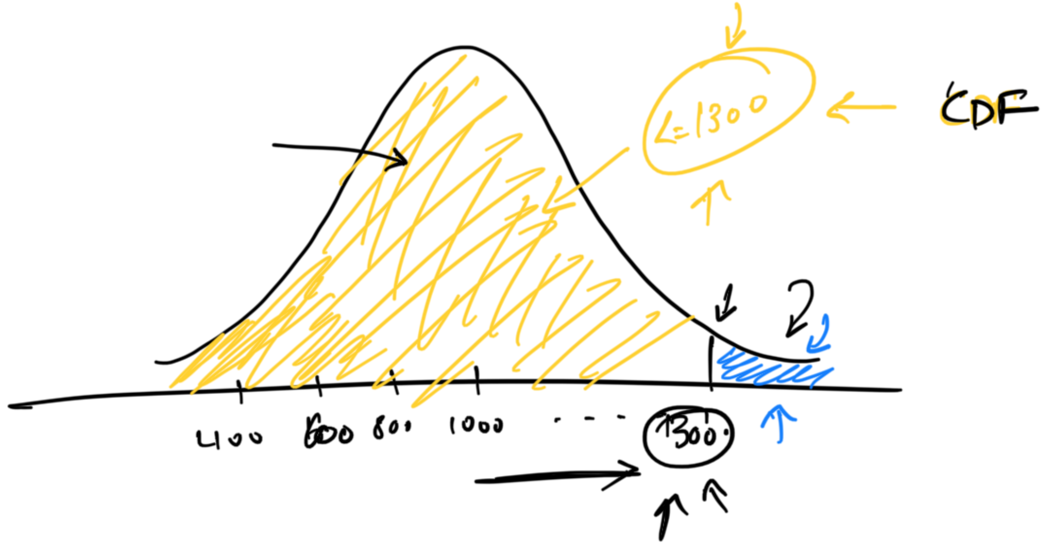
Central Limit Theorem

Q: 1000 toothpastes $\rightarrow$ $\mu = 1000$

$\sigma = 200$

$\rightarrow$ stock $= 1300 \leftarrow x$

what fraction (prob.) of weeks we will go out of stock?



