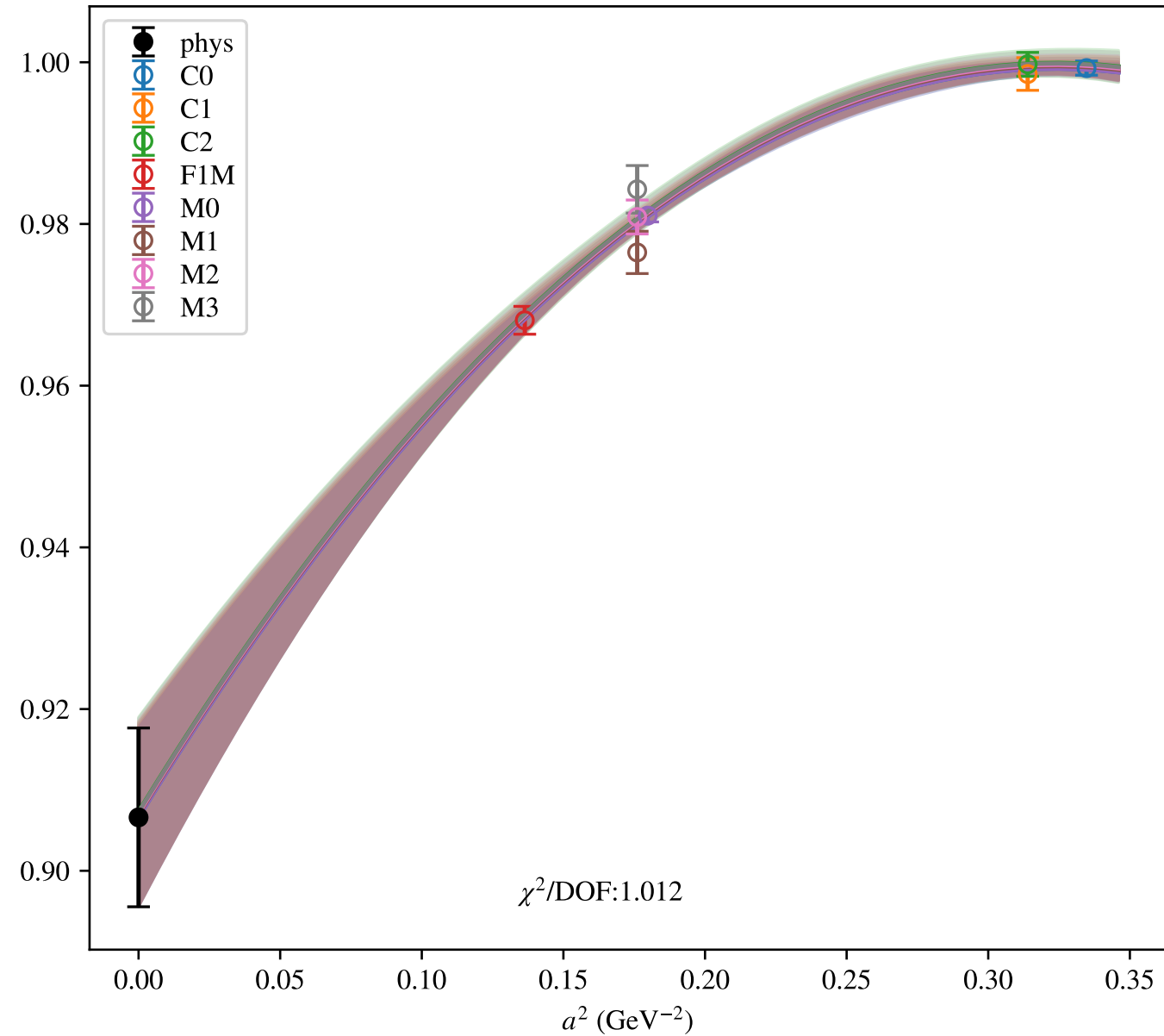


SSmPP

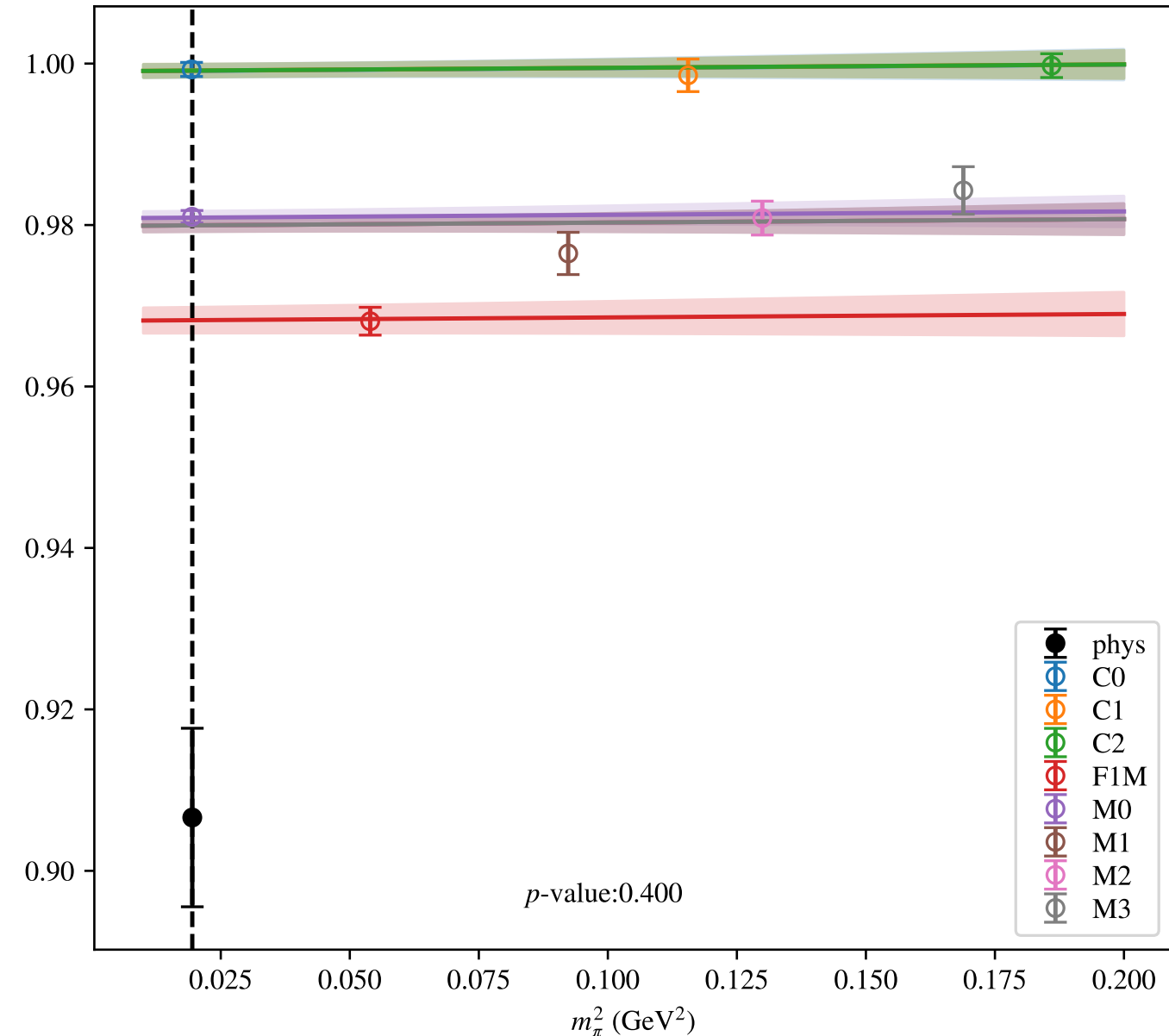


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

SSmPP

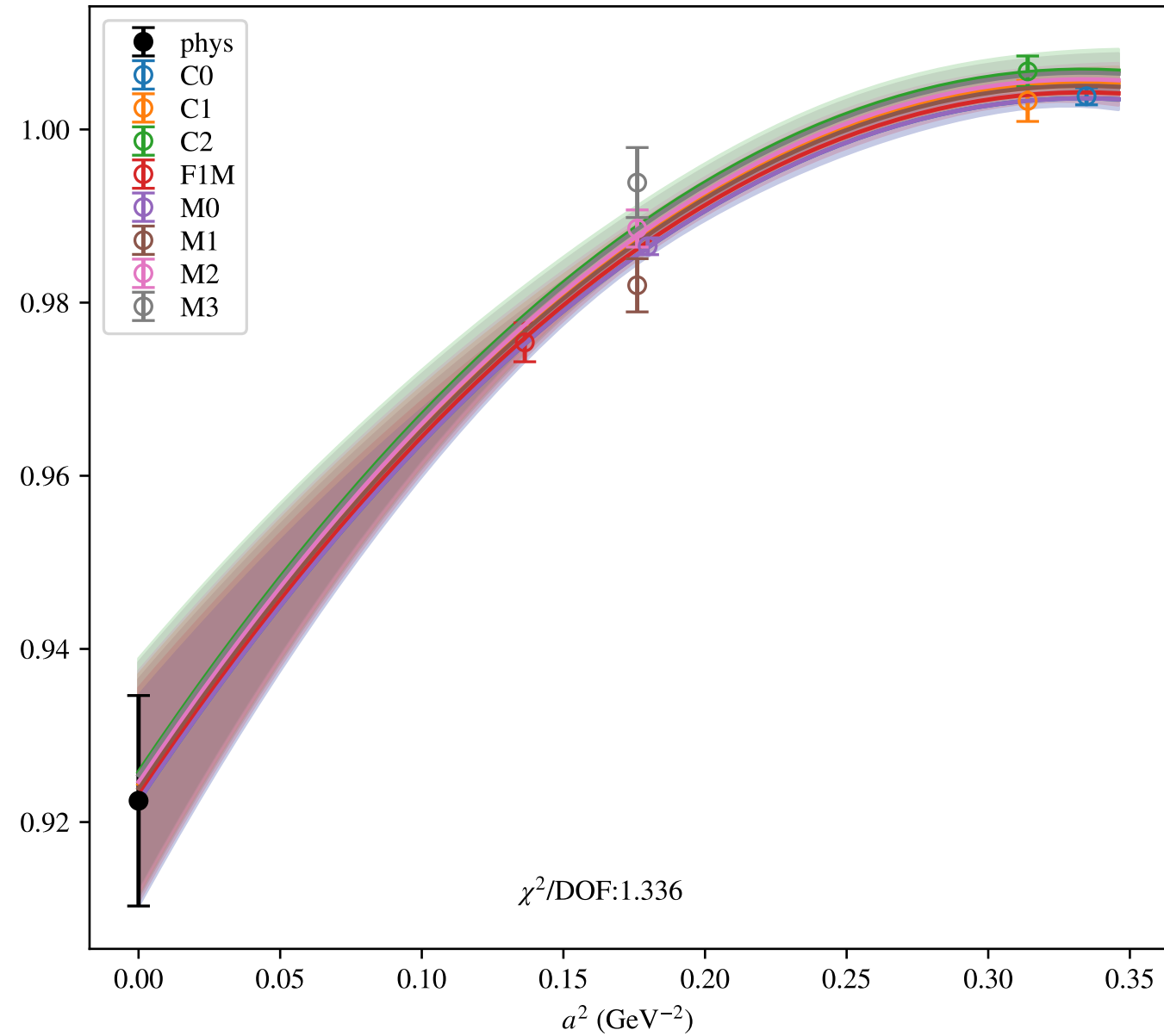


SSmPP

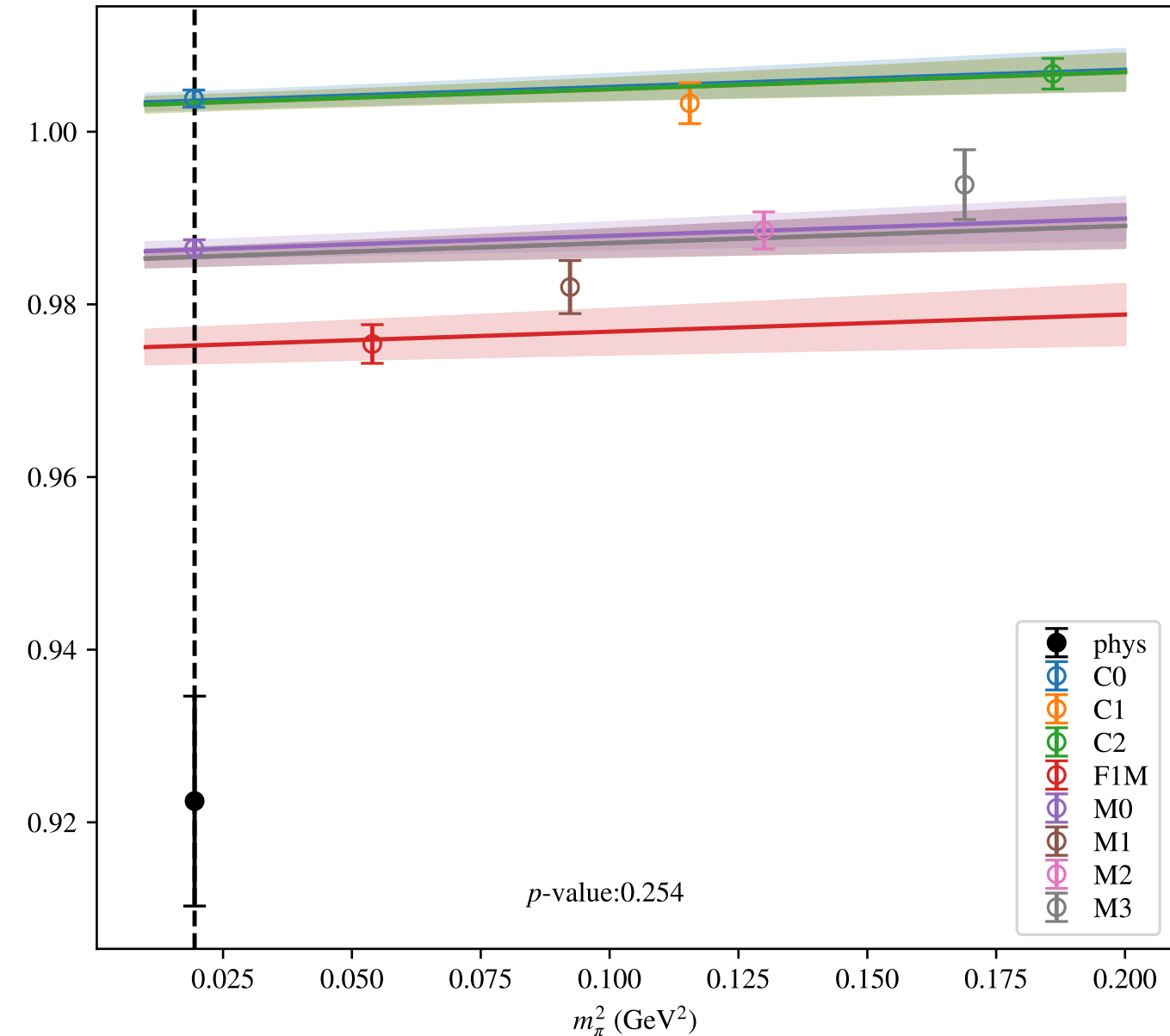


$$a^2, a^4, m_\pi^2, \mu = 1.8 \text{ GeV}$$

SSmPP

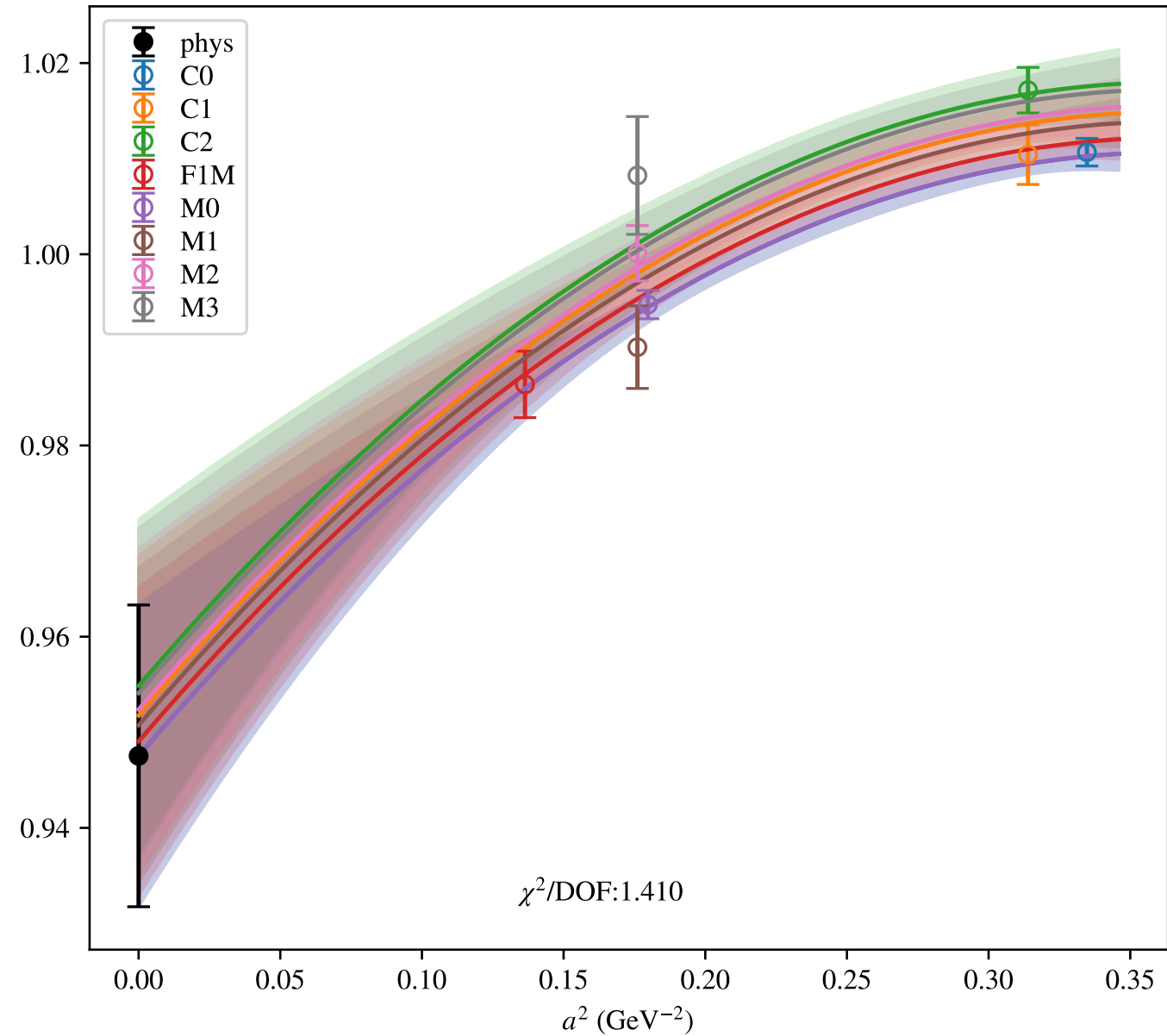


SSmPP

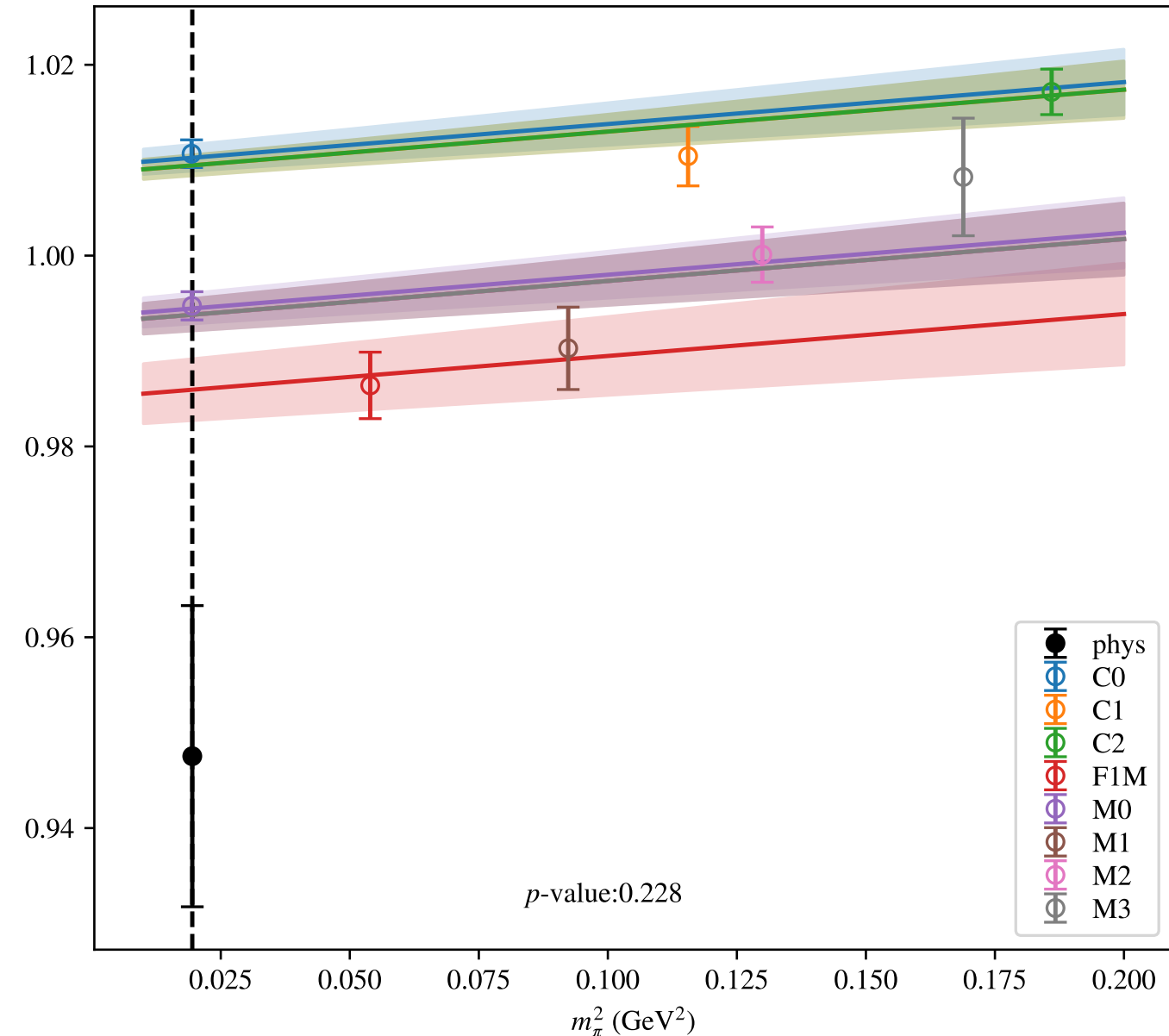


$$a^2, a^4, m_\pi^2, \mu = 1.5 \text{ GeV}$$

SSmPP



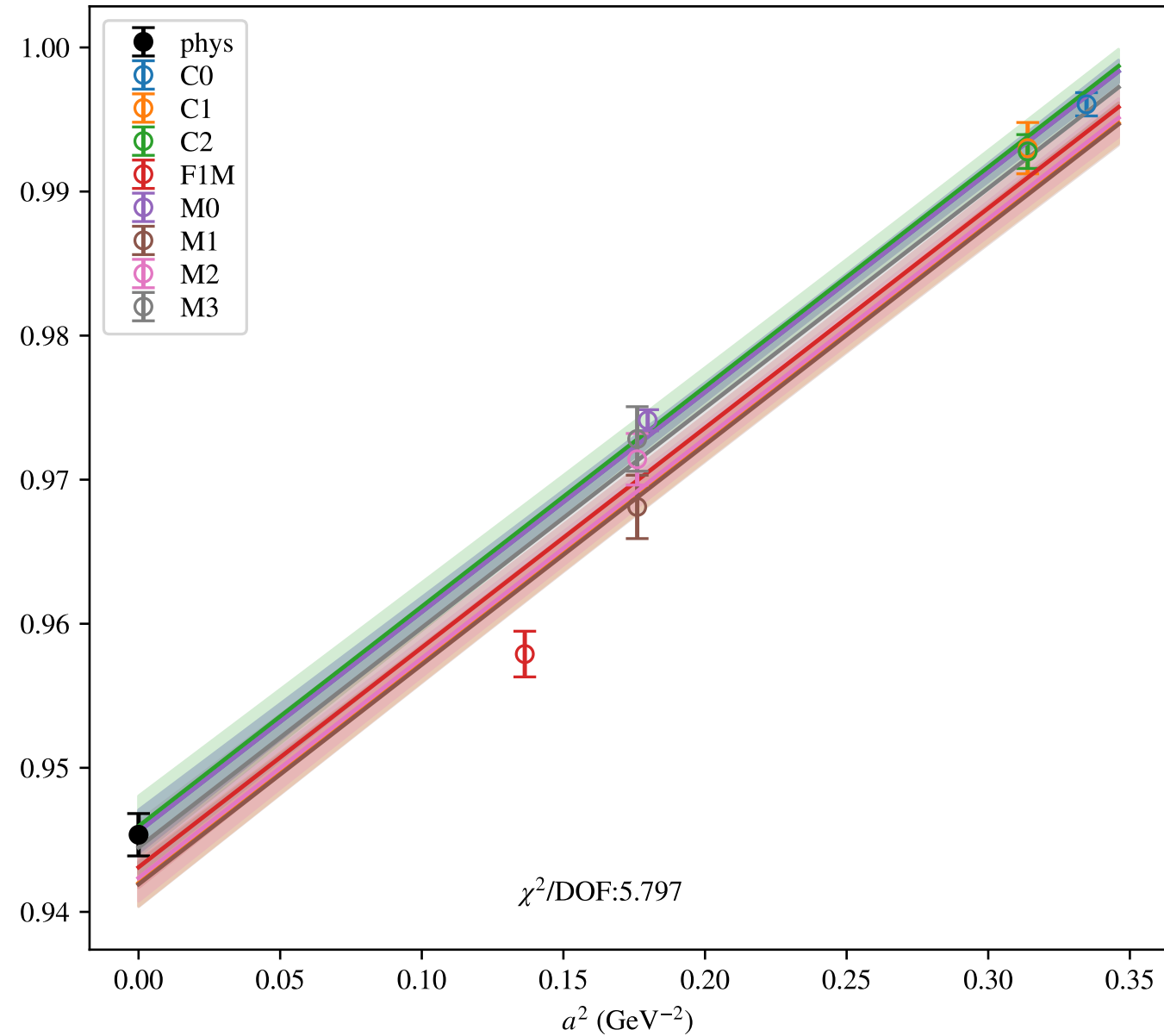
SSmPP



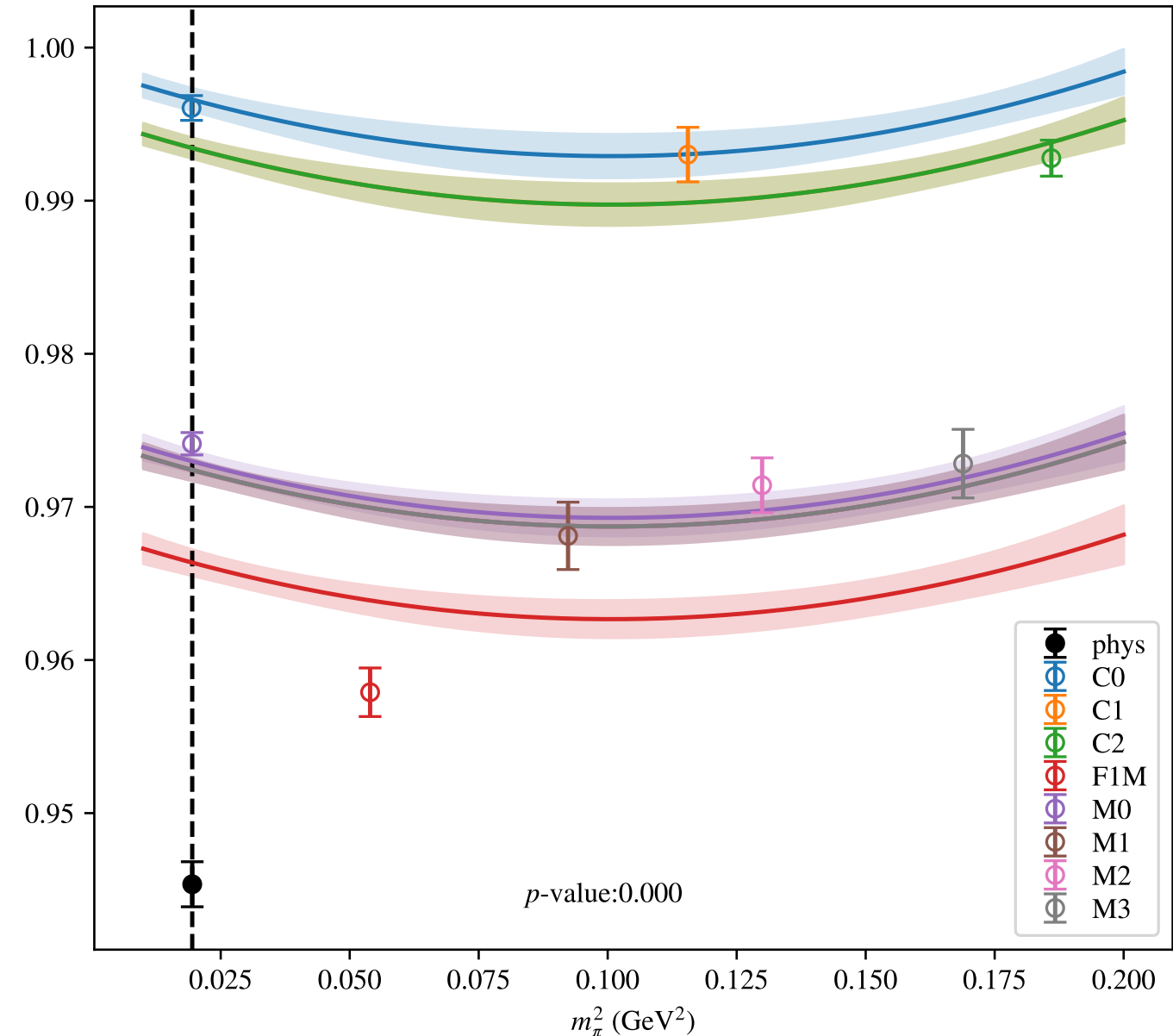


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.4 \text{ GeV}$$

SSmPP

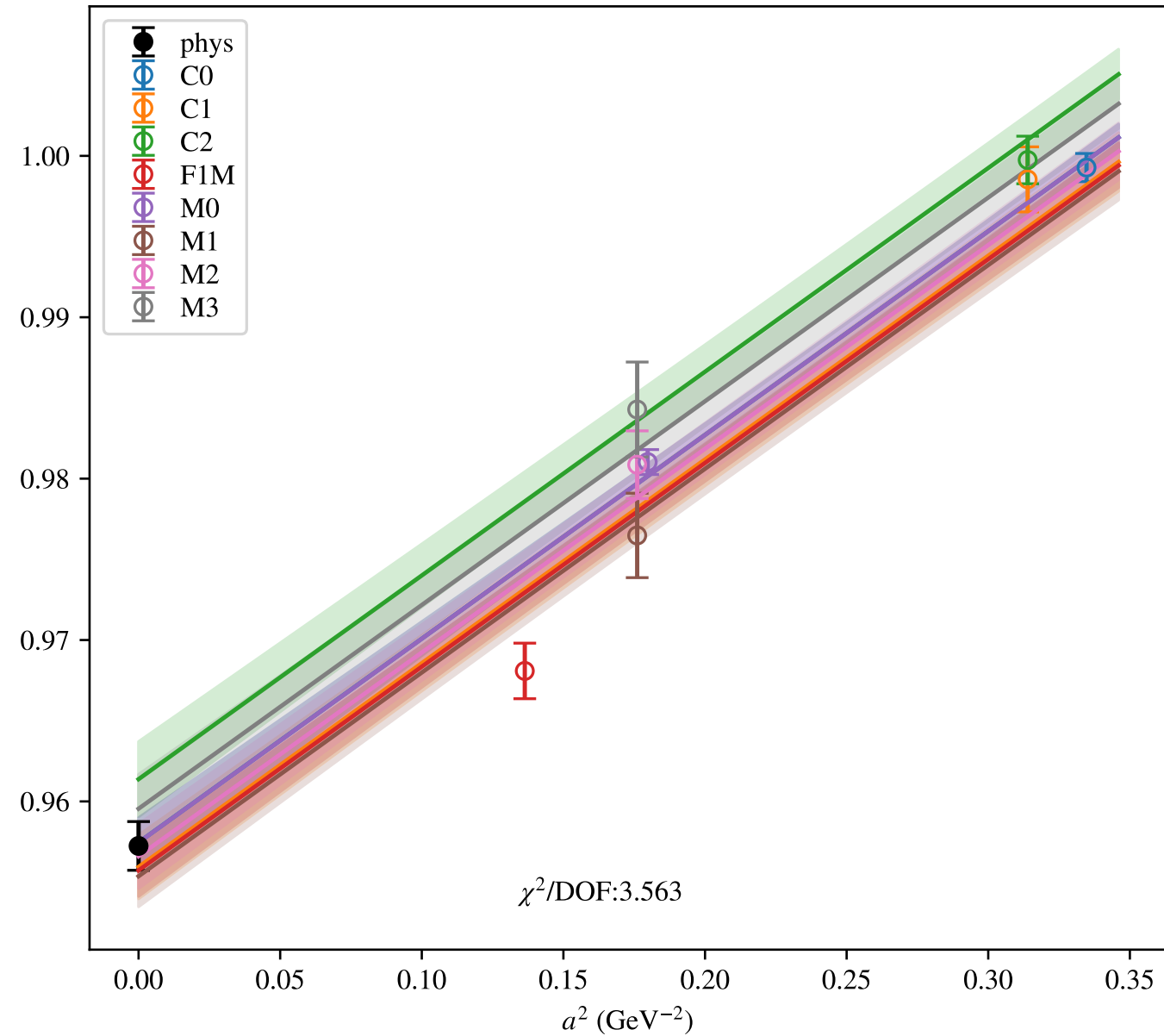


SSmPP

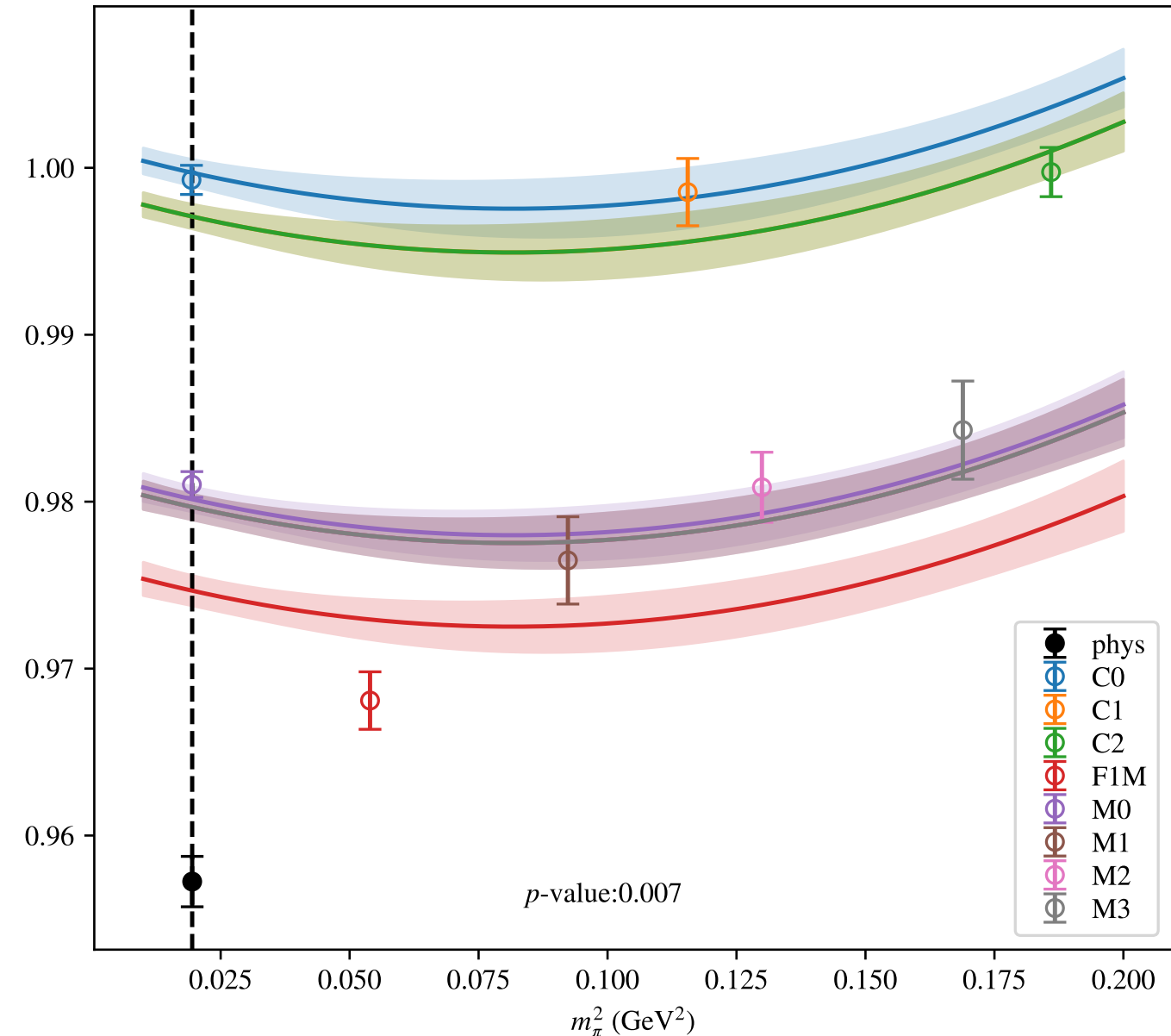


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

SSmPP

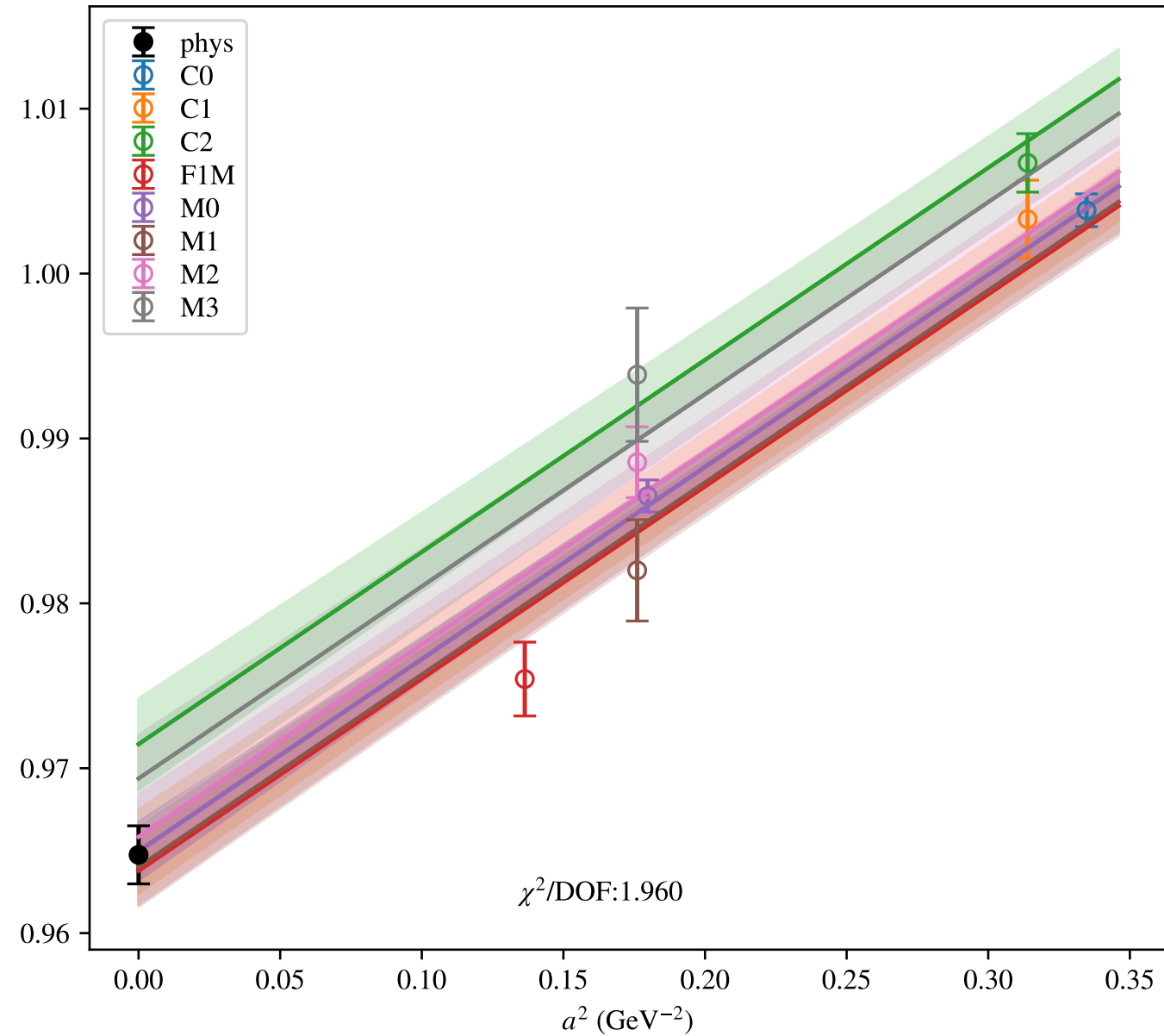


SSmPP

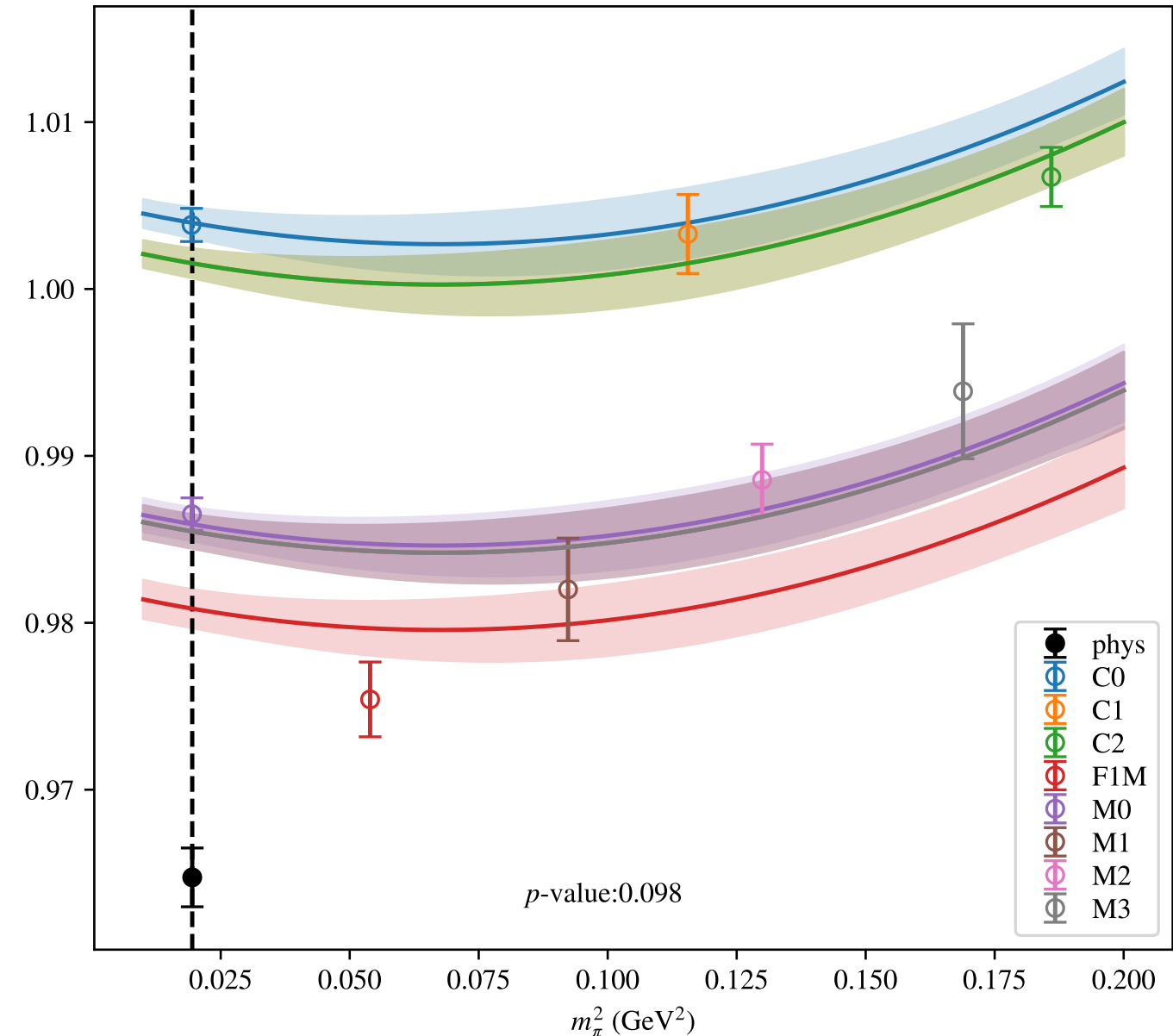


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.8 \text{ GeV}$$

SSmPP

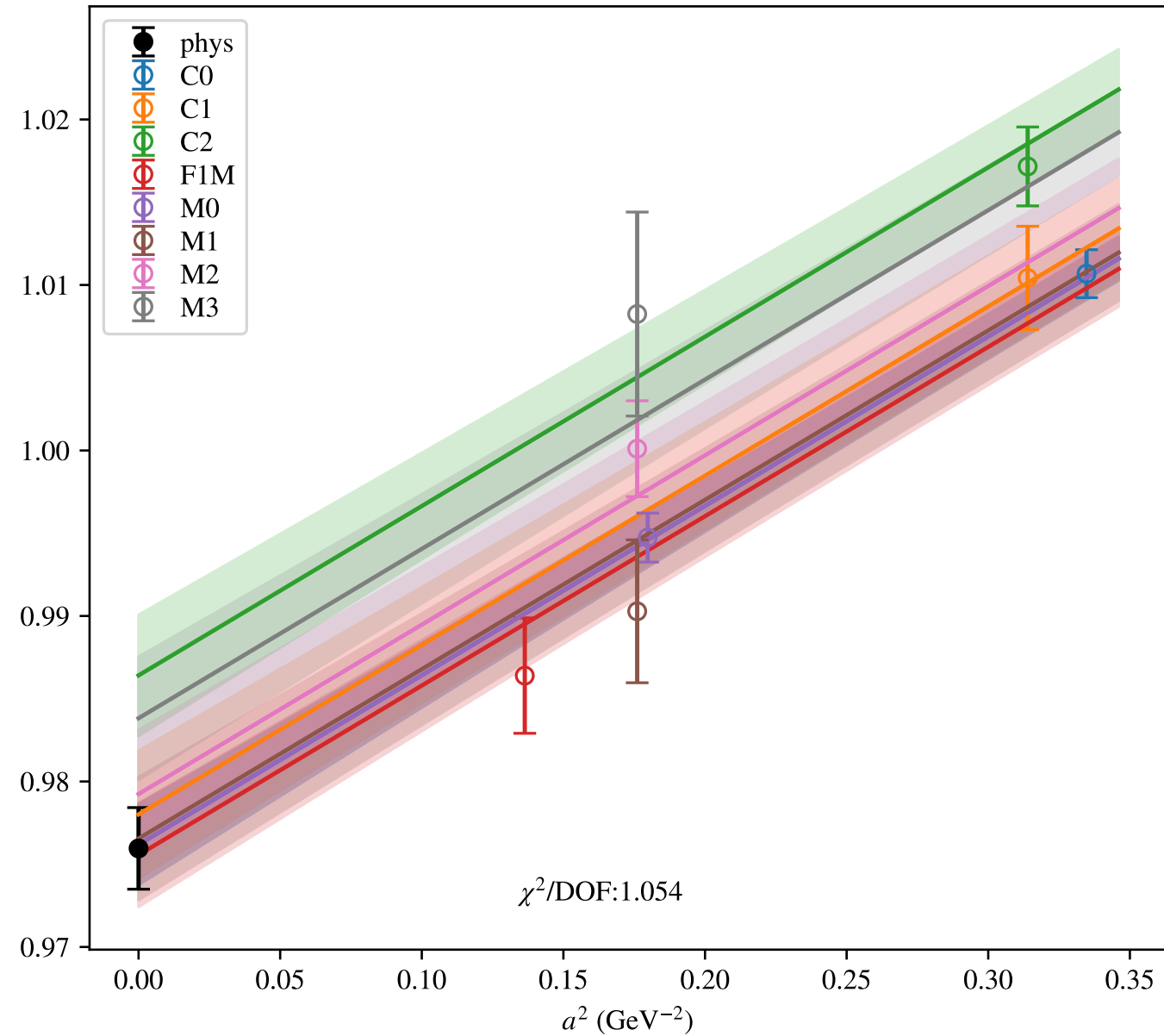


SSmPP

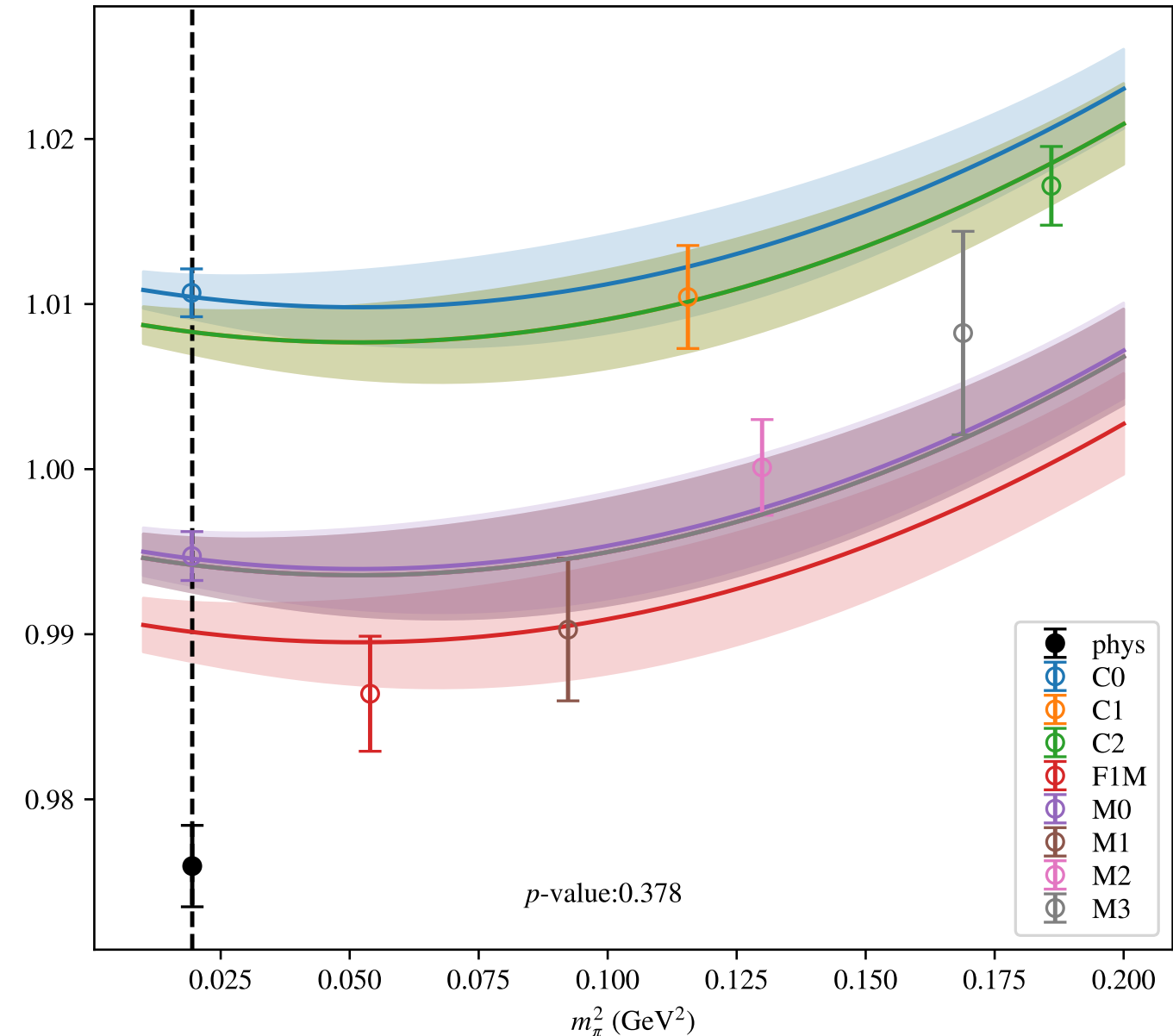


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.5 \text{ GeV}$$

SSmPP

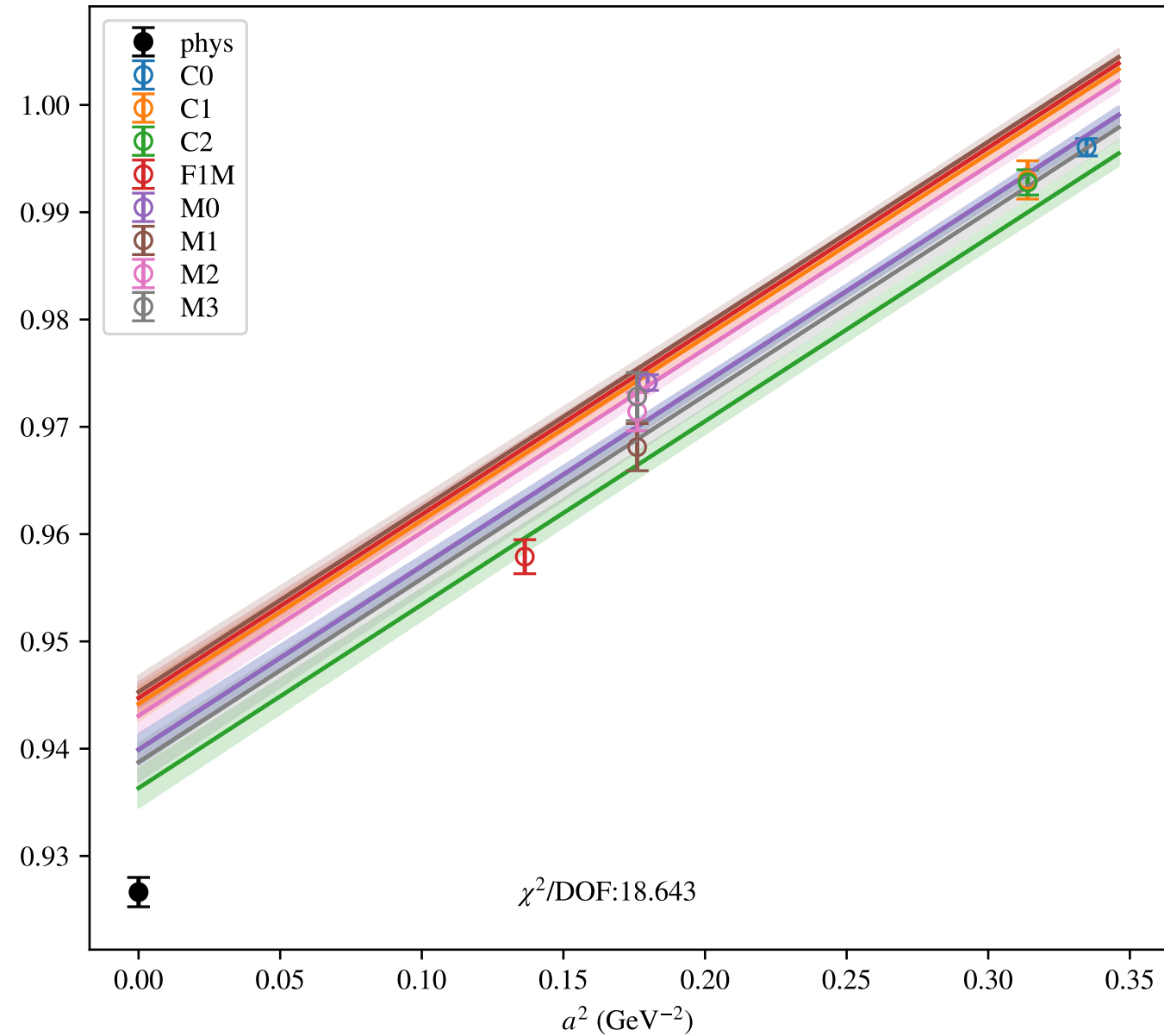


SSmPP

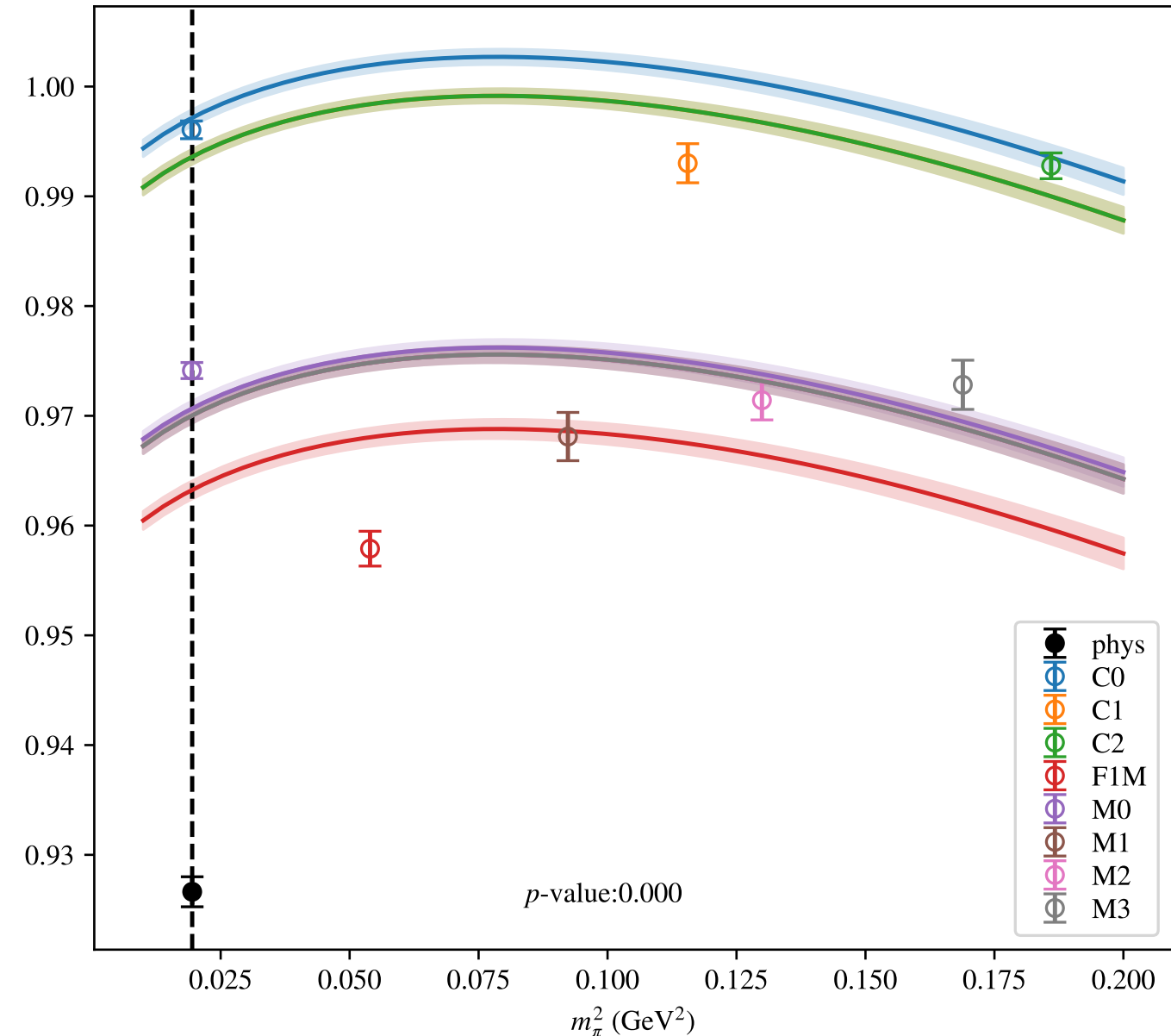


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 2.4 \text{ GeV}$$

SSmPP

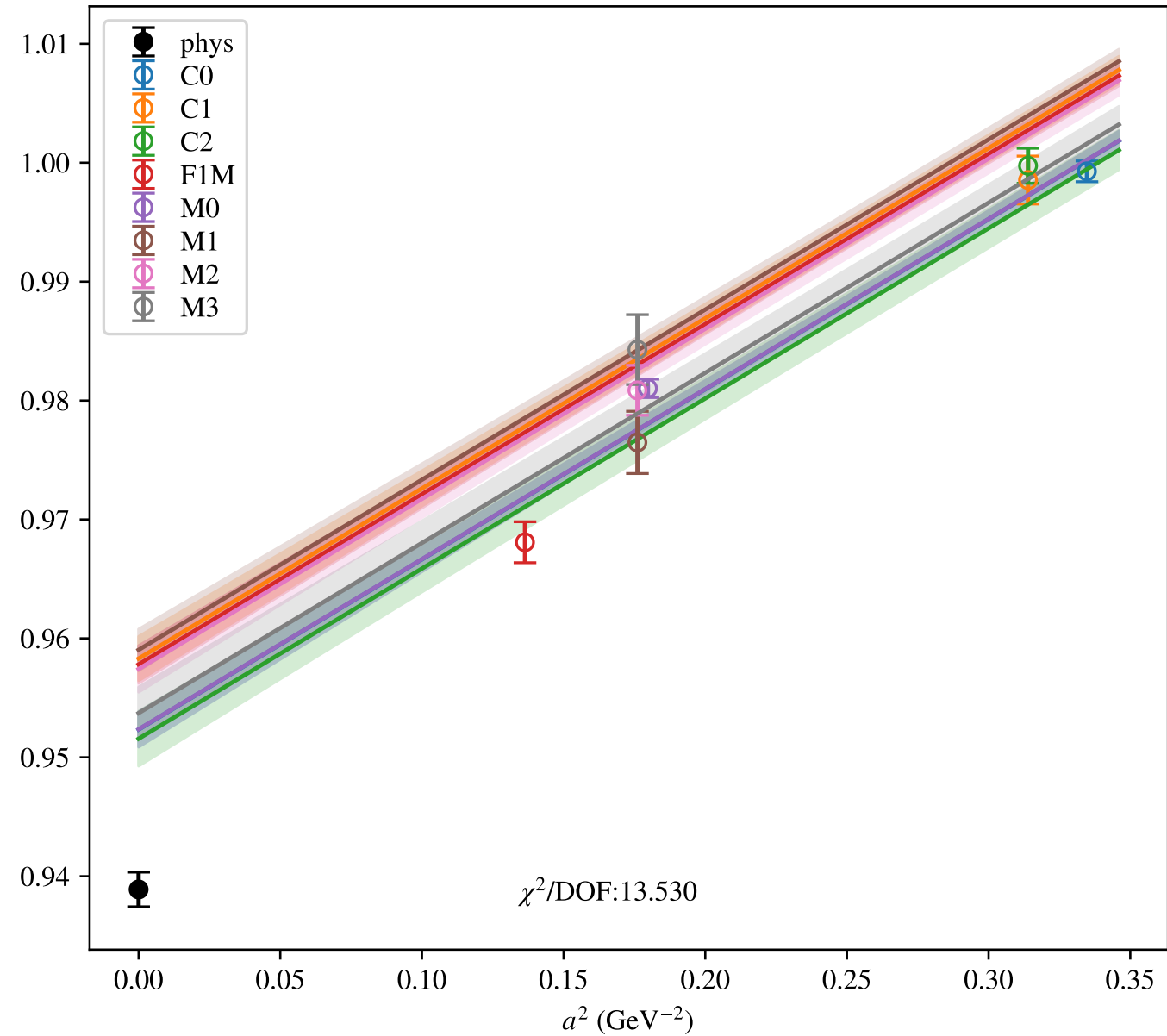


SSmPP

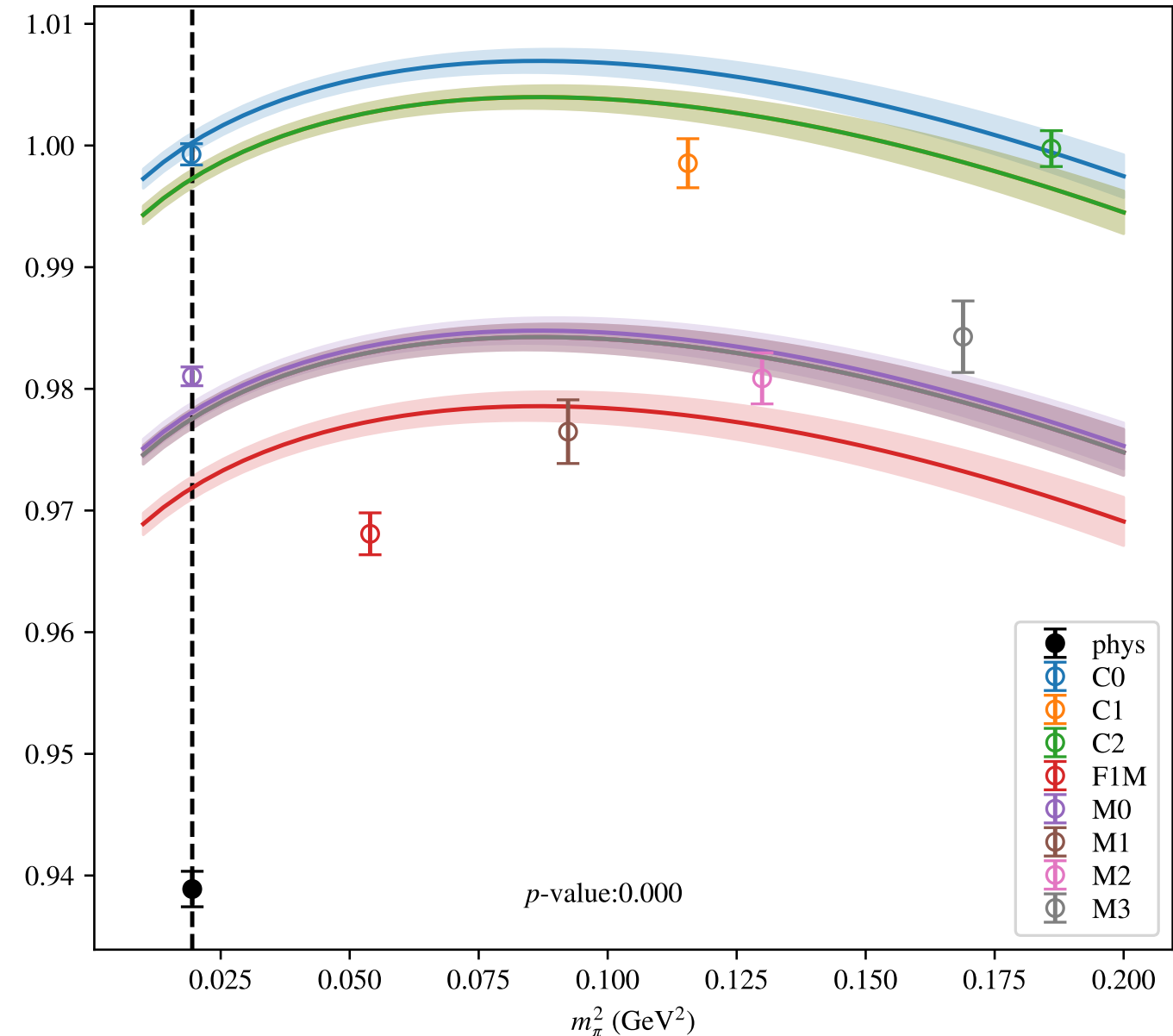


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 2.0 \text{ GeV}$$

SSmPP

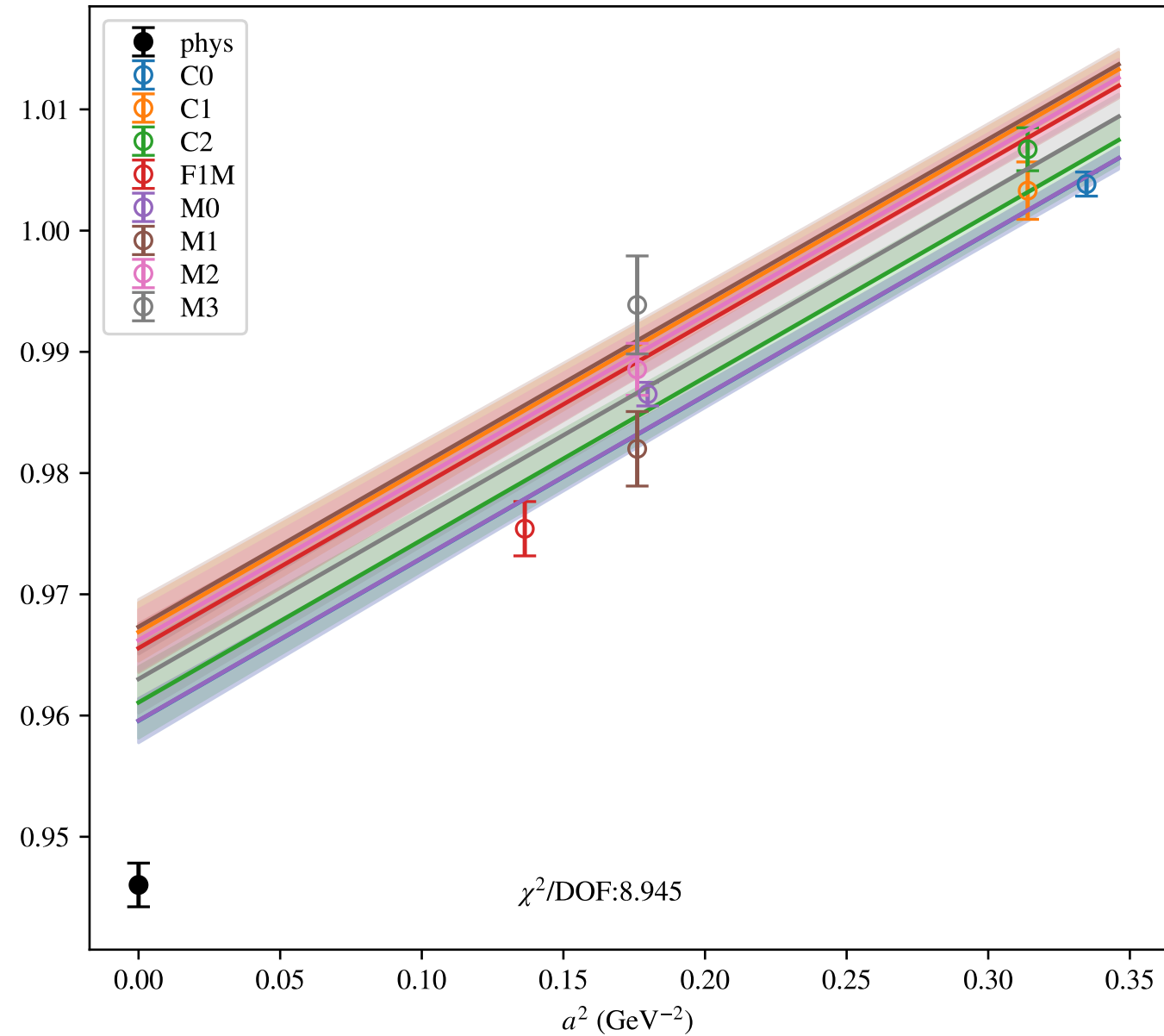


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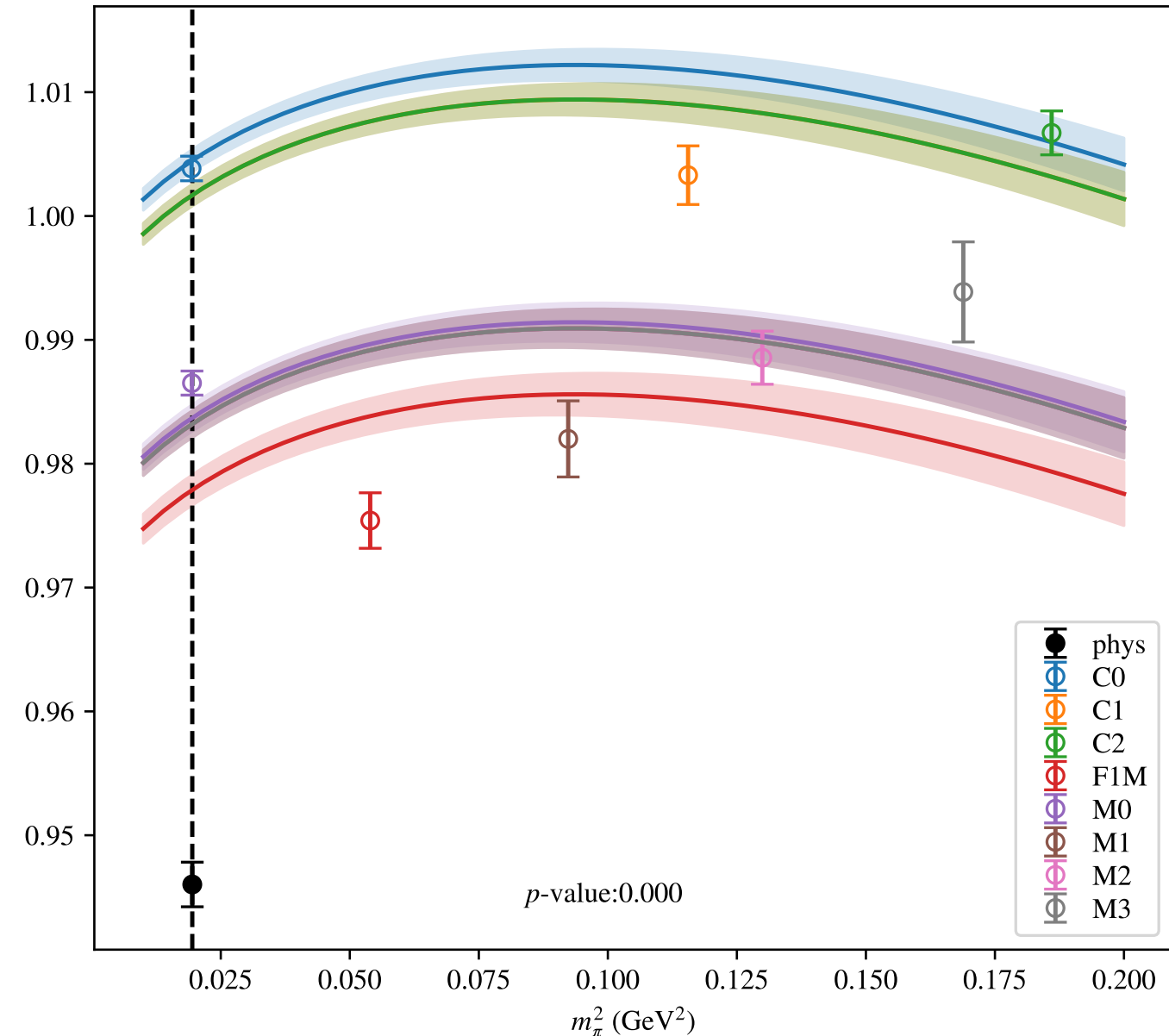


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 1.8 \text{ GeV}$$

SSmPP

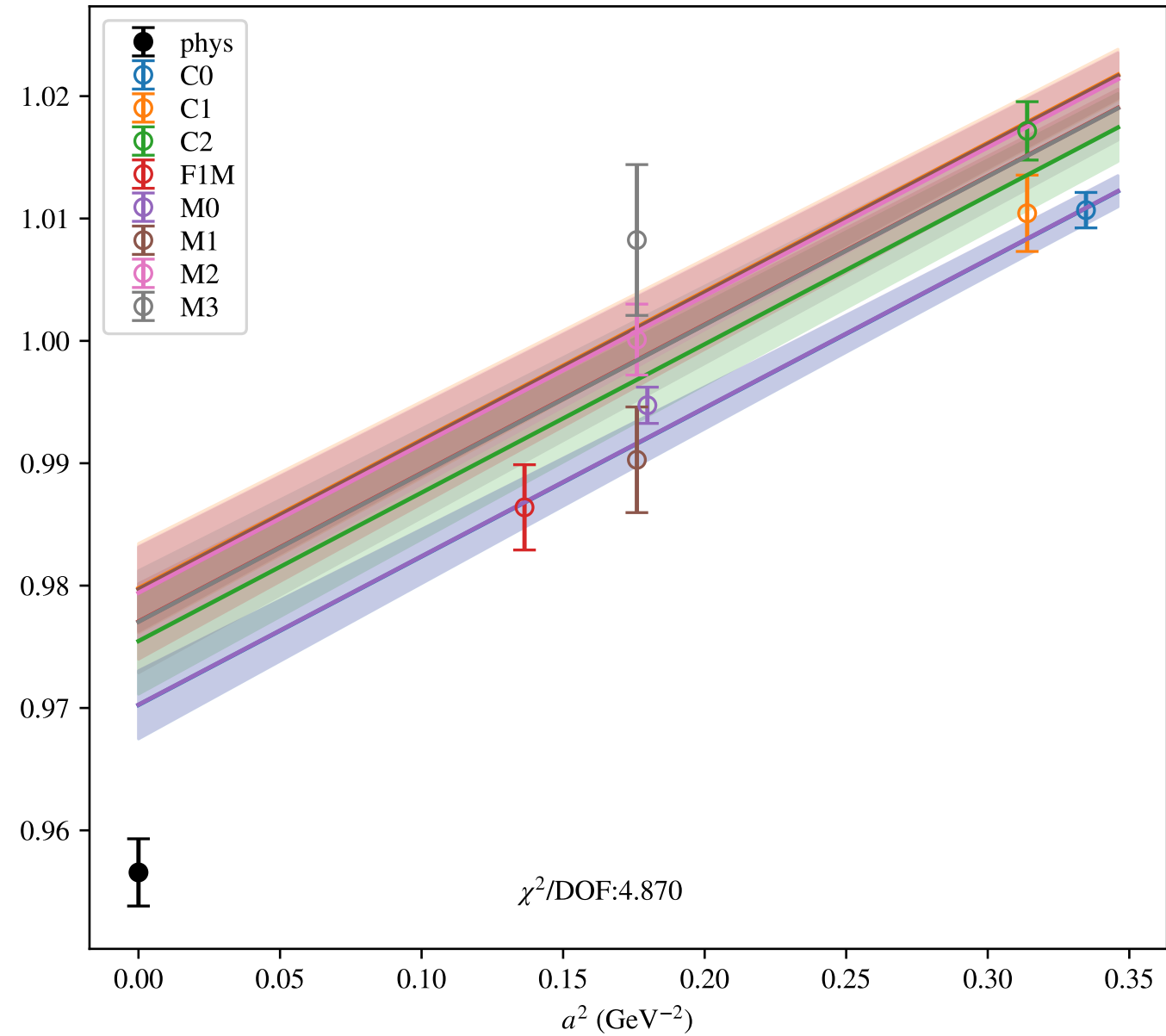


SSmPP

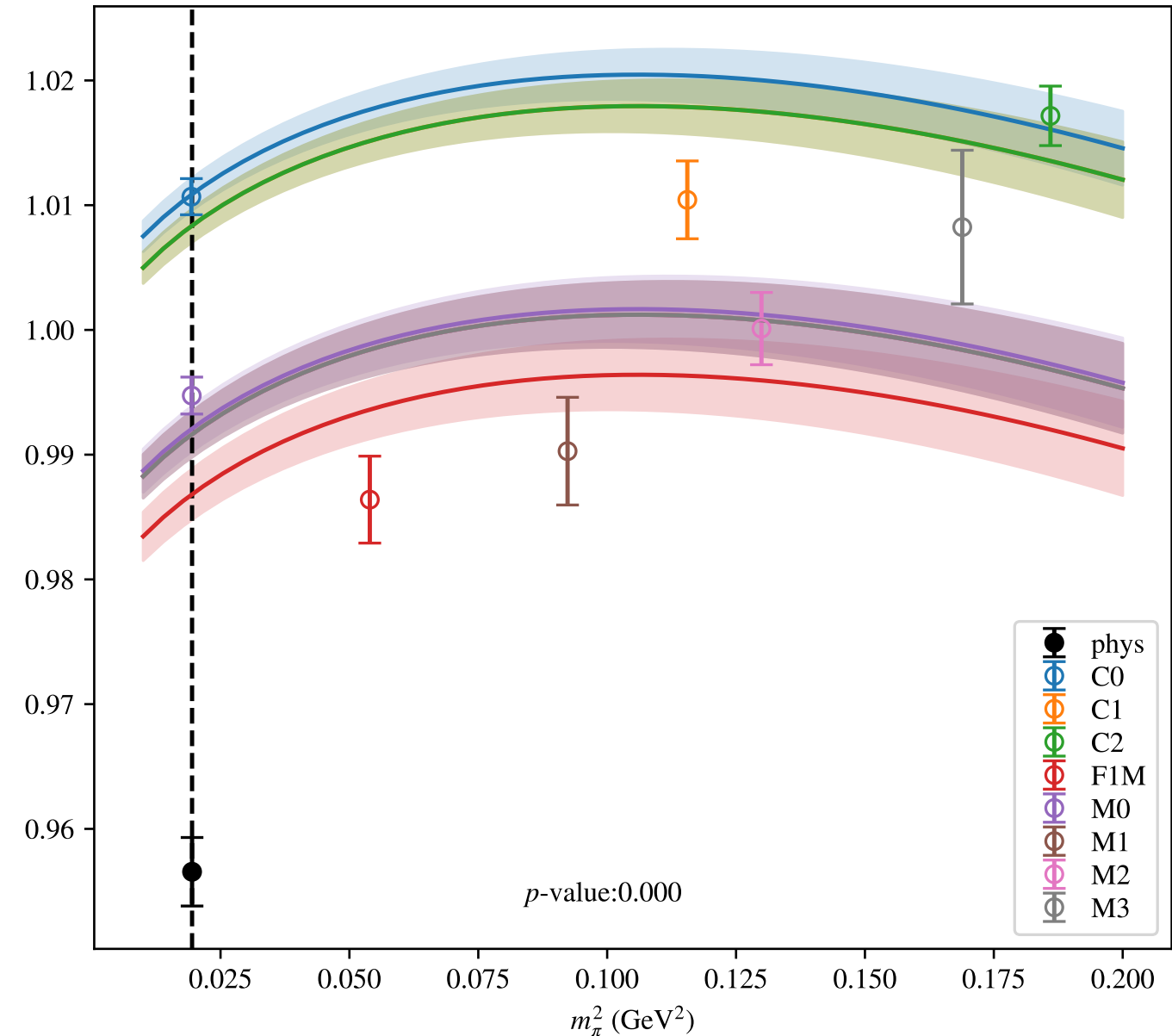


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 1.5 \text{ GeV}$$

SSmPP



SSmPP





