Waves(20) 11.15

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Question :-

A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of $10ms^{-1}$, (b) recedes from the platform with a speed of $10ms^{-1}$? (ii) What is the speed of sound in each case ? The speed of sound in still air can be taken as $340ms^{-1}$.

This problem requires knowledge of *Doppler Effect* , So first we will learn Doppler effect and then we will solve our problem . Before learning Doppler effect , we will also understand Sound Waves .

Symbol	Description
$y(t)$	instantaneous displacement of wave
f_n	natural frequency
f_r	received frequency
A	amplitude of wave
ϕ	phase difference of wave
t	time
λ	wavelength of wave
T	time period of wave

TABLE I

Equation of Sound Wave :-

Sound Wave is transmission of energy ; sound wave depends on many parameters . A general equation of sound wave is shown below

$$y(t) = A\sin(2\pi ft + \phi) \tag{1}$$

From equation 1 , equation of sound wave when whistle is blown by train is

Symbol	Description	Value
f	frequency of source	400Hz
v	velocity of air	$340ms^{-1}$
v_o	velocity of observer	$0ms^{-1}$
v	velocity of source	$10ms^{-1}$

TABLE II

$$y(t) = A\sin(2\pi \times 400 \times t + \phi)$$

$$\text{for this case } f = 400Hz$$

Doppler Effect for Sound Waves :-

Doppler effect for sound wave refers to change in frequency or pitch of sound wave observed by an observer when there is a relative motion between observer and source .

General Equation of Doppler :-

Now we will see formulas for Doppler effect in different situations .

$$f' = \frac{v'}{\lambda'} \tag{2}$$

$$f_n = \frac{v}{\lambda} \tag{3}$$

$$f' = \frac{v \pm v_o}{\lambda \pm v_s T} \tag{4}$$

$$f_n = \frac{1}{T} \tag{5}$$

From equations 3 and 5 , then substituting in equation 4

$$f' = \frac{(v \pm v_o)(f_n)}{v \pm v_s} \tag{6}$$

Above equation is general equation for any case in doppler effect . Signs of equation 6 depends on velocities of both observer and source .

Effect of observer's velocity on frequency :-

If observer is moving towards source and source is stationary ($v_s = 0$), then sound will reach observer faster , so observed frequency will increase. It means

$$f' > f_n \tag{7}$$

From equation 6

$$f' = \frac{(v \pm v_o) f_n}{v} \tag{8}$$

From equations 3 and 8 ,

$$\frac{(v \pm v_o) f_n}{v} > f_n \tag{9}$$

$$v \pm v_o > v \tag{10}$$

There must be (+) sign to satisfy equation 10 . And for vice-versa case(observer is moving away) , so frequency will be less than f_n So ,

$$f' < f_n \tag{11}$$

From equation 8 ,

$$\frac{(v \pm v_o) f_n}{v} < f_n \tag{12}$$

$$v \pm v_o < v \tag{13}$$

There must be (-) sign to satisfy equation 13 .

Effect of source's velocity on frequency :-

If source is moving towards stationary observer($v_o = 0$), so λ' in equation 2 will compress and denominator will decrease , so f' will increase ,

$$f' > f_n \tag{14}$$

From equation 6

$$f' = \frac{(v) f_n}{v \pm v_s} \tag{15}$$

From equations 8 and 3

$$\frac{v f_n}{v \pm v_s} > f_n \tag{16}$$

$$\frac{v}{v \pm v_s} > 1 \tag{17}$$

There must be (-) sign to satisfy equation 17 . And for vice-versa case(source is moving away) , so wavelength will increase and denominator in equation 4 increases so f' will decrease

$$f' < f_n \tag{18}$$

From equation 6

$$f' = \frac{(v) f_n}{v \pm v_s} \tag{19}$$

From equations 19 and 3 ,

$$\frac{v f_n}{v \pm v_s} < f_n \tag{20}$$

$$\frac{v}{v \pm v_s} < 1 \tag{21}$$

There must be (+) sign to satisfy equation 21.

v_o and v_s directly affect equation 6 , independent of each other . We can change signs of numerator and denominator independently by analysing situation .

A table is given below , which includes all cases with different situations .

frequencies observed in Different cases			
Doppler Shift	Stationary Observer	Observer moving towards Source	Observer moving away from Source
Stationary Source	$f' = f_n$	$f' = \frac{(v+v_o)f_n}{v}$	$f' = \frac{(v-v_o)f_n}{v}$
Source moving towards Observer	$f' = \frac{vf_n}{v-v_s}$	$f' = \frac{(v+v_o)f_n}{v-v_s}$	$f' = \frac{(v-v_o)f_n}{v-v_S}$
Source moving away from Observer	$f' = \frac{vf_n}{v+v_s}$	$f' = \frac{(v+v_o)f_n}{v+v_s}$	$f' = \frac{(v-v_o)f_n}{v+v_s}$

TABLE III

Let's get back to our problem solution
(i).a When the train approaches the platform (i.e., the observer at rest),
Solution :-
So here source is approaching , wavelength will decrease and frequency will increase , so we have to increase f'
From Table:3

$$f' = \frac{v f_n}{v - v_s} \tag{22}$$

On Substituting in equation 22

$$f'_a = 400(\frac{340}{340 - 10})$$
$$f' = 412.1212$$

b. When the train recedes the platform (i.e., the

observer at rest),
Solution :-
It is vice-versa of above, From Table:3

$$f' = f_n(\frac{v}{v + v_s}) \tag{23}$$
$$f' = 400(\frac{340}{340 + 10})$$
$$f' = 388.5714$$

(ii) The speed of sound in each will be same.It is $340ms^{-1}$ in each case.

From equation 6 we will interchange v_o *and* v_s

Transmitted Signal	Received Signal
Source transmits a signal with frequency	Source will receive reflection of its transmitted Signal
$A\sin(2\pi f t + \phi)$	$A\sin(2\pi f_r t + \phi)$

TABLE IV

$$f_r = \frac{(v \pm v_s)(f')}{v \pm v_o} \tag{24}$$

For signs , we will use same logic, if someone is approaching another one , so definitely it will increase frequency(f_r) and if receding so it will decrease frequency(f_r) .

In our problem if , train approaches with $v_s 10ms^{-1}$ and if platform is taken as obstacle with $v_o = 0$,
Received frequency by platform is f' 412.1212 ,
Here train(source) is approaching , so v_s will try to increase f_r
On substituting in equation 24

$$f_r = \frac{(v \pm v_s)(f')}{v}$$

To increase f_r , there must be (+) sign ;

$$f_r = \frac{(340 + 10)(412.1212)}{340}$$

f_r=424.2424

So Received signal will be ,
From table Table:4

$$y_r(t) = A\sin(2\pi(424.2424)t + \phi)$$

if train is receding with $10ms^{-1}$, so v_s will try to decrease f_r so we will use (-) sign in equation 24 with $v_o = 0$

$$f_r = \frac{(v - v_s)(f')}{v}$$

f'=388.5714
on substituting in equation 24

$$f_r = \frac{(340 - 10)(388.5714)}{340}$$

$f_r = 377.1428$
So received signal will be ,
From table Table:4

$$y_r(t) = A\sin(2\pi(377.1428)t + \phi)$$