

Binomial & Geometric Distribution

To day \rightarrow You have an interview

- \rightarrow 1. Give 10 interviews - pick the best offer.
- \rightarrow 2. Keep giving interviews till you succeed.

Binomial

Geometric



100th

$n=100$

$(1-p)^{99} p$

Strategy 1:

- \rightarrow Fixed no. of interviews (n) ✓
- \rightarrow Success prob. is constant (p) ✓
- \rightarrow Each interview is independent of another.

$\rightarrow X$: number of successes

Q:

1 interview (1 trial)

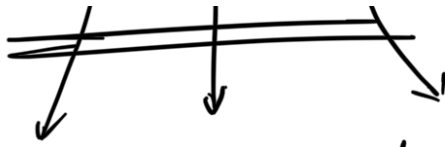
$X \leftarrow$ no. of successes $\leftarrow \{0, 1\}$

$X \rightarrow 0$ ✓

$X \rightarrow 1$

\Rightarrow 2 interviews (2 trials)

$X \in \{0, 1, 2\}$



Sample space $\Rightarrow \{ \underline{ff}, \boxed{fs}, \underline{sf}, ss \}$

$p = 0.1, n = 2$

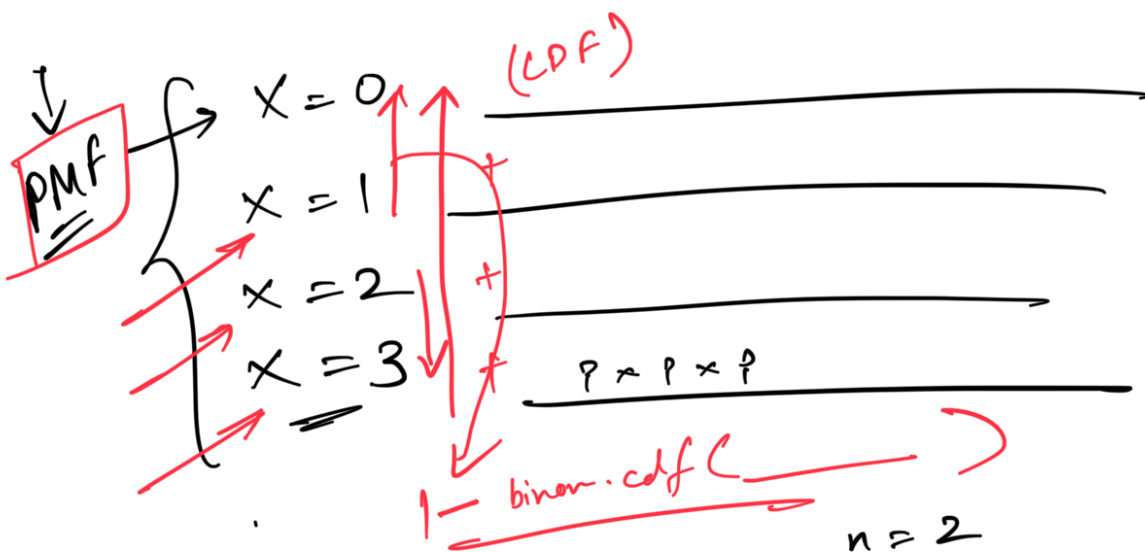
$$\rightarrow P(X=0) = \underline{(1-p)} * \underline{(1-p)}$$

$$\underline{P(X=1)} = \underline{(1-p) \times p} + \underline{p \times (1-p)}$$

$$P(X=2) = p \times p$$

\Rightarrow 3 interviews

ss $\Rightarrow \{ sss, ffs, fsf, sff, ssf, fss, sfs, fff \}$



$n = \underline{3}$

$X = 4$

$X = 3$

1. . . . 1.

$$X = (n+1) \text{ vals}$$

$$X_{\text{vals}} = \text{np.arange}(0, n+1)$$

n ✓

p ✓

N trials

Sample Space size $\rightarrow 2^n$

Out of 2^n outcomes, how many map to \textcircled{k}

$n=3 \rightarrow \{ \text{sss}, \text{ssf}, \text{sfs}, \text{fss}, \text{stt}, \text{fst}, \text{fts}, \text{ftt} \}$
↑
successes.

