1

Progressions (7) 11.9.5

EE23BTECH11051-Rajnil Malviya

Question:-

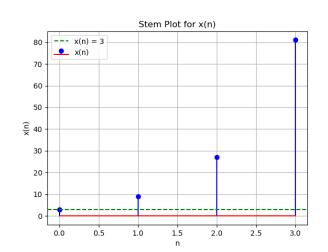
If a function Satisfying f(x + y) = f(x) f(y) for all $x, y \in N$ such that f(1) = 3 and $\sum_{x=1}^{n} f(x) = 120$, find the value of n.

Solution: Putting y=1, we get

$$\frac{f(x+1)}{f(x)} = 3\tag{1}$$

| Symbol | Description | Value |
|--------------|--------------------|-----------------|
| <i>x</i> (0) | first term | 3 |
| r | common ratio | 3 |
| y(n) | sum of all n terms | 120 |
| x(n) | $x(0) r^n u(n)$ | depends on n |

TABLE I



$$x(n) = x(0) r^n u(n)$$
 (2)

From (??)

$$X(z) = \frac{3}{1 - 3z^{-1}} \quad |z| > |3| \tag{3}$$

$$Y(z) = \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \tag{4}$$

$$Y(z) = \frac{x(0)}{r - 1} \left(\frac{r}{1 - rz^{-1}} - \frac{1}{1 - z^{-1}} \right) \quad |z| > |r| \quad (5)$$

applying inverse z transform;

$$y(n) = x(0) \left(\frac{r^{n+1} - 1}{r - 1} \right) u(n)$$
 (6)

$$120 = 3\left(\frac{3^{n+1} - 1}{3 - 1}\right) \tag{7}$$

$$\implies n = 3$$
 (8)

