

Progressions (7) 11.9.5

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Question :-

If a function Satisfying $f(x+y) = f(x)f(y)$ for all $x, y \in N$ such that $f(1) = 3$ and $\sum_{x=1}^n f(x) = 120$, find the value of n .

Solution:- $x = 1$ and $y = 1$, we get

$$f(2) = f(1+1) \quad (1)$$

$$= f(1)f(1) \quad (2)$$

$$= [f(1)]^2 \quad (3)$$

$$f(3) = f(2+1) \quad (4)$$

$$= f(2)f(1) \quad (5)$$

$$= [f(1)]^3 \quad (6)$$

Using induction, we get ;

$$f(x) = [f(1)]^x \quad (7)$$

so it is a GP with common ratio $r = 3$;

Symbol	Value	Description
$x(0)$	3	first term
r	3	common ratio
$y(n)$	120	sum of all n terms
$x(n)$	$x(0)r^n u(n)$	$n+1^{th}$ term

TABLE I

Applying z-transformation on $x(n)$;

$$\Rightarrow X(z) = \frac{3}{1-3z^{-1}} \quad |z| > |3| \quad (8)$$

$$Y(z) = X(z)U(z) \quad (9)$$

$$\Rightarrow Y(z) = \left(\frac{3}{1-3z^{-1}} \right) \left(\frac{1}{1-z^{-1}} \right) \quad |z| > |r| \quad (10)$$

Using contour integration ;

$$y(n) = \frac{1}{2\pi j} \oint_C \frac{3}{(1-3z^{-1})(1-z^{-1})} dz \quad (11)$$

We can observe two non repeated poles $z = 3$ and $z = 1$

$$R_1 = \lim_{z \rightarrow 3} (z-3) \frac{3z^{n+1}}{(z-3)(z-1)} \quad (12)$$

$$= \frac{3^{n+2}}{2} \quad (13)$$

$$R_2 = \lim_{z \rightarrow 1} (z-1) \frac{3z^{n+1}}{(z-3)(z-1)} \quad (14)$$

$$= \frac{-3}{2} \quad (15)$$

$$y(n) = R_1 + R_2 \quad (16)$$

$$= \frac{3^{n+2}}{2} + \frac{-3}{2} \quad (17)$$

$$\Rightarrow 120 = \frac{3^{n+2} - 3}{2} \quad (18)$$

$$\Rightarrow n = 3 \quad (19)$$

Ans . n take values from $n = 0$ to $n = 3$, so there are total four terms .

