DYNAMICS MODEL OF THE SPACE SYSTEM

Representation of a position vector in spherical coordinates as

$$\boldsymbol{r} = (r, \theta, \phi)$$

or

$$\boldsymbol{r} = r\hat{\boldsymbol{r}} \tag{1}$$

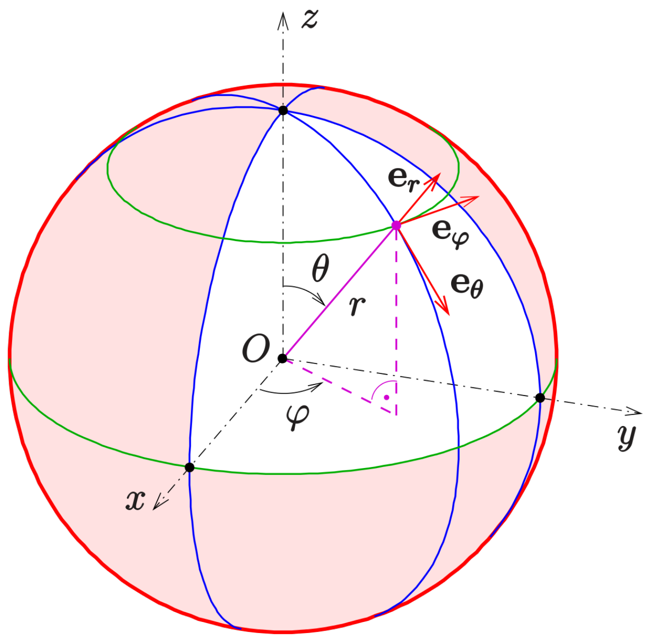

Figure 1. Spherical Coordinate

Velocity vector is given as:

$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta p\hat{h}i \tag{2}$$

and acceleration is given as:

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\theta} + (r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)\hat{\phi} \tag{3}$$

In case of a satellite orbiting earth

$$F = ma = \begin{bmatrix} U_r \\ U_\theta \\ U_\phi \end{bmatrix} + \mu/r^2\hat{r}\ , \tag{4}$$

Comparing with equation 3 we get,

$$\ddot{r} = (r\dot{\theta}^2 + r\dot{\phi}^2\sin^2\theta - \mu/r^2 + U_r)/m \tag{5}$$

$$\ddot{\theta} = (-2 * \dot{r}\dot{\theta}/r + \dot{\phi}^2\sin\theta\cos\theta + U_\theta/r)/m \tag{6}$$

$$\ddot{\phi} = (-2\dot{r}\dot{\phi}/r - 2\dot{\theta}\dot{\phi}\cos\theta/\sin\theta)/m \tag{7}$$

Euler Equations

For an space system performing a rotation motion in body frame of reference is governed by the following equations:

$$\dot{\boldsymbol{\omega}} = J^{-1}(-[\boldsymbol{\omega}^{\times}]J\boldsymbol{\omega} + \boldsymbol{U}\ , \tag{8}$$

Where $J\epsilon\boldsymbol{R}^3$ is the moment of Inertia matrix of the space system, $\boldsymbol{\omega}\epsilon\boldsymbol{R}^3$ is the angular velocity of the space system, $\boldsymbol{U}\epsilon\boldsymbol{R}^2$ is the control torque vector, $\boldsymbol{u_d}\epsilon\boldsymbol{R}^3$ is the external disturbance torque vector,

$$\boldsymbol{U} = \begin{bmatrix} u_1 \\ u_2 \\ u_3, \end{bmatrix}$$

,

$$\boldsymbol{u_d} = \begin{bmatrix} u_{d1} \\ u_{d2} \\ u_{d3} \end{bmatrix}$$

and $\boldsymbol{\omega}^{\times}$ is defined as

$$[\boldsymbol{\omega}^{\times}] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}\ , \tag{9}$$

Kinematic model

Kinematic equation of a spacecraft is a set of first order equation which determines the variation of spacecraft attitude parameters over time. In this paper we used quaternions to describe the motion of the attitude. The kinematic equation to describe the attitude of the space system in terms of quaternion attitude parameters can be defined as:

$$\dot{\boldsymbol{q}} = \frac{1}{2}E(\boldsymbol{q})\dot{\boldsymbol{\omega}}\ , \boldsymbol{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}\ , \tag{10}$$

Where $E(\boldsymbol{q})$ is the conversion matrix from angular velocity $\boldsymbol{\omega}$ to quatenion kinematics $\dot{\boldsymbol{q}}$.

$$E(\mathbf{q}) = \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix}\ , \tag{11}$$

Relative motion in Space mechanics

The satellite about which all other satellite motions are referenced is called the chief satellite. The remaining satellites, referred to as the deputy satellites, are to fly in formation with the chief.

