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Parameter estimation

1) Let $(X_1, X_2, \ldots)$ be random Sample of Size n from normal population with parameter mean $= \theta_1$ and variance $= \theta_2$. Find the maximum likelihood estimate of the two population parameter

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Sample Size $= n$

$$L(x_1, x_2, \ldots x_n) = f(x_1) \times f(x_2) \times \ldots f(x_n)$$

$$= \left(\frac{1}{2\pi\sigma^2} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{2\pi\sigma^2} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) - - - -$$

Taking ln on both Sides we get

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

Taking derivative w.r.t μ

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} \left(-\frac{(2(x_i-\mu))}{2\sigma^2}\right) = 0$$

$$\sum_{i=1}^{n} (x_i - \mu) = 0$$

$$n\bar{X} - n\mu = 0$$

$$\bar{X} = \mu$$

Hence $\theta_1 = \bar{X}$

$$\ln(L) = -\frac{n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n}\left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

Taking derivative w.r.t σ^2

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n}\left(-\frac{(x_i-\mu)^2}{2(\sigma^2)^2}\right) = 0$$

$$-n + \sum_{i=1}^{n}\left(-\frac{(x_i-\mu)^2}{\sigma^2}\right) = 0$$

$$n = \sum_{i=1}^{n}\left(\frac{-(x_i-\mu)^2}{\sigma^2}\right)$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$Q_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

2) Let $X_1, X_2, X_3 --- X_n$ be random sample from $B(m, \theta)$ distribution where $\theta \in \Theta = (0,1)$ is unknown & is +ve integer, compute value of θ using MLE

$$\text{Binomial Dist}^n = {}^nC_{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$$

$$L = \prod_{i=1}^{n} {}^nC_{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$$

log on both sides

$$\log(L) = \sum_{i=1}^{n} \left(\log({}^nC_{x_i}) + \log(\theta^{x_i}) + \log(1-\theta)^{n-x_i} \right)$$

$$\log(L) = \sum_{i=1}^{n} \left(\log({}^nC_{x_i}) + \log(\theta \sum_{i=1}^{n} x_i) + \log\left((1-\theta)(\sum(n-x_i)\right)\right)$$

Diffrentiate wrt θ

$$\frac{d \log L}{\partial \theta} = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{1}{1-\theta} \sum (n-x_i) = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum (x_i) = 0$$

$$\frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta}$$

$$\boxed{\theta = \frac{\sum x_i}{n^2}}$$