Waves(20) 11.15

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Question :-

A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of $10ms^{-1}$, (b) recedes from the platform with a speed of $10ms^{-1}$? (ii) What is the speed of sound in each case ? The speed of sound in still air can be taken as $340ms^{-1}$.

This problem requires knowledge of *Doppler Effect* , So first we will learn Doppler effect and then we will solve our problem . Before learning Doppler effect , we will also understand Sound Waves .

Equation of Sound Wave :-

Sound Wave is transmission of energy ; sound wave depends on many parameters . A general equation of sound wave is shown below

$$y(t) = Asin(2\pi ft + \phi) \tag{1}$$

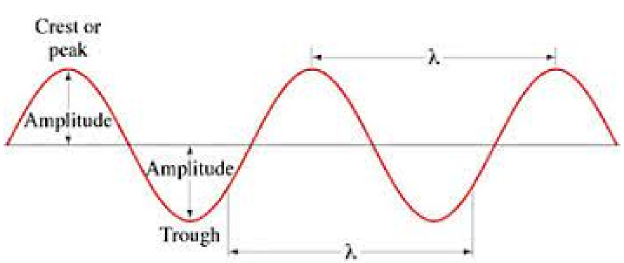
y(t) is instantaneous displacement of wave at time t;

A is amplitude of wave;

$$f\ is\ frequency\ of\ wave;$$

$$t\ is\ time;$$

$$\phi\ is\ phase\ angle;$$



$$\lambda\ is\ wavelength\ of\ wave;$$

$$crest\ is\ peak(highest\ point)\ of\ wave;$$

$$trough\ is\ dip(lowest\ point)\ of\ wave;$$

$$2\pi f\ is\ called\ angular frequency;$$

On comparing our problem with equation (3) , equation for different cases are given

equation of sound wave when whistle is blown by train is

$$y(t) = Asin(2\pi \times 400 \times t + \phi)$$

$$\text{for this case } f = 400Hz$$

equation of sound wave observed by observer when train is approaching observer

$$y(t) = Asin(2\pi \times 412.1212 \times t + \phi)$$

$$\text{for this case } f = 412.1212Hz$$

equation of sound wave observed by observer when train is receding observer

$$y(t) = Asin(2\pi \times 388.5714 \times t + \phi)$$

$$\text{for this case } f = 388.5714Hz$$

Doppler Effect for Sound Waves :-

Doppler effect for sound wave refers to change in frequency or pitch of sound wave observed by an observer when there is a relative motion between observer and source .

Derivation of Doppler :-

To derive Doppler , we can write equation of sound as shown

$$f = \frac{v}{\lambda} \tag{2}$$

using equation (2) , we get

$$y(t) = Asin(2\pi\frac{v}{\lambda}t + \phi) \tag{3}$$

$$v\ is\ speed\ of\ sound\ in\ that\ medium$$

1. Source is moving toward stationary Observer-

Now consider the relative motion in which source is moving towards observer , in that case effective wavelength λ' observed by observer will be compressed ,

$$v_s\ is\ velocity\ of\ source$$

$$v_o\ is\ velocity\ of\ observer$$

$$v_s = v_s$$

$$v_o = 0$$

$$\lambda' = \lambda - v_s T \tag{4}$$

T is time period(time taken by source wave to complete one revolution) and effective frequency f' observed by observer will be

$$f' = \frac{v}{\lambda'} \tag{5}$$

using equations (2) , we get

$$f' = \frac{v}{\lambda - v_s T} \tag{6}$$

$$f' = \frac{vf}{f(\lambda - v_s T)}$$

we know ,

$$T = \frac{1}{f} \tag{7}$$

using equation (7)

$$f' = \frac{vf}{v - v_s} \tag{8}$$

2. Source is moving away from stationary Observer-

Similarly , if source is receding from observer than λ,will be increased

$$v_s = v_s$$

$$v_o = 0$$

$$\lambda' = \lambda + v_s T \tag{9}$$

using equations (5) and (9) , we get

$$f' = \frac{v}{\lambda + v_s T} \tag{10}$$

$$f' = \frac{vf}{f(\lambda + v_s T)}$$

using equation (2) and (7)

$$f' = \frac{vf}{v + v_s} \tag{11}$$

Doppler effect depends on relative velocity , so we will use this concept to prove frequencies for different cases depending on situation .

3. Observer is moving towards Stationary Source-

In this case , the velocity at which sound is approaching observer will increase .

$$v_s = 0$$

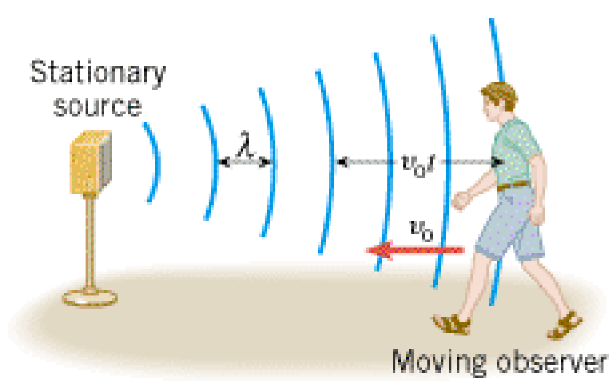
$$v_o = v_o$$

$$f' = \frac{v'}{\lambda'} \tag{12}$$

$$v' = v + v_o \tag{13}$$

$$But\ what\ about\ wavelength??$$

It's answer is , wavelength will be same .