## 4 SSpPP

$\mu$ (GeV)	$a^2, m_\pi^2$	$a^2, m_\pi^2$ (no C)	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$	$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2)$
2.4	<b>0.53833(89)</b> : 5.538 (0.0)	<b>0.5585(52)</b> : 1.377 (0.252)	<b>0.5565(86)</b> : 5.756 (0.0)	<b>0.53751(95)</b> : 5.517 (0.0)	<b>0.54787(91)</b> : 14.558 (0.0)
2.0	<b>0.56754(98)</b> : 4.716 (0.0)	<b>0.5922(55)</b> : 0.797 (0.451)	<b>0.5969(91)</b> : 3.427 (0.008)	<b>0.5666(10)</b> : 4.48 (0.001)	<b>0.57742(99)</b> : 12.333 (0.0)
1.8	<b>0.5840(12)</b> : 3.149 (0.008)	<b>0.6114(59)</b> : 0.41 (0.664)	<b>0.6210(99)</b> : 1.37 (0.242)	<b>0.5832(11)</b> : 3.371 (0.009)	<b>0.5943(12)</b> : 8.019 (0.0)
1.5	<b>0.6089(17)</b> : 1.887 (0.093)	<b>0.6400(71)</b> : 0.233 (0.792)	<b>0.657(12)</b> : 0.272 (0.896)	<b>0.6084(15)</b> : 2.247 (0.061)	<b>0.6199(18)</b> : 4.156 (0.001)

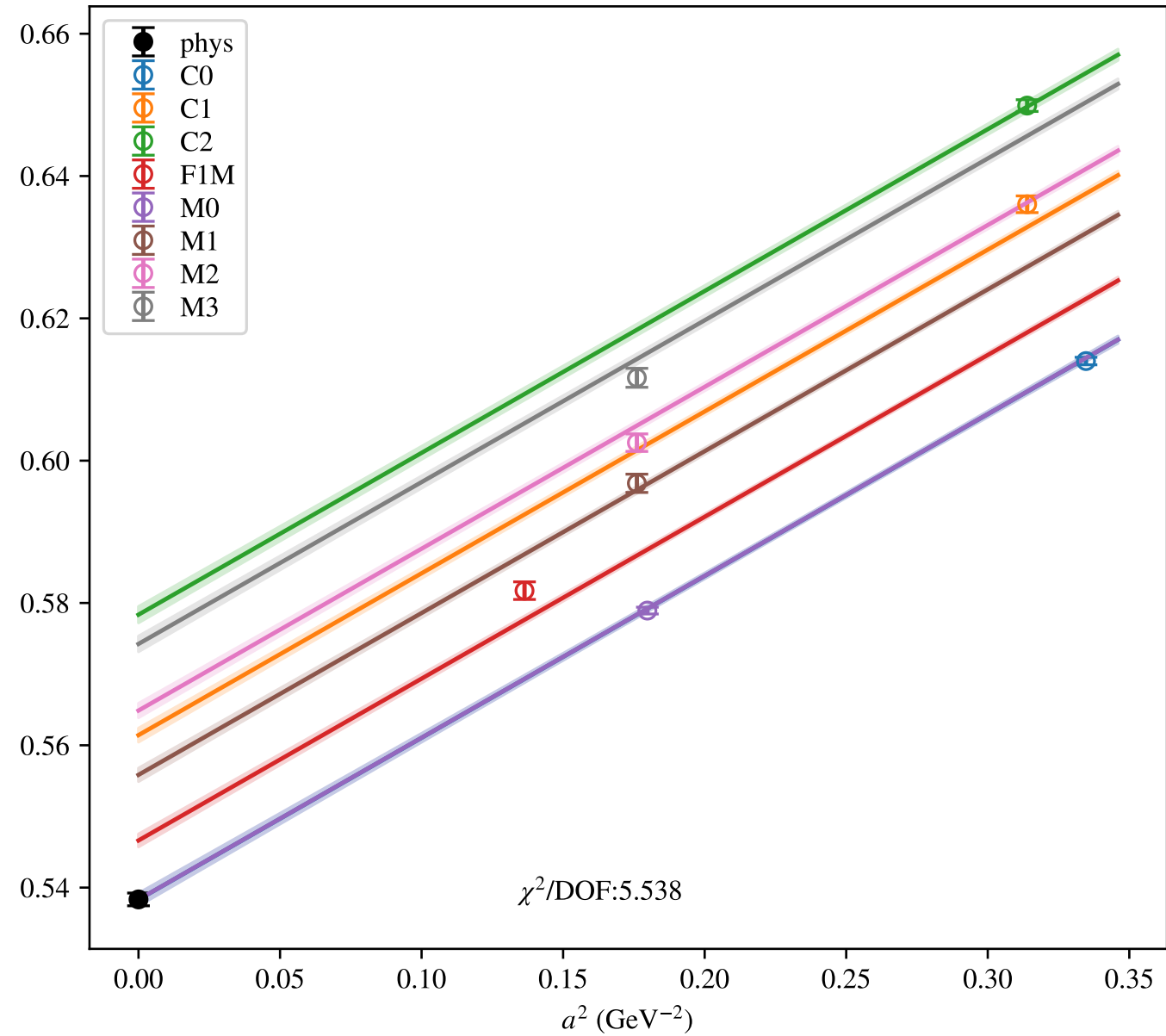
Table 7: Physical point value from chiral and continuum extrapolation at renormalisation scale  $\mu$ . Entries are **value(error)**:  $\chi^2/\text{DOF}$  ( $p$ -value).

$\mu$ (GeV)		$a^2, m_\pi^2$	$a^2, m_\pi^2$ (no C)	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$	$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2)$
2.4	$\alpha$	0.4261(69)	0.205(56)	0.11(14)	0.4334(77)	0.4100(68)
	$\beta$	0.00765(14)	0.00685(23)	0.00751(15)	0.00953(77)	0.01234(14)
2.0	$\alpha$	0.3507(67)	0.096(54)	-0.1(13)	0.3581(75)	0.3364(67)
	$\beta$	0.00799(18)	0.00723(30)	0.00774(17)	0.01010(85)	0.01272(17)
1.8	$\alpha$	0.3254(70)	0.052(56)	-0.2(13)	0.3312(75)	0.3104(69)
	$\beta$	0.00812(19)	0.00748(31)	0.00782(18)	0.00967(87)	0.01287(19)
1.5	$\alpha$	0.2889(77)	-0.008	-0.3(15)	0.2926(76)	0.2734(76)
	$\beta$	0.00829(22)	0.00784(37)	0.00792(18)	0.0092(10)	0.01303(22)

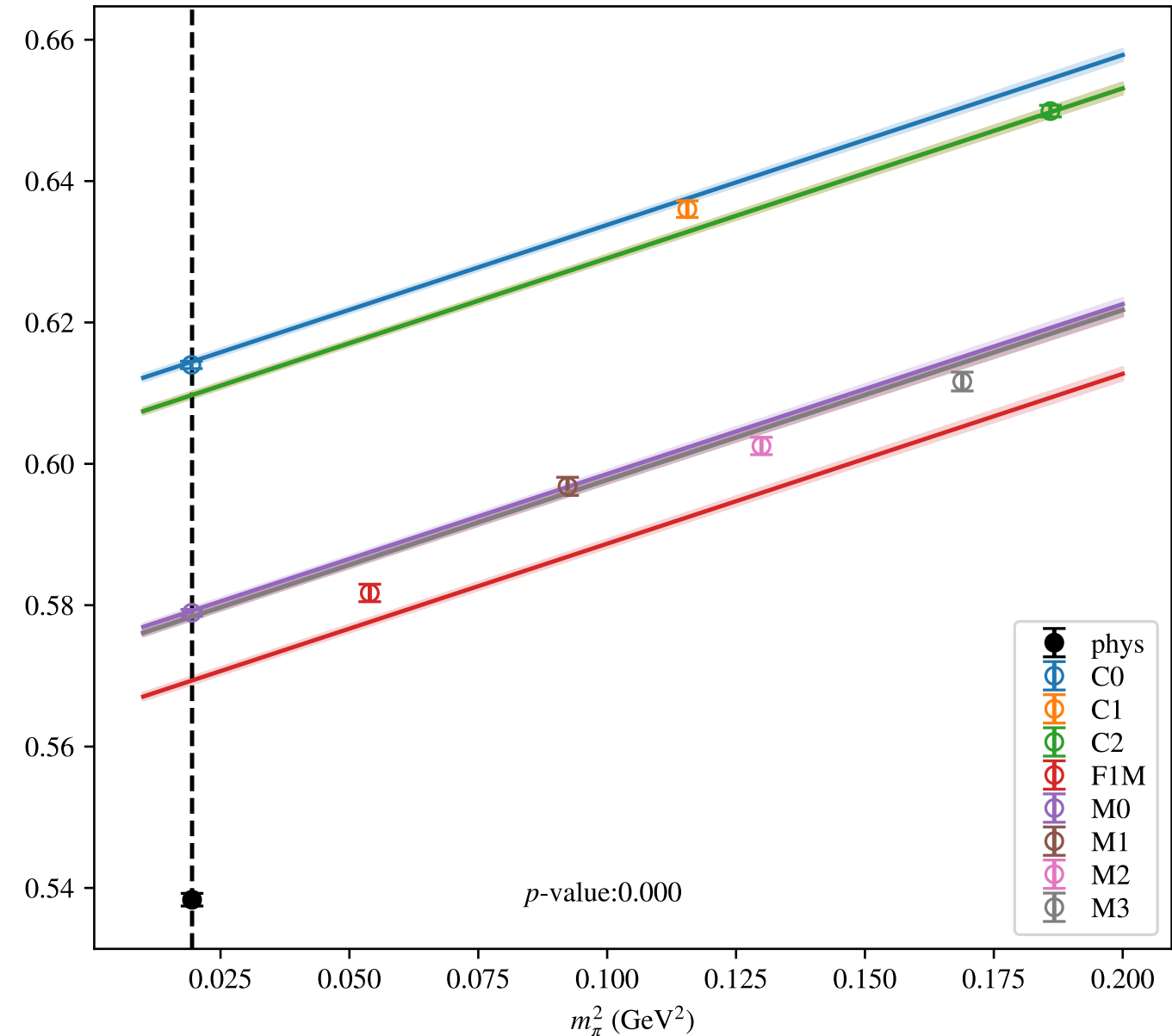
Table 8: Fit values of coefficients in  $B = B_0(1 + \alpha a^2 + \beta \frac{m_\pi^2}{f_\pi^2} + \dots)$ .

