# Progressions (7) 11.9.5

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*Question :-*

If a function Satisfying f$(x+y)$ = f$(x)$ f$(y)$ for all $x, y \in N$ such that f(1) =3 and $\sum_{x=1}^{n} f(x) = 120$ , find the value of n .

*Solution:-* Using induction $x = 1$ and $y = 1$ , we get

$$f(2) = f(1)[f(1)] \tag{1}$$
$$f(3) = f(1)[f(1)]^2 \tag{2}$$
$$f(4) = f(1)[f(1)]^3 \tag{3}$$
$$\implies f(x) = [f(1)]^x \tag{4}$$

so it is a GP with common ratio $r = 3$;

$$x(n) = [x(0) r^n] u(n) \tag{5}$$

| Symbol | Value | Description |
|---|---|---|
| $x(0)$ | 3 | first term |
| $r$ | 3 | common ratio |
| $y(n)$ | 120 | sum of all n terms |
| $x(n)$ | $x(0) r^n u(n)$ | $n+1^{th}$ term |

TABLE I

Applying z-transformation on x$(n)$ and using (**??**)

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |z| > |r| \tag{6}$$
$$\implies X(z) = \frac{3}{1 - 3z^{-1}} \quad |z| > |3| \tag{7}$$
$$y(n) = x(n) * u(n) \tag{8}$$
$$Y(z) = X(z) U(z) \tag{9}$$
$$\implies Y(z) = \left(\frac{3}{1 - 3z^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right) \quad |z| > |r| \tag{10}$$

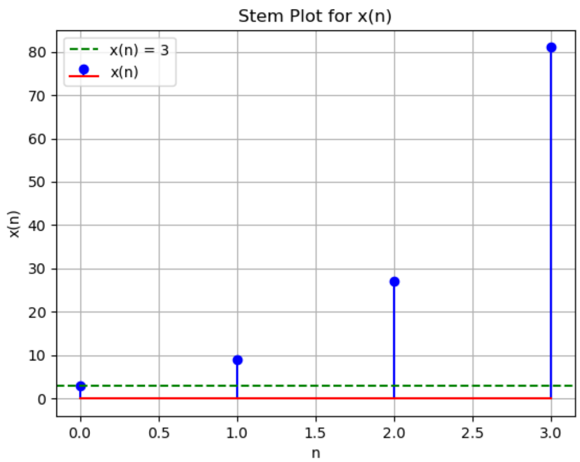
Using contour integration ;

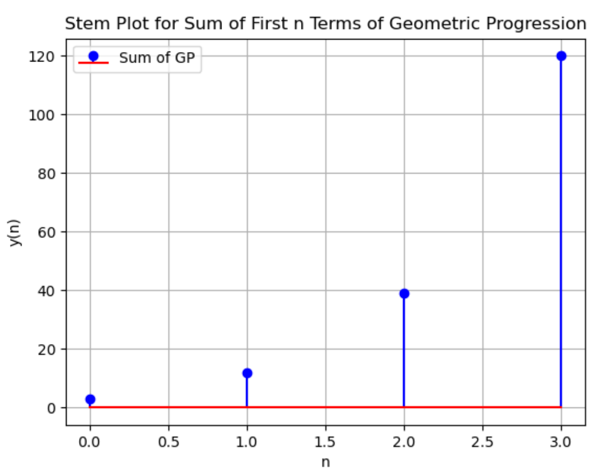
$$y(n) = \frac{1}{2\pi j} \oint_C \frac{3}{(1 - 3z^{-1})(1 - z^{-1})} \, dz \tag{11}$$

We can observe two non repeated poles $z = 3$ and $z = 1$

$$R_1 = \lim_{z \to 3} (z - 3) \frac{3z^{n+1}}{(z-3)(z-1)} \tag{12}$$
$$R_1 = \lim_{z \to 3} \frac{3z^{n+1}}{(z-1)} \tag{13}$$
$$\implies R_1 = \frac{3^{n+2}}{2} \tag{14}$$
$$R_2 = \lim_{z \to 1} (z - 1) \frac{3z^{n+1}}{(z-3)(z-1)} \tag{15}$$
$$R_2 = \lim_{z \to 1} \frac{3z^{n+1}}{(z-3)} \tag{16}$$
$$\implies R_2 = \frac{-3}{2} \tag{17}$$
$$y(n) = R_1 + R_2 \tag{18}$$
$$y(n) = \frac{3^{n+2}}{2} + \frac{-3}{2} \tag{19}$$
$$120 = \frac{3^{n+2} - 3}{2} \tag{20}$$
$$243 = 3^{n+2} \tag{21}$$
$$\implies n = 3 \tag{22}$$

Ans . We are also assuming $n = 0$ , so there are total four terms .


Stem Plot for x(n)
x(n) = 3
x(n)
80
70
60
50
40
30
20
10
0
x(n)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
n


Stem Plot for Sum of First n Terms of Geometric Progression
Sum of GP
120
100
80
60
40
20
0
y(n)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
n