The inertial chief position is expressed through the vector $\boldsymbol{r_c(t)}$, while the deputy satellite position is given by $\boldsymbol{r_d(t)}$. The relative orbit position vector $\boldsymbol{\rho}$ is expressed in $\boldsymbol{O}$ frame components as

$$\rho = (x, y, z)^T$$

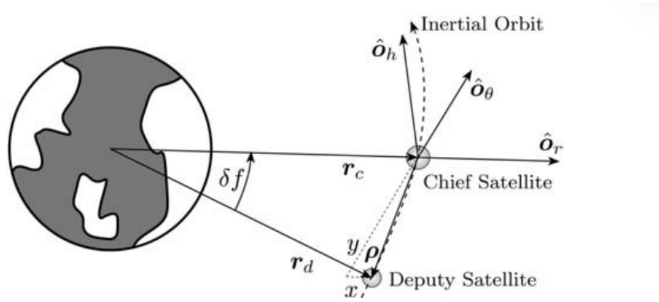

Illustration of a leader follower type of a two-spacecraft formation.

Figure 2.

Its origin is at the chief satellite position and its orientation is given by the vector triad $\hat{\boldsymbol{O}}_{\boldsymbol{r}}; \hat{\boldsymbol{O}}_{\boldsymbol{\theta}}, \hat{\boldsymbol{O}}_{\boldsymbol{h}}$ shown in Fig. The vector $\hat{\boldsymbol{O}}_{\boldsymbol{r}}$ is in the orbit, radius direction, while $\hat{\boldsymbol{O}}_{\boldsymbol{h}}$ is parallel to the orbit momentum vector in the orbit normal direction. The vector $\hat{\boldsymbol{O}}_{\boldsymbol{\theta}}$ then completes the right-hand coordinates system. The deputy satellite position vector is written as,

$$\boldsymbol{r_d} = \boldsymbol{r_c} + \boldsymbol{\rho} = (r_c + x)\hat{\boldsymbol{O}}_{\boldsymbol{r}} + y\hat{\boldsymbol{O}}_{\boldsymbol{\theta}} + z\hat{\boldsymbol{O}}_{\boldsymbol{h}} \tag{12}$$

Considering the motion as Keplerian and assuming that there is no disturbance acting, the relative equation of motion between the chief and the deputy satellite is written as,

$$\ddot{x} - x(\dot{\theta}^2 + 2\mu/r_c^3) - y\ddot{\theta} - 2\dot{y}\dot{\theta} = U_x \tag{13}$$

$$\ddot{y} - x\ddot{\theta} + 2\dot{x}\dot{\theta} - y(\dot{\theta}^2 - \mu/r_c^3) = U_y \tag{14}$$

$$\ddot{z} + z\mu/r_c^3 = U_z \tag{15}$$

$$\boldsymbol{\omega_{O/N}} = \dot{f}\hat{\boldsymbol{O}}_{\boldsymbol{h}};, \theta = \omega + f \tag{16}$$

$$\ddot{f} = -2\frac{\dot{r_c}}{r_c}\dot{f} \tag{17}$$

Relative motion Kinematics

Let r_c and r_d represent the state variables of the chief satellite and the deputy satellite respectively in inertial frame as shown in figure 2. Where,

$$\boldsymbol{r_c} = [r_c, \theta_c, \phi_c]^T$$

, and

$$\boldsymbol{r_d} = [r_d, \theta_d, \phi_d]^T$$

then,

$$X_c = [T]\boldsymbol{r_c}$$

$$X_d = [T]\boldsymbol{r_d}$$

Where, $[T]$ is the transformation from Spherical coordinate to Cartesian coordinate frame.

$$\boldsymbol{X_d} = \boldsymbol{X_c} + [ON]^T\boldsymbol{\rho}$$

$$\boldsymbol{\rho} = [ON](\boldsymbol{X_d} - \boldsymbol{X_d})$$

ρ is the relative position of deputy satellite in chief satellite frame(hill's frame) and $[ON]$ is the direction cosine matrix that relates inertial frame component to hill frame component

$$\dot{\boldsymbol{\rho}} = [ON](\dot{\boldsymbol{X}}_{\boldsymbol{d}} - \dot{\boldsymbol{X}}_{\boldsymbol{c}}) - [\omega^X][ON](\boldsymbol{X_d} - \boldsymbol{X_d})$$