

$$\underline{\underline{x^0 = 1}}$$

$$\text{pow}(a, 6)$$

$$\begin{aligned} \text{do st (recursive call)} &= \text{pow}(a, 6-1) \\ &= a^{(6-1)} \times a \\ &= \underline{\underline{a^6}} \end{aligned}$$

$T(N)$

```
public static int power_basic(int a, int b) {
    //  $\rightarrow a^1$ 
    int smallAns = power_basic(a, b - 1); //  $\rightarrow T(N-1)$ 
    return smallAns * a; //  $\rightarrow a^1$ 
}
```

$$\begin{aligned} T(N) &= 2 + T(N-1) \\ T(N) &= 2 + 1 + T(N-2) \\ T(N-1) &= 2 + T(N-2) \\ T(N-2) &= 2 + T(N-3) \\ T(N-3) &= 2 + T(N-4) \\ &\vdots \\ T(1) &= 2 + T(0) \end{aligned}$$

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$$T(N) = N \times 2 + T(0)$$

$$\begin{aligned} T(N) &= O(N) \\ \boxed{T(N) \approx O(N)} \end{aligned}$$

$$\begin{aligned} 2^{1000} \\ 2^{10^6} &\approx \underline{\underline{(10^6 \text{ unit second})}} \end{aligned}$$

$p(2,0)$	<del><math>(2,0)</math></del>	$1 \times 2$
$p(2,1)$	<del><math>(2,1)</math></del>	$2 \times 2$
$p(2,2)$	<del><math>(2,2)</math></del>	$4 \times 2$
$p(2,3)$	<del><math>(2,3)</math></del>	$8 \times 2$
$p(2,4)$	<del><math>(2,4)</math></del>	$16 \times 2$
$p(2,5)$	<del><math>(2,5)</math></del>	$32 \times 2$
$p(2,6)$	<del><math>(2,6)</math></del>	$64 \times 2$
$p(2,7)$	<del><math>(2,7)</math></del>	$128 \times 2$
$p(2,8)$	<del><math>(2,8)</math></del>	$256 \times 2$
$p(2,9)$	<del><math>(2,9)</math></del>	$512 \times 2$
$p(2,10)$	<del><math>(2,10)</math></del>	$1024$
main()	<del><math>\text{power}(2,10)</math></del>	$\underline{\underline{1024}}$

$\text{pow}(9, 6)$   
 $\mu e \rightarrow \text{do} \rightarrow \text{Recursion}$   
 $\text{smallAns} = \text{pow}(9, 6/2)$   
 $(\text{smallAns} \times \text{smallAns})$

$3 \xrightarrow{\mu e} \text{do} \rightarrow 3^4$   
 $(3^4)(3^4) = 3^8$

$9/2 = 4$

$3^9 \xrightarrow{\mu e} \text{do} \rightarrow (3^4)(3^4)(3^1)$

$3^9 = 3^{4.5} \cdot 3^{4.5}$   
 $= 3^4 \cdot 3^1 \cdot 3^{4.5} \cdot 3^{4.5}$   
 $= (3^4)(3^4) \cdot 3^1$

```

T(N)
public static int power_opti(int a, int b) {
    if (b == 0) {
        return 1;
    }

    int smallAns = power_opti(a, b/2);
    smallAns *= smallAns;
    return b % 2 == 0 ? smallAns : smallAns * a;
}

```

$P(2,0)$	$P(2,1)$	$1 \times 1 \times 2$
$P(2,0)$	$P(2,1)$	$2 \times 2$
$P(2,1)$	$P(2,2)$	$4 \times 4 \times 2$
$P(2,0)$	$P(2,1)$	$2 \times 2$
$P(2,0)$	$P(2,1)$	$1 \times 1$

$N, N/2, N/4, N/8, \dots, 1$

$a_0 = N$   
 $a_k = 1$   
 $r = (1/2)$

$a_k = a_0 \cdot r^{k-1}$   
 $1 = N \left(\frac{1}{2}\right)^{k-1}$   
 $2^{k-1} = N$   
 $\log_2 2^{k-1} = \log_2 N$

$(k-1) \log_2 2 = \log_2 N$   
 $k-1 = \log_2 N$   
 $k = \log_2(N+1)$

$T(N) = 2 + T(N/2)$   
 $T(N) = \lambda + T(N/2)$   
 $T(N/2) = \lambda + T(N/4)$   
 $T(N/4) = \lambda + T(N/8)$   
 $T(N/8) = \lambda + T(N/16)$   
 $T(N/16) = \lambda + T(N/32)$   
 $T(N) = K \cdot \lambda + T(0)$   
 $T(N) = \lambda (\log_2(N+1)) + T(0)$   
 $T(N) = O(\log_2(N))$   
 $T(N) = O(\log_2(N))$