Waves(20) 11.15

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Question :-

A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of $10ms^{-1}$, (b) recedes from the platform with a speed of $10ms^{-1}$? (ii) What is the speed of sound in each case ? The speed of sound in still air can be taken as $340ms^{-1}$.

Solution :-
frequency of whistle, $F = 400Hz$
speed of sound in air, $a = 340ms^{-1}$
speed of train, $u = 10ms^{-1}$

(i) a. When the train approaches the platform (i.e., the observer at rest),

F'_a is frequency observed by observer when train is approaching platform ,

$$F'_a = F \times \frac{a}{a-u}$$

$$F'_a = 400 \times \frac{340}{340-10}$$

$$F'_a = 412.1212$$

b. When the train recedes the platform (i.e., the observer at rest),

F'_a is frequency observed by observer when train is receding platform,

$$F'_r = F \times \frac{a}{a+u}$$

$$F'_r = 400 \times \frac{340}{340+10}$$

$$F'_r = 388.5714$$

(ii) The speed of sound in each will be same.It is $340ms^{-1}$ in each case.

Equation of Sound Wave :-

Sound Wave is transmission of energy ; sound wave depends on many parameters . A general equation of sound wave is shown below

$$y(t) = Asin(2\pi ft + \phi)$$

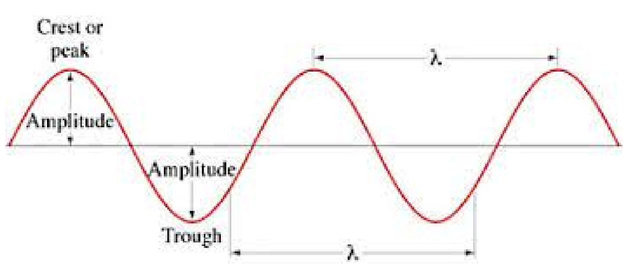
y(t) is instantaneous displacement of wave at time t;

$$A \text{ is amplitude of wave;}$$

$$f\ is\ frequency\ of\ wave;$$

$$t\ is\ time;$$

$$\phi\ is\ phase\ angle;$$



$$\lambda\ is\ wavelength\ of\ wave;$$

$$crest\ is\ peak(highest\ point)\ of\ wave;$$

$$trough\ is\ dip(lowest\ point)\ of\ wave;$$

$$2\pi f = \omega;$$

$$\omega\ is\ called\ angular\ frequency;$$

if this is replaced in above equation ; so it reduced to

$$y(t) = Asin(\omega t + \phi)$$

On comparing above equation with our problem , equation for different cases are given

equation of sound wave when whistle is blown by train is

$$y(t) = A sin(2\pi \times 400 \times t + \phi)$$

$$\text{for this case } f = 400Hz$$

equation of sound wave observed by observer when train is approaching observer

$$y(t) = A sin(2\pi \times 412.1212 \times t + \phi)$$

$$\text{for this case } f = 412.1212Hz$$

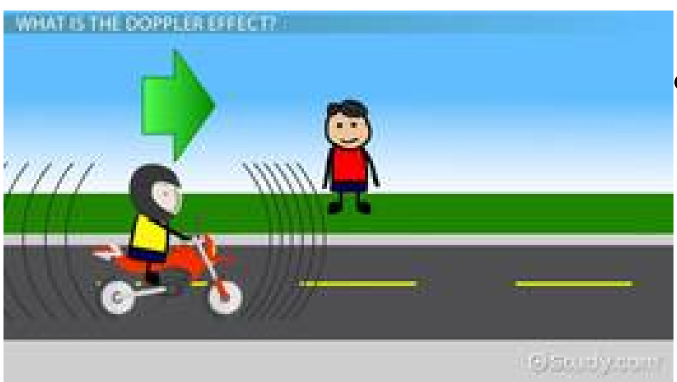
equation of sound wave observed by observer when train is receding observer

$$y(t) = A sin(2\pi \times 388.5714 \times t + \phi)$$

$$\text{for this case } f = 388.5714Hz$$

Doppler Effect for Sound Waves :-

Doppler effect for sound wave refers to change in frequency or pitch of sound wave observed by an observer when there is a relative motion between observer and source .



Derivation of Doppler :-

To derive Doppler , we can write equation of sound as shown

$$f = \frac{v}{\lambda}$$

$$y(t) = A sin(2\pi \frac{v}{\lambda} t + \phi)$$

$$v \textit{ is speed of sound in that medium}$$

Now consider the relative motion in which source is moving towards observer , in that case effective wavelength λ' observed by observer will be compressed ,

$$\lambda' = \lambda - v_o T$$

$$v_o \textit{ is relative velocity of observer}$$

T is time period(time taken by source wave to complete one revolution) and effective frequency f' observed by observer will be

$$f' = \frac{v}{\lambda'}$$

on substituting values from above we get

$$f' = \frac{v}{\lambda - v_o T}$$

$$f' = \frac{vf}{f(\lambda - v_o T)}$$

we know ,

$$T = \frac{1}{f}$$

$$f' = \frac{vf}{v - v_o}$$

Similarly , if source is receding from observer than

λ,will be increased

$$\lambda' = \lambda + v_o T$$

on substituting this , we get

$$f' = \frac{v}{\lambda + v_o T}$$

$$f' = \frac{vf}{f(\lambda + v_o T)}$$

$$f' = \frac{vf}{v + v_o}$$

Above equations are suggesting , if source approaches observer or observer approaches source than frequency will increase and if they recedes than frequency will decrease .

Doppler effect depends on relative velocity , so we are providing a table in which different frequencies are given depending on situation .

frequencies observed in Different cases			
Doppler Shift	Stationary Observer	Observer moving towards Source	Observer moving away from Source
Stationary Source	$f' = f$	$f' = \frac{(v + v_o)f}{v}$	$f' = \frac{(v - v_o)f}{v}$
Source moving towards Observer	$f' = \frac{vf}{v - v_s}$	$f' = \frac{(v + v_o)f}{v - v_s}$	$f' = \frac{(v - v_o)f}{v - v_S}$
Source moving away from Observer	$f' = \frac{vf}{v + v_s}$	$f' = \frac{(v + v_o)f}{v + v_s}$	$f' = \frac{(v - v_o)f}{v + v_s}$

v_s *is velocity of Source velocity of Source and Observer are measured from same frame of reference.*