

# Sequence(19) 10.5.3

EE23BTECH11051-Rajnil Malviya

*Question :-*

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on . In how many rows are the 200 logs placed and how many logs are in the top row?

Symbol	Description
$n$	term number
$a_1$	first term( $n=1$ ) of A.P
$a_n$	$n_{th}$ term of A.P
$d$	common difference of A.P
$S_n$	Sum upto $n_{th}$ of A.P

TABLE I

For an Arithmetic Progression :-

$$x(n) - x(n-1) = d \quad (1)$$

$$x(n-1) - x(n-2) = d \quad (2)$$

$$x(n-2) - x(n-3) = d \quad (3)$$

.

.

.

$$x(2) - x(1) = d \quad (4)$$

adding all these equations :-

$$x(n) - x(1) = (n-1)d \quad (5)$$

$$x(n) = x(1) + (n-1)d \quad (6)$$

Sum upto  $n$  terms of A.P :-

$$S_n = x(1) + x(2) + \dots + x(n) \quad (7)$$

$$S_n = x(n) + x(n-1) + \dots + x(1) \quad (8)$$

adding equations 7 and 8

$$2S_n = [x(1) + x(n)] + [x(2) + x(n-2)] + \dots$$

$$\dots + [x(n) + x(1)] \quad (9)$$

Substituting by using equation 6

$$2S_n = [x(1) + (x(1) + (n-1)d)] + \dots$$

$$\dots + [(x(1) + (n-1)d) + x(1)] \quad (10)$$

$$S_n = \frac{n}{2} (2x(1) + (n-1)d) \quad (11)$$

*Solution :-*

Symbol	Value
$a_1$	20 (logs in bottom row)
$a_2$	19
$a_3$	18
$S_n$	200 (total number of logs)

TABLE II

Using equation 4

$$d = x(2) - x(1) \quad (12)$$

Using equation 6

$$x(n) = (21 - n) \quad (13)$$

*Z Transformation :-*

The relation between  $x(n)$  and  $u(n)$ :

$$x(n) = (21 - n) u(n) \quad (14)$$

$$u(n) \xleftrightarrow{ZT} \frac{1}{(1 - z^{-1})} \quad [\text{ROC: } |z| > 1] \quad (15)$$

$$U(z) = \frac{1}{(1 - z^{-1})} \quad (16)$$

Z-transform of  $n^k u(k)$  in terms of the  $k$ -th derivative of  $U(z)$ :

$$n^k u(n) \xleftrightarrow{ZT} (-1)^k z^k \frac{d^k}{dz^k} U(z) \quad (17)$$

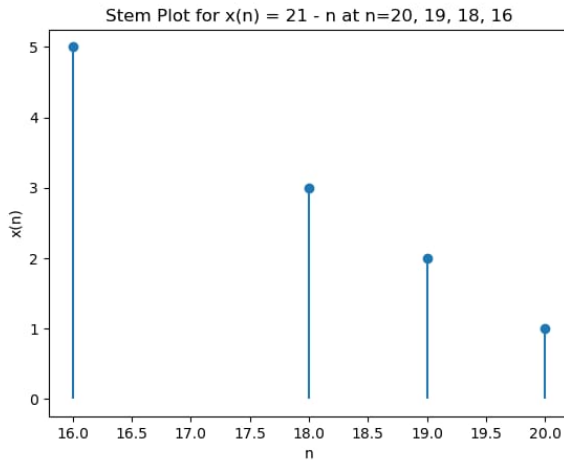
$$nu(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \quad (18)$$

$$n^2 u(n) \xleftrightarrow{Z} \frac{(z^{-1})(1 + z^{-1})}{(1 - z^{-1})^3} \quad |z| > 1 \quad (19)$$

$$n^3 u(n) \xleftrightarrow{Z} \frac{(z^{-1})(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4} \quad |z| > 1 \quad (20)$$

Referencing the equations from 14,18,19

$$x(n) \xleftrightarrow{Z} \frac{21}{(1 - z^{-1})} + \frac{(z^{-1})(1 + z^{-1})}{(1 - z^{-1})^3} \quad |z| > 1 \quad (21)$$



Substituting from Table:II

$$d = -1 \quad (22)$$

For Practical reasons

Symbol	Description
$a_n$	number of logs in top most row

TABLE III

$$x(n) > 0 \quad (23)$$

Using equation 13 and substituting in equation 6

$$20 + (n - 1)(-1) > 0 \quad (24)$$

$$n < 21 \quad (25)$$

Using equation 11 and substituting from Table:2

$$n^2 - 41n + 400 = 0 \quad (26)$$

$$n = 16, 25 \quad (27)$$

From equation 16

$$n = 16 \quad (28)$$

Substituting in equation 6

$$x(n) = 20 + (16 - 1)(-1) \quad (29)$$

$$a_n = 5 \quad (30)$$

Ans . There are 16 rows and 5 logs in top row