Sequence(19) 10.5.3

EE23BTECH11051-Rajnil Malviya

Question :-

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on . In how many rows are the 200 logs placed and how many logs are in the top row?

Symbol	Description	Value
$x(0)$	bottom row	20
d	common difference	-1
S_n	total number of logs	200
x(n)	number of logs in n row	depends on n

TABLE I

For an Arithmetic Progression :-

$$x(n) = [x(0) + nd]\,u(n) \tag{1}$$
$$x(n) = [20 - n]\,u(n) \tag{2}$$
$$X(Z) = \frac{20 - 21z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \tag{3}$$
$$y(n) = x(n) * u(n) \tag{4}$$
$$Y(Z) = X(Z)\,U(Z) \tag{5}$$
$$Y(Z) = \frac{20 - 21z^{-1}}{(1 - z^{-1})^3} \tag{6}$$

Using Contour Integration to find the inverse Z-transform,

$$y(n) = \frac{1}{2\pi j}\oint_C Y(z)\,z^{n-1}\,dz \tag{7}$$
$$= \frac{1}{2\pi j}\oint_C \frac{(20 - 21z^{-1})z^{n-1}}{(1 - z^{-1})^3}\,dz \tag{8}$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{9}$$
$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left(20z^{n+2} - 21z^{n+1}\right) \tag{10}$$
$$= \frac{1}{2}\lim_{z\to 1}(20(n+2)(n+1)z^n - 21(n)(n+1)z^{n-1}) \tag{11}$$
$$= \frac{1}{2}[20(n+2)(n+1) - 21(n)(n+1)] \tag{12}$$

$$R = S_n \tag{13}$$
$$200 = \frac{1}{2}\left[20\,(n+2)\,(n+1) - 21\,(n)\,(n+1)\right] \tag{14}$$
$$n = 15\ ,\ 24 \tag{15}$$

For Practical reasons
x(n) is number of logs in top row

$$x(n) > 0 \tag{16}$$

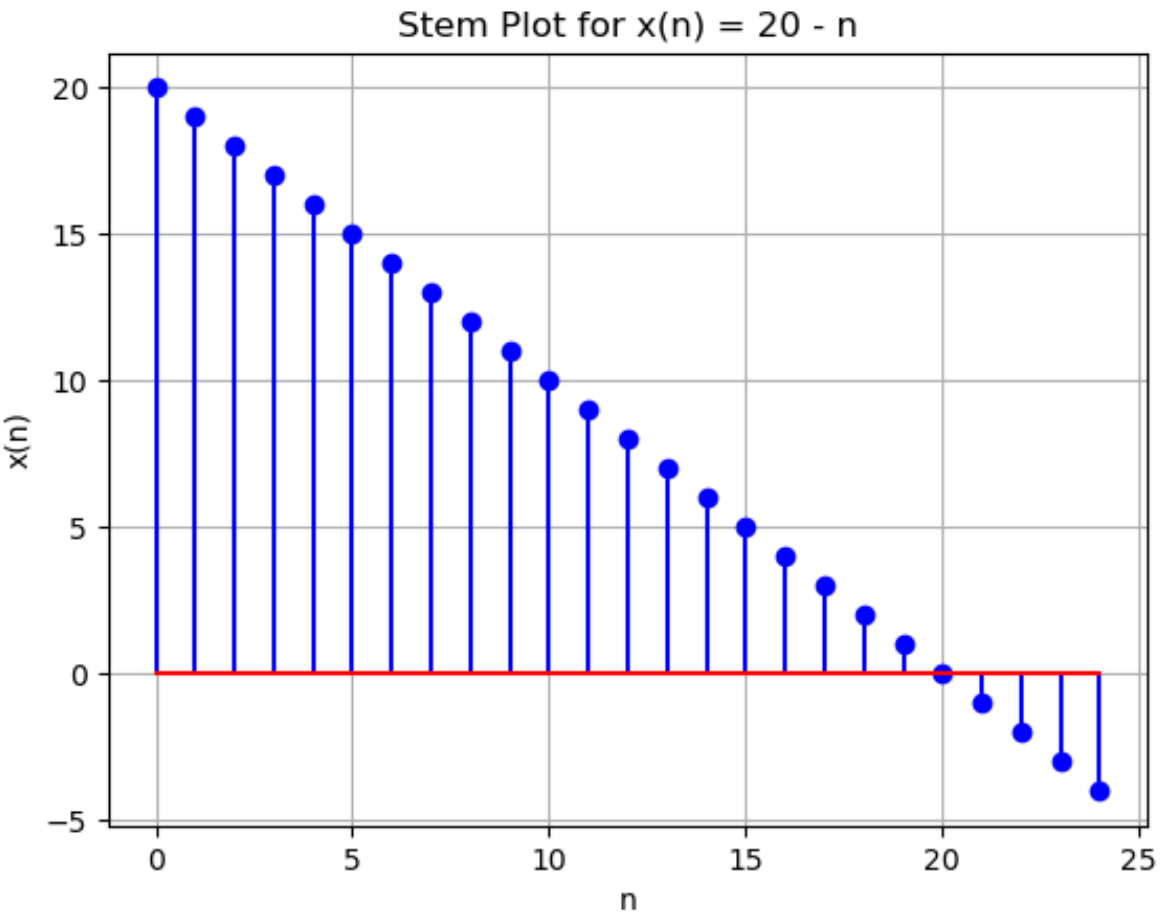
Using equation 2

$$x(15) = 5 \tag{17}$$
$$x(24) = -4 \tag{18}$$

x(24) is rejected because it is negative

$$x(15) = 5 \tag{19}$$

Ans. There are 16 rows with 5 logs in top row


Stem Plot for x(n) = 20 - n
x(n)
n
20
15
10
5
0
−5
0
5
10
15
20
25