$$a^2, m_\pi^2, \mu = 2.4 \text{ GeV}$$

SSpPP

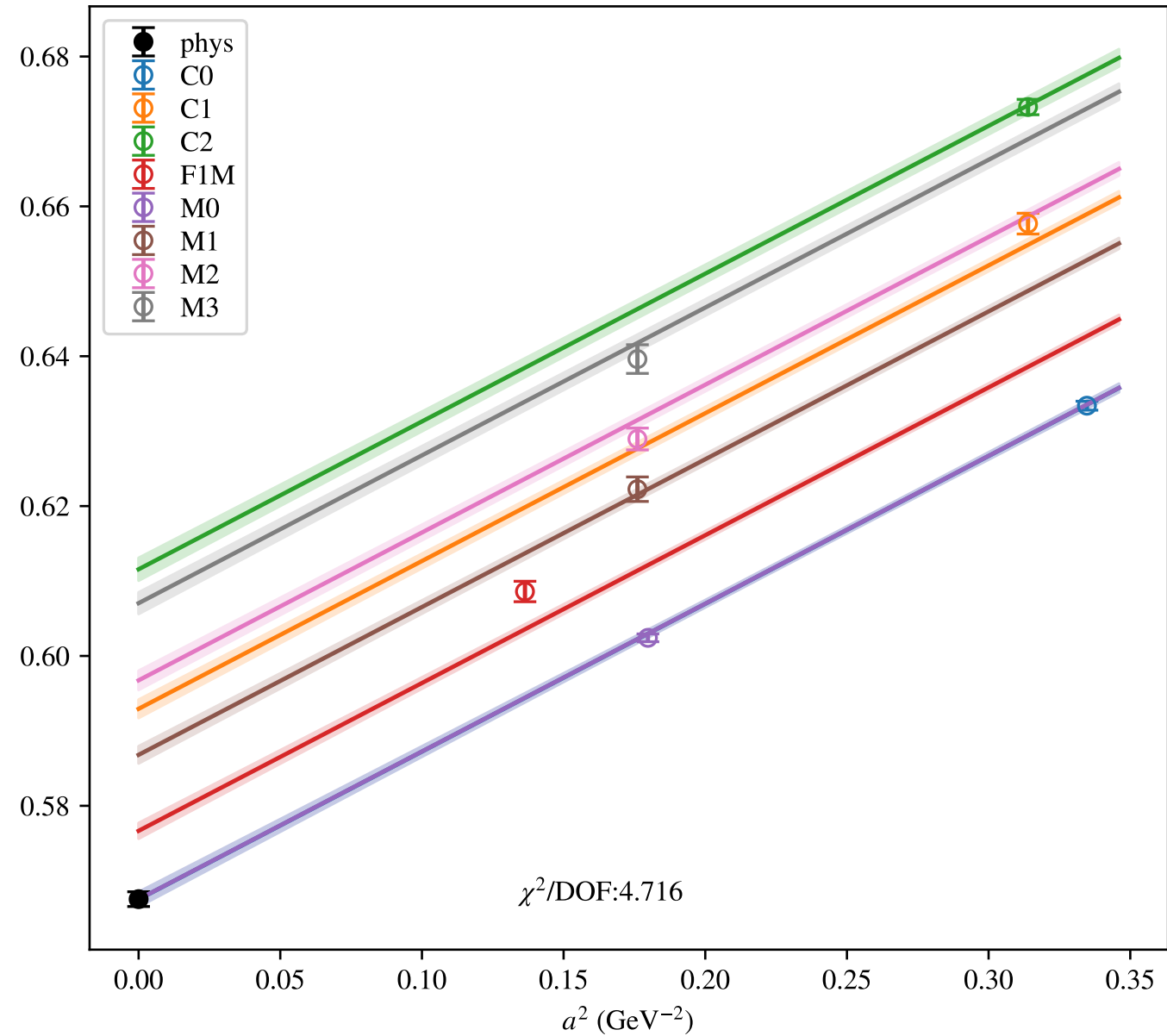


SSpPP

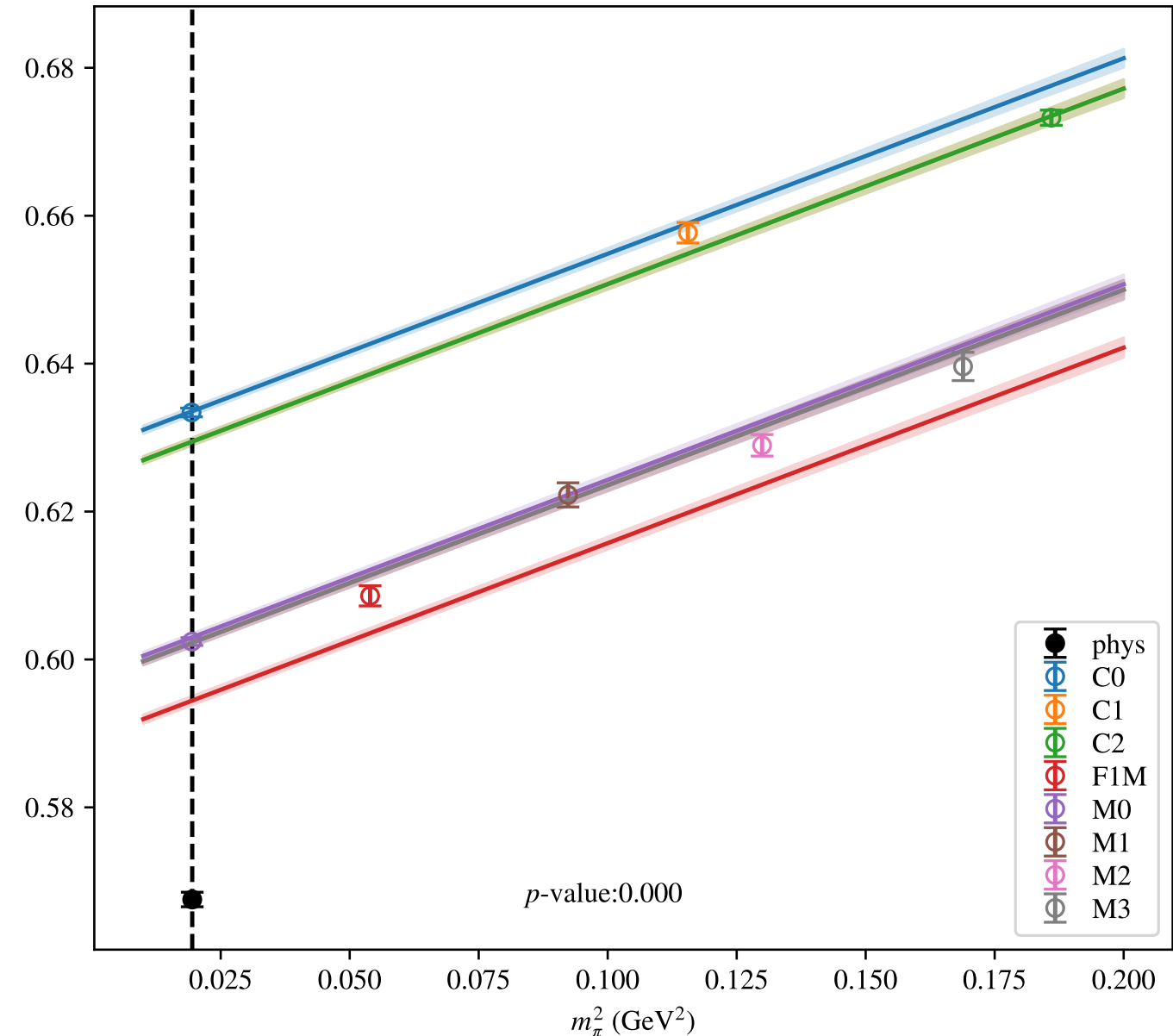


$$a^2, m_\pi^2, \mu = 2.0 \text{ GeV}$$

SSpPP

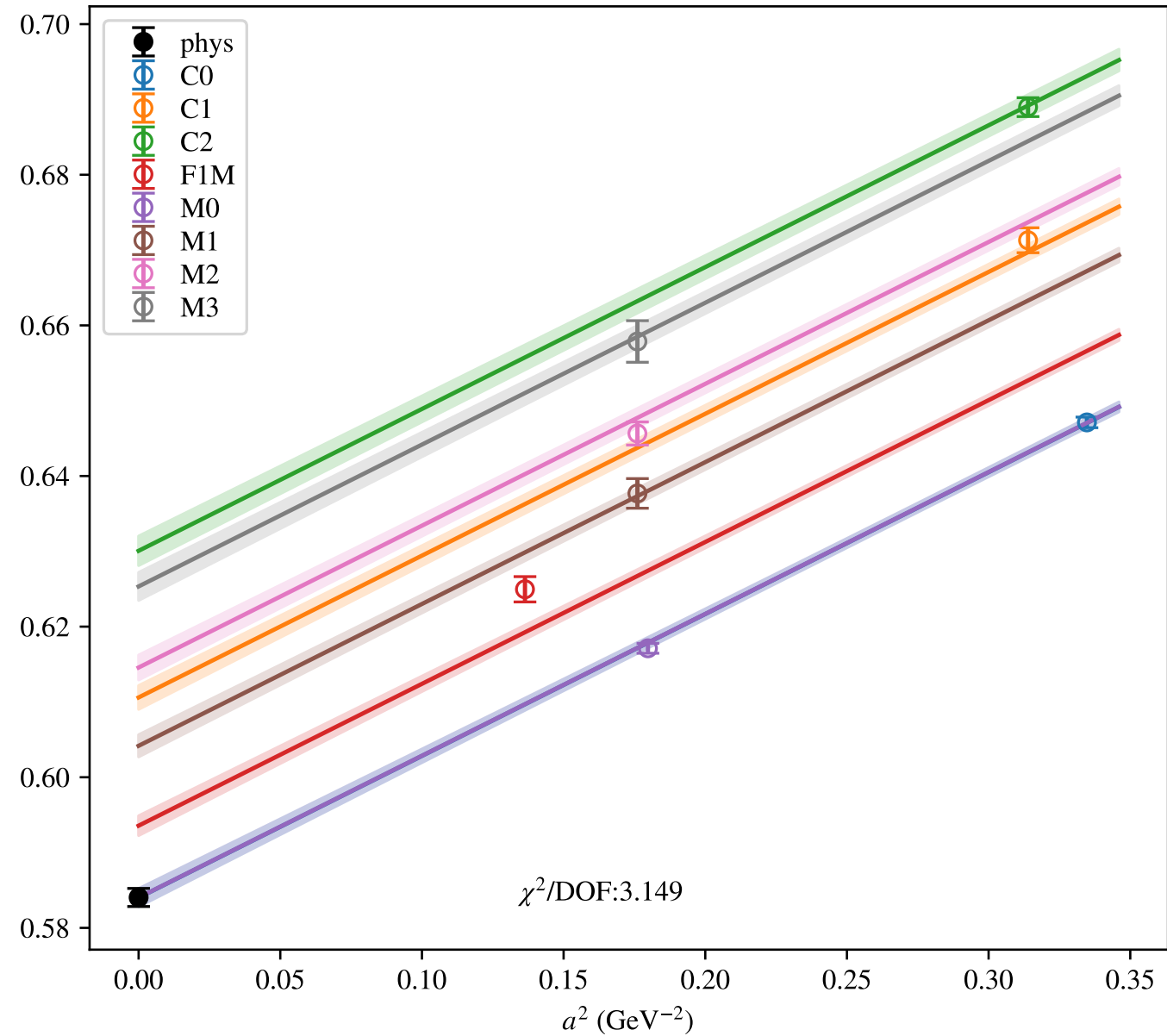


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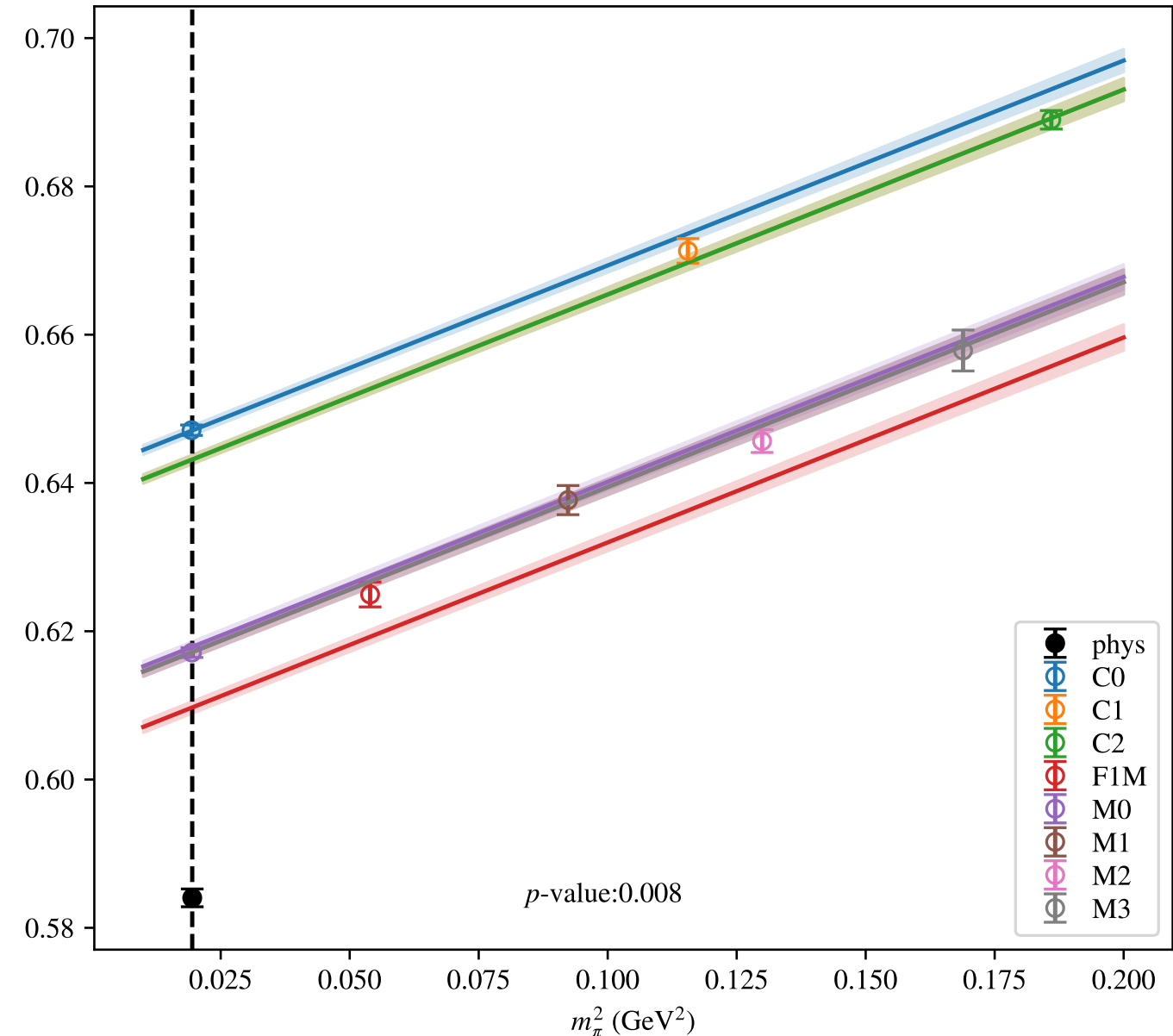


$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$

SSpPP

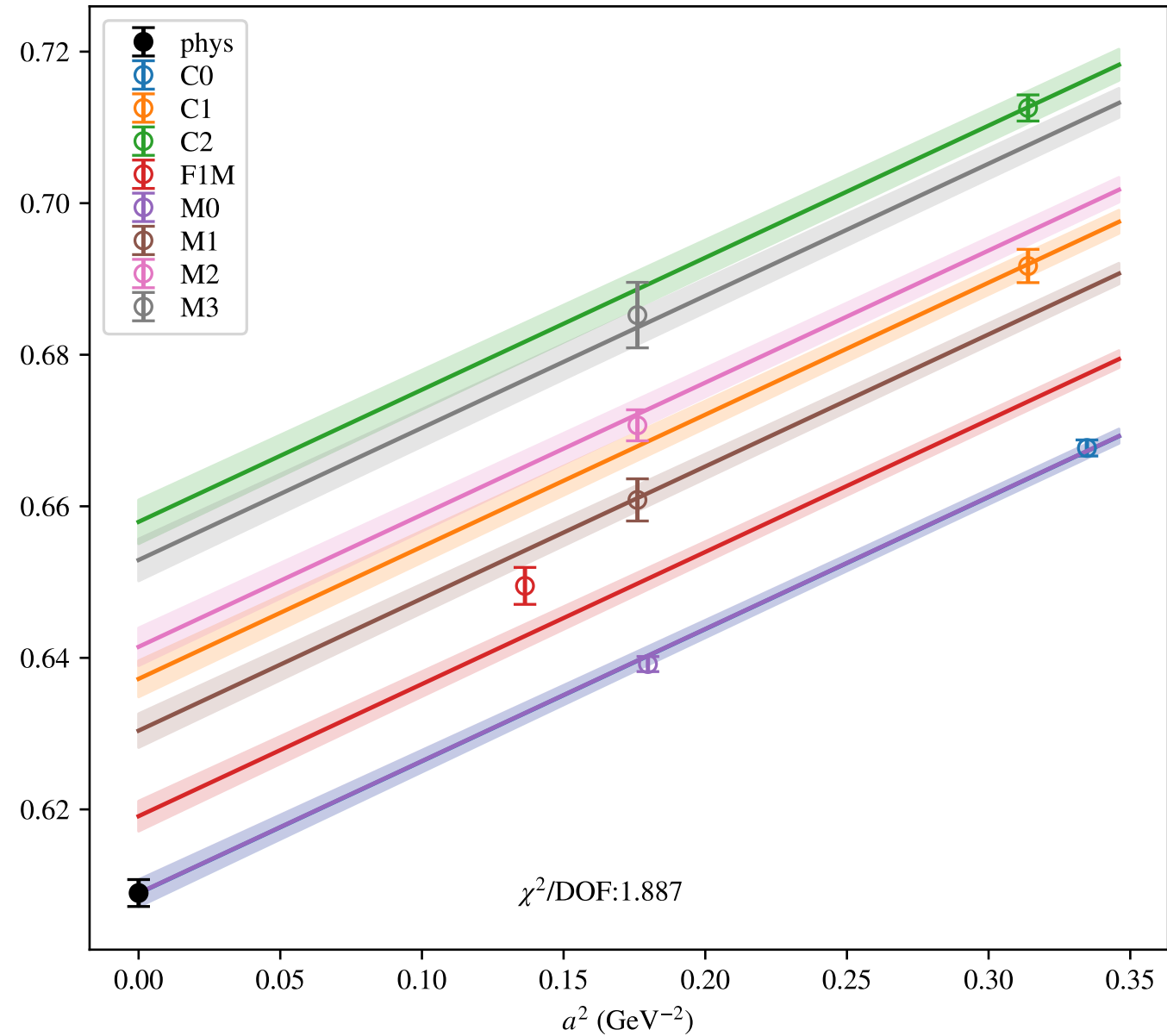


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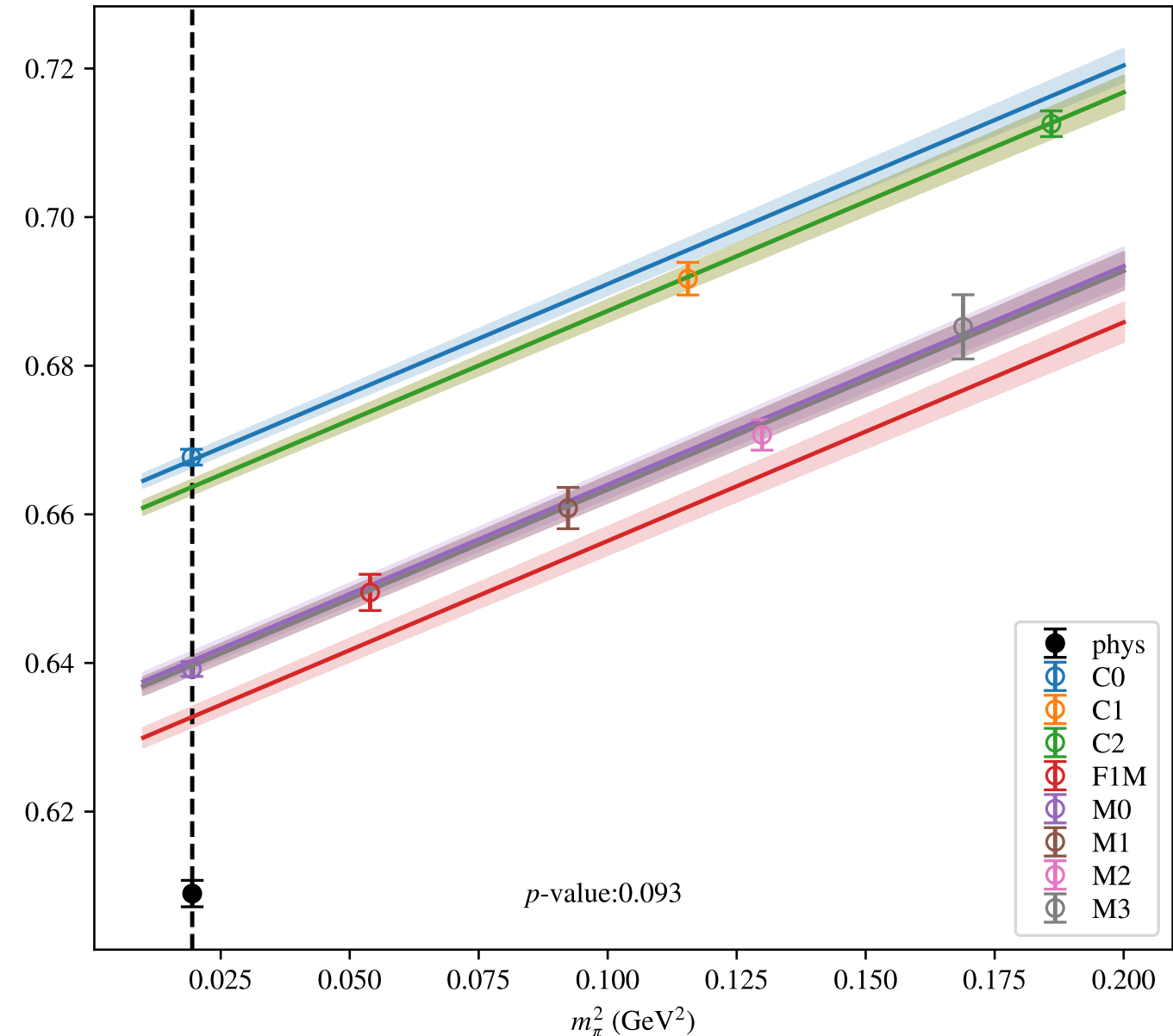


$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$

SSpPP

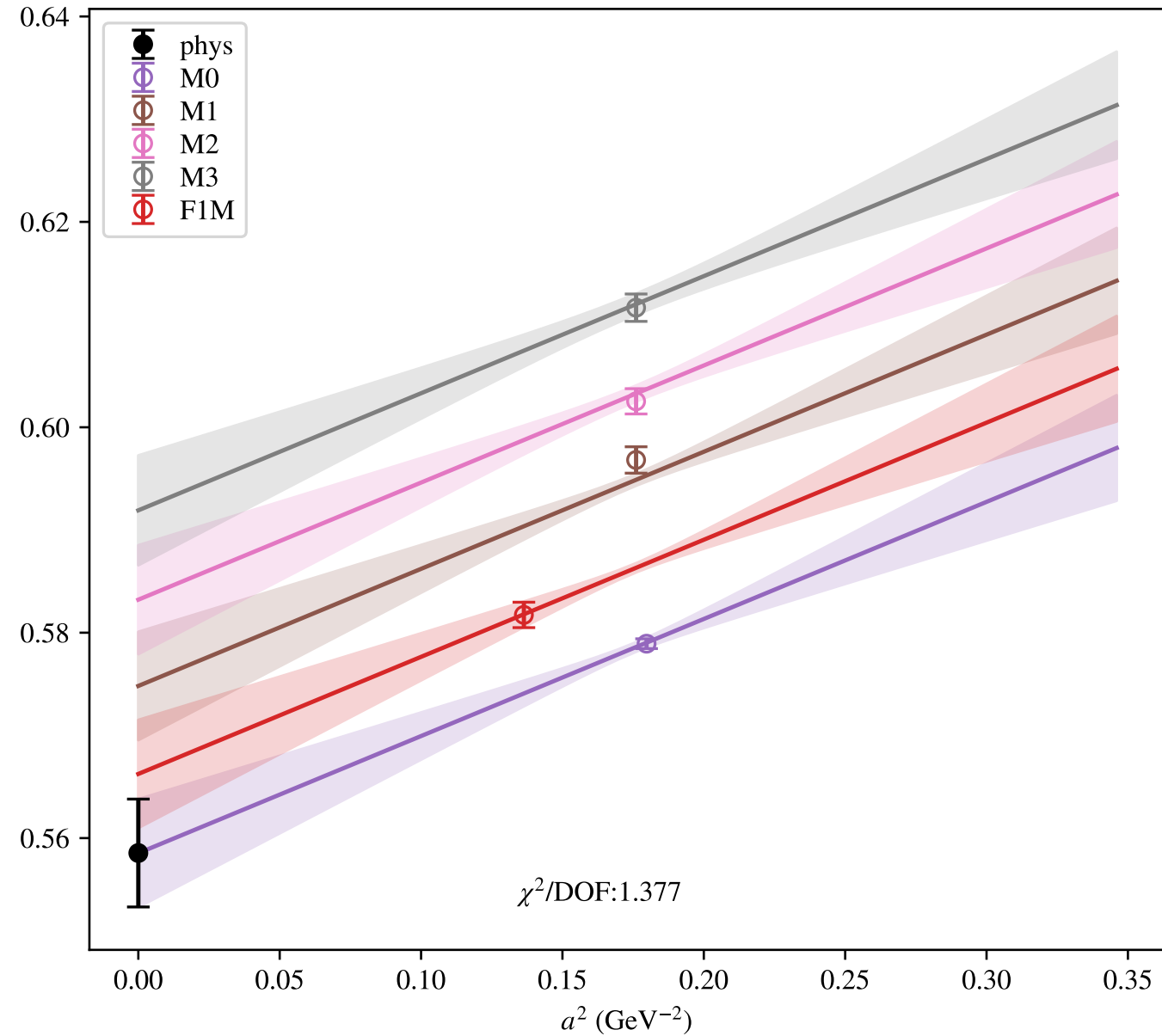


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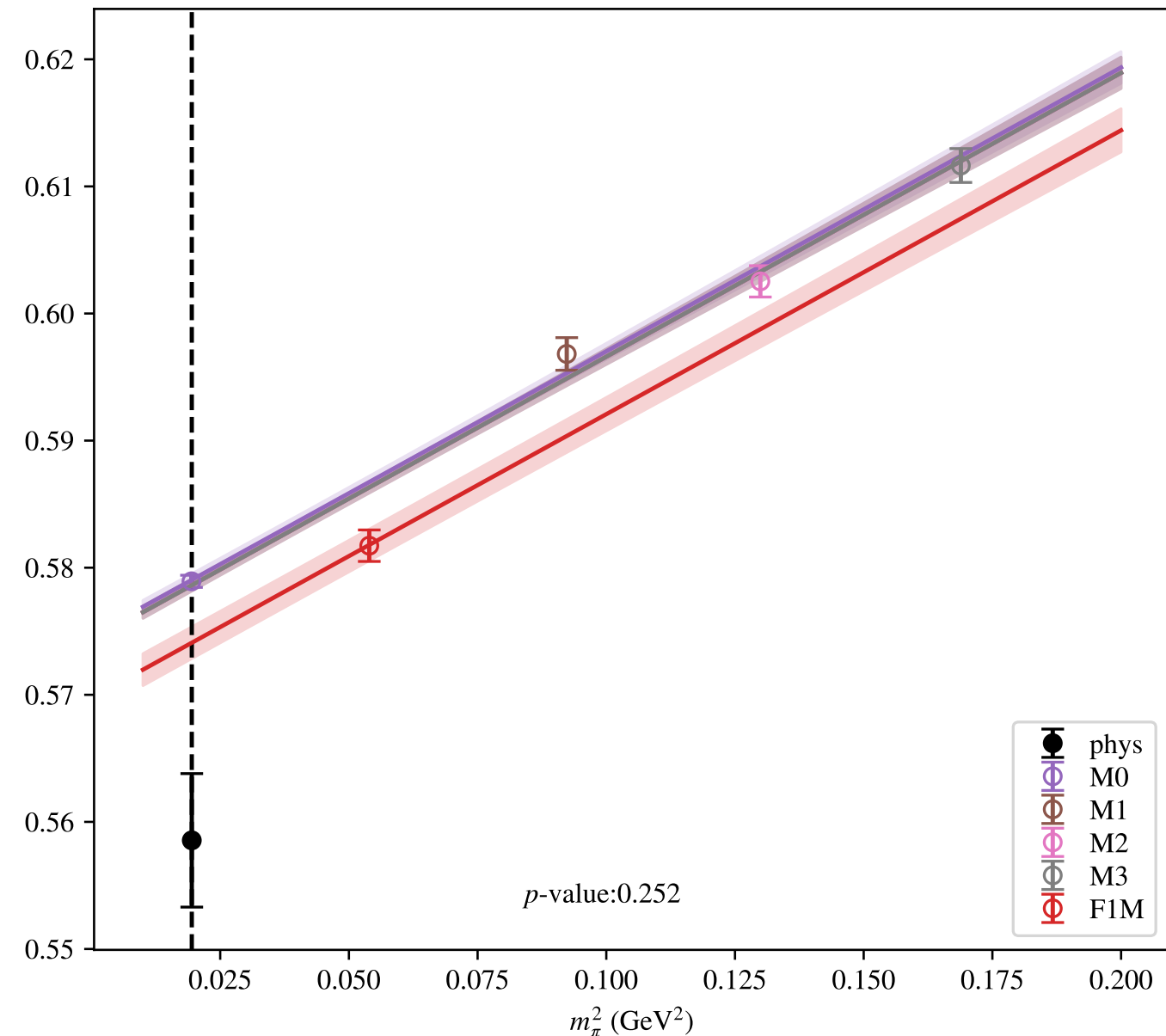


$a^2, m_\pi^2$  (no C),  $\mu = 2.4$  GeV

SSpPP

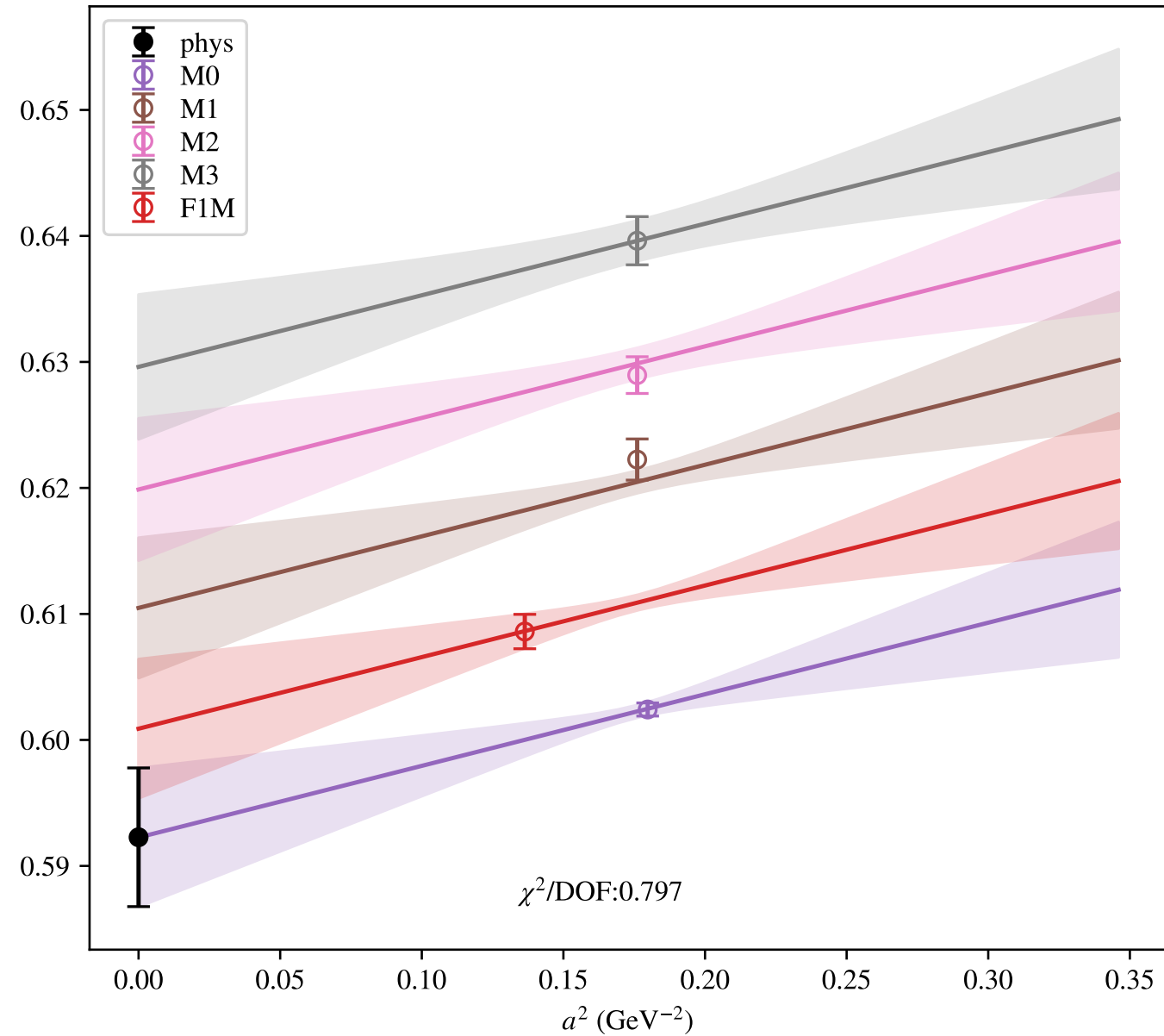


SSpPP

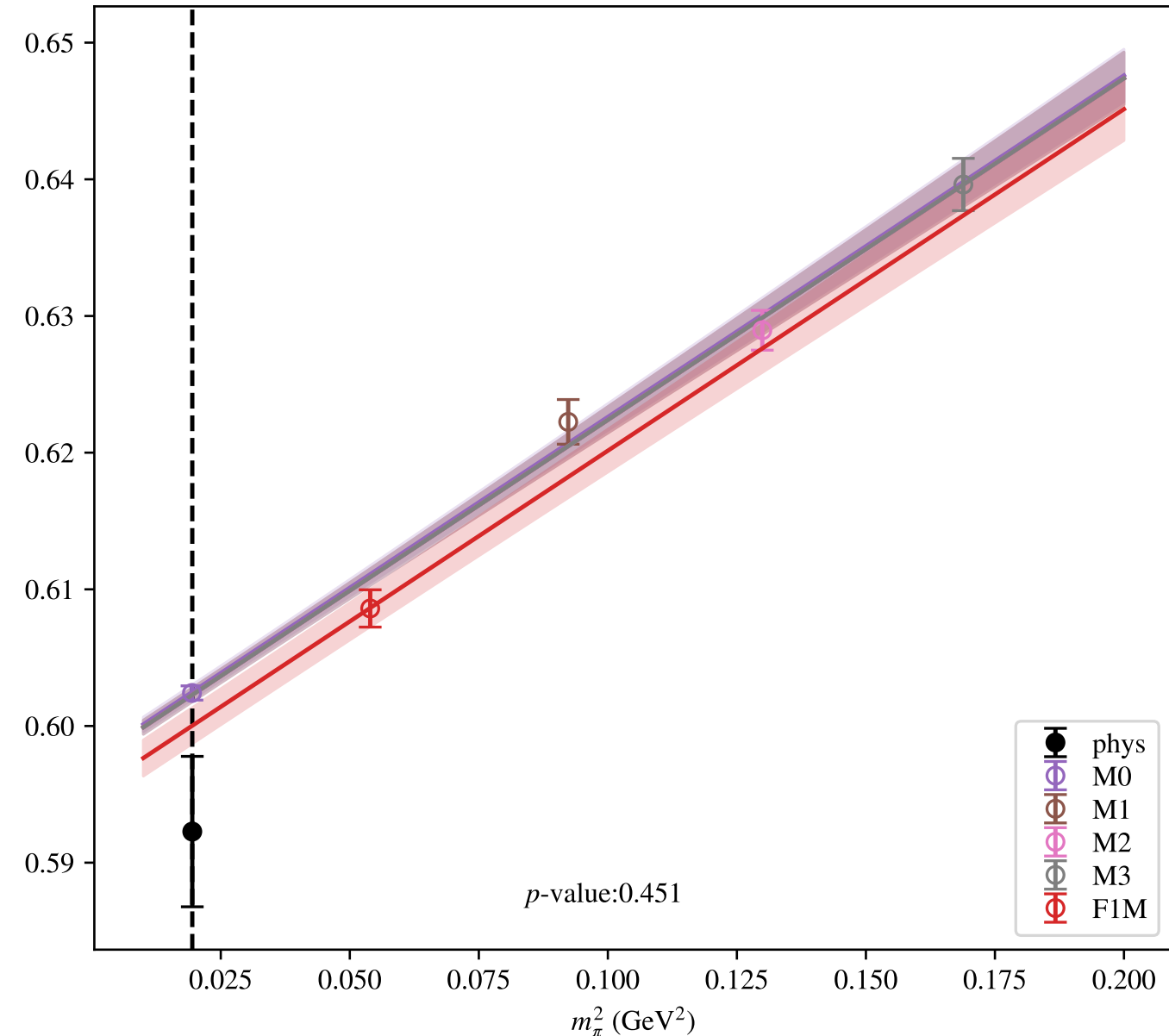


$a^2, m_\pi^2$  (no C),  $\mu = 2.0$  GeV

SSpPP

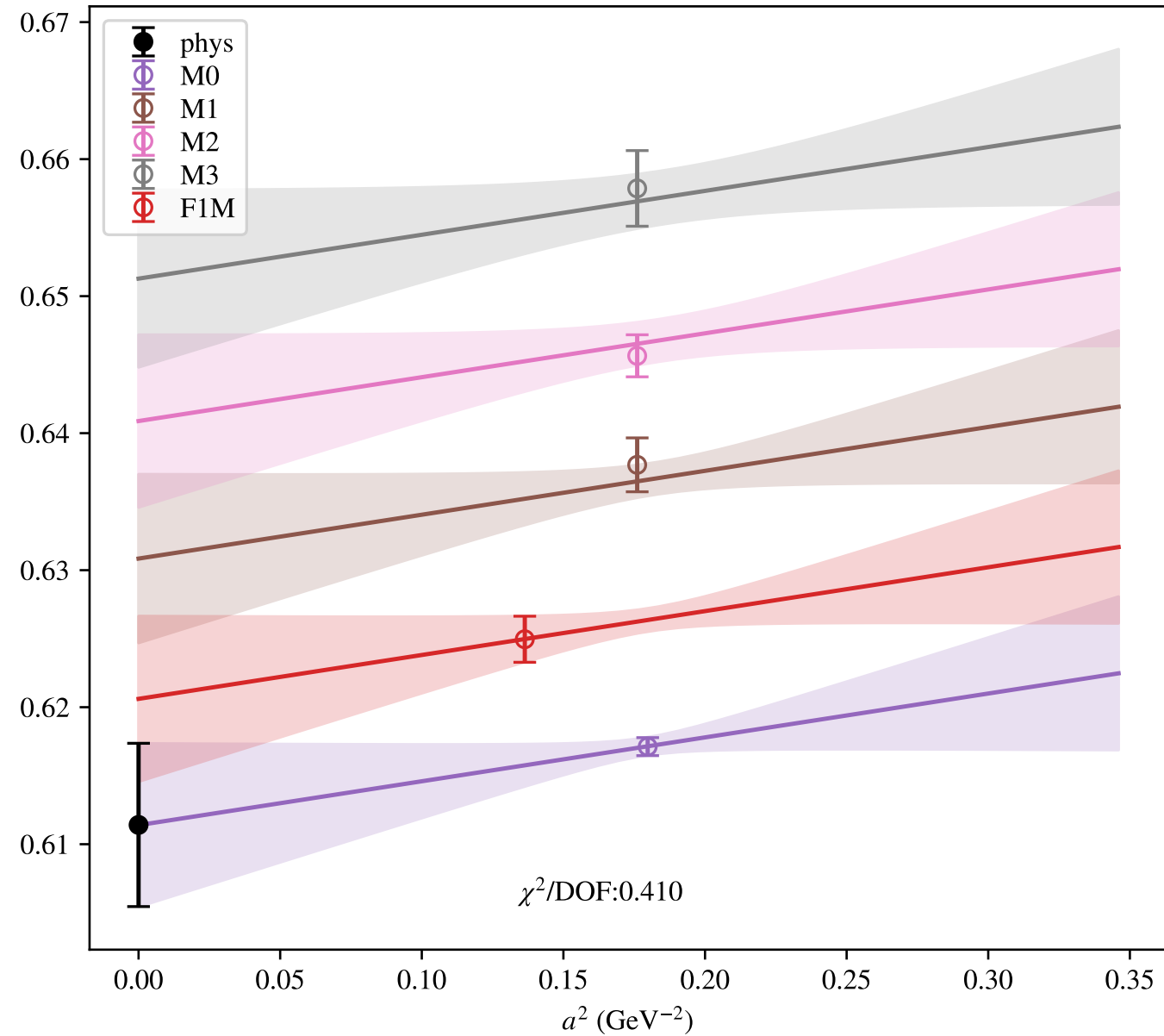


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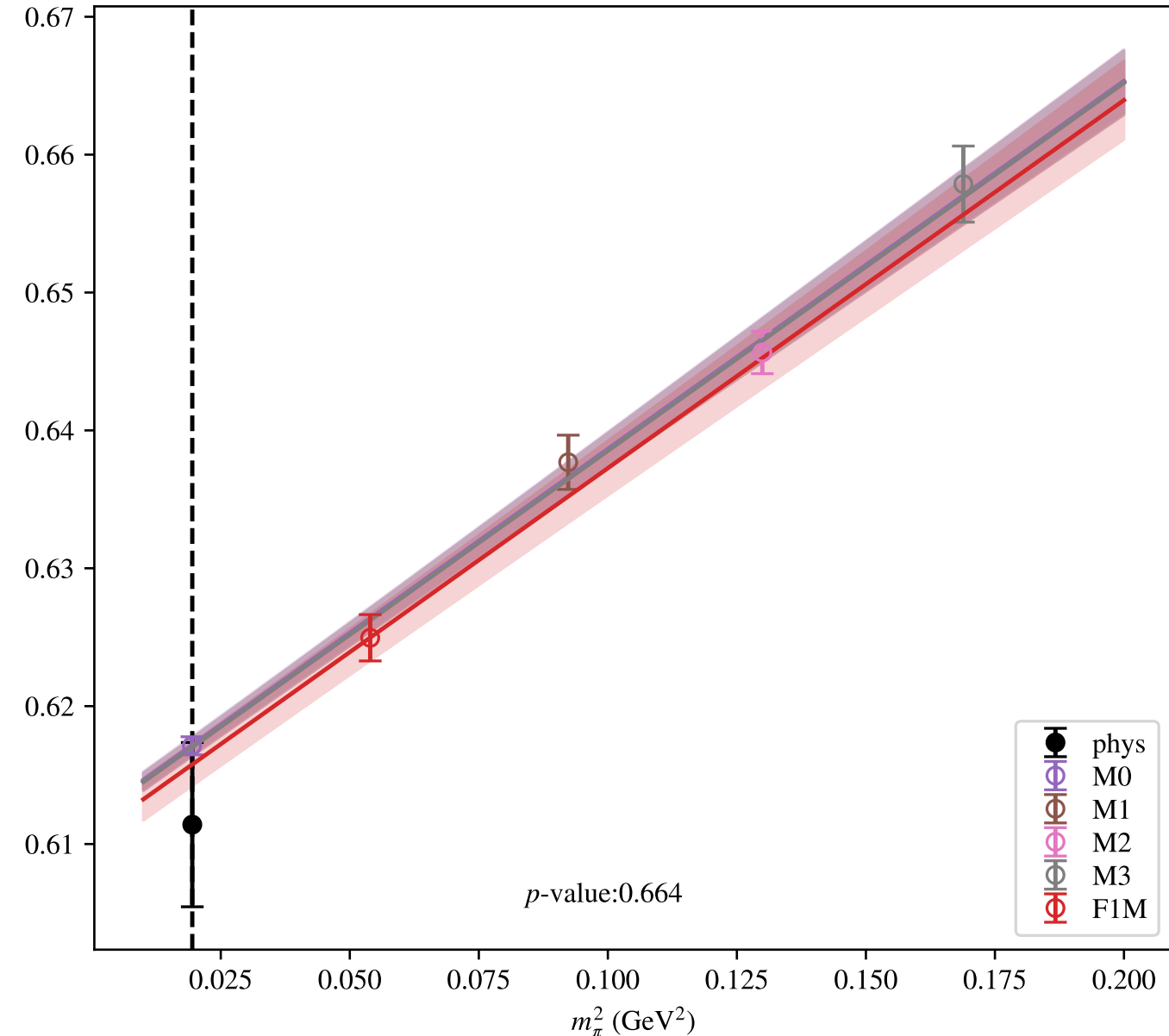