Sound properties only depends on situation of source and not observer .

$$\lambda' = \lambda \tag{14}$$

using equations (13) and (14) , and substituting in equation (12)

$$f' = \frac{v + v_o}{\lambda} \tag{15}$$

using equation (2) , we get

$$f' = \frac{(v + v_o)f}{v} \tag{16}$$

4. Observer is moving away from Stationary Source-

In this case , the velocity at which sound is approaching observer will decrease .

$$v_s = 0$$

$$v_o = v_o$$

$$v' = v - v_o \tag{17}$$

In this case also , wavelength will not change.

$$\lambda' = \lambda \tag{18}$$

using equations (17) and (18) , and substituting in equation (12)

$$f' = \frac{v - v_o}{\lambda} \tag{19}$$

using equation (2) , we get

$$f' = \frac{(v - v_o)f}{v} \tag{20}$$

5. Source and Observer are both moving towards each other-

In this case , the velocity at which sound is approaching observer will increase and wavelength will compress .

$$v_s = v_s$$

$$v_o = v_o$$

$$v' = v + v_o \tag{21}$$

$$\lambda' = \lambda - v_s T \tag{22}$$

using equations (21) and (22) , and substituting in equation (12)

$$f' = \frac{v + v_o}{\lambda - v_s T} \tag{23}$$

using equation (2) , we get

$$f' = \frac{(v + v_o)f}{v - v_s} \tag{24}$$

6. Source and Observer are both moving away from each other-

In this case , the velocity at which sound is approaching observer will decrease and wavelength will stretch .

$$v_s = v_s$$

$$v_o = v_o$$

$$v' = v - v_o \tag{25}$$

$$\lambda' = \lambda + v_s T \tag{26}$$

using equations (25) and (26) , and substituting in equation (12)

$$f' = \frac{v - v_o}{\lambda + v_s T} \tag{27}$$

using equation (2) , we get

$$f' = \frac{(v - v_o)f}{v + v_s} \tag{28}$$

7. Source is moving towards Observer and Observer moving away from Source-

In this case , the velocity at which sound is approaching observer will decrease and wavelength will compress .

$$v_s = v_s$$

$$v_o = v_o$$

$$v' = v - v_o \tag{29}$$

$$\lambda' = \lambda - v_s T \tag{30}$$

using equations (29) and (30) , and substituting in equation (12)

$$f' = \frac{v - v_o}{\lambda - v_s T} \tag{31}$$

using equation (2) , we get

$$f' = \frac{(v - v_o)f}{v - v_s} \tag{32}$$

8. Source is moving away from Observer and Observer is moving towards Source-

In this case , the velocity at which sound is approaching observer will increase and wavelength will stretch.

$$v_s = v_s$$

$$v_o = v_o$$

$$v' = v + v_o \tag{33}$$

$$\lambda' = \lambda + v_s T \tag{34}$$

using equations (33) and (34) , and substituting in equation (12)

$$f' = \frac{v + v_o}{\lambda + v_s T} \tag{35}$$

using equation (2) , we get

$$f' = \frac{(v + v_o)f}{v + v_s} \tag{36}$$

9. Both Source and Observer are stationary-

If both Source and Observer are stationary , it means

$$v_s = 0$$

$$v_o = 0$$

also there will be no change in wavelength

$$\lambda' = \lambda \tag{37}$$

$$v' = v \tag{38}$$

using equation (37) and (38), and substituting in equation (12)

$$f' = \frac{v}{\lambda} \tag{39}$$

using equation (2) , we get

$$f' = f \tag{40}$$

So , Doppler effect depends on relative velocity of Observer and Source with respect to same frame and also velocity of Sound in that medium .

Let's get back to our problem solution

Solution :-

Symbol	Meaning of Symbol
f	actual frequency of source
f'_a	frequency observed by observer when train is approaching observer
f'_r	frequency observed by observer when train is receding observer
v	velocity of air in that medium
v_s	velocity of source which is train
v_o	velocity of observer

(i) a. When the train approaches the platform (i.e., the observer at rest), so this situation is similar to our **(1.)** derivation of Doppler effect , so we will use equation (8). If we compare with equation (8) which is given below

$$f' = \frac{vf}{v - v_s}$$

On Substituting

Symbol for Problem	Symbol in equation 8
f_o	f
f'_a	f'
v	v
v_t	v_s
v_o	v_o

$$f'_a = f_o \times \frac{v}{v - v_t} \tag{41}$$

$$f'_a = 400 \times \frac{340}{340 - 10}$$

$$f'_a = 412.1212$$

b. When the train recedes the platform (i.e., the observer at rest), it is similar to our **2.** in Doppler derivation and equation (11)

$$f' = \frac{vf}{v + v_s}$$

f_r' is frequency observed by observer when train is receding platform,
On comparing with equation (11)

$$f_r' = f'$$

substituting in equation (11)

$$f_r' = f_o \times \frac{v}{v + v_t} \tag{42}$$

$$f_r' = 400 \times \frac{340}{340 + 10}$$

$$f_r' = 388.5714$$

(ii) The speed of sound in each will be same.It is $340ms^{-1}$ in each case. We are providing a table in which various formulas of frequencies are written depending on situation .

frequencies observed in Different cases			
Doppler Shift	Stationary Observer	Observer moving towards Source	Observer moving away from Source
Stationary Source	$f' = f$	$f' = \frac{(v + v_o)f}{v}$	$f' = \frac{(v - v_o)f}{v}$
Source moving towards Observer	$f' = \frac{vf}{v - v_s}$	$f' = \frac{(v + v_o)f}{v - v_s}$	$f' = \frac{(v - v_o)f}{v - v_S}$
Source moving away from Observer	$f' = \frac{vf}{v + v_s}$	$f' = \frac{(v + v_o)f}{v + v_s}$	$f' = \frac{(v - v_o)f}{v + v_s}$