$\mu = 1000$

$\sigma = 200$

$x = 1300$

$$z = \frac{x - \mu}{\sigma} = \frac{1300 - 1000}{200} = \frac{300}{200} = 1.5$$

1 − norm.cdf(1.5) ←

#

10kg

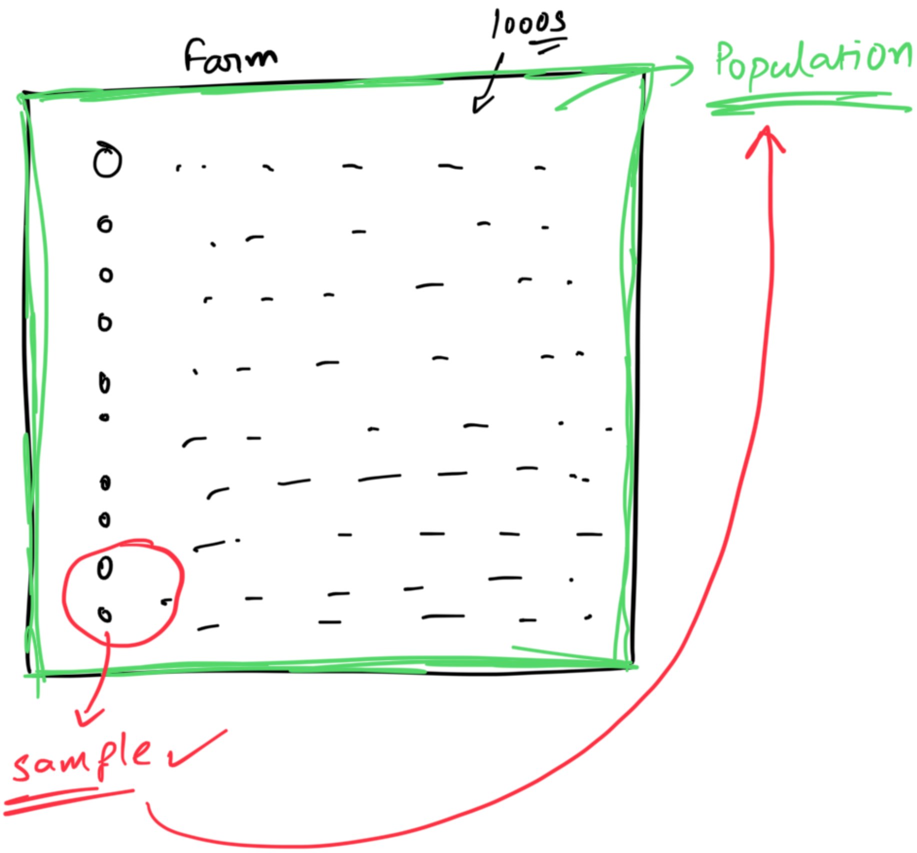
Logical Approach

↓

Random sampling

↓

Avg. of samples as estimates.



Catch

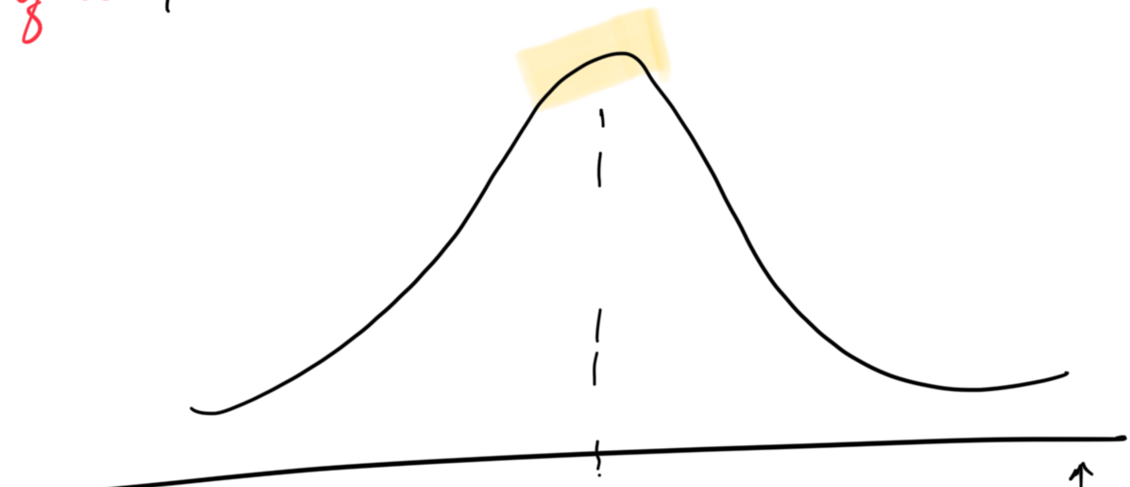
→ How do you ensure that the samples that you've picked represent the entire field.

→ What if you just pick the 30 heaviest or 30 lightest?

→ CLT → Central Limit Theorem.

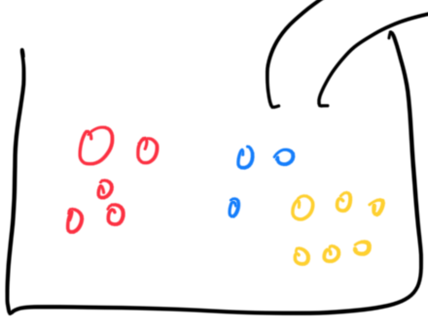
→ If you take a bunch of samples (like groups

→ of [illegible] (… of 30 or 40 pumpkins) and calculate the average weight for each sample, then the distribution of these sample averages will be $\boxed{\text{normal}}$ even if the original weights of all pumpkins are not distributed normally



Sample average wt.