$a^2, m_\pi^2$  (no C),  $\mu = 1.8$  GeV

SSpPP



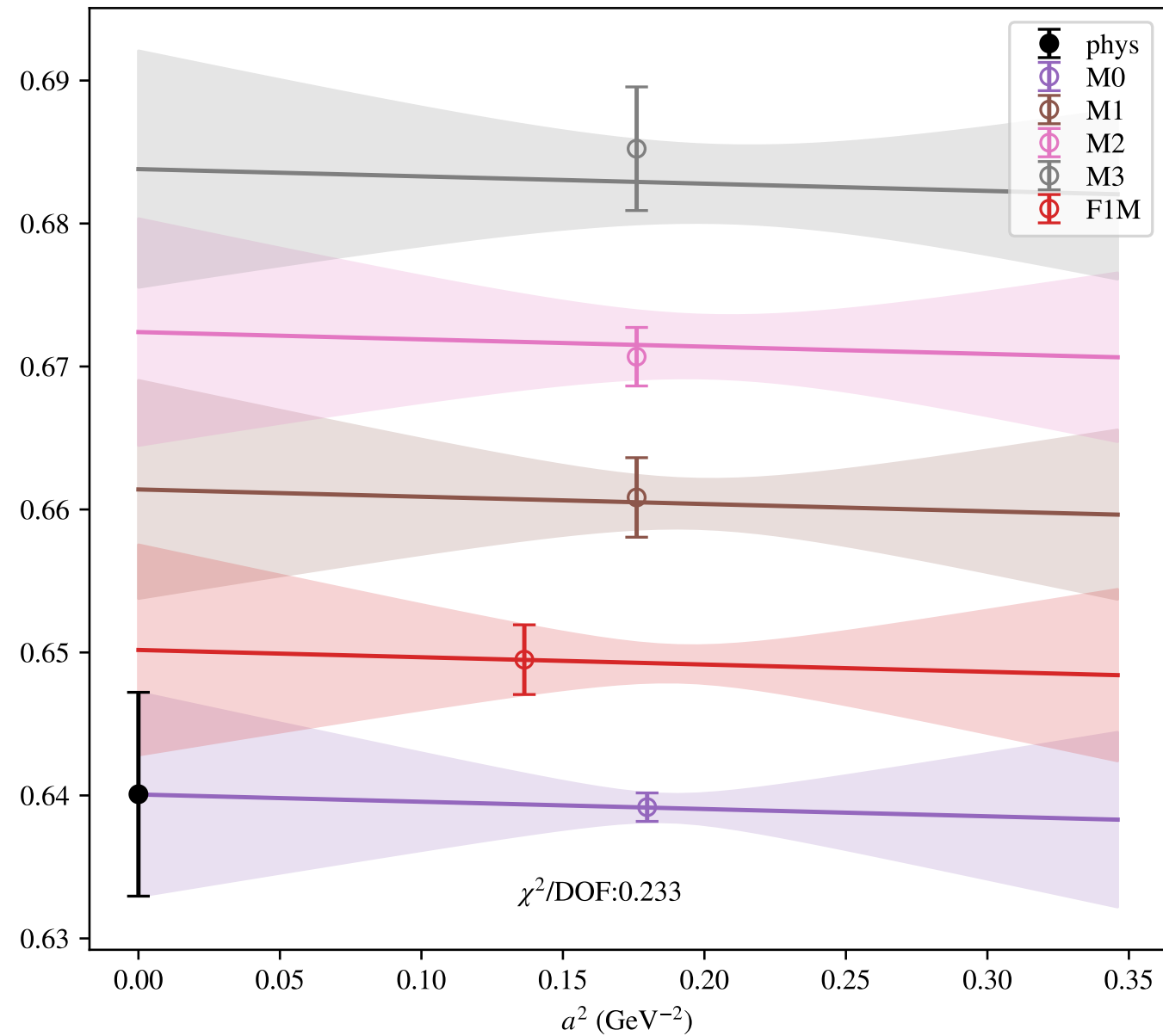
SSpPP



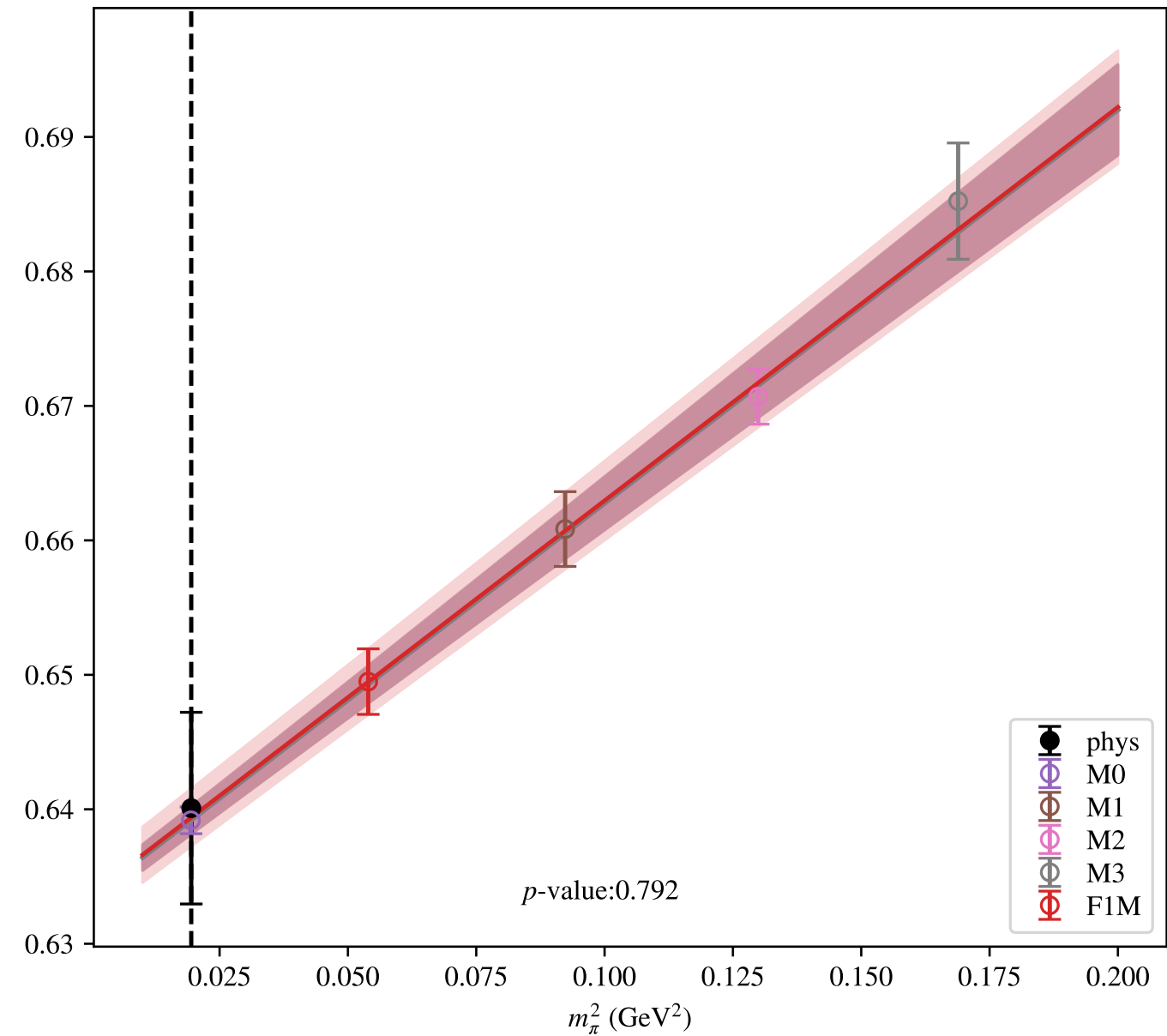


$a^2, m_\pi^2$  (no C),  $\mu = 1.5$  GeV

SSpPP

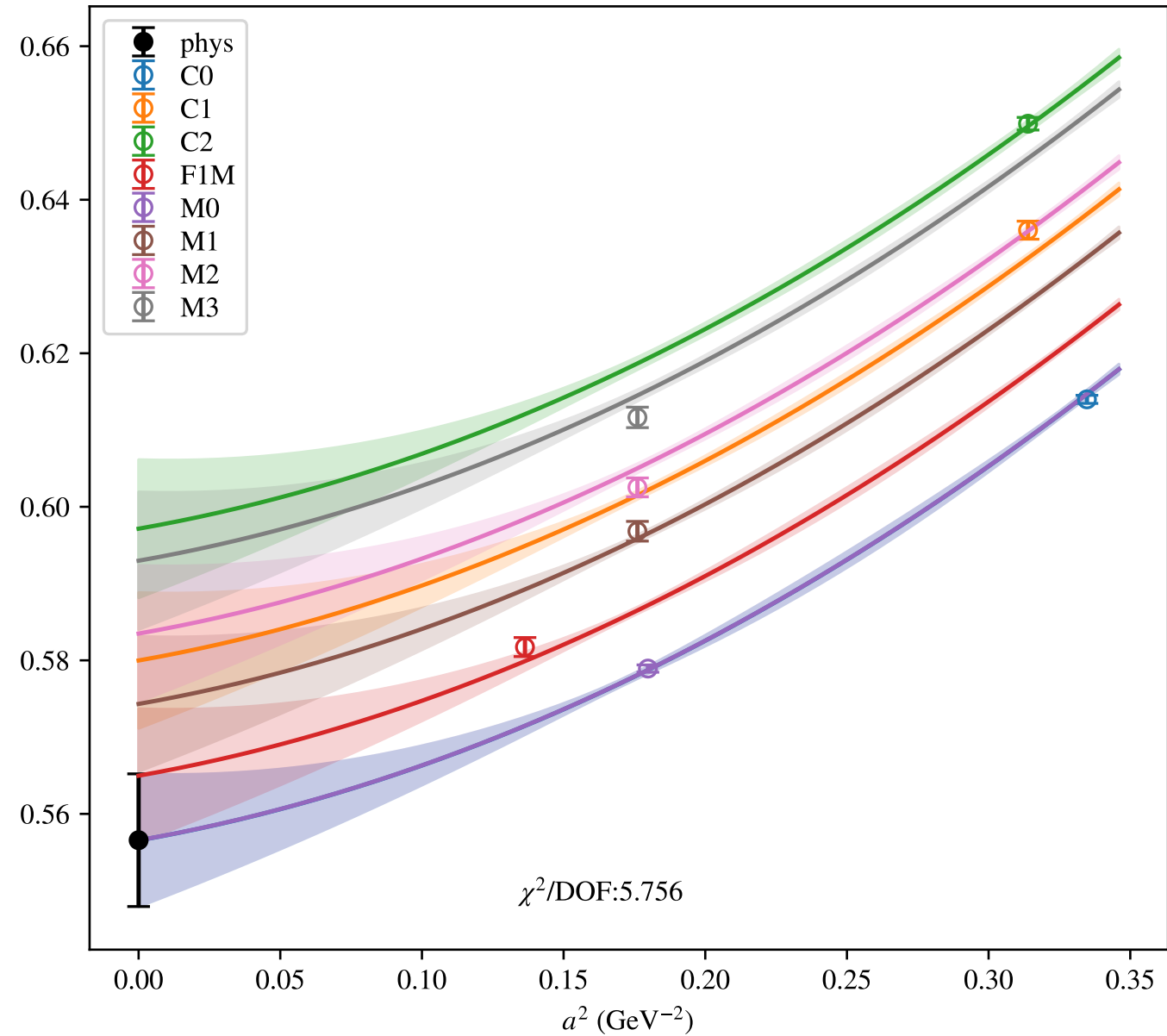


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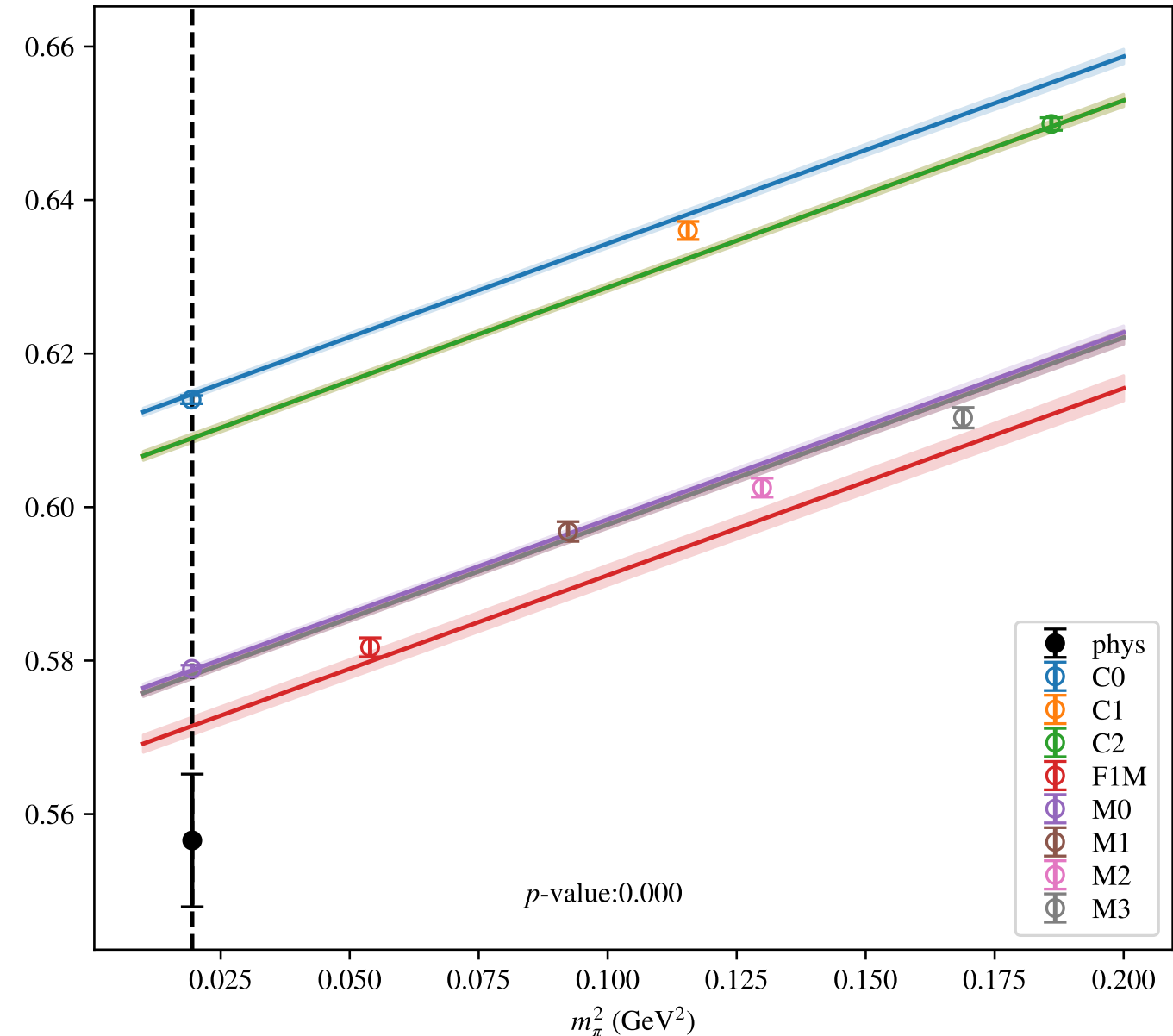


$$a^2, a^4, m_\pi^2, \mu = 2.4 \text{ GeV}$$

SSpPP

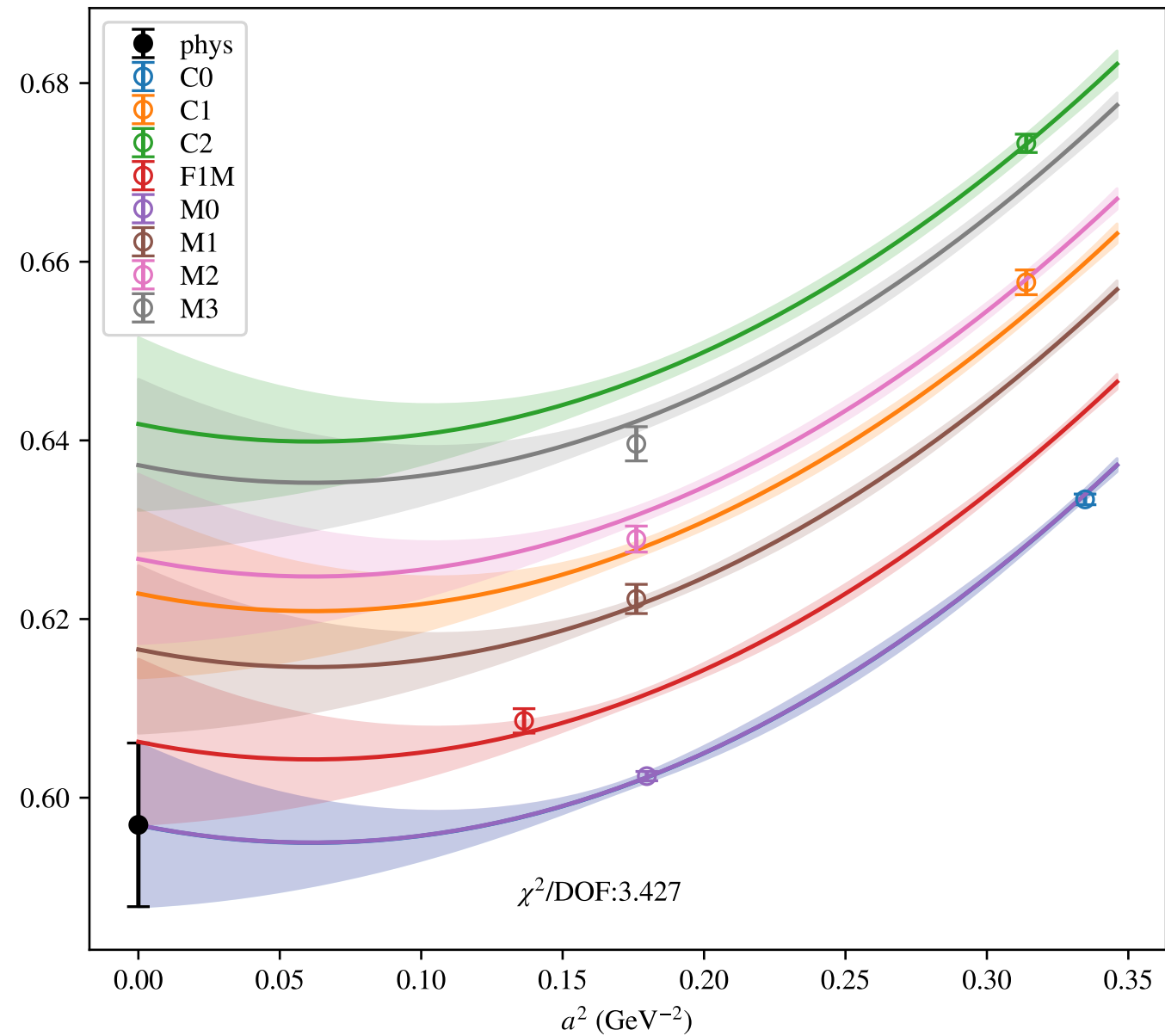


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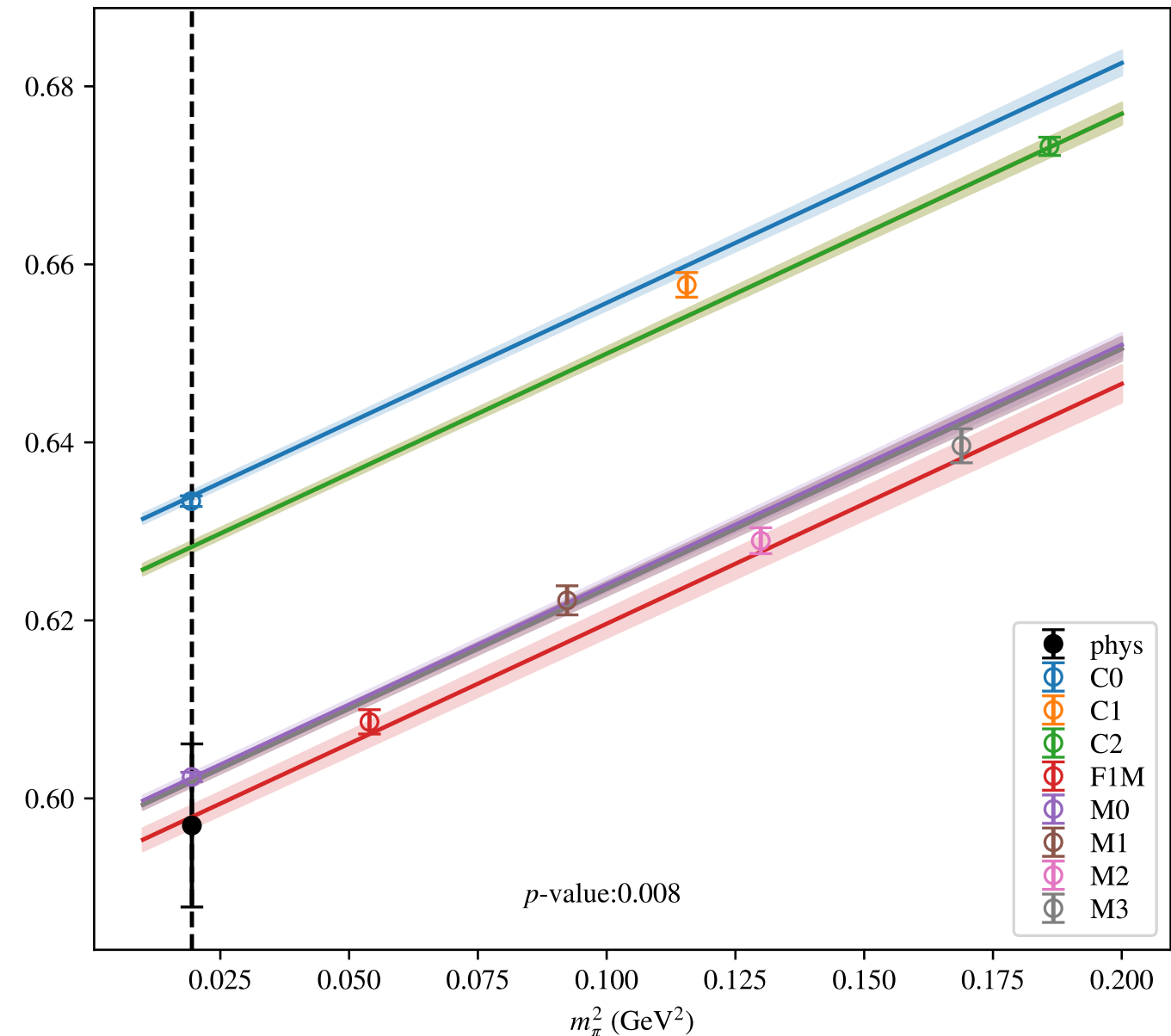


$$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$$

SSpPP

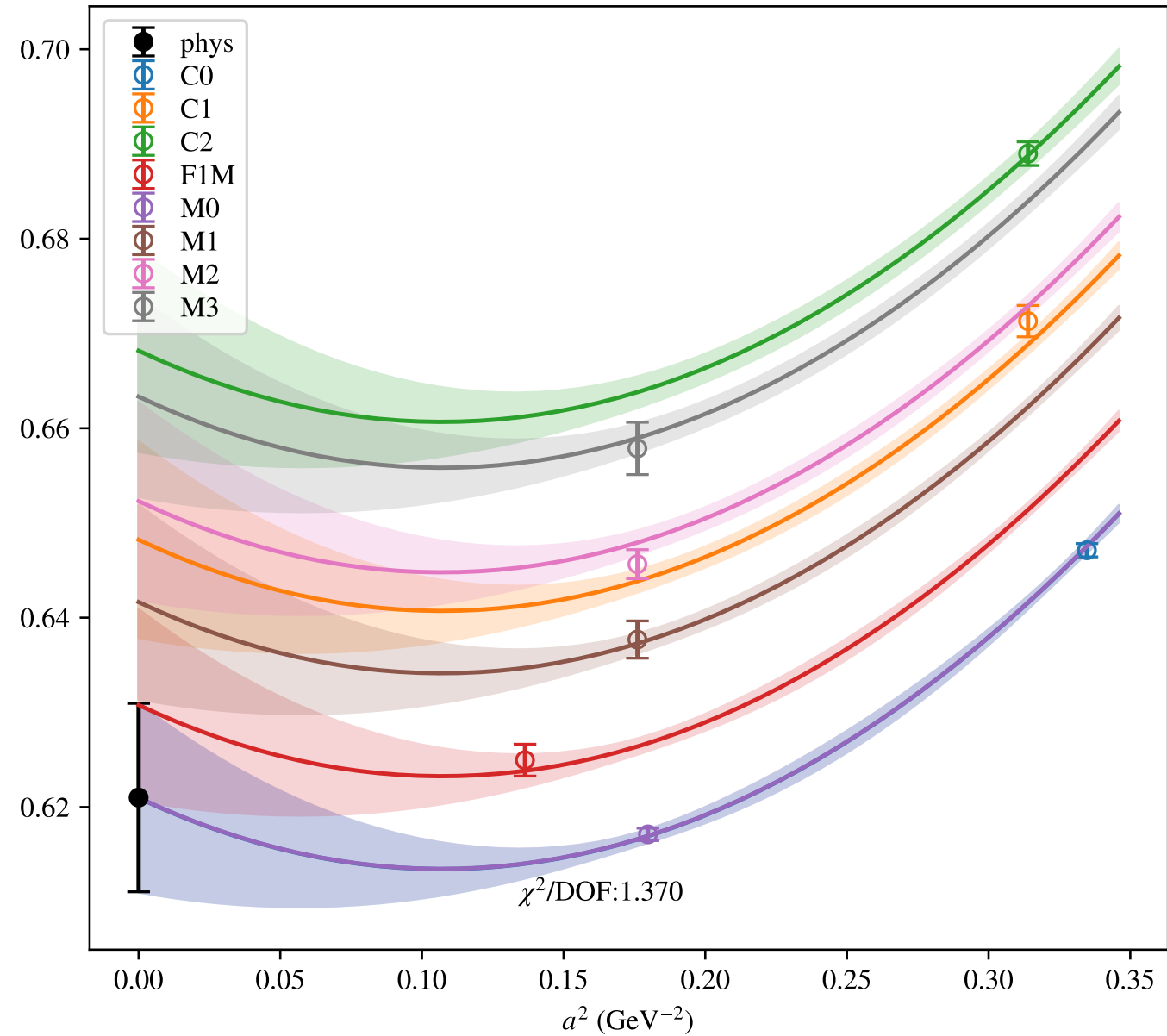


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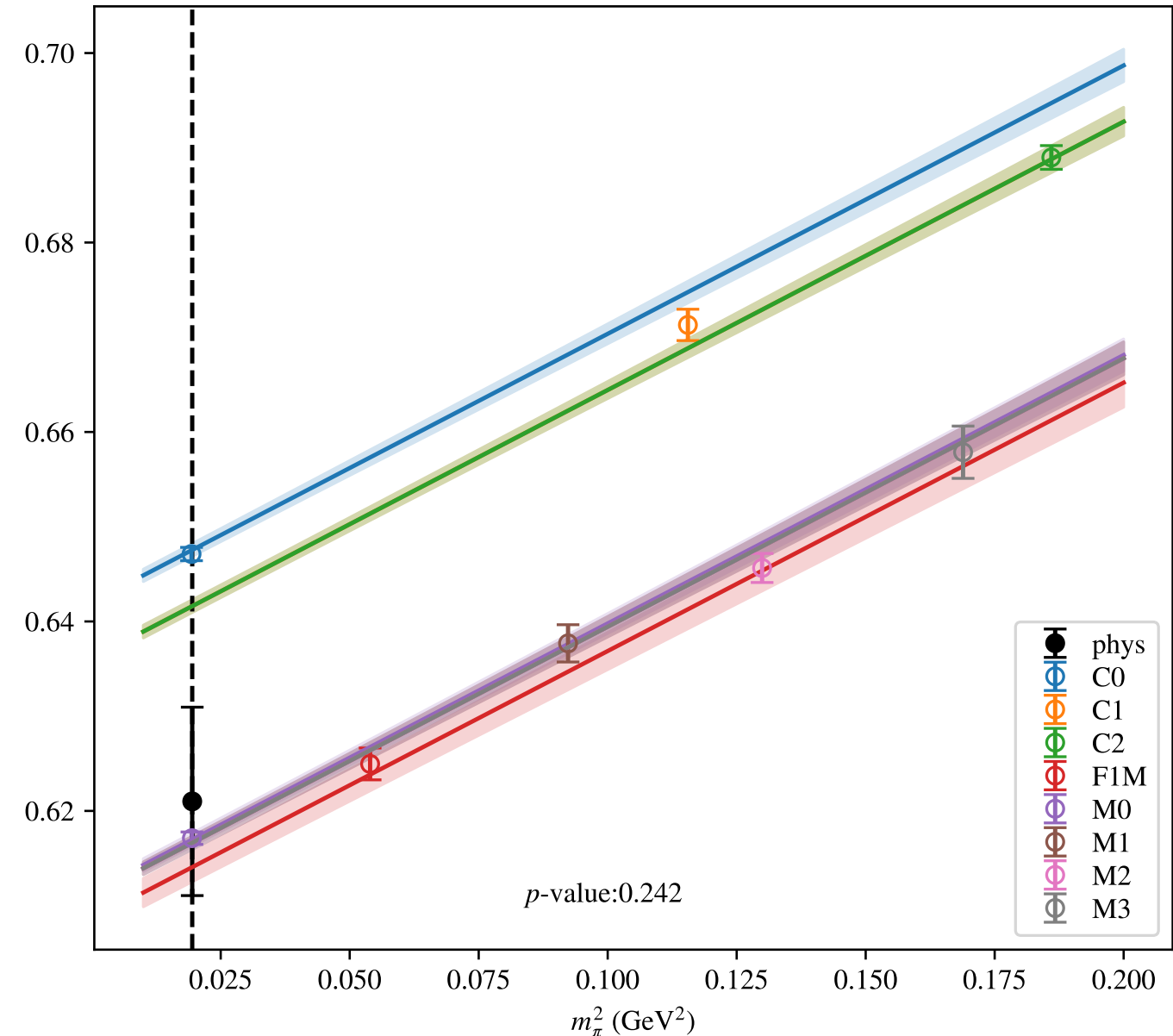


$$a^2, a^4, m_\pi^2, \mu = 1.8 \text{ GeV}$$

SSpPP

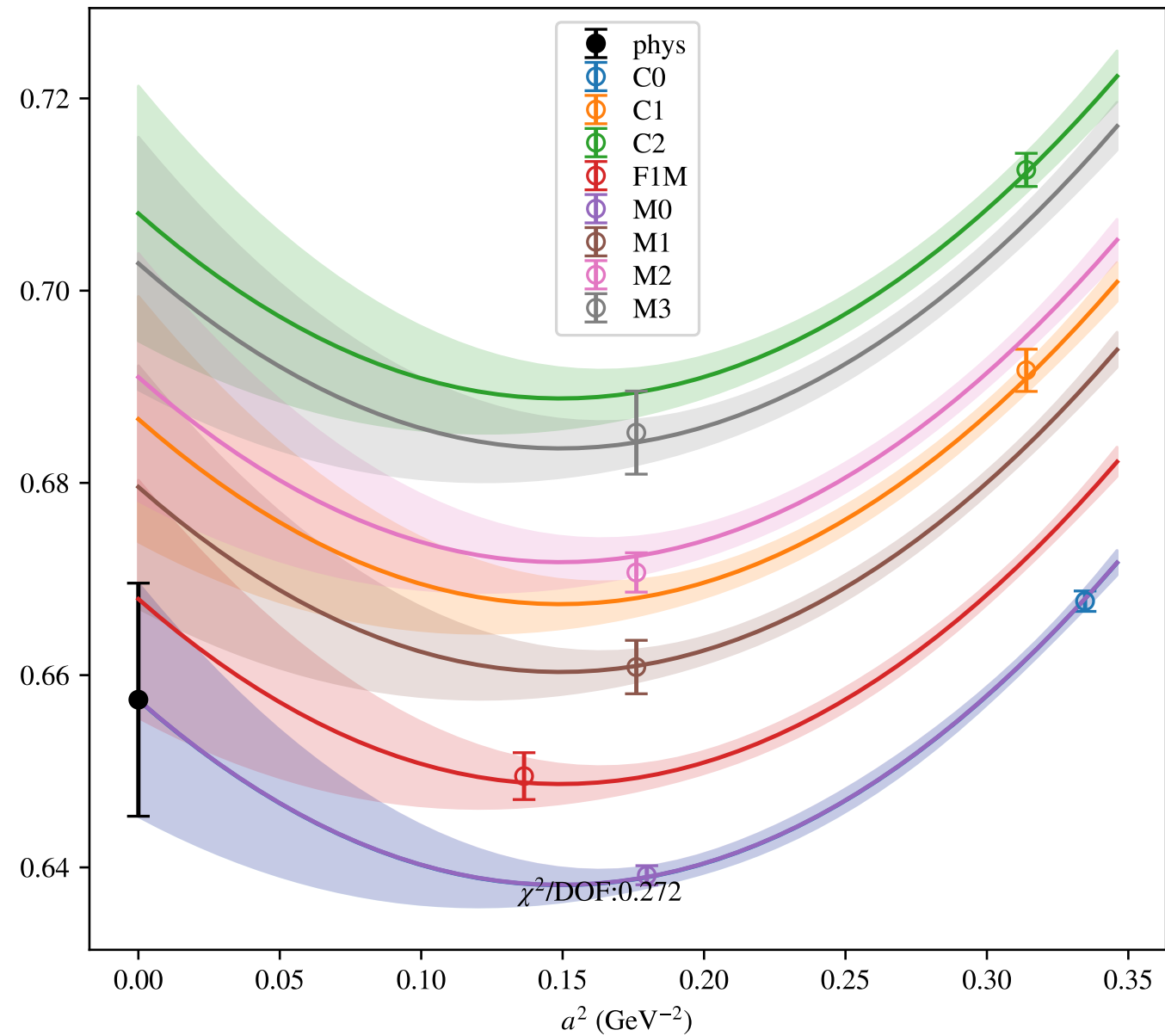


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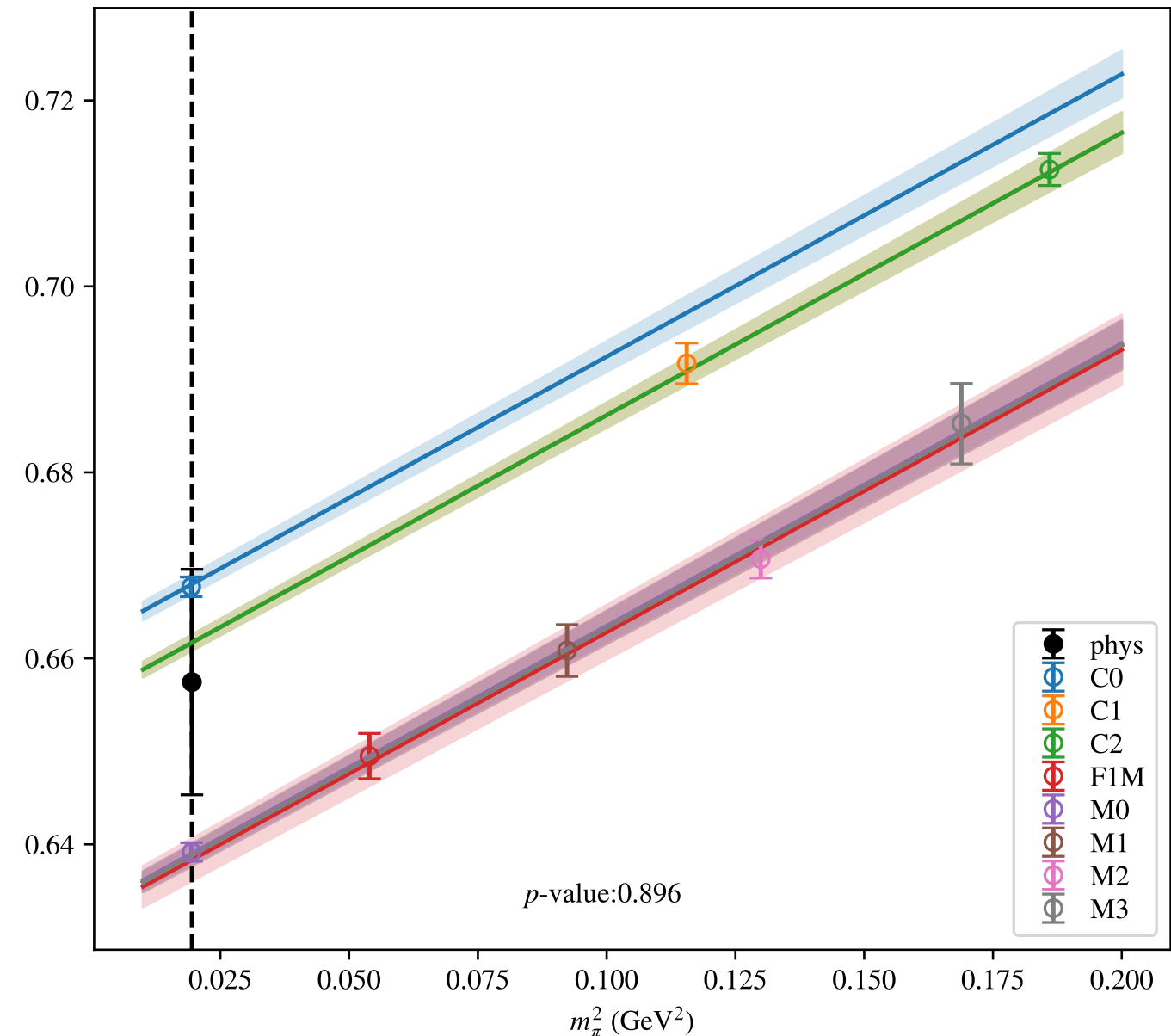


$$a^2, a^4, m_\pi^2, \mu = 1.5 \text{ GeV}$$

SSpPP

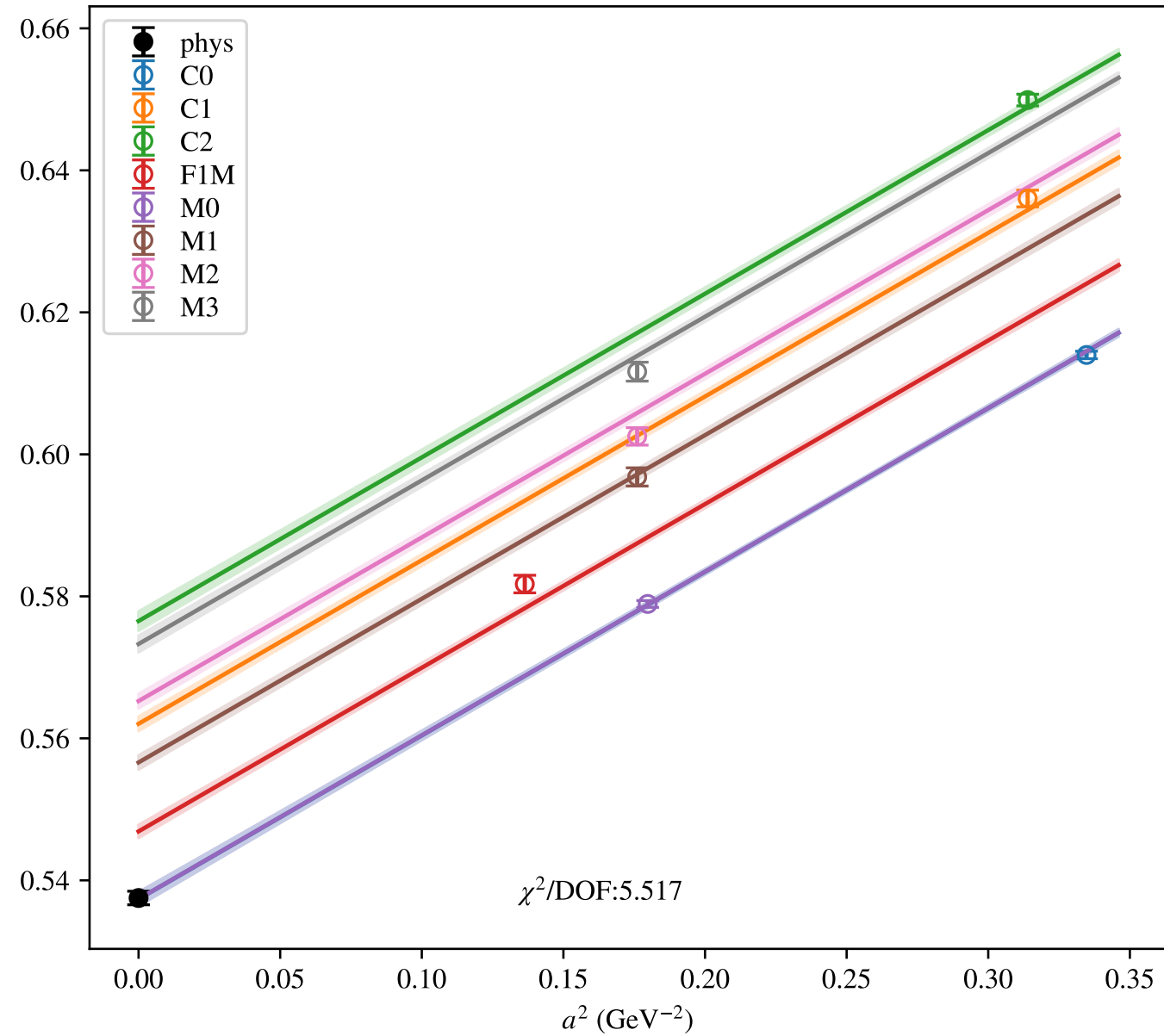


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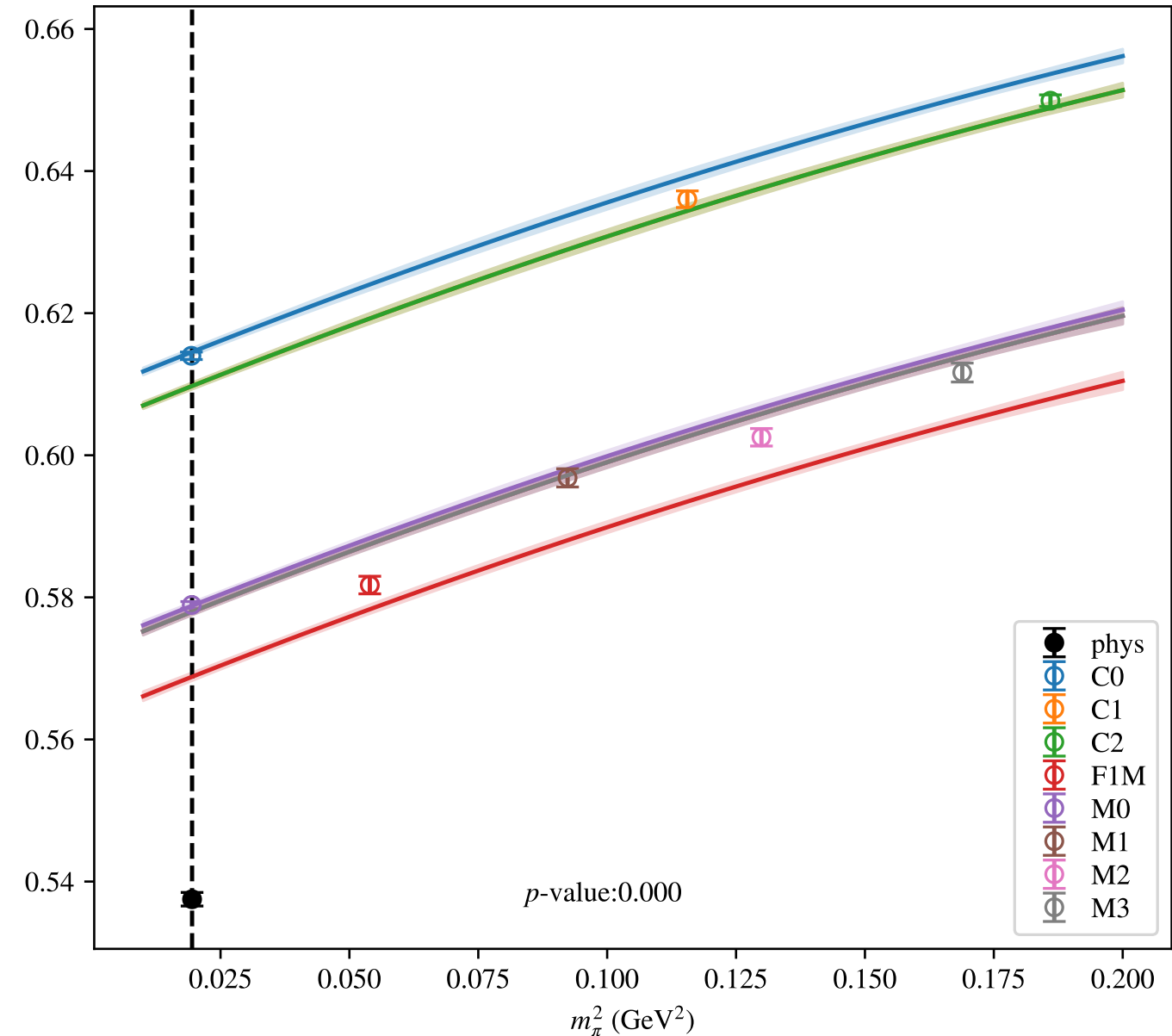


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.4 \text{ GeV}$$

SSpPP

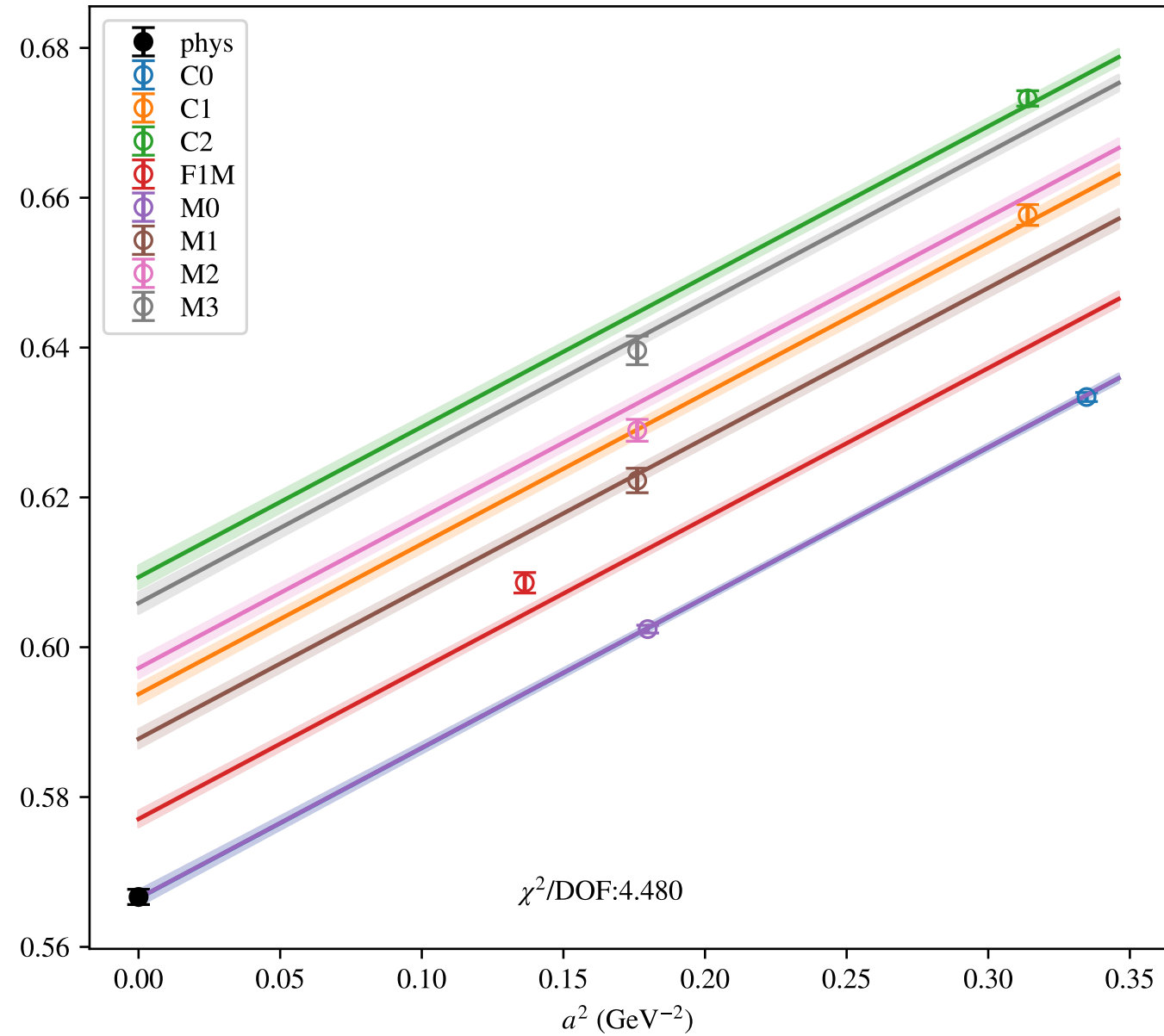


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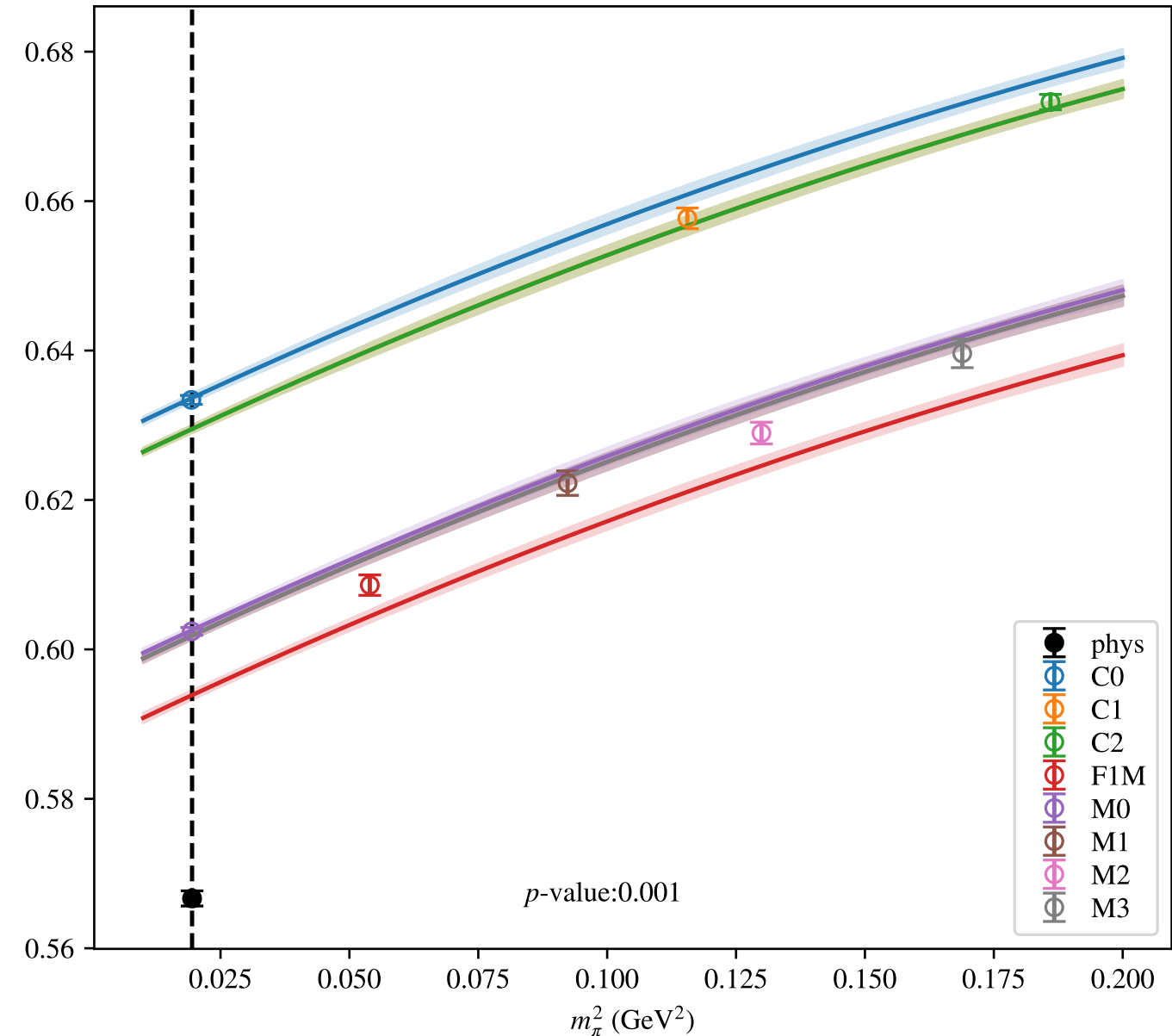


