

Progressions (7) 11.9.5

EE23BTECH11051-Rajnil Malviya

Question :-

If a function Satisfying $f(x+y) = f(x) f(y)$ for all $x, y \in N$ such that $f(1) = 3$ and $\sum_{x=1}^n f(x) = 120$, find the value of n .

Solution:- Putting $y=1$, we get

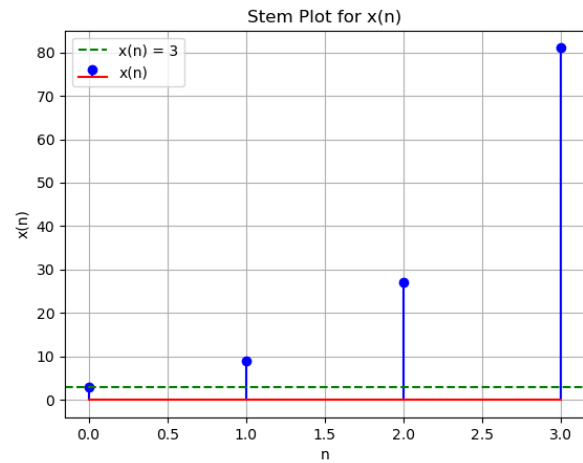
$$\frac{f(x+1)}{f(x)} = 3 \quad (1)$$

so common ratio $r = 3$;

$$x(n) = x(0) r^n \quad (2)$$

Symbol	Description	Value
$x(0)$	first term	3
r	common ratio	3
$y(n)$	sum of all n terms	120
$x(n)$	n^{th} term	$x(0) r^n$

TABLE I



$$x(n) = x(0) r^n u(n) \quad (3)$$

From (??)

$$X(z) = \frac{3}{1 - 3z^{-1}} \quad |z| > |3| \quad (4)$$

$$Y(z) = \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \quad (5)$$

$$Y(z) = \frac{x(0)}{r-1} \left(\frac{r}{1 - rz^{-1}} - \frac{1}{1 - z^{-1}} \right) \quad |z| > |r| \quad (6)$$

applying inverse z transform ;

$$y(n) = x(0) \left(\frac{r^{n+1} - 1}{r - 1} \right) u(n) \quad (7)$$

$$120 = 3 \left(\frac{3^{n+1} - 1}{3 - 1} \right) \quad (8)$$

$$\Rightarrow n = 3 \quad (9)$$