Sampling → Bootstrapping



→ 1 ball → $\frac{3}{14}$ ✓

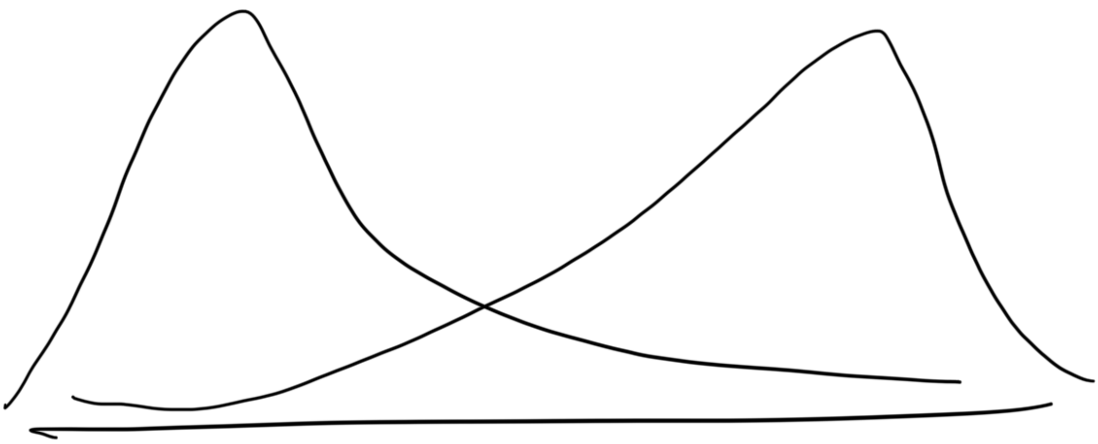
→ 2nd ball ⇒ $\frac{3}{14}$ or $\frac{2}{13}$

→ 14 balls

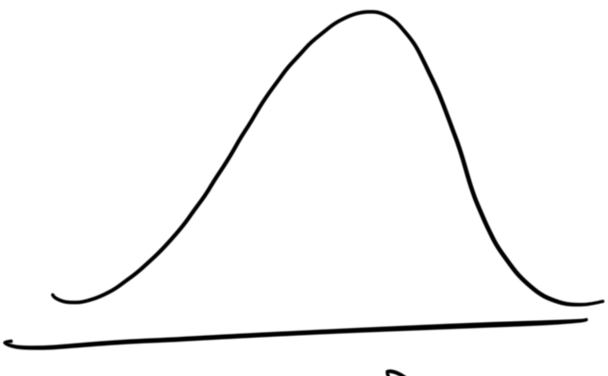
$$\frac{{}^{3}C_{2}}{{}^{14}C_{2}}$$

Sample size → 5

# of samples → 1000



⇓



Population → μ → pop. avg.

σ → pop. std.

CLT

↪ $\bar{x} = \dfrac{x_1 + x_2 + x_3 + \dots + x_n}{n}$ ←

x_i is a random

Sample mean

sample from same population.

$\bar{x}$ will follow gaussian distribution.

with $\boxed{E[\bar{x}] = \mu}$ pop. mean.

std. dev of $\bar{x}$ = $\frac{\sigma}{\sqrt{n}}$

$\Rightarrow \bar{x} \sim N\left(\mu, \frac{\sigma}{\sqrt{N}}\right)$ { if n is large ($n > 30$)

$\rightarrow$ σ is finite.

Original distribution need not be gaussian. }

$\boxed{n = 5}$ sample size $\boxed{n = 20}$

standard deviation of sample mean

pop. std. = σ

$\Rightarrow$ $\boxed{\boxed{\text{sample std}} = \frac{\sigma}{\sqrt{n}}}$ Imp.

$\longrightarrow \frac{\sigma}{\sqrt{5}}$, $\frac{\sigma}{\sqrt{20}}$

$\sqrt{5} \quad <$

Q:

$\bar{x} = 122$

$\sigma = 10$

16 people

n

$$std(\bar{x}) = \frac{10}{\sqrt{16}} = \frac{10}{4} = 2.5$$

$$z\text{-score} = \frac{125 - 122}{2.5} = 1.2$$

$$1 - \text{norm.cdf}(1.2) = 0.115$$