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

SSpPP

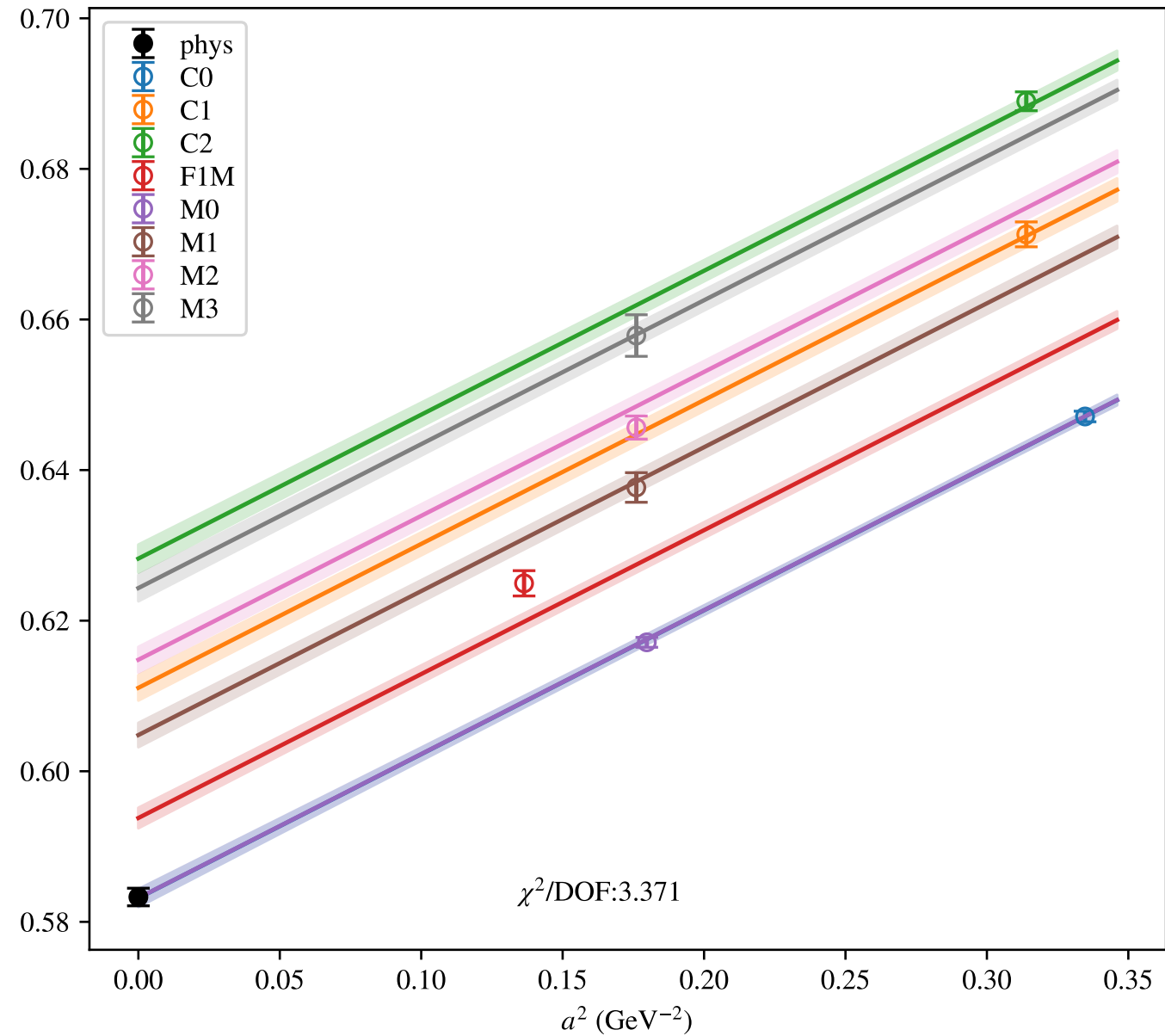


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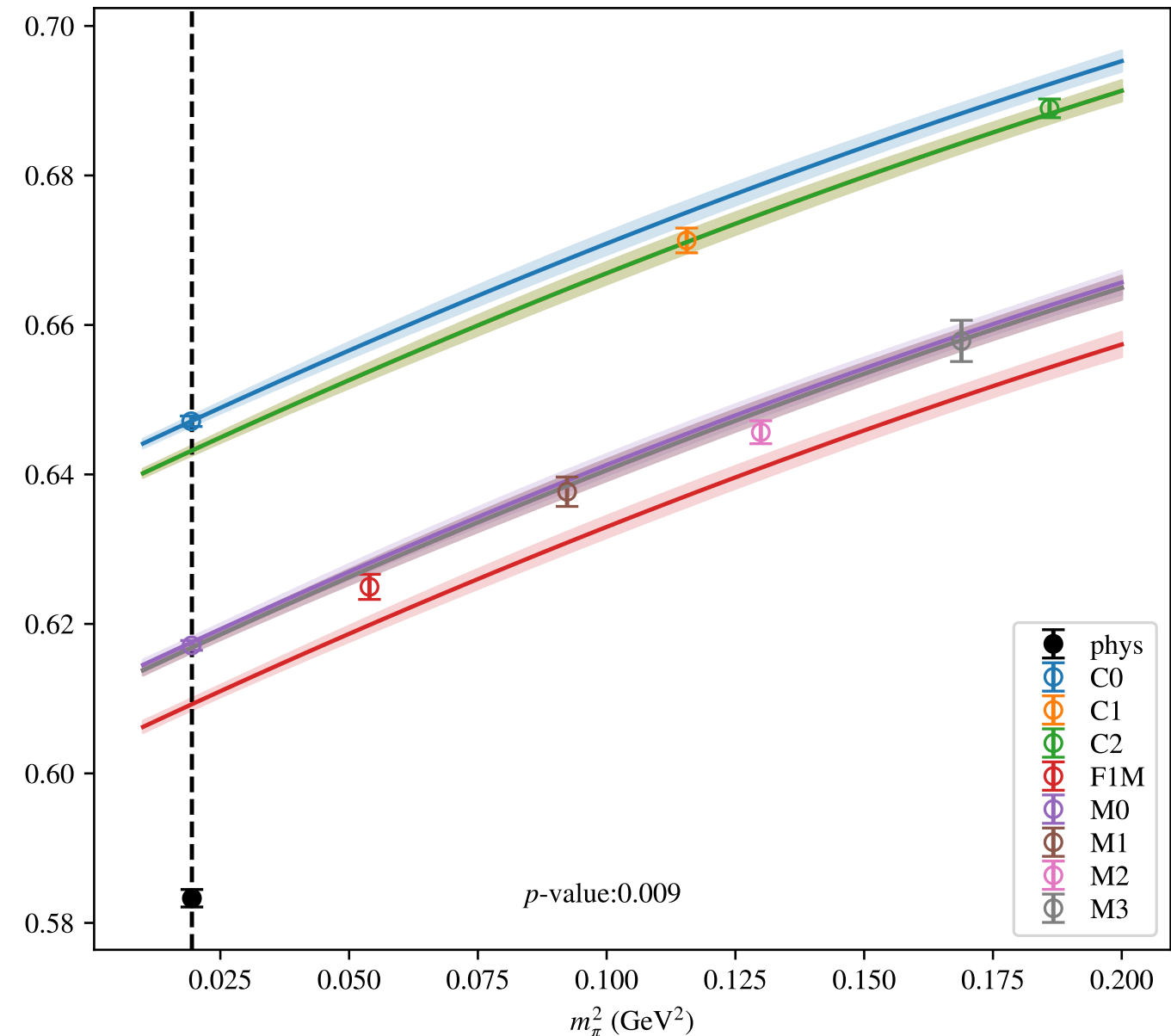


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.8 \text{ GeV}$$

SSpPP



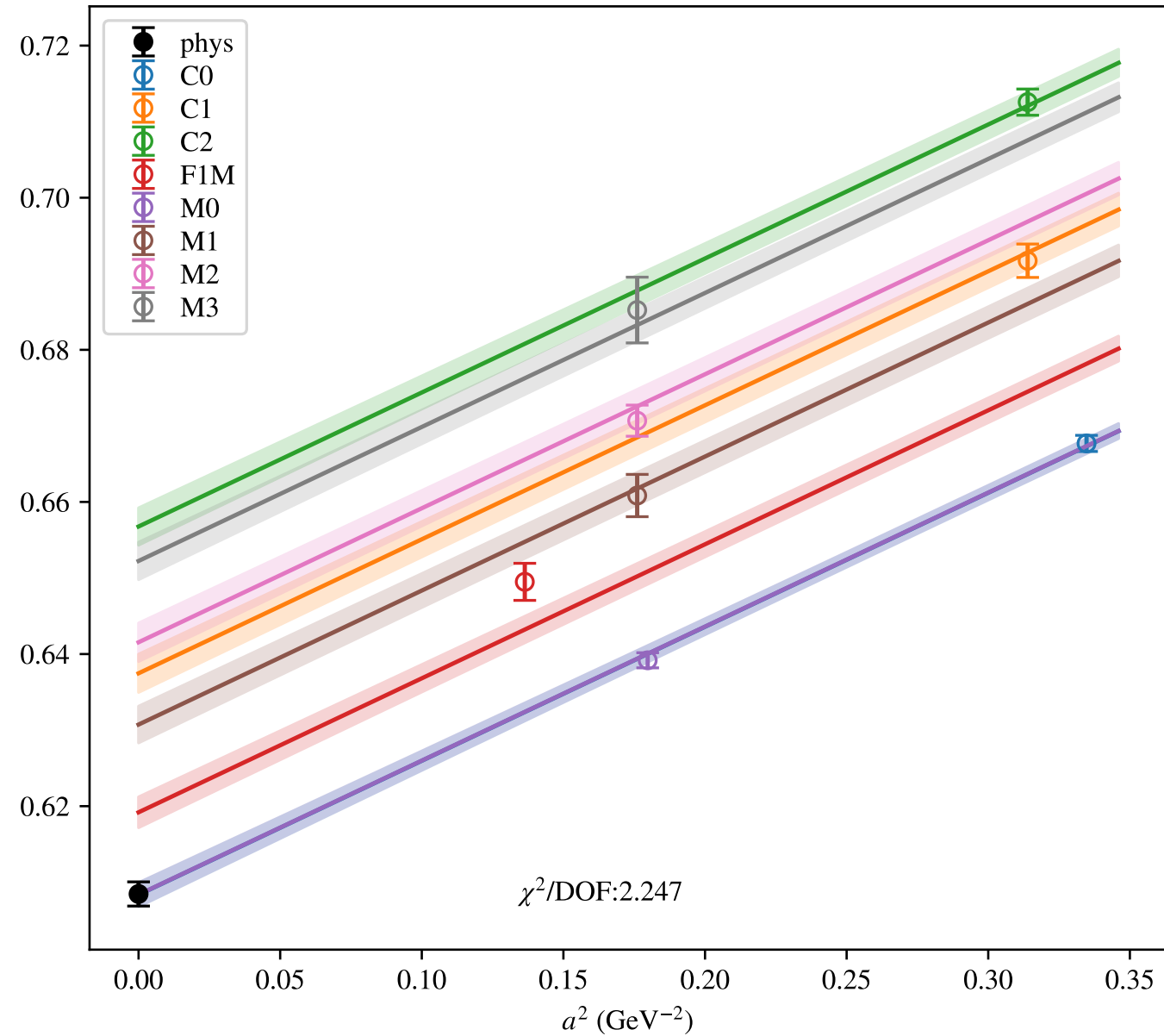
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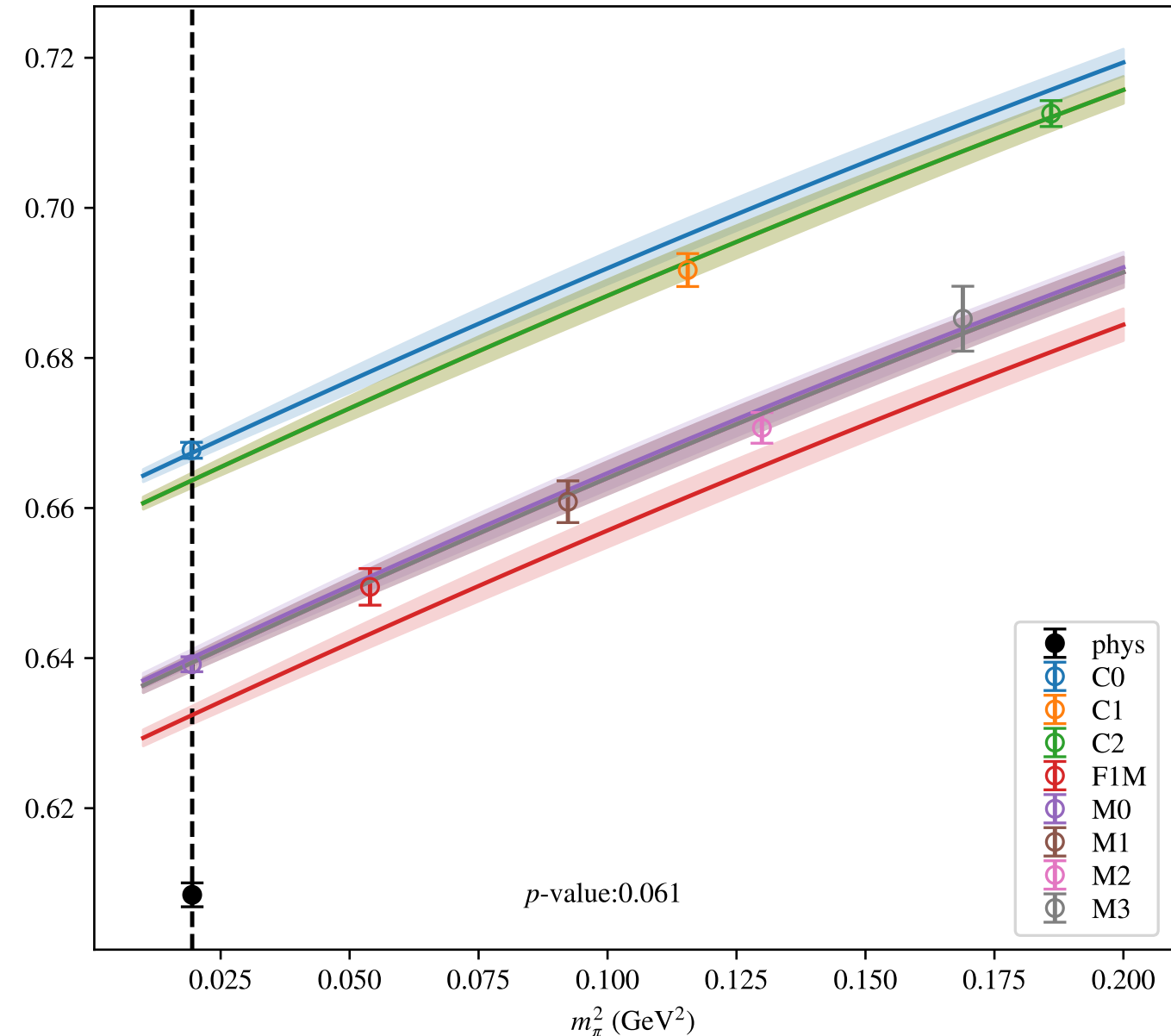


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.5 \text{ GeV}$$

SSpPP

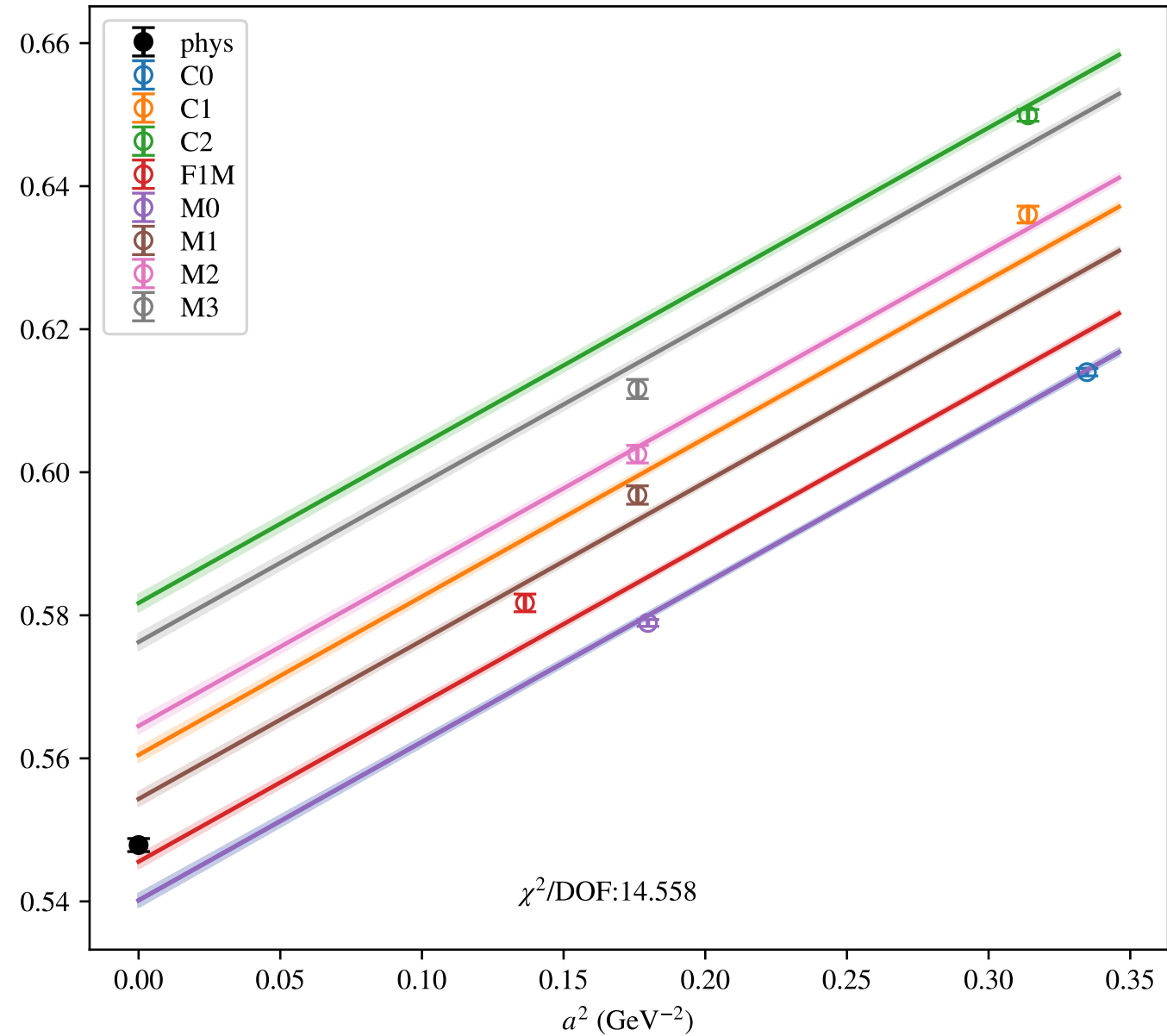


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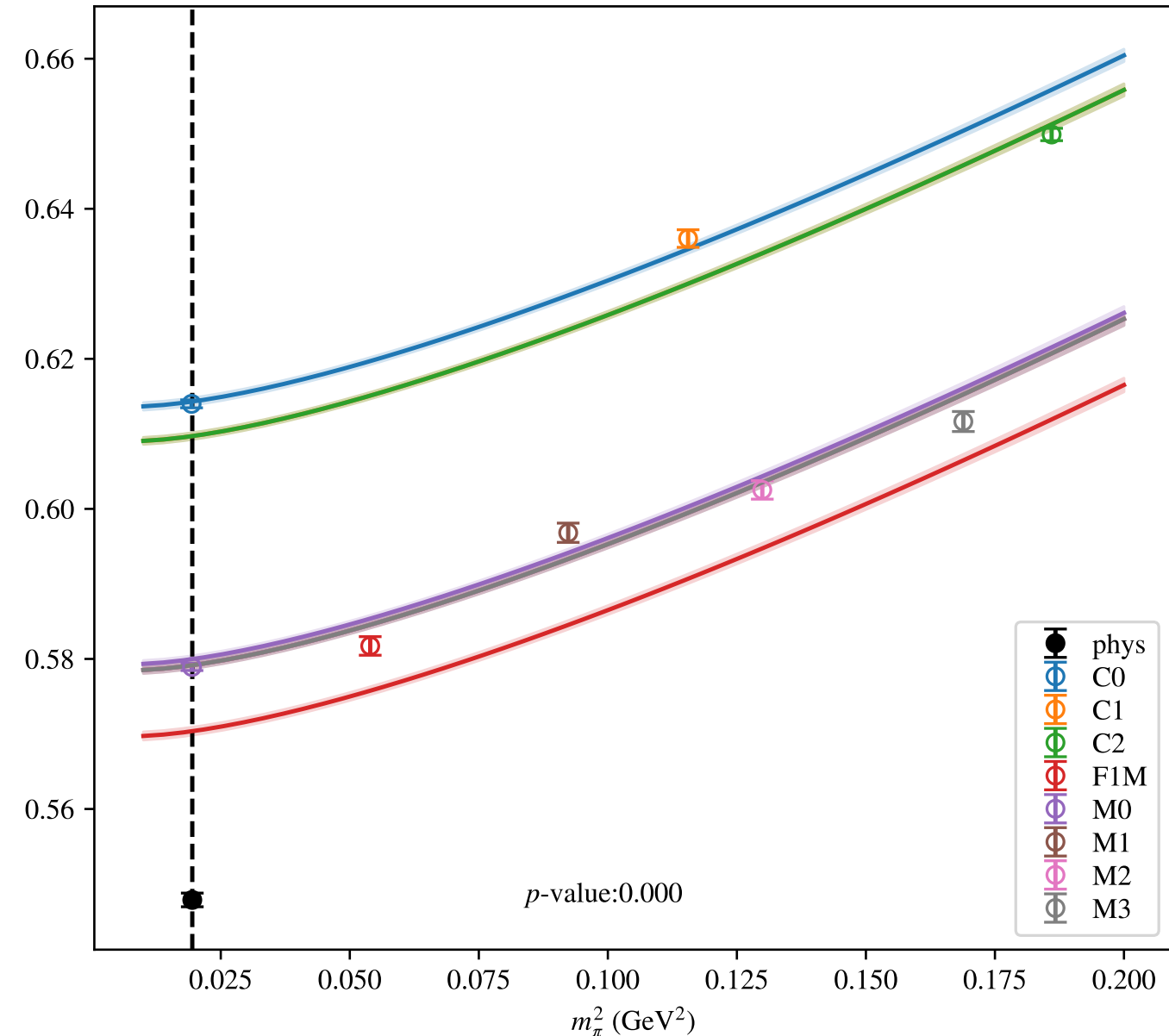


$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 2.4 \text{ GeV}$

SSpPP

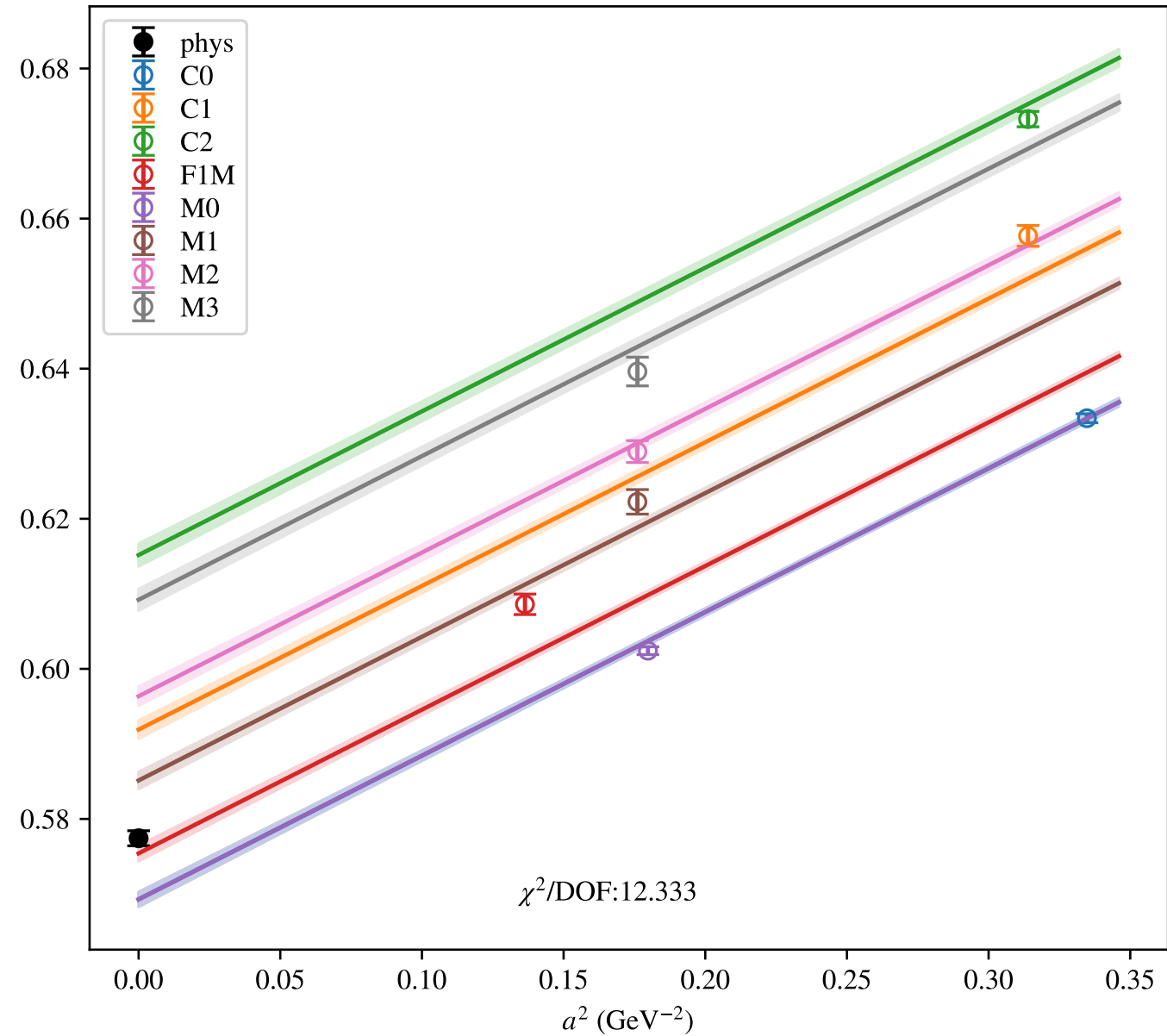


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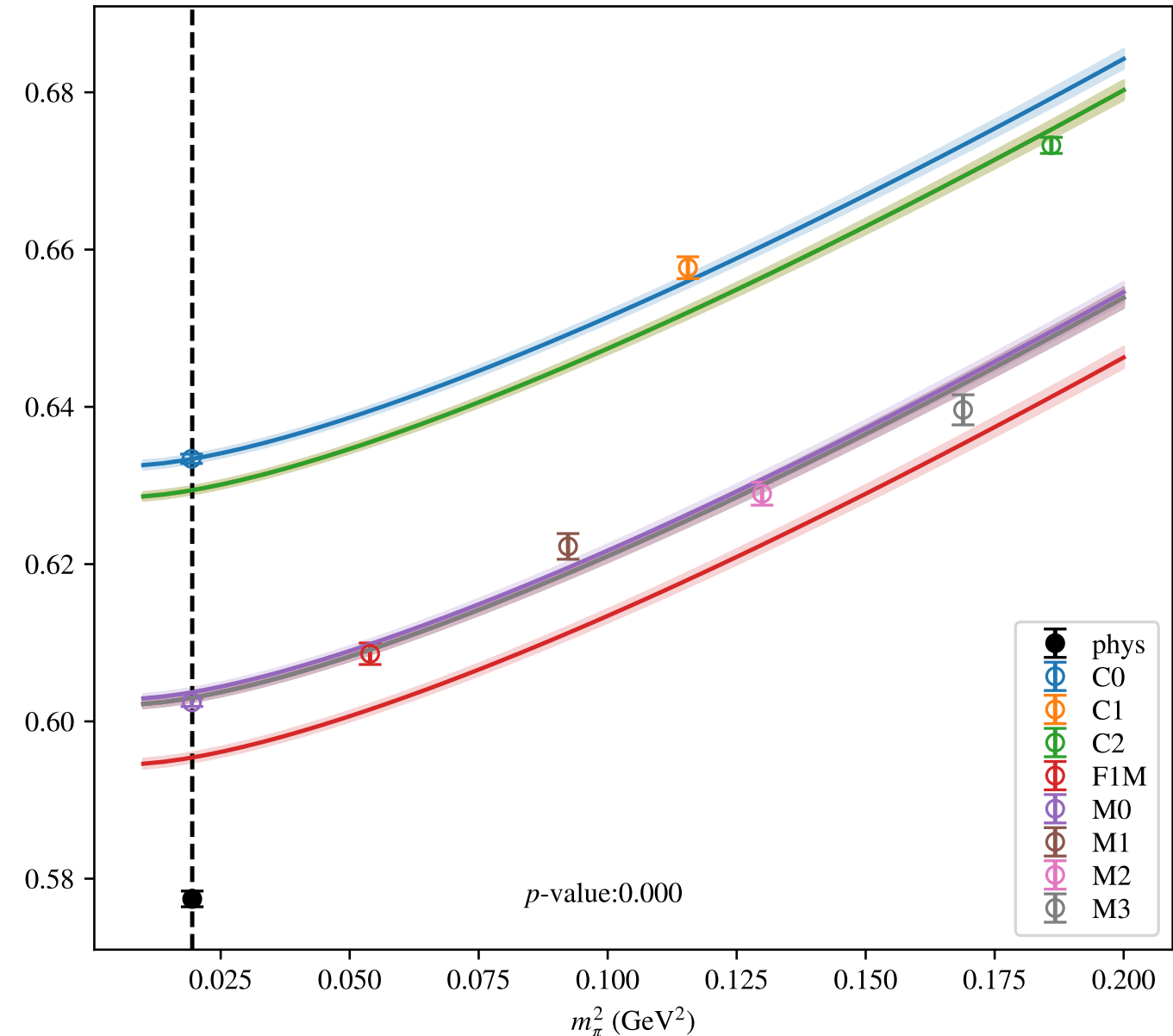


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 2.0 \text{ GeV}$$

SSpPP

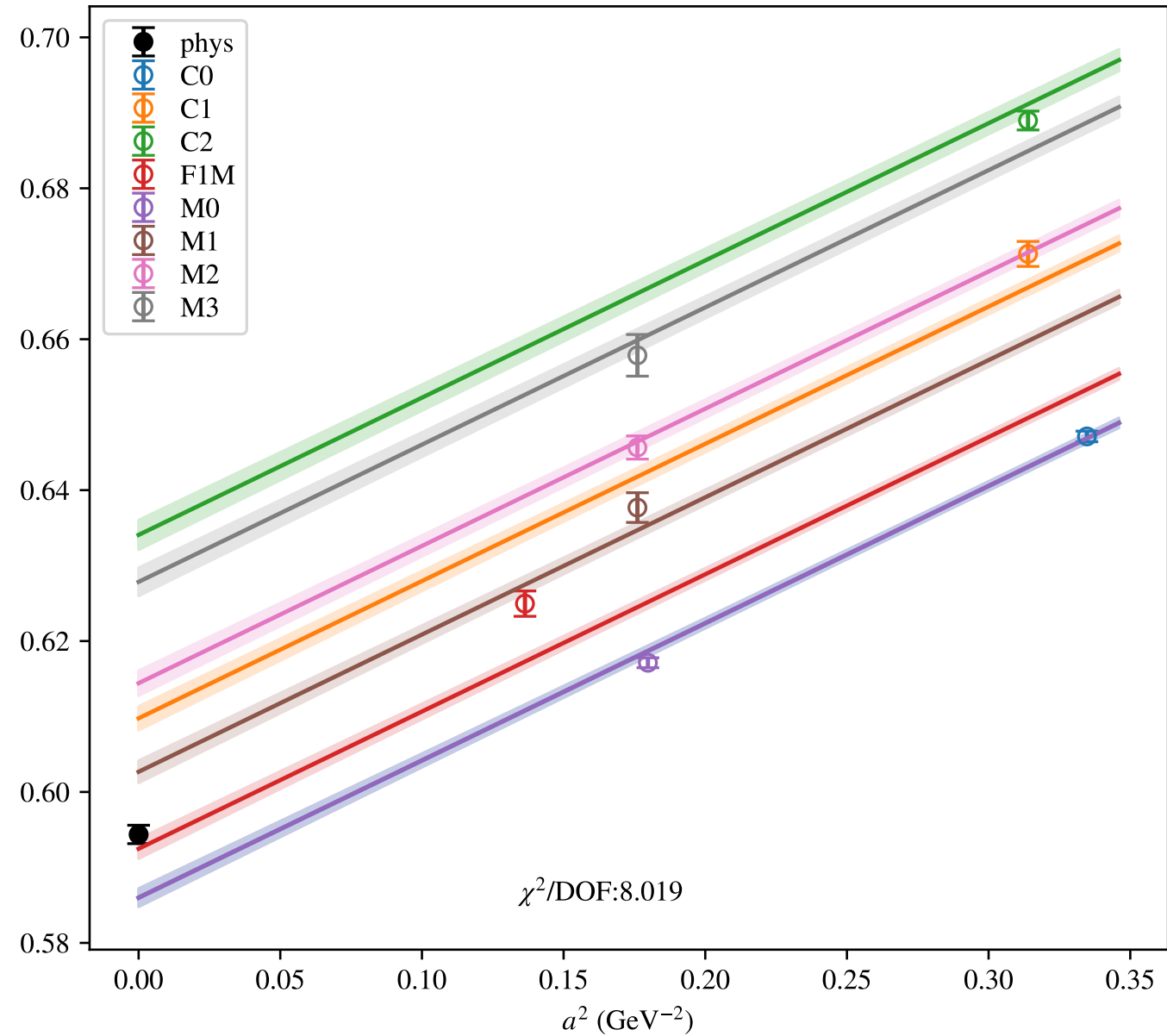


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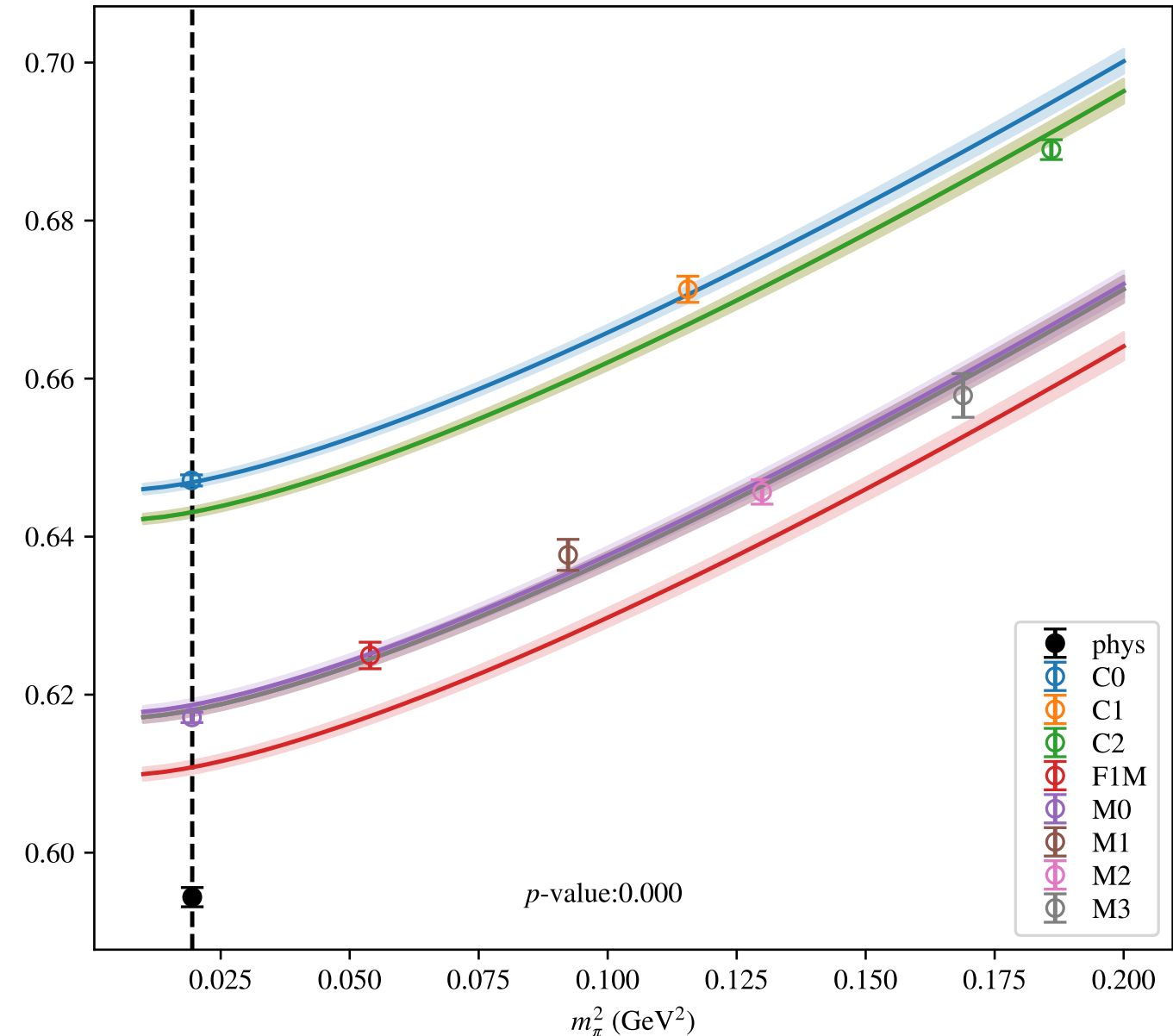


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 1.8 \text{ GeV}$$

SSpPP

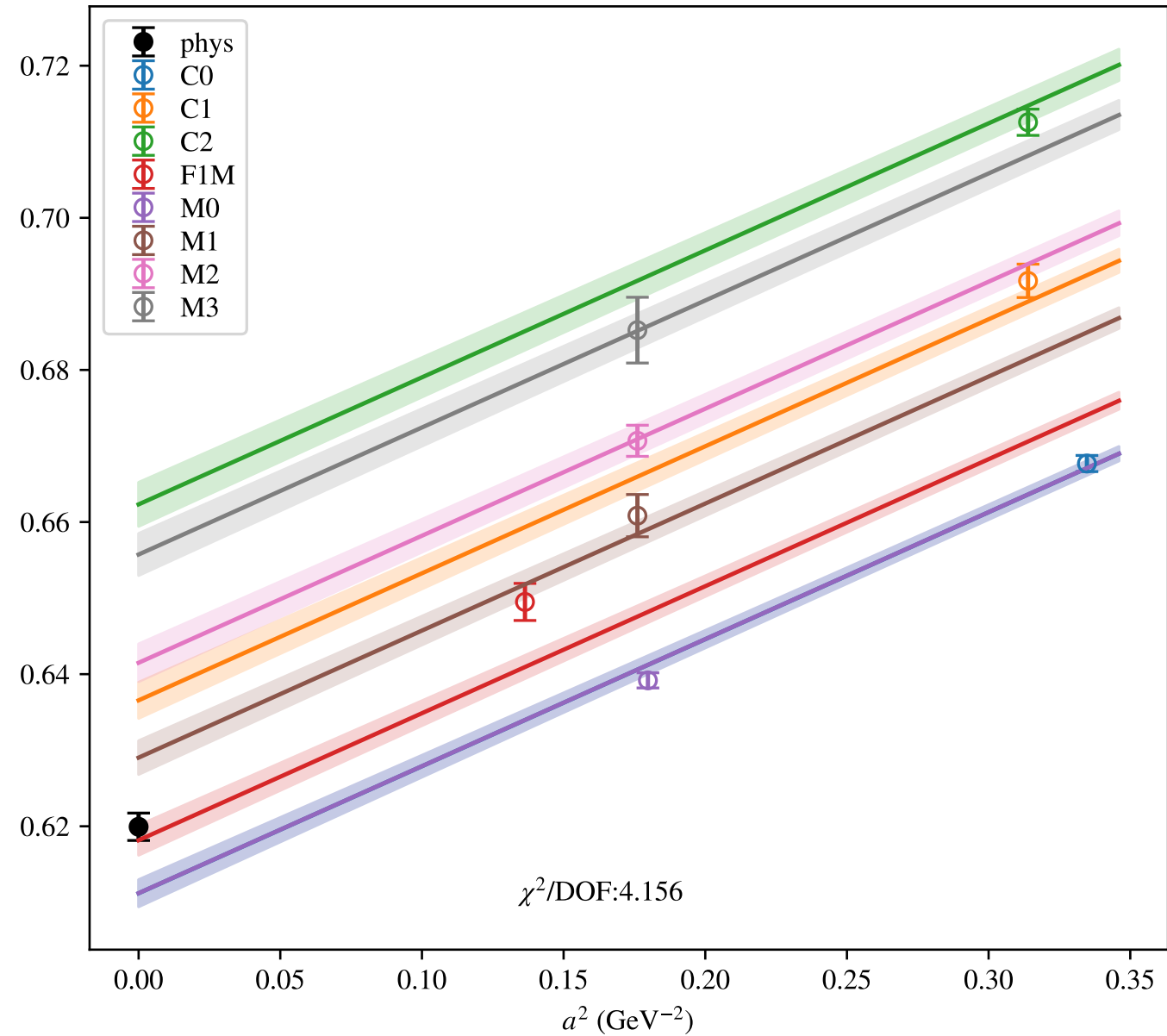


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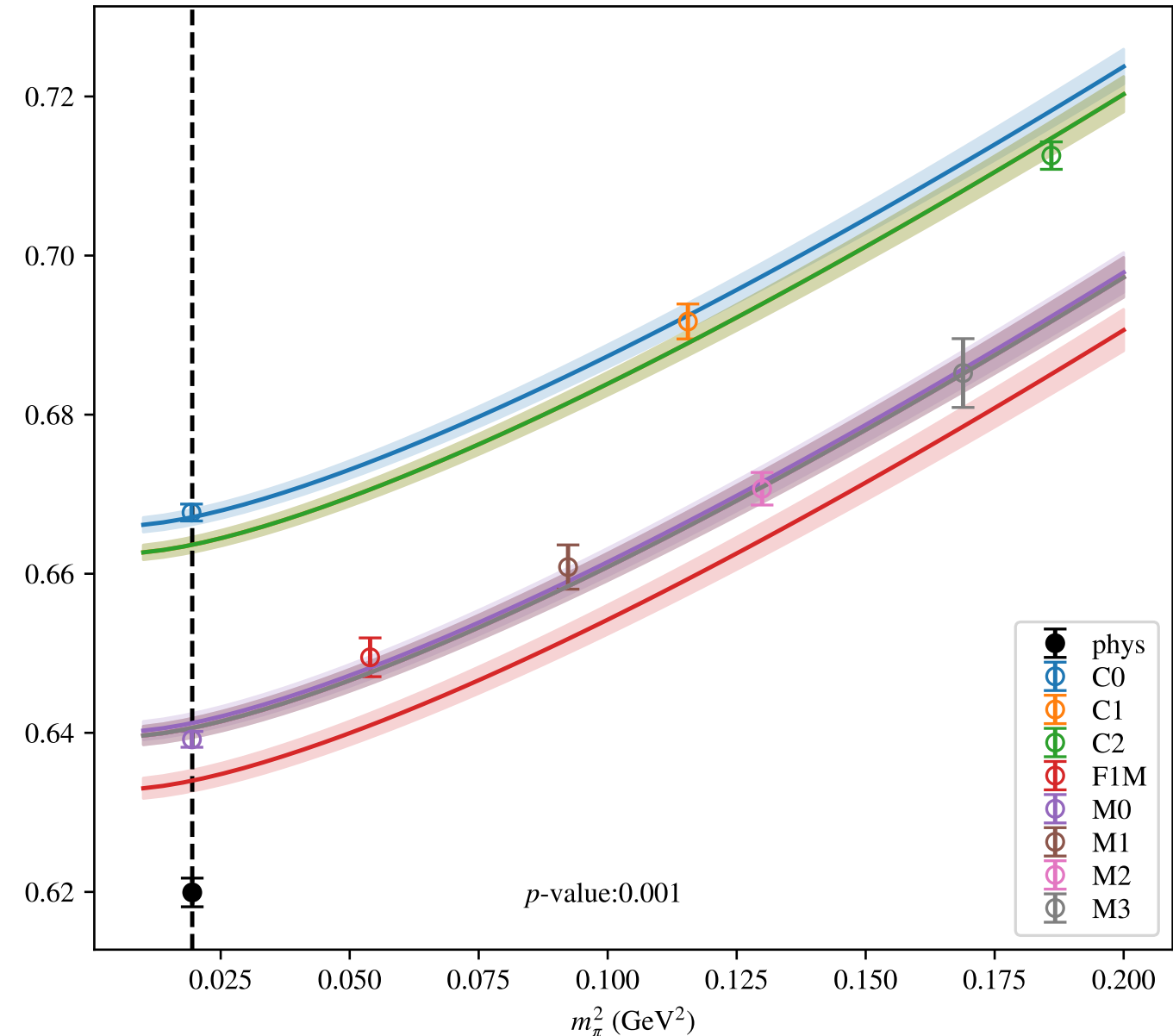


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 1.5 \text{ GeV}$$

SSpPP



SSpPP



## 5 TT

$\mu$ (GeV)	$a^2, m_\pi^2$	$a^2, m_\pi^2$ (no C)	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$	$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2)$
2.4	<b>0.52399(82)</b> : 8.212 (0.0)	<b>0.5570(52)</b> : 0.839 (0.432)	<b>0.5693(86)</b> : 3.317 (0.01)	<b>0.52309(82)</b> : 6.953 (0.0)	<b>0.53288(83)</b> : 20.113 (0.0)
2.0	<b>0.5940(10)</b> : 7.141 (0.0)	<b>0.6355(60)</b> : 0.563 (0.569)	<b>0.658(10)</b> : 1.281 (0.275)	<b>0.5930(10)</b> : 6.036 (0.0)	<b>0.6039(10)</b> : 14.63 (0.0)
1.8	<b>0.6358(17)</b> : 3.741 (0.002)	<b>0.6796(79)</b> : 0.363 (0.695)	<b>0.711(13)</b> : 0.386 (0.819)	<b>0.6345(15)</b> : 3.542 (0.007)	<b>0.6468(17)</b> : 7.548 (0.0)
1.5	<b>0.6990(32)</b> : 2.017 (0.073)	<b>0.745(12)</b> : 0.194 (0.823)	<b>0.791(20)</b> : 0.255 (0.907)	<b>0.6975(26)</b> : 2.13 (0.074)	<b>0.7116(32)</b> : 3.696 (0.002)

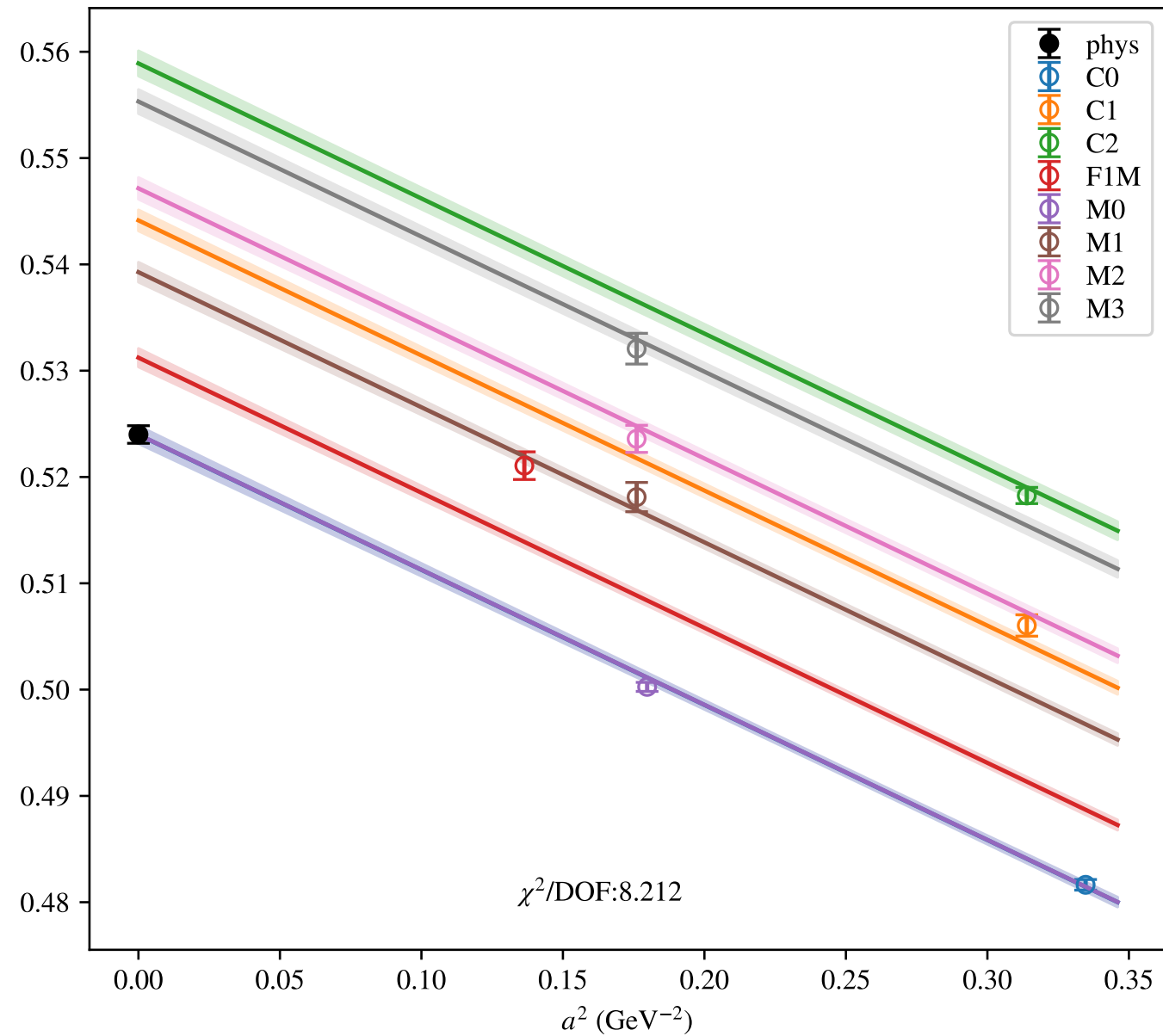
Table 9: Physical point value from chiral and continuum extrapolation at renormalisation scale  $\mu$ . Entries are **value(error)**:  $\chi^2/\text{DOF}$  ( $p$ -value).

