Progressions (7) 11.9.5

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Question :-

If a function Satisfying f$(x + y)$ = f(x) f(y) for all $x, y \in N$ such that f(1) =3 and $\sum_{x=1}^{n} f(x)$ = 120 , find the value of n .

Solution:- Using induction x=1 and y=1 , we get

$$f(2) = f(1)^2 \tag{1}$$

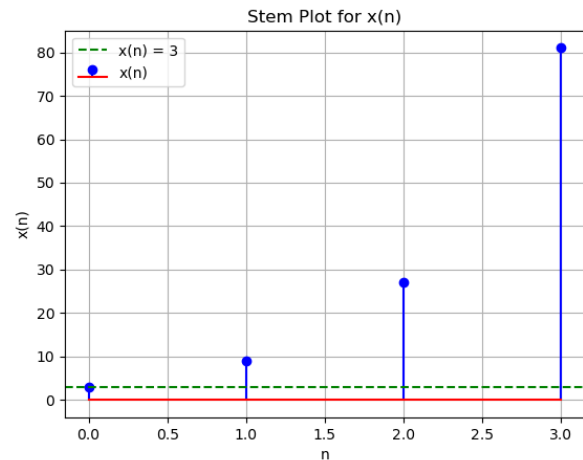$$f(3) = f(1) f(1)^2 \tag{2}$$

$$f(4) = f(1) f(1)^3 \tag{3}$$

$$\implies f(x) = f(1)^x \tag{4}$$

so it is a GP with common ratio $r = 3$;

$$x(n) = x(0) r^n \tag{5}$$

Symbol	Description	Value
$x(0)$	first term	3
r	common ratio	3
$y(n)$	sum of all n terms	120
$x(n)$	n^{th} term	$x(0) r^n$

TABLE I

Stem Plot for x(n)

$$x(n) = x(0) r^n u(n) \tag{6}$$

From (**??**)

$$X(z) = \frac{3}{1 - 3z^{-1}} \quad |z| > |3| \tag{7}$$

$$Y(z) = \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \tag{8}$$

$$Y(z) = \frac{x(0)}{r - 1} \left(\frac{r}{1 - rz^{-1}} - \frac{1}{1 - z^{-1}} \right) \quad |z| > |r| \tag{9}$$

applying inverse z transform ;

$$y(n) = x(0) \left(\frac{r^{n+1} - 1}{r - 1} \right) u(n) \tag{10}$$

$$120 = 3 \left(\frac{3^{n+1} - 1}{3 - 1} \right) \tag{11}$$

$$\implies n = 3 \tag{12}$$

Stem Plot for Sum of First n Terms of Geometric Progression