$n=3, k=2 \rightarrow \{ \text{ssf}, \text{sfs}, \text{fss} \}$

$$\textcircled{n} \quad \textcircled{k} \quad \rightarrow \quad \{ {}^n C_k \}$$

These many outcomes will map to k successes out of 2^n outcomes for n trials

$n=5$
 $x = \{0, 1, 2, 3, 4, 5\}$ $(k) \rightarrow \text{success}$
 $(n-k) \rightarrow \text{failures}$

$$\Rightarrow \boxed{P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}}$$

\uparrow \uparrow

Binomial Formula

$$n = 2$$

$$k = 0$$

$$\begin{aligned}
 P(X = 0) &= {}^2C_0 p^0 (1-p)^{2-0} \\
 &= 1 \times (1-p)^2
 \end{aligned}$$

$$n = \checkmark$$

$$k = \checkmark$$

$$n \Rightarrow (5) \leftarrow n$$

$$k \quad (3) \quad k$$

$$\rightarrow 2$$

$$\underline{\underline{n-k}}$$

PMF \rightarrow —

Manufacturing \rightarrow

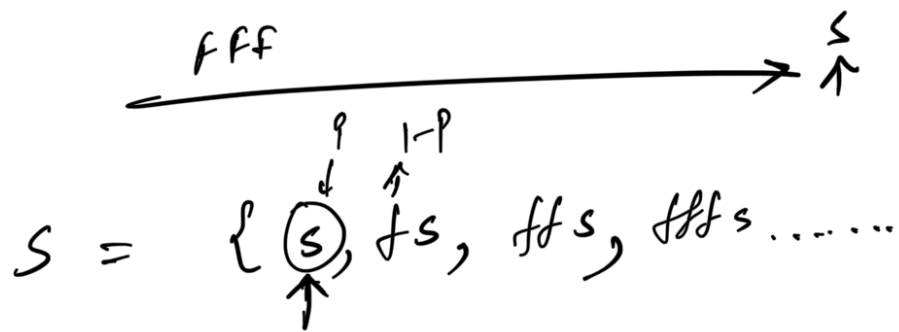
Bottle

at most
 1 defective bottle
 in 50 bottles
 sample

$$K = 3$$



Geometric : What is the prob. that you'd succeed in your 25th interview?



Here, $X = \underline{\text{no of interviews}}$
 $P = \underline{\text{prob. of success.}}$

$$P(X=1) = p \checkmark$$

$$P(X=2) = (1-p) \times p$$

$$P(X=3) = (1-p) \times (1-p) \times p$$

$$\rightarrow P(X=4) = (1-p)^3 p \checkmark$$

$$P(X=5) = (1-p)^4 \times p$$

$$\rightarrow P(X=K) = (1-p)^{K-1} \times p$$

Bottles \rightarrow 1 by 1

Find the probability that the first defective bottle shows up on the 25th bottle

[Signature]

Messi's penalty success rate: 80%

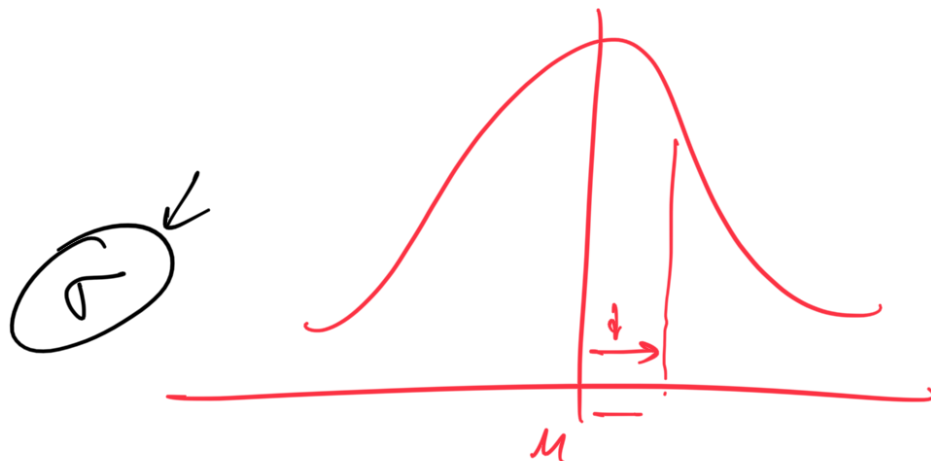
He kicks 10 times. What is the prob. of having 7 or less successes?

$$\left. \begin{array}{l} p = \underline{\underline{0.8}} \\ n = 10 \end{array} \right\} X = \{ \underline{\underline{0, 1, 2, 3, \dots}} \}$$

$$P(X \leq 7) = P(X=0) + P(X=1) + P(X=2) + \dots + P(X=7)$$
 CDF

$$= {}^{10}C_0 (0.8)^0 (0.2)^{10} +$$

=



Variance (σ^2) = $\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$

Scores = $\frac{[85, 89, 76, 92, 90]}{5}$

$\bar{x} = \frac{85 + \dots + 90}{5} = 87$

$\frac{(85 - 87)^2}{5-1}$
 \vdots
 $\frac{(76 - 87)^2}{5-1}$

Std.