$\mu$ (GeV)		$a^2, m_\pi^2$	$a^2, m_\pi^2$ (no C)	$a^2, a^4, m_\pi^2$	$a^2, m_\pi^2, m_\pi^4$	$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2)$
2.4	$\alpha$	-0.244(51)	-0.57(48)	-0.9(12)	-0.239(53)	-0.249(50)
	$\beta$	0.00685(14)	0.00633(25)	0.00656(12)	0.00962(72)	0.01156(13)
2.0	$\alpha$	-0.284(51)	-0.64(48)	-1.1(12)	-0.278(53)	-0.288(51)
	$\beta$	0.00728(23)	0.00685(43)	0.00685(19)	0.01082(99)	0.01205(22)
1.8	$\alpha$	-0.290(59)	-0.64(55)	-1.2(13)	-0.284(56)	-0.296(59)
	$\beta$	0.00713(24)	0.00708(40)	0.00674(19)	0.0100(10)	0.01189(24)
1.5	$\alpha$	-0.302(78)	-0.65(72)	-1.3(17)	-0.296(63)	-0.310(76)
	$\beta$	0.00700(26)	0.00741(46)	0.00661(19)	0.0094(13)	0.01173(26)

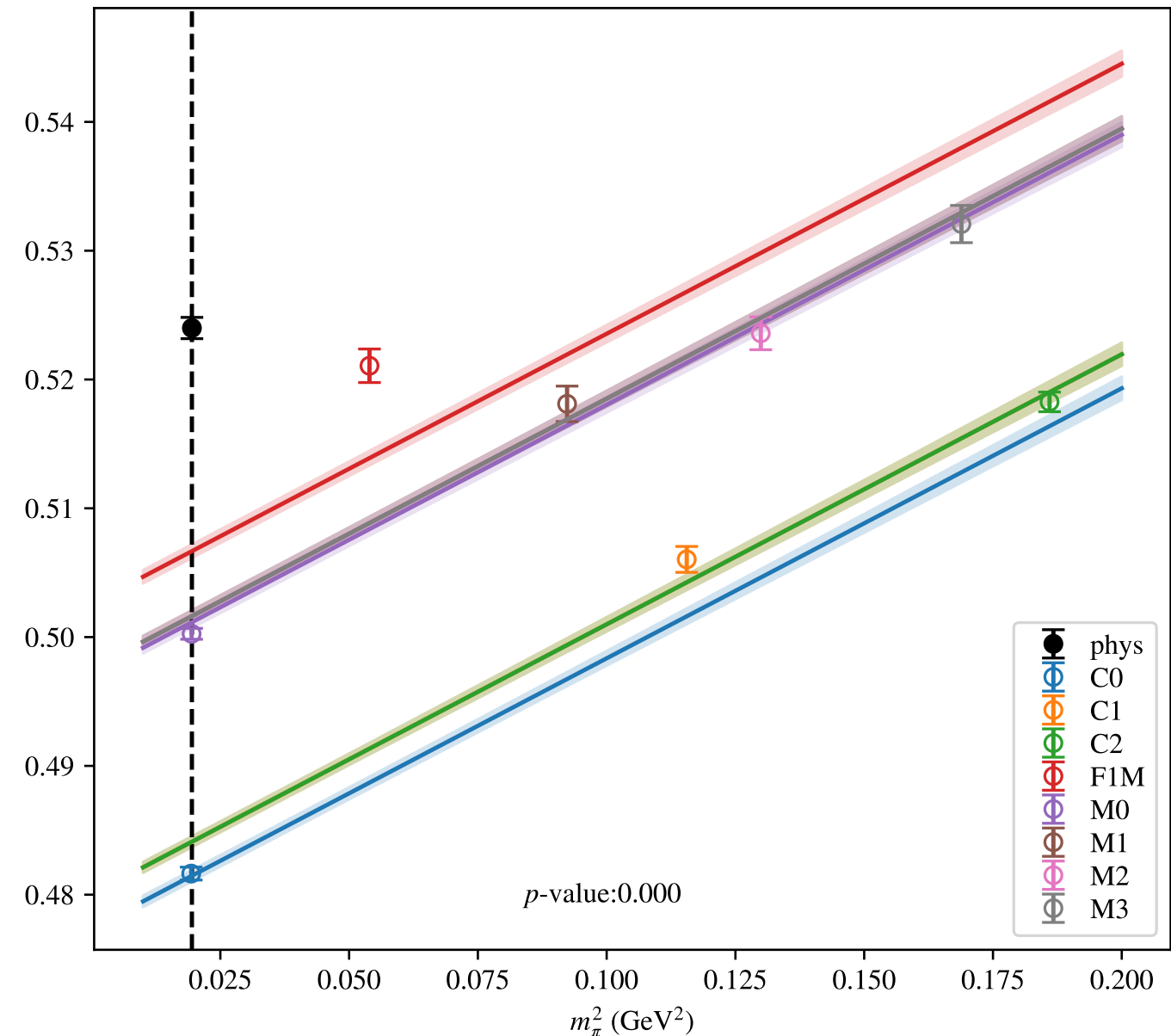
Table 10: Fit values of coefficients in  $B = B_0(1 + \alpha a^2 + \beta \frac{m_\pi^2}{f_\pi^2} + \dots)$ .

$$a^2, m_\pi^2, \mu = 2.4 \text{ GeV}$$

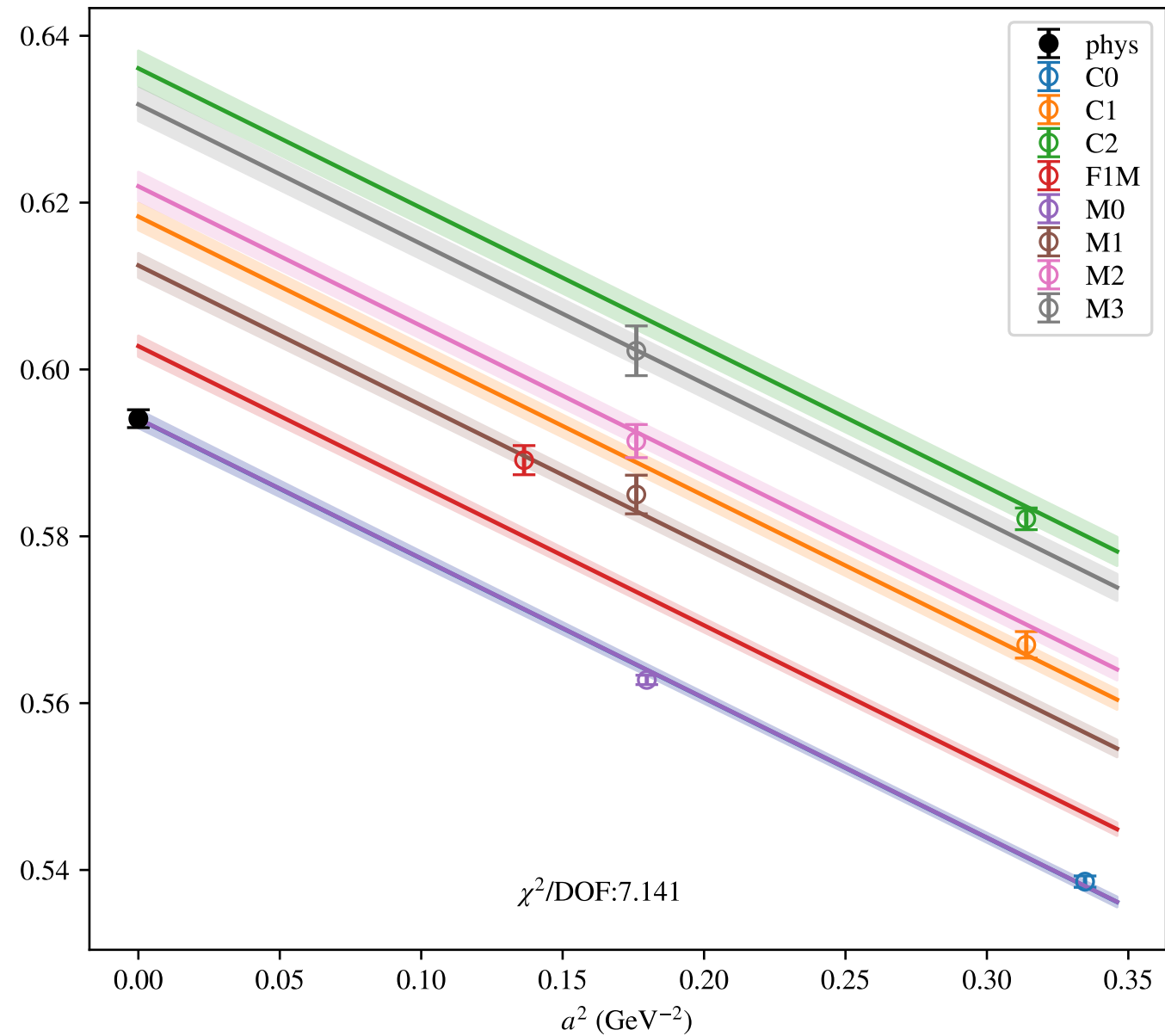
TT



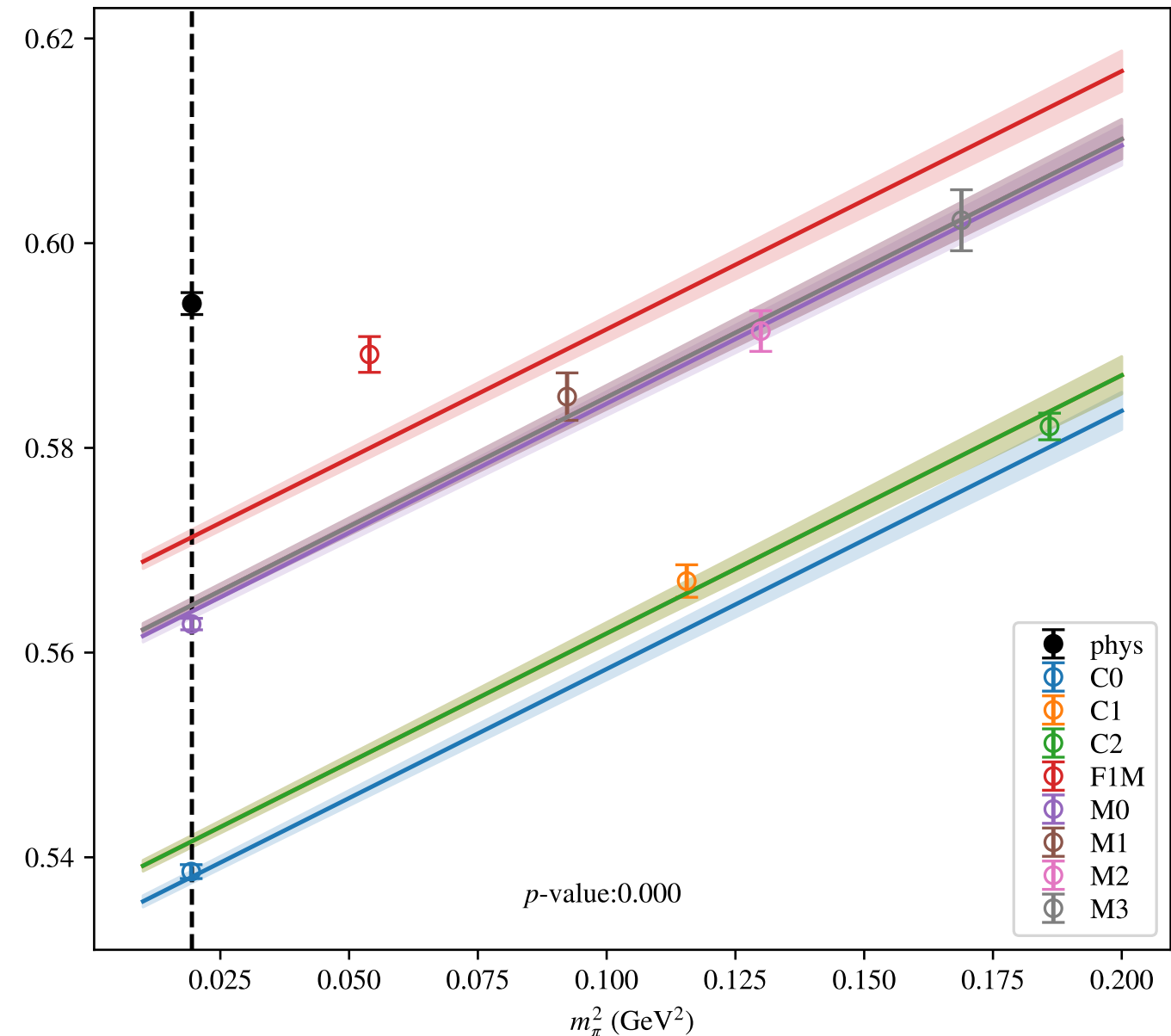
TT



TT



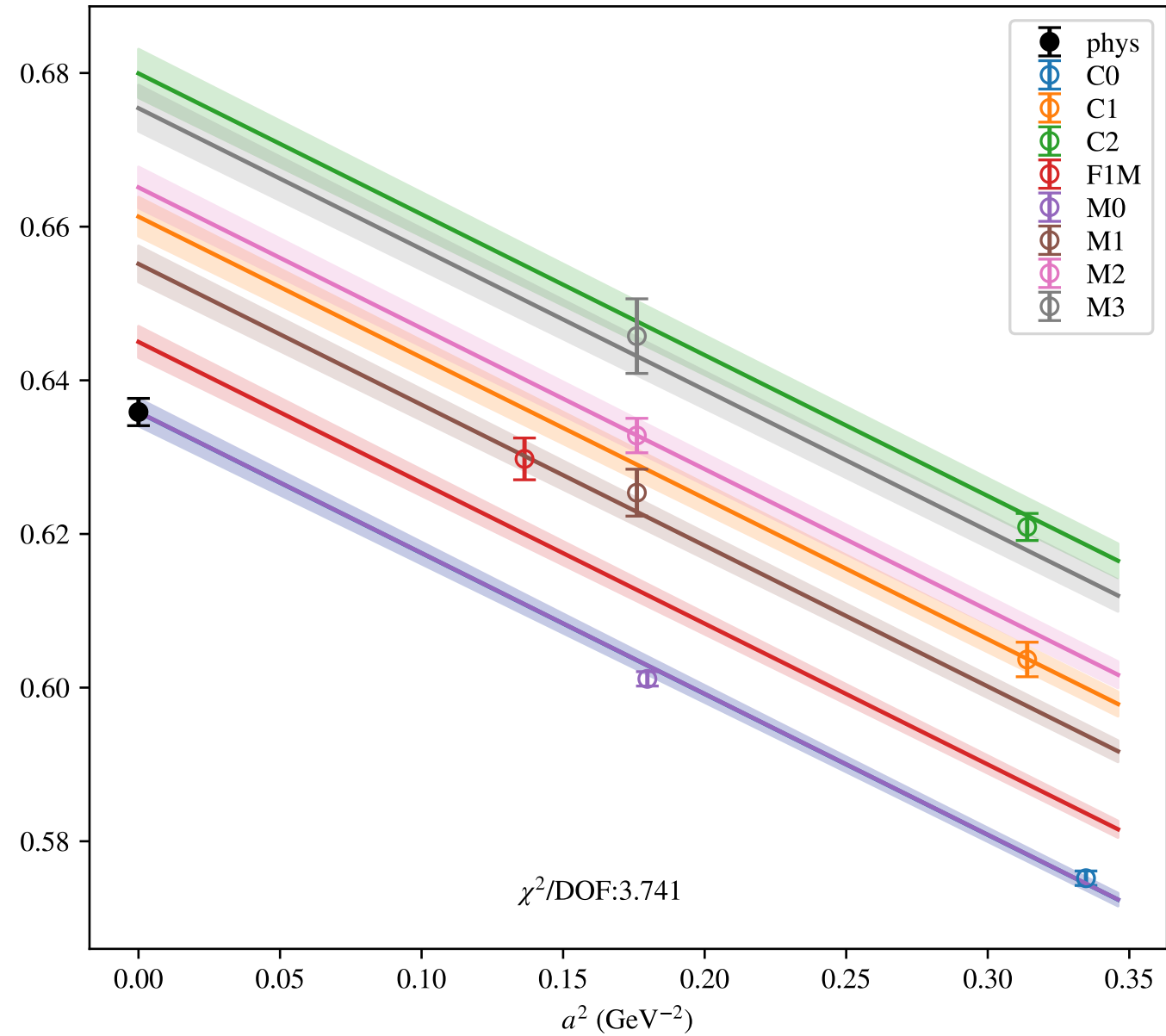
TT



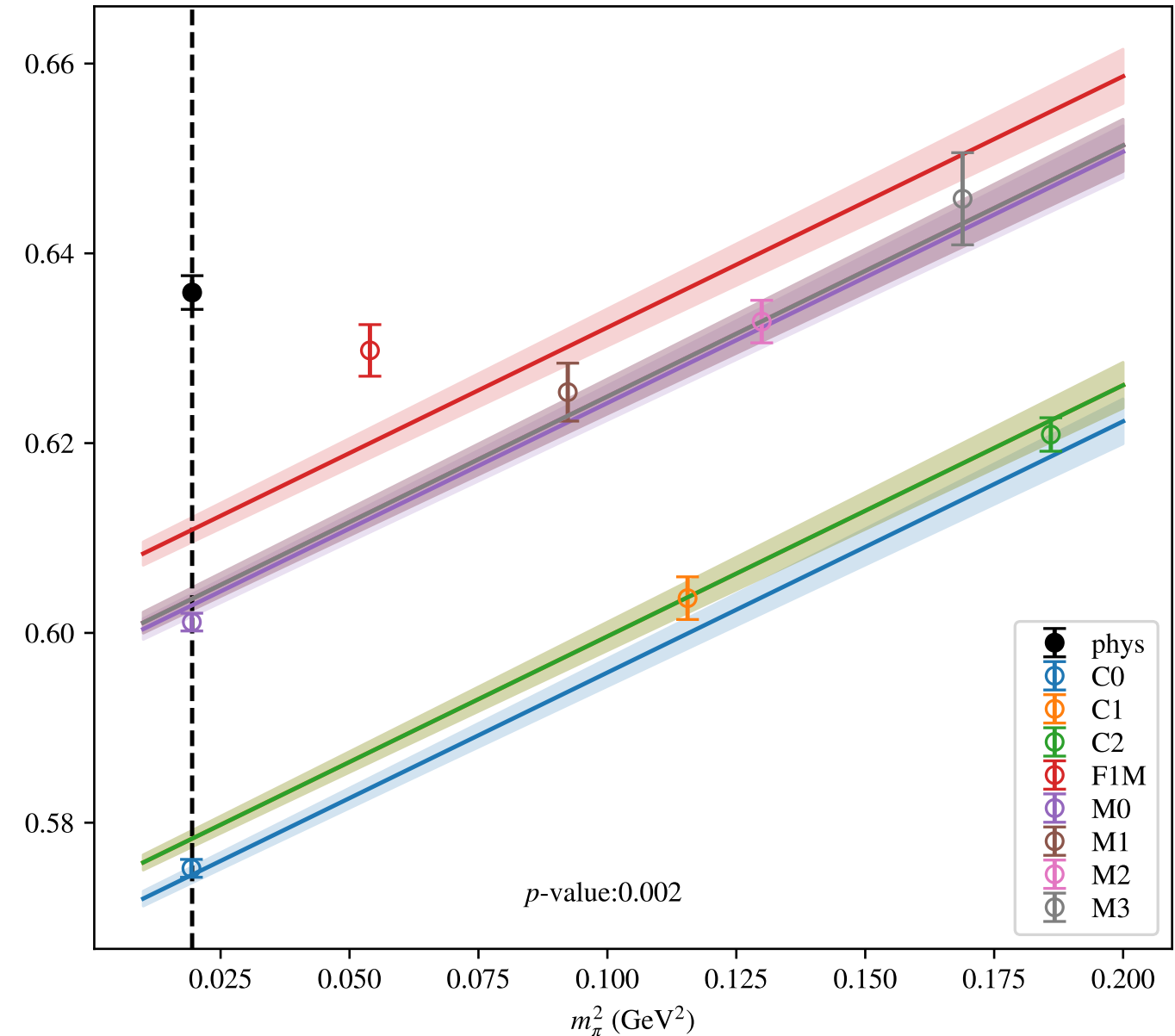


$a^2, m_\pi^2, \mu = 1.8 \text{ GeV}$ 

TT

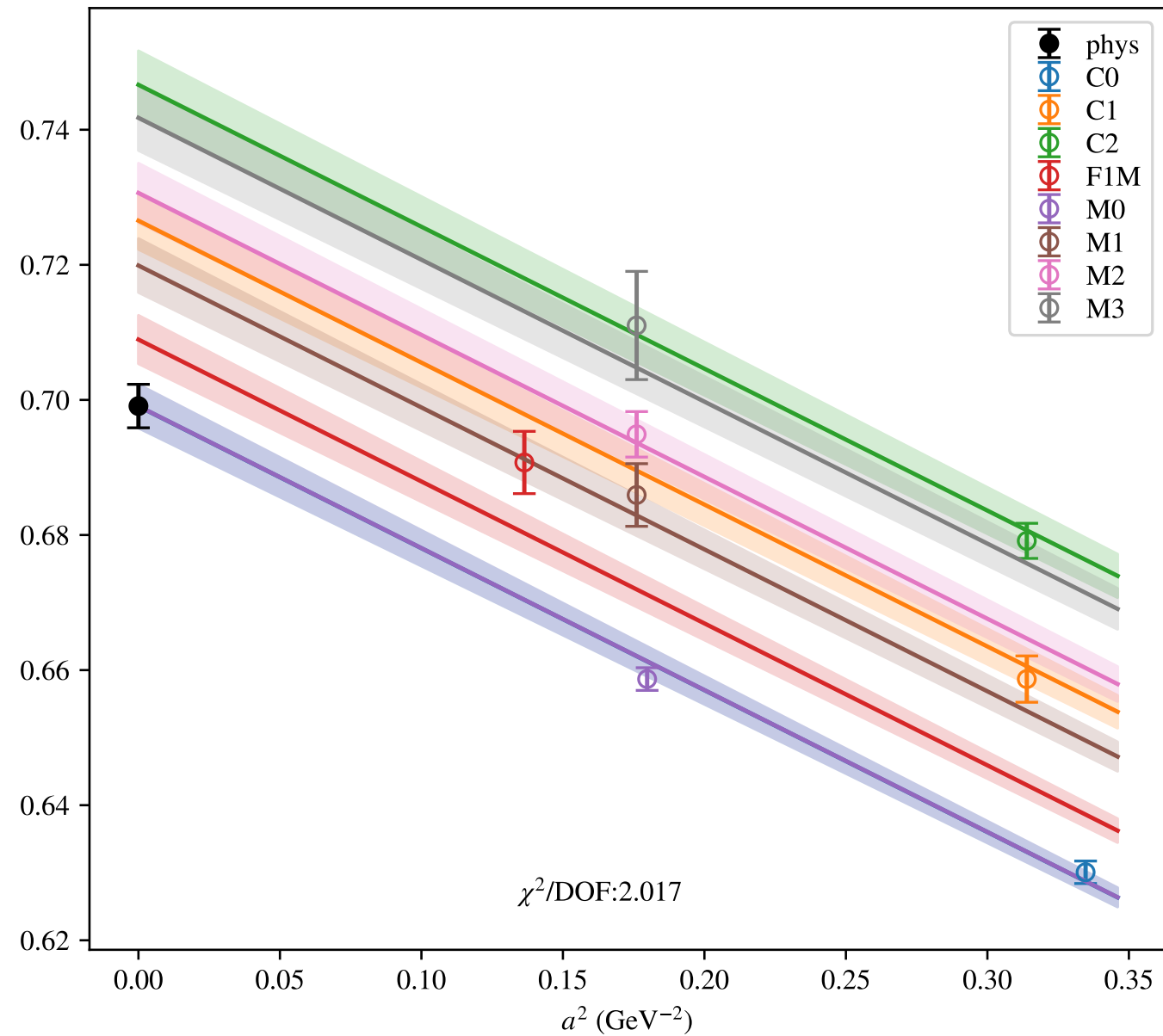


TT

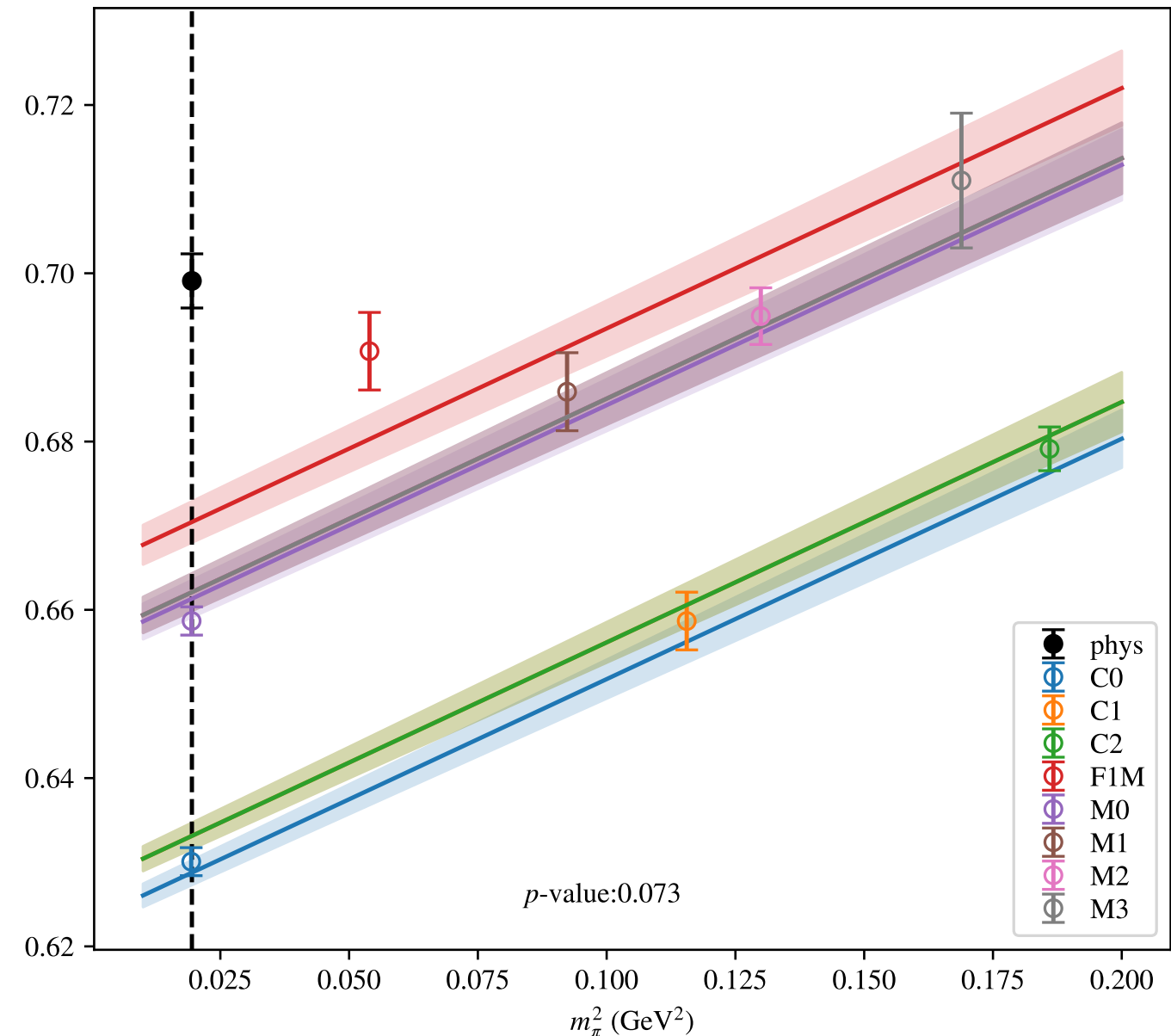


$$a^2, m_\pi^2, \mu = 1.5 \text{ GeV}$$

TT

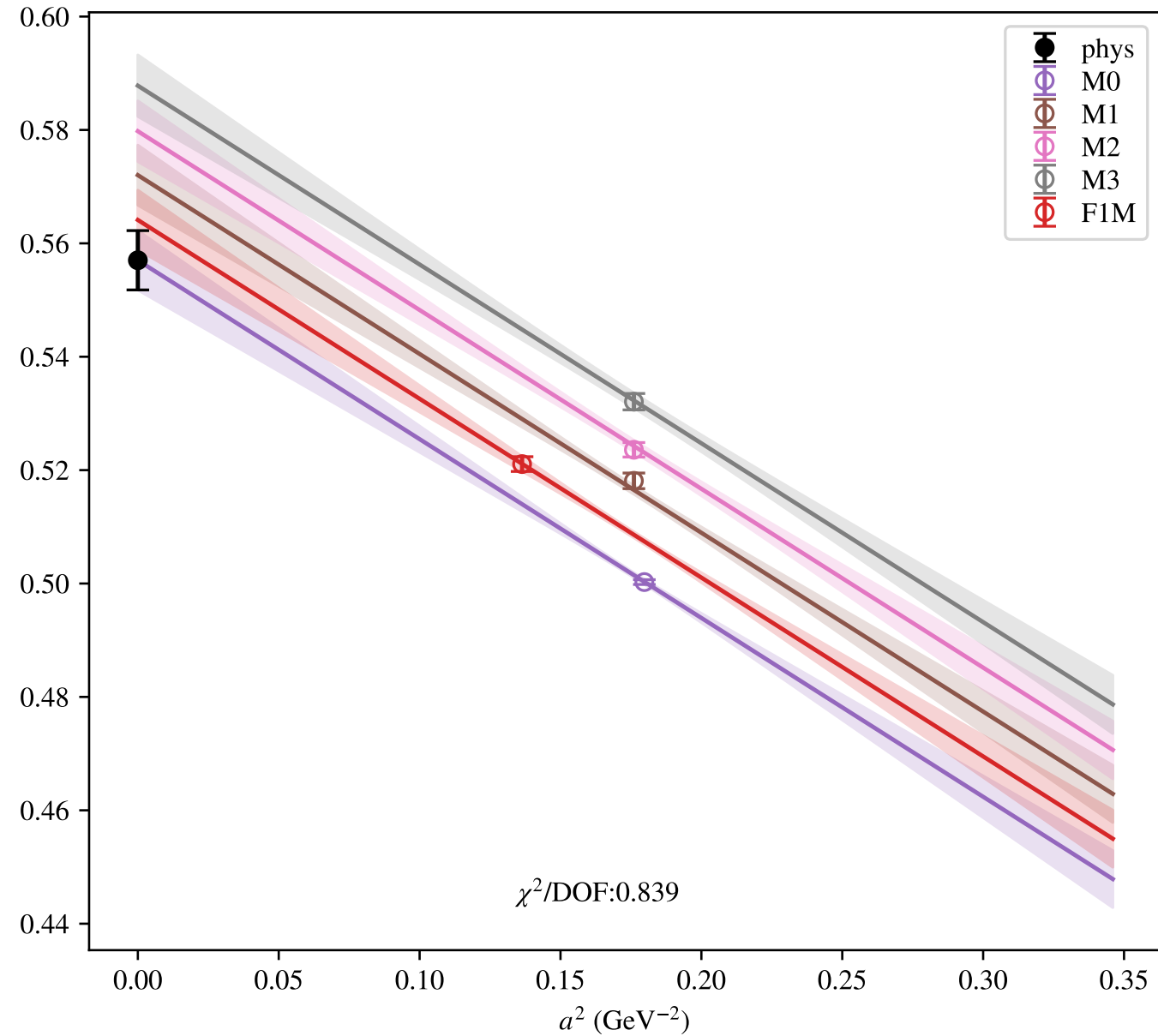


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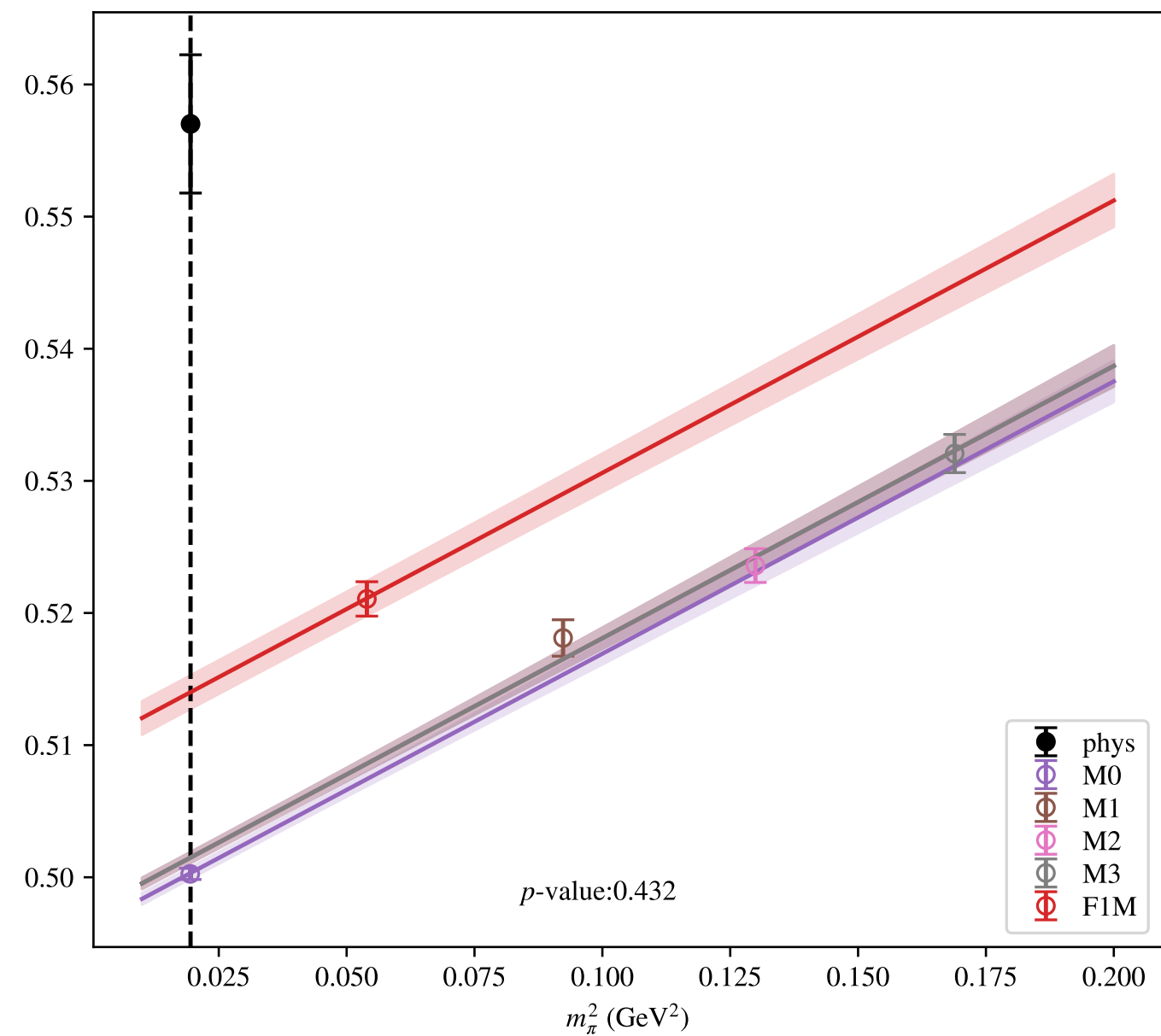


$a^2, m_\pi^2$  (no C),  $\mu = 2.4$  GeV

TT

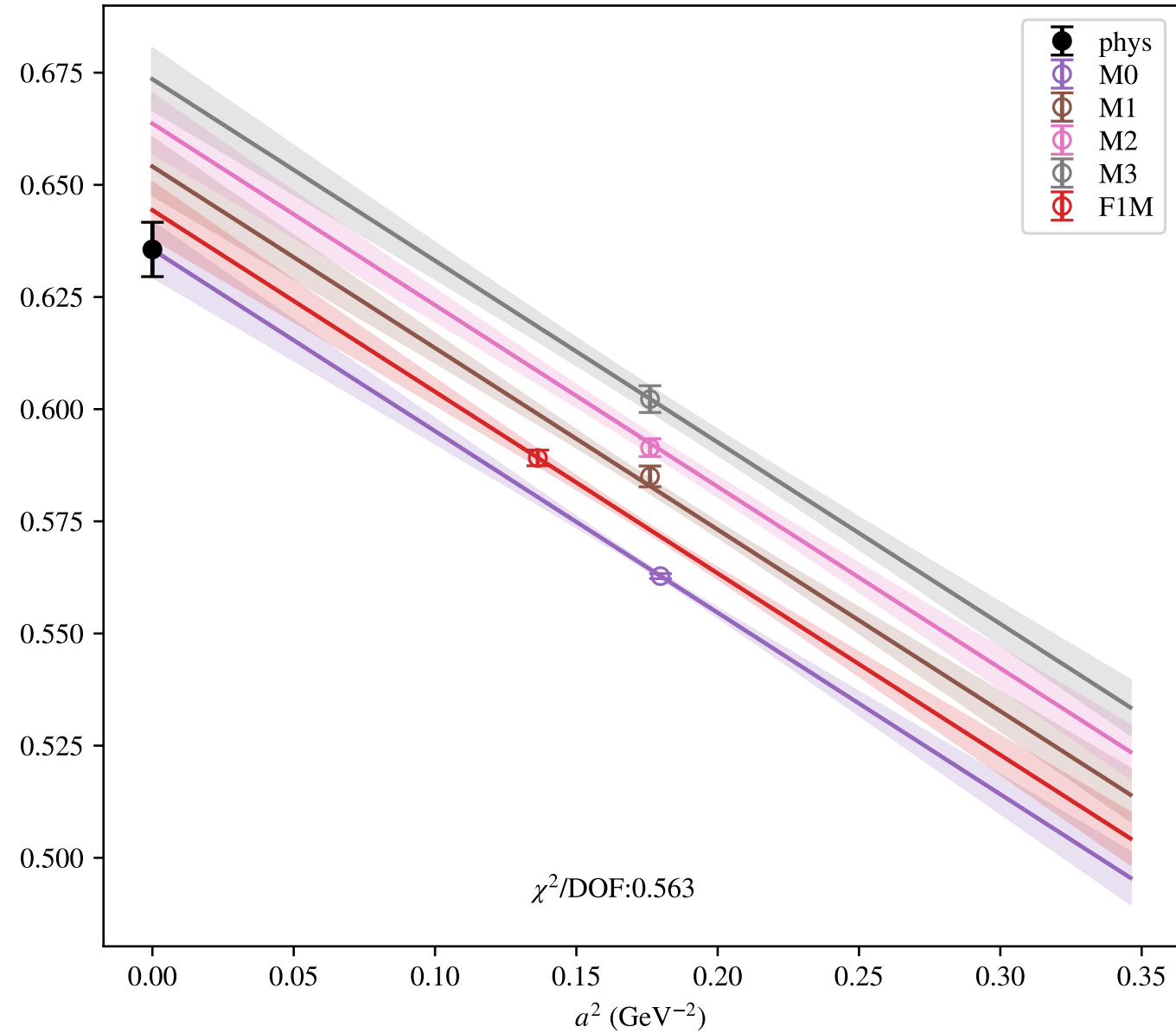


TT

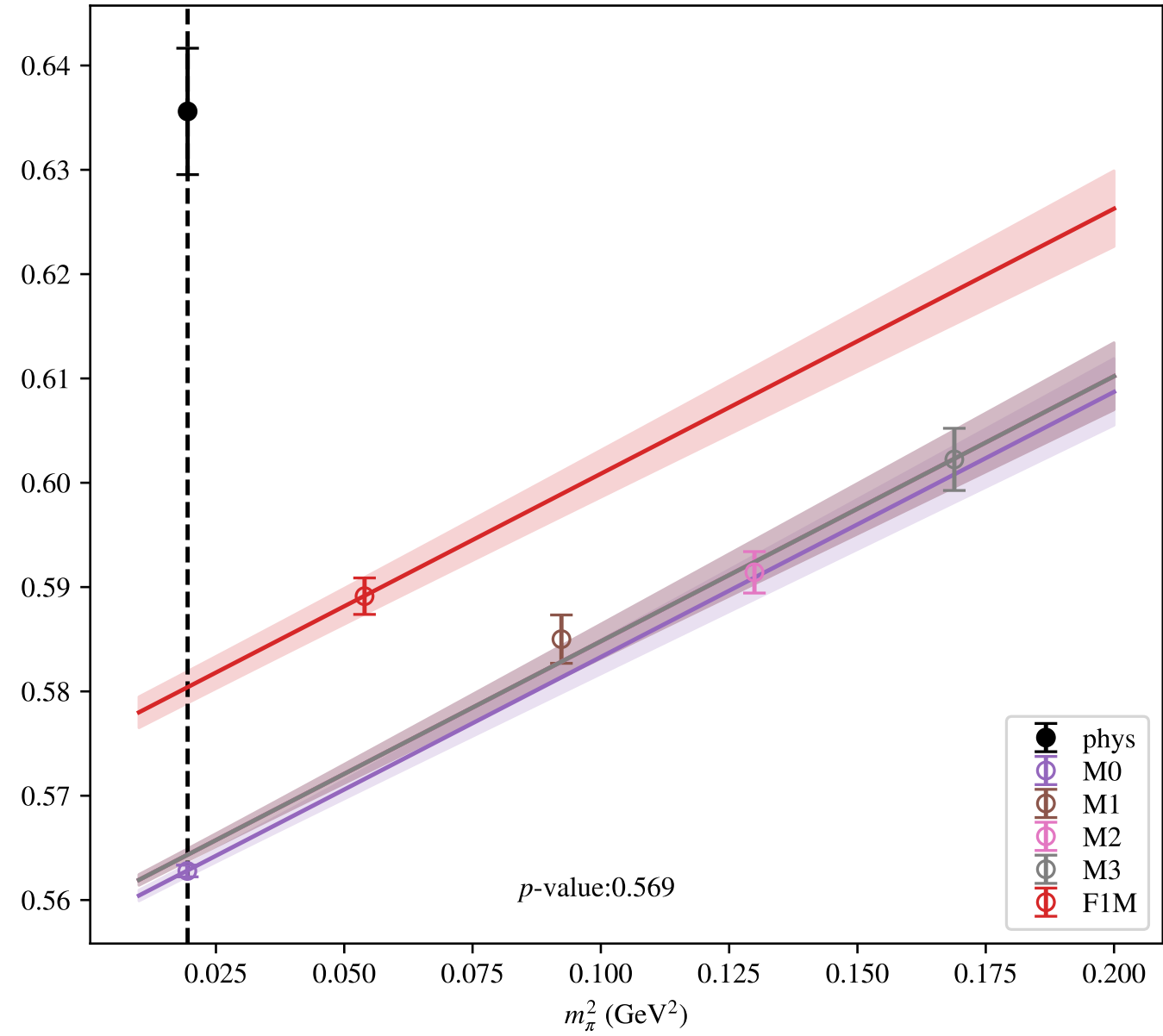


$a^2, m_\pi^2$  (no C),  $\mu = 2.0$  GeV

TT

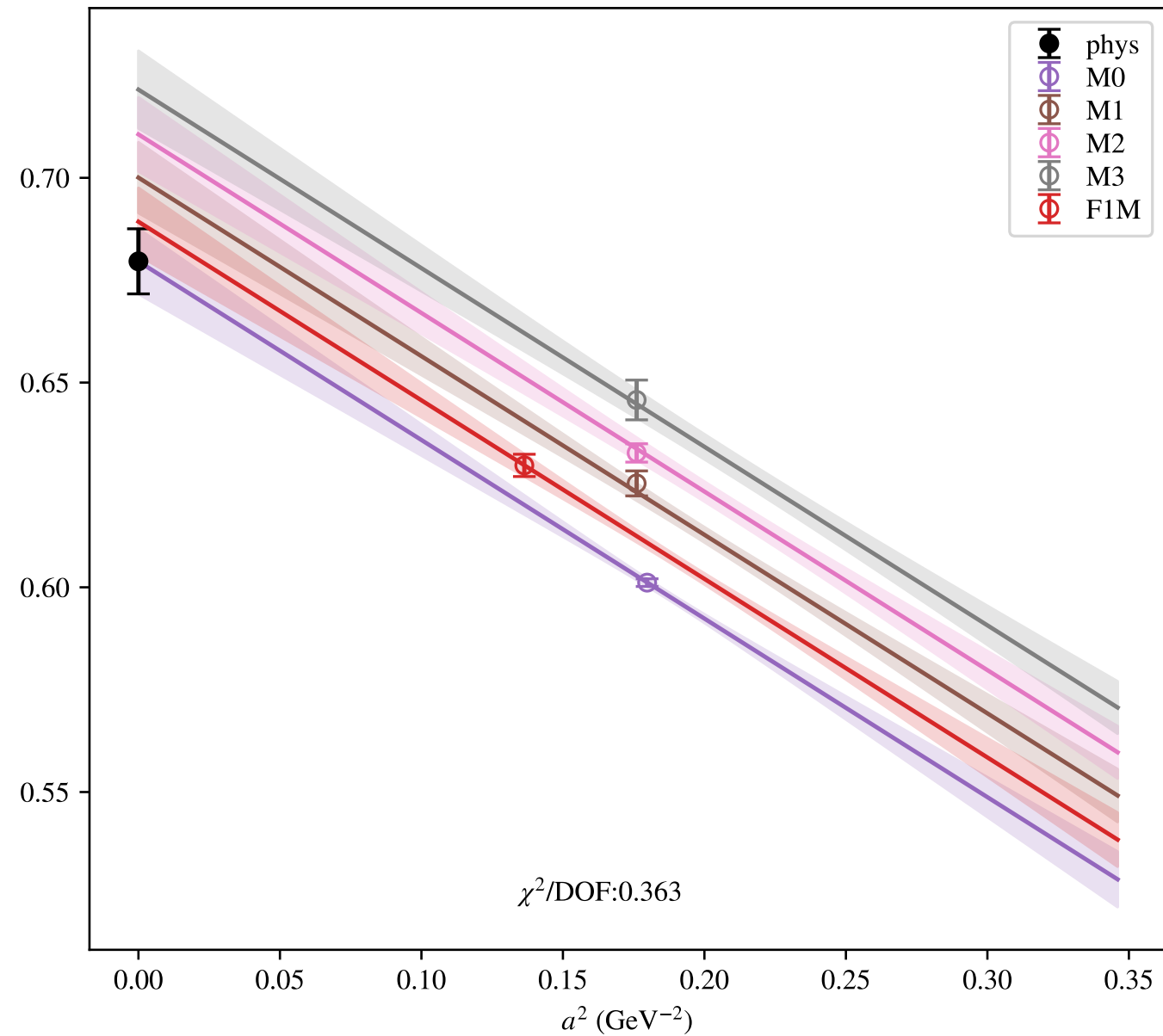


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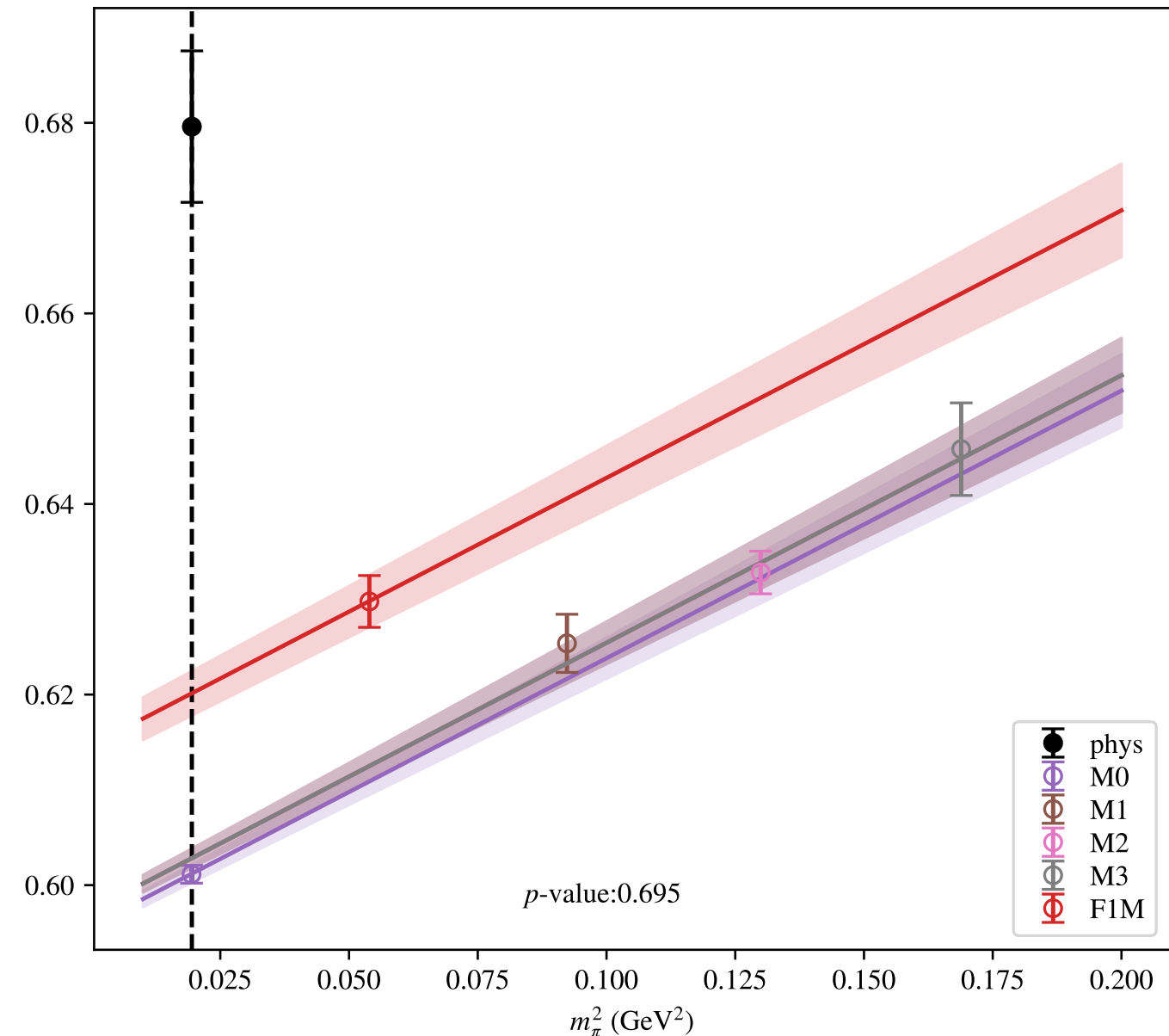


