Sequence(19) 10.5.3

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Question :-

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on . In how many rows are the 200 logs placed and how many logs are in the top row?

Symbol	Description
n	term number
a_1	first term(n=1) of A.P
a_n	n_{th} term of A.P
d	common difference of A.P
S_n	Sum upto n_{th} of A.P

TABLE I

Solution :-

For an Arithmetic Progression :-

$$a_n - a_{n-1} = d \tag{1}$$

$$a_{n-1} - a_{n-2} = d \tag{2}$$

$$a_{n-2} - a_{n-3} = d \tag{3}$$

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$$a_2 - a_1 = d \tag{4}$$

adding all these equations :-

$$a_n - a_1 = (n-1)\,d \tag{5}$$

$$a_n = a_1 + (n-1)\,d \tag{6}$$

Sum upto n terms of A.P :-

$$S_n = a_1 + a_2 + ... + a_n \tag{7}$$

$$S_n = a_n + a_{n-1} + ... + a_1 \tag{8}$$

adding equations 7 and 8

$$2S_n = (a_1 + a_n) + (a_2 + a_{n-2}) + ... + (a_n + a_1) \tag{9}$$

Substituting by using equation 6
$2S_n = [a_1 + (a_1 + (n-1)d)] + ...$

$$.... + [(a_1 + (n-1)d) + a_1] \tag{10}$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d) \tag{11}$$

Using equation 4

Symbol	Value
a_1	20 (logs in bottom row)
a_2	19
a_3	18
S_n	200 (total number of logs)

TABLE II

$$d = a_2 - a_1 \tag{12}$$

Substituting from Table:II

$$d = -1 \tag{13}$$

For Practical reasons

Symbol	Description
a_n	number of logs in top most row

TABLE III

$$a_n > 0 \tag{14}$$

Using equation 13 and substituting in equation 6

$$20 + (n-1)(-1) > 0 \tag{15}$$

$$n < 21 \tag{16}$$

Using equation 11 and substituting from Table:2

$$n^2 - 41n + 400 = 0 \tag{17}$$

$$n = 16\ ,\ 25 \tag{18}$$

From equation 16

$$n = 16 \tag{19}$$

Substituting in equation 6

$$a_n = 20 + (16 - 1)(-1) \tag{20}$$

$$a_n = 5 \tag{21}$$

Ans . There are 16 rows and 5 logs in top row .