$a^2, m_\pi^2$  (no C),  $\mu = 1.8$  GeV

TT

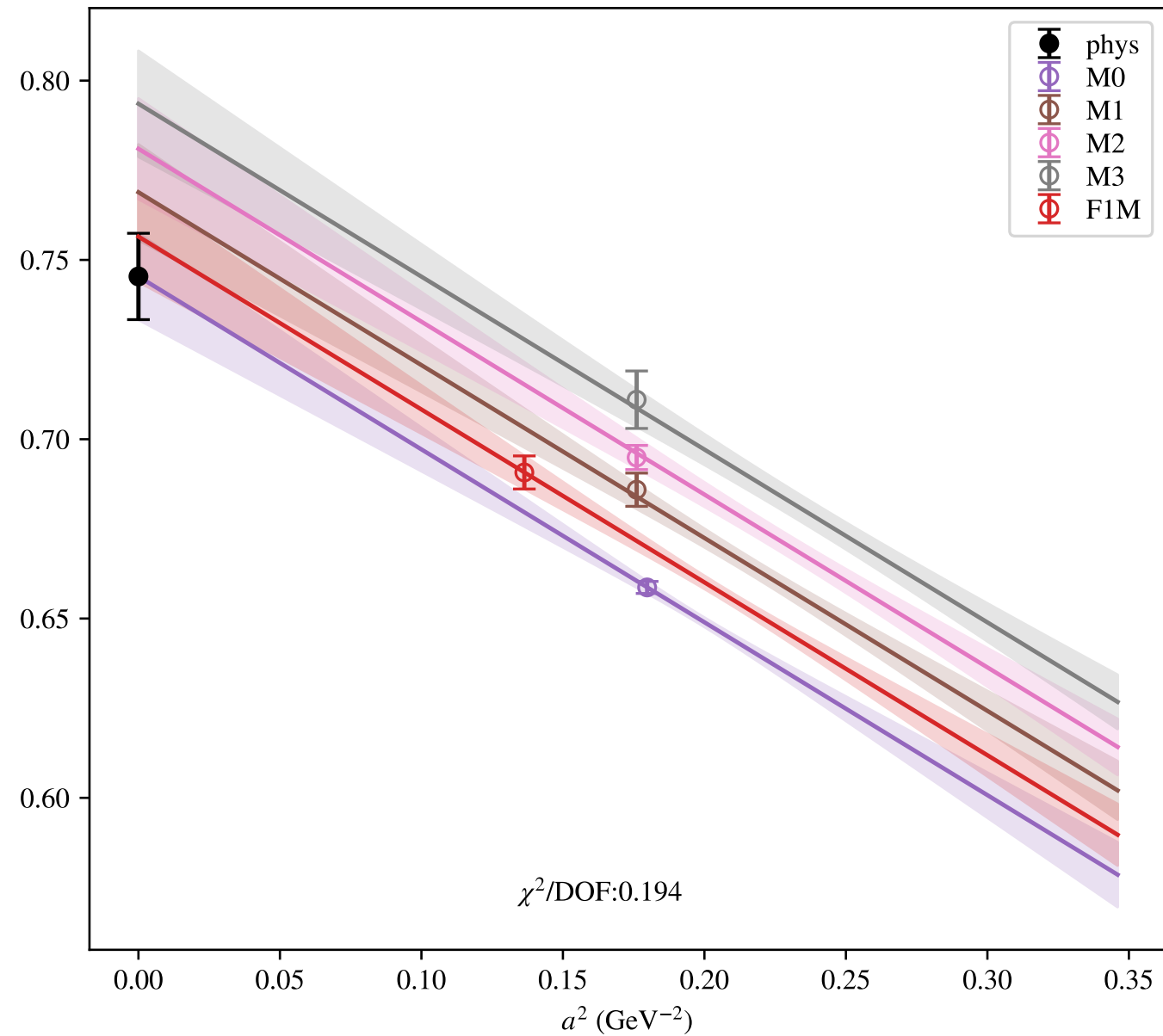


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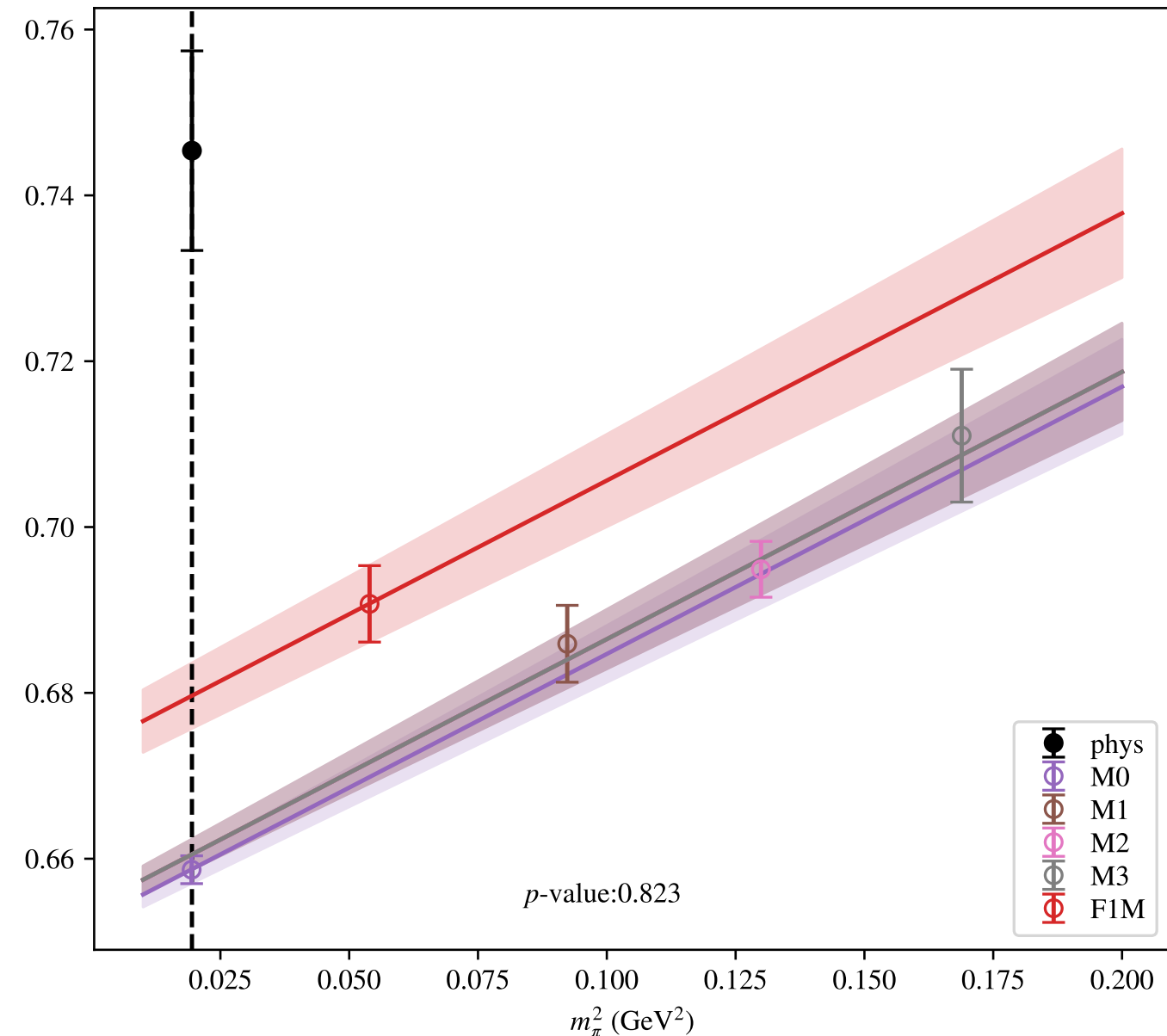


$a^2, m_\pi^2$  (no C),  $\mu = 1.5$  GeV

TT

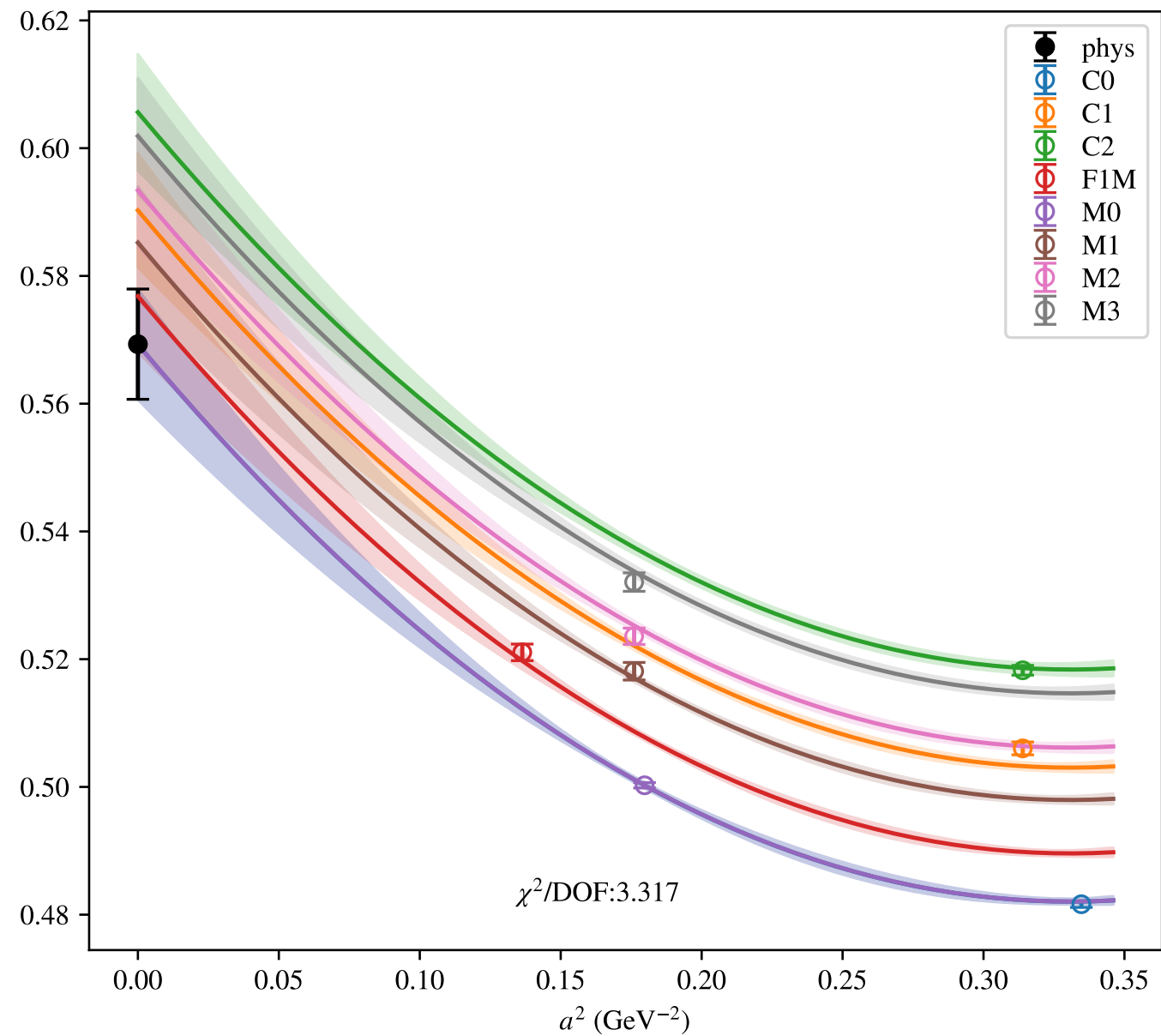


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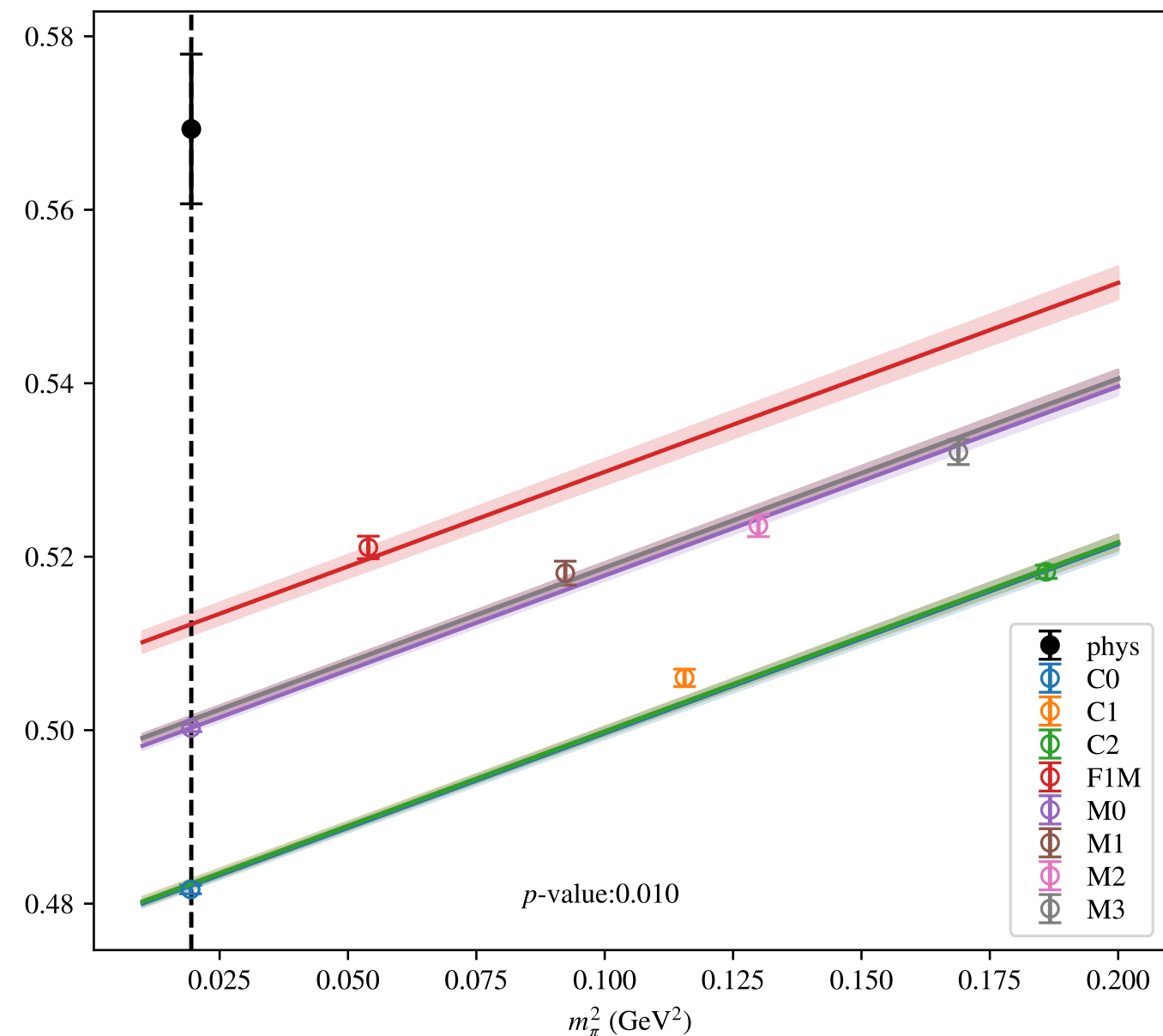


$$a^2, a^4, m_\pi^2, \mu = 2.4 \text{ GeV}$$

TT

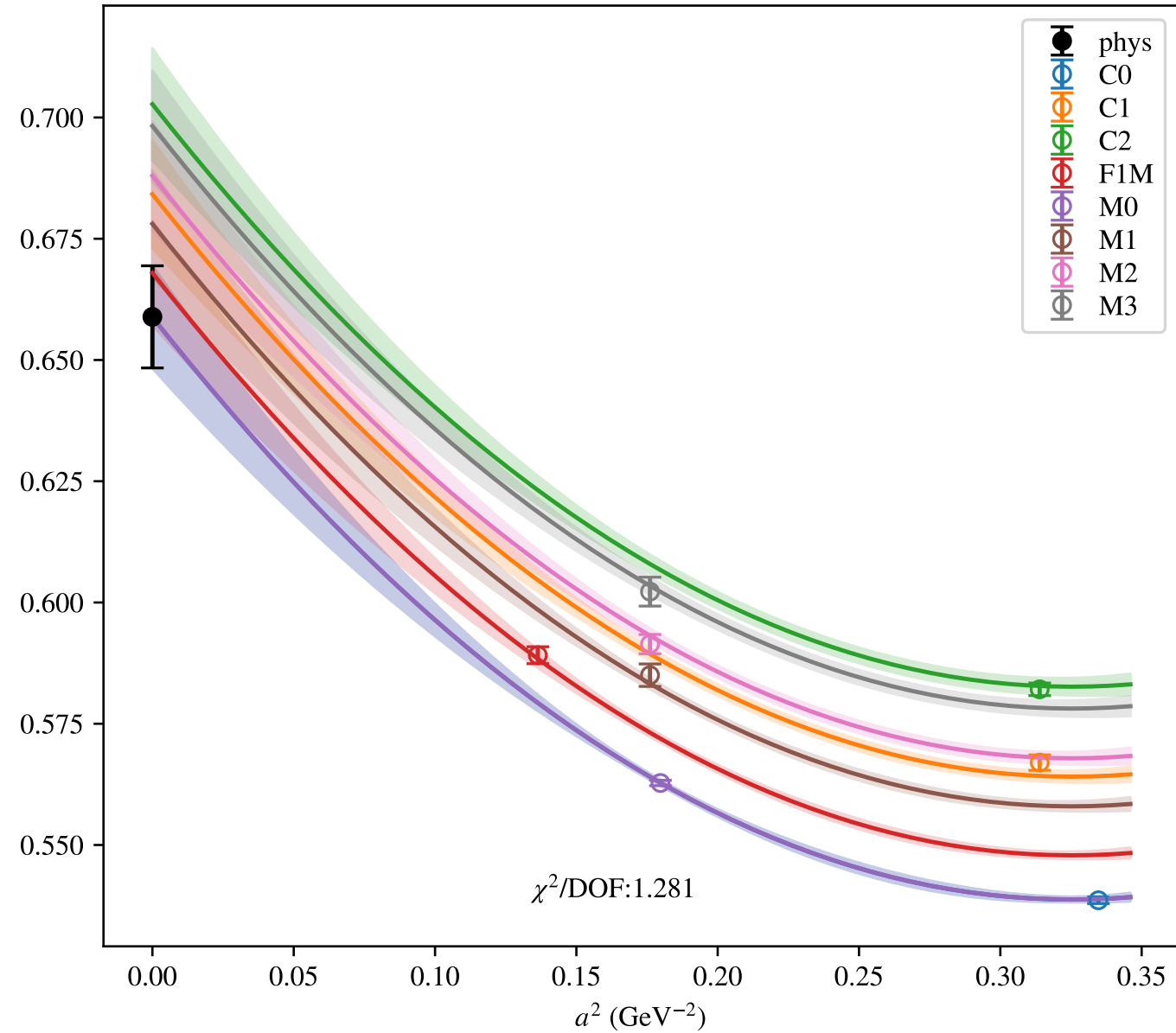


TT

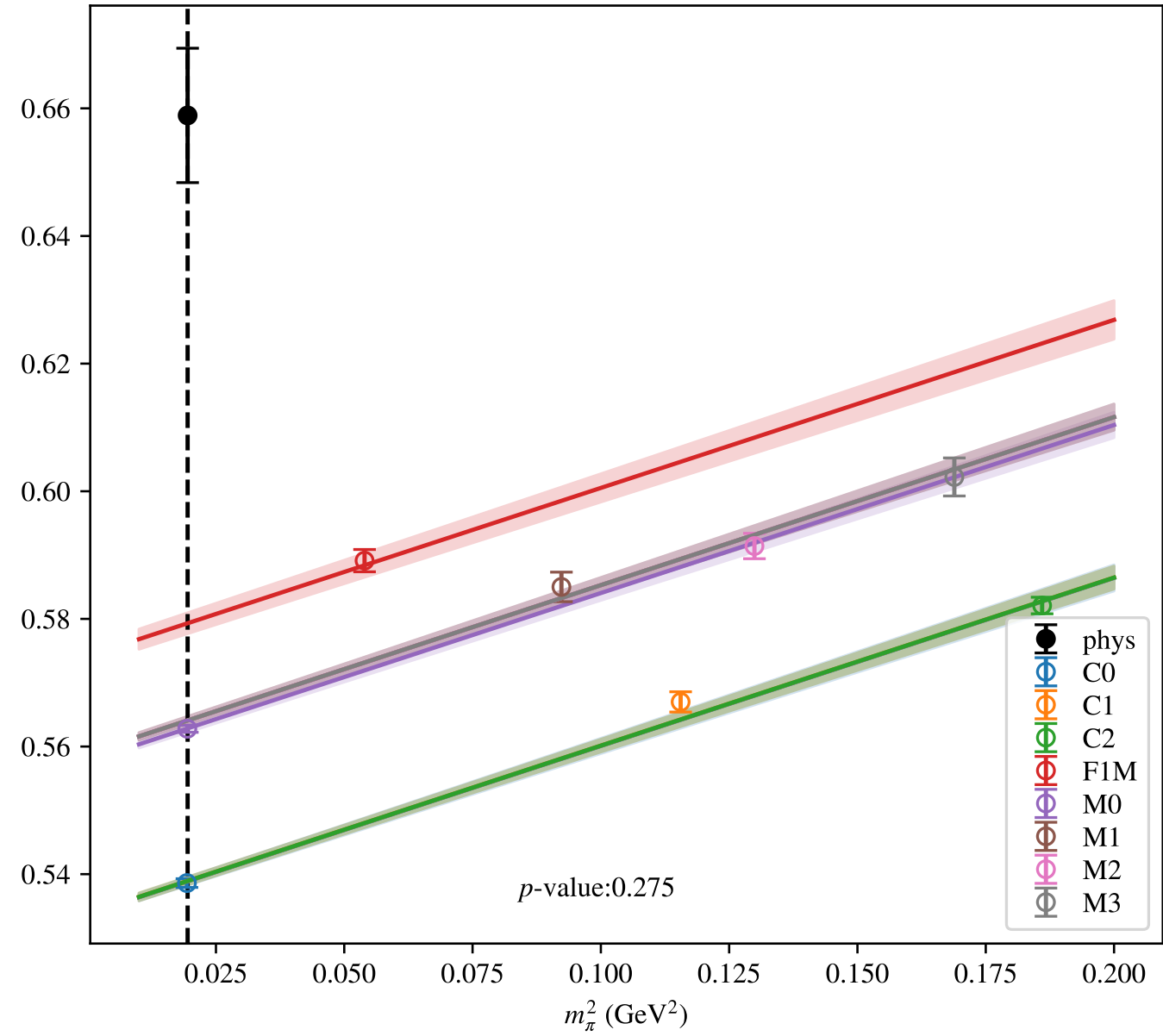


$a^2, a^4, m_\pi^2, \mu = 2.0 \text{ GeV}$

TT



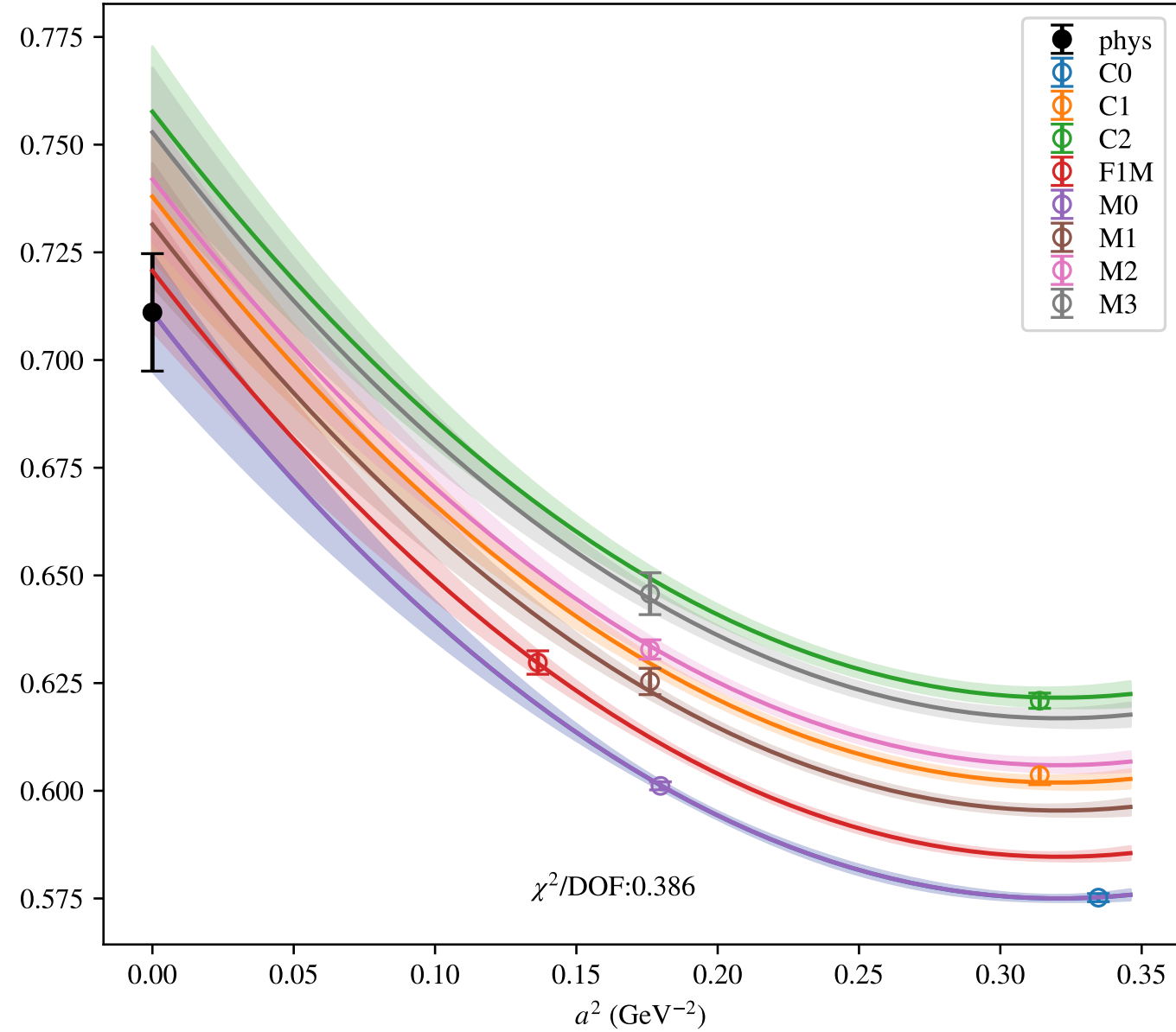
TT



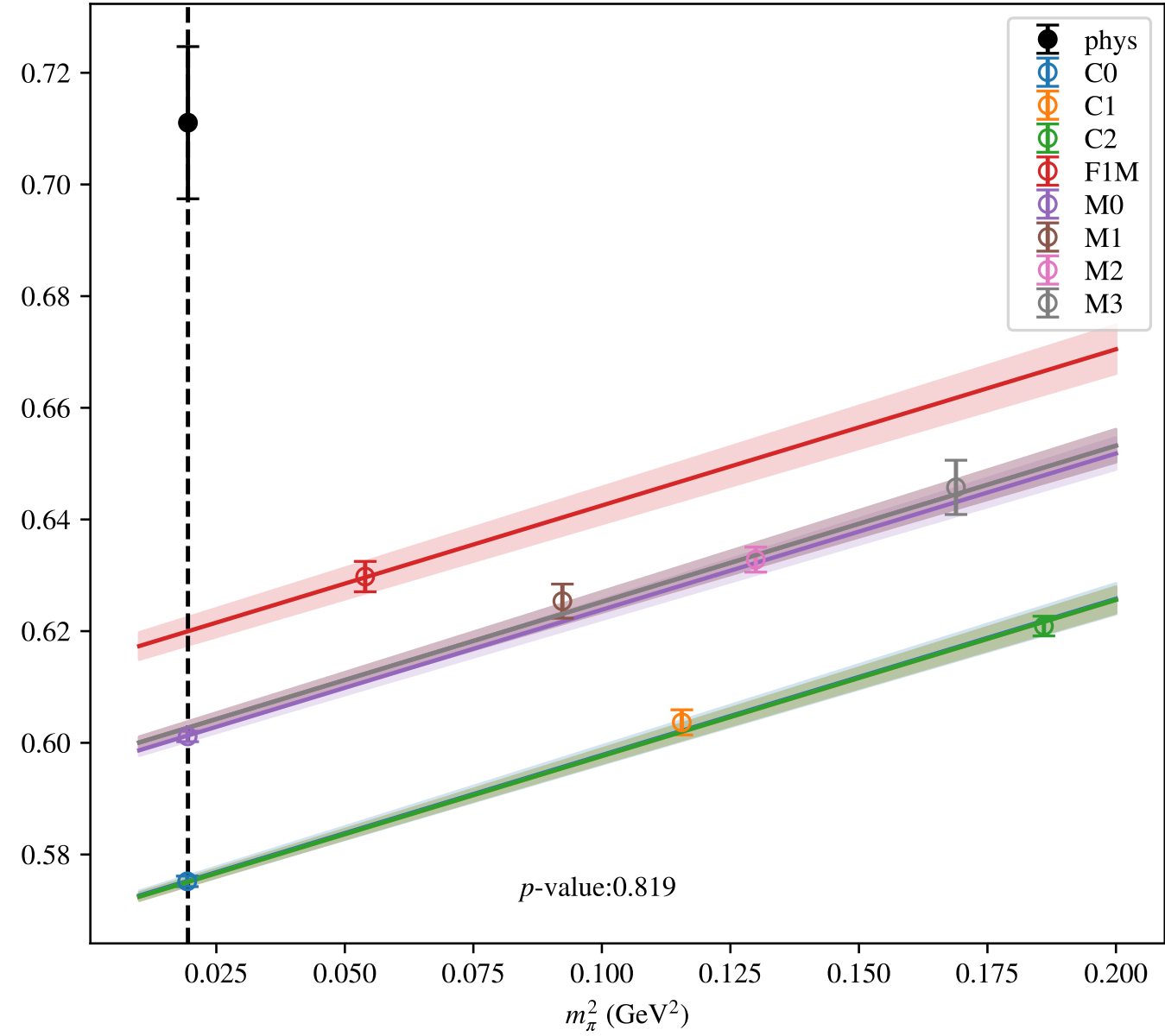


$$a^2, a^4, m_\pi^2, \mu = 1.8 \text{ GeV}$$

TT

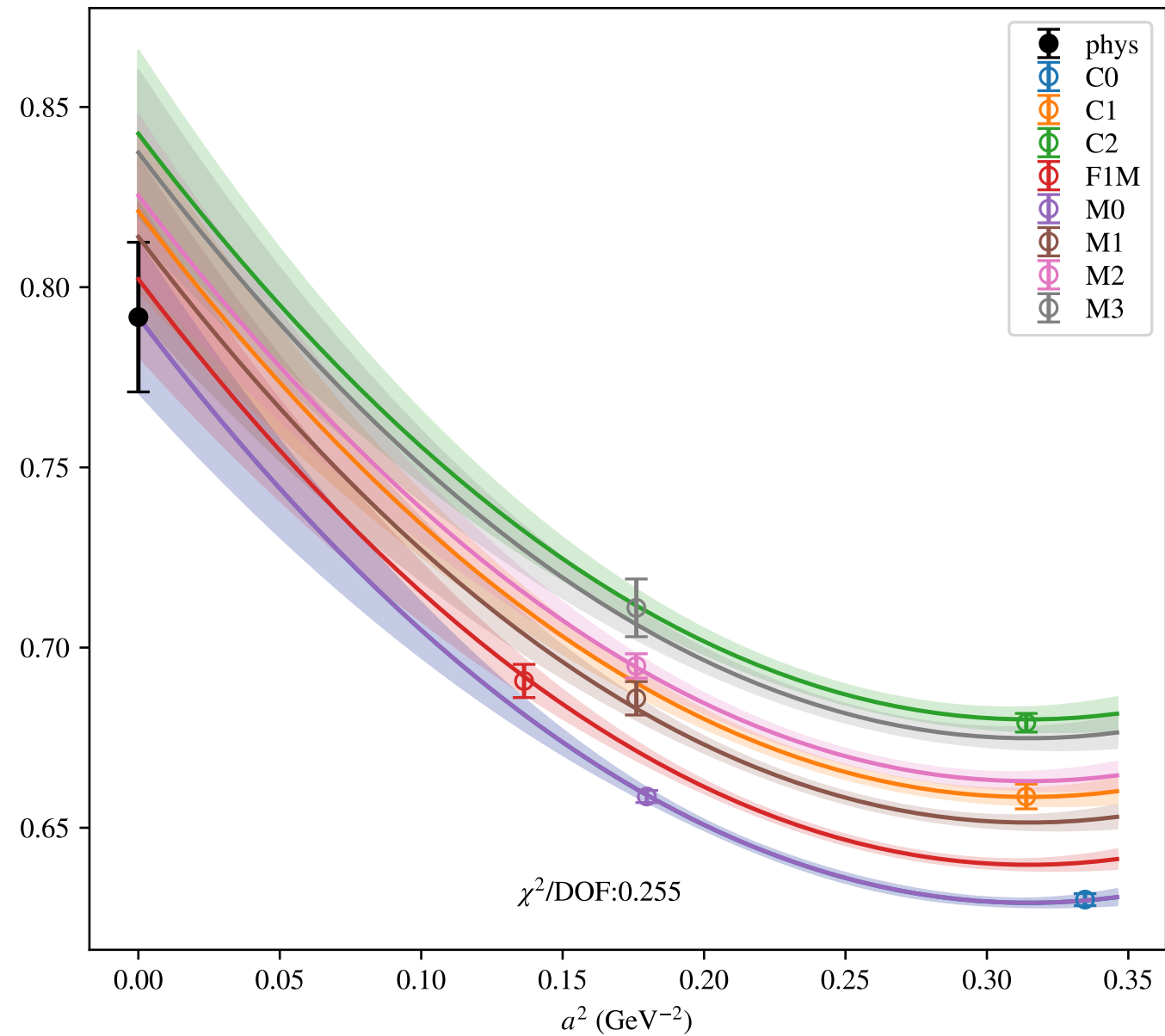


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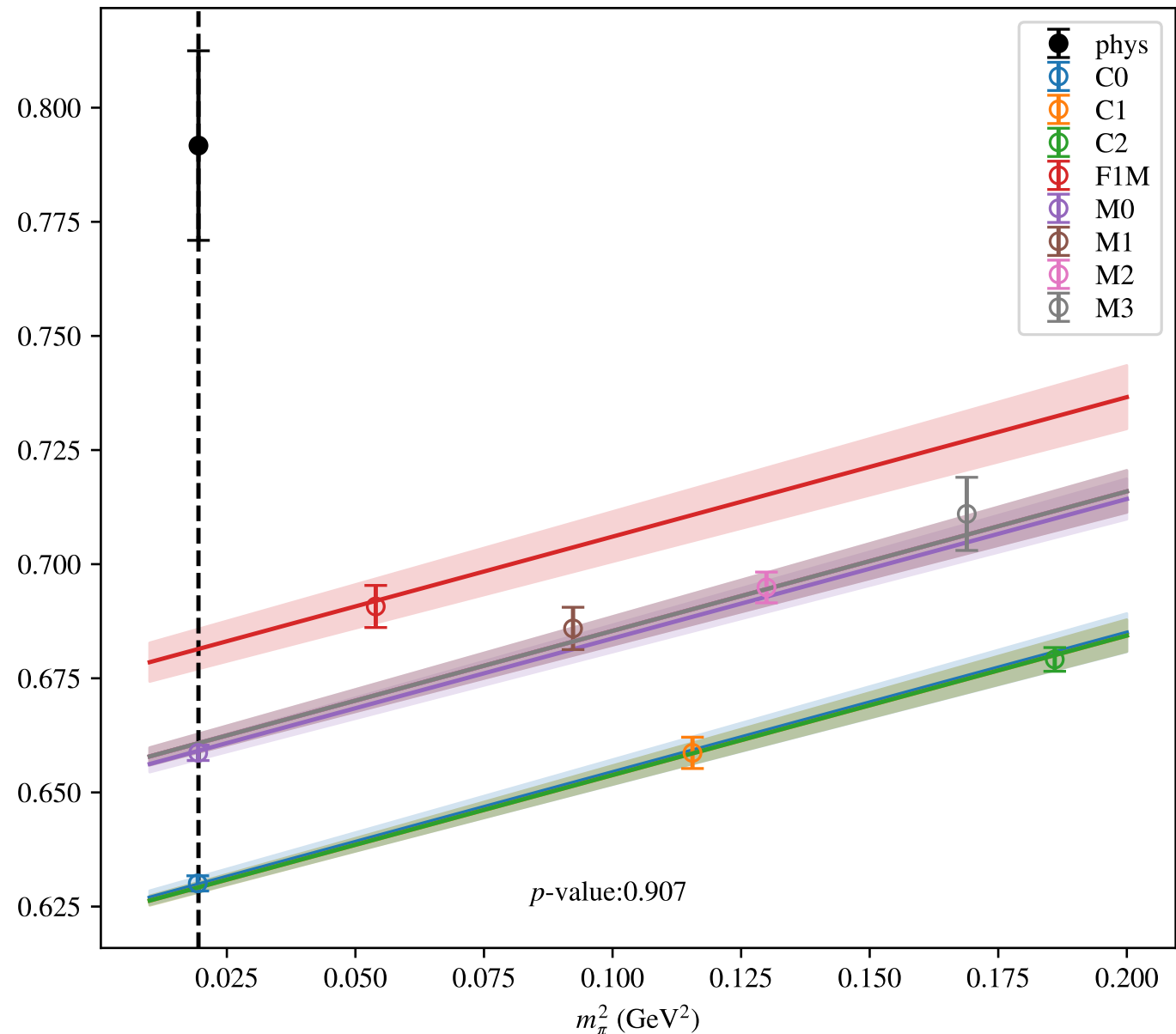


$$a^2, a^4, m_\pi^2, \mu = 1.5 \text{ GeV}$$

TT

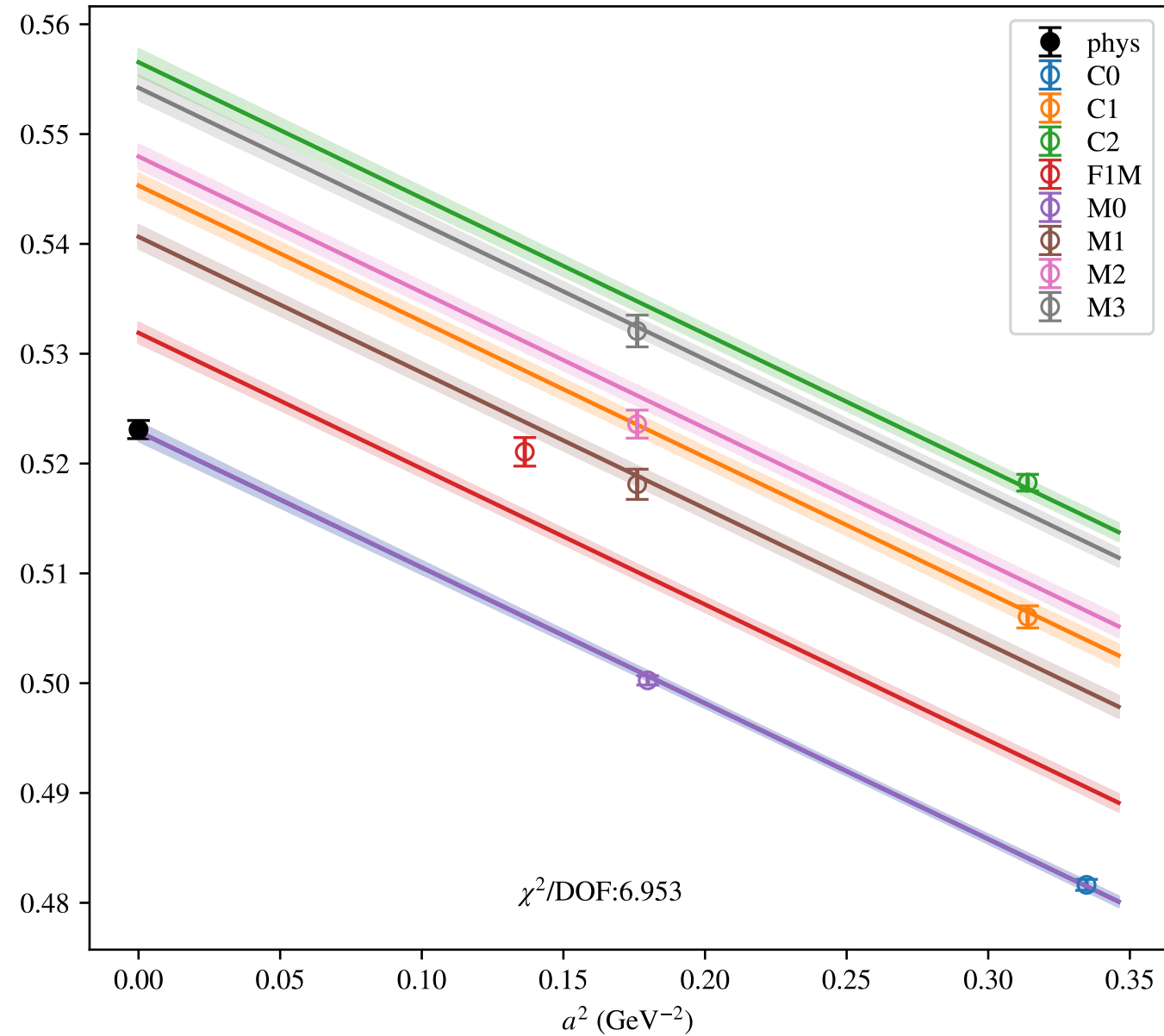


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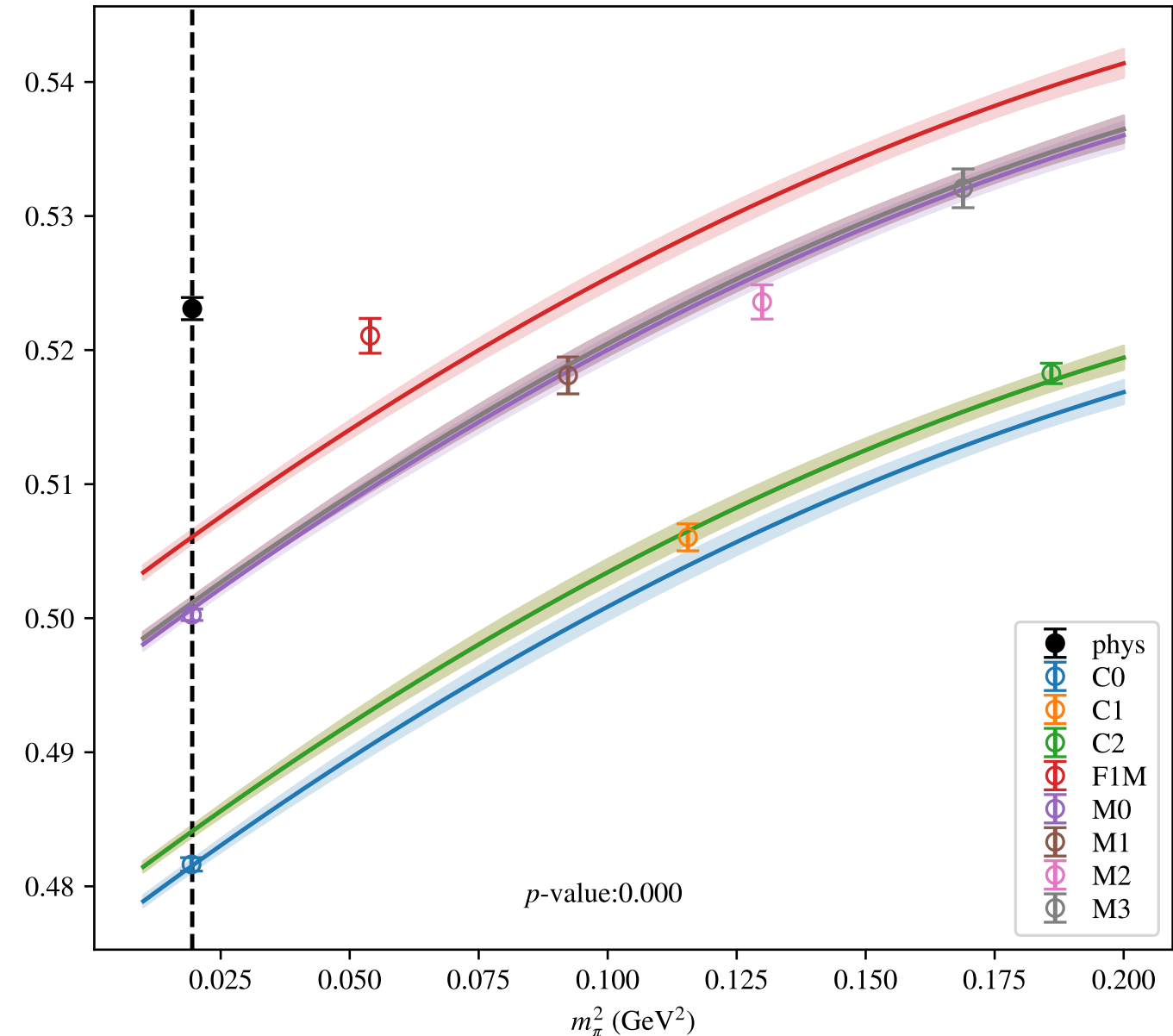


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.4 \text{ GeV}$$

TT

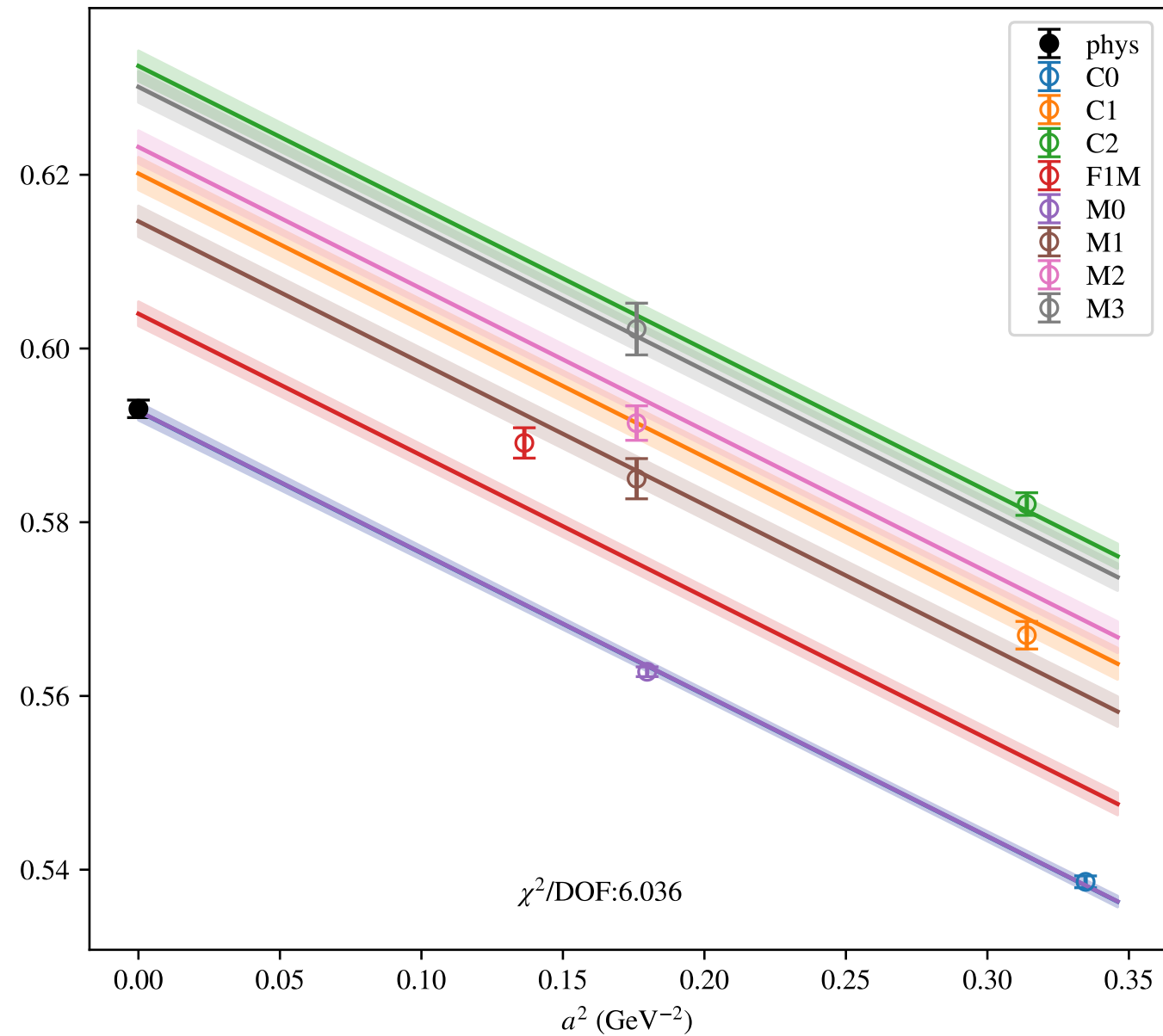


TT

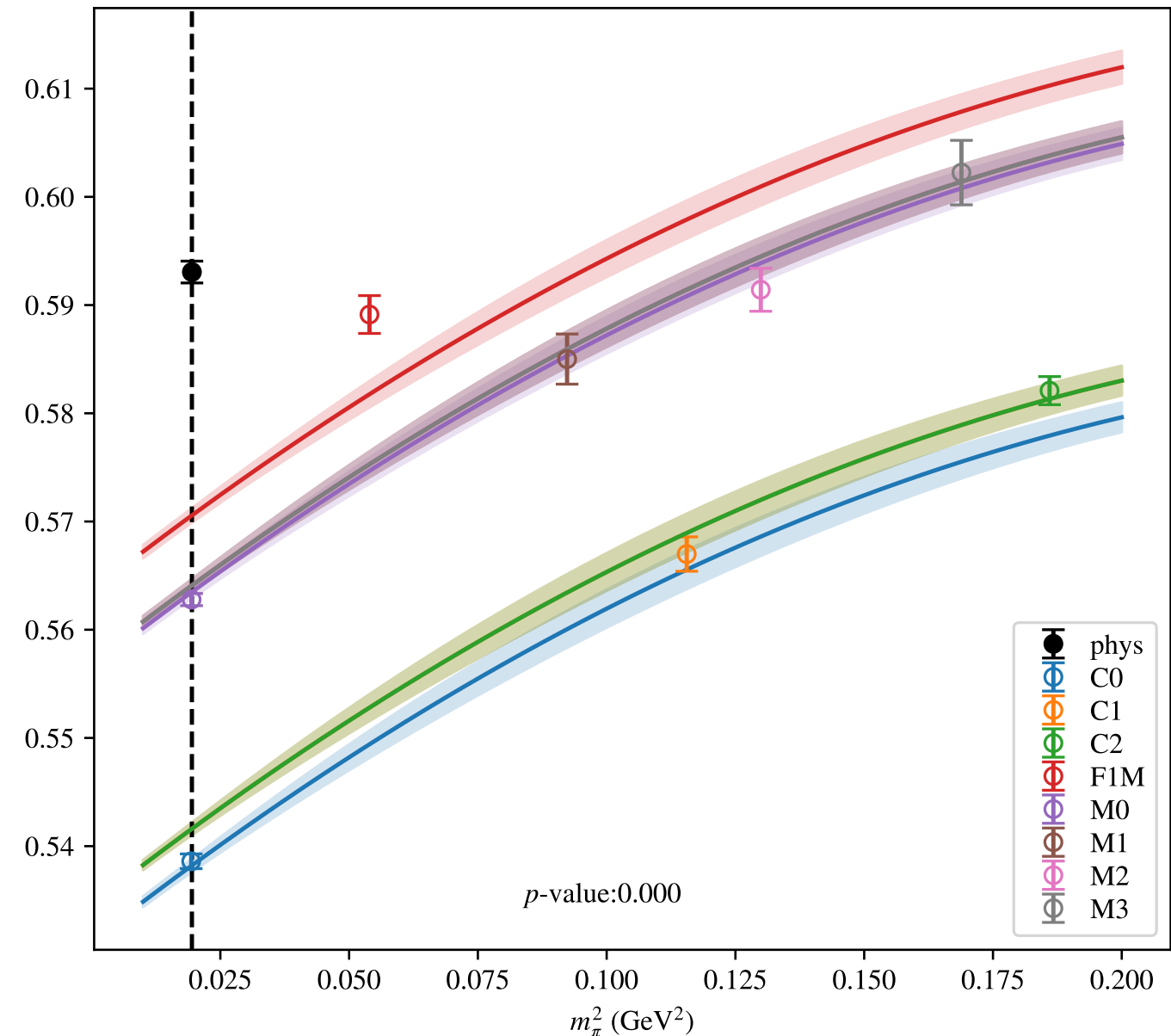


$$a^2, m_\pi^2, m_\pi^4, \mu = 2.0 \text{ GeV}$$

TT

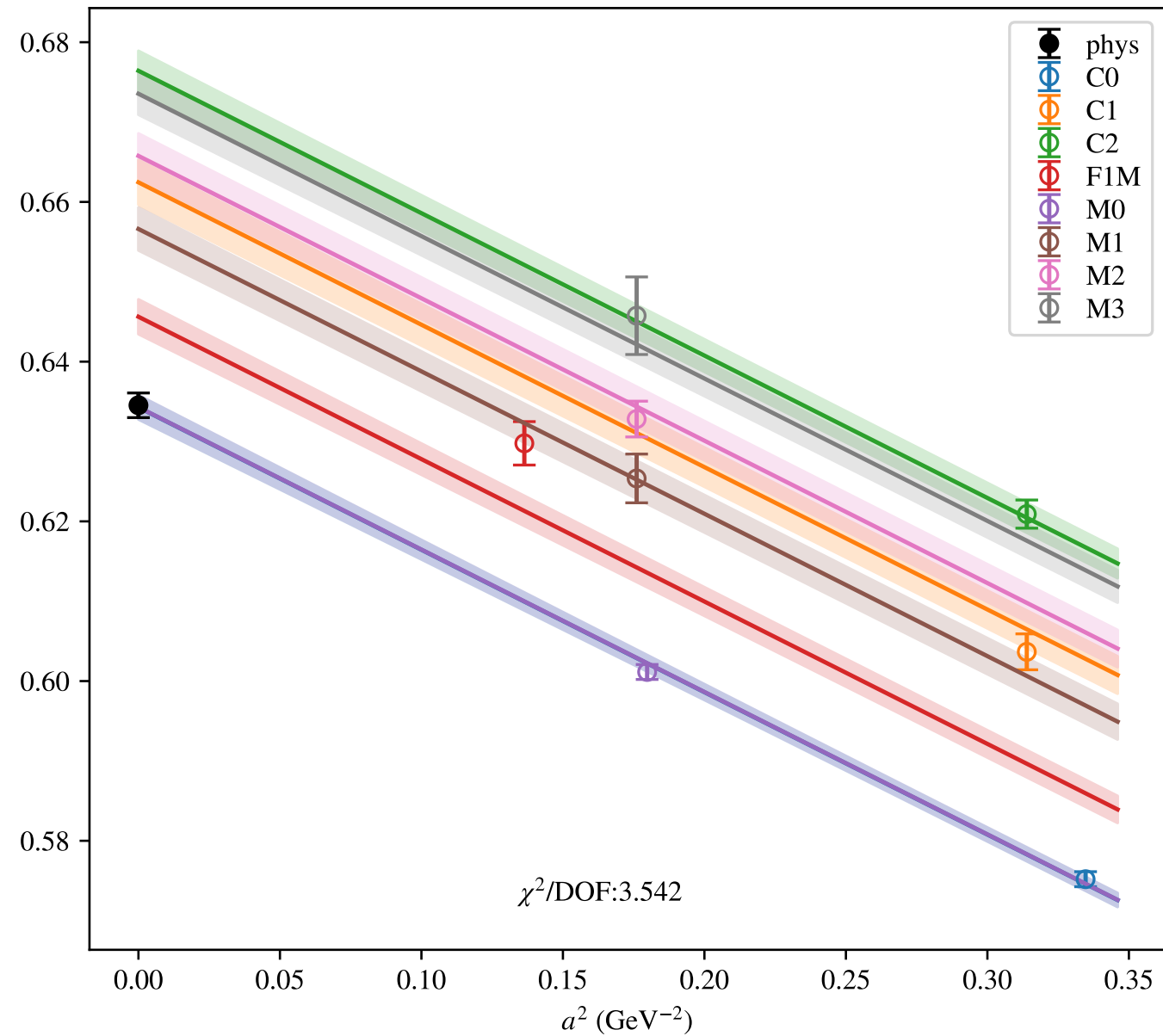


TT

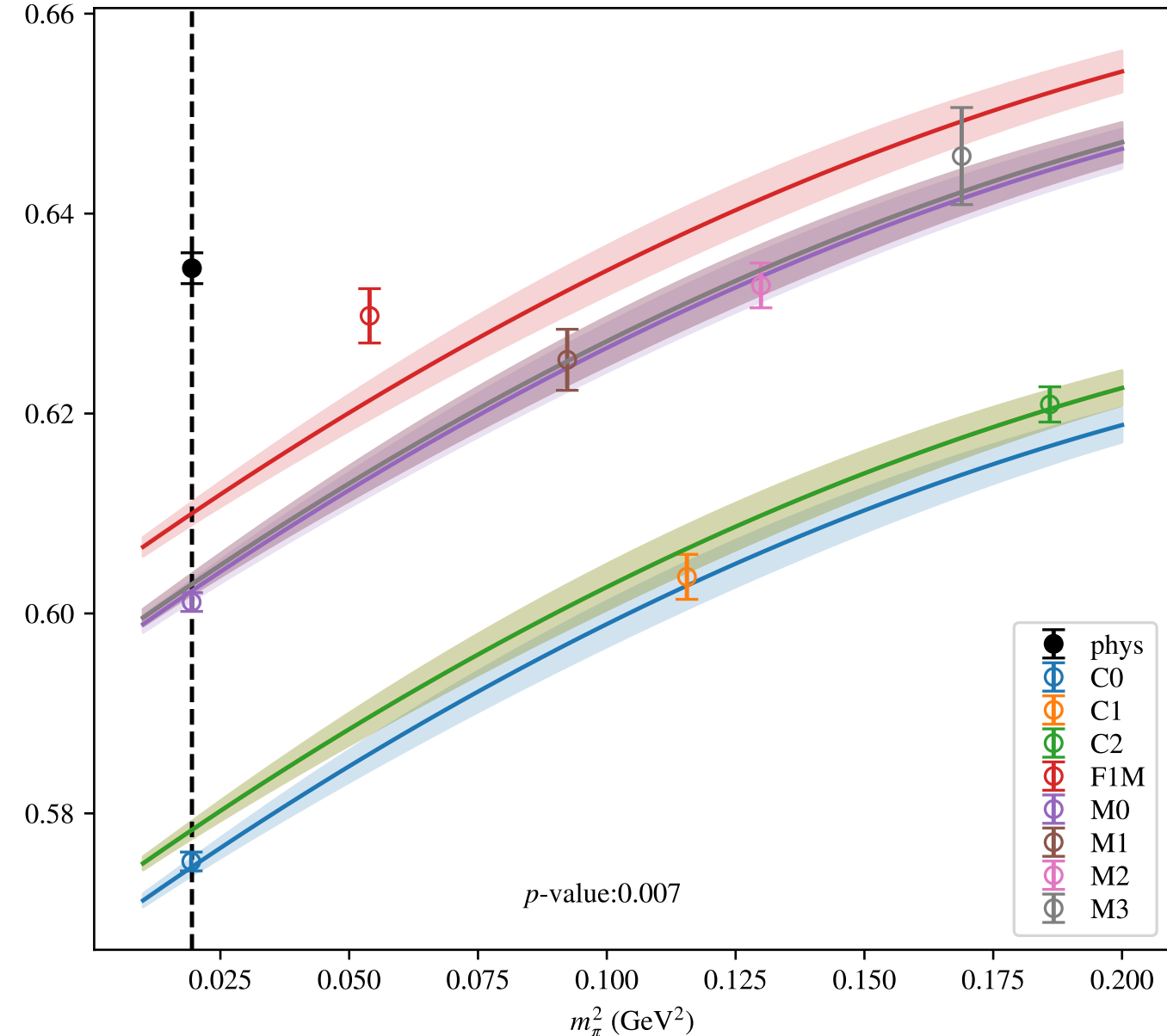


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.8 \text{ GeV}$$

TT

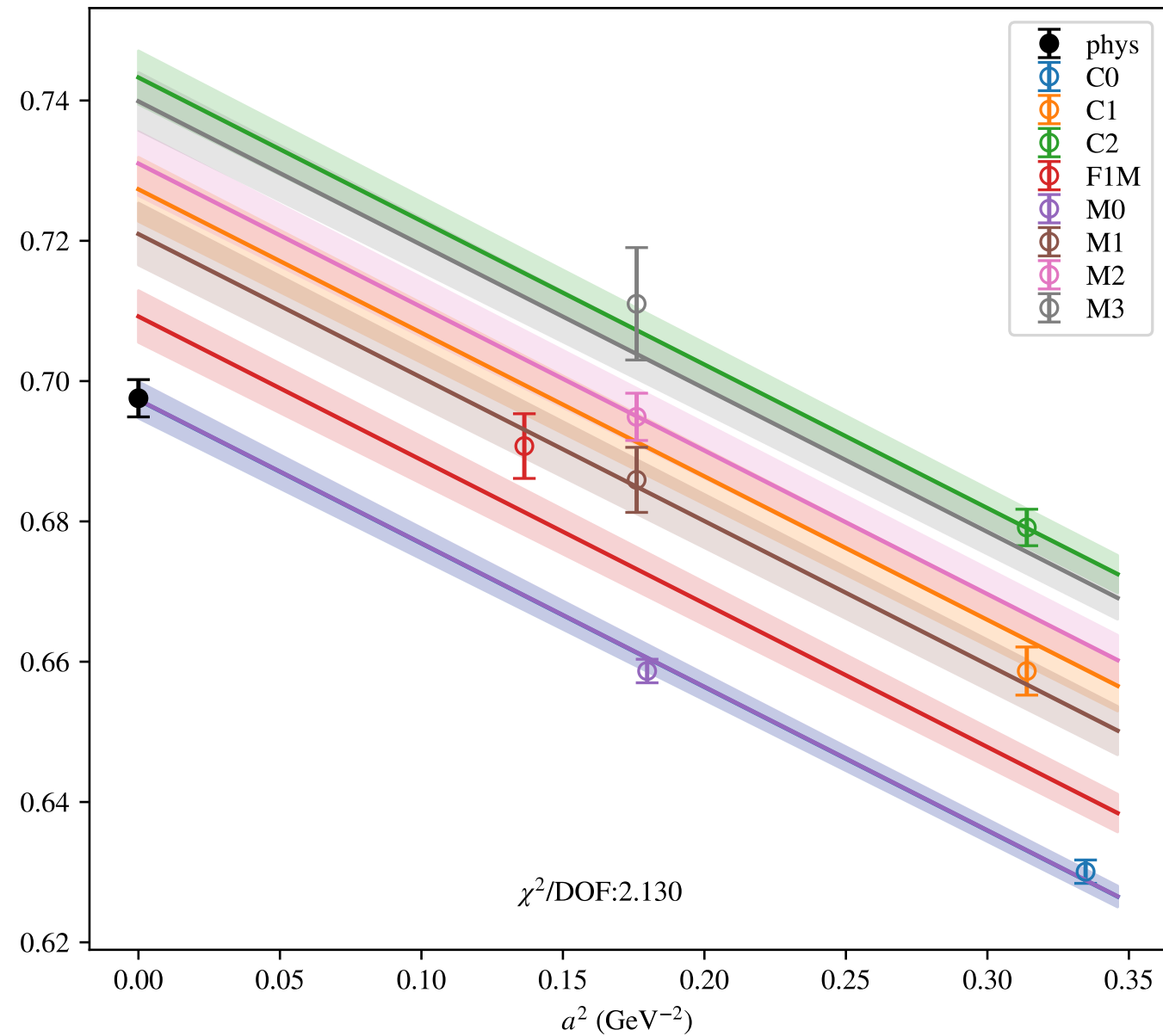


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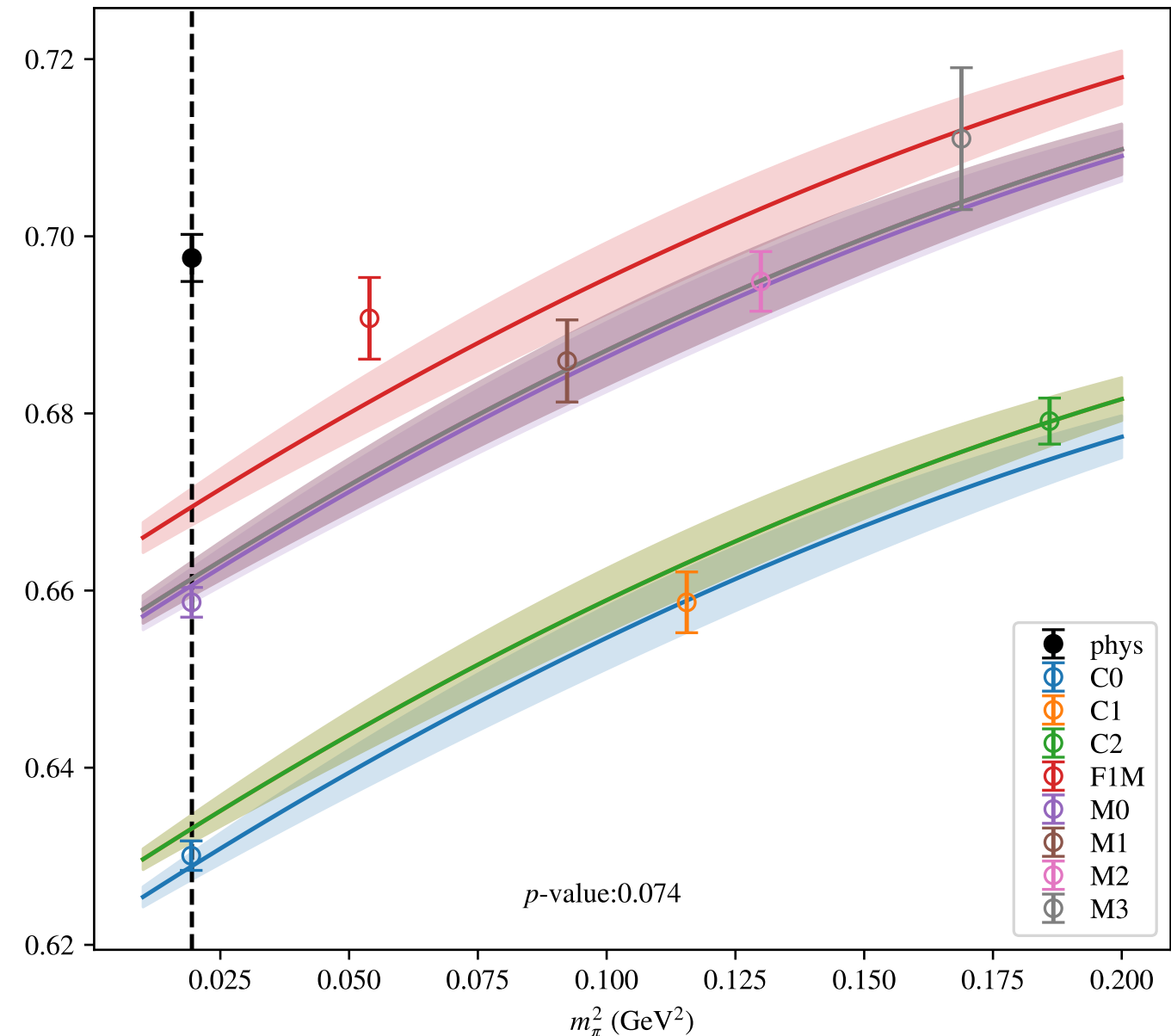


$$a^2, m_\pi^2, m_\pi^4, \mu = 1.5 \text{ GeV}$$

TT

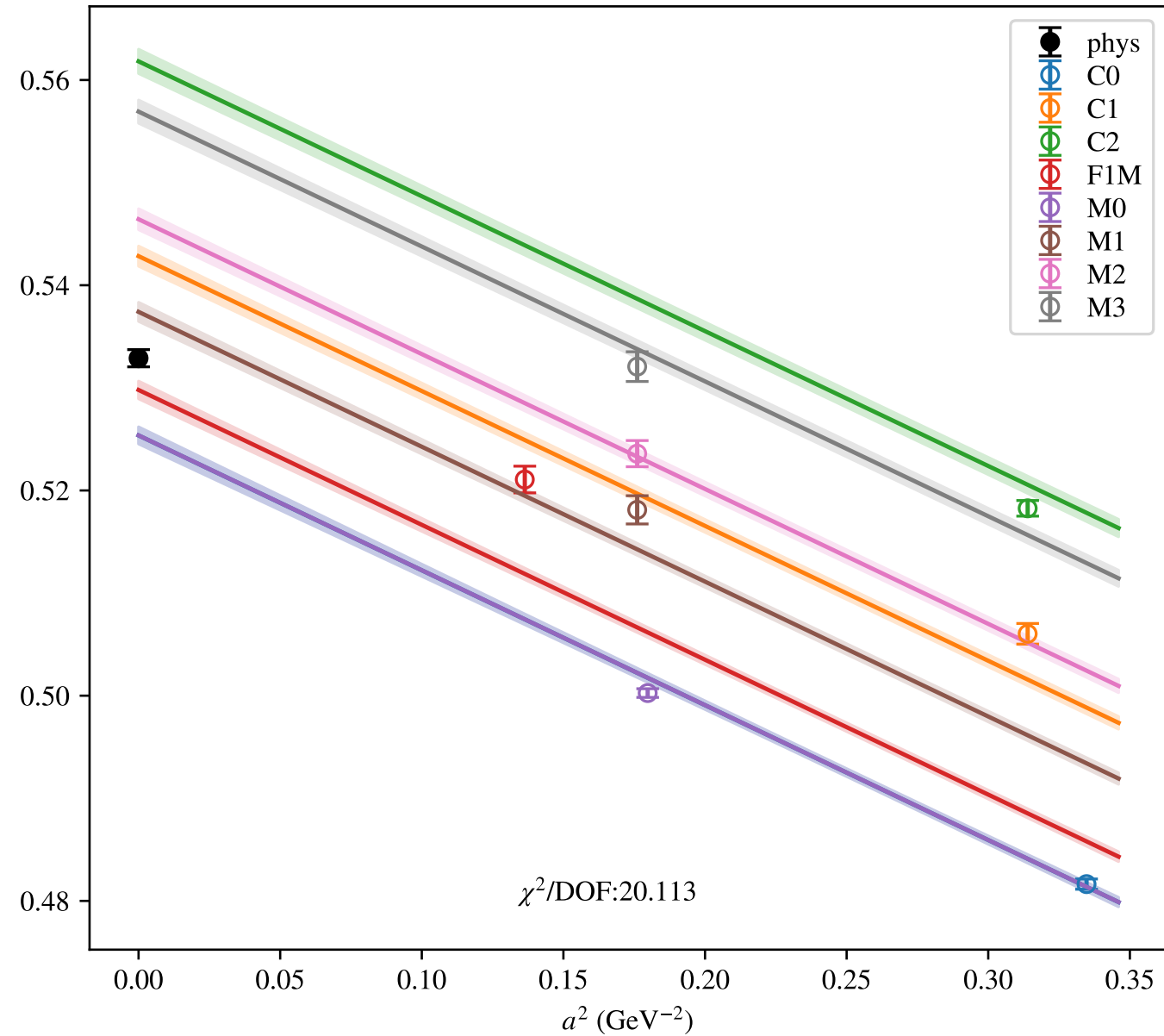


TT

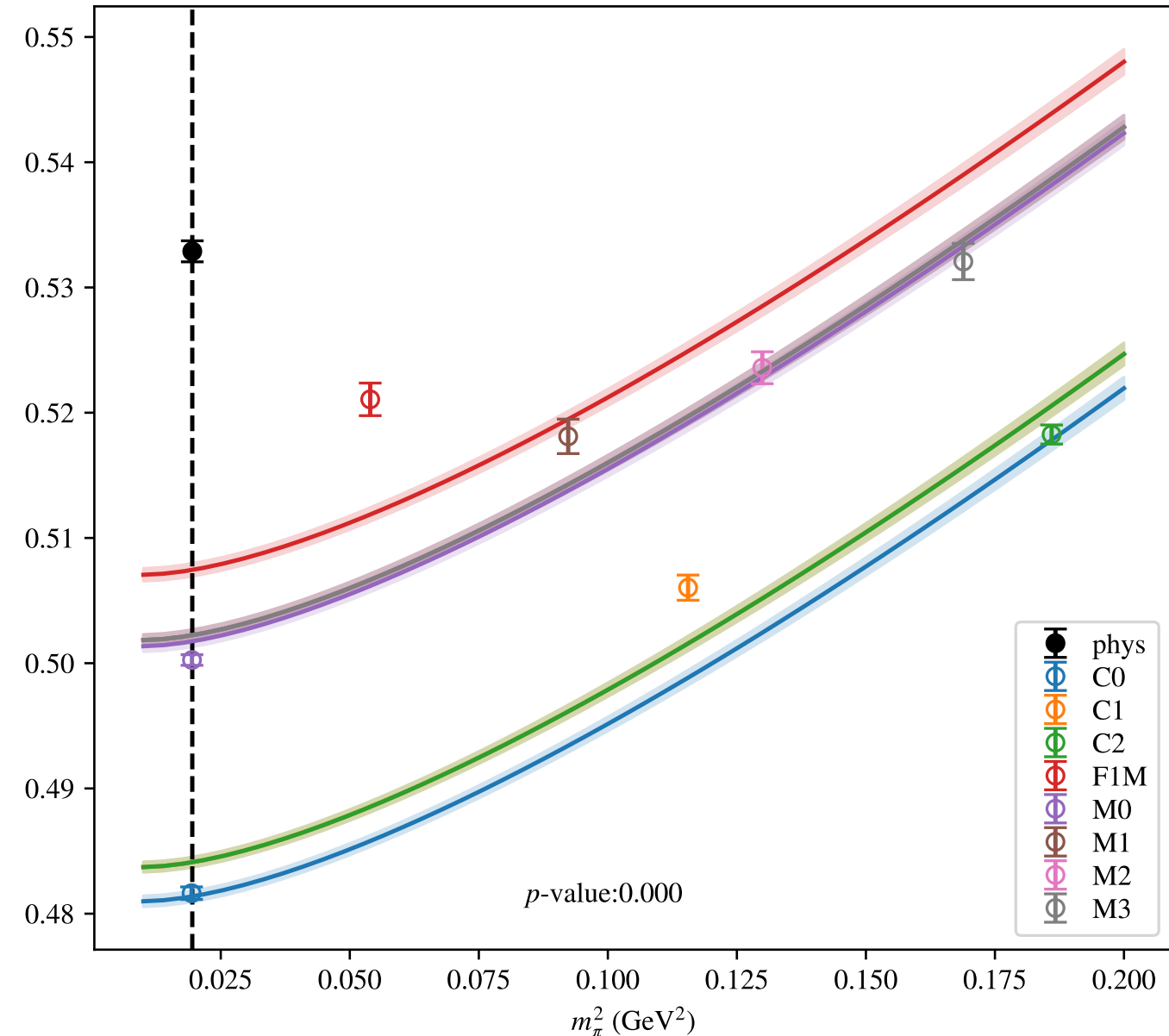


$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 2.4 \text{ GeV}$ 

TT

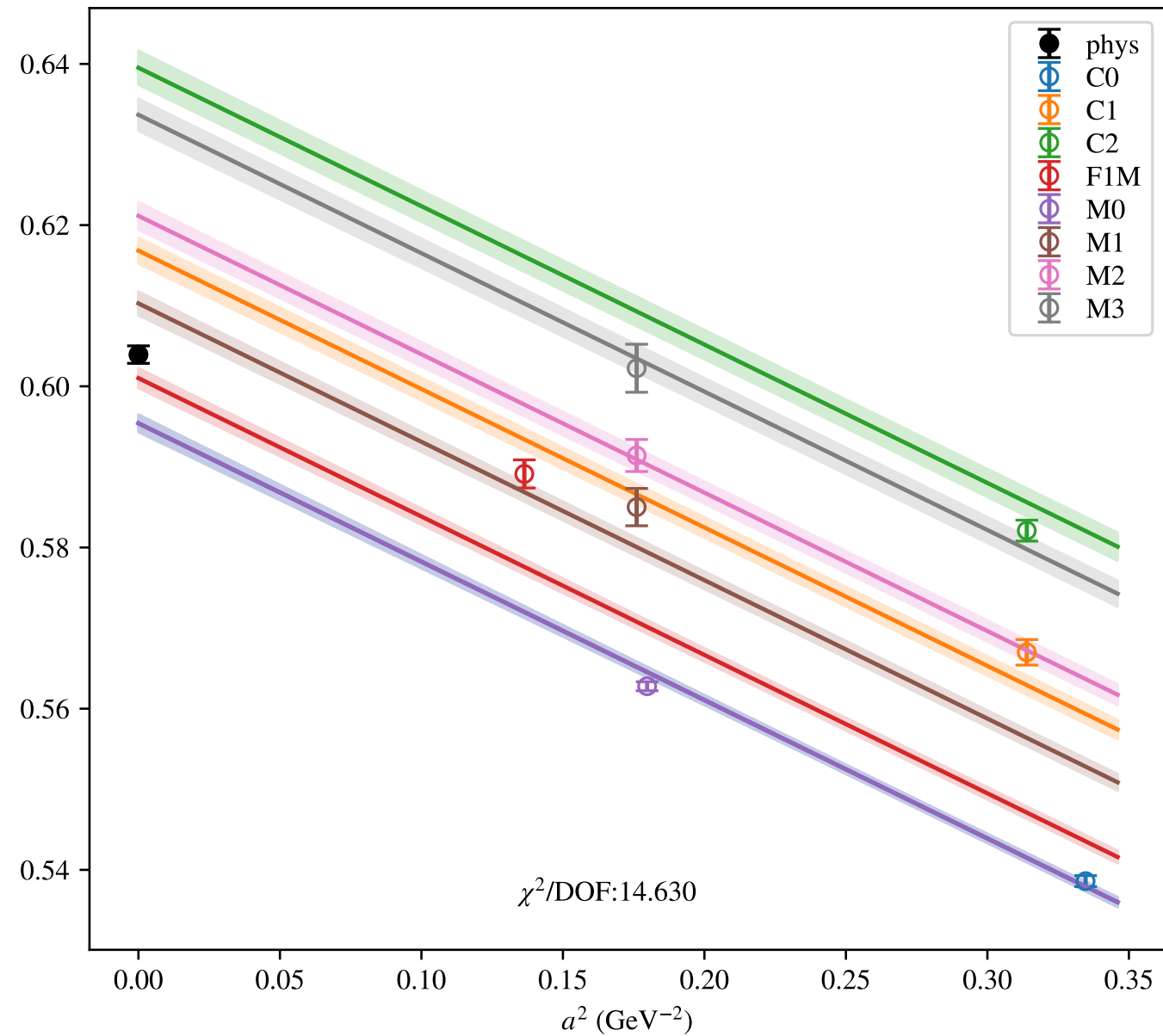


TT

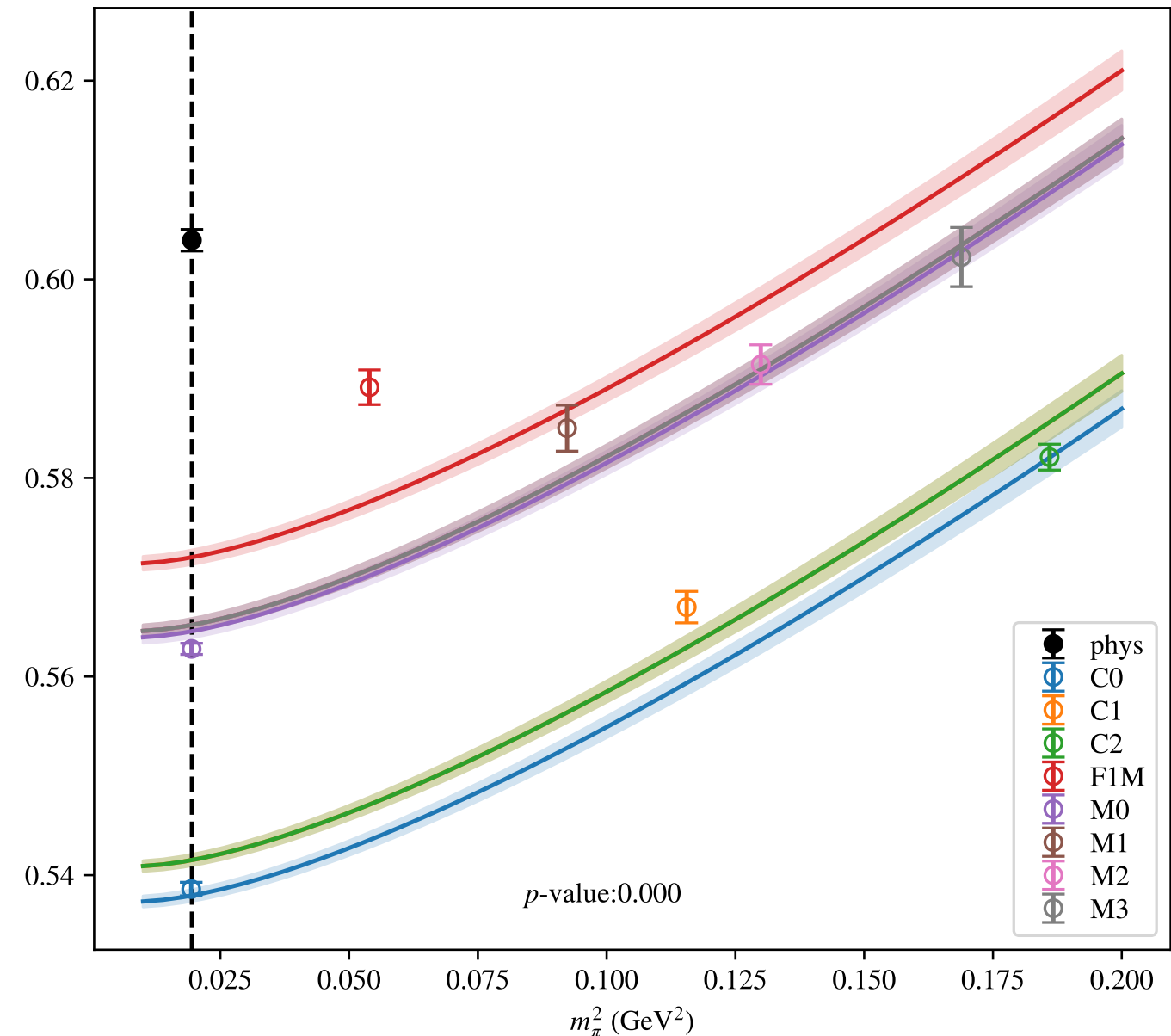


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 2.0 \text{ GeV}$$

TT



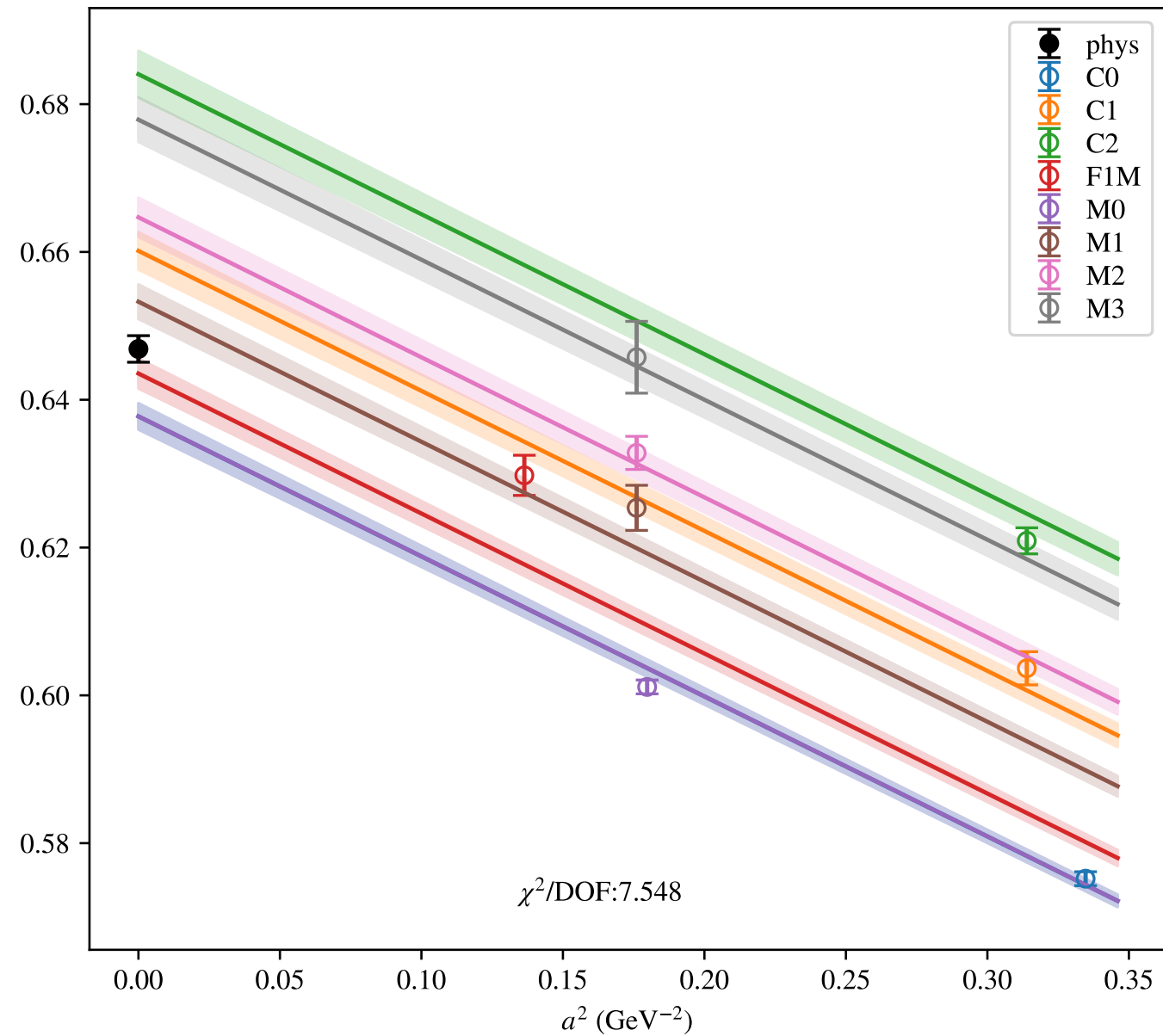
TT



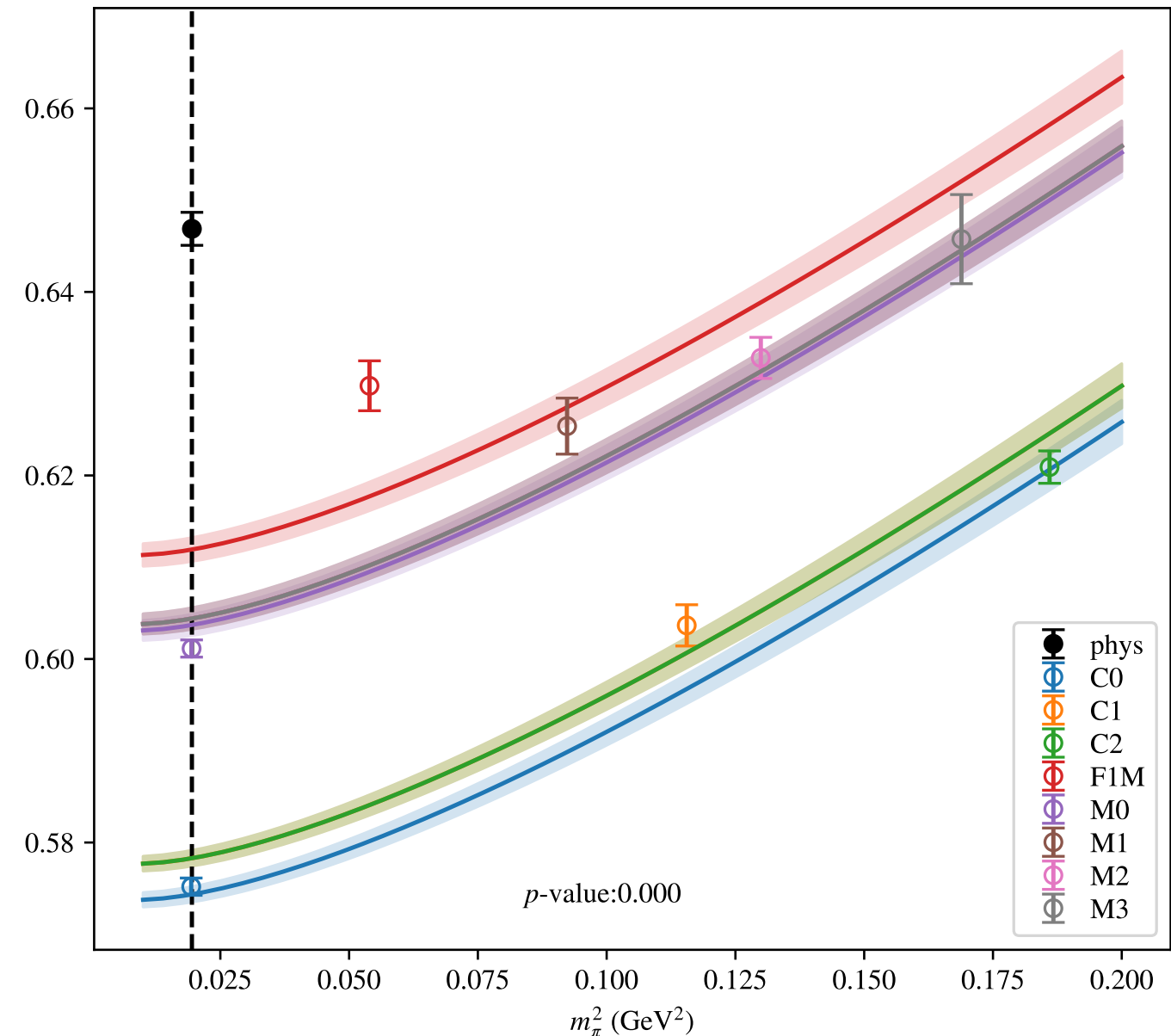


$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 1.8 \text{ GeV}$$

TT

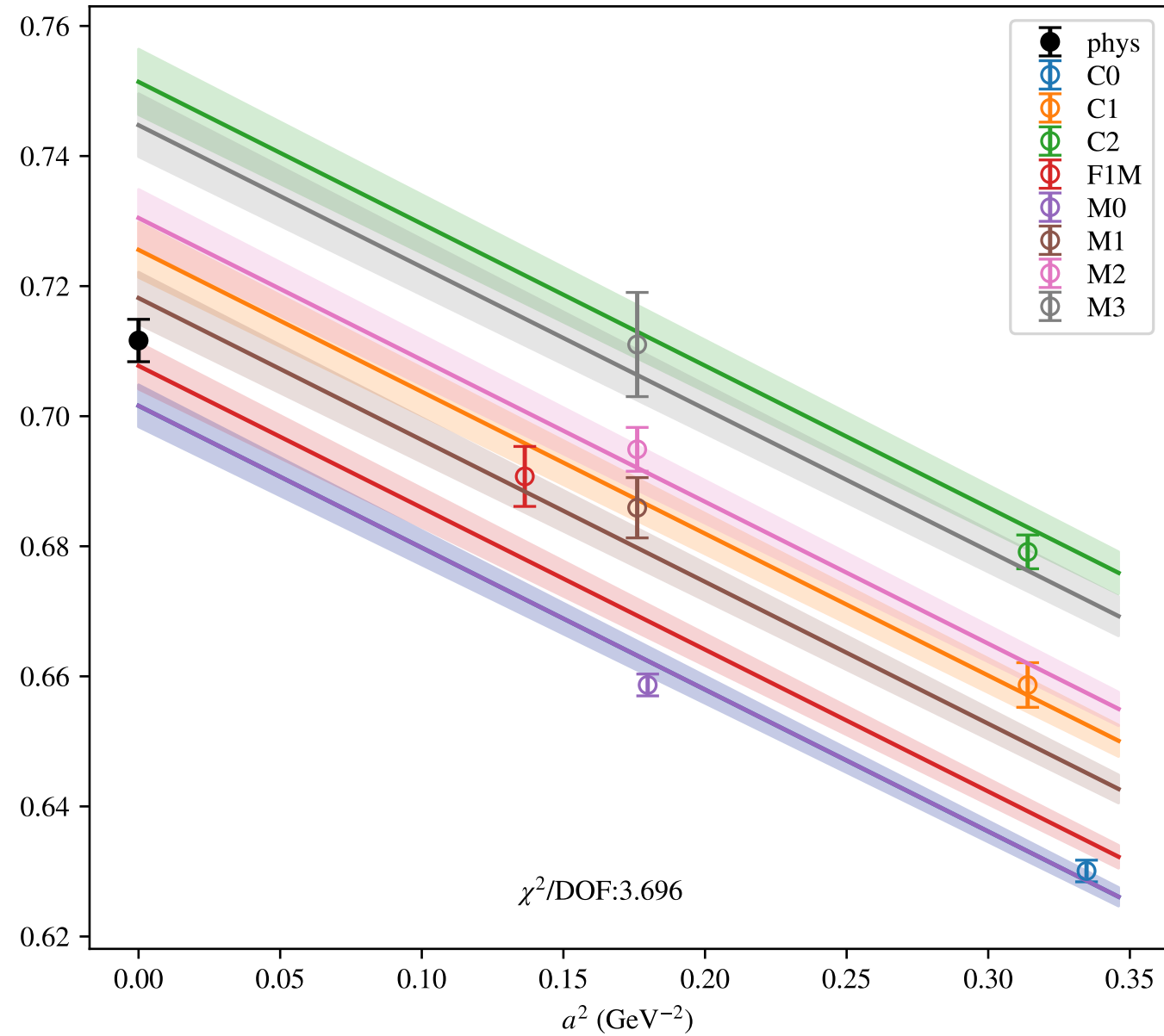


TT



$$a^2, m_\pi^2, \log(m_\pi^2/\Lambda^2), \mu = 1.5 \text{ GeV}$$